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## **Financing New Investments under Asymmetric Information: a General Approach**

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**Abstract:** We study the efficiency of credit market equilibria when financial intermediaries cannot observe the riskiness or the returns of potential investment projects. With loan financing, there is over-investment in high-return, high-risk projects and under-investment in low-return, low-risk projects relative to the social optimum. If firms have the choice of equity finance, there is unambiguously over-investment under reasonable conditions. The well-know cases of Stiglitz and Weiss and of the Meza and Webb emerge as special cases. Policy implications are considered, and the results are extended to allow for signaling and screening equilibria.

**Keywords:** Credit Markets, Asymmetric Information

**JEL Classification:** G14, G18

## 1. Introduction

It is well known that if lending intermediaries are forced by imperfect information to pool high-quality and low-quality investment projects, an inefficient volume of loans will be made and there will be an a priori case for public intervention. However, whether too many or too few loans will be extended depends upon the distribution of project characteristics within the pool. The literature has focused on two special cases, which yield opposing outcomes and policy implications. In one, due to Stiglitz and Weiss (1981), intermediaries know the expected returns of projects, but not their riskiness. Projects of a given expected return but different risk must therefore be pooled. In equilibrium, too few projects of any given expected return will be undertaken, leading to a policy prescription of a subsidy on loans. In the other, due to de Meza and Webb (1987), intermediaries know the level of returns that projects will yield if successful, but not the probability of success. Projects with a given return if successful must be pooled. In this case, too many projects of a given return will be funded, calling for a tax on loans. Furthermore, the Stiglitz-Weiss case can potentially lead to credit rationing (excess demand for loans) in equilibrium, whereas in the de Meza-Webb case credit rationing cannot arise. And, if equity contracts were available as an alternative to debt contracts, de Meza and Webb argue that they would be used in the Stiglitz-Weiss case, but not the de Meza-Webb case. One is left with no clear policy implications of the effects of adverse selection in credit markets.

There is, however, no particular reason to suppose that intermediaries can distinguish projects ex ante either by their expected return or by their return if successful. In general, they might be unable to distinguish among projects with very different patterns of returns and riskiness. The purpose of this paper is to investigate the consequence of asymmetric information in credit markets under a more general mix of characteristics than those considered by Stiglitz and Weiss (1981) and de Meza and Webb (1987). In particular, we suppose that pooled projects can have any general mix of returns and probabilities of success. In this context, we study the efficiency properties of competitive financial markets and the implications for policy. When these more general project mixes are considered, there will be over-investment in certain types of projects and under-investment in others when finance takes the form of loans. However, we also show that this ambiguity can disappear when firms are free to opt for either debt or equity finance. Under reasonable assumptions, there will be unambiguously too many projects undertaken, with the implication that a tax on projects financed will be efficiency improving.

In this more general setting, we then consider some extensions to the simply pooling equilibrium models, all of which involve introducing mechanisms that enhance the information available to banks. The possibility of entrepreneurs signaling the quality of their projects at a cost is analyzed. As well, the possibility of intermediaries separating projects by offering price-quantity contracts is considered.

## 2. The Basic Model

The model we use has a number of simple features, common to those used in the established literature (e.g. de Meza and Webb, 1987). There is a continuum of potential entrepreneurs, each with a project for which finance is needed. All projects require a given amount of capital  $K$ , which is the only input. A project's return depends upon which of two states of nature occurs—a good one or a bad one. The good state occurs with probability  $p$  and yields a return  $R$ , where  $0 \leq p \leq 1$ ,  $0 \leq R < \infty$ . The bad state occurs with probability  $1 - p$  and, for simplicity only, yields a return of zero. Projects are characterized by a  $\{p, R\}$  combination that can differ among projects. The distribution of project returns among entrepreneurs is given by joint distribution function  $G(p, R)$  and its associated density function  $g(p, R)$ . The distribution of  $p$  and  $R$  can take arbitrary forms. In particular, there may be ranges where the density is zero,  $g(p, R) = 0$ . The characteristics of individual projects are private information to the entrepreneur associated with each project, but the distribution  $G(p, R)$  is known both to the financial intermediaries and to the government.

Entrepreneurs differ only in their project characteristics  $\{p, R\}$ . All have the same initial wealth  $W \geq 0$ , and all have the same alternative income prospects  $y$  that they forgo if they undertake the project. They require an amount of finance  $K - W = B$  to undertake their projects, and they obtain that from financial intermediaries. Finance may take the form of debt or equity depending on the circumstances. Intermediaries, in turn, obtain their finance at a rate of return (or deposit rate)  $\rho$ , which is taken as fixed by each intermediary but may be variable for the capital market as a whole. All agents in the economy—entrepreneurs, financial intermediaries and the government—are risk-neutral. Intermediaries are fully competitive, so earn zero expected profits in equilibrium. Stiglitz and Weiss (1981) showed that in the kinds of models we are considering, credit rationing is a possibility. In fact, that was the main focus of their paper. Since that is not our interest, we assume that credit rationing does not in fact occur.

If project returns are not freely observable, it will be costly for intermediaries to verify these returns. We assume that intermediaries can learn a project's ex post return perfectly by incurring a fixed monitoring cost of  $c \geq 0$  per project monitored. With debt contracts, monitoring will occur only if firms declare bankruptcy since in the absence of bankruptcy the intermediary's return is independent of the project return  $R$ . Indeed, the existence of costly verification can itself be the rationale for debt financing (Williamson, 1987).

## 3. Debt Finance

We begin, following Stiglitz and Weiss (1981) and de Meza and Webb (1987), with the case where projects are financed by debt. To avoid issues of internal financing and collateral, which we take up later, we assume in this section that entrepreneurs have no initial wealth

( $W = 0$ ). To undertake their projects, they require an amount of loan finance  $B = K$ , which they can borrow at the interest rate  $r$  from an intermediary, here a bank. The net expected profit of an entrepreneur, given  $r$ , is:

$$\pi(p, R, r) = p(R - (1 + r)B) - y \quad (1)$$

This is increasing in  $p$  if  $R > (1 + r)B$ —which will be the case for all funded projects—and in  $R$  (and decreasing in  $r$ ). Given  $r$ , there will be a set of marginal entrepreneurs whose projects satisfy  $\pi(p, R, r) = 0$ . This equation can be depicted as an *iso-profit curve* for the marginal entrepreneurs in  $(p, R)$ -space. The slope and curvature of this iso-profit curve are obtained by differentiation:

$$\left. \frac{dR}{dp} \right|_{\pi=0} = -\frac{R - (1 + r)B}{p} < 0, \quad \left. \frac{d^2R}{dp^2} \right|_{\pi=0} = -\frac{1}{p} \left. \frac{dR}{dp} \right|_{\pi=0} + \frac{R - (1 + r)B}{p^2} > 0 \quad (2)$$

where the signs follow from the fact that  $\pi(p, R, r) = 0$  implies  $R > (1 + r)B$ . All entrepreneurs whose projects have combinations of  $p$  and  $R$  to the northeast of the iso-profit curve will demand a loan, while those southwest will not. Note that since  $\pi$  is decreasing in  $r$ , an increase in  $r$  will cause the iso-profit curve to shift outwards.

With debt finance, entrepreneurs will be unable to repay their loans in a bad state, so will go bankrupt. If project outcomes cannot be observed by the banks, projects declaring bankruptcy will be monitored. Since we assume that monitoring perfectly reveals the project's outcome, only truly bankrupt firms will declare bankruptcy.<sup>1</sup> Assuming that the government has no better information than the banks, these ex post monitoring costs will be socially necessary expenditures. Given that, the net social benefit of undertaking a project with characteristics  $\{p, R\}$  is as follows:

$$S(p, R) = pR - (1 - p)c - (1 + \rho)B - y \quad (3)$$

This is increasing in both  $p$  and  $R$ , and is independent of the interest rate  $r$ . A project will be worth undertaking from a social point of view if  $S(p, R) \geq 0$ . An *iso-benefit curve* for the marginally socially profitable project is defined by  $S(p, R) = 0$ . Differentiating, we obtain its slope and curvature as:

$$\left. \frac{dR}{dp} \right|_{S=0} = -\frac{R + c}{p} < 0, \quad \left. \frac{d^2R}{dp^2} \right|_{S=0} = -\frac{1}{p} \left. \frac{dR}{dp} \right|_{S=0} + \frac{R + c}{p^2} > 0 \quad (4)$$

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<sup>1</sup> In a more general analysis, the possibility of imperfect monitoring could be considered. This would affect the nature of contracts offered to firms, which must be generous enough to preclude false declarations of bankruptcy.

Thus, the iso-benefit curve also slopes downward and is convex to the origin. All projects to its northeast ought to be undertaken from a social point of view, while those southwest should not be.

Comparing (4) with (2), we obtain:

$$\left. \frac{dR}{dp} \right|_{S=0} = -\frac{R+c}{p} < -\frac{R-(1+r)B}{p} = \left. \frac{dR}{dp} \right|_{\pi=0}$$

so the iso-benefit curve is steeper than the iso-profit curve. Intuitively, since entrepreneurs only bear the cost of finance in the event of success while society bears it in either state, the increment in  $R$  needed to compensate them for a reduced probability of success is less than what society must be compensated. That is, entrepreneurs face no downside risk so discount the costs of a lower probability of success. The intersection point can be found by subtracting the iso-benefit expression from the iso-profit expression to obtain:

$$p^* = \frac{(1+\rho)B+c}{(1+r)B+c} \quad (5)$$

The value of  $p^*$  will decrease with an increase in  $r$ : the iso-profit curve shifts outwards while the iso-benefit curve is unaffected.

Figure 1 illustrates the determination of  $p^*$  for a given value of  $r$ , with the cross-hatched areas representing project-types that either should be undertaken but are not (the under-investment area), or projects that are undertaken but should not be (the over-investment area). For  $p > p^*$ , there are socially profitable projects that do not enter, while for  $p < p^*$ , there will be entry of socially unprofitable projects. This reflects the fact that increments of  $p$  are of relatively more value socially than to entrepreneurs since a share of the proceeds of successful projects accrues to banks. The set of available projects among which intermediaries (and the government) are unable to distinguish—so they must be pooled at the same interest rate—can be anywhere in  $(p, R)$ -space. To the extent that they are found in the two cross-hatched areas, inefficiency will result.<sup>2</sup>

The equilibrium in Figure 1 depends on the value of  $r$ . To complete the description of the market outcome, the equilibrium value of  $r$  is determined by the zero-expected-profit condition applying to the competitive banks (all of whom are identical):<sup>3</sup>

$$\Pi = \bar{p}(1+r)B - (1-\bar{p})c - (1+\rho)B = 0 \quad (6)$$

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<sup>2</sup> Mankiw (1986) makes a similar observation for a model in which there are no monitoring costs. He points out moreover that loan markets may ‘collapse’ in the sense that there may be no equilibrium in which loans are made.

<sup>3</sup> We are implicitly assuming that banks do not incur any real costs of operation, as is common in the literature. Adding bank operating costs would have no effect on the analysis.

where  $\bar{p}$  is the expected probability of success among all projects being funded by the banks. By rearranging this zero-profit condition and using (5), we obtain:

$$\bar{p} = \frac{(1 + \rho)B + c}{(1 + r)B + c} = p^* \quad (5')$$

Thus, the equilibrium interest rate will be set such that the value of the expected probability of success of all accepted projects  $\bar{p}$  is equal to the value of  $p$  at which the iso-benefit and iso-profit curves intersect. The iso-profit curve adjusts with a change in  $r$  until this is the case. This relation proves useful in what follows.

Consider now the policy implications of these results. To get some insight into the kinds of policies that would ensure that all socially profitable projects are undertaken, we first characterize sets of policies that could in principle convert the expression for private profits (1) into that for social profits (3). One such set of policies can be obtained by letting  $\tau_1$  be a common tax imposed on all successful firms and  $\tau_2$  be a common tax imposed if successful or not. With such a tax system, after-tax expected project profits become, using (1):

$$\pi(p, R, r) = p(R - (1 + r)B - \tau_1) - \tau_2 - y$$

Then, this expression reduces to social profits (3) by setting:

$$\tau_1 = -c - (1 + r)B \quad \text{and} \quad \tau_2 = c + (1 + \rho)B$$

Note that these taxes would in principle be easy to implement since they do not require information on individual probabilities of success  $p$  or returns  $R$ . The government only needs to know whether a project has been successful. Under such a scheme, all entrepreneurs would be taxed ex ante and subsidized in the event of success at the same rates  $\tau_1$  and  $\tau_2$ . Unfortunately, the ex ante tax that the government would have to extract from all entrepreneurs up front would exceed the value of the loan ( $\tau_2 > B$ ). If the entrepreneurs were able to pay this tax, they would not need outside financing in the first place. Therefore, this informationally parsimonious policy is not feasible in the current circumstances.

An optimal policy that avoids this infeasibility problem would be one that taxes or subsidizes profitable firms ex post based on their project characteristics, assuming that the government can acquire that information, say, by firms reporting it to the tax authorities. To make the iso-profit curve for the marginal project coincide with the iso-benefit curve, a non-linear progressive tax would have to be imposed on projects with a high  $R$  (low  $p$ )—for whom there is over-investment—and progressive subsidy on those with a low  $R$  (high  $p$ ). To be more precise, let  $T(p, R)$  be the tax imposed on a firm with given  $(p, R)$ , so after-tax profits become  $\pi(p, R, r) = p(R - (1 + r)B - T(p, R)) - y$ . If  $T(p, R)$  is set so that after-tax profits equal social profits in (3), we have  $p(R - (1 + r)B - T(p, R)) - y = pR - (1 - p)c - (1 + \rho)B - y$ . This yields an optimal tax of

$$T(p, R) = T(p) = [(1 - p)c + (1 + \rho - p(1 + r))B]/p$$

with  $T'(p) < 0$  and  $T(p^*) = 0$ , so  $T(p) > 0$  for all  $p < p^*$ , and  $T(p) < 0$  for all  $p > p^*$ .

This optimal tax is informationally demanding. If the tax authorities cannot learn  $p$  or  $R$ , the best that can be done is a general tax or subsidy at the same rate on all loans or deposits. Whether it is a tax or subsidy depends on the mix of available projects (and that may be hard for the government to know as well). Given that there will be over-investment in some projects and under-investment in others, applying a tax at a common rate will generally not yield the first-best optimum (except in special cases such as the one considered next).

The above analysis applies for any general distribution of feasible projects  $G(p, R)$ . Let us now consider two special cases that restrict the set of available projects: those of de Meza and Webb (1987) and Stiglitz and Weiss (1981). The equilibrium outcomes and policy implications for the latter case are not as straightforward and unambiguous as the existing literature seems to imply.

### The de Meza-Webb Case

In this case, project returns are assumed to be common knowledge, so intermediaries are able to classify projects according to their project return  $R$ . To consider this case, we can focus on a particular value of  $R$ . The same analysis will apply for other sets of projects with a different return  $R$ , although of course different interest rates  $r$  will apply for different values of  $R$ . Geometrically, the available projects are distributed along a horizontal line for the given  $R$ . The form of the distribution does not matter for our analysis, although it is convenient to assume  $g(p, R) > 0$  for all  $p$  over the range of interest.

Figure 1 illustrates this case for projects with a given return  $\bar{R}$ . The locus of available projects intersects the equilibrium iso-profit locus at  $\tilde{p}$ . To the right of the intersection line, projects enter, and to the left they do not. It is clear that when the banks' zero-profit condition is satisfied, this intersection point  $\tilde{p}$  is to the left of  $p^*$ , so there is a range where some projects are funded that are socially unprofitable—the de Meza-Webb (1987) over-investment result. To see this, recall that the entrepreneurs' profits  $\pi(p, R, r)$  are increasing in  $p$ . Given that  $R$  is fixed, it follows that there will be a marginal project with probability of success  $\tilde{p}$ , and all projects with  $p \geq \tilde{p}$  will be undertaken. This implies that the average probability of success of funded projects satisfies  $\bar{p} > \tilde{p}$ , implying that  $p^* > \tilde{p}$ , so the intersection point of the fixed- $R$  line lies to the left of  $p^*$ .

Algebraically, this over-investment result can be shown as follows. The marginal entrepreneur's zero-net-profit condition and the banks' zero-expected-profit condition are by (1) and (6) respectively:

$$\begin{aligned}\tilde{p}(R - (1 + r)B) - y &= 0 \\ \bar{p}(1 + r)B - (1 - \bar{p})c - (1 + \rho)B &= 0\end{aligned}$$



Adding these expressions and rearranging we obtain an expression for the social profitability of the marginal project,  $S(\tilde{p})$ :

$$\underbrace{\tilde{p}R - (1 - \tilde{p})c - (1 + \rho)B - y}_{S(\tilde{p})} = (\tilde{p} - \bar{p})((1 + r)B - c) \quad (7)$$

Since  $\bar{p} > \tilde{p}$  in the de Meza-Webb case, the marginal project has negative social benefits. Since social benefits are increasing in  $p$ , there is too much entry of low- $p$  projects. This excess entry can be corrected by a uniform tax applied either on all successful projects (of this given  $R$  class), on bank loans, or on bank deposits provided the government has the same information about projects as do the banks. In fact, setting the optimal tax is not straightforward. Available projects can take on many different values of  $R$ , and the banks will be able to pool them into separate classes, each with an equilibrium of the sort illustrated in Figure 1. The size of the distortion between social and private returns can vary by  $R$  class, so the optimal tax rates on loans in each class will vary. Moreover, they will vary in a non-systematic way depending on the distribution of projects in each  $R$  class.

### The Stiglitz-Weiss Case

In this case, intermediaries are assumed to be able to identify projects by their expected return. Within a given class of such projects, the set of available projects satisfies  $pR = k$ , where  $k$  is a constant. The distribution of available projects within a class  $pR = k$  can take any form. For convenience, we again assume that  $g(p, R) > 0$  for all projects over the relevant range on  $p$ .

The slope of the locus of available projects is given by:

$$\left. \frac{dR}{dp} \right|_{SW} = -\frac{R}{p} < 0$$

so, by (2) and (4) and assuming  $c > 0$ :

$$\left. \frac{dR}{dp} \right|_{S=0} < \left. \frac{dR}{dp} \right|_{SW} < \left. \frac{dR}{dp} \right|_{\pi=0} < 0 \quad (8)$$

Therefore, the slope of the locus of projects available in the Stiglitz-Weiss case with  $c > 0$  is between the slope of the iso-profit locus and the iso-benefit locus, although its exact position depends on the value of  $k$ , that is, the class of projects under consideration.

Figure 2 illustrates this case for a particular value of  $k = pR$ .<sup>4</sup>In this case where  $pR$  is fixed, entrepreneur's expected profits  $\pi(p, R, r)$ —given by (1)—are decreasing in  $p$ , so all

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<sup>4</sup> In this figure, we are assuming that  $k$  is such that  $\tilde{p}$  is in the interior.

projects with  $p \leq \tilde{p}$  are undertaken, where  $\tilde{p}$  is again the point of intersection between the Stiglitz-Weiss locus of available projects and the iso-profit locus of the marginal projects. Therefore, it must be that  $\bar{p} < \tilde{p}$ . Thus, since  $\bar{p} = p^*$ , the marginal project lies to the right of  $p^*$  along the iso-profit locus. This implies that the locus of available projects intersects the iso-profit locus to the right of  $p^*$ . We can infer the following from Figure 2. There are three ranges along the locus of available projects delineated by where it intersects the iso-profit locus ( $\tilde{p}$ ) and where it intersects the iso-benefit locus, denoted  $p^o$ . From a social point of view, only projects with  $p \geq p^o$  should be undertaken. Consider the social profitability of projects in the three ranges:

- i.  $p > \tilde{p}$ : Projects in this range are socially profitable, but are not undertaken.
- ii.  $p^o \leq p \leq \tilde{p}$ : Projects in this range are socially optimal and are undertaken
- iii.  $p < p^o$ : Projects in this range are socially unprofitable, but are undertaken.<sup>5</sup>

Thus, too few high- $p$  projects are undertaken—all those the highest  $p$ —and too many low- $p$  projects are undertaken—those with the lowest  $p$ . The policy implications are obviously ambiguous.

It is worth contrasting these results with the case when there are no ex post monitoring costs ( $c = 0$ ), as was assumed by Stiglitz-Weiss (1981) and recounted in both Mankiw (1986) and de Meza and Webb (1987). In this case, the slopes of the iso-benefit locus (4) and the locus of available projects  $pR = k$  would be the same,  $-R/p$ . Both are rectangular hyperbolas so are parallel to one another in  $p, R$ -space. The iso-profit locus is again given by  $\pi(p, R, r) = p(R - (1 + r)B) - y = 0$ , so by (2), (4), and (8):

$$\left. \frac{dR}{dp} \right|_{S=0} = \left. \frac{dR}{dp} \right|_{SW} = -\frac{R}{p} < -\frac{R - (1 + r)B}{p} = \left. \frac{dR}{dp} \right|_{\pi=0}$$

Since the available projects locus is steeper negative than the iso-profit locus, it intersects the latter from above. Since the iso-benefit locus and the locus of available projects are now parallel, the range of over-investment in Figure 2 ( $p < p^o$ ) disappears. Projects with lower  $p$  will still be undertaken, while those with higher  $p$  will not. The expected return on projects financed  $\bar{p}$ —still given by (5'), the intersection point of the iso-benefit and iso-profit curves—is less than that for the marginal project  $\tilde{p}$ . Since all projects should be undertaken, there will be under-investment, as shown by de Meza and Webb (1987). Algebraically,  $S(\tilde{p}) > 0$  by (7) since  $\tilde{p} > \bar{p}$ .

The possibility of these various outcomes makes policy prescriptions difficult. Whatever the magnitude of monitoring costs  $c$ , Figures 1 and 2 indicate that in general, it would

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<sup>5</sup> The existence of this range depends on  $c$  being high enough. As shown below, if  $c$  is zero, the locus of available projects and the iso-benefit lines are parallel.

be desirable to tax high- $R$ , low- $p$  projects and subsidize low- $R$ , high- $p$ . However, even if the government has the same information as the banks and can observe the expected project returns  $pR$ , it cannot distinguish within any given category projects with different characteristics  $(p, R)$ , so it cannot target taxes and subsidies appropriately.

#### 4. Equity Finance

Suppose now that intermediaries provide finance to entrepreneurs in exchange for a share of the project returns. Let  $\sigma$  be the share of a project's returns going to the entrepreneur, so the remaining  $(1 - \sigma)$  goes to the intermediary. Now all projects must be monitored since returns must be verified in both the good and the bad states. The expected net profits of entrepreneurs are:

$$\pi(p, R, \sigma) = \sigma pR - y$$

Since this is increasing in the expected return  $pR$ , entrepreneurs with the higher expected returns will enter. The iso-profit condition for the marginal entrepreneur becomes  $\sigma pR - y = 0$ , and the expected returns on marginal projects are denoted  $\widetilde{pR}$ , where:

$$\widetilde{pR} = \frac{y}{\sigma} \tag{9}$$

All projects with  $pR \geq \widetilde{pR}$  apply for financing.

The iso-benefit locus in  $(R, p)$ -space is now given by  $S(p, R) = pR - c - (1 + \rho)B - y = 0$ , which differs slightly from (3) since now all projects must be monitored. Along this locus, expected returns are constant and denoted  $(pR)^o$ , where:

$$(pR)^o = (1 + \rho)B + c + y \tag{10}$$

All projects with  $pR \geq (pR)^o$  should be undertaken from a social point of view. Note that both the iso-profit and the iso-benefit loci are rectangular hyperbolas defined by (9) and (10). We need then only to compare  $\widetilde{pR}$  with  $(pR)^o$  to see if too many or too few projects are undertaken. To do so, we use the intermediaries' zero-expected-profit condition to obtain the equilibrium share  $\sigma$  offered by intermediaries in return for providing finance  $B$ :

$$(1 - \sigma)\overline{pR} - (1 + \rho)B - c = 0 \implies \sigma = \frac{\overline{pR} - (1 + \rho)B - c}{\overline{pR}} \tag{11}$$

Substituting this into the iso-profit condition (9) and rearranging, we obtain:

$$\widetilde{pR} = y + \frac{\widetilde{pR}}{\overline{pR}}((1 + \rho)B + c) < y + (1 + \rho)B + c = (pR)^o \tag{12}$$

Therefore, the iso-profit curve lies uniformly inside the iso-benefit curve, so too many projects are unambiguously undertaken in the market equilibrium (unlike with the case of debt finance).

Note that in order to implement equity finance, it must be possible to observe  $R$  ex post. That being so, the equity contract could in principle be made conditional on  $R$ . We return to the consequences of that below.

## 5. Choice of Debt versus Equity Finance

If returns can be observed ex post, the choice between debt and equity finance should be endogenous. In this section, we assume that competition among financial intermediaries determines which projects are financed by debt and which by equity. Suppose (following Hellmann and Stiglitz, 2000<sup>6</sup>) that there are two sorts of financial intermediaries: those that provide debt finance and those that provide equity finance. Both are risk-neutral and competitive. Entrepreneurs can choose their most preferred form of finance.

Consider first entrepreneurs' expected net profits under debt and equity finance. Analogous to the above, they are as follows (with superscripts added to indicate debt and equity finance):

$$\pi^D(p, R, r) = p(R - (1 + r)B) - y, \quad \pi^E(p, R, \sigma) = \sigma pR - y \quad (13)$$

As before, define iso-profit curves for debt and equity finance, given  $r$  and  $\sigma$ , by  $\pi^D(p, R, r) = 0$  and  $\pi^E(p, R, \sigma) = 0$ . Differentiating these iso-profit curves, we obtain:

$$\left. \frac{dR}{dp} \right|_{\pi^D=0} = -\frac{R - (1 + r)B}{p} > -\frac{R}{p} = \left. \frac{dR}{dp} \right|_{\pi^E=0}$$

The iso-profit curve for debt is therefore flatter than that for equity, so intersects it from below (assuming, as we do, the intersection is in the interior). Given  $r$  and  $\sigma$ , the intersection point will satisfy  $\pi^D(p, R, r) = \pi^E(p, R, \sigma) = 0$ , or, using (13):

$$\widehat{R} = \frac{(1 + r)B}{1 - \sigma} \quad (14)$$

If firms have a choice between forms of finance given  $r$  and  $\sigma$ , they will choose debt if:

$$\pi^D(p, R, r) - \pi^E(p, R, \sigma) = p((1 - \sigma)R - (1 + r)B) > 0 \implies R > \widehat{R}$$

For  $R < \widehat{R}$ , equity finance will be chosen.

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<sup>6</sup> The focus of Hellmann and Stiglitz (2000) was on credit rationing rather than inefficiency in the number of projects financed. As mentioned, we are ignoring the possibility of credit rationing.

Figure 3 illustrates the projects that will apply for finance of one sort or another, and which type of finance will be preferred. The horizontal line at  $\widehat{R} = (1+r)B/(1-\sigma)$  divides the project space into those that would be financed by debt and those by equity. All projects that are above the horizontal line and to the right of the debt iso-profit curve are financed by debt. All projects that are below the horizontal line and to the right of the iso-profit curve for equity are financed by equity. Those to the southwest of both curves are not financed at all. The intuition is that projects with high  $R$  (and low  $p$ ) take debt finance because all the residual of a good outcome goes to the entrepreneur. Low risk-low return projects take equity finance.

The social net benefits are defined as before, taking account of the fact that with debt finance, monitoring is only necessary in the event of bankruptcy. Iso-benefit curves for debt and equity finance are:

$$\begin{aligned} S^D(p, R) &= pR - (1-p)c - (1+\rho)B - y = 0 \\ S^E(p, R) &= pR - c - (1+\rho)B - y = S^D(p, R) - pc = 0 \end{aligned} \quad (15)$$

The slopes of these two iso-benefit curves are given by:

$$\left. \frac{dR}{dp} \right|_{S^E=0} = -\frac{R}{p} > -\frac{R+c}{p} = \left. \frac{dR}{dp} \right|_{S^D=0}$$

Thus,  $S^E(p, R) = 0$  will be a rectangular hyperbola, like the iso-profit locus of entrepreneurs under equity financing. By the same argument as in Section 4, the iso-benefit lies outside the iso-profit locus for equity financed projects. The iso-benefit curve is steeper for debt financed projects than for equity-financed projects. Moreover, as (15) implies, if monitoring costs  $c$  are not too large, the iso-benefit curve  $S^D = 0$  will intersect the iso-profit curve  $\pi^D = 0$  at  $R < \widehat{R}$  as shown in Figure 3.<sup>7</sup> If that is the case, the iso-benefit curve for debt financed projects  $S^D = 0$  will lie outside the iso-profit curve  $\pi^D = 0$  whenever debt is chosen ( $R > \widehat{R}$ ).

The implication is that, for  $c$  not too large, there will be too many projects financed both by debt and equity. Thus, it seems that allowing for both debt and equity finance

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<sup>7</sup> More generally, at the intersection point  $\pi^D = 0 = S^D$ ,  $R^* = (1+r)B + [(1+r)B + c]y/[(1+\rho)B + c]$ . Combining this with expression (14) for  $\widehat{R}$ , and using (11) to determine  $\sigma$ , we obtain after some manipulation:

$$R^* = \widehat{R} \frac{(pR)^{Eo}}{\overline{pR}^E} + \frac{cy}{(1+\rho)B + c}$$

where  $(pR)^{Eo}$  is the expected return along the iso-benefit curve  $S^E = 0$  and  $\overline{pR}^E$  is expected profits along the iso-profit curve  $\pi^E = 0$ . Since  $(pR)^{Eo} < \overline{pR}^E$ ,  $R^* < \widehat{R}$  if  $cy$  is small enough.

removes the ambiguity in the direction of the distortion we found earlier, at least with small  $c$ . There will too many projects financed by both debt and equity, calling for a tax on the financing of projects. Of course, the optimal tax will not be uniform across projects of different  $p, R$  characteristics, but this cannot be implemented without the government having better information about projects than intermediaries. Failing that, the best we can say is that some tax on projects of all types will be welfare-improving.

## 6. Contracts Contingent on Ex Post Returns

If intermediaries can monitor project returns, they should be able to offer contracts that are contingent on  $R$ . In this section, we investigate the consequences of these contracts.<sup>8</sup> Our analysis is simplified by continuing to assume that each project has only two possible outcomes, zero and  $R$ . If there is more than one state with positive returns, the analysis becomes much more complicated.

To allow for general contract forms, let  $\alpha + \beta R$  be the return to the bank ex post, given  $R$ . (Of course,  $\alpha$  and  $\beta$  will vary with  $R$ , but since our analysis is for any given  $R$ , our notation ignores that dependency for simplicity.) Thus,  $\alpha$  could be like a debt component and  $\beta$  an equity one. Social net benefits are given by  $S(p, R) = pR - c - (1 + \rho)B - y$  since all projects must be monitored. Expected entrepreneurial profits are:

$$\pi(p, R, \beta, \alpha) = p((1 - \beta)R - \alpha) - y \quad (16)$$

Note that this is increasing in  $p$  for projects with non-negative profits. For projects of given  $R$ , iso-profit curves in  $(\beta, \alpha)$ -space have the slope:

$$\left. \frac{d\alpha}{d\beta} \right|_{\pi} = -R$$

The contracts offered by intermediaries will in equilibrium satisfy the zero-expected-profit condition for any given  $R$ :

$$\Pi = \bar{p}(\alpha + \beta R) - c - (1 + \rho)B = 0 \implies \alpha + \beta R = \frac{c + (1 + \rho)B}{\bar{p}} \quad (17)$$

Therefore, given  $R$  and  $\bar{p}$ , we obtain:

$$\left. \frac{d\alpha}{d\beta} \right|_{\Pi=0, \bar{p}} = -R$$

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<sup>8</sup> There may be an implementation problem with contracts contingent on  $R$  since there is no assurance that banks will reveal  $R$  truthfully to third parties, such as the courts. We ignore this problem.

Since  $d\alpha/d\beta$  are the same for banks and firms, *debt and equity are perfect substitutes*. Banks will choose  $\alpha + \beta R$  until  $\Pi = 0$ , and this determines  $\bar{p}$ . Any combination of  $\alpha$  and  $\beta$  giving the required equilibrium value of  $\alpha + \beta R$  will suffice.

Consider now the efficiency of the marginal project  $\tilde{p}$ . The zero-expected-profit condition for this project from (16) is:

$$\pi(\tilde{p}, R, \alpha, \beta) = \tilde{p}((1 - \beta)R - \alpha) - y = 0$$

Combining this with the intermediaries' zero-expected-profit condition (17) yields:

$$\underbrace{\tilde{p}R - c - (1 + \rho)B - y}_{S(\tilde{p})} = (\tilde{p} - \bar{p})(\alpha + \beta R)$$

Since expected profits are increasing in  $p$ ,  $\bar{p} > \tilde{p}$ . Therefore,  $S(\tilde{p}) < 0$  indicating that there is overinvestment, as in the de Meza-Webb case. Efficiency would be increased by taxing projects. In the full optimum, the tax would have to vary with  $R$ , but not generally monotonically. The value of  $\alpha + \beta R$  is contingent on  $R$  but not in a straightforward way. In particular,  $\bar{p}$  will differ by  $R$  depending on the distribution of firms over  $p$ , given  $R$ .

## 7. Extensions

In this section, we consider some extensions to the basic model. Most of the extensions involve enhancing the information available to the intermediaries. For simplicity, attention is restricted to loan financing. We begin with the case where entrepreneurs are able to signal the quality of their projects to banks at a cost. We then turn to cases in which the banks may be able to separate projects of different types by separating contracts.

### Signaling by Entrepreneurs

Suppose entrepreneurs can signal their types *perfectly* by incurring a fixed cost  $f$ . Signaling of this sort is considered by Fuest et al (2003) for the de Meza-Webb and Stiglitz project distributions. Here we consider it for the general case. Potential entrepreneurs now choose from three options: signal and take a loan, not signal and take a loan, and not take a loan. For entrepreneurs that signal, banks learn perfectly their type and can charge a type-specific interest rate. The bank zero-expected-profit condition—the analogue of (6)—implies that the interest rate charged to them (assuming for simplicity  $c = 0$ ) satisfies  $1 + r = (1 + \rho)/p$ , so  $r$  is decreasing in  $p$ . For those who choose not to signal, the pooled interest rate obtained from the bank zero-expected-profit condition (6) satisfies  $(1 + r)\bar{p}^n = 1 + \rho$ , where  $\bar{p}^n$  is the average probability of success of funded projects of entrepreneurs that do not signal.

Entrepreneurial expected profits for those who do and do not signal are then given by:

$$\text{With signaling :} \quad \pi^s(p, R) = pR - (1 + \rho)B - y - f$$

$$\text{With no signaling :} \quad \pi^n(p, R, r) = p(R - (1 + \rho)B/\bar{p}^n) - y$$

Entrepreneurs who take a loan will signal if  $\pi^s \geq \pi^n$ , or

$$(1 + \rho)(p/\bar{p}^n - 1)B - f \geq 0 \quad (18)$$

Denote the probability of success of the marginal entrepreneur who signals by  $\hat{p}$ , which is the value of  $p$  such that (18) is satisfied with equality, or:

$$\hat{p} = \bar{p}^n \left( 1 + \frac{f}{(1 + \rho)B} \right) > \bar{p}^n$$

Entrepreneurs with  $p \geq \hat{p} > \bar{p}^n$  will signal and obtain expected profits  $\pi^s(p, R)$ , while those with  $p < \hat{p}$  who take a loan will not signal and will obtain expected profits  $\pi^n(p, R, r)$ .

Figure 4 illustrates this for the general case. In the absence of signaling, the average probability of success of funded projects is  $\bar{p}$ . Signaling by the high- $p$  entrepreneurs causes the interest rate to rise for non-signaled projects, since this pool now has a lower expected probability of success,  $\bar{p}^n < \bar{p}$ . The iso-profit curve for pooled projects—those with  $p < \hat{p}$ —shifts parallel outward from  $\pi(p, R) = 0$  as shown. However, for  $p > \hat{p}$ , the iso-profit curve to  $\pi^s(p, R) = 0$  is a rectangular hyperbola  $pR = (1 + \rho)B + y + f$ , so parallel to the iso-benefit curve,  $S(p, R) = 0$ . The area of over-investment will fall, but the area of under-investment may well rise. Moreover, the deadweight cost of signaling is incurred. On balance, therefore, the ability of entrepreneurs to signal their quality may or may not be welfare-improving. Equivalently, there may be too much or too little signaling compared with the socially optimal amount. Nonetheless, the qualitative results of the non-signaling case carry over. There will be too many low- $p$  and too few high- $p$  projects undertaken, and, unless the government has enough information to be able to target tax rates by  $p$ , the optimal direction of intervention is ambiguous.

The de Meza-Webb case follows from this general analysis, and is depicted in Figure 4 for a given return  $\bar{R}$ . In the absence of signaling, we have  $\bar{p} > \tilde{p}$ . Signaling will peel off the highest  $p$  projects, causing  $\bar{p}$  fall to  $\bar{p}^n$  and the pooled interest rate  $r$  to rise. As long as some projects do not signal in equilibrium, we have  $\hat{p} > \bar{p}^n > \tilde{p}^n$ , where  $\tilde{p}^n$  is the probability of success of the marginal project. Since the pooled interest rate rises,  $\tilde{p}^n > \tilde{p}$ , so there will be fewer projects funded. However, as long as some pooled projects remain, there will be more projects than is socially optimal, by the same reasoning as before. Signaling itself will have ambiguous effects on welfare. Fewer projects will be



funded, which is a good thing in the de Meza-Webb case. But, costs of signaling will be incurred, which is wasteful: those who signal would have been funded anyway. The effect of signaling on them is simply to reduce the interest rate they face, which is a pure transfer from the banks. On balance, the welfare effect of signaling is ambiguous.<sup>9</sup> However, the policy implications are not affected by signaling. As in the de Meza-Webb case without signaling, it will be efficient to tax loans or funded projects. As Fuest et al (2003) argue, a tax will reduce the number of lowest- $p$  projects funded, which is welfare-improving. It will also reduce the number of entrepreneurs who chose to signal, which reduces social costs. The reason is that with the lowest- $p$  projects leaving the pool, the interest rate on pooled projects falls, causing  $\hat{p}$  to rise.

In the Stiglitz-Weiss case (assuming that all projects are socially beneficial), either  $\hat{p} > \tilde{p} > \bar{p}$  or  $\tilde{p} > \hat{p} > \bar{p}$ . If  $\tilde{p} > \hat{p} > \bar{p}$ , all projects are funded, which is good. The need for policy to discourage entry of projects is supplanted. However, the cost of signaling is incurred so the social optimum is not achieved. In fact, as in the de Meza-Webb case, signaling might be either welfare improving or welfare deteriorating.

### Different Internal Financing Requirements

Suppose now that all entrepreneurs have some amount of initial wealth  $W$ . We consider first whether it is possible to separate entrepreneurs by requiring them to provide different amounts of own finance in return for different interest rates. To avoid multi-dimensional screening problems, we assume that firms differ in  $p$  alone (analogous to the de Meza-Webb case). It turns out that a pooling equilibrium will be the outcome even in this simple case.

Let  $K - B \leq W$  be the amount of internal finance, so the loan is  $B \geq K - W$ . The expected profits of becoming an entrepreneur for a type- $p$  entrepreneur are now:

$$\pi(r, B) = p(R - (1 + r)B) - (1 + \rho)(K - B) - y \quad (19)$$

where  $(1 + \rho)(K - B)$  is the opportunity cost of the entrepreneur's own funds used in the firm. Note first that:

$$\frac{\partial \pi}{\partial B} = -p(1 + r) + (1 + \rho) \gtrless 0 \quad \text{as} \quad \frac{1 + \rho}{p} \gtrless 1 + r$$

Thus, if  $1 + r = (1 + \rho)/p$ , entrepreneurs are indifferent between internal finance and loans, while they prefer loans over internal finance if the interest rate is less than that, and vice versa. Define  $r^*$  by  $1 + r^* = (1 + \rho)/p$ . This would be the interest rate offered to the firm

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<sup>9</sup> Fuest et al (2003) show that if firms are distributed uniformly over  $p$ , signaling will be excessive. A tax on signaling would be welfare-improving, if it could be implemented.

in a full information (first-best) setting. The slope and curvature of iso-profit curves is obtained by differentiating (19):

$$\begin{aligned}\frac{dr}{dB}\Big|_{\pi} &= -\frac{\pi_B}{\pi_r} = -\frac{p(1+r) - (1+\rho)}{pB} \geq 0 \quad \text{as } r^* \geq r \\ \frac{d^2r}{dB^2}\Big|_{\pi} &= -\frac{1}{B} \frac{\partial r}{\partial B}\Big|_{\pi} + \frac{p(1+r) - (1+\rho)}{pB^2} \geq 0 \quad \text{as } r \geq r^*\end{aligned}$$

Figure 5 depicts four indifference curves for each of two firms with probabilities of success  $p_2 > p_1$ , with those for  $p_1$  drawn as dashes. In the full information outcome, the firms face interest rates  $r_1^*$  and  $r_2^*$ , where their indifference curves are parallel indicating that the level of loans within the admissible bounds  $K \geq B \geq K - W$  is irrelevant. However, this is not implementable if banks cannot distinguish the two types of firms. Moreover, a separating equilibrium will not be possible in this case since any contract offered to type 2's at the interest rate  $r = r_2^*$  will be preferred by the type 1's to their first best contract at the interest rate  $r_1^*$  (i.e., any contract below the indifference curve  $r_1 = r_1^*$  is preferred by the type 1's). Instead a pooling contract must be offered, and the only pooling contract that is an equilibrium will be that involving the minimal loan  $B = K - W$ . A pooling contract with a higher level of  $B$ , such as that  $B_1$  will be broken a bank offering a contract with lower  $B$ , thereby siphoning off the high- $p$  entrepreneurs and leaving the previous contract with a loss for the banks. Thus, simply allowing entrepreneurs to have a given initial amount of wealth  $W > 0$  will not change the previous results.

### Different Collateral Requirements

On the other hand, if entrepreneurs have initial wealth, it is possible for the banks to separate projects using differential collateral requirements, at least in the simple case where firms differ in one dimension only. Consider again the de Meza-Webb case where firms differ in  $p$ , but have the same  $R$ . Let  $C$  be the amount of collateral that the firm must pay in the event of bankruptcy. Given wealth  $W$ , collateral is limited by  $0 \leq C \leq W$ . The expected profits of the firm become:

$$\pi(r, C) = p(R - (1+r)B) - (1-p)C - y \quad (20)$$

Note that  $\partial\pi/\partial C < 0$ , so firms dislike collateral. The possibility of a separating equilibrium then revolves around which types of firms dislike it more. The slope of an iso-profit curve now becomes:

$$\frac{dr}{dC}\Big|_{\pi} = -\frac{(1-p)}{pB} < 0$$

Since this slope is independent of both  $r$  and  $C$ , iso-profit lines are straight ones. To verify a single-crossing property, differentiate with respect to  $p$  to obtain:

$$\frac{d}{dp} \left[ \frac{dr}{dC}\Big|_{\pi} \right] = \frac{1}{p^2 B} > 0$$

Thus, the higher- $p$  firm has flatter iso-profit curves. This gives rise to the possibility of separating contracts.

Figure 6 illustrates this for the case of two types with  $p_2 > p_1$ . With no collateral, type 1's would prefer the interest rate appropriate for the type 2's,  $r_2$ . It is possible to design a contract for the high- $p$  types to separate them from the low- $p$  potential mimickers. The optimal separating contract is given by  $(r_2^*, C_2^*)$  such that the type 1's are just indifferent between this contract and their no-collateral first-best interest rate  $r_1$ . The interest rates charged to the entrepreneurs depend on the banks' zero-expected-profit conditions. For the low- $p$  types who have no collateral,  $p_1(1 + r_1) = 1 + \rho$ . For the high- $p$  types, the interest rate satisfies  $p_2(1 + r_2^*)B + (1 - p_2)C_2^* - (1 + \rho)B = 0$ . This interest rate is less than the full information one  $r_2^*$  because of the effect of collateral. However, this separating contract may or may not be an equilibrium in the Rothschild and Stiglitz (1976) sense. Only if the pooling interest rate exceeds  $\hat{r}$ —that is, if there is a high enough proportion of high-risk types—will a separating equilibrium exist.

Suppose now there are a large number of potential types of entrepreneurs, who differ in their probability of success. As above, the single-crossing property will apply between any pair of probability types, with iso-profit curves being flatter for higher  $p$ 's. Assuming that separating equilibria exist, as many types of contracts will be offered as probability types if there is no bunching (partial pooling).<sup>10</sup> Incentive constraints for a given  $p$ -type entrepreneur will be binding only on the adjacent entrepreneur with the next lowest  $p$ . Profits will be increasing monotonically with  $p$ . If there is a pool of potential entrepreneurs, entry will occur until the profits of the last one fall to zero. Since the lowest- $p$  entrepreneurs face no incentive constraint (analogous to type-1 in Figure 6), the marginal entrepreneurs obtain their first-best level of finance, and the earn zero expected profits. Since the banks are able to offer them a separate contract, the probability of success of the marginal entrepreneur will also reflect the probability of repayment to the bank, or  $\tilde{p} = \bar{p}$  in the notation of basic model. This implies that social profitability of the marginal entrepreneur's project will also be zero ( $S(\tilde{p}) = 0$ ). There is therefore no need to intervene to ensure that the optimal number of entrepreneurs receive project financing.

### Different Capital Requirements

Suppose now that project size  $K$  is variable, and that project returns are given by the function  $R(K)$ , where  $R' > 0 > R''$ . If entrepreneurs differ only by their probability of success  $p$ , all would choose the same level of  $K$  for any given  $r$  so could not be separated

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<sup>10</sup> For an analysis of separating equilibria with many different types of agents, see Guesnerie and Seade (1982). We assume away bunching since it has no effect on the qualitative results of interest to us.

by the banks. To make the case of variable capital size interesting, we assume that entrepreneurs also vary by an ability parameter  $a$ . And, to avoid multi-dimensional screening problems, we assume that abilities and probabilities of success are perfectly correlated: all entrepreneurs of a given ability  $a$  have the same probability of success  $p$ . For expositional purposes, suppose initially there are only two types of entrepreneurs, differing in their probability of success  $p_i$  ( $i = 1, 2$ ) and their ability  $a_i$ , both of these characteristics being private information to the entrepreneur. Type 2's are taken to be more able, so that  $a_2 > a_1$ , but alternative possibilities on their relative chances of success ( $p_1 \gtrless p_2$ ) can apply. Later we consider the case where entrepreneurs can take on many different abilities in order to address the issue of whether the an efficient number of projects financed.

Output of type  $i$  if successful is  $a_i R(K)$ . In the event of failure, output is zero, as above. Assuming entrepreneurs have no initial wealth (so  $K = B$ ), the expected profit of a type  $i$  can be written:

$$\pi_i = p_i(a_i R(B) - (1 + r)B) - y \quad (21)$$

Preferences of the two types over  $(r, B)$  contracts can then be represented by iso-profit contours. By differentiating (21), these have an inverted-U shape, with the direction of increasing profit being downwards and with the curves for the high-ability types peaking, at any given  $r$ , to the right of those of with low ability. Moreover, the single-crossing property applies such that at each point in  $(r, B)$ -space, the slope of the iso-profit contour of the high-ability type exceeds that of the low-ability type. Given this, the possibility arises of a separating equilibrium in which the two types are offered, and select, different  $(r, B)$  contracts (as in Boadway *et al*, 1998). In any such equilibrium, each contract must earn zero expected profits for the bank (otherwise it would be vulnerable to entry). Since expected profits on a type  $i$  contract are simply  $p_i(1 + r_i)B_i - (1 + \rho)B_i$  (assuming no ex post monitoring costs for simplicity), this requires that

$$1 + r_i = \frac{1 + \rho}{p_i} \quad i = 1, 2$$

where  $r_i$  is the actuarially fair interest rate associated with  $p_i$ .

Equilibrium is characterized by the banks offering contracts  $(r_i, B_i)$  which maximize expected profits subject to this zero-expected condition for each type, the participation constraint that  $\pi_i \geq 0$  for both types, and the incentive constraints that each type prefer the contract intended for it, so that

$$a_i R(B_i) - (1 + r_i)B_i \geq a_i R(B_k) - (1 + r_k)B_k \quad i, k = 1, 2, k \neq i$$

Note that in the first-best allocation, each type maximizes  $p_i a_i R(B_i) - (1 + \rho)B_i$ , so that  $a_i R'(B_i) = (1 + \rho)/p_i$ , with strict concavity of  $R(K)$  ensuring that the participation constraint is satisfied. Diagrammatically, this corresponds to a point of tangency between

an iso-profit contour of type  $i$  and a horizontal line at the corresponding actuarially fair interest rate  $r_i$ .

Suppose, for illustrative purposes, that the high-ability types are also the more likely to succeed, so that  $p_2 > p_1$ . And, assume that the probabilities of success are sufficiently different that the incentive constraints are not all slack. The relevant separating equilibrium—assuming there are enough type 1's so it exists—is shown in Figure 7. The low-ability types are offered their first-best contract at  $\alpha$  but the high-ability types are required to over-invest at  $\beta$  relative to their first-best level at  $\gamma$ . Policy can improve on this outcome. To see how, note first that there would be a social gain if the high-ability types could be induced to accept a contract at  $\delta$  (slightly north-west of  $\beta$ , and on the same iso-profit contour of the low-ability type) rather than  $\beta$ , since this would bring their level of investment closer to the first-best level. The difficulty is that although the pair  $(\alpha, \delta)$  is incentive compatible it cannot be sustained as a no-intervention equilibrium because  $\delta$  (since it involves an interest rate higher than  $r_2$ ) leaves the banks with positive profits. But the government can soak up these profits by imposing a tax on the interest payments associated with the high-ability contract. More precisely, by charging a tax on the low-interest loans equal to the vertical distance between  $\delta$  and  $\beta$ , the government can induce banks, solving the problem described above, to offer the contract at  $\delta$ , and they will earn zero net profits by doing so. This tax makes high-ability entrepreneurs worse off, of course, so that this argument will fail if it leads to a violation of their participation constraint. It is readily verified, however, that their participation constraint is not binding in the no-intervention equilibrium, and so will continue to hold for a sufficiently small tax. The participation constraint may make it impossible to sustain the first-best as an equilibrium in this way by charging a high enough tax to induce the contract at  $\zeta$ , directly above  $\gamma$ , but a small tax is sure to be improve welfare (and, of course, raise revenue in the process).

Suppose now that there are a large number of potential ability-types of entrepreneurs, with higher ability types having higher probabilities of success. Following the same reasoning as above, it is clear that the optimal number of entrepreneurs will be financed. The lowest ability entrepreneurs receive their first-best level of finance, and the marginal one will obtain zero profits. Since the banks are able to offer them a separate contract, the probability of success of the marginal entrepreneur will also reflect the probability of repayment to the bank. This implies that the social profitability of the marginal entrepreneur's project will also be zero, so again the optimal number will enter.<sup>11</sup>

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<sup>11</sup> More formally, the zero-profit condition of the marginal entrepreneur implies:  $\tilde{p}(\tilde{a}R(\tilde{B}) - (1 + \tilde{r})\tilde{B}) - y = 0$ , where tildes refer to values for the marginal entrepreneur. Since  $\tilde{p}(1 + \tilde{r}) = (1 + \rho)$ , this zero-profit condition becomes  $\tilde{p}\tilde{a}R(\tilde{B}) - (1 + \rho)\tilde{B} - y = 0 = S(\tilde{p})$ . Thus, the marginal entrant earns zero social profit.

A parallel analysis applies for the case in which it is the low-ability types that are more likely to succeed, so that  $p_2 < p_1$ . The separating equilibrium is in this case as shown in Figure 8, with the low-ability types credit-constrained in the sense that their contract implies under-investment. By essentially the same argument as above, the outcome can again be improved by imposing a tax on the low interest loans, shifting the low-ability types from  $\beta$  to a point like  $\zeta$ . The participation constraint raises somewhat more delicate issues in this case, since there is no assurance that it does not bite for the low-ability types in the no-intervention equilibrium. In particular, if there is a spectrum of ability types, entry will occur until the least profitable type just earns zero profits. The least profitable type can be of high or a low ability since ability and probability of success work in opposite directions. In the low-ability entrepreneurs also earn the least profits, correcting for the under-investment by imposing a tax on credit-constrained entrepreneurs can cause an offsetting distortion by forcing some of them out of business. With that proviso, however, the case for a tax on low-interest loans carries through essentially unchanged.

It is striking that the qualitative nature of the optimal policy is the same irrespective of whether it is the low- or high-ability types that have the greater prospect of success. Since the inefficiency being addressed involves over-investment in one case and under-investment in the other, one might have expected a tax and subsidy to be optimal in the two cases. The key point, however, is that the inefficiency is being induced not by setting an inappropriate interest rate but by a distortion in the amount of credit being made available. The tax acts not directly on investment, but on the self-selection constraint that induces the inefficiency. By making the low interest loans less attractive, the tax weakens the incentive constraint—making the low interest loan less attractive to the low- $p$  type—and so shifts the equilibrium to the social optimum.

### Moral Hazard

Suppose finally that entrepreneurs can affect the probability of success of their projects by exerting variable amounts of effort. For simplicity, projects all earn the same return  $R$  if successful (the de Meza-Webb case). Let the probability of success be given by  $\theta(p, e)$ , where  $p$  is the project's 'endowed' probability and  $e$  is effort. The function  $\theta(p, e)$  is increasing and strictly concave. If the cost per unit of effort is constant and normalized to unity, entrepreneurial profits, analogous to (1) in the basic model, become:

$$\pi(p) = \max_{\{e\}} \{\theta(p, e)[R - (1 + r)B] - e - y\} \quad (22)$$

The first-order condition for the entrepreneur's effort decision is:

$$\theta_e[R - (1 + r)B] - 1 = 0 \quad (23)$$

Social surplus generated by a given entrepreneur's project is given by:

$$s(p) = \theta(p, e)R - (1 - \theta(p, e))c - e - (1 + \rho)B - y$$

which is the analogue to (3). Therefore, socially optimal effort satisfies:

$$\frac{\partial s(p)}{\partial e} = \theta_e[R + c] - 1 = 0$$

Comparing this with (23), we see that the entrepreneur takes too little effort compared to the socially optimal level. However, since the government cannot observe effort, it is unable to implement a corrective policy.

There will again be a marginal project now with success probability  $\theta(\tilde{p}, e)$ , where  $e$  is determined endogenously by the entrepreneur. Using (22), the marginal project satisfies:

$$\pi(\tilde{p}) = \theta(\tilde{p}, e)[R - (1 + r)B] - e - y = 0$$

where  $e$  is chosen optimally. Using the banks' zero-expected-profit condition,  $\bar{\theta}(1 + r)B - (1 - \bar{\theta})c - (1 - \rho)B = 0$ , the social profitability of the marginal project becomes:

$$s(\tilde{p}) = \theta(\tilde{p}, e)R - (1 - \theta(\tilde{p}, e))c - e - (1 + \rho)B - y = [\theta(\tilde{p}, e) - \bar{\theta}]((1 + r)B - c) < 0$$

where  $\theta(\tilde{p}, e) - \bar{\theta} < 0$ , analogous to the basic de Meza and Webb case. Therefore, the addition of variable effort does not change the qualitative results about over-investment. The government is unable to correct for variable effort since effort cannot be observed.

## 8. Conclusions

The starting point for this paper was the finding that when banks fund projects of unknown quality at a common interest rate, either under- or over-investment could occur depending on whether the pooled projects have the same expected return (the Stiglitz-Weiss case) or the same return if successful (the de Meza-Webb case). Depending on which case prevails, policy intervention can call for a subsidy or a tax on loans. We have investigated the nature of inefficiency and the case for policy intervention in credit markets under alternative, more general assumptions. When project characteristics in a given loan pool can include various combinations of return  $R$  and probability of success  $p$ , there will generally be under-investment in projects with low  $R$  and high  $p$ , and over-investment in projects with high  $R$  and low  $p$ . In principle, a progressive tax system on ex post returns will be efficiency improving, although it is difficult to implement. Once entrepreneurs are offered the choice of loan and equity finance, the ambiguity disappears. High- $R$  projects opt for debt finance, low- $R$  projects opt for equity finance, and there will be unambiguously too many projects of each type undertaken. In these circumstances, a tax on all new projects would be

efficiency-improving. This will also be the case if financial contracts are contingent on ex post returns.

If banks are able to separate projects of various qualities, these inefficiencies are mitigated. The mitigation is partial when higher quality projects are able to signal their quality to the banks at a cost. There will, however, still be an inefficient number of projects funded among those projects that do not signal and are pooled together. If the banks are able to offer separating contracts by requiring either varying amounts of collateral or variable sizes of loans, there will no longer be an inefficient number of projects funded. Instead, there will be inefficiency in the size of the project, and this inefficiency can be reduced by imposing a tax on low-interest loans.

These results depend on a number of simplifying assumptions that are common to the literature and designed to focus attention on the consequences of imperfect information about project quality. For example, both entrepreneurs and creditors are assumed to be risk-neutral thereby ruling out risk-sharing as a function of credit markets. As well, for the most part we have assumed away effort on the part of either the entrepreneurs or the intermediaries. Moreover, we have ruled out ex ante monitoring activities of the banks as a means of improving their information. These would all be interesting extensions.



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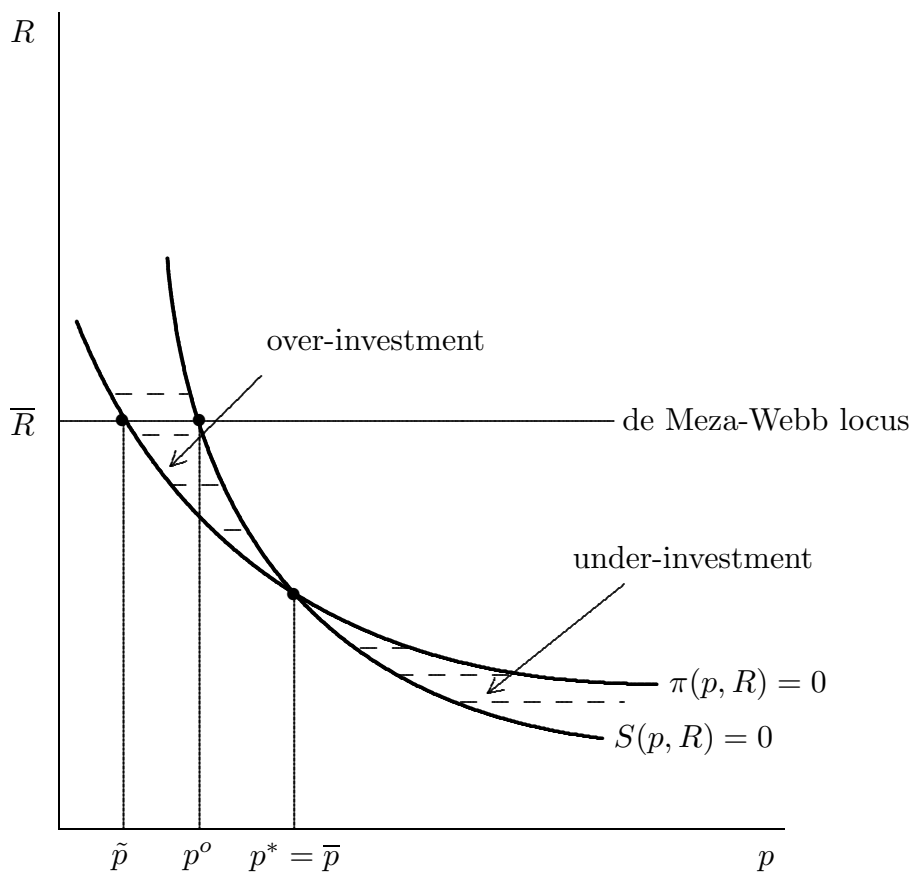


Figure 1. Credit Market Inefficiency with Debt Finance

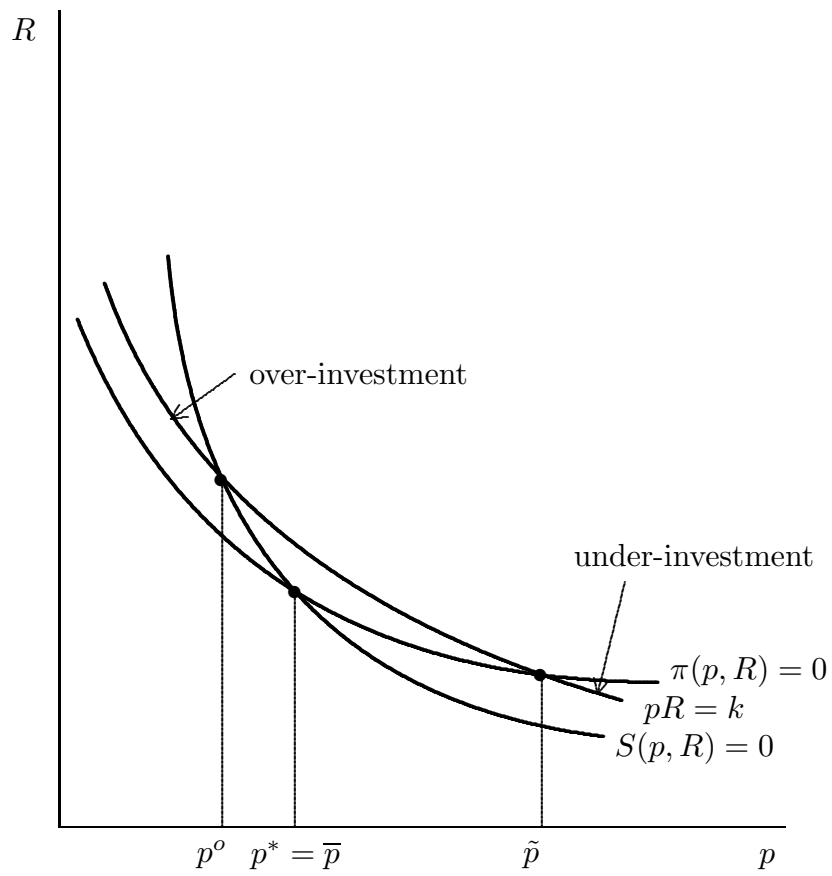


Figure 2. The Stiglitz-Weiss Case with  $c > 0$

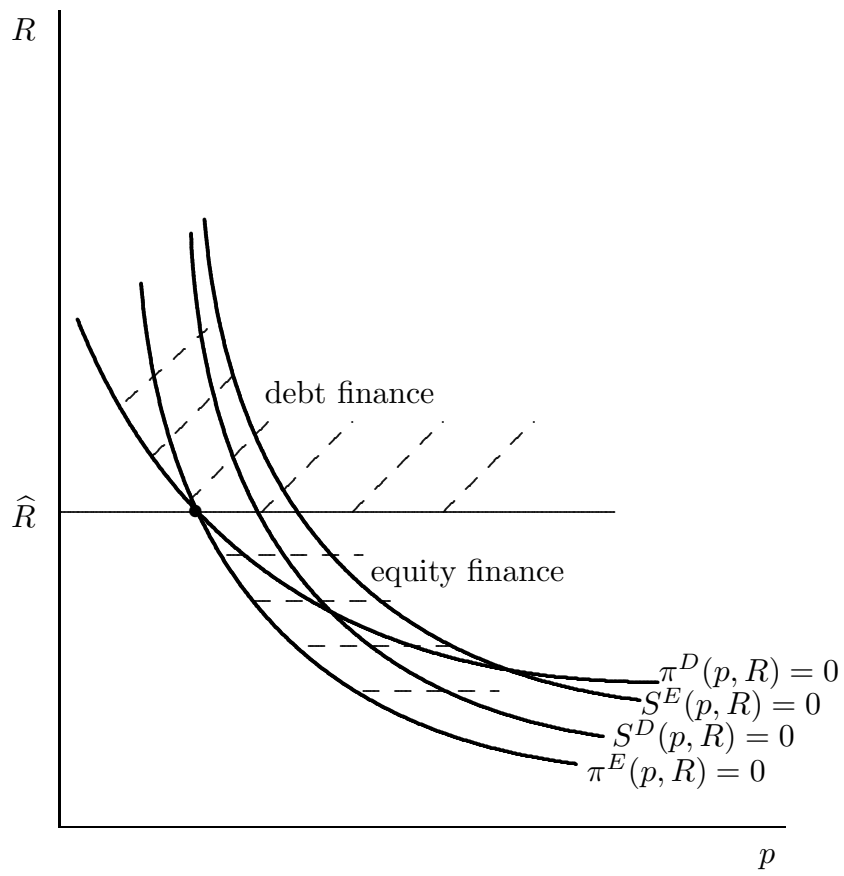


Figure 3. Debt and Equity Finance

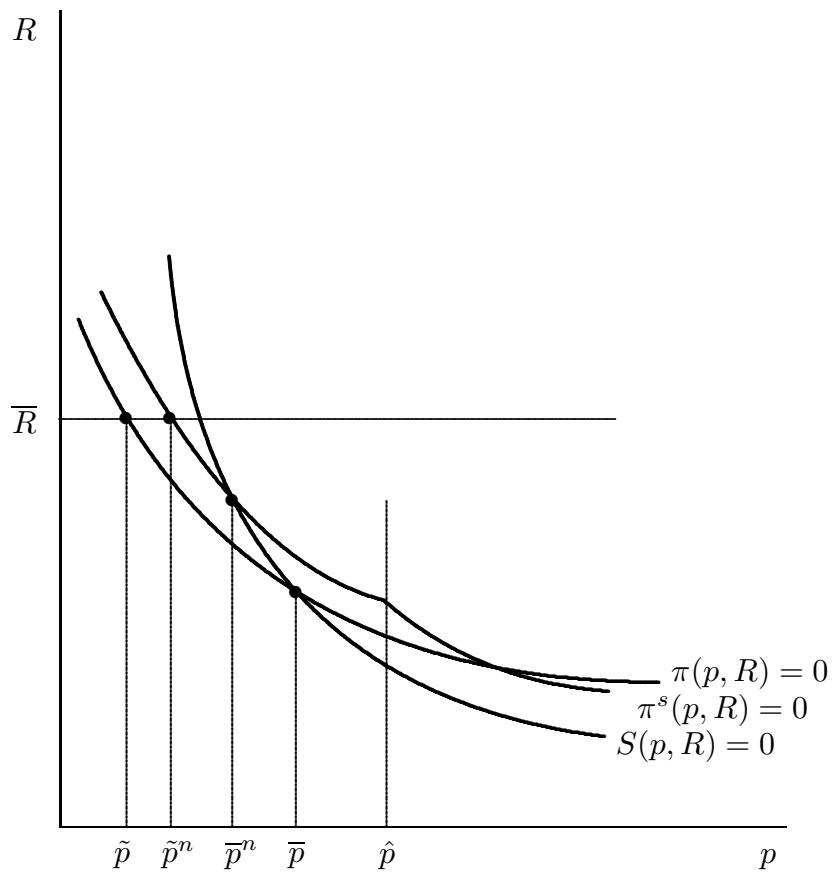


Figure 4. The Signaling Case with Debt Finance

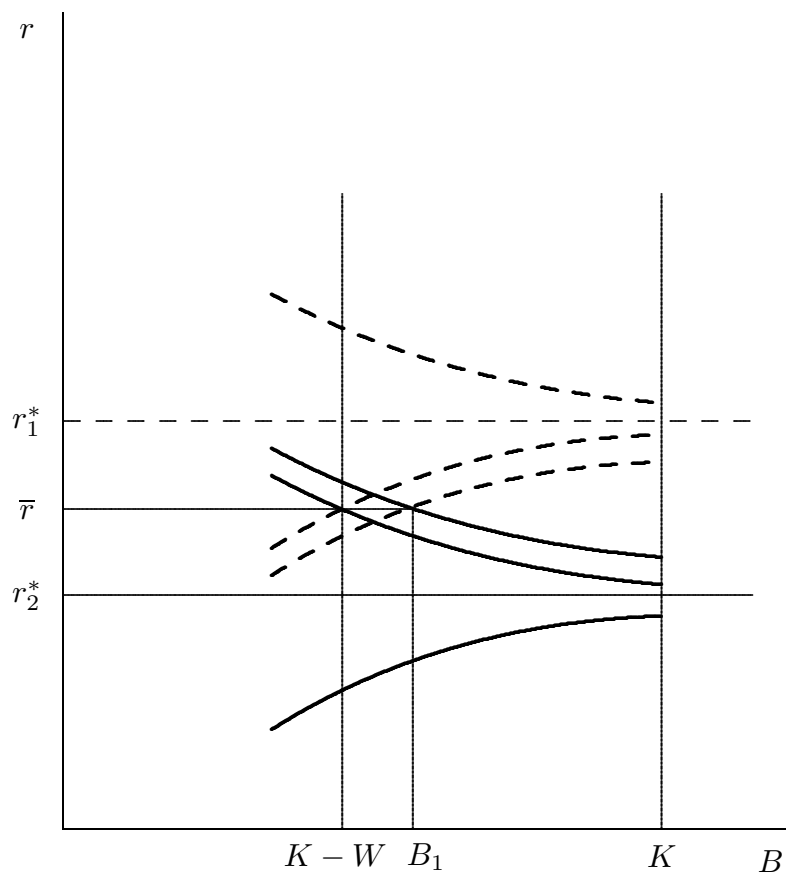
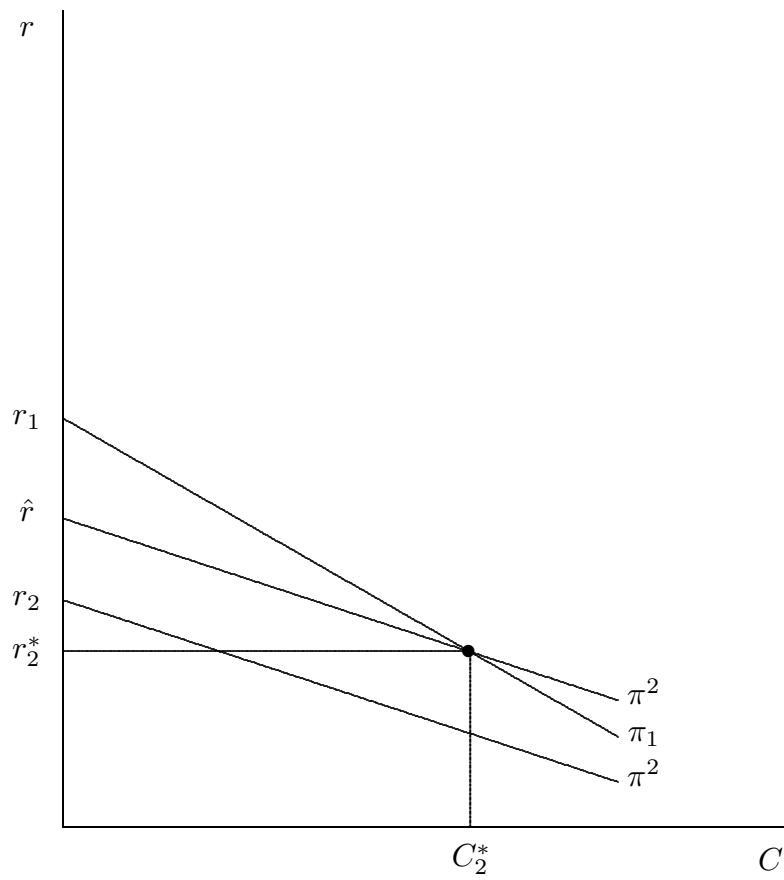
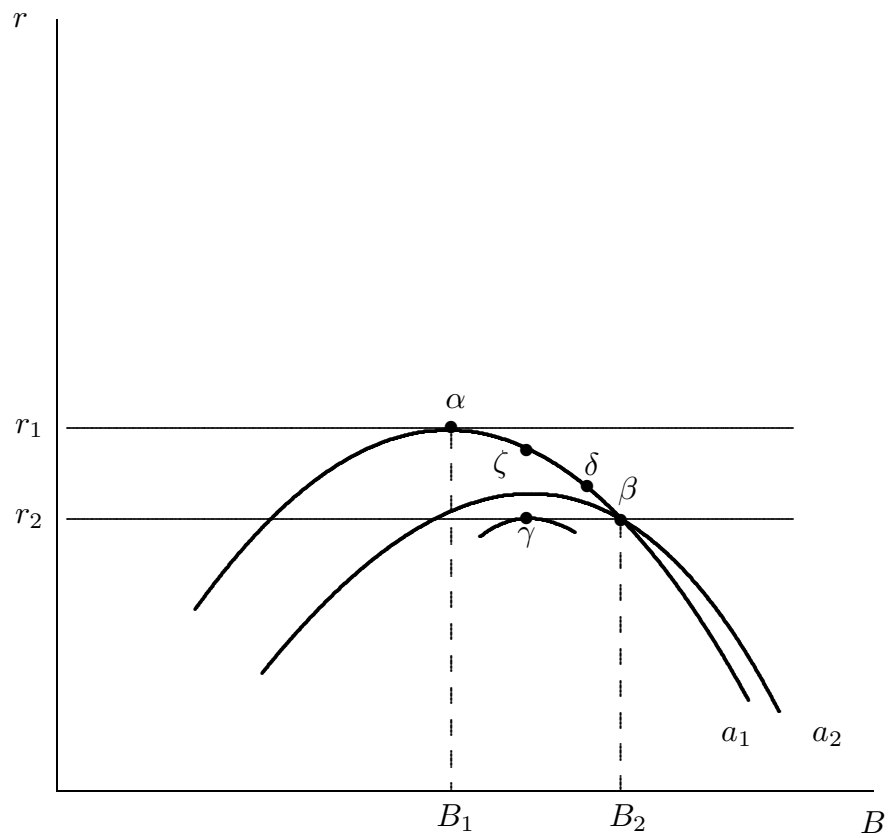


Figure 5. Variable Internal Finance

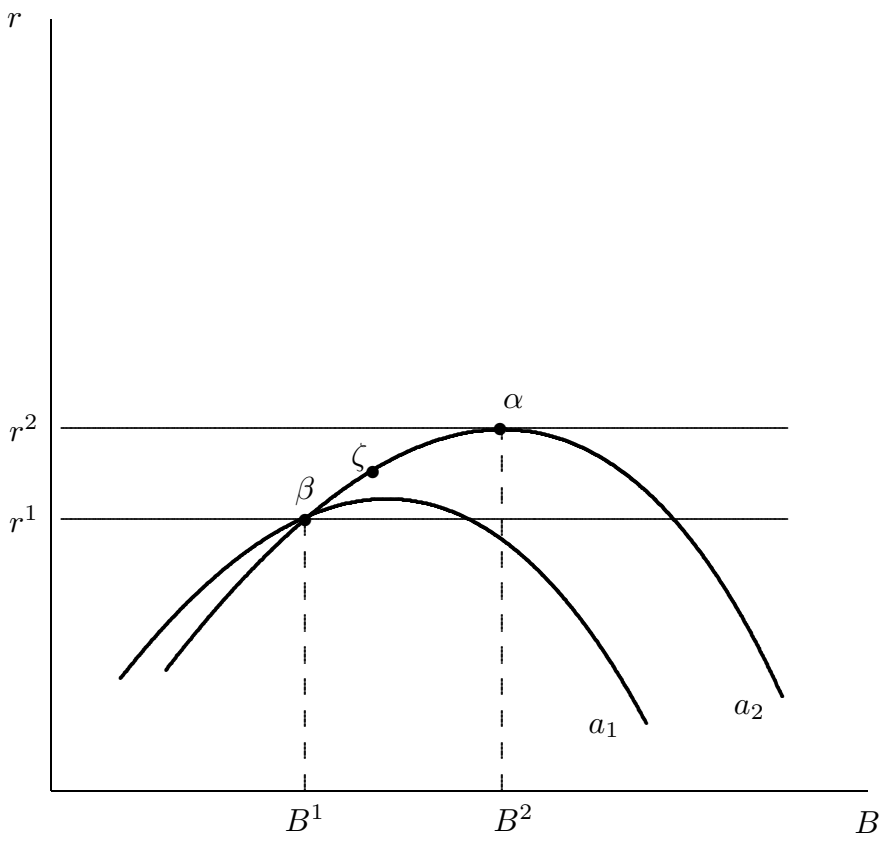


**Figure 6. Variable Collateral Requirements**



**Figure 7. Ability and Success Probability Positively Correlated**





**Figure 8. Ability and Success Probability Negatively Correlated**