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Robust Multidimensional Poverty Comparisons with Discrete Indicators of Well-being

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Abstract:
This paper provides a method to make robust multidimensional poverty comparisons when one or more of the dimensions of well-being or deprivation is discrete. Sampling distributions for the statistics used in these poverty comparisons are provided. Several examples show that the methods are both practical and interesting in the sense that they can provide richer information than do univariate poverty comparisons.

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JEL Classification: D31, D63, I31, I32

Most poverty analysts agree that poverty is multidimensional, in theory. In practice, empirical poverty studies are overwhelmingly univariate, with most economists limiting their attention to income or expenditures. This paper is part of a larger research agenda that aims to bring the empirical literature closer to the widely accepted theory. In particular, we show that it is both interesting and practicable to make general poverty comparisons when poverty is measured using more than one dimension of well-being. This study follows previous work that addressed this same issue for the case of two or more continuous measures of well-being. The innovation here is to consider multidimensional poverty comparisons when one or more of the indicators of well-being is discrete. This is of considerable practical importance, since important dimensions of well-being such as literacy and political enfranchisement are usually considered as discrete. In addition, discrete data are often collected for dimensions that are in fact continuous: income may be grouped into ranges rather than reported directly; respondents may be asked to rank their health status on a scale from 1 to 5, etc. As it happens, the methods that we use also provide a way to avoid the arbitrary choice of household equivalence scales in standard univariate poverty comparisons, a result first developed in Atkinson (1992) and Atkinson and Bourguignon (1987).

The approach in this paper is genuinely multidimensional. We do not aggregate several dimensions of well-being into a single index as is sometimes done elsewhere in the literature. Such aggregation involves arbitrary value judgments about the relative importance of each dimension of well-being which we prefer to avoid.[1]

Our intellectual debt to Anthony Atkinson is obvious throughout the paper. Atkinson (1987) pioneered the use of stochastic dominance techniques in poverty analysis. Atkinson and Bourguignon (1982) is a seminal contribution to the literature on comparisons of well-being in multiple dimensions. And Atkinson (1992) and Atkinson and Bourguignon (1987) develop a specific example of the general problem that we consider, in which welfare is measured in the dimensions of income, a continuous variable, and household size, a discrete one.

Section 1 provides our main theoretical results for robust poverty comparisons when well-being is measured in two dimensions, one continuous and the other discrete. We consider two cases, depending on whether or not the class of poverty measures of interest is continuous at the poverty line. Proofs may be found in the appendix. Section 2 provides an estimator and sampling distribution for the tests proposed in section 1. Section 3 expands the results of section 1 to the case of one continuous and two (or more) discrete measures of well-being. Section 4 provides examples, and section 5 concludes.

1. Multivariate poverty comparisons with one discrete and one continuous indicator of well-being

Suppose that a population can be split into K exhaustive and exclusive population subgroups, whose population share is denoted by $\phi(k), k=1,...,K$. Hence, $\sum_{k=1}^{K}\phi(k)=1$. We can define these subgroups based on a discrete welfare measure such as literacy, political enfranchisement, access to a public service, or physical capabilities. Alternatively, we can differentiate households by their relative needs, based on size and composition, type of activities, or area of residence. In either case, the important point is that these discrete differences in the characteristics of households or individuals suggest that, for a given value of the continuous measure of well-being, certain groups have lower overall well-being than others. This can be because the discrete variable is itself a measure of well-being (being illiterate is worse than being literate), or because it indicates differences in needs, prices, or poverty lines. In addition, we can suppose that there is some uncertainty as to the precise value of these differences.

We will assume below that the K subgroups can be ordered in increasing value of a discrete measure of well-being, in such a way that at common values of another, continuous, measure of well-being, individuals in subgroup 1 are more deprived than individuals in subgroup 2, who are more deprived than individuals in subgroup 3, and so on. For now, we assume that there is only one discrete variable (and thus a one-dimensional ordinal ranking of the K subgroups at some common value of the continuous measure), but we will later generalize the analysis to the case of several such discrete variables.

As is standard in the literature, for simplicity we limit our attention to poverty measures that are additive, so that poverty in each of the population subgroups can be defined as:

$$P(k;z(k)) = \int_0^{z(k)} \pi_k(x) f(x;k)dx \tag{1}$$

where x is a continuous measure of well-being, $f(x;k)$ is subgroup k's density of living standards at x normalized such that $\int f(x;k)dx = \phi(k)$ (the share of group k in the population), and $z(k)$ is subgroup k's poverty line in the dimension of x. $\pi_k(x)$ is the contribution to subgroup k's poverty of an individual in that subgroup with living standard equal to x. Since the non-poor do not, by definition, contribute to total poverty, we have that $\pi_k(x) = 0$ if $x > z(k)$. Total poverty in the population is given by:

$$\begin{aligned} P(z(1),...,z(K)) &= \sum_{k=1}^{K} \int_0^{z(k)} \pi_k(x) f(x;k)dx \\ &= \sum_{k=1}^{K} P(k;z(k)). \end{aligned} \tag{2}$$

For expositional simplicity, we will sometimes denote $P(z(1),...,z(K))$ simply by P. One such poverty index is the sum of FGT indices (see Foster, Greer, and Thorbecke (1984)) across subgroups, each with its own poverty line. Denote the FGT(α) index for subgroup k and parameter α, times the population share of group *k*, by

$$P^{\alpha}(k;z(k)) = \int_0^{z(k)} (z(k)-x)^{\alpha} f(x;k)dx. \tag{3}$$

Total poverty as measured by the FGT index is then:

$$P^{\alpha}(z(1),...,z(K))=\sum_{k=1}^{K}P(k;z(k))^{\alpha}. \tag{4}$$

Note that $P^{0}(z(1),...,z(K))$ is the population headcount, with each subgroup k being assigned its specific poverty line $z(k)$. Similarly, $P^{1}(z(1),...,z(K))$ is the average poverty gap in the population, again with each subgroup k being assigned its specific poverty line $z(k)$. Other multidimensional additive poverty indices can be defined along similar lines, extending, for instance, the unidimensional Watts (1968) or Chakravarty (1983) poverty indices.

Rather than focus on any one poverty index, however, we want to establish conditions under which multivariate poverty will be lower in one group than in another for any poverty index in a broad class of indices, in the tradition of the stochastic dominance approach to poverty comparisons (Atkinson, 1987; Foster and Shorrocks, 1988a, b, c). The conditions for such 'poverty dominance' differ for poverty indices that are continuous *vs*. discontinuous at the poverty line. We treat the continuous case first because it is somewhat simpler.

1.1. Continuous poverty indices

Define $\Pi^{1}(z(1),...,z(K))$ to be a class of multidimensional first-order poverty indices. $\Pi^{1}(z(1),...,z(K))$ includes all of the additive P indices defined in equation (2) that satisfy three conditions:

$$z(1)\geq z(2)\geq ...\geq z(K) \tag{5}$$

$$\pi_{1}^{(1)}(x)\leq \pi_{2}^{(1)}(x)\leq ...\leq \pi_{K}^{(1)}(x)\leq 0,\forall x \tag{6}$$

$$\pi_{k}(z(k))=0,\forall k=1,...,K. \tag{7}$$

where $\pi_1^{(1)}(x)$ is the first-order derivative of $\pi_1(x)$ with respect to *x*. The first condition says that the poverty lines in the continuous dimension for the subgroups can be ordered from the poorest (neediest) to the richest (least needy) group. This is a sensible ordering since we assume that for the same value of x, group k has lower well-being than group $k+1$, etc.

The second condition orders the first derivatives of $\pi_k(x)$ with respect to x. This assumption says that an increase in x causes at least as much poverty reduction the poorer is a person in the discrete dimension k. Roughly speaking, this assumption says that x and k are substitutes in the production of well-being. In most circumstances, this is a reasonable assumption – improving one dimension of well-being for those who are poorer in another dimension should generate greater poverty reduction than the same improvement for those who are richer in that dimension. However, it is possible that complementarity in the production of two dimensions of well-being might force a reversal. As an extreme case of complementarity, imagine poverty indices defined over continuous income and two discrete states, alive or dead. Hamlet aside, we probably want to rank 'alive' as better than 'dead', yet a reasonable poverty measure would probably fall by more if we increased the incomes of the living than the dead.[2] But we do not wish to consider such cases in the paper, in part because we feel that they are less empirically relevant.

The third condition assumes continuity of the poverty measure at the poverty line for each subgroup k.

Define $\Delta P(z(1)...,z(K)) = P_A(z(1)...,z(K)) - P_B(z(1)...,z(K))$, and $\Delta P^\alpha(z(1)...,z(K))$ and $\Delta P^\alpha(k;z(k))$ analogously. The above assumptions lead to the following equivalence for all poverty measures in the class $\Pi^1(z(1),...,z(K))$:

Theorem 1. (First-order poverty dominance for heterogeneous populations)

$$\Delta P(\zeta(1),...,\zeta(K)) > 0,$$

$$\forall P(\zeta(1),...,\zeta(K)) \in \Pi^1(\zeta(1),...,\zeta(K))$$

$$\text{and } \forall \zeta(k) \in [0, z(k)], k = 1,...,K$$

$$\text{iff } \sum_{k=1}^{i} \Delta P^0(k;\zeta) > 0, \forall \zeta \in [0, z(i)] \text{ and } \forall i = 1,...,K. \quad (8)$$

Proof: See Appendix.

Recall that $P^0(k;\zeta)$ is the FGT(0) measure, or headcount, for subgroup k and poverty line ζ, times the population share of subgroup k. $\sum_{k=1}^{i} P^0(k;\zeta)$ thus gives as a proportion of the total population the number of individuals below ζ in subgroups 1 to i, that is, in the i most deprived, or neediest, subgroups. $\sum_{k=1}^{i} P^0(k;\zeta)$ can then be termed the cumulative headcount index at ζ for the i most deprived subgroups. The first-order dominance condition (8) requires that this cumulative headcount be greater in A than in B, whatever the number i of groups we wish to include, and at all common poverty lines $0 \le \zeta \le z(i)$. Note, however, that it does not require that each subgroup k have independently more poor in A than in B, nor does it require that the population headcount (with each subgroup being assigned its own particular poverty line) be greater in A than in B.[3]

To see this more clearly, consider the case of poverty comparisons involving only two groups of individuals, $K = 2$, with $z(1)$ being the poverty line of the more deprived group and $z(2)$ the poverty line of the less deprived. Multidimensional dominance is checked first by comparing the headcount of those in group 1 whose value of x falls below poverty lines ζ between 0 and $z(1)$, and then by comparing the combined poverty headcounts of the two groups at all common poverty lines between 0 and $z(2)$. This is illustrated in Figure 1, where $\zeta(1)$ and $\zeta(2)$ denote the poverty lines at which poverty in each of the two subgroups is assessed. For $\Pi^1(z(1), z(2))$ dominance, we need to compare the global poverty headcount at all of the combinations of poverty lines on the $\zeta(1)$ axis up to $z(1)$ (that is, up to point G), and on the 45-degree line (up to point E). Comparing poverty for the combination of poverty lines on the $\zeta(1)$ axis amounts to checking the sign of $\Delta P^0(1;\zeta)$ for $\zeta \in [0, z(1)]$. Comparing poverty for the combination of poverty lines on the 45-degree line (until point E) amounts to checking the sign of $\sum_{k=1}^{2} \Delta P^0(k;\zeta)$ for $\zeta \in [0, z(2)]$.

<Figure 1 about here>

If the dominance conditions in (8) are met, then we obtain a very robust ordering of multidimensional poverty. Indeed, we can then assert with confidence that all of the multidimensional poverty indices contained in $\Pi^1(\zeta(1),...,\zeta(K))$ will show more poverty in A than in B, and this, regardless of the selection of any particular combination of poverty lines, so long as they belong to the set defined by $\zeta(k) \in [0, z(k)], k = 1,..., K$.

1.2. Discontinuous poverty indices

The dominance condition becomes more stringent, however, if we include in the analysis the headcount and other indices that are discontinuous at the poverty line (in the manner, for instance, of Bourguignon and Fields (1997)) and replace assumption (7) by the following:

$$\pi_1(x) \geq \pi_2(x) \geq ... \geq \pi_K(x) \geq 0, \forall x. \tag{9}$$

This condition requires that the poverty measure for group k evaluated at a given x be at least as great as the poverty measure in the next neediest group $k+1$ evaluated at the same value of x. This must hold for all k and for all x. A larger class $\tilde{\Pi}^1(z(1),...,z(K))$ of additive poverty indices then includes all the P indices defined in equation (2) that satisfy assumptions (5), (6), and (9). The 'traditional' headcount index, by which total poverty is measured by assigning each subgroup its *own* poverty line, belongs to $\tilde{\Pi}^1$ but not to Π^1. We thus expect the dominance conditions for $\tilde{\Pi}^1$ to be correspondingly more demanding. The definition of $\tilde{\Pi}^1(z(1),...,z(K))$ leads to the following equivalence:

Theorem 2. (First-order poverty dominance without continuity)

$$\begin{aligned}
&\Delta P(z(1),...,z(K)) > 0,\\
&\forall P(z(1),...,z(K)) \in \tilde{\Pi}^1(z(1),...,z(K))\\
&\text{iff } \begin{cases} \sum_{k=1}^{i} \Delta P^0(k;\zeta) > 0, \forall \zeta \in [0, z(i)]\, and\, \forall i = 1,...K \\ and \\ \sum_{k=1}^{i} \Delta P^0(k; z(k)) > 0,\ \forall i = 1,...K. \end{cases}
\end{aligned} \tag{10}$$

Proof**:** See Appendix.

The first condition in (10) is identical to the one already discussed in (8). In addition, we must check that the cumulative headcount differences are positive when each group k has its specific poverty line set to $z(k)$. That is the second condition in (10). In the two-group case of Figure 1, this adds to the previously-discussed test locations one more test at point F on the figure. The dominance conditions are thus more demanding than before. More importantly, however, note also that the combinations of poverty lines over which the $\tilde{\Pi}^1(z(1),...,z(K))$ ranking is robust are far more restricted than for the previous result: in fact, dominance by (10) ensures robustness only at the exact combination of lines $\{z(1),...,z(K)\}$. To extend the results to all of the poverty lines $\zeta(k)$ contained in $[0, z(k)]$ as in (8), we must also check the sign of the cumulative headcount when each subgroup is assigned its specific poverty line, instead of a common value ζ. This new condition would need to be checked for all combinations of poverty lines (other than $\{z(1),...,z(K)\}$) for which we would wish the poverty ordering $\tilde{\Pi}^1$ to be robust. For the 2-group case, this requires checking for dominance at all of the combinations of poverty lines defined by the shaded area of Figure 1. This is clearly a more stringent condition than that stated in Theorem 1, and it explains why we might wish to limit the generality of poverty orderings to a continuous class such as $\Pi^1(\zeta(1),\ldots,\zeta(K))$[4].

1.3 Higher-order dominance comparisons

It is possible to extend the above reasoning to any arbitrary order of dominance. For a given order of dominance s, we would assume continuity at the poverty line up to the $(s-1)^{\text{th}}$ order. We would also require conditions on the s^{th} order derivative $\pi_k^{(s)}(y;z(k))$, and on the ranking of these derivatives across population subgroups. Ordering conditions would use the sums of the $P^{s-1}(k;\zeta)$ curves.

For second-order dominance, this would require that $\pi_1^2(x)\geq\ldots\geq\pi_K^2(x)\geq 0,\forall y$. Indices in Π^2 would then be convex in y and thus decreasing in mean-preserving equalizing transfers of living standards; that is, they would obey the Pigou-Dalton principle of transfers within each group. The convexity of $\pi_k(x)$, and thus the importance of the Pigou-Dalton principle of transfers, would also be assumed to be decreasing in k and hence increasing in the needs of the subgroups. At a given x, the greater the needs of a subgroup of individuals, the greater the poverty-relieving effect of a mean-preserving equalizing transfer within that subgroup. The dominance conditions would then use $P^1(k;\zeta)$ -- which is the average poverty gap in subgroup k for a poverty line ζ, times the population share of subgroup k -- and would cumulate it across the i most deprived subgroups to give $\sum_{k=1}^{i}P^1(k;\zeta)$. When this cumulative average poverty gap is greater in A than in B, whatever the number i of neediest subgroups included, and at all common poverty lines $0\leq\zeta\leq z(i)$, poverty in A is unambiguously greater than in B for all of the indices in $\Pi^2(\zeta(1),\ldots,\zeta(K))$ and at all of the poverty lines $\zeta \in [0, z(i)]$. For the 2-group case, the graphical combinations of poverty lines over which this condition must be tested are the same as in the discussion of Figure 1 for condition (8).[5]

2. Estimation

Suppose that we have two random samples of N independently and identically distributed observations drawn from the joint distribution of membership into group k and of the indicator of well-being x, one sample from each of two paired distributions A and B (from panel data for instance). We can write these observations, drawn from a population $L = A, B$, as (k_i^L, x_i^L), $i = 1,\ldots,N$. A natural estimator of the sum of the dominance curves $\sum_{k=1}^{j} P_L^\alpha(k; z(k))$ is then given by:

$$\sum_{k=1}^{j} \hat{P}_L^\alpha(k; z(k)) = \frac{1}{N} \sum_{i=1}^{N} \left(z(k_i^L) - x_i^L \right)_+^\alpha I(k_i^L \le j) \tag{11}$$

where $I(\cdot)$ equals 1 if the argument is true and 0 otherwise. Expression (11) has the convenient property of being a simple sum of IID variables. We can then state:

Theorem 3. *For $L, M = A, B$, let the joint population moments of order 2 of $(z(k^L) - x^L)_+^\alpha I(k^L \le j)$ and $(z(k^M) - x^M)_+^\alpha I(k^M \le j)$ be finite, for all $j = 1,\ldots,K$. Then $N^{1/2}\left(\sum_{k=1}^{j} \hat{P}_L^\alpha(k; z(k)) - \sum_{k=1}^{j} P_L^\alpha(k; z(k)) \right)$ and $N^{1/2}\left(\sum_{k=1}^{j} \hat{P}_M^\alpha(k; z(k)) - \sum_{k=1}^{j} P_M^\alpha(k; z(k)) \right)$ are asymptotically normal with mean zero and with asymptotic covariance structure given by:*

$$\begin{aligned} &\lim_{N\to\infty} N \cdot \operatorname{cov}\left(\sum_{k=1}^{j} \hat{P}_L^\alpha(k; z(k)), \sum_{k=1}^{j} \hat{P}_M^\alpha(k; z(k)) \right) = \\ &E\left((z(k^L) - x^L)_+^\alpha I(k^L \le j)(z(k^M) - x^M)_+^\alpha I(k^M \le j) \right) - \\ &\sum_{k=1}^{j} P_L^\alpha(k; z(k)) \sum_{k=1}^{j} P_M^\alpha(k; z(k)) . \end{aligned} \tag{12}$$

Proof: See Appendix.

If A and B are independent, then we can use (12) by setting L equal to M, and replace N by either N_A or N_B. The covariance between the independent estimators of A and B will then be zero.

3. Multiple discrete variables

In principle, it is straightforward to extend Theorem 1 or 2 to the case of one continuous and multiple discrete measures of well-being: we only need to combine the discrete variables into one grouping. So, for example, if we have a literacy indicator and an enfranchisement indicator, both with only two possible values, then we create a combination of these variables with four values: illiterate/disenfranchised, literate/disenfranchised, illiterate/enfranchised, and literate/enfranchised. The only problem with this strategy is that the ordering of the four outcomes from poorest to richest (or most to least needy) is not always clear. While illiterate/disenfranchised is obviously the worst outcome, and literate/enfranchised is the best, we cannot say which of the other two is better or worse. To overcome this, we must check the dominance conditions of Theorem 1 or 2 using both of the possible orderings. Formally, the result is obtained for Theorem 1 by supposing two discrete indicators, say k and k^*, with K and K^* different possible values, and by assuming that the poverty indices $P(z(1,1), z(1,2), \dots, z(1,K^*,), z(2,1), \dots, z(K,K^*))$ defined over these two discrete and one continuous indicators satisfy the following conditions:

$$z(k,k^*) \geq z(l,k^*) \text{ if } k < l \tag{13a}$$

$$z(k,k^*) \geq z(k,l^*) \text{ if } k^* < l^* \tag{13b}$$

$$\pi^{(1)}_{k,k^*}(x) \leq \pi^{(1)}_{k+1,k^*} \leq 0, \forall x, k, k^* \tag{14}$$

$$\pi^{(1)}_{k,k^*}(x) \leq \pi^{(1)}_{k,k^*+1} \leq 0, \forall x, k, k^* \tag{15}$$

$$\pi_k(z(k,k^*)) = 0, \forall k = 1,\dots,K \text{ and } \forall k^*, = 1,\dots,K^*. \tag{16}$$

Define the class of such poverty indices as $\dot{\Pi}^1((z(1,1),\dots,z(K,K^*)))$, and as above let

$$P^\alpha(k,k^*;z) = \int_0^z (z-x)^\alpha f(x;k,k^*)dx. \tag{17}$$

where $f(x;k,k^*)$ is the density of those at x and in groups k and k^* normalized such that its integral over x gives the population share of those in groups k and k^*. We then have:

Theorem 4. (First-order poverty dominance for two discrete indicators and one continuous one)

$$\Delta P(\zeta(1,1),\dots,\zeta(K,K^*)) > 0,$$

$$\forall P(\zeta(1,1),\dots,\zeta(K,K^*)) \in \dot{\Pi}^1(\zeta(1,1),\dots,\zeta(K,K^*))$$

$$\text{and } \forall \zeta(k,k^*) \in [0, z(k,k^*)], k = 1,\dots,K, k^* = 1,\dots,K^* \tag{18}$$

$$\text{iff } \sum_{k=1}^{i} \sum_{k^*=1}^{j} \Delta P^0(k,k^*;\zeta) > 0, \forall \zeta \in [0, z(i,j)]$$

$$\text{and } \forall i = 1,\dots,K, j = 1,\dots,K^*.$$

The proof is analogous to that of Theorem 1. Again, this is equivalent to checking the dominance conditions of Theorem 1 using all of the possible orderings of the discrete indicators.

4. Examples

Our first example for this section obviates the equivalence scale problem by using the method first suggested by Atkinson (1992). We ask the question, ‘which type of transfer payment reduces poverty more in Romania, child allowances or social security pensions?’ Because the answer can easily be influenced by the choice of equivalence scale, we will avoid that choice altogether. Instead, we use bivariate dominance tests where the second dimension of well-being is household size, an indicator of greater needs. The neediest group is households with six or more people,[6] the next neediest contains households with five people, etc. The data come from the Romania Integrated Household Survey (Government of Romania 1994). The other well-being variable is household income, plus the relevant transfer payment (child allowances or social security pensions). We have standardized these payments so that they have the same mean, thus ensuring that the tests do not merely reflect the fact that one program is very large relative to the other.

Table 1 gives the *t*-statistics for the differences in the dominance curves of the neediest group, the two neediest groups, *etc*., up to the entire sample, as required by Theorem 1. The difference is the dominance curve for income plus child allowances minus that for income plus social security pensions, so a positive *t*-statistic indicates that social security pensions are more poverty-reducing than child allowances, and vice-versa. For large households, child allowances clearly reduce poverty by more than social security payments, regardless of the poverty line chosen. But this result is reversed once we include households with only two people, where the dominance curves now cross, and where social security payments appear to be more beneficial to poorer households. The same pattern holds for $s=2$ and $s=3$ (involving the 'poverty gap' and 'poverty severity' curves, respectively), suggesting that we cannot make any robust statement as to the comparative poverty-reducing impact of these two transfer payments without excluding households of size 2 and 1. Thus, any dominance result derived with a particular equivalence scale will not be robust to the choice of that scale in this case, a result that is not too surprising given the very different demographic profile of households receiving these two transfers.

<Table 1 about here>

Our second example considers a case in which poverty is measured on two dimensions, household expenditures per capita and adult literacy, the latter of which is discrete. We consider the change in poverty in Peru between 1985 and 1994, as measured by the *Encuesta Nacional de Hogares sobre Medición de Niveles de Vida* in those two years. This period spanned a significant economic crisis, including the hyperinflation of 1990–1991. Table 2 is similar to Table 1, but the groups are now defined by literacy.[7] We assume that, for a given level of expenditure, adults who cannot read and write have lower well-being than those who can. Thus, the first group is the illiterate population. The *t*-statistics are for the 1985 dominance curve minus the 1994 curve. For the entire sample, illiterate and literate together, column 3 shows a clear worsening of poverty due to the economic crisis. However, for poverty lines above the 35th-percentile of the expenditure distribution, the conditions of Theorem 1 are not met in this case. Even though poverty of illiterate Peruvians increased unambiguously during this period, the theorem requires that poverty normalized by the population share of the cumulative groups increase. Illiteracy actually fell from 18 to 13 percent between 1985 and 1994, which means that $P^0(k;\zeta)$ (which includes the normalization) for the illiterate group must be greater in 1985 than 1994 at very high poverty lines, yielding an indeterminant comparison.

<Table 2 about here>

In the previous example, using a bivariate poverty comparison impedes our ability to get a clear dominance between distributions even when the univariate expenditure distribution does show a statistically significant difference in poverty. While the stricter conditions for multivariate dominance might make us think that this is usually the case, it is also possible that when there is no univariate dominance in the continuous dimension, bivariate comparisons may produce useful dominance results, as we demonstrate in our next example.

Atkinson and Bourguignon (1982) distinguish 'union' from 'intersection' definitions of poverty. In the former, one is considered poor if s/he falls below the poverty line in *either* dimension. In that latter, one is considered poor only if s/he falls below *both* poverty lines.

Duclos, Sahn, and Younger (2006a, 2006b) show that union poverty comparisons require univariate dominance in both dimensions, but intersection poverty comparisons do not. They give examples of cases in which there is no univariate dominance in one or both dimensions, but there is bivariate dominance for a set of intersection poverty measures. A similar result is possible in the present case of one discrete and one continuous measure of well-being. Table 3 compares poverty in Ecuador between 1998 and 1999, also a period of significant macroeconomic turmoil. The comparison is in two dimensions: real household expenditures per capita and area of residence, where we suppose that well-being is lower in rural than in urban areas. The second column shows that there is neither univariate dominance in the dimension of household expenditures nor bivariate dominance as defined in Theorem 1. However, Column 1 shows that income poverty did decline unambiguously in rural areas for any choice of poverty line. Thus, we do have a dominance result for intersection poverty measures if the poverty line in the discrete dimension includes only rural residents, *i.e.* if only rural residents are considered to be poor in the dimension of area of residence.

<Table 3 about here>

Our last example illustrates the case of poverty comparisons over one continuous variable and two discrete ones. The data come from the fourth and twelfth waves of the British Household Panel Survey, carried out in 1994 and 2002, respectively. We measure well-being in the dimensions of real income per adult equivalent, education status, and health status. Data for the latter two variables are discrete. Education status is the highest qualification attained, and health status is whether or not the respondent reports that her/his daily activities are limited by her/his health. As Theorem 4 shows, poverty dominance requires comparisons across all the possible cumulative combinations of education and health status, regardless of the order of accumulation. Table 4 gives these comparisons. Despite the rather demanding criteria, we do find that multidimensional poverty declined over this period in Britain for poverty lines up to the 55th percentile of income distribution. Actual poverty in this period was about 19 percent, so it is safe to conclude that multivariate poverty declined during this period for all reasonable poverty lines.

<Table 4 about here>

As a more general check of this correspondence, we made trivariate poverty comparisons for each wave of the first 12 waves of the BHPS (except the ninth) for these combinations of variables: income/education/health; income/education/happiness; and income/health/happiness.[8] We set the maximum reasonable poverty line at the median of the joint income distribution for all waves. For the income/education/health comparisons similar to the one in Table 4, the multivariate comparison rejects the null in favor of dominance in 15 of the 43 cases in which the univariate comparison for income per adult equivalent rejects the null.[9] For the income/happiness/education comparisons, we reject the null in the multivariate tests in 26 of 43 cases. But in the income/health/happiness comparisons, we reach the same conclusion in only 7 of 43 cases.[10] Thus, while use of the multivariate comparisons does make it more difficult to find that poverty differs significantly over time, it certainly does not make such a conclusion impossible.

5. Conclusion

This paper has drawn on two literatures to which Tony Atkinson was an early and influential contributor: one uses stochastic dominance methods to make very general poverty comparisons (Atkinson 1987), and another studies poverty comparisons on multiple dimensions (Atkinson 1992; Atkinson and Bourguignon 1982). Drawing on both those literatures, we have shown that it is possible to make robust multidimensional poverty comparisons when one or more of the dimensions of well-being is discrete. Practically, this is useful because many measures of well-being are either inherently discrete or are recorded as such. We have also seen that to make such comparisons, it is important to distinguish between discontinuous headcount-like multidimensional poverty indices and continuous ones. The importance of this distinction is well understood in the univariate poverty literature, being linked *inter alia* to whether poverty indices obey the Pigou-Dalton principle. Finally, we have derived the sampling distributions for our multivariate poverty comparisons, so that they can be stated in a statistically meaningful way.

The examples presented highlight several key points about multivariate poverty comparisons that distinguish them from the standard univariate case. First, because multivariate comparisons appear to be more demanding than univariate ones, there is a concern that these tests will not be able to reject the null of non-dominance in practice. While this is true for most of the comparisons that we consider, we found many cases in which the null is rejected using surveys with typical sample sizes of a few thousand households. Second, there are cases when a multivariate poverty comparison rejects the null for an intersection definition of poverty even when the univariate income comparison does not. This can occur when income poverty declines for a subset of the poorest groups of households but does not for all households. Third, for a lower range of poverty lines, it is also possible for multivariate poverty to increase even if the share of households in the poorest subset of discrete groups declines. This happens if income poverty of the poorest groups rises so much that the number of poor in those groups increases even though their overall number declines. Each of these examples shows how a multivariate analysis can be richer and more subtle than poverty comparisons based on income alone.

6. Appendix

Proof of Theorem 1.

The proof follows the line of Atkinson (1992) and Jenkins and Lambert (1993). We first use (2) to integrate by parts the difference ΔP. We find:

$$\begin{aligned}\Delta P \quad &= \sum_{k=1}^{K} \pi_k(z(k))\Delta P^0(k;z(k)) \\ &- \sum_{k=1}^{K}\int_0^{z(k)} \pi_k^{(1)}(x)\Delta P^0(k;x)dx.\end{aligned} \tag{19}$$

Recall the continuity assumption that $\pi_k(z(k)) = 0, \forall k$. For $\Delta P > 0$, we thus need to show that

$$\sum_{k=1}^{K}\int_0^{z(k)} \pi_k^{(1)}(x)\Delta P^0(k;x)dx < 0. \tag{20}$$

Recall that $\pi_k^{(1)}(x) = 0$ if $x > z(k)$; combined with (5), we can then rewrite (20) as:

$$\int_0^{z(1)} \sum_{k=1}^{K} \pi_k^{(1)}(x)\Delta P^0(k;x)dx < 0. \tag{21}$$

The inner sum in (21) can be rewritten as:

$$\sum_{k=1}^{K} \pi_k^{(1)}(x)\Delta P^0(k;x) = \tag{22}$$

$$\pi_K^{(1)}(x)\sum_{l=1}^{K} \Delta P^0(l;x) + \left(\pi_{K-1}^{(1)}(x) - \pi_K^{(1)}(x)\right)\sum_{l=1}^{K-1} \Delta P^0(l;x) \tag{23}$$

$$+\ldots+\left(\pi_1^{(1)}(x) - \pi_2^{(1)}(x)\right)\Delta P^0(1;x) \tag{24}$$

Denoting $\pi_{K+1}^{(1)}(x) \equiv 0$, we can thus rewrite the right-hand-side of (21) as

$$\int_0^{z(1)} \sum_{i=1}^{K}\left[\left(\pi_i^{(1)}(x) - \pi_{i+1}^{(1)}(x)\right)\sum_{k=1}^{i} \Delta P^0(k;x)\right]dx. \tag{25}$$

Note that by the definition of the class of indices $\Pi^1(\zeta(1),...,\zeta(K))$, $\pi_i^{(1)}(x) - \pi_{i+1}^{(1)}(x) \le 0, \forall i = 1,...,K$, with strict inequality for some values of i over some range of $x \in [0,\zeta(i)]$ (for the indices to be non-degenerate). Hence, if $\sum_{k=1}^{i} \Delta P^0(k;\zeta) > 0, \forall \zeta \in [0, z(i)]$ and $\forall i = 1,...,K$, then it must be that (21) holds for all

$P(z(1),...,z(K)) \in \Pi^1(z(1),...,z(K))$. But this also implies that

$\Delta P(\zeta(1),...,\zeta(K)) > 0, \forall P(\zeta(1),...,\zeta(K)) \in \Pi^1(\zeta(1),...,\zeta(K))$, and

$\forall \zeta(k) \in [0, z(k)], k = 1,..., K.$ This proves the sufficiency of condition (8).

For the necessity part, it suffices to consider any particular case in which $\sum_{k=1}^{i} \Delta P^0(k;\zeta) \le 0,$ for some $\zeta \in [z^-(i), z^+(i)]$ and for some value of i. Consider then a poverty index that belongs to $\Pi^1(z(1),...,z(K))$ such that $\pi_k^{(1)}(x) - \pi_{k+1}^{(1)}(x) = 0$ everywhere, except for $k = i$ and over that range $\zeta \in [z^-(i), z^+(i)]$ over which $\sum_{k=1}^{i} \Delta P^0(k;\zeta) \le 0$. Then, by (25), $\Delta P \le 0$ for that index, which therefore shows the necessity of condition (8).

Proof of Theorem 2.

Consider again equation (19):

$$\begin{aligned}\Delta P \quad &= \sum_{k=1}^{K} \pi_k(z(k)) \Delta P^0(k; z(k)) \\ &- \sum_{k=1}^{K} \int_0^{z(k)} \pi_k^{(1)}(x) \Delta P^0(k; x) dx.\end{aligned} \tag{26}$$

The second line of condition (10) guarantees the non-negativity of the second line of (27), as shown before in the proof of Theorem 1. Denoting again $\pi_{K+1}^{(1)}(y) \equiv 0$, rewrite the first term on the right-hand side of (26) as:

$$\sum_{i=1}^{K} \left[\left(\pi_i(z(i)) - \pi_{i+1}(z(i+1)) \right) \sum_{k=1}^{i} \Delta P^0(k; z(i)) \right]. \tag{27}$$

Note that by the definition of the class of indices $\tilde{\Pi}^1(z(1),...,z(K))$,

$\pi_i(z(i)) - \pi_{i+1}(z(i+1)) \le 0, \forall i = 1,..., K$. Hence, if $\sum_{k=1}^{i} \Delta P^0(k; z(k)) > 0, \forall i = 1,..., K$, then the first part on the right-hand-side of (26) is also non-negative. The combination of the first and of the second parts of condition (10) guarantees that $\Delta P > 0$.

The necessity of condition (10) proceeds as for the proof of Theorem 1.

Proof of Theorem 3.

For each distribution, the existence of the appropriate population moments of order 1 lets us apply the law of large numbers to (11), thus showing that $\sum_{k=1}^{j} \hat{P}_L^{\alpha}(k; z(k))$ is a consistent estimator of $\sum_{k=1}^{j} P_L^{\alpha}(k; z(k))$. Given also the existence of the population moments of order 2, the central limit theorem shows that the estimator in (11) is root-*N* consistent and asymptotically normal with asymptotic covariance matrix given by (12).

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Figure 1. Domain for dominance testing

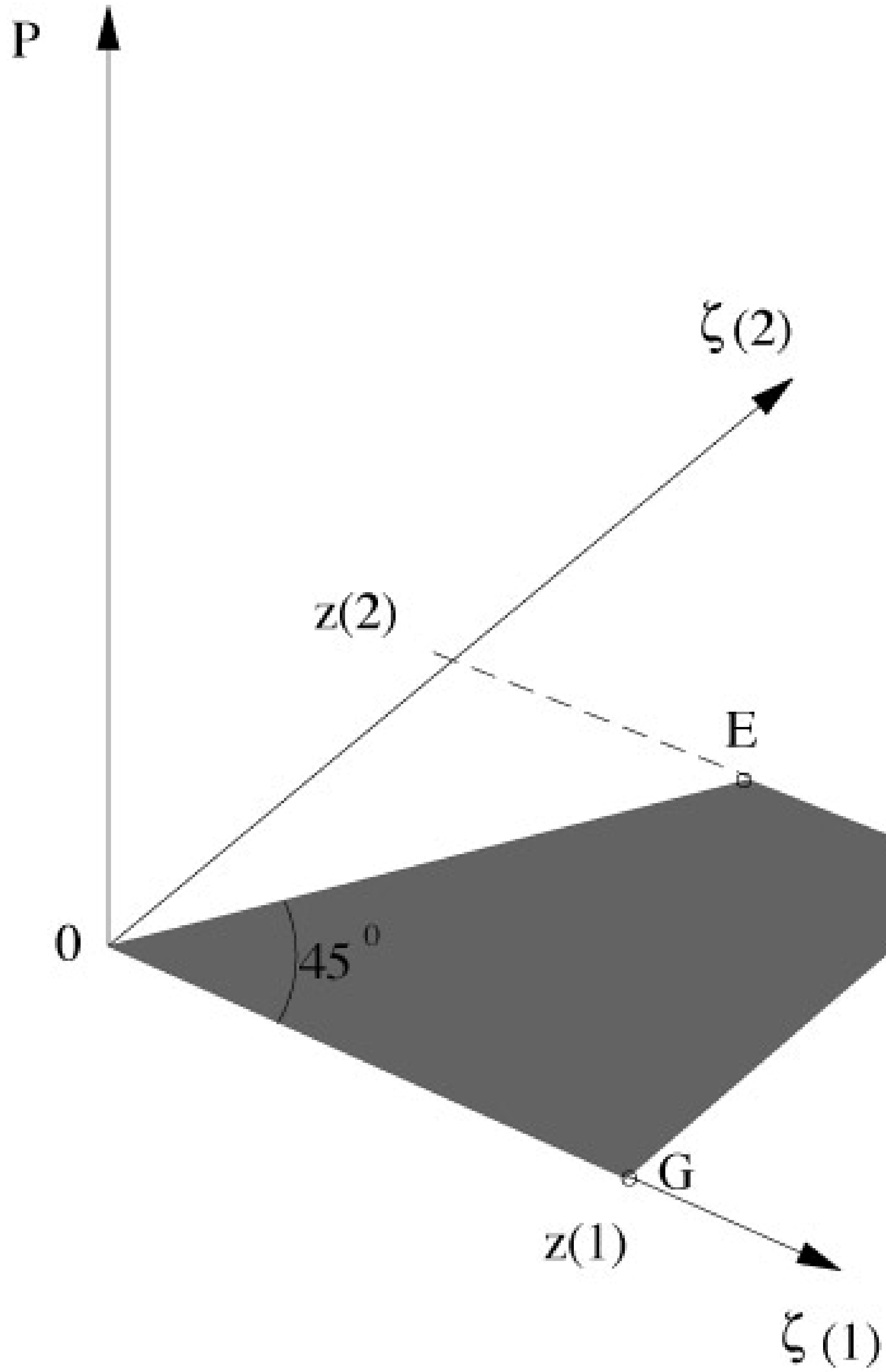

Table 1. ***t*** **-statistics for differences between household income and size with child allowances vs. with social security (Romania)**

	Household size					
Household income	**6 or more**	**5 or more**	**4 or more**	**3 or more**	**2 or more**	**1 or more**
43,527	-34.18	-29.58	-23.42	-11.92	20.13	31.76
63,000	-41.75	-36.41	-30.21	-16.64	17.37	26.29
78,602	-48.06	-42.47	-35.26	-20.58	13.33	18.16
94,671	-52.78	-46.22	-38.96	-23.84	8.02	10.49
112,460	-55.70	-50.17	-42.04	-27.11	2.93	4.43
129,630	-57.68	-50.80	-42.56	-26.96	-0.19	1.10
147,520	-58.83	-51.50	-44.16	-28.86	-5.58	-4.08
165,140	-59.24	-52.32	-45.61	-30.11	-10.49	-8.79
181,640	-53.43	-49.80	-45.08	-30.20	-13.23	-11.33
201,020	-48.50	-44.31	-42.38	-29.71	-15.46	-13.32
221,510	-45.84	-42.16	-41.40	-29.37	-16.76	-14.44
243,760	-41.58	-39.64	-38.43	-28.22	-17.54	-15.34
268,200	-38.29	-36.61	-35.07	-27.00	-17.57	-15.48
293,170	-35.85	-33.74	-30.97	-23.61	-16.06	-14.15
327,070	-29.78	-28.32	-27.53	-20.95	-14.29	-12.65
367,370	-20.72	-21.82	-22.94	-17.49	-12.31	-10.78
423,820	-15.39	-16.53	-16.28	-12.36	-7.98	-6.72
521,180	-17.38	-13.10	-11.24	-7.34	-4.40	-3.75
756,240	-8.35	-8.37	-7.48	-5.87	-2.24	-1.41

Notes: Values in column 1 are each 5th percentile in the distribution of expenditures.
A positive *t*-statistic in columns 2 through 7 indicates greater poverty for income plus child allowances than for income plus social security pensions.
Source: Authors' calculation from the Romania Integrated Household Survey, 1994.

Table 2. ***t*** **-statistics for differences between per capita expenditures for literate and illiterate Peruvians, 1985 minus 1994**

Expenditure p.c.	Illiterate	All
915	-15.25	-17.41
1,166	-15.86	-24.62
1,436	-16.41	-29.66
1,663	-14.57	-33.91
1,883	-10.89	-36.70
2,122	-7.94	-39.51
2,350	-5.29	-41.82
2,634	-0.82	-43.15
2,926	5.79	-45.07
3,232	9.67	-48.03
3,575	15.31	-47.82
3,974	20.72	-49.01
4,401	30.47	-50.86
4,909	42.42	-49.67
5,543	57.29	-49.30
6,324	80.30	-46.59
7,364	116.09	-41.50
9,187	162.57	-37.62
12,842	287.58	-27.85

Notes: Values in column 1 are each 5th percentile in the distribution of deflated expenditures per capita, measured in nuevo soles per capita per month.
A positive t-statistic in columns 2 and 3 indicates greater poverty in 1985 than in 1994.

Source: *Encuesta Nacional de Hogares sobre Medición de Niveles de Vida*, 1985 and 1994.

Table 3 - *t*-statistics for differences between household expenditures per capita for rural and urban residents in Ecuador, 1998 vs. 1999

Expenditure p.c.	Rural	All
35,401	2.47	1.02
48,323	4.03	2.50
56,506	3.91	2.87
66,447	5.00	1.60
75,234	6.86	1.15
84,304	6.45	-0.03
92,862	6.16	0.20
101,830	5.48	-1.17
111,330	7.40	-1.23
123,420	7.95	-1.39
135,690	7.57	-1.46
149,940	7.69	-1.33
167,140	7.05	-2.11
186,530	6.80	-1.70
208,850	8.60	-0.72
242,520	10.35	-0.98
288,660	11.83	-1.30
365,330	14.62	-0.76
541,410	19.60	-1.12

Notes: Values in column 1 are each 5th percentile in the distribution of real expenditures per capita, measured in sucres per capita per month.
A positive t-statistic in columns 2 and 3 indicates greater poverty in 1998 than in 1999.

Source: *Encuesta sobre Condiciones de Vida*, 1998 and 1999.

Table 4 - *t*-statistics for differences between household income per adult equivalent by education level and whether or not health restricts normal daily activities, Britain, 1994 and 2002

Income p.a.e.	Limited < O levels	Not Limited O levels	Limited A levels	Not Limited > A levels	Limited >= 1st degree	Not Limited < O levels	Limited O levels	Not Limited A levels	Limited > A levels	Not Limited >= 1st degree
6,260	11.20	14.00	13.20	12.50	7.31	14.50	13.10	12.80	12.80	10.10
7,460	30.30	31.60	28.80	27.10	20.70	28.10	25.90	23.80	22.70	18.10
8,580	35.60	34.90	30.20	28.90	23.00	31.10	28.90	26.20	24.80	19.80
9,670	41.10	38.10	30.90	29.20	22.20	32.70	30.40	27.00	25.40	19.90
10,600	39.90	35.00	27.30	24.50	19.00	32.40	30.60	27.30	25.90	20.10
11,700	36.90	33.20	26.00	22.90	17.50	33.20	31.40	27.80	26.30	20.10
12,700	32.90	28.50	20.40	17.80	11.90	32.40	31.30	27.50	26.20	19.10
13,700	33.20	28.70	20.40	17.30	10.70	33.20	32.70	30.00	28.30	20.00
14,700	32.20	28.80	18.40	14.10	6.01	31.60	31.60	29.50	27.70	18.40
15,800	32.50	28.80	17.20	13.60	4.14	31.80	31.90	30.40	28.30	18.20
16,800	31.30	27.00	14.00	10.50	2.07	33.70	34.60	33.00	30.20	18.10
18,000	24.30	20.20	7.05	4.08	-4.37	32.90	34.30	32.80	29.90	16.70
19,300	26.30	21.30	4.56	1.08	-8.70	33.80	36.20	33.20	30.40	15.40
20,800	32.60	25.10	1.98	-2.28	-13.80	34.30	37.50	34.70	30.50	13.40
22,500	40.80	30.20	0.11	-4.33	-18.30	37.00	39.90	37.10	32.00	11.60
24,700	41.10	25.60	-7.82	-11.30	-28.20	35.70	40.80	39.00	32.60	8.95
27,400	45.30	29.10	-11.90	-16.10	-38.80	39.90	46.00	43.80	36.80	7.24
31,700	54.10	34.10	-17.80	-27.00	-54.50	45.50	54.10	52.60	43.10	4.28
39,100	66.00	45.50	-19.90	-33.20	-77.00	63.30	74.70	77.00	61.50	2.02

Table 4 (continued)

Income p.a.e.	< O levels Limited	< O levels Not Limited	O levels Limited	O levels Not Limited	A levels Limited	A levels Not Limited	> A levels Limited	> A levels Not Limited	>= 1st degree Limited	>= 1st degree Not Limited
6,590	11.20	17.20	17.70	15.50	15.20	14.40	14.20	14.00	12.80	10.10
8,070	30.30	32.40	32.70	29.30	28.60	25.60	25.30	24.00	22.70	18.10
9,420	35.60	36.50	36.20	32.60	31.40	27.80	27.60	25.90	24.80	19.80
10,600	41.10	40.60	39.70	35.50	33.60	29.00	28.70	26.80	25.40	19.90
11,700	39.90	41.90	40.20	36.40	34.20	29.50	28.90	27.10	25.90	20.10
12,800	36.90	43.10	41.60	37.60	35.40	30.20	29.50	27.70	26.30	20.10
13,600	32.90	44.50	42.50	38.80	36.10	30.30	29.60	27.90	26.20	19.10
14,700	33.20	46.60	44.50	41.20	38.30	33.40	32.60	30.40	28.30	20.00
15,800	32.20	47.50	45.60	42.20	38.60	33.90	32.70	30.30	27.70	18.40
16,900	32.50	49.80	47.60	43.70	39.70	35.20	34.10	31.40	28.30	18.20
18,100	31.30	54.20	51.50	48.00	43.20	38.20	37.00	33.30	30.20	18.10
19,400	24.30	56.70	53.40	49.60	44.20	38.70	37.40	33.50	29.90	16.70
20,800	26.30	62.80	58.90	55.00	48.20	40.00	38.60	34.60	30.40	15.40
22,300	32.60	69.50	65.10	60.60	52.00	42.90	41.20	35.50	30.50	13.40
24,300	40.80	80.50	75.30	67.90	57.70	46.90	45.20	38.00	32.00	11.60
26,600	41.10	87.10	80.40	74.60	62.40	50.90	49.10	40.10	32.60	8.95
29,300	45.30	103.00	95.90	87.90	73.20	58.80	56.50	46.40	36.80	7.24
33,800	54.10	132.00	122.00	110.00	91.30	73.30	70.30	56.20	43.10	4.28
41,500	66.00	164.00	154.00	142.00	121.00	106.00	101.00	79.00	61.50	2.02

Notes: Values in column 1 are each 5th percentile in the distribution of income per adult equivalent.
A positive t-statistic indicates greater poverty for 1991 than for 1999.
The equivalence scale is due to McClements (1977).

Endnotes

[1] The best-known of these is the Human Development Index (UNDP 1990), which also involves arbitrary aggregations across individuals. A recent WIDER conference on Inequality, Poverty, and Human Well-Being examined many more such indices. See http://www.wider.unu.edu/conference/conference-2003-2/conference2003-2.htm.

[2] Bourguignon and Chakravarty (2002) and Duclos, Sahn, and Younger (2006a, 2006b) have more detailed discussions of this assumption for the continuous case.

[3]Condition (8) is the discrete analogue of condition (8) in Duclos, Sahn, and Younger (2006b) for the continuous case. Here, the joint distribution is bunched in one dimension to a finite set of mass points.

[4]See Atkinson (2002) for a discussion of this.

[5]More discussion of this can be found in Duclos and Makdissi (2005).

[6] There are very few households with more than six people in the sample.

[7] The comparison is for those fifteen years old or older.

[8] The happiness question is 'Have you recently been feeling reasonably happy, all things considered?' with possible responses: more so than usual, about the same as usual, less so than usual, much less than usual. Wave 9 did not include this question, so we did not use that wave in our comparisons.

[9] These are the only interesting comparisons. If the univariate comparison fails to reject the null, the multivariate comparisons must also fail to do so, so the correspondence is perfect in that case.

[10] In the latter two cases, the correspondence is considerably higher if we lower the maximum reasonable poverty line to the 40th percentile.