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Berardi, Michele
University of Manchester

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Beyond the static money multiplier
In search of a dynamic theory of money

Michele Berardi
University of Manchester

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Abstract

In this paper, we analyze the process of money creation in a credit economy. We start from the consideration that the traditional money multiplier is a poor description of this process and present an alternative and dynamic approach that takes into account the heterogeneity of agents in the economy and their interactions. We show that this heterogeneity can account for the instability of the multiplier and that it can make the system path-dependent. By using concepts and techniques borrowed from network theory and statistical mechanics, we then try to shed some light on the actual process by which money is endogenously created in an economy.

Money, Money multiplier, Network theory, Statistical mechanics.

1 Introduction

Though we all live in a monetary economy where credit money plays a fundamental role, the process through which money is created in the economy is largely neglected by modern macroeconomic theory. A common approach maintains that the process starts with an exogenous increase in the monetary base made by the central bank, and that this, through a fixed multiplier, gives rise to a proportional increase in the amount of money in the economy. The multiplier is usually taken as constant in this process, at least on short time scales, and most importantly, independent from the money creation process itself. The result is essentially a static, aggregate theory, with very poor behavioural microfoundations, that completely neglects the *process* through which money is generated in an economy.

As a consequence of this representation, money is taken to be exogenously determined and its quantity explained through changes in the monetary base magnified proportionally by the fixed multiplier. Unfortunately, this theory is not able to provide any insights about the process that generates money in a credit economy, apart from assuming that changes in the monetary stock are originated by central bank interventions, and proportional to them. It misses completely the idea that money is created and destroyed endogenously, through the interactions of the many actors (mainly banks, households and firms) participating in the monetary and credit markets.

An important drawback of the traditional theory, as represented by the static multiplier,[1] is that it does not allow for a proper theory of endogenous money creation that many economists think would be necessary.[2] Presenting the whole process of money creation as a pure deterministic response of the monetary stock to an exogenous change in the monetary base is deeply misleading. In the words of Goodhart (1984), the standard multiplier theory of money creation is "... such an incomplete way of describing the process of the determination of the stock of money that it amounts to misinstruction".

In modern economies, where the central bank wants to control the interest rate, money is necessarily endogenous to the system as the policymaker must provide enough monetary base so that the equilibrium interest rate on the market is the desired one. Though this fact is often recognized even in standard macroeconomic textbooks, then an exogenous and fixed multiplier is still considered to be the link between the monetary base and the amount of money available in the economy. It is completely neglected the fact that the ratio between these two aggregates can vary according to the behaviour of the system and must not be assumed fixed *a priori*.[3]

In this work we take a narrow perspective regarding the creation of money in a credit economy and focus our attention only on its *process*. In particular, our analysis should help explain the short term variability in the amount of money, for the part that can be imputed to the volatility in the multiplier.[4] Our work does not try to analyze the determinants of the behaviour of banks and households but puts emphasis on the heterogeneity of the actors involved in the monetary and credit market and tries to provide a better understanding of the dynamics of the process of money creation, stripped down to its mechanics and deprived of any behavioural content. Still, we believe that this approach can provide useful insights and help build a more comprehensive theory of money in a credit economy.

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[1] We dub the traditional multiplier as static, to emphasize its lack of attention to the dynamics involved in the process of money creation.

[2] Post-Keynesian economists, in particular, have long argued about the need of an endogenous theory of money, one that recognizes the fact that the financial system is able to generate monetary liabilities in response to real sector's needs. But also on the other side of the macroconomics spectrum (see, e.g., Kydland and Prescott, 1990) there is support for the view of endogenous money.

[3] These issues are somewhat related to the debate between verticalists and horizontalists that was popular in the 1970s. For a detailed exposition and analysis of the two positions, see Moore (1988).

[4] Moore (1988) shows that variations in the monetary base can explain only about 40% of the variability in the M1 aggregate on a monthly base, while this proportion raises to about 65% with quarterly data and to 90% over horizons of one year. Over short time horizons, therefore, a lot of variability in M1 is left unexplained by the standard theory.

2 Models of money creation

2.1 The static multiplier

Standard macroeconomic theory explains the amount of money available in an economy starting from the monetary base (H), which is composed of currency held by the public (CU) and reserves held by the banking sector (R).[5] The money multiplier is simply derived as the ratio between the monetary base provided by the central bank and a monetary aggregate (M), composed of currency (CU) and deposits (D):[6]

$$H = CU + R \tag{1}$$
$$M = CU + D, \tag{2}$$

from which, dividing everything by D and defining cu = CU/D, re = R/D, it follows that

$$m = \frac{M}{H} = \frac{1 + cu}{cu + re}. \tag{3}$$

The standard money multiplier represents therefore an aggregate characteristic of the economy, with essentially no behavioural content. Nevertheless, the ratios re and cu are often taken to represent agents' individual preferences, assumed constant and homogeneous. The whole approach is essentially static and neglects completely the process through which money is created.

2.2 A dynamic version of the multiplier

We present here a different way to obtain the multiplier: instead of using ratios of aggregate quantities, we consider the dynamic process that unfolds through monetary and credit transactions. We start with an increase in monetary base, in the form of an increase in funds available to the public. Suppose we are in a situation where households have exactly the proportion of cash/deposits (cu) that they wish, and banks have the proportion of reserve/deposit (re) that they want to hold. Therefore households will split the additional funds they receive between deposits and cash, in the proportion cu. Banks in turn will keep a fraction (re) of the additional deposits they receive as reserves and use the rest to extend new loans (L) to the public, who will split them again into cash and deposits, and the process continues.[7]

From the definitions above, we get that at each step i of the process:[8]

[5] It is customary not to distinguish between households and firms, and consider them as an aggregate entity (the public). We will follow here this simplification as well.

[6] In this work we will refer to a generic monetary aggregate M, which could be understood as M1 in US or Europe.

[7] The following restrictions apply: $0 \leq re \leq 1$, $cu \geq 0$.

[8] Here CU_i is the additional amount of cash available at time i with respect to time i-1, not the total cash available at time i. The same for the other variables here used.

$$CU_i = \frac{cu}{1+cu} L_i \quad (4)$$

$$D_i = \frac{1}{1+cu} L_i \quad (5)$$

$$L_{i+1} = (1-re) D_i \quad (6)$$

which lead to

$$M_i = \left(\frac{1-re}{1+cu}\right)^i M_0 \quad (7)$$

and therefore

$$m = \frac{\sum_{i=0}^{\infty} M_i}{M_0} = \sum_{i=0}^{\infty} \left(\frac{1-re}{1+cu}\right)^i = \frac{1}{1-\frac{1-re}{1+cu}} = \frac{1+cu}{cu+re}, \quad (8)$$

where M_0 is the original increase in monetary base, in the currency component. This alternative derivation of the static multiplier shows its microfoundations when the behavioural parameters cu and re are constant and homogeneous. But once we introduce heterogeneity in those individual parameters, the system changes significantly its behaviour.

To better analyze the importance of heterogeneity, the aggregate description for the process (4)-(8) must be replaced with a distributed one, where each single bank and household are represented and explicitly considered. This implies that in general a closed form solution for the multiplier will not exist, and computer simulations will be used to gain insights into the behaviour of the system.

2.3 Introducing heterogeneity

In a heterogeneous setting, each bank has its own reserve/deposit ratio and each household its own currency/deposit ratio. If we assume that each agent (bank or household) in linked to only one agent of the other type, so that the flow of money is never split into different streams, it is then possible to express the multiplier (for a unitary increase in the monetary base) as

$$m_d = 1 + \sum_{i=1}^{\infty} \left(\prod_{j=1}^{i} \frac{1-re_j}{1+cu_j} \right), \quad (9)$$

where the index i refers to a “round” in the process (i.e., household i deposits money in bank i; bank i extends a loan to household $i+1$, who will deposit money into bank i+1). A bank or household can be activated in more than one round during the process, as the index does not identify an agent uniquely, only the action of an agent.

We can see that if $\mathrm{re}_z = 1$, or $\mathrm{cu}_z = \infty$, for some generic z, then the terms in (9) for $i \geq z$ are all zero, because agent z acts as an absorbing state in the

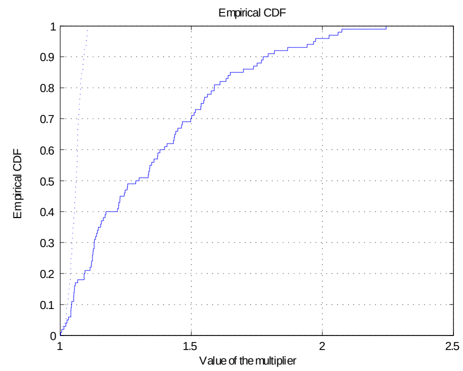

Figure 1: Empirical cdf of average (dotted line) and dynamic (solid line) multipliers.

system and interrupts the multiplicative process of money creation. This implies that heterogeneity is important, and can not be simply averaged out. In fact, the value of the multiplier computed with (9) is different from the one we would obtain by using averages of all the reserve/deposit and currency/deposit ratios:

$$m_a = \frac{1 + \frac{1}{n}\sum_{h=1}^{n} cu_h}{\frac{1}{n}\sum_{h=1}^{n} cu_h + \frac{1}{k}\sum_{b=1}^{k} re_b}, \tag{10}$$

where k is the number of banks and n the number of households in the economy. Here indexes represent individual banks and households. Under homogeneity ($\forall$b, re$_b$ = re; $\forall$h, cu$_h$ = cu), (8) = (9) = (10). But with heterogeneous agents, this is not in general true, as it can be seen from a simple experiment. We create 100 different economies, each characterized by 1000 banks and 1000 households with randomly drawn individual ratios and derive the empirical cumulative distribution function (cdf) for the dynamic multiplier computed using (9) and for the one computed using averages as in (10). As can be seen in Fig. 1, the average multiplier m_a varies over a restricted range of values, as much of the variability is washed out by the averaging.

When the behavioural parameters are heterogeneous, the value of the dynamic multiplier depends, among other things, on the position where the process starts (for an exogenous intervention, where the CB "drops" the monetary base). The system is in fact path dependent and the order by which agents take part in the process becomes relevant. This is confirmed by our simulations when we compute the dynamic multiplier 1000 times for the same economy, each time changing the order by which agents are activated. Results show that the multiplier can vary over a wide range of values, *for the same economy*, depending on the order by which agents take part in the process.[9]

The standard way to represent the multiplier is therefore misleading, as in that representation the coefficients re and cu are not really behavioural parameters, as it may appear by their definitions, but simply ratios of aggregate quantities.

Note then that equation (9) is valid only when all the money remains in a unique stream and never gets split into different branches. If we allow each agent (bank or household) to be connected with more than one counterpart, we then need to keep track of all the streams of money that get generated, and the analytic formula becomes intractable.

2.4 Monetary network

We therefore build an artificial economy and try to gain some insights into the process of money creation by means of simulations. We abstract from any considerations involving the real side of the economy and only model the structure of monetary and credit transactions, considering different possible network topologies at the base of the system and their impact on the multiplicative process.

The network composed of banks and households is a bipartite network, where edges exist only between nodes belonging to different classes. In the process that we describe, each node (bank or household) receives some money from its incoming links, keeps part of it (as reserves or cash holdings) and passes along the rest through the outgoing edges. We can uniquely define each node by its ratio of reserve/deposit or currency/deposit, and build two matrices, one for the links from banks to households (where the edges of this network represent the flow of credit that banks extend to households), and one for the links from households to banks (where the edges represent the flow of deposits from households to banks).

We will consider three different network topologies and try to understand how they impact on the size distribution of the multiplier: a random graph, a regular graph and star graph. Other topologies of course could be considered (e.g., small-world á la Watts and Strogatz or scale-free á la Barabasi), but we restrict for now to these more common structures.

We start by considering a random network, where banks and households are assigned random behavioural ratios (cu_h and re_b)[10] and are randomly linked to each other. The system is composed of 5 banks and 100 households, with each bank receiving money from and extending loans to a random number of households. We simulate 100 economies and compute for each the average and the dynamic multiplier. In Fig. 2 we show the distributions (as histograms) of the two measures. We can see that the variability in the dynamic multiplier is much higher than in the average one, where the part due to heterogeneity gets washed out.

[9] In one of the experiments that we ran, the dynamic multiplier showed a distribution of values in the interval 1-2.5. Of course m_a was instead constant (and equal to 1.06).

[10] With re_b and $\frac{cu_h}{1+cu_h}$ uniformly distributed between 0 and 1.

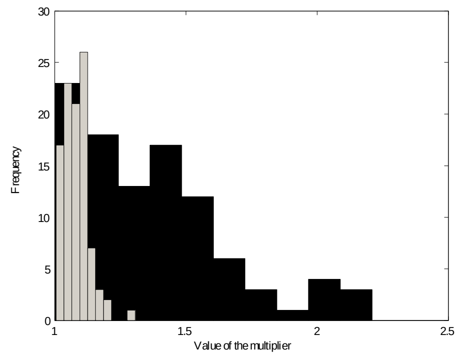

Figure 2: Histograms of average (grey) and dynamic (black) multiplier with a random network of monetary transactions.

We then consider one single economy with a fixed set of parameters (thus fixing the average multiplier) and simulate 1000 different processes of money creation by randomly inject money on different sites. Fig. 3 shows the empirical cdf of the resulting dynamic multiplier: as it can be seen, the monetary system is path dependent and the final size of the money multiplier depends, among other things, on the position where money is injected into the economy. This means that the multiplier could change even when behavioural ratios for banks' reserves and households' currency remain fixed, an aspect that is completely neglected by the standard theory.

The next topology that we consider is a regular structure, where banks and households are laid down on a bi-dimensional lattice. Each bank is linked to four households, and each household to four banks. Each link is bi-directional, for deposits and loans (though some can have zero weight). We simulate the process of money creation on a lattice composed by 18 banks and 18 households, and show the distribution (histograms) for the average and the dynamic multipliers in Fig. 4. Compared with the case of a random graph, the variability in the dynamic multiplier is now reduced, as the presence of absorbing states does not disconnect entire regions of the system.

To conclude, we look at the extreme case of a star topology, where all households are linked to one single bank which receives deposits and extends loans to them. We simulate the process of money creation on a structure of this type with 100 households and one bank, and show the results in Fig.5. As we can see, the variability in the dynamic multiplier increases again now, because the presence of only one bank makes the whole system dependent on the behaviour of that bank.

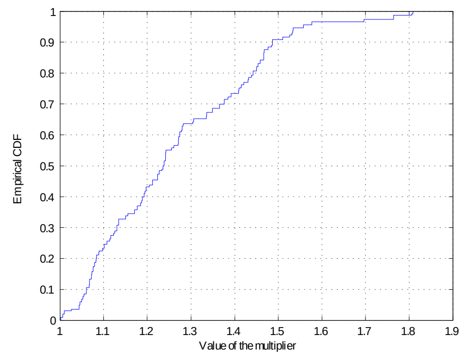

Figure 3: Empirical cdf of the dynamic multiplier in a random economy with different paths of propagation.

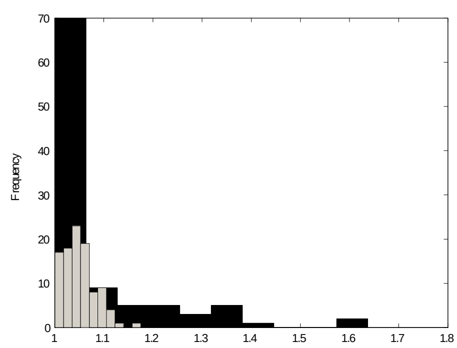

Figure 4: Histograms of average (grey) and dynamic (black) multiplier with a regular network of monetary transactions.

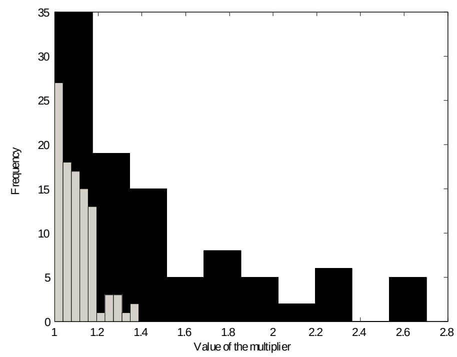

Figure 5: Histograms of average (grey) and dynamic (black) multiplier with a star network of monetary transactions.

2.5 Monetary cascades and the sandpile model: an attempt at perspective

We try to suggest here an alternative but somewhat complementary interpretation of the process through which money is created in a credit economy, viewing it as an avalanche that propagates across the economy through monetary and credit transactions.

An interesting phenomenon that has been studied in physics is that of self-organised criticality (SOC), where a system drives itself on the edge of a critical state, right between stability and instability.[11] The classical example is that of the sandpile model developed by Bak et al (1987).

We think that this interpretation could provide useful insights for the explanation of the process of money creation in a credit economy. If the system operates right on the edge of a critical state, the introduction of new monetary base could have a final effect on the monetary aggregate that is unpredictable and can vary across a wide range of values.

Suppose that banks try to keep an average reserve/deposit ratio in line with legislation requirements, but take actions and extend new loans only when their individual reserve/deposit ratio reaches a fixed threshold; and that households try to keep an average currency/deposit ratio according to their individual needs/preferences, but take actions and deposit funds into a bank only when their ratio reaches a certain upper bound. So that when banks extend new loans and households make new deposits, they will do it for an amount that exceeds the marginal availability of funds beyond their own threshold.[12] In this way, as

[11] For a review of the concept, see Turcotte (1999).

[12] Technically, these behaviours prevent the system from reaching a stationary state of equilibrium, where all agents have just the desired reserve and currency ratios and simply pass along any additional funds they receive.

time passes, the system could drive itself towards a critical state, on the edge between stability and instability.

Once in this critical state, for each increase in monetary base we could see a final increase in the monetary aggregate M of any size. At times, the process of money creation would end soon, when money reaches an agent that is below its threshold and therefore hoards the additional money he receives; but at times the process could spread out and generate an avalanche, if many nodes involved reach their own threshold and pass along money to others.

This interpretation could provide a good explanation of the variability observed in the multiplier, and if the analogy with the sandpile model is correct, the size of monetary cascades should be distributed according to a power-law.[13] [14]

We now turn to data to see if a power law characterizes the size of the multiplier. In this respect, there are a number of issues to keep in mind. First, the central bank does not "drop" monetary base constantly and regularly in fixed amounts in the economy; secondly, the temporal scale is such that different avalanches may overlap, as there is no guarantee that the time between one central bank intervention and the next is enough for the system to fully respond and adjust to the first intervention; third, we have data available at regularly intervals (bi-weekly or monthly), but an avalanche of money may take different lengths of time to reach its full extent at different times; finally, we detrend the multiplier, as its trend is likely to derive from long-run changes in behaviours that we do not try to explain here and want to abstract from.[15] Having all these limitations in mind, we test for the presence of a power law in the size distribution of the multiplier.[16] Fig. 7 (in a log-log scale) shows the best fit of the estimated Pareto distribution for the right tail (dashed-dotted line) with the vertical dotted line showing the point from which the Pareto distribution has been identified. Out of the 568 observations available (bi-weekly data for US, February 1984–November 2005),[17] only 157 were identified to be distributed according to a power-law, and the estimated coefficient is 2.55.

According to this test, the evidence for a Pareto distribution in the data for

[13] A feature that is crucial in the sandpile model is the dispersion of the sand involved in the avalanche. In the monetary system, of course, there is no dispersion of money, so that the "pile" of money keeps growing in absolute size, but the relative size with respect to deposits, that is what matters here, remains constant.

[14] While earlier studies of the sandpile model were done using a regular lattice to represent the interactions among sand grains, Goh et al (2003) study the avalanche dynamics of the sandpile model on a scale-free network with heterogeneous thresholds and find that the avalanche size distribution still follows a power law.

[15] The series is detrended using the Hodrick-Prescott filter.

[16] We apply a procedure that first tests for the presence of a Pareto distribution in the data, identifies a region that with a 95% confidence interval follows such a distribution and then applies bootstrapping techniques to find the Hill estimator for the coefficient of the distribution.

[17] We also applied the same procedure to a constructed series for the multiplier, obtained as the ratio between the monetary aggregate M1 and the monetary base, using US monthly data for the period 01/1959-08/2006, with the resulting multiplier then detrended using the HP filter. We obtained similar results in terms of the proportion of data appearing to be Pareto distributed, though the estimate for the coefficient was lower, about 2.25.

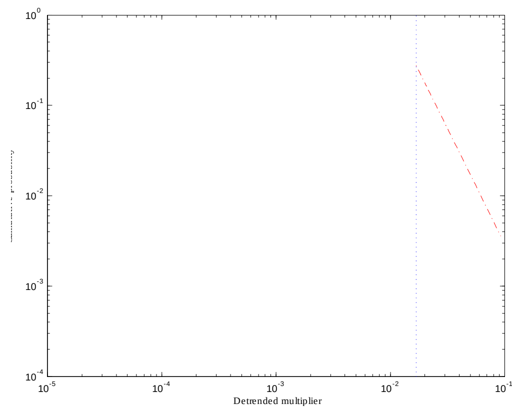

Figure 6: Empirical disribution of the detrended money multiplier. The dashed-dotted line indicates the best fit for a power-law.

the multiplier seems rather weak so far, though we believe that a more careful analysis is required. In particular, it has to be identified the measure that better captures the avalanche style behaviour of the system, since the multiplier, suffering from the limitations described above, might be a poor indicator of such a behaviour.

3 Conclusions

This paper is a tentative contribution in the field of monetary economics and offers a representation of the money creation process in a credit economy that is alternative to the standard one provided by the static multiplier. We have focused our attention on the mechanics of the process, and we have shown the importance of the role played by the heterogeneity of the actors involved and their interactions. An important feature that has been shown here is the path dependence of the system, which implies that position and timing of CB's interventions on the money market will have an impact on their effectiveness. Finally, the structure of the monetary system has been shown to affect the variability of the multiplier and therefore the process of money creation. It is therefore important that some effort be devoted in order to understand the empirical structure of monetary and credit transactions.

The approach we have adopted in this work, we believe, is well suited for supporting a theory of endogenous money, as it does not imply a deterministic and causal relationship between the monetary base and the quantity of money. Emphasis is placed on the monetary and credit transactions, and though we did not try to link these transactions to the economic activity, the two aspects are clearly interrelated.

Our analysis is just an initial step and much road has still to be covered in order to develop a theory that can properly account for the process of money creation, but we hope that our work will stimulate others to join the ride.

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