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Forecasting Canadian Time Series with the New-Keynesian Model

Ali Dib
Mohamed Gammoudi
Kevin Moran

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Dib: Bank of Canada
Gammoudi: Bank of Canada
Moran: Corresponding author. Département d'économique, Université Laval, Québec, QC Canada G1K 7P4
kmoran@ecn.ulaval.ca

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Abstract:
This paper documents the out-of-sample forecasting accuracy of the New Keynesian Model for Canada. We repeatedly estimate our variant of the model on a series of rolling subsamples, forecasting out-of-sample one to eight quarters ahead at each step. We then compare these forecasts to those arising from simple VARs, using econometric tests of forecasting accuracy. Our results show that the forecasting accuracy of the New Keynesian model compares favourably to that of the benchmarks, particularly as the forecasting horizon increases. These results suggest that the model can become a useful forecasting tool for Canadian time series. The principle of parsimony is invoked to explain our findings.

Keywords: New Keynesian Model, Forecasting accuracy

JEL Classification: C53, E37

1 Introduction

New-Keynesian models are becoming standard tools in applied macroeconomic analysis.[1] They are used widely to study the impact of shocks on economic activity and inform the decisions of monetary policy makers in several central banks worldwide. These models are attractive because their optimizing environment provides a coherent determination of the time paths of aggregate variables in a framework suitable for monetary policy analysis.

It is now common to estimate, rather than calibrate, the parameters of these models, using aggregate time series and standard econometric techniques.[2] However, the models are seldom used to generate out-of-sample forecasts: evidence on the quality of these forecasts thus remains scarce.

To strengthen this evidence, this paper documents the out-of-sample forecasting properties of New Keynesian models for Canada. Specifically, we develop a variant of the model, repeatedly estimate it on a series of rolling subsamples, and compute out-of-sample forecasts one to eight quarters ahead at each step. We then compare these forecasts to those arising from Vector Autoregressions (VARs) using econometric tests of forecasting accuracy.

We find that the model's forecasting accuracy compares favorably to that of the VAR benchmarks, particularly as the forecasting horizon increases. Specifically, the model can forecast output, interest rates and money as well or better than the benchmarks, and forecasts inflation no worse. Further, our results also suggests that a combination of the two sets of forecasts may have superior forecasting power to each taken alone. Overall, these findings indicate that the New Keynesian class of model has the potential to become a useful forecasting tool for Canadian time series.

Using VARs as the benchmarks for comparing forecasts is natural be-

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[1] New Keynesian models are DSGE (Dynamic Stochastic General Equilibrium) environments where monopolistically competitive firms set prices subject to various adjustment costs. They are built around a core consisting of a price-setting equation (the 'New Phillips Curve'), an equation linked to intertemporal consumption smoothing, and a monetary policy rule. Although derived from the Real Business Cycle methodology, their emphasis on nominal rigidities and monetary features make them well suited to monetary policy analysis. Woodford (2003) provide a synthesis of the model's implications for monetary policy analysis.

[2] For example, Ireland (1997, 2001a, 2003, 2004), Dib (2003a,b, forthcoming) and Bouakez et al. (forthcoming) estimate parameters using maximum likelihood; Christiano et al. (2005) do so by minimizing the distance between the model's impulse responses following monetary policy shocks and those computed with VARS; Smets and Wouters (2003) and Del Negro and Schorfheide (2004) employ a Bayesian strategy to compute the posterior distribution for the parameters.

cause the New Keynesian model itself can be written as a VAR whose parameters are restricted by non-linear constraints linked to the model's structure. Our forecasting experiments thus compare the out-of-sample forecasting properties of a restricted model to those of an unrestricted counterpart. Clements and Hendry (1998) discuss conditions under which better forecasting accuracy may be attained by the restricted, or parsimonious, model. This requires the trade-off between squared inconsistency (how 'wrong' are the restrictions) and sampling uncertainty (estimating a large number of parameters lowers precision) to favour the parsimonious specification.[3] Situations where this is the case are more likely when the sample size for estimation is small and the forecasting horizon is high, as in monetary policy practice.

Evidence about the practical value of parsimony for forecasting is emerging. For example, Doan et al. (1984), Ingram and Whiteman (1994) and more recently Del Negro and Schorfheide (2004) demonstrate that constraining the estimation of a VAR by employing a Bayesian strategy and priors linked to structural models[4] improves the VARs' forecasting ability. Working within the classical perspective, Ireland (2004) shows that a version of the Real Business Cycle model estimated with maximum likelihood can have better forecasts than simple VARs; Dolar and Moran (2002) verify that his results hold for Canada. Recent papers by Smets and Wouters (2004), Del Negro et al. (2004), and Boivin and Giannoni (2005) contribute to the emerging evidence about the good forecasting properties of New Keynesian models.

The present paper contributes and strengthens this evidence in the following manner. First, it follows standard econometric methodology by repeatedly estimating competing models on rolling subsamples and applying standard tests of forecasting accuracy to compare forecasts. Second, it presents evidence that these models can forecast well Canadian time series in addition to American or European data.

The rest of the paper is organized as follows. Section 2 develops our variant of the New Keynesian model. Section 3 discusses the model's estimation and presents estimation results for the first subsample. Section 4 describes the forecasting experiment we conduct and presents our results about the accuracy of the model's forecasts. Section 5 assesses results and concludes.

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[3] In addition, problems associated with the small samples properties of the more complex non-linear estimation must be relatively small.

[4] While Doan et al. (1984) use the 'Minnesota Prior' (random walks for all variables), Ingram and Whiteman (1994) derive priors from the basic Real Business Cycles model, while those in Del Negro and Schorfheide (2004) arise from a simple New Keynesian model.

2 Model

This section develops our variant of the New-Keynesain class of models. The structure of the model is similar to the one in Dib (forthcoming) and Ireland (2003). Time is discrete and one model period represents a quarter. There are two sectors of production. The first sector, producing final goods, is competitive: firms take input prices as given and produce an homogenous good which they sell at flexible prices. Final good production is divided between consumption and investment. Capital adjustment costs restrict the accumulation of capital and thus influence investment choices. The firms in the second sector, which produce intermediate goods, operate under monopolistic competition. Each firm produces a distinct good for which it chooses the market price. Changes to the price of these goods are constrained by the Calvo (1983) mechanism, so that these prices are 'sticky'. Intermediate good production requires capital and labour services, inputs for which the firms act as price takers. The economy is closed.[5] Finally, the monetary authority's policy rule manages movements in the short-term nominal interest rate to respond to inflation deviations from its target as well as deviations of output and money growth from their trends.

2.1 Household

There exist a continuum of identical, infinitely-lived households that derive utility from consumption c_t, detention of real money balances M_t/P_t, and leisure $(1 - h_t)$, where h_t represents hours worked. A representative household's expected lifetime utility is described as follows:

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, M_t/P_t, h_t), \tag{1}$$

where $\beta \in (0,1)$ is the discount factor and the single-period utility function is specified as:

$$u(\cdot) = \frac{\gamma z_t}{\gamma - 1} \log \left(c_t^{\frac{\gamma-1}{\gamma}} + b_t^{\frac{1}{\gamma}} (M_t/P_t)^{\frac{\gamma-1}{\gamma}} \right) + \zeta \log(1 - h_t), \tag{2}$$

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[5] While this assumption deprives the model from the ability to capture information related to external factors, we believe it does not invalidate our forecasting experiments for the two following reasons. First, the VAR benchmarks we compare the model to are also run using only Canadian time series. Further, Dib (2003b) shows that most estimates unrelated to open-economy features do not change when a model extended to comprise open-economy features is estimated using Canadian data.

where γ and ζ are positive structural parameters, and z_t and b_t are serially correlated shocks. As shown by McCallum and Nelson (1999), the preference shock z_t resembles, in equilibrium, a shock to the IS curve of more traditional Keynesian analysis. On the other hand, b_t is interpreted as a shock to money demand. These shocks follow the first-order autoregressive processes

$$\log(z_t) = \rho_z \log(z_{t-1}) + \varepsilon_{zt}, \tag{3}$$

and

$$\log(b_t) = (1 - \rho_b)\log(b) + \rho_b \log(b_{t-1}) + \varepsilon_{bt}, \tag{4}$$

where $\rho_z, \rho_b \in (-1, 1)$ and the serially uncorrelated innovations ε_{zt} and ε_{bt} are normally distributed with zero mean and standard deviations σ_z and σ_b, respectively.

The representative household enters period t with k_t units of physical capital, M_{t-1} units of nominal money balances, and B_{t-1} units of bonds. During period t, the household supplies labour and capital to the intermediate-good-producing firms, for which it receives total factor payment $R_{kt}k_t + W_t h_t$, where R_{kt} is the rental rate for capital and W_t is the economy-wide wage. Further, the household receives a lump-sum transfer from the monetary authority, X_t, as well as dividend payments D_t from intermediate-good-producing firms.[6] The household allocates these funds to consumption purchases C_t and investment in capital goods I_t (both priced at P_t), to money holdings M_t and to bond holdings B_t, priced at $1/R_t$ (R_t denotes the gross nominal interest rate between t and $t+1$). The following budget constraint therefore applies:

$$P_t\left(C_t + I_t\right) + M_t + B_t/R_t \leq R_{kt}k_t + W_t h_t + M_{t-1} + B_{t-1} + X_t + D_t. \tag{5}$$

Investment i_t increases the capital stock over time according to

$$K_{t+1} = (1-\delta)K_t + I_t - \Psi\left(K_{t+1}, K_t\right), \tag{6}$$

where $\delta \in (0, 1)$ is the constant capital depreciation rate and $\Psi(.,.)$ is a capital-adjustment cost function specified as $\frac{\psi}{2}\left(\frac{K_{t+1}}{K_t} - 1\right)^2 K_t$, where $\psi > 0$ is the capital-adjustment cost parameter. With this specification both total and marginal costs of adjusting capital are zero in the steady-state equilibrium.

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[6] The transfer X_t is related to the monetary authority's managements of short term interest rates through its policy rule (described below)

The representative household chooses C_t, M_t, h_t, K_{t+1} and B_t in order to maximize expected lifetime utility (1) subject to the budget constraint (5) and the investment constraint (6). The first-order conditions for this problem are as follows:

$$\begin{aligned}
\frac{z_t C_t^{-\frac{1}{\gamma}}}{C_t^{\frac{\gamma-1}{\gamma}} + b_t^{\frac{1}{\gamma}} (M_t/P_t)^{\frac{\gamma-1}{\gamma}}} &= \lambda_{t;} & (7)\\
\frac{z_t b_t^{\frac{1}{\gamma}} (M_t/P_t)^{-\frac{1}{\gamma}}}{C_t^{\frac{\gamma-1}{\gamma}} + b_t^{\frac{1}{\gamma}} (M_t/P_t)^{\frac{\gamma-1}{\gamma}}} &= \lambda_t - \beta E_t \left(\frac{P_t \lambda_{t+1}}{P_{t+1}} \right); & (8)\\
\frac{\zeta}{1-h_t} &= \lambda_t \frac{W_t}{P_t}; & (9)\\
\lambda_t \left[\psi \left(\frac{K_{t+1}}{K_t} - 1 \right) + 1 \right] &= \beta E_t \left[\lambda_{t+1} \left(\frac{R_{kt+1}}{P_{t+1}} + 1 - \delta \right) \right] & \\
&\quad + \beta E_t \left[\lambda_{t+1} \psi \left(\frac{K_{t+2}}{K_{t+1}} - 1 \right) \frac{K_{t+2}}{K_{t+1}} \right] & (10)\\
\frac{1}{R_t} &= \beta E_t \left[\frac{P_t \lambda_{t+1}}{P_{t+1}} \right]; & (11)
\end{aligned}$$

where λ_t is the Lagrange multiplier associated with constraint (5).

As shown in Ireland (1997), combining conditions (7), (8) and (11) yields the following optimization-based money-demand equation:

$$\log(M_t/P_t) \simeq \log(C_t) - \gamma \log(r_t) + \log(b_t), \tag{12}$$

where $r_t = R_t - 1$ denotes the net nominal interest rate between t and $t+1$, γ is the interest rate-elasticity of money demand, and b_t is the serially correlated money-demand shock described above.

2.2 The final goods-producing firm

The final good, Y_t, is produced by assembling a continuum of intermediate goods $y_{jt}, j \in (0,1)$ that are imperfect substitutes with a constant elasticity of substitution θ. The aggregation function is defined as

$$Y_t \leq \left(\int_0^1 y_{jt}^{\frac{\theta-1}{\theta}} \mathrm{d}j \right)^{\frac{\theta}{\theta-1}}, \qquad \theta > 1. \tag{13}$$

Final good-producing firms behave competitively, maximizing profits and taking the market price of the final good P_t as well as the intermediate-good prices $p_{jt}, j \in (0,1)$ as given. The maximization problem of a representative, final good-producing firm is therefore

$$\max_{\{y_{jt}\}_{j=0}^{1}} \left[P_t Y_t - \int_0^1 p_{jt} y_{jt} \mathrm{d}j \right],$$

subject to (13). The resulting input demand function for the intermediate good j is

$$y_{jt} = \left(\frac{p_{jt}}{P_t} \right)^{-\theta} Y_t, \tag{14}$$

and represents the economy-wide demand for good j as a function of its relative price and of the economy's total output of final good Y_t. Competition in the sector and the constant returns to scale production (13) implies that these firms make zero profits. Imposing the zero-profit condition leads to the following description of the final-good price index P_t:

$$P_t = \left(\int_0^1 {p_{jt}}^{1-\theta} \mathrm{d}j \right)^{\frac{1}{1-\theta}}. \tag{15}$$

2.3 The intermediate goods-producing firm

The intermediate good-producing firm j uses capital and labor services k_{jt} and h_{jt} to produce y_{jt} units of good j, according to the following constant-returns-to-scale technology:

$$y_{jt} \leq k_{jt}^{\alpha} \left(A_t \eta^t h_{jt} \right)^{1-\alpha}, \quad \alpha \in (0,1), \tag{16}$$

where η_t represents the effect of trend productivity growth. The presence of such growth implies on a the balanced growth path, output, investment, consumption, the real wage, capital and real money balances all grow at the same rate η. Further A_t describes an aggregate technology shock common to all firms. This shock follows a stationary first-order autoregressive process:

$$\log A_t = (1-\rho_A)\log(A) + \rho_A \log(A_{t-1}) + \varepsilon_{At}, \tag{17}$$

where $\rho_A \in (-1,1)$ is an autoregressive coefficient, $A > 0$ is a constant, and ε_{At} is normally distributed with mean zero and standard deviation σ_A.

Each intermediate good-producing firm sells its output under monopolistic competition; the economy-wide demand for the good produced by producer j is given by (14). Following Calvo (1983), assume that each firm is only allowed to re-optimize its output price at specific times. Specifically, with probability ϕ the firm must charge the price that was in effect in the preceding period, indexed by the steady-state rate of inflation π; with probability $1-\phi$, the firm is free to re-optimize and choose an unrestricted new price. On average, each firm therefore re-optimizes every $1/(1-\phi)$ periods.[7]

At time t, if firm j receives the signal to reoptimize, it chooses a price $\widetilde{p}_{jt}$, as well as contingency plans for h_{jt+k}, k_{jt+k}, for all $k \geq 0$ that maximize its discounted, expected (real) total profit flows for the period where it will not be able to reoptimize. The profit maximization problem is the following (θ^k represents the probability that $\widetilde{p}_{jt}$ remains in effect at $t+k$:

$$\max_{\{k_{jt}, h_{jt}, \widetilde{p}_{jt}\}} E_0 \left[\sum_{k=0}^{\infty} (\beta\phi)^k \lambda_{t+k} D_{jt+k}/P_{t+k} \right],$$

with D_{jt+k}/P_{t+k}, the real profit flow at time $t+k$ and

$$D_{jt+k} = \widetilde{p}_{jt}\pi^k y_{jt+k} - R_{kt+k}k_{jt+k} - W_{t+k}h_{jt+k}. \tag{18}$$

Profit maximization is subject to the demand for good j (14) and the production function (16) (to which the Lagrange multiplier $\xi_t > 0$ is associated). The first-order conditions for for k_{jt+k}, h_{jt+k}, and $\widetilde{p}_{jt}$ are:

$$\frac{R_{kt}}{P_t} = \alpha q_t \frac{y_{jt}}{k_{jt}}; \tag{19}$$

$$\frac{W_t}{P_t} = (1-\alpha) q_t \frac{y_{jt}}{k_{jt}}; \tag{20}$$

$$\widetilde{p}_{jt} = \frac{\theta}{\theta-1} \frac{E_t \sum_{k=0}^{\infty} (\beta\phi\pi^{-\theta})^k \lambda_{t+k} Y_{t+k} q_{t+k} P_{t+k}^{\theta}}{E_t \sum_{k=0}^{\infty} (\beta\phi\pi^{1-\theta})^k \lambda_{t+k} Y_{t+k} P_{t+k}^{\theta-1}}; \tag{21}$$

where $q_t \equiv \xi_t/\lambda_t$ is the real marginal cost of the firm.

The symmetry in the demand for their good implies that all firms allowed to reoptimize choose the same price $\widetilde{p}_{jt}$, which we denote $\widetilde{p}_t$. Considering the definition of the price index in (15) and the fact that at the economy's level, a fraction $1-\phi$ of intermediate-good producing firms reoptimize, the aggregate price index P_t evolves according to

$$P_t^{1-\theta} = \phi(\pi P_{t-1})^{1-\theta} + (1-\phi)(\widetilde{p}_t)^{1-\theta}. \tag{22}$$

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[7] This specification of the Calvo mechanism follows Yun (1996). Alternatively, Christiano et al. (2005) assume that when the re-optimization signal is not received, the price is increased by the *preceding period*'s rate of inflation. Smets and Wouters (2003) implement a flexible specification that nests the two cases.

Equations (19) and (20) state that firms choose production inputs in order for their costs to equal marginal product weighted times real marginal costs. Equation (21) relates the optimal price to the expected future price of the final good and to expected future marginal costs. Taking a first-order approximation of this condition, of (22), and combining them leads us to derive the model's New Keynesian Phillips curve:

$$\widehat{\pi}_t = \beta\widehat{\pi}_{t+1} + \frac{(1-\phi)(1-\beta\phi)}{\phi}\widehat{q}_t, \tag{23}$$

where a hatted variables denotes its deviation from steady-state value. This expression relates the present period's inflation rate to the expectation of its future values as well as to today's marginal costs, an indicator of the strength of economic activity.

2.4 The monetary authority

As in Ireland (2003) and Dib (forthcoming), assume that the monetary authority conducts manages the short-term nominal interest rate R_t to respond to deviations of inflation, $\pi_t \equiv P_t/P_{t-1}$, output, Y_t, and money growth, $\mu_t \equiv \overline{M}_t/\overline{M}_{t-1}$ from their target (or trend) levels. This monetary policy rule is given by:

$$\log(R_t/R) = \varrho_\pi \log(\pi_t/\pi) + \varrho_y \log(Y_t/\eta^t Y) + \varrho_\mu \log(\mu_t/\mu) + \log(v_t), \tag{24}$$

where R, π, and μ are the steady-state or target values of R_t, π_t, and μ_t, respectively, while $\eta^t Y$ is trend output. Further, v_t is a monetary policy shock that evolves according to

$$\log(v_t) = \rho_v \log(v_{t-1}) + \varepsilon_{vt}, \tag{25}$$

where $\rho_v \in [0, 1)$ is an autoregressive coefficient and ε_{vt} is a zero-mean, serially uncorrelated shock with standard deviation σ_v. The monetary authority implements this rule with the appropriate lump-sum injection/withdrawal of money X_t

The policy coefficients ϱ_π, ϱ_y, and ϱ_μ are chosen by the monetary authorities. When $\varrho_\pi > 0$, $\varrho_y > 0$, and $\varrho_\mu = 0$, monetary policy follows a Taylor (1993) rule, in which nominal interest rates increase in response to deviations of inflation and output from trend.

In contrast, (24) states that monetary policy follows a modified Taylor (1993) rule that adjusts short-term nominal interest rates in response to changes in money-growth as well as to deviations of inflation and output.

In that case, a unique equilibrium exists as long as the sum of ϱ_π and ϱ_μ exceeds one. The money-growth rate can be interpreted as an indicator of expected inflation or as a proxy for some omitted variables, such as the exchange rate or financial variables, to which monetary policy responds. Alternatively, Ireland (2003) interprets such a rule as a combination policy that influences a linear combination of the interest rate and the money-growth rate to control inflation.

2.5 Symmetric equilibrium

Let $r_{kt} \equiv R_{kt}/P_t$, $w_t \equiv W_t/P_t$, and $m_t \equiv M_t/P_t$ denote the real capital rental rate, the real wage, and real money balances, respectively. A symmetric equilibrium for this economy consists in a sequence of allocations $\{Y_t, C_t, I_t, m_t, h_t, K_t\}_{t=0}^{\infty}$ a sequence of prices and co-state variables $\{w_t, r_{kt}, R_t, \pi_t, \lambda_t, q_t\}_{t=0}^{\infty}$ and the stochastic processes for preference, money demand, technology, and monetary policy shocks. These allocations, prices, and shocks are such that (i) households, final good-producing firms, and intermediate good-producing firms optimize, (ii) the monetary policy rule (24) is satisfied, and (iii) the following market-clearing conditions are satisfied:

$$
\begin{aligned}
K_t &= \int_0^1 k_{jt}\, dj; && (26)\\
h_t &= \int_0^1 h_{jt}\, dj; && (27)\\
M_t &= \overline{M_t} = \overline{M_{t-1}} + X_t; && (28)\\
B_t &= 0; && (29)\\
Y_t &= C_t + I_t. && (30)
\end{aligned}
$$

Allowing for trend productivity growth in the production process (13) implies that Y_t, C_t, I_t, K_t, w_t and m_t all grow at the same rate η in equilibrium. In the solution of the model, all growing variables are divided by η^t (so $y_t \equiv Y_t/\eta^t$, for example) in order for the solution to be expressed in terms of stationary variables.

Next, the steady-state of the system is computed, a first-order linear approximation of the equilibrium system around these steady-state values is formed, and Blanchard and Kahn (1980)'s procedure is used to transform this forward-looking model into the following state-space solution:

$$
\begin{aligned}
\widehat{s}_{t+1} &= \mathbf{\Phi}_1 \widehat{s}_t + \mathbf{\Phi}_2\, \varepsilon_{t+1}, && (31)\\
\hat{d}_t &= \mathbf{\Phi}_3 \widehat{s}_t, && (32)
\end{aligned}
$$

where $\widehat{s}_t$ is a vector of state variables that includes predetermined and exogenous variables; $\hat{d}_t$ is the vector of control variables; and the vector ε_{t+1} contains the random innovations.[8] The elements of matrices $\mathbf{\Phi}_1, \mathbf{\Phi}_2$, and $\mathbf{\Phi}_3$ depend on the model's structural parameters.

3 Estimation

3.1 Methodology

It is usual in this literature to calibrate the values of some of the model's parameters, before estimating the values of the remaining ones. In light of this, we set the weight on leisure in the utility function ζ to 1.35, which implies that households spend around one third of non-sleeping time in market activities (work). The share of capital in production, α, and the depreciation rate, δ, are assigned values of 0.33 and 0.025, respectively; these values are commonly used in the literature. The degree of monopoly power in intermediate-goods markets θ is equal to 6, which implies a markup of 20% in steady state: this matches values usually used in similar studies. Finally, both Ireland (2001a) and Dib (2003b) remark that the capital adjustment parameter ψ is difficult to estimate without data on capital stock. We fix this parameter to 15, as in Dib (2003b).

The remaining 18 parameters[9] are estimated using maximum likelihood. This requires us to select a subset of the control variables $\hat{d}_t$ in (32) for which data is available, as well as selecting the appropriate rows of $\mathbf{\Phi}_3$. Next, the likelihood of the sample $\{\hat{d}_t\}_{t=1}^{T}$ is computed recursively using the Kalman filter (Hamilton, 1994, Chap. 13). Finally, the parameter values that maximize the likelihood are found using standard numerical procedures.[10]

Since the model is driven by four shocks, we estimate the model using data for four series, to avoid problems of stochastic singularity. We

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[8] For any stationary variable x_t, $\hat{x}_t = \log(x_t/x)$ denotes the deviation of x_t from its steady-state value x. In our specification, $\widehat{s}_t = \left(\hat{k}_t, \widehat{m}_{t-1}, \hat{z}_t, \hat{b}_t, \hat{A}_t, \widehat{v}_t,\right)'$, $\hat{d}_t = \left(\hat{\lambda}_t, \hat{q}_t, \widehat{m}_t, \widehat{y}_t, \hat{R}_t, \hat{r}_{kt}, \hat{c}_t, \hat{i}_t, \widehat{\pi}_t, \widehat{w}_t, \hat{h}_t, \hat{\mu}_t\right)'$, and $\varepsilon_{t+1} = (\varepsilon_{zt+1}, \varepsilon_{bt+1}, \varepsilon_{At+1}, \varepsilon_{vt+1})'$. Appendix A presents a full list of the equilibrium conditions of the model, the steps involved in finding the steady-state, and the linearized equations introduced into the Blanchard and Kahn (1980) algorithm.

[9] These are $\beta, \gamma, \varrho_\pi, \varrho_\mu, \varrho_y, \rho_v, \sigma_v, \phi, A, \rho_A, \sigma_A, b, \rho_b, \sigma_b, \rho_z, \sigma_z, \pi$ and η.

[10] In addition to Dib (2003a,b, forthcoming) and Ireland (2003, 2004), this estimation method is used by Bergin (2003) and Bouakez et al. (forthcoming), and several others. Ireland (2004) provides some of the details about the estimation procedure. We employ the simplex algorithm, as implemented by *Matlab*.

use Canadian data on output, inflation, a short-term interest rate and real money balances. Output is measured by real, final domestic demand. Inflation is the gross rate of increase in the GDP deflator. The nominal interest rate is the three-month Treasury Bill rate. Finally, real money balances are measured by dividing the M2 money stock by the GDP deflator. Output and real money balances are expressed in per-capita terms using the civilian population aged 16 and over.[11]

Note that we directly estimate the parameter η, which describes the rate at which output and real money balances grow in the steady state. This trend is not shared by inflation, and we assume that inflation is trendless, which also fixes a constant steady-state nominal interest rate through the Fisher relation (see below for a discussion of potential alternatives to assuming trendless inflation). Our treatment of trends is thus different from the one pursued usually (Smets and Wouters, 2003) where authors typically render data stationary before estimation by linearly detrending all series and using the detrended series in estimation. We believe the strategy pursued in this paper is particularly attractive in the context of a forecasting exercise. It enables us to produce forecasts for the log-levels of the data directly, rather than forecasts for detrended series that must then be transformed into forecasts for log levels.

3.2 Estimation Results

Parameter Estimates

We first estimate the model's parameters for the sample running from 1981:3 through 1995:4. Tables 1 reports the maximum-likelihood estimates of the parameters, alongside their standard errors and t-statistics. Almost all of the estimated parameters are statistically significant and economically meaningful. The estimate of the discount rate β is 0.99, which implies an annual steady-state real interest rate of just over 4 per cent. The estimates of b, determining the steady-state ratio of real balances to consumption, is 0.5; while the constant elasticity of substitution between consumption and real balances γ is around 0.06, similar to that estimated by Dib (2003a) for the Canadian economy. The estimate of ϕ, the probability of not adjusting prices in the next period, is 0.63. Thus, on average, firms keep their prices unchanged, except for indexation, for about two quarters and a half. This estimate is very close to those obtained in the closed-economy estimates of Dib (2003b).

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[11] Appendix B contains additional details, notably the mnemonics, about the data.

The estimates of the monetary policy parameters are statistically significant, with the exception of ϱ_y. Specifically, the responses of monetary policy to inflation, output, and money growth (ϱ_π, ϱ_y, and ϱ_μ) are 0.75, 0.02, and 0.48, respectively.[12] The estimates of ρ_v and σ_v, the persistence coefficient and standard deviation of monetary policy shocks, are 0.20 and 0.006, respectively. Overall, the estimates of monetary policy parameters are similar to those previously estimated by Dib (2003b, forthcoming) for the Canadian economy. They indicate that, to achieve its objectives, the Canadian monetary authorities have responded significantly to inflation and money growth and hardly (if at all) to output deviations from trend.

The autoregressive coefficient estimates indicate that the technology, money demand, and preference shocks are relatively persistent, with the money demand shock being the most persistent ($\rho_z = 0.994$). The standard deviation estimates suggest that the the aggregate demand-side shocks (money demand and preferences) are the most volatile.

Next, to assess the model's performance, we provide a brief analysis of the impulse response functions drawn from the estimated model and its variance-decomposition.[13]

Impulse response functions

Figures 1 to 4 display the economy's response following the four types of exogenous shocks, at the estimated parameter values. The response of output is measured as a deviation from its steady-state value, whereas the responses of the other variables are in net (annualized) percentage points.

Figure 1 plots the economy's response to a monetary policy tightening, i.e. setting the innovation ε_{vt} to 0.01, a value close to its estimated standard deviation. Following the tightening, interest rates increases and return to steady state moderately fast (recall that the estimated serial correlation in monetary policy shocks, ρ_v, is 0.20.) Output, inflation and money growth by contrast, fall sharply on impact. Output and inflation return gradually to steady state, while money growth overshoots slightly in the following periods, converging back to steady state from above. This gradual return to steady state reflects the actions of the Calvo (1983) mechanism and the serial correlation of the shock. Notice that the negative, contemporaneous correlation between interest rates and money growth –the liquidity effect– is consistent with the evidence.[14]

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[12] Recall that indeterminate equilibria do not occur as long as $\varrho_\pi + \varrho_\mu > 1$.

[13] Similar analysis are available elsewhere; see Dib (2003a) for example.

[14] Evidence also suggests the responses of inflation and output following monetary policy shocks should be characterized by hump-shaped patterns, where the maximum impact on

Figure 2 shows the economy's responses to a money demand shock (setting the innovation ε_{bt} to 0.01). The shock has only a small impacts on output and inflation, which decrease slightly on impact. Money growth increases sharply, however, to accommodate the increase in demand. Since the rule followed by the monetary authority includes a response to money growth increases, the nominal interest rate increases slightly, which is at the source of the slight output decreases. These responses roughly match Poole's (1970) classic analysis, in which the monetary policy authority changes the short-term nominal interest rate to react to exogenous demand-side disturbances.

Figure 3 presents responses following a shock to technology (an increase of 0.01 in ε_{At}. Output jumps on impact, while the nominal interest rate and inflation fall below their steady-state levels. Money growth responds positively to the shock before falling below its steady-state level after two quarters. The deflationary pressure brought about by the shock leads to a sustained easing of monetary policy; recall the monetary policy rule in (24). This mechanism serves to accommodate the shock and gradually increase output, which peaks three quarters after the shock. Therefore, the monetary authority's response helps the economy to adjust to the supply-side disturbances.

Finally, Figure 4 shows the impulse responses to a 1 per cent increase in the preference shock, an disturbance to households' marginal utility of consumption. In response to this shock, output, the nominal interest rate, inflation, and money growth jump immediately above their steady-state levels before returning gradually to those levels. Because the estimates of the preference autoregressive coefficient, ρ_z, are relatively large, the computed impulse responses are highly persistent. To control the rises in output and inflation, the monetary authority increases short-term interest rates slightly, but persistently.

Variance decomposition

Table 2 reports the standard deviations, expressed in percentage terms, of output, real balances, inflation and the nominal interest rate as computed from the data and from the estimated model. In the data, output and real balances have standard deviations of 3.44 and 2.78 per cent, respectively. Inflation and the short-term nominal interest rate are less volatile; their

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the variables is attained several periods after the shock. Christiano et al. (2005) show that adding several additional features to the model enables it to display these patterns. Because out emphasis in on the out-of-sample forecasting ability of the model and we want to keep the model parsimonious, we do not conduct our experiments with such a model.

standard deviations are less than 0.6 per cent. The table shows that the model (i) underpredicts the volatility of output, (ii) generates real balance-volatility close to that observed in the data, and (iii) slightly overpredicts the volatility of inflation and the nominal interest rate.

To understand which of the four shocks are driving the results, Table 3 decomposes the forecast-error variances of output, real balances, inflation and the nominal interest rate into the fractions that can be attributed to each of the shocks. The table shows that preference and technology shocks are the most important source of fluctuation in output, both in the short and long term. Monetary policy shocks also account for a smaller but significant fraction of output fluctuations in the short term. Monetary policy and technology shocks are the most important factors determining fluctuations in the inflation rate. Together, they account for around 80 per cent of fluctuations at the one-quarter-ahead horizon. Preference shocks do contribute to some of inflation volatility, particularly at loner horizons. The great majority of interest rate fluctuations are attributable to preferences shocks; the contribution of the other shocks, particulary that of monetary policy shocks, is not significant. Finally, technology and money demand shocks explain more than 90 per cent of the fluctuations in real money balances, while, once again, monetary policy shocks explain about 10 per cent of the short-term fluctuations.

Overall, the estimation results indicate that the New Keynesian model can provide a coherent explanation for how several types of shocks affect the economy. Next, we assess the out-of-sample forecasting properties of the model.

4 Forecasting Properties of the Model

4.1 The Experiment

We compute out-of-sample forecasts for the model and for several VAR benchmarks.[15] Specifically, we begin by estimating both models using data from 1981:3 through 1995:4. These estimates are used to produce forecasts one- to eight-quarters-ahead, i.e. for 1996:1 to 1997:4, for the four variables used in the estimation. Next, the sample used becomes 1981:4 to 1996:1, the estimates are updated, and then used to produce another set of forecasts, for 1996:2 through 1998:1. Estimates and forecasts are updated in this manner

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[15] The VAR benchmarks differ in the number of lags they contain, whether they include trend parameters for all variables, and whether the Minnesota prior of Doan et al. (1984) is imposed.

until the end of the available sample; at this point, we have time series for one to eight quarter ahead forecasts spanning the range 1996:1 to 2004:1. Table 4 illustrates graphically the experiment conducted. These forecasts are then ready to be compared to realized data over the same period.

Figure 5 to 8 give a graphical illustration of the results. First, Figure 5 first compares the model's forecasts to realized data. It shows that the model provides what appears to be a relatively good characterization of output fluctuations, for the one-quarter-ahead, four-quarter-ahead and also eight-quarter ahead horizons. The model also maintains a reasonably balanced forecast for inflation, although realized data exhibits some transitory fluctuations that are not well captured by the model. Further, the model is slow to incorporate the interest rate decreases of 2001 in its forecasts. Finally, the model's forecasts for money track reasonably well this variable.

Figures 6 to Figure 8 then depict the forecasting errors of the model (the solid line) with those arising from the VAR(2) benchmark[16] (the dotted lines) for the case of one-quarter-ahead (Figure 6), four-quarter-ahead (Figure 7), and eight-quarter-ahead forecasts (Figure 8). In Figure 6, the two models appear to give forecasts that are roughly equivalent, apart from the case of output, where the VAR benchmark may produce smaller errors. Next, at the four-quarter-ahead horizon (Figure 7), the NK model seems to outperform the benchmark for output interest rates and real money balances, whereas the inflation forecasts appear very close. Finally, at the eight-quarter horizon (Figure 8), important differences appear, with the model forecasting better than the benchmark once again for output, interest rates and money, while the inflation forecasts remain close.

The first column of Table 5 synthesizes the information contained in Figures 6-8. It reports the Mean Square Error (MSE) of the New Keynesian model, relative to that of the VAR benchmark. Values smaller than one therefore suggest that the NK model possesses the superior forecasting accuracy, while values bigger than one favour the VAR benchmark. As suggested above, the MSEs tend to favour the NK model, particularly as the forecasting horizon increases. In particular, at the eight-quarter ahead horizon, the model's MSE is only between 17% that of the VAR benchmark for output and less than 30% for interest rates. Note, however, that this favourable performance does not apply to inflation, for which the VAR benchmark has slightly lower MSEs. Further, the table shows that for very short term horizons, the advantage for the NK models vanishes: forecasting

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[16] This VAR contains a constant and a trend, as well as two lags of each variables; the Minnesota prior is not used (but see below).

one-quarter ahead output appears to be easier with the VAR benchmark.

4.2 Econometric Tests of Forecasting Accuracy

To test whether these improvements in MSE are statistically significant, we first use Diebold and Mariano (1995)'s test. To compute the test, define the forecast errors of the New Keynesian model as $\{e_t^M\}_{t=1}^T$ and those from the VAR benchmark as $\{e_t^B\}_{t=1}^T$. Further, define a sequence of 'loss differentials' $\{l_t\}_{t=1}^T$ where $l_t = (e_t^B)^2 - (e_t^M)^2$. If the NK model is a better forecasting tool, one would expect that on average, the loss differentials l_t would be positive. Conversely, one would expect negative values if the VAR benchmark is superior. Following this intuition, the Diebold and Mariano (1995) test considers the null hypothesis $H_0 : E[l_t] = 0$; positive values of the statistic suggest that the forecasts from the New Keynesian model have lower mean-squared errors, while negative values favour the VAR benchmark. The test statistic (denoted DM) is asympotically normal and standard critical values are used.[17] Harvey et al. (1997) propose a corrected Diebold and Mariano (1995) statistic, in order to reduce size distortions that might be significant in small samples. The corrected statistic is compared to a Student's t distribution with $N-1$ degrees of freedom, where N is the number of forecasted data.

The last two columns of Table 5 thus report the Diebold and Mariano (1995) and Harvey et al. (1997) statistics, as well as their p-value in parenthesis. Due to the small number of forecasts available (30 for the one-quarter-ahead forecasts, and 22 for the eight-quarter ahead one, it is not surprising that many test statistics are not significant. Nevertheless, the NK model does appear to forecast better than the VAR benchmark for some variables and at some horizons; notably for interest rates, money, and output for the longer-term horizons. The overall message of the table is thus similar to the images shown in Figures 6-8: the New Keynesian model's forecasting ability compares very favourably to that of the VAR benchmark, performing better for output, interest rates and money at some horizons and not worse for inflation.[18] The dimension along which the New Keynesian model performs less well is thus when forecasting inflation is concerned. This fact probably arises because as estimated, the model does not allow trends to affect inflation, when ample anecdotal or econometric evidence suggests that

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[17] The statistic is computed as $DM = l/\hat{\sigma}(l)$ where l is the sample average of l_t and $\hat{\sigma}(l)$ is an (HAC-consistent) estimate of the standard deviation of l.

[18] This favourable performance also obtains when the New Keynesian model is compared to a VAR with one lag in each variable. These additional results are available on request.

structural breaks have affected inflation over the last two decades. Section 5 below discusses one possibility for future research on New Keynesian model to tackle this important issue.

Even if all tests in Table 5 were conclusively identifying the superior model, forecasts from the lesser model may still contain information not present in those from the first model; combining both forecasts could therefore reduce further the forecasting errors.[19] A more stringent test of whether one model dominates another in forecasting may be to test whether the second model contains *any* information not contained in the first model's forecasts.

In this context, Granger and Newbold (1973) define the forecasts from one model as "conditionally efficient" when combining them with those from another model does not lead to an overall decrease in forecast accuracy. Chong and Hendry (1986) define the same situation as one where the first set of forecasts "encompass" those from the second model: there is no need to keep the second model's forecasts because the information they contain is encompassed by those of the first model.

To implement the test for forecast encompassing, we follow Harvey et al. (1998), which propose test statistics similar to those in Diebold and Mariano (1995) and its Harvey et al. (1997) correction. The null hypothesis is that the New Keynesian model' forecasts contain no information that isn't already contained in those from the VAR.[20]

Table 6 presents the results. The first column presents the test statistic as proposed by Diebold and Mariano (1995) and the second the correction proposed by Harvey et al. (1997). Recall that high values of the test statistic reject the hypothesis that no value can be gained from using the NK forecasts when the VAR model is available. Similar to what was discussed from Table 5, the results in Table 6 suggest that VAR forecasts for output, interest rates, and money are improved when those from the NK model are put into contribution, whereas one cannot reject the hypothesis of no value from the

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[19] For example, the lesser model might outperform the first in specific times, such as when the economy is in recession.

[20] Specifically, assume the following regression equation:

$$e_t^B = \gamma(e_t^B - e_t^M) + \epsilon_t,$$

where e_t^B and e_t^M represent the forecasting errors from the VAR benchmark model and the NK model, respectively. The null hypothesis is $H_0 : \gamma = 0$. Under the null, the errors made by the VAR benchmark cannot be explained (and thus potentially reduced) by information arising from the NK model. Conversely, one can test whether the NK forecasts encompass those from the VAR, whether there is *any* information in the VAR forecasts that is not present in the NK forecasts.

NK model for inflation.

As discussed in the introduction, researchers have often pointed out that imposing the Minnesota prior –all variables follow simple random walks– on the Bayesian estimation of simple VARs delivers superior forecasting accuracy. In that context, Table 7 repeats the results of Table 5, but such a Bayesian VAR is now the benchmark to which the New Keynesian model is compared. Comparing the two tables, one can see that the forecasting accuracy of the benchmark VAR indeed is often superior to what it was without the priors (the MSE of the NK model is often higher than it was in Table 5). Nevertheless, similar observations about the model's forecasting properties can made: in particular, the NK model can forecast output, interest rates, and money well compared to the benchmark, and as the forecasting horizon increases, several of these differences become statistically significant.

Finally, note that the comparisons conducted so far have been between a model where inflation and interest rates were restricted to have no trends (the NK model) against one where such trends were present (the VAR benchmark). A better suited comparison might be between two models for which inflation and interest rates are restricted to contain no significant trends. Repeating the analysis by using a VAR where the trend components have been taken out of the equations for inflation and interest rates reinforces the results presented in Table 5 to Table 7.[21]

5 Discussion and Conclusion

Since the coming of age of the RBC methodology, researchers have often identified dimensions along which these structural models seemed at odds with features of observed data.[22] Further, researchers that extend the simple RBC structure to produce models that feature nominal rigidities and multiple sources of volatility also have had difficulties replicating observed features of the data, like the strong autocorrelation properties of inflation or output.

In such a context, the evidence that structural models like the New Keynesian may possess comparable or even better out-of-sample forecasting ability than unrestricted VARs is surprising.[23] Taken generally, this

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[21] Further, using the Sign test to compare the forecasts from the two models does not modify our overall conclusions.

[22] For example, Cogley and Nason (1995) showed that the simple RBC model could not match the autocorrelation function of output or the impulse responses of Blanchard and Quah (1989).

[23] As mentioned in the introduction, Ingram and Whiteman (1994) and DeJong, Ingram

evidence suggests that restricted or parsimonious specifications, although at odds with some features of the data, may often outperform unrestricted alternatives in out-of-sample exercises. Clements and Hendry (1998, 1999) assess this conjecture. The main trade-off discussed is that of sampling variability (introduced in the unrestricted specification by the estimation of numerous parameters) versus inconsistency (introduced in parsimonious models by imposing possibly false restrictions).

Overall, the results presented in this paper do suggest that this tradeoff may be favourable to parsimonious specifications similar to the New Keynesian model. Such findings are encouraging for researchers working with models of this type. The econometric tests we report make clear that at a minimum, restricting a VAR by appealing to the structure of the New Keynesian model has no negative impact on its forecasting performance. In the case of output, interest rates and money, the restricted model may in fact be the one with the superior forecasting accuracy, particularly as the horizon one is interested in increases. As shown in the results, the forecasting properties of the model for inflation are not as strong, although there are not significantly worse than those of the simple benchmark VARs.

The forecasting properties of the model for inflation could likely be improve if trends in the inflation target of the monetary authority authority could be introduced. This would allow the model to better track the apparent downward trend in inflation over the last 20 years. To introduce such a feature, the regime switching environment in Erceg and Levin (2003), where the inflation target of monetary authorities is periodically modified, could be employed.

It would also be interesting to study whether basing the growth of technology on a difference stationary process, rather than the trend-stationary one used, would improve the forecasting ability of the model. Such a specification of trend growth would imply that real variables like output would be differenced to make them stationary. The natural benchmark to compare forecasts would then be a VAR in differences.[24]

Finally, using an open-economy specification would allow the model to capture information related to external (principally American) data and

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and Whiteman (2000) display such evidence. In an earlier paper, Ireland (1995) reports that, once translated into a bivariate VAR, the simple version of the permanent income theory is rejected within-sample but helps the model to better forecast out-of-sample.

[24] In the experiments of Stock and Watson (1999, 2002) the variables used where for the most part differenced. Ireland (2001b), however, reports in a formal comparison between estimating trend-stationary or difference-stationary RBC models that the better out-of-sample forecasting accuracy arises from the trend stationary specification.

the various channels by which they affect the Canadian economy, providing potentially important help for forecasting Canadian time series ahead.[25]

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[25] It is doubtful whether such an open economy specification would modify significantly the estimation of other parameters, however: Dib (2003b) shows that most estimates unrelated to open-economy features do not change when a model extended to comprise open-economy features is estimated using Canadian data.

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Table 1: Maximum-likelihood estimates and standard errors

(1981Q3 to 1995Q4)

Parameter	Estimate	Std. Deviation	t-statistic
β	0.990	0.002	642.0
γ	0.055	0.022	2.45
ϱ_π	0.754	0.165	4.56
ϱ_μ	0.481	0.105	4.57
ϱ_y	0.018	0.026	0.68
ρ_v	0.194	0.094	2.06
σ_v	0.006	0.001	7.46
ϕ	0.630	0.060	10.47
A	3.532	0.200	18.13
ρ_A	0.950	0.062	15.38
σ_A	0.013	0.002	5.51
b	0.498	0.060	8.40
ρ_b	0.994	0.010	99.22
σ_b	0.012	0.001	8.35
ρ_z	0.917	0.050	18.48
σ_z	0.017	0.004	4.5
π	1.010	0.004	221.13
η	1.002	0.002	550.98
LL	898.7011		

Note: LL is the maximum log-likelihood value.

Table 2: Volatility

Variable	$\hat{y}_t$	$\hat{\pi}_t$	$\hat{R}_t$	$\hat{m}_t$
Data	3.44	0.60	0.48	2.78
Model	2.33	0.74	0.61	2.87

Table 3: Forecast-error variance decompositions

		Percentage owing to:			
Quarters	Variance in %	Tech.	Mon.dem.	Policy	Pref.
		A. Output			
1	0.0121	49.71	6.94	12.09	31.27
2	0.0248	58.18	3.84	7.29	30.68
4	0.0476	66.09	2.06	3.98	27.88
10	0.0904	75.56	1.10	2.10	21.24
		C. Inflation			
1	0.0023	31.52	17.20	48.61	2.66
2	0.0029	30.46	15.12	46.08	8.34
4	0.0034	28.56	13.03	40.47	17.94
10	0.0040	26.91	11.09	34.43	27.56
		D. Nominal interest rate			
1	0.0010	6.30	4.10	0.5	95.22
2	0.0017	2.72	2.78	2.5	94.25
4	0.0026	5.89	5.89	2.2	92.03
10	0.0038	10.72	10.72	1.5	87.84
		B. Real balances			
1	0.0125	41.25	41.83	7.93	9.00
2	0.0324	42.56	44.80	4.26	8.38
4	0.0757	42.70	48.42	1.98	6.91
10	0.1887	39.69	55.49	0.80	4.01

Table 4. The Forecasting Experiment (1996:1 - 2004:1)

Estimate	Forecast k periods ahead				
	$k = 1$	$k = 2$	$k = 3$	$\cdots$	$k = 8$
$1981:3 \longrightarrow 1995:4$	$1996:1$	$1996:2$	$1996:3$	$\cdots$	$1997:4$
$1981:3 \longrightarrow 1996:1$	$1996:2$	$1996:3$	$1996:4$	$\cdots$	$1998:1$
$1981:3 \longrightarrow 1996:2$	$1996:3$	$1996:4$	$1997:1$	$\cdots$	$1998:2$
$1981:3 \longrightarrow 1996:3$	$1996:4$	$1997:1$	$1997:2$	$\cdots$	$1998:3$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$1981:3 \longrightarrow 2003:2$	$2003:3$	$2003:4$	$2004:1$	$---$	$---$
$1981:3 \longrightarrow 2003:3$	$2003:4$	$2004:1$	$---$	$---$	$---$
$1981:3 \longrightarrow 2003:4$	$2004:1$	$---$	$---$	$---$	$---$

Table 5: Testing for Equal Forecasting Accuracy: Model and VAR(2) Benchmark; 1997:1 - 2004:1

Variable	Relative MSE (NK Model)[a]	DM stat.[b] (p-value)	HLN stat.[c] (p-value)
Forecasting One Period Ahead			
Output	1.73	-2.18(0.03)	-2.14(0.04)
Inflation	1.21	-0.51(0.61)	-0.50(0.62)
Interest Rate	0.73	1.57(0.12)	1.54(0.13)
Money	0.67	2.29(0.02)	2.25(0.03)
Forecasting Two Periods Ahead			
Output	1.26	-0.74(0.45)	-0.70(0.49)
Inflation	1.28	-0.63(0.53)	-0.60(0.56)
Interest Rate	0.60	2.78(0.00)	2.64(0.01)
Money	0.49	2.44(0.02)	2.31(0.03)
Forecasting Four Periods Ahead			
Output	0.57	0.70(0.48)	0.61(0.55)
Inflation	1.28	-0.46(0.64)	-0.40(0.69)
Interest Rate	0.53	2.72(0.00)	2.36(0.03)
Money	0.30	1.87(0.06)	1.62(0.12)
Forecasting Six Periods Ahead			
Output	0.33	1.22(0.22)	0.94(0.36)
Inflation	1.17	-0.12(0.91)	-0.09(0.93)
Interest Rate	0.45	5.07(0.00)	3.91(0.00)
Money	0.18	1.41(0.16)	1.08(0.29)
Forecasting Eight Periods Ahead			
Output	0.26	1.20(0.23)	0.79(0.44)
Inflation	1.32	-0.81(0.42)	-0.54(0.60)
Interest Rate	0.40	NA(NA)	NA(NA)
Money	0.13	1.24(0.22)	0.82(0.42)

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[a] MSE (NK Model) / MSE (VAR Benchmark); values smaller than 1 therefore suggest superior performance by the NK model

[b] Test statistic from the Diebold and Mariano (1995) test. The null hypothesis if of equal forecasting accuracy between the two models. The statistic is asymptotically normal.

[c] Harvey et al. (1997)'s correction of the Diebold and Mariano (1995) test. The statistic follows a t_{N-1} distribution, with N the number of forecasts.

Table 6: Forecast Encompassing: Does the NK model provide *any* information not contained in the VAR Benchmark?

Variable	DM stat.[a] (p-value)	HLN stat.[b] (p-value)
	Forecasting One Period Ahead	
Output	2.94 (0.00)	2.88 (0.00)
Inflation	0.54 (0.30)	0.53 (0.30)
Interest Rate	3.42 (0.00)	3.36 (0.00)
Money	2.40 (0.01)	2.36 (0.01)
	Forecasting Two Periods Ahead	
Output	3.03 (0.00)	2.87 (0.00)
Inflation	0.41 (0.34)	0.39 (0.35)
Interest Rate	4.31 (0.00)	4.09 (0.00)
Money	2.52 (0.01)	2.38 (0.01)
	Forecasting Four Periods Ahead	
Output	1.74 (0.04)	1.51 (0.07)
Inflation	0.60 (0.28)	0.52 (0.31)
Interest Rate	6.00 (0.00)	5.18 (0.00)
Money	1.90 (0.03)	1.56 (0.04)
	Forecasting Six Periods Ahead	
Output	1.42 (0.08)	1.09 (0.14)
Inflation	1.27 (0.10)	0.98 (0.17)
Interest Rate	26.0 (0.00)	20.7 (0.00)
Money	1.58 (0.06)	1.22 (0.12)
	Forecasting Eight Periods Ahead	
Output	1.29 (0.10)	0.85 (0.20)
Inflation	0.61 (0.27)	0.40 (0.35)
Interest Rate	NA (NA)	NA (NA)
Money	1.39 (0.08)	0.91 (0.19)

---

[a] Test statistic proposed by Harvey et al. (1998). The null hypothesis is that the forecasts from the NK model provide no information not already contained in those from the VAR benchmark.

[b] Harvey et al. (1997)'s correction. The statistic follows a t_{N-1} distribution

Table 7: Testing for Equal Forecasting Accuracy: Model and BVAR(2) Benchmark; 1997:1 - 2004:1

Variable	Relative MSE (NK Model)[a]	DM stat.[b] (p-value)	HLN stat.[c] (p-value)
	Forecasting One Period Ahead		
Output	2.31	-2.44 (0.02)	-2.40 (0.02)
Inflation	1.01	-0.04 (0.97)	-0.04 (0.97)
Interest Rate	0.90	0.70 (0.49)	0.68 (0.50)
Money	0.78	1.64 (0.10)	1.61 (0.12)
	Forecasting Two Periods Ahead		
Output	1.56	-0.91 (0.36)	-0.86 (0.40)
Inflation	0.93	0.36 (0.72)	0.34 (0.74)
Interest Rate	0.79	0.93 (0.35)	0.88 (0.39)
Money	0.72	1.53 (0.13)	1.45 (0.16)
	Forecasting Four Periods Ahead		
Output	0.69	0.69 (0.49)	0.60 (0.58)
Inflation	0.94	0.32 (0.75)	0.28 (0.78)
Interest Rate	0.65	1.24 (0.22)	1.07 (0.30)
Money	0.56	1.07 (0.29)	0.93 (0.36)
	Forecasting Six Periods Ahead		
Output	0.35	1.87 (0.06)	1.44 (0.16)
Inflation	0.80	2.45 (0.01)	1.89 (0.07)
Interest Rate	0.48	2.35 (0.02)	1.81 (0.08)
Money	0.42	1.18 (0.24)	0.91 (0.27)
	Forecasting Eight Periods Ahead		
Output	0.24	2.17 (0.03)	1.43 (0.17)
Inflation	1.06	-0.42 (0.68)	-0.27 (0.79)
Interest Rate	0.38	7.80 (0.00)	5.14 (0.00)
Money	0.30	1.44 (0.15)	0.95 (0.35)

---

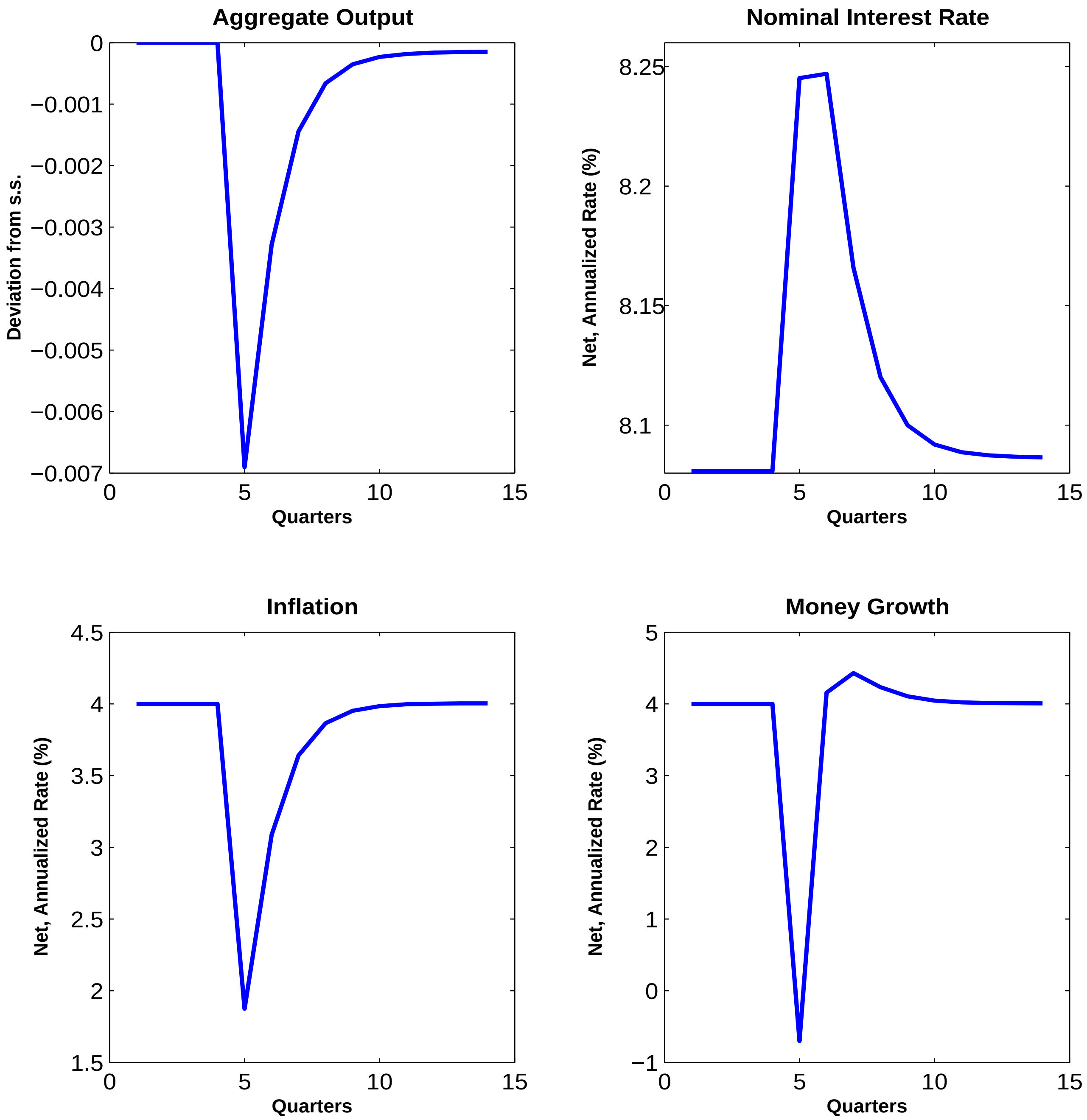
[a] MSE (NK Model) / MSE (VAR Benchmark); values smaller than 1 therefore suggest superior performance by the NK model

[b] Test statistic from the Diebold and Mariano (1995) test. The null hypothesis if of equal forecasting accuracy between the two models. The statistic is asymptotically normal.

[c] Harvey et al. (1997)'s correction of the Diebold and Mariano (1995) test. The statistic follows a t_{N-1} distribution, with N the number of forecasts.

Figure 1: A Monetary Policy Tightening

(Shock occurs at $t = 5$)



Aggregate Output

Nominal Interest Rate

Inflation

Money Growth

Figure 2: The Economy's Response to a Positive Money-Demand Shock

(Shock occurs at $t = 5$)

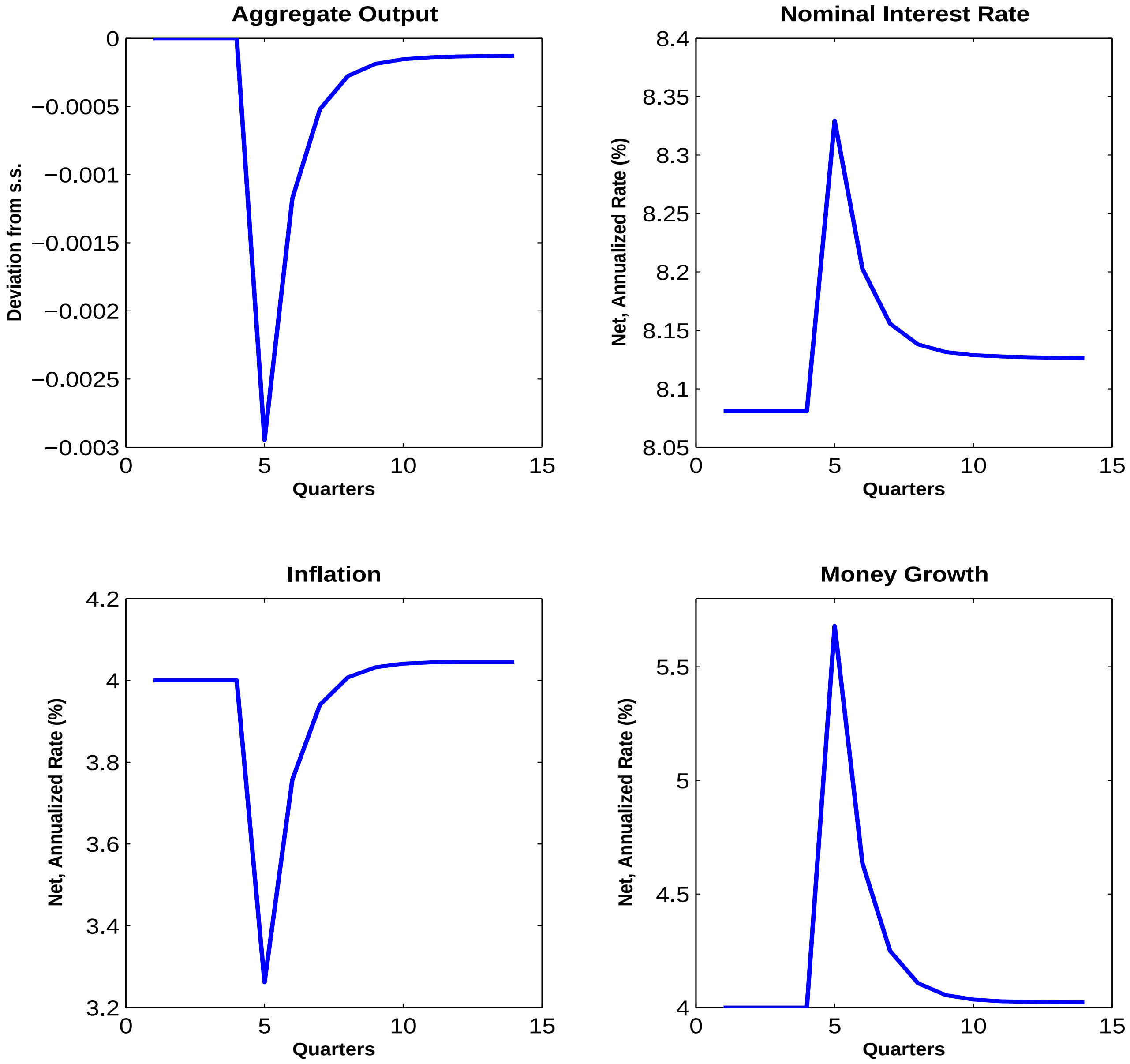


Figure 3: The Economy's Response to a Positive Technology Shock

(Shock occurs at $t = 5$)

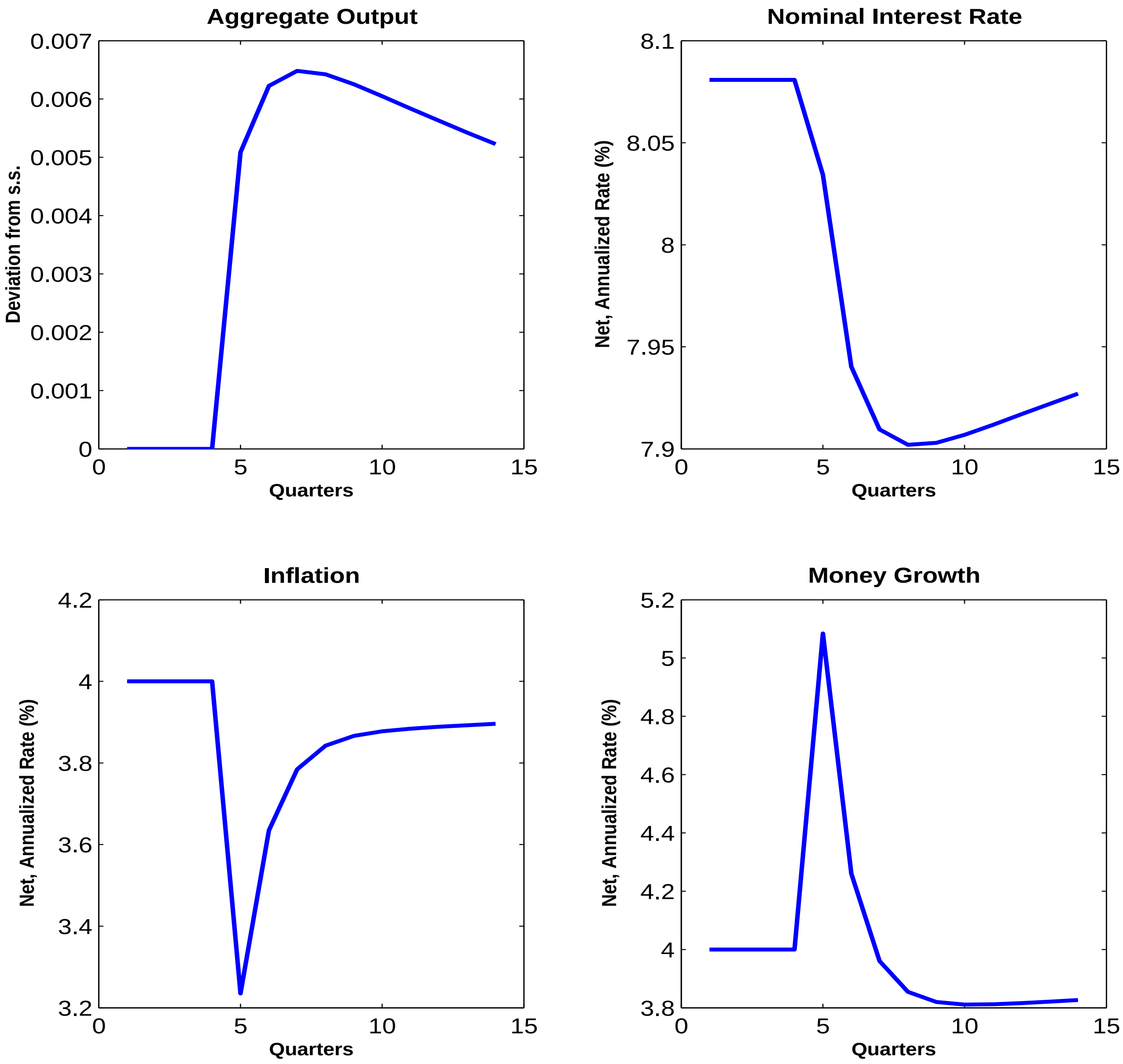


Figure 4: The Economy's Response to a Positive Preference Shock

(Shock occurs at $t = 5$)

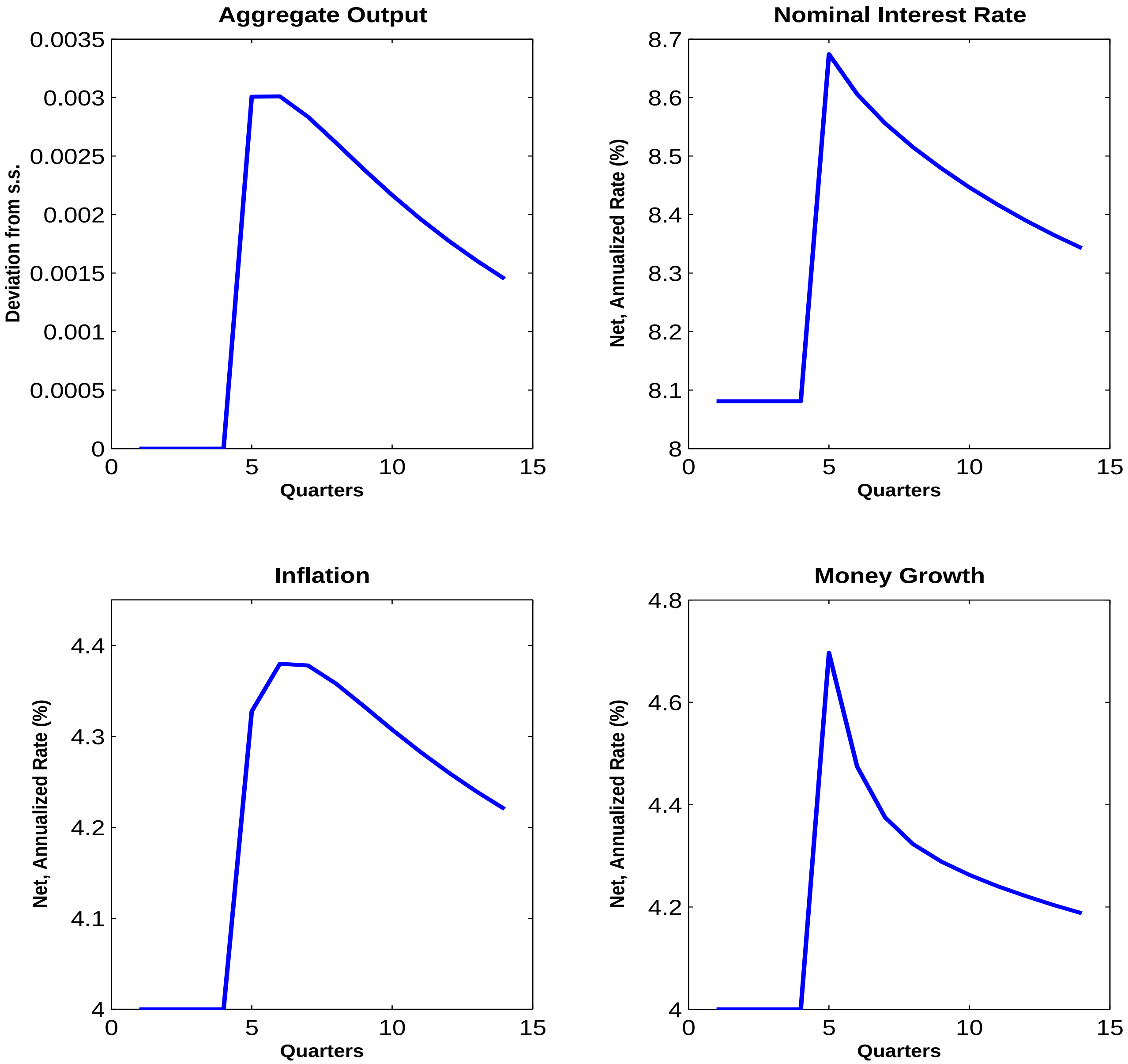


Figure 5. Actual Data and Forecasts from the New Keynesian Model

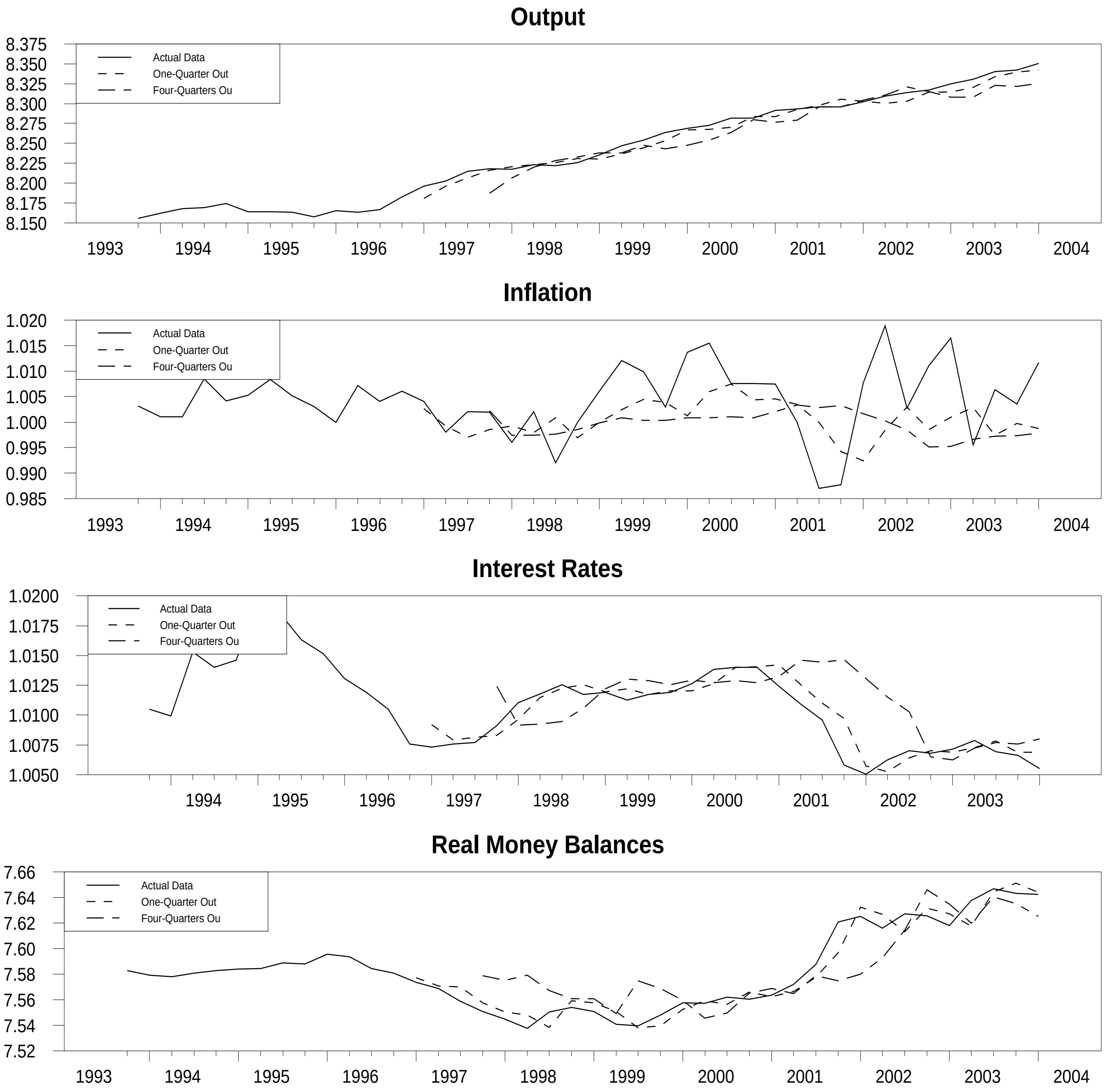


Figure 6. Forecast Errors: One-Quarter Ahead

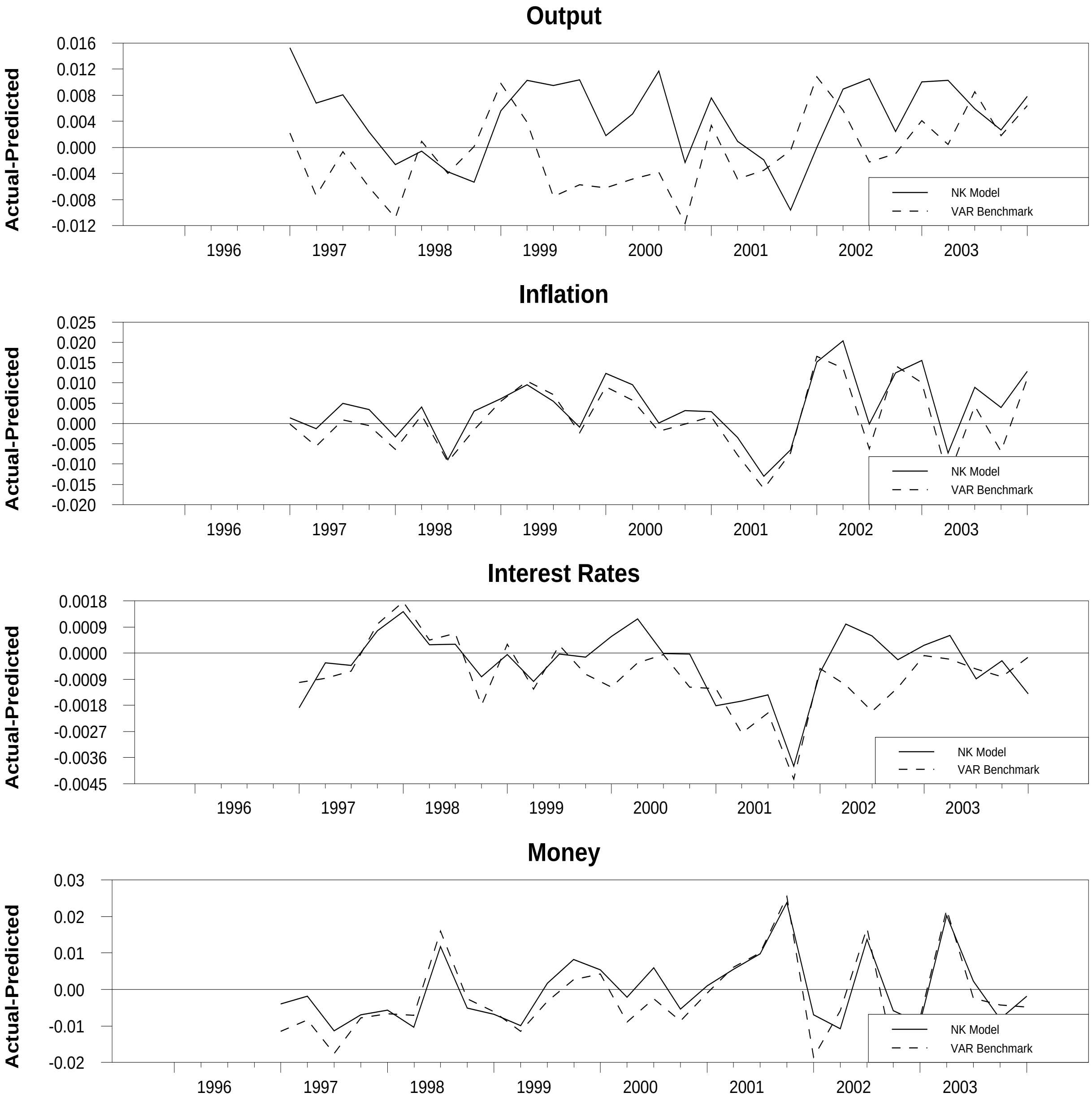


Figure 7. Forecast Errors: Four Quarters Ahead

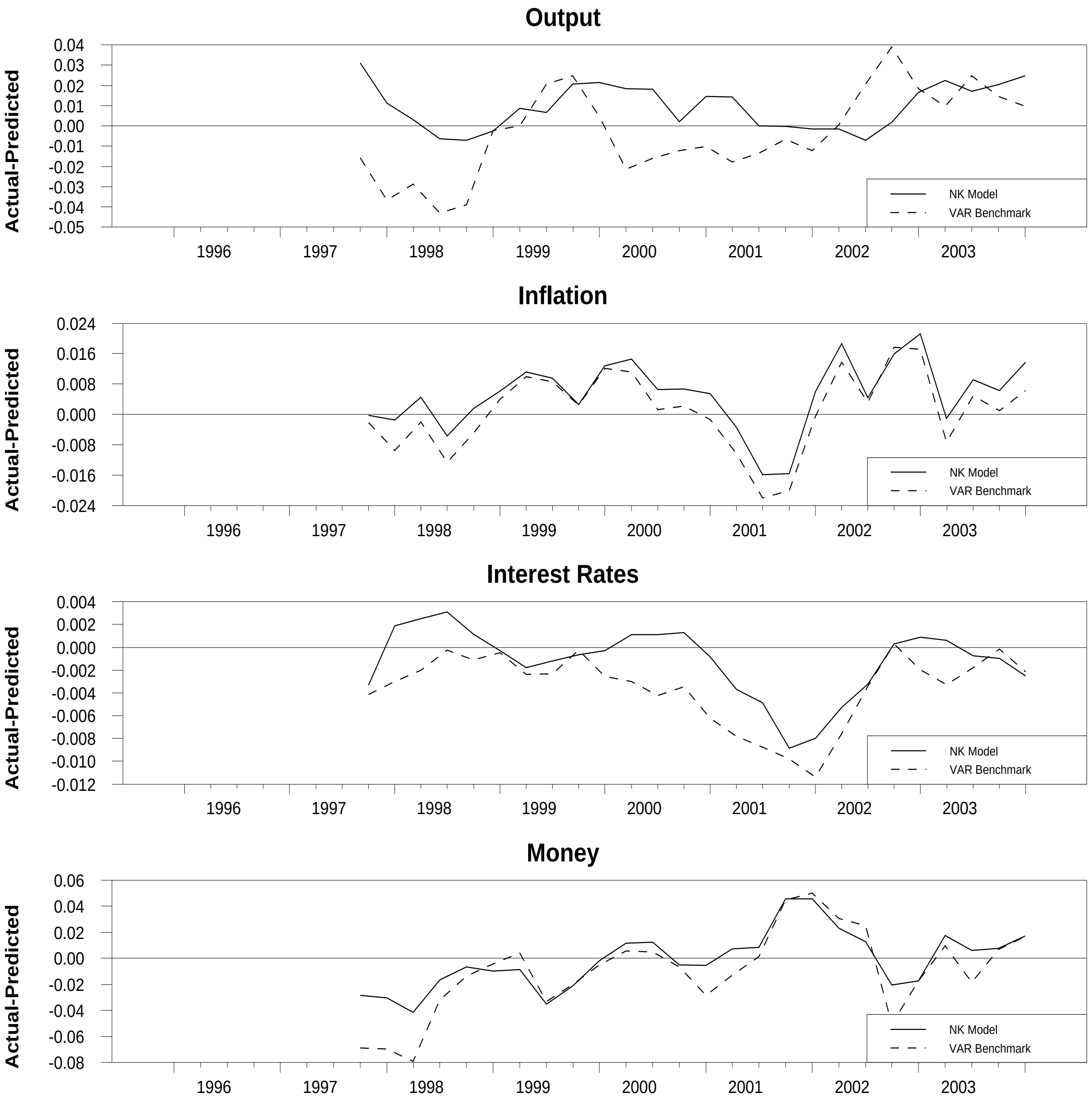
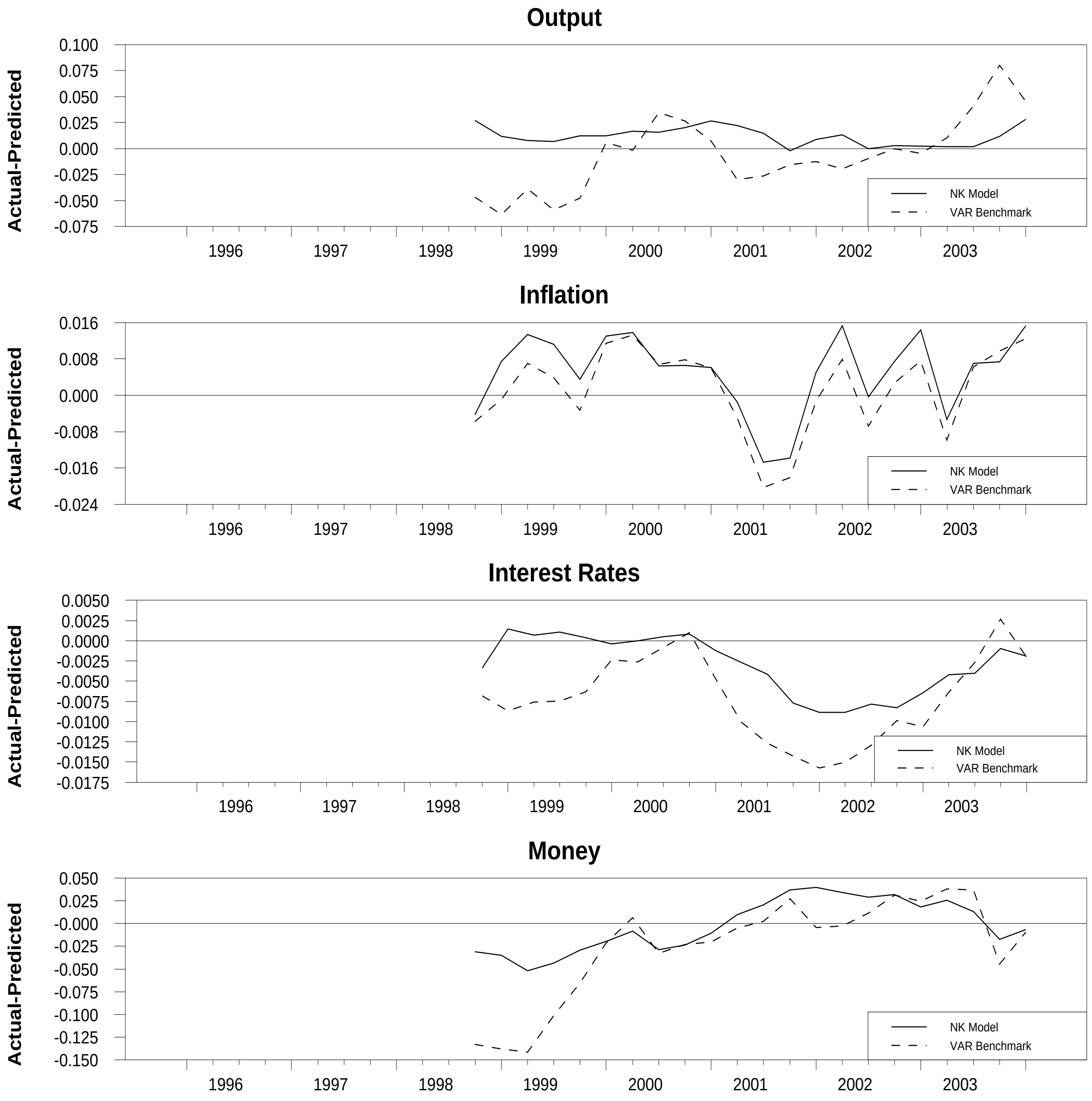


Figure 8. Forecast Errors: Eight Quarters Ahead



A Solving the New Keynesian Model

A.1 The symmetric equilibrium

$$\frac{z_t c_t^{-\frac{1}{\gamma}}}{c_t^{\frac{\gamma-1}{\gamma}} + b_t^{1/\gamma} m_t^{\frac{\gamma-1}{\gamma}}} = \lambda_t; \tag{A.1}$$

$$\frac{z_t b_t^{1/\gamma} m_t^{-\frac{1}{\gamma}}}{c_t^{\frac{\gamma-1}{\gamma}} + b_t^{1/\gamma} m_t^{\frac{\gamma-1}{\gamma}}} = \lambda_t - \beta E_t\left(\frac{\lambda_{t+1}}{\pi_{t+1}}\right); \tag{A.2}$$

$$\frac{\zeta}{1-h_t} = \lambda_t w_t; \tag{A.3}$$

$$\begin{aligned}&\beta E_t\left[\lambda_{t+1}\left(r_{kt+1} + 1 - \delta + \psi\left(\frac{k_{t+2}}{k_{t+1}} - 1\right)\frac{k_{t+2}}{k_{t+1}} - (\psi/2)(\frac{k_{t+2}}{k_{t+1}} - 1)^2\right)\right]\\ &\qquad = \eta\lambda_t\left[1 + \psi\left(\frac{k_{t+1}}{k_t} - 1\right)\right];\end{aligned} \tag{A.4}$$

$$\frac{1}{R_t} = \beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t \pi_{t+1}}\right] \tag{A.5}$$

$$y_t = k_t^\alpha (A_t h_t)^{1-\alpha}; \tag{A.6}$$

$$\frac{\alpha y_t}{k_t} = q_t r_t; \tag{A.7}$$

$$\frac{(1-\alpha) y_t}{h_t} = q_t w_t; \tag{A.8}$$

$$\widetilde{p}_t = \frac{\theta\pi_t}{\theta-1}\frac{E_t\sum_{k=0}^{\infty}(\beta\phi\pi^{-\theta})^k\lambda_{t+k}y_{t+k}q_{t+k}(\prod_{s=1}^{k}\pi_{t+s}^{\theta})}{E_t\sum_{k=0}^{\infty}(\beta\phi\pi^{1-\theta})^k\lambda_{t+k}y_{t+k}(\prod_{s=1}^{k}\pi_{t+s}^{\theta-1})}; \tag{A.9}$$

$$\pi_t^{1-\theta} = (1-\phi)\widetilde{p}_t^{1-\theta} + \phi\pi; \tag{A.10}$$

$$y_t = c_t + \eta k_{t+1} - (1-\delta)k_t - (\psi/2)(\frac{k_{t+1}}{k_t} - 1)^2; \tag{A.11}$$

$$\mu_t = \frac{m_t \pi_t}{m_{t-1}}; \tag{A.12}$$

$$\log(R_t/R) = \varrho_\pi \log(\pi_t/\pi) + \varrho_y \log(y_t/y) + \varrho_\mu \log(\mu_t/\mu) + \log(v_t); \tag{A.13}$$

$$\log(A_t) = (1-\rho_A)\log(A) + \rho_A \log(A_{t-1}) + \varepsilon_{At}; \tag{A.14}$$

$$\log(b_t) = (1-\rho_b)\log(b) + \rho_b \log(b_{t-1}) + \varepsilon_{bt}; \tag{A.15}$$

$$\log(z_t) = \rho_z \log(z_{t-1}) + \varepsilon_{zt}; \tag{A.16}$$

$$\log(v_t) = \rho_v \log(v_{t-1}) + \varepsilon_{vt}. \tag{A.17}$$

A.2 Finding The Non-stochastic Steady-State

Setting all shocks to their mean, the economy converges to a steady state in which all variables in (A.1) to (A.17) are constant. Removing the time subscripts to denote the steady-state values of these variables, one is lead to the following system:

$$\begin{aligned}
\mu &= \pi; && \text{(A.18)}\\
R &= \frac{\pi}{\beta}; && \text{(A.19)}\\
r_k &= \frac{1}{\beta} - 1 + \delta; && \text{(A.20)}\\
q &= \frac{\theta - 1}{\theta}; && \text{(A.21)}\\
\lambda c &= \left[1 + b\left(\frac{\mu}{\mu - \beta}\right)^{\gamma - 1}\right]^{-1}; && \text{(A.22)}\\
\lambda m &= \lambda c b \left(\frac{\mu}{\mu - \beta}\right)^{\gamma}; && \text{(A.23)}\\
\frac{k}{y} &= \frac{\alpha q}{r_k}; && \text{(A.24)}\\
\frac{c}{y} &= 1 - \delta\left(\frac{k}{y}\right); && \text{(A.25)}\\
wh\lambda &= \frac{q(1-\alpha)(\lambda c)}{(c/y)}; && \text{(A.26)}\\
h &= \frac{wh\lambda}{\zeta + wh\lambda}; && \text{(A.27)}\\
y &= hA\left(\frac{k}{y}\right)^{\frac{\alpha}{1-\alpha}}. && \text{(A.28)}
\end{aligned}$$

A.3 Linearized System

The next step in the solution is to compute a first-order approximation of (A.1) to (A.17) around the steady state. A hatted variable denotes the deviation relative to the steady-state value of the variable. The equations are divided between *Static* and *Dynamic* equations.

A.3.1 Static equations

$$\hat{Y}_t + (\alpha - 1)\hat{h}_t = \alpha\hat{k}_t + (1-\alpha)\hat{A}_t; \tag{A.29}$$

$$\hat{\mu}_t - \hat{\pi}_t = \hat{m}_t - \widehat{m_{t-1}}; \tag{A.30}$$

$$\hat{Y}_t - \hat{r}_{kt} = \hat{k}_t - \hat{q}_t; \tag{A.31}$$

$$\begin{aligned}(-1-(\gamma-1)\lambda c)\hat{C}_t &= \gamma\hat{\lambda}_t + ((\gamma-1)\lambda((\mu-\beta)/\mu)m)\hat{m}_t \\ &\quad +(\lambda((\mu-\beta)/\mu)m)\hat{b}_t - \gamma\hat{z}_t;\end{aligned} \tag{A.32}$$

$$\begin{aligned}(-\beta/(\mu-\beta))\hat{R}_t - (\lambda((\gamma-1)/\gamma)c)\hat{C}_t &= \hat{\lambda}_t + (\lambda((\gamma-1)/\gamma)(\mu-\beta)/\mu)m) + 1/\gamma)\hat{m}_t \\ &\quad +((\lambda((\mu-\beta)/\mu)m - 1)/\gamma)\hat{b}_t - \hat{z}_t;\end{aligned} \tag{A.33}$$

$$(h/(1-h))\hat{h}_t - \hat{w}_t = \hat{\lambda}_t; \tag{A.34}$$

$$\hat{Y}_t - \hat{w}_t - \hat{h}_t = -\hat{q}_t; \tag{A.35}$$

$$\hat{R}_t - \varrho_\mu\hat{\mu}_t - \varrho_\pi\hat{\pi}_t - \varrho_y\hat{y}_t = \hat{v}_t; \tag{A.36}$$

These equations can be rewritten compactly in matrix form as

$$AZ_t = BX_t + CU_t$$

where A, B, and C are 8x8, 8x5, and 8x4 matrices, respectively and we have $Z_t = (\hat{y}_t, \hat{R}_t, \widehat{r_{kt}}, \hat{C}_t, \hat{\pi}_t, \hat{w}_t, \hat{h}_t, \hat{\mu}_t)'$ (a vector of endogenous variables), $X_t = (\hat{k}_t, \widehat{m_{t-1}}, \hat{\lambda}_t, \hat{q}_t, \hat{m}_t)'$ (a vector of state and co-state variables) and $U_t = (\hat{A}_t, \hat{b}_t, \hat{v}_t, \hat{z}_t)'$ (a vector of shock variables).

A.3.2 Dynamic equations

The dynamic equations are the following:

$$\beta\hat{\pi}_{t+1} = \hat{\pi}_t - \frac{(1-\beta\phi)(1-\phi)}{\phi}\hat{q}_t; \tag{A.37}$$

$$\begin{aligned}(\psi(\beta(1-\delta)-(1+\beta)))\hat{k}_{t+1} + (\beta(1+r_k-\delta))\hat{\lambda}_{t+1} \\ +\beta r_k\hat{r}_{kt+1} + (\beta\psi y/k)\hat{y}_{t+1} - (\beta\psi c/k)\hat{c}_{t+1} = -\psi\hat{k}_t + \hat{\lambda}_t;\end{aligned} \tag{A.38}$$

$$k\hat{k}_{t+1} = (1-\delta)k\hat{k}_t + y\hat{y}_t - c\hat{C}_t; \tag{A.39}$$

$$\hat{\lambda}_{t+1} - \hat{\pi}_{t+1} = \hat{\lambda}_t - \hat{R}_t; \tag{A.40}$$

$$\hat{m}_t = \hat{m}_t. \tag{A.41}$$

Again, this can be rewritten in matrix form as

$$DX_{t+1} + EZ_{t+1} = FX_t + GZ_t + HU_t$$

where D, E, F, G, and H are 5x5, 5x8, 5x5, 5x8, and 5x4 matrices, respectively.

Using the static and dynamic equations, we can solve the model using the methodology of Blanchard and Kahn (1980), which leads us to the following first-order state-space solution of the system:

$$\begin{aligned} \widehat{s}_{t+1} &= \mathbf{\Phi}_1 \widehat{s}_t + \mathbf{\Phi}_2 \varepsilon_{t+1}, \\ \hat{d}_t &= \mathbf{\Phi}_3 \widehat{s}_t, \end{aligned}$$

where the matrices $\mathbf{\Phi}_1$, $\mathbf{\Phi}_2$ and $\mathbf{\Phi}_3$ are functions of the structural parameters of the model.

B Data

The model is estimated using data that spans the period 1981:3 to 2004:1. The data is taken from Statistics Canada's CANSIM database, for which we list the associated mnemonics. Output Y_t is *final domestic demand* [V1992068], of quarterly frequency and in chained 1987 dollars. We convert this series into per-capita terms using the population of age 15 and over.

The interest rate R_t is the three month treasury bill rate ([V122531]), a series of daily frequency, for which we take a quarterly average.

Finally, the money stock M_t is *M2* [B1630] which is of monthly frequency; we take a quarterly average and convert the resulting series into real, per-capita terms by dividing it with the GDP implicit price deflator ([D100465]) and the population age 15 and over. Output and money data are logged before estimation.