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# A theory of sharecropping: the role of price behavior and imperfect competition

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## Abstract

This paper proposes a theory of sharecropping on the basis of price behavior in agriculture and imperfectly competitive nature of rural product markets. We consider a contractual setting between one landlord and one tenant with seasonal variation of price, where the tenant receives a low price for his output while the landlord can sell his output at a higher price, and show the superiority of sharecropping over fixed rental contracts. Then we consider more general interlinked contracts to show that there are multiple optimal interlinked contracts. Finally, proposing an equilibrium refinement that incorporates imperfect competition in the rural product market, it is shown that the unique contract that is robust to this refinement results in sharecropping.

**Keywords:** Sharecropping, price variation, imperfect competition, interlinkage, the  $\varepsilon$ -agent

**JEL Classification:** D02, D23, J43, O12, O17, Q15

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# 1 Introduction

Over the years, sharecropping has remained a widely prevalent, and perhaps the most controversial, tenurial system in agriculture. While writings on this institution can be traced back earlier, modern economic theories of sharecropping are centered around its criticism of Alfred Marshall (1920). The essence of the Marshallian critique is that sharecropping is an inefficient system. Under a share contract, the tenant-cultivator pays the landlord a stipulated proportion of the output. This leads to suboptimal application of inputs: even though there is gain in surplus from employing additional inputs, it does not pay the tenant to do so since he keeps only a fraction of the marginal product. In contrast, the tenant has the incentive to maximize the surplus under a fixed rental contract where he keeps the entire output and pays only a fixed rent to the landlord. The landlord, who usually has the bargaining power, can then extract the entire additional surplus by appropriately determining the rent. Thus, apart from being inefficient, sharecropping is also apparently suboptimal for the landlord. The wide prevalence of this institution has therefore remained a puzzle and several theories have been put forward to explain its existence. In particular, it has been argued that sharecropping can be explained by the trade-off between risk-sharing and incentive provision (Stiglitz, 1974; Newbery, 1977; Newbery and Stiglitz, 1979), informational asymmetry (Hallagan, 1978; Allen, 1982; Muthoo, 1998), moral hazard (Reid, 1976; Eswaran and Kotwal, 1985; Laffont and Matoussi, 1995; Ghatak and Pandey, 2000) or limited liability (Shetty, 1988; Basu, 1992; Sengupta, 1997; Ray and Singh, 2001).<sup>1</sup>

This paper is motivated by an aspect of agriculture that has not received much attention in the theoretical literature of sharecropping. Given that the core of the contention here is sharing of the agricultural product between the contracting parties, a natural question is: does the price behavior in agriculture influence the resulting tenancy contracts? This question is usually sidestepped in the existing literature as it is always implicitly assumed that price is competitively determined in agriculture and the contracting parties take the same price as given. While price in agriculture is often regarded to be competitive, it is also well-known that it does exhibit variation—seasonal, spatial or both. The seasonal variation has a broad pattern: the price is the lowest right after the harvest, then it rises and finally reaches its peak just before the next harvest. In less-developed agrarian economies, a landlord can take advantage of price variations by ‘hoarding’ (i.e., storing the output for a few months and sell it when the price is high) or transporting the produce to a location that offers a better price (e.g., from the village to the town market). A tenant-farmer, on the other hand, has to sell the output at low price immediately after the harvest due to various reasons such as not having enough buffer wealth to pay for essential commodities for immediate consumption, urgency for clearing his debts or the lack of necessary storage and transportation facilities. Generally speaking, one can say that a landlord has better access to the market and as a result the price that he receives for the produce is higher than the price received by the tenant. We argue that that this innate difference of the two parties can explain sharecropping even in the absence of factors such as risk aversion or

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<sup>1</sup>See also Cheung (1969), Bardhan and Srinivasan (1971), Bardhan (1984), Binswanger and Rosenzweig (1984), Hayami and Otsuka (1993) and recent papers of Ray (1999) and Roy and Serfes (2001). The literature of sharecropping is enormous and we do not attempt to summarize it here. We refer to Singh (1989) for a comprehensive survey.

informational asymmetry. The underlying intuition is simple. A fixed rental contract leaves the entire output with the tenant. Since the tenant receives a low price for the output, the revenue and consequently the rent to the landlord is low. The landlord may prefer a share contract because it enables him to take advantage of price variation by allowing him to keep a proportion of the output.

We formalize the intuition above in a landlord-tenant model with seasonal variation of price, where the tenant receives a low price for his output while the landlord can sell his output at a higher price, and show the superiority of sharecropping over fixed rental contracts. We subsequently consider more general contracts where the landlord specifies the shares for both parties, a rental transfer and a price at which he offers to buy the tenant's share of output. These are *interlinked* contracts that enable the landlord to interact with the tenant in two markets: land (through share and rent) and product (through his offer of price).<sup>2</sup> We show that the landlord has multiple optimal interlinked contracts. The intuition behind the multiplicity is simple. The tenant's incentive is determined by (i) his share and (ii) the price he receives for his share, so the optimal level of incentive can be sustained by multiple combinations of these two variables. To resolve this multiplicity, we appeal to the nature of the rural product markets and propose an equilibrium refinement that takes into consideration the fact that although the landlord has monopoly power over the land he owns, this is not necessarily the case in the product market, where he could face competition from other entities (e.g., traders, intermediaries) who might be interested in trading with the tenant. In fact, a rural product market closely resembles what one might call a situation of *imperfect competition*, along the lines suggested by Stiglitz (1989: 25):

“There is competition; inequality of wealth itself does not imply that landlords can exercise their power unbridled. On the other hand, markets in which there are a large number of participants...need not be highly competitive...transaction costs and, in particular, information costs imply that some markets are far better described by models of imperfect competition than perfect competition.”

The refinement criterion we propose incorporates imperfect competition as follows. Suppose there is a small but positive probability that a third agent emerges in the end of production to compete with the landlord as a potential buyer for the tenant's share of output. Then the question is, out of the multiple contracts obtained before, which ones will the landlord choose when he anticipates such a possibility? We show that the unique contract that is robust to this refinement criterion is a sharecropping contract. To see the intuition, observe that incentive provision to the tenant demands that a relatively high share for the landlord has to be compensated by a relatively high price at which the landlord offers to buy the tenant's share. The possibility of a third agent as another potential buyer enables the landlord to have a high share of output for himself without incurring the loss of buying the tenant's output at high price. We show that competition in the product market generates a Pareto improving subset of share contracts out of the multiple contracts obtained before. It is optimal for the landlord to choose that specific contract in this subset where his own share

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<sup>2</sup>The theoretical literature on interlinkage has mainly focused on credit contracts, considering (i) land-credit linkage (e.g., Bhaduri, 1973; Braverman & Stiglitz, 1982; Mitra, 1982; Basu, 1983; Bardhan, 1984; Gangopadhyay & Sengupta, 1986; Ray & Sengupta, 1989; Banerji, 1995; Basu et al., 2000) and (ii) product-credit linkage (e.g., Gangopadhyay & Sengupta, 1987; Bell & Srinivasan, 1989). See also Chapter 14 of Basu (1998) and Chapter 9 of Bardhan and Udry (1999).

is maximum. The upshot is that the unique robust contract results in sharecropping where the tenant's share is high enough to ensure that the third agent trades with the tenant and just breaks even.

While the specific aim of this paper is to provide a theoretical analysis of sharecropping, the paper relates to some of the more general themes of development economics. Rural economies of poor countries are subject to volatilities of different kinds such as in weather, prices and wages that severely effect the people living there [see, e.g., Bliss and Stern (1982), Rosenzweig and Binswanger (1993), Rosenzweig and Wolpin (1993), Jayachandran (2006)]. It is also important to note that the effect of these volatilities are different across agents. In their study of Indian villages for 1975-84, Rosenzweig and Binswanger (1993) find evidence that facing possible income volatilities, wealthier households engaged in significantly more risky production activities and on the average obtained a much higher return than poorer households. Studying the effect of productivity shocks on agricultural workers using wage data from India for 1956-87, Jayachandran (2006) finds support for her theoretical prediction that such shocks cause higher wage fluctuations for poor workers that make them worse off, but in contrast, rich landowners are better off since negative productivity shocks are compensated by lower wages. Thus, our basic premise that landlords can take advantage of price fluctuations while the tenant-farmers cannot, is part of a much broader phenomenon of agrarian economies that shows that rich and poor agents respond differently to volatilities.

Our theoretical conclusion that tenancy contracts could be endogenous to the nature of price fluctuations is consistent with the well observed aspect of rural economies that institutions and contractual forms often emerge to cope with the volatilities mentioned above. Specifically, various formal and informal rural insurance systems in this regard have been extensively studied in a large literature [see, e.g., Platteau and Abraham (1987), Udry (1990), Townsend (1994), Ravallion and Chaudhuri (1997), Munshi and Rosenzweig (2007)]. It should be mentioned that interlinked contracts in our model play the role of implicitly providing insurance to the tenant-farmer. When the landlord specifies a price to buy the tenant's share of output, the tenant is insured against two contingencies: (i) if the immediate post-harvest price is even lower than expected, he is assured of a higher price from the landlord and (ii) the already standing offer from the landlord improves the tenant's position as a seller vis-à-vis another potential buyer (e.g., a third agent of the kind described before). The landlord needs to provide such insurance to make sure that the tenant's incentive stays at its optimal level. If there are other entities (e.g., the government or a big outside firm that does not have a stake at small village-level competition) that can reliably assure the tenant of a high price, the price differential between the two contracting parties will be reduced and the resulting tenancy contracts will also evolve. This is similar in spirit to the conclusion of Jayachandran (2006) who finds evidence that access to financial services such as banks reduces wage fluctuations for agricultural workers.

The paper is organized as follows. In Section 2 we present a few case studies to provide support for our premises that landlords store output to take advantage of price variation and rural product markets are imperfectly competitive in nature. In Section 3 we present the benchmark model and derive the optimal contracts in the class of tenancy contracts. In Section 4 we study the model with interlinked contracts and obtain multiple optimal contracts. Section 5 constructs the equilibrium refinement by modeling imperfect competition in the product market. We conclude in Section 6. Most proofs are relegated to the Appendix.

## 2 Empirical evidence

### 2.1 Price variation and product storage by landlords

One basic premise of our proposed theory is that landlords store the agricultural output in order to take advantage of price fluctuations. Some evidence of this is given below.

The first evidence is taken from Myers (1984) who studies four villages in north China for the period 1890-1949. The village Ssu pei ch'ai, located at Luan-che'eng county, was one of the villages covered in this study. Two main tenurial systems of this village were *shao-chung-ti* (a form of share tenancy) and *pao-chung-ti* (a form of fixed rent). Cotton was the main marketed crop and the large market located in the county seat of Luan-che'eng was the major outlet for landlords and traders. The immediate post-harvest market there is described as follows (ibid: 79):

“On the supply side, absentee landlords also sold cotton to the market, but their percentage of total supply marketed was very small. They naturally preferred to sell long after the harvest when cotton prices resumed their rise...Cotton prices were high during the winter months and low during the summer period...landlords retained their cotton and sold during the early spring...”

The source of the second evidence is Baker (1984) who studies three sub-regional economies of the south Indian region of Tamilnad from 1880-1955. Landlords having customary rights in land were called *mirasidars* in this region. The mirasidars usually leased their lands using a specific form of sharecropping called *waram*. The description of the paddy market there makes it clear that not only did landlords store the produce, but also their crop-sharing decision was influenced by such marketing activities:

“...[T]here was a distinct pattern to the annual marketing cycle...The first stage came immediately after the main harvest in the months from January to April. This was the time when cultivators had to pay their government revenue and service their debts. Many cultivators, particularly the smaller ones, were obliged to unload their produce immediately. Perhaps half of the entire crop was sold at this point and naturally enough the prices were low...Substantial mirasidars...would procure stocks of rice in order to store against an expected price rise. They accumulated stocks through crop-shares they received from waram tenants; the mirasidars who were really interested in the market would have provided the seed and the cattle for the waram tenant in order that they might take away a very substantial crop-share (p.239)...in the final stage of the marketing year...mirasidars...would release stocks on the eve of the next harvest when prices reached their peak. (p.241)”

The next evidence is from Bolivia. In the pre-land reform Bolivia during 1920-50, different forms of land tenurial systems such as sharecropping and *colonato* (a kind of labor-rent system) existed [see, e.g., Mendelberg (1965: 46), Jackson (1994: 162-163), Assies (2006: 580)]. In his study of pre-reform agriculture markets of the north highlands of Bolivia, Clark (1968) finds that most landlords there were absentees, who lived in the city of La Paz that was also the major marketing center of the highlands area. It is clearly documented that landlords engaged in storing and marketing of the produce in a fairly organized manner:

“At the time of the harvest the landlord visited the firm to make sure that he received the agricultural produce that was due him (p.157)...In the last seven to ten years before 1952 many landlords began to use their own or rented trucks to bring produce to La Paz...Once in La Paz agricultural produce was stored and subsequently sold in the store or *aljería* owned by the landlord...The person who worked in the store was called an *aljiri*...The specific obligations of an *aljiri* were to go and tell the retailers in the city markets who had done business with the landlord previously of the arrival of products from the farm...If the buyer was interested the *aljiri* would call the landlord...to come and make a sale...These sales were usually made in large quantities to established retailers in the La Paz markets...when sales were difficult to make in large quantities at a good price, the landlord would sell directly to consumers in small quantities (p. 158).”

The last evidence is taken from Sharma (1997) whose study is based on fieldworks of a village in the Indian state of Uttar Pradesh, conducted in the early nineties. Sharecropping was the dominant form of tenurial system in this village. It is reported that the rich landlords there stored output to take advantage of price variation (ibid: 270-271):

“Two of the rich peasant households in the village each own a large diesel-operated machine for wheat-threshing and winnowing and rice-shelling which enables them...to process and bag much of their grains in the village (eliminating the middlemen and the cost of transport to the mills), and to sell it directly to grain merchants in Aligarh and Delhi for a much higher return. The imposing brick-made *godown* (grain-storage barn) in the centre of the village...not only acts as a storage bin, but also allows the rich land-owners periodically to withhold grain from the market until prices improve.”

## 2.2 Imperfect competition in rural product markets

Now we present some evidence that supports our premise that rural product markets are often imperfectly competitive in nature.

The first evidence is taken from Rudra (1992). In his village-level survey in two states of India, the rural product market is described as follows:

“Our investigations in more than 200 villages in West Bengal and Bihar indicate the following ranking among different categories of traders in terms of prices paid by them as purchasers of grains.

1. village retail shops.
2. big farmers acting as traders.
3. village wholesalers.
4. travelling traders (or itinerant merchants) and other village level traders.
5. *hats* (that is, non-permanent markets centres functioning on a number of days per month or per week), market wholesalers, and rice mills.

The lowest prices are paid by the village retail shops and the highest prices are paid by the rice mills, market wholesalers, and *hats*.” (Rudra: 53-54)

The second evidence is taken from Hayami et al (1999) who surveys rice marketing in Laguna province, Philippines for 1995-1997:

“To whom did farmers sell their marketable surplus of paddy? A part was shipped directly to rice mills, and the rest was assembled by small traders called ‘collectors’ for procurement by rice mills...Two types of collectors can be distinguished. One type is what we call a ‘commission agent’ who identifies farmers willing to sell paddy, makes contract of sale with them at a price approved by the miller, and then let the miller make the hauling of paddy and the payment to farmers. For this task she receives a certain commission. Another type is an ‘independent trader’ who purchases paddy by her own risk and finance, and assemble them into a bulk for shipment for sale to the mill...75% of paddy procured from farmers in East Laguna Village was assembled by collectors, of which about two-thirds were handled by those of the independent trader-type...None of the private dealers had a disproportionately large share...Therefore, they are bound to compete each other strongly...” (pp. 81-83)

The studies above show that price variation, product storage by landlords and imperfect competition in the product market are commonly observed features in agriculture. Given that, it is plausible that they may play a role in determining tenurial institutions.

### 3 The benchmark model

Consider a small village consisting of one landlord and many potential tenants. The landlord owns a piece of land that can grow only one crop. The landlord leases out his land to a tenant to carry out production.

- *The Production Process:* There is only one input of production: labor ( $\ell$ ). In the land leased out by the landlord, the production function is  $f(\ell)$ , where  $f(0) = 0$ . We assume that  $f$  is twice continuously differentiable with  $f'(\ell) > 0$  and  $f''(\ell) < 0$  for  $\ell > 0$ , i.e.,  $f$  is strictly increasing and strictly concave. Moreover,  $\lim_{\ell \downarrow 0} f'(\ell) = \infty$  and  $\lim_{\ell \rightarrow \infty} f'(\ell) = 0$ . The cost of  $\ell$  units of labor is  $w(\ell)$ , where  $w(0) = 0$ . It is assumed that  $w$  is twice continuously differentiable, strictly increasing and convex, i.e.,  $w'(\ell) > 0$  and  $w''(\ell) \geq 0$  for  $\ell > 0$ .

- *Price Behavior:* The market price of the product exhibits variation that could be seasonal, spatial, or a combination of both. There are two seasons 1 and 2. Season 1 corresponds to the immediate post-harvest period, while season 2 corresponds to a future period sometime after the harvest, but before the next harvest. The price is  $p_1$  in season 1 and  $p_2$  in season 2, where  $p_2 > p_1 > 0$ . To capture seasonal as well as spatial variation in a simple way, we also assume that  $p_1$  is the low price that a seller of the village receives for the produce at the local level (e.g., at village retail shops), while  $p_2$  corresponds to the high price that is obtained at a different location (e.g., at town markets or rice mills).<sup>3</sup> It is assumed that these prices are determined by economy-wide demand-supply conditions. The price in season 1 is low due to large aggregate supply immediately after the harvest and in season 2, price rises due to a fall in the aggregate supply. We normalize  $p_1 = 1$  and denote  $p_2 \equiv p > 1$ .

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<sup>3</sup>This is the simplest way to model price variation that has both seasonal and spatial components. Alternatively, one can model spatial variation *within* a season by assuming price variation across locations in the same season. This will generate more levels of prices which complicates our analysis, but does not give any additional insights.



The landlord has storage/transportation facilities with finite capacity  $\bar{Q} > 0$ . He can store any output  $q \leq \bar{Q}$  in season 1 and sell it later in season 2 at price  $p > 1$ . The tenant, on the other hand, sells any output at his disposal in season 1 at low price 1. There are two main reasons behind this difference in the selling behavior of the two parties: (i) the tenant lacks storage and transportation facilities and (ii) unlike the landlord, the tenant does not have enough buffer wealth, so he has to sell his output in season 1 to pay for essential commodities for immediate consumption.

We assume that the output held by any agent of the village is very small compared to the aggregate supply. So in any season, an agent of the village can sell his output at the existing market price of that season without affecting the price. This assumption is reasonable for season 1 as the aggregate supply immediately after the harvest is large. Regarding season 2, it can be seen from the empirical evidence given in the last section that landlords who seek to take advantage of price fluctuations usually sell their produce in large town markets (e.g., markets in the county seat of Luan-che'eng or in cities like La Paz or Aligarh). It is assumed that although the aggregate supply falls in season 2, still it is very large in a town market and a landlord, being a small player in such a market, does not affect the price.

- *The Set of Contracts:* The landlord leases out his land to the tenant through *linear tenancy contracts*. A typical contract is a pair  $(\alpha, \beta)$  where  $\alpha \in [0, 1]$  is the share of the output of the tenant and  $\beta \in \mathbb{R}$  is the fixed rental transfer from the tenant to the landlord. If the tenant works under the contract  $(\alpha, \beta)$  and produces output  $Q$ : (i) he keeps  $\alpha Q$  and leaves the rest  $(1 - \alpha)Q$  with the landlord and (ii) makes the rental transfer  $\beta$  to the landlord. The mode of the rental transfer  $\beta$  could be either cash or kind. Accordingly, the tenancy contracts can be classified into two sets:

$$\mathbb{C} = \{(\alpha, \beta) | \alpha \in [0, 1], \beta \in \mathbb{R}, \beta \text{ is a transfer in cash}\} \text{ and}$$

$$\mathbb{K} = \{(\alpha, \beta) | \alpha \in [0, 1], \beta \in \mathbb{R}, \beta \text{ is a transfer in kind}\} \quad (1)$$

A contract  $(\alpha, \beta) \in \mathbb{C} \cup \mathbb{K}$  is a *share contract* if the landlord and the tenant share the output, i.e., if  $0 < \alpha < 1$ . A share contract is a *share plus cash rental* contract if  $\beta$  is a cash transfer and it is a *share plus kind rental* contract if  $\beta$  is a transfer in kind. If  $0 < \alpha < 1$  and  $\beta = 0$ , we have a *pure share* contract.

A contract  $(\alpha, \beta)$  is a *fixed rental* contract if the tenant keeps the entire output and pays only the fixed rent  $\beta$  to the landlord, i.e., if  $\alpha = 1$  and  $\beta > 0$ . A contract with  $\alpha = 1$  and  $\beta > 0$  is a *cash rental* contract if  $\beta$  is a cash transfer and a *kind rental* contract if  $\beta$  is a transfer in kind.

- *The Strategic Interaction:* The strategic interaction between the landlord and the tenant is modeled as a game  $G$  in extensive form that has the following stages. In the first stage, the landlord offers a contract  $(\alpha, \beta)$  to the tenant, which could be either a contract in cash, or a contract in kind. In the second stage, the tenant either rejects the contract in which case the game terminates with both parties get their reservation payoffs, or he accepts in which case the game moves to the third stage where the tenant chooses the amount of labor for carrying out production and output is realized. In the fourth stage, the tenant pays the landlord in accordance with the contract. Finally payoffs are realized and the game terminates. The solution concept is the notion of Subgame Perfect Equilibrium (SPE).

### 3.1 The tenant's problem

Consider a contract  $(\alpha, \beta) \in \mathbb{C} \cup \mathbb{K}$ . When the tenant chooses labor input  $\ell$ , the output is  $f(\ell)$  and his cost is  $w(\ell)$ .

If  $(\alpha, \beta) \in \mathbb{C}$ , the output at the tenant's disposal is his share  $\alpha f(\ell)$ . He sells this output in season 1 at price 1 to obtain the revenue  $\alpha f(\ell)$ . After making the cash transfer  $\beta$  to the landlord, his payoff is  $\alpha f(\ell) - w(\ell) - \beta$ .

If  $(\alpha, \beta) \in \mathbb{K}$ , then after paying the landlord the kind transfer  $\beta$ , the output at the tenant's disposal is  $\alpha f(\ell) - \beta$ . Selling this output in season 1 at price 1 yields the revenue  $\alpha f(\ell) - \beta$  for the tenant, so his payoff is  $\alpha f(\ell) - \beta - w(\ell)$ .

Therefore, regardless of whether the mode of the rental payment is in cash or kind, the tenant's payoff under the contract  $(\alpha, \beta)$  when he employs  $\ell$  units of labor is  $\alpha f(\ell) - w(\ell) - \beta$  and  $\beta$  being a constant, his problem reduces to choosing  $\ell$  to maximize

$$\phi^\alpha(\ell) := \alpha f(\ell) - w(\ell) \quad (2)$$

Since  $f'' < 0$  and  $w'' \geq 0$ , by (2),  $\phi^\alpha(\ell)$  is strictly concave in  $\ell$  for  $\alpha > 0$ . For  $\alpha \geq 0$ , let  $\ell(\alpha)$  be the unique maximizer of  $\phi^\alpha(\ell)$ . Clearly  $\ell(0) = 0$ . For  $\alpha > 0$ ,  $\ell(\alpha)$  is obtained from the first-order condition  $\alpha f'(\ell) = w'(\ell)$ . Hence

$$\ell(0) = 0 \text{ and } \alpha f'(\ell(\alpha)) = w'(\ell(\alpha)) \text{ for } \alpha > 0. \quad (3)$$

Now define the composite functions  $F, \Phi : \mathbb{R}_+ \rightarrow \mathbb{R}_+$  as

$$F(\alpha) := f(\ell(\alpha)) \text{ and } \Phi(\alpha) := \phi^\alpha(\ell(\alpha)) = \alpha F(\alpha) - w(\ell(\alpha)). \quad (4)$$

The following lemma, which characterizes the solution to the tenant's problem under different classes of contracts, follows from (2)-(4) and by the envelope theorem.

**Lemma 1** (i) *Let  $(\alpha, \beta) \in \mathbb{C} \cup \mathbb{K}$ . Regardless of whether the mode of the rental transfer is cash or kind, the tenant chooses labor input  $\ell(\alpha)$ , the output produced is  $F(\alpha)$  and the tenant obtains the payoff  $\Phi(\alpha) - \beta$ .*

(ii)  $\ell(0) = 0$ ,  $F(0) = 0$  and  $\Phi(0) = 0$ .

(iii)  $\ell'(\alpha) > 0$ ,  $F'(\alpha) > 0$  and  $\Phi'(\alpha) = F(\alpha) > 0$  for  $\alpha > 0$ .

Having characterized the solution of the tenant's problem under any contract offered by the landlord, we are in a position to solve the landlord's problem of determining his optimal contracts. Before solving that problem, we qualify two more aspects of our model. First, we impose more structure to the model by making an additional assumption and second, we specify the reservation payoff of the tenant in terms of the function  $\Phi(\cdot)$ .

#### 3.1.1 Assumption: Concavity of $F(x)$

Consider the function  $F(\alpha) = f(\ell(\alpha))$ . As  $F'(\alpha) = f'(\ell(\alpha))\ell'(\alpha)$ , we have

$$F''(\alpha) = f''(\ell(\alpha))[\ell'(\alpha)]^2 + f'(\ell(\alpha))\ell''(\alpha).$$

Since  $f'' < 0$ , the first term of the expression above is negative, but the sign of the second term is ambiguous. We make the following additional assumption, which is a sufficient condition to ensure that the landlord's problem will have a unique solution.

**Assumption A1** The functions  $f(\ell)$  and  $w(\ell)$  are such that  $F(\alpha)$  is concave, i.e.,  $F''(\alpha) \leq 0$ .

Assumption A1 holds for  $f(\ell) = \ell^a$  and  $w(\ell) = k\ell^b$  for  $k > 0$ ,  $a < 1 \leq b$  and  $a/b \leq 1/2$ .

### 3.1.2 Reservation payoff of the tenant

We observe that if a tenant cultivates the land leased out by the landlord without any contractual obligation and sells the output at price 1, under his optimal choice of labor, the profit that he obtains is  $\Phi(1)$  (take  $\alpha = 1$ ,  $\beta = 0$  in Lemma 1). We assume that the reservation payoff  $\underline{\Phi} > 0$  of the tenant is less than  $\Phi(1)$ . Since  $\Phi(0) = 0$  and  $\Phi(\cdot)$  is strictly increasing (Lemma 1), there is a constant  $\alpha_0 \in (0, 1)$  such that  $\underline{\Phi} = \Phi(\alpha_0)$ . For the rest of the paper we assume that the reservation payoff of the tenant is  $\Phi(\alpha_0)$  for some  $\alpha_0 \in (0, 1)$ .

## 3.2 The landlord's problem

### 3.2.1 Revenue from the product market

Let  $\Psi^p(Q)$  be the revenue of the landlord from the product market when he has output  $Q$  at his disposal. Recall that the landlord has a storage capacity  $\bar{Q} > 0$ . If  $Q \leq \bar{Q}$ , he can store the entire output in season 1 and sell it in season 2 at price  $p > 1$  to obtain the revenue  $pQ$ .<sup>4</sup> If  $Q > \bar{Q}$ , he can store output  $\bar{Q}$  across seasons to obtain the revenue  $p\bar{Q}$ . The remaining output  $Q - \bar{Q}$  cannot be stored and has to be sold in season 1 at price 1 that yields revenue  $Q - \bar{Q}$ . Therefore for  $Q > \bar{Q}$ , his revenue is  $p\bar{Q} + Q - \bar{Q} = Q + (p - 1)\bar{Q}$ . So we have

$$\Psi^p(Q) = \begin{cases} pQ & \text{if } Q \leq \bar{Q}, \\ Q + (p - 1)\bar{Q} & \text{if } Q > \bar{Q} \end{cases} \quad (5)$$

The next lemma summarizes the properties of  $\Psi^p(Q)$ .

**Lemma 2** (i)  $\Psi^p(Q)$  is continuous and strictly increasing in  $Q$ . It is differentiable at all  $Q$  except at  $Q = \bar{Q}$ .

(ii) Let  $p > 1$  and  $Q > 0$ . Then  $Q < \Psi^p(Q) \leq pQ$ .

(iii) (Decreasing returns) Let  $Q_1, Q_2 > 0$ . Then  $\Psi^p(Q_1 + Q_2) = \Psi^p(Q_1) + \Psi^p(Q_2)$  if  $Q_1 + Q_2 \leq \bar{Q}$  and  $\Psi^p(Q_1 + Q_2) < \Psi^p(Q_1) + \Psi^p(Q_2)$  if  $Q_1 + Q_2 > \bar{Q}$ .

**Proof** See the Appendix. ■

### 3.2.2 Optimal tenancy contracts

Consider a tenancy contract  $(\alpha, \beta) \in \mathbb{C} \cup \mathbb{K}$ . By Lemma 1, under this contract, the tenant's optimal choice of labor is  $\ell(\alpha)$  that yields the output  $F(\alpha)$  and the tenant's payoff is  $\Phi(\alpha) - \beta$ . Since his reservation payoff is  $\Phi(\alpha_0)$ , he will accept the contract  $(\alpha, \beta)$  only if

$$\Phi(\alpha) - \beta \geq \Phi(\alpha_0). \quad (6)$$

Define  $H : [0, 1] \rightarrow R_+$  as

$$H(\alpha) := (1 - \alpha)F(\alpha). \quad (7)$$

The landlord's share of the output under the contract  $(\alpha, \beta)$  is  $H(\alpha)$ . Lemma 3 lists the properties of the function  $H(\cdot)$ . The proof is standard and hence omitted. Assumption A1 [concavity of  $F(\cdot)$ ] is used to prove the strict concavity of  $H(\cdot)$ .

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<sup>4</sup>If  $Q < \bar{Q}$ , the landlord potentially has the option of buying the output  $\bar{Q} - Q$  in season 1 from alternative sources such as other farmers. However, gathering output from these sources involves searching and traveling, and the opportunity cost of such activities is likely to be high for a landlord (particularly, an absentee one). For this reason, we rule out this option for the landlord.

**Lemma 3** (i)  $H(0) = H(1) = 0$ .

(ii)  $H'(\alpha) = (1 - \alpha)F'(\alpha) - F(\alpha)$  and  $H(\alpha)$  is strictly concave for  $\alpha \in [0, 1]$ .

(iii) There is a constant  $\tilde{\alpha} \in (0, 1)$  such that  $H'(\alpha) \geq 0 \Leftrightarrow \alpha \leq \tilde{\alpha}$ , i.e.,  $\tilde{\alpha}$  is the unique maximizer of  $H(\alpha)$  over  $\alpha \in [0, 1]$ .

Observe that under a contract  $(\alpha, \beta)$ , the output produced is  $F(\alpha)$  and by the monotonicity of  $F(\cdot)$  (Lemma 1), the output produced can be at most  $F(1)$ . By Lemma 3(iii), the landlord's share of output under a contract can be at most  $H(\tilde{\alpha}) < F(\tilde{\alpha}) < F(1)$ . The following assumption imposes a lower and an upper bound on the storage capacity of the landlord. It is not a crucial assumption, but it simplifies our analysis.

**Assumption A2**  $H(\tilde{\alpha}) < \bar{Q} < F(1)$

**Pure share contracts:** It will be useful to first separately consider the set of pure share contracts  $\mathbb{S} = \{(\alpha, 0) | \alpha \in (0, 1)\}$ . Under a pure share contract  $(\alpha, 0)$ , the landlord and the tenant share the output ( $0 < \alpha < 1$ ) without any rental transfer ( $\beta = 0$ ). Then the output at the landlord's disposal is  $H(\alpha)$  and his payoff is simply the revenue from this output. Taking  $Q = H(\alpha)$  in (5), his payoff is

$$\Pi_S^p(\alpha) = \Psi^p(H(\alpha)) \quad (8)$$

Taking  $\beta = 0$  in (6), the tenant's participation constraint is  $\Phi(\alpha) \geq \Phi(\alpha_0)$ , which reduces to  $\alpha \geq \alpha_0$  (by the monotonicity of  $\Phi(\cdot)$ , Lemma 1). Therefore under the set of contracts  $\mathbb{S}$ , the landlord's problem is to choose  $\alpha \in [\alpha_0, 1)$  to maximize (8). Since  $\Psi^p(\cdot)$  is monotonic, by (8), the landlord's problem reduces to choosing  $\alpha \in [\alpha_0, 1)$  to maximize  $H(\alpha)$ . The following result is then immediate from Lemma 3(iii).

**Proposition 1** For any  $p \geq 1$ , the landlord has a unique optimal pure share contract  $(\alpha_S, 0)$  that has the following properties.

- (i) If  $\alpha_0 < \tilde{\alpha}$ , then  $\alpha_S = \tilde{\alpha}$ . The landlord obtains  $\Psi^p(H(\tilde{\alpha}))$  and the tenant obtains  $\Phi(\tilde{\alpha}) > \Phi(\alpha_0)$  (more than his reservation payoff).
- (ii) If  $\alpha_0 \geq \tilde{\alpha}$ , then  $\alpha_S = \alpha_0$ . The landlord obtains  $\Psi^p(H(\alpha_0))$  and the tenant obtains his reservation payoff  $\Phi(\alpha_0)$ .

**Tenancy contracts with rental transfers:** Now we consider general tenancy contracts  $(\alpha, \beta) \in \mathbb{C} \cup \mathbb{K}$ .

- $(\alpha, \beta) \in \mathbb{C}$  (Rental transfer in cash): For this contract, the output at the landlord's disposal is  $H(\alpha)$ . Taking  $Q = H(\alpha)$  in (5), his revenue from this output is  $\Psi^p(H(\alpha))$ . As he obtains the cash transfer  $\beta$  from the tenant, his payoff is

$$\Pi_C^p(\alpha, \beta) = \Psi^p(H(\alpha)) + \beta \quad (9)$$

Under the set of contracts  $\mathbb{C}$ , the landlord's problem is to choose  $(\alpha, \beta)$  to maximize (9) subject to (6). For any  $\alpha$ , the optimal  $\beta$  for the landlord is

$$\bar{\beta}(\alpha) = \Phi(\alpha) - \Phi(\alpha_0) \quad (10)$$

that binds the tenant's participation constraint (6). So the landlord's problem reduces to choosing  $\alpha \in [0, 1]$  to maximize

$$\Pi_C^p(\alpha) = \Psi^p(H(\alpha)) + \Phi(\alpha) - \Phi(\alpha_0) \quad (11)$$

•  $(\alpha, \beta) \in \mathbb{K}$  (Rental transfer in kind): For this contract, the landlord obtains the rent  $\beta$  in kind from the tenant in addition to his share of the output. Therefore the output at his disposal is  $H(\alpha) + \beta$ . Taking  $Q = H(\alpha) + \beta$  in (5), his payoff is

$$\Pi_K^p(\alpha, \beta) = \Psi^p(H(\alpha) + \beta) \quad (12)$$

Under the set of contracts  $\mathbb{K}$ , the landlord's problem is to choose  $(\alpha, \beta)$  to maximize (12) subject to (6). Since  $\Psi^p(\cdot)$  is monotonic, it follows from (12) that for any  $\alpha$ , the optimal  $\beta$  for the landlord is again  $\bar{\beta}(\alpha)$ , given in (10). Hence the landlord's problem reduces to choosing  $\alpha \in [0, 1]$  to maximize

$$\Pi_K^p(\alpha) = \Psi^p(H(\alpha) + \Phi(\alpha) - \Phi(\alpha_0)) \quad (13)$$

**Optimal contracts in the absence of price variation:** When there is no price variation (i.e.  $p = 1$ ), by (5),  $\Psi^p(Q) = \Psi^1(Q) = Q$ . In this case, whether the rental transfer  $\beta$  is collected in cash or kind makes no difference to the landlord's payoff and by (11) and (13), his problem under both sets  $\mathbb{C}$  and  $\mathbb{K}$  is to choose  $\alpha \in [0, 1]$  to maximize  $s(\alpha) - \Phi(\alpha_0)$  where

$$s(\alpha) = H(\alpha) + \Phi(\alpha) \quad (14)$$

When  $p = 1$ ,  $s(\alpha)$  is the total surplus (sum of payoffs of the landlord and the tenant) at  $\alpha$  and the landlord chooses  $\alpha$  to maximize this surplus by leaving the tenant his reservation payoff  $\Phi(\alpha_0)$ . Since  $\Phi'(\alpha) = F(\alpha)$  (Lemma 1) and  $H'(\alpha) = (1 - \alpha)F'(\alpha) - F(\alpha)$  (Lemma 3), by (14) it follows that  $s'(\alpha) = (1 - \alpha)F'(\alpha)$ , so  $s(\alpha)$  is strictly increasing for  $\alpha \in [0, 1]$  and

$$s(1) = \Phi(1) > s(\alpha) \text{ for } \alpha \in [0, 1) \quad (15)$$

Therefore, it is optimal for the landlord to choose  $\alpha = 1$  (a fixed rental contract). Taking  $\alpha = 1$  in (6), the rent is  $\bar{\beta}(1) = \Phi(1) - \Phi(\alpha_0)$ . The inequality in (15) shows that when  $p = 1$ , a share contract always results in a lower total surplus compared to a fixed rental contract. This presents the Marshallian inefficiency argument against sharecropping.

**Optimal contracts in the presence of price variation:** The optimal contracts when there is price variation ( $p > 1$ ) are presented in next proposition.

**Proposition 2** (I) *For any  $p > 1$ , the landlord has a unique optimal contract under the set  $\mathbb{K}$ . This contract is the fixed rental contract in kind  $(1, \bar{\beta}(1))$  where the landlord obtains  $\Pi_K^p(1) = \Psi^p(\bar{\beta}(1))$  and the tenant obtains  $\Phi(\alpha_0)$ .*

(II) *For any  $p > 1$ , the landlord has a unique optimal contract  $(\alpha_p^*, \bar{\beta}(\alpha_p^*))$  under the set  $\mathbb{C}$ . The optimal contract is a share contract  $(0 < \alpha_p^* < 1)$  with cash rent  $\bar{\beta}(\alpha_p^*) = \Phi(\alpha_p^*) - \Phi(\alpha_0)$ . It has the following properties.*

(a) *The landlord obtains  $\Pi_C^p(\alpha_p^*) = pH(\alpha_p^*) + \bar{\beta}(\alpha_p^*)$  and the tenant obtains  $\Phi(\alpha_0)$ .*

(b)  $\alpha_p^* > \tilde{\alpha}$  for all  $p > 1$ ,  $\alpha_p^*$  is strictly decreasing,  $\lim_{p \rightarrow \infty} \alpha_p^* = \tilde{\alpha}$  and  $\lim_{p \downarrow 1} \alpha_p^* = 1$ .

(III) *There are parameters of the model under which a share contract with rental transfer in cash is better for the landlord than the fixed rental contract in kind  $(1, \bar{\beta}(1))$ . Specifically, if  $H(\tilde{\alpha}) > \bar{\beta}(1)$ , then  $\exists \bar{p} > 1$  such that for all  $p > \bar{p}$ , the contract  $(\tilde{\alpha}, \bar{\beta}(\tilde{\alpha})) \in \mathbb{C}$  yields higher payoff for the landlord than the contract  $(1, \bar{\beta}(1)) \in \mathbb{K}$ .*

**Proof** See the Appendix. ■

Now we provide some intuition for the results of Proposition 2. When the landlord collects the rental transfer in kind, he can keep the entire produced output by leaving the tenant with his reservation payoff. For this reason, the presence of price variation does not affect the landlord's problem. Accordingly, under the set of contracts  $\mathbb{K}$ , it is optimal for the landlord to choose the fixed rental contract and the Marshallian inefficiency argument against share contracts stays intact (Prop 2(I)).

When rental transfers are obtained in cash, the Marshallian inefficiency argument does not have its unequivocal force in the presence of price variation. Since the tenant receives a low price for the product, a fixed rental contract in cash generates a low revenue. In contrast, a share contract allows the landlord to keep a proportion of output and enables him to take advantage of the price variation. This explains why a share contract is optimal under the set of contracts  $\mathbb{C}$  (Prop 2(II)).

The fraction  $\tilde{\alpha}$  (the tenant's share under the optimal unconstrained pure share contract) forms a lower bound of his share under the optimal contract in  $\mathbb{C}$  (Prop 2(II)(b)). To see the intuition, note from (11) that the landlord's payoff has two components: (i) the revenue from the product market  $\Psi^p(H(\alpha))$  and (ii) the cash rent  $\Phi(\alpha) - \Phi(\alpha_0)$ . The rent is increasing in  $\alpha$ . If  $\alpha < \tilde{\alpha}$ ,  $H(\alpha)$  is also increasing (Lemma 3), resulting both components of the payoff to move in the same direction. The landlord is then better off raising the tenant's share until it reaches  $\tilde{\alpha}$ . When  $\alpha \geq \tilde{\alpha}$ , there is a trade-off:  $H(\alpha)$  then starts falling, so a higher rent can be obtained only at the cost of a lower revenue from the product market. This trade-off is settled by the extent of price variation. As  $p$  increases, the revenue  $\Psi^p(H(\alpha))$  has a relatively higher weight in the landlord's payoff and he chooses a relatively small value of  $\alpha$  that raises  $H(\alpha)$ . In the two extremes, the optimal contract converges to two "pure" contractual forms: towards the unconstrained optimal pure share contract for large values of  $p$  and the fixed rental contract when  $p$  is close to 1 (Prop 2(II)(b)).

Part (III) of the proposition identifies situations under which a share contract with rental transfer in cash dominates the optimal fixed rental contract in kind. Observe that under the latter contract, the landlord has output  $\bar{\beta}(1) = \Phi(1) - \Phi(\alpha_0)$  at his disposal. For a contract  $(\alpha, \beta) \in \mathbb{C}$ , the output at the landlord's disposal is  $H(\alpha)$  which is bounded above by  $H(\tilde{\alpha})$  (Lemma 3). Therefore, if  $H(\tilde{\alpha}) > \bar{\beta}(1)$ , there are share contracts in the set  $\mathbb{C}$  that enable the landlord to keep a higher volume of output at his disposal compared to the fixed rental contract in kind. A high share for the landlord (i.e. low  $\alpha$ ) results in low cash rent, but for relatively large values of  $p$ , the gains from higher volume of output offsets the losses from lower rents. Consequently, for large values of  $p$ , there are share contracts in  $\mathbb{C}$  that dominate the fixed rental contract in kind.

It can be noted that if  $H(\tilde{\alpha}) > \bar{\beta}(1)$ , then for the share contract  $(\tilde{\alpha}, \bar{\beta}(\tilde{\alpha})) \in \mathbb{C}$ , which dominates the fixed rental contract in kind, the rent  $\bar{\beta}(\tilde{\alpha}) = \Phi(\tilde{\alpha}) - \Phi(\alpha_0)$  is negative. Thus, in return for a high share of output, the landlord makes a net positive cash transfer to

the tenant. There is empirical support for such contracts. Sometimes it is the case that the landlord provides the tenant with starting capital or equipments before the production, which effectively play the role of positive cash transfer to the tenant. In his study of post-bellum sharecropping contracts in early twentieth century South Carolina, Taylor (1943) provides several such instances. For example, the contract signed between Susan J. Hill of Greenville [the landlord] and Alvin Thompson of Laurens County [the tenant] had the following terms:

“The said party of the first part [landlord] furnishes to the said party of the second part [tenant], One mule and all necessarily tools and impliments to be used by said party of the second part in making and gathering his crop; and also agrees to furnish him Seventy (70) dollars to be paid in the following payments: Ten (10) dollars to be paid on the 1st of every month until all is paid, and the said party of the second part agrees to to give to the party of the first part a first lien on his entire crop to secure the same.

And the party of the second part agrees to pay as rent to the party of the first part one half ( $\frac{1}{2}$ ) of his entire crop, that is made on the place and he is to pay all debts to the 1st party out of the 1st proceeds.

The mule and all the farming tools, are to be returned to the party of the 1st part, after the crop is made and gathered in good condition.” (Taylor, 1943:125)

The passage above also illustrates that in actual contractual settings, the rental transfer from one party to another may not necessarily be a one-shot side payment and can be made via multiple transactions at different stages of production. In such cases, both parties may prefer to have at least some transfers in cash, as carrying out such transactions entirely in kind may not be feasible.

The fraction  $\tilde{\alpha}$ , as well as the functions  $\Phi$  and  $H$  are determined by  $f$  (the production function) and  $w$  (the cost of labor), which depend on the characteristics of the contracting agents. For example, the production functions for two landlords who own lands of different quality are likely to be different. So it is possible that while one landlord finds it optimal to offer a share contract, the other may prefer a fixed rental contract. Thus, our results suggest that tenancy contracts in a region may vary across agents depending on agent-specific characteristics. This is similar in spirit to the conclusion of the screening models of sharecropping [e.g., Newbery and Stiglitz (1979), Hallagan (1982), Allen (1982), Muthoo (1998)] that argue that tenants of different skills may be offered types of contracts. Like the screening models, our theory may also provide an explanation of the coexistence of different forms of tenancy contracts in a given region [see, e.g., Taylor (1943: 124-128), Myers (1970: 227-229), Rudra (1992: 293)].

Apart from the agent-specific variation of contracts discussed above, there is another kind of variation that is also observed. It is crop-specific. For example, in his study of West Godavari district of the state of Andhra Pradesh in India, Rao (1971: 584-585) finds that:

“...[W]ithin the same district, share-lease and cash-lease arrangements coexist, the latter being negligible in the rice zone and predominant in the tobacco zone...Also, the rice crop, for which the share-lease system is extensive, is a major marketed or cash-crop of the region, so that the share-lease system cannot readily be explained in terms of the subsistence nature of the crop.”

Since foodgrains like rice are more likely to exhibit seasonal fluctuations of price compared to non-food crops like tobacco, our theory may provide a plausible explanation of the kind of crop-specific variation described above. Thus, our model is consistent with some of the stylized facts of tenancy contracts. However, further empirical work is needed to identify the extent to which price variation plays a role in specific contexts.

To conclude this section, we note that the issue of the mode of the rental payment has received some attention in Marshall (1920: 534-535):

“The question whether the payments made by the cultivator for the use of his land should be reckoned in money or in produce is of growing interest with reference to both India and England. But we may pass it by for the present and consider the more fundamental distinction between the “English” system of rental and that of holding land on “shares”...”

The existing literature of sharecropping generally does not distinguish between rental payments in kind and cash. This paper shows that this distinction is important in the presence of price variation.

## 4 Interlinked contracts

In this section we consider more general contracts where in addition to share and rent, the landlord specifies a price at which he offers to buy the tenant’s share of the output. A contract offered by the landlord is now a triplet  $(\alpha, \beta, \gamma)$ , where  $\alpha \in [0, 1]$  is the tenant’s share of output,  $\beta \in \mathbb{R}$  is the rental transfer from the tenant to the landlord and  $\gamma \in [1, p]$  is the unit price at which the landlord offers to buy the tenant’s output. Such a contract is an *interlinked contract*, as it enables the landlord to interact with the tenant in two markets: the land market (through share  $\alpha$  and rent  $\beta$ ) and the product market (through price  $\gamma$ ). We shall restrict to interlinked contracts where the rent  $\beta$  is a *cash transfer* from the tenant to the landlord.<sup>5</sup> The set of interlinked contracts is given by

$$\mathbb{I} = \{(\alpha, \beta, \gamma) | \alpha \in [0, 1], \beta \in \mathbb{R}, \gamma \in [1, p]\}.$$

The strategic interaction is modeled as a three-stage extensive-form game  $G_1$ . In the first stage, the landlord offers a contract  $(\alpha, \beta, \gamma) \in \mathbb{I}$  to the tenant. In the second stage, the tenant can reject the contract, in which case the game terminates with both parties getting their reservation payoffs, or he can accept, in which case the game moves on to the third stage where the tenant decides on the amount of labor for carrying out production and output is realized. If the output is  $Q$ : (i) the tenant keeps  $\alpha Q$  and leaves the rest  $(1 - \alpha)Q$  with the landlord, (ii) makes the rental transfer  $\beta$  to the landlord and (iii) sells his share of output  $\alpha Q$  to the landlord at price  $\gamma$ . The solution concept is Subgame Perfect Equilibrium (SPE).

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<sup>5</sup>Our qualitative conclusions stay unaltered under interlinked contracts with rental transfers in kind. Considering such contracts does not provide any additional insights to the problem, but entails a different set of analysis. So for clarity of presentation, we restrict to cash transfers.



## 4.1 The tenant's problem

Under the contract  $(\alpha, \beta, \gamma)$ , the tenant's payoff has two components: (i) the profit from his share  $\alpha$  of the output that he sells at price  $\gamma$  and (ii) the cash transfer  $\beta$ . If the tenant chooses labor input  $\ell$ , the output is  $f(\ell)$  and he obtains the revenue  $\gamma\alpha f(\ell)$  by selling his share  $\alpha f(\ell)$  to the landlord at price  $\gamma$ . As the cost of  $\ell$  units of labor is  $w(\ell)$ , the profit of the tenant from his share is  $\gamma\alpha f(\ell) - w(\ell)$ . As he has to make the cash transfer  $\beta$  to the landlord, his payoff is  $\gamma\alpha f(\ell) - w(\ell) - \beta$ . Defining  $\theta := \gamma\alpha$ , this payoff is

$$\theta f(\ell) - w(\ell) - \beta$$

and  $\beta$  being a constant, the tenant's problem is to choose  $\ell$  to maximize

$$\phi^\theta(\ell) = \theta f(\ell) - w(\ell)$$

Note that  $\theta$  is the *effective unit price* of the output for the tenant when he works under the contract  $(\alpha, \beta, \gamma)$ . As  $\alpha \in [0, 1]$  and  $\gamma \in [1, p]$ , we have  $\theta \in [0, p]$ . The following lemma, which characterizes the tenant's optimal choice, follows by replacing  $\alpha$  by  $\theta$  in (2), (3) and (4).

**Lemma 4** *Consider a contract  $(\alpha, \beta, \gamma) \in \mathbb{I}$  and let  $\theta = \gamma\alpha$ .*

- (i) *Under this contract, the tenant's optimal labor input is  $\ell(\theta)$  where  $\ell(0) = 0$  and  $\theta f'(\ell(\theta)) = w'(\ell(\theta))$  for  $\theta > 0$ .*
- (ii) *The output produced is  $F(\theta)$  and the tenant obtains the payoff  $\Phi(\theta) - \beta$  where  $F(\theta) = f(\ell(\theta))$  and  $\Phi(\theta) = \phi^\theta(\ell(\theta)) = \theta F(\theta) - w(\ell(\theta))$ .*
- (iii)  *$F(0) = \Phi(0) = 0$ ,  $F'(\theta) > 0$  and  $\Phi'(\theta) = F(\theta) > 0$  for  $\theta > 0$ .*

## 4.2 The landlord's problem

By Lemma 3, when the tenant acts optimally under the contract  $(\alpha, \beta, \gamma)$ , the output produced is  $F(\theta)$  where  $\theta = \gamma\alpha$ . The payoff of the landlord has the following components.

- (a) The landlord has his share of output  $(1 - \alpha)F(\theta)$ . Moreover the tenant sells his share  $\alpha F(\theta)$  to the landlord. So the landlord has the total output  $F(\theta)$  at his disposal. Taking  $Q = F(\theta)$  in (5), his revenue from the product market is  $\Psi^p(F(\theta))$ .
- (b) The tenant sells his share of output  $\alpha F(\theta)$  to the landlord at price  $\gamma$ . So the landlord pays  $\gamma\alpha F(\theta) = \theta F(\theta)$  to the tenant.
- (c) The landlord obtains the cash transfer  $\beta$  from the tenant.

By (a)-(c), the payoff of the landlord is

$$\Pi^p(\alpha, \beta, \gamma) = \Pi^p(\theta, \beta) = \Psi^p(F(\theta)) - \theta F(\theta) + \beta$$

By Lemma 6, the tenant's payoff under his optimal labor input is  $\Phi(\theta) - \beta$ . As the tenant's reservation payoff is  $\Phi(\alpha_0)$ , his participation constraint is  $\Phi(\theta) - \beta \geq \Phi(\alpha_0)$ . For any  $\theta$ , the optimal  $\beta$  for the landlord is the one that binds this constraint:

$$\bar{\beta}(\theta) = \Phi(\theta) - \Phi(\alpha_0) \tag{16}$$

So it is sufficient to consider contracts  $(\alpha, \bar{\beta}(\theta), \gamma)$  where  $\alpha \in [0, 1]$ ,  $\gamma \in [1, p]$  and  $\theta = \gamma\alpha \in [0, p]$ . Under such a contract, the landlord's payoff is a function of  $\theta$  only, given by

$$\Pi^p(\theta) = \Psi^p(F(\theta)) - \theta F(\theta) + \Phi(\theta) - \Phi(\alpha_0) \quad (17)$$

Therefore the landlord's problem reduces to choosing  $\theta \in [0, p]$  to maximize (17).

**Proposition 3** *Consider the set  $\mathbb{I}$  of all interlinked contracts. For any  $p > 1$ , the landlord has multiple optimal interlinked contracts and any optimal contract has  $\theta = \gamma\alpha = 1$ . Specifically the set of all optimal contracts is*

$$\mathbb{I}_p^* = \{(\alpha, \bar{\beta}(1), \gamma) \mid \alpha \in [1/p, 1], \gamma \in [1, p], \gamma\alpha = 1\} \quad (18)$$

where  $\bar{\beta}(1) = \Phi(1) - \Phi(\alpha_0)$  that binds the tenant's participation constraint. The landlord's payoff under any optimal contract is  $(p-1)\bar{Q} + \Phi(1) - \Phi(\alpha_0)$ .

**Proof** See the Appendix. ■

The reason behind the multiplicity of optimal contracts is clear. As the tenant's incentive depends on his effective unit price  $\theta = \gamma\alpha$ , the optimal level of incentive can be sustained by multiple combinations of  $\gamma$  and  $\alpha$ . Observe from (18) that the fixed rental contract ( $\alpha = 1, \beta = \bar{\beta}(1), \gamma = 1$ ) (i.e. the landlord leaves the entire output with the tenant, sets the rental transfer  $\bar{\beta}(1)$  and offers to buy the output at low price 1) is an optimal contract. In addition, there are share contracts ( $0 < \alpha < 1$ ) that can be optimal as well.

Since any optimal contract has  $\theta = \gamma\alpha = 1$ , if  $\alpha \in (0, 1)$ , then  $\gamma > 1$ . So in this model a share contract is necessarily accompanied by interlinkage (the landlord buying the tenant's output at a price  $\gamma$  that is higher than price 1 of season 1). This contract can be viewed alternatively as follows: if the output is  $Q$ , the landlord effectively provides a subsidy of  $(\gamma - 1)\alpha Q$  to the tenant. Thus, an interlinked transaction in our model can be interpreted as a cost-sharing arrangement. Under this broader interpretation, our theory has some empirical support as share contracts in practice often involve cost-sharing [see, e.g., Bardhan and Rudra (1978: 99-100), Rudra (1992: 293-294), Reddy (1996: 52-53), Sharma and Drèze (1996: 8)]. So far as empirical support for general interlinked contracts is concerned, one problem is the lack of sufficient empirical work on this, as recently pointed out by Bardhan (2005: 88):

“There is now quite a bit of theoretical literature...on interlinked contracts in a poor agrarian economy, but there is even now very little empirical work on the subject. Our dataset [Bardhan & Rudra (1978)] is one of the earliest and still the largest that exists on such interlinked contracts.”

Evidence of tenancy-credit linkage can be found in Bardhan and Rudra (1978: 99). Jodha (1984) finds that under a “fairly broad” definition of interlinked operations, between 6 to 21 percent of tenancy transactions of his survey had some form of interlinkage (ibid: 110). The following is a specific evidence from Akola district in the state of Maharashtra in India (ibid: 111):

“In the Akola villages, the few interlinked transactions concerned primarily land lease, credit and marketing. One of the reasons for this pattern was the public intervention in the form of the monopoly purchase of cotton by the Cotton Marketing Federation

in Maharashtra...Small farmers with a limited holding capacity sometimes had to use large farmers as informal intermediaries to do their cotton marketing, a practice that led to interlinked tenancy credit and market transactions.”

Although the tenancy-marketing interlinkage above arose out of special circumstances, nevertheless the nature of the transaction is very similar to the one considered in this paper where the tenant, lacking storage facility, sells his output to the landlord.

## 5 Imperfect competition in the product market: the perturbed game $G_1(\varepsilon)$

In the last section, it was implicitly assumed that the landlord is a monopolist in the land market and a monopsonist in the product market. While the landlord can exercise monopoly power over the land he owns, empirical evidence presented in Section 2 suggests that this is not necessarily the case in the rural product market, which closely resembles a situation of imperfect competition. Therefore, in trading with the tenant, the landlord might face competition from other agents. Suppose such an agent appears with some small but positive probability. Then the question is, out of the multiple contracts obtained in Proposition 3, what are the ones that the landlord will choose once he anticipates such a possibility? It will be shown that there is a unique contract that satisfies this refinement criterion and it results in a sharecropping contract.

The possibility of competition in the product market is formally modeled by the perturbed game  $G_1(\varepsilon)$  that has the following stages. In the first stage, the landlord offers a contract  $(\alpha, \beta, \gamma)$  to the tenant. In the second stage, the tenant either rejects the contract, in which case the game terminates with both parties getting their reservation payoffs, or he accepts, in which case the game moves to the third stage where the tenant carries out production and output is realized. If the output is  $Q$ : (i) the tenant keeps  $\alpha Q$  and leaves the rest  $(1 - \alpha)Q$  with the landlord and (ii) makes the rental transfer  $\beta$  to the landlord. At the end of this stage: (a) with probability  $\varepsilon \in (0, 1)$ , a third agent, who we call *the  $\varepsilon$ -agent*, emerges and (b) with probability  $1 - \varepsilon$ , he does not emerge.

If the  $\varepsilon$ -agent does not emerge, then  $G_1(\varepsilon)$  proceeds like the unperturbed game  $G_1$ . If the  $\varepsilon$ -agent emerges, the game moves to the fourth stage where the  $\varepsilon$ -agent decides whether to buy the tenant's output or not, and accordingly he offers a price to buy the tenant's share of output. The tenant then decides whether to sell his output to the landlord or the  $\varepsilon$ -agent and trade takes place between one of the following buyer-seller pairs: (landlord—tenant) or ( $\varepsilon$ -agent—tenant). Finally payoffs are obtained and the game terminates. The solution concept is Subgame Perfect Bayesian Equilibrium (SPBE).<sup>6</sup>

**Definition:** Let  $p > 1$ . Consider the set of all optimal interlinked contracts  $\mathbb{I}_p^*$  of the unperturbed game  $G_1$ . We say that a contract  $(\alpha, \beta, \gamma) \in \mathbb{I}_p^*$  is *robust to the emergence of the  $\varepsilon$ -agent* if there is a sequence  $\{(\alpha(\varepsilon), \beta(\varepsilon), \gamma(\varepsilon))\}$  such that: (i) for  $\varepsilon \in (0, 1)$ ,  $(\alpha(\varepsilon), \beta(\varepsilon), \gamma(\varepsilon))$

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<sup>6</sup>To keep our analysis simple, in this game we do not allow the  $\varepsilon$ -agent to trade with the landlord. However, such a possibility can be ruled out *endogenously*. Specifically, it can be shown that if the landlord has a slight bargaining edge (no matter how small) over the  $\varepsilon$ -agent, then there is no SPBE where they trade with each other.

is the contract offered by the landlord in an SPBE of  $G_1(\varepsilon)$  and (ii)  $(\alpha(\varepsilon), \beta(\varepsilon), \gamma(\varepsilon)) \rightarrow (\alpha, \beta, \gamma)$  as  $\varepsilon \rightarrow 0+$ .

## 5.1 The problem of the $\varepsilon$ -agent

To determine SPBE of  $G_1(\varepsilon)$ , we begin from the fourth stage of the game and solve the  $\varepsilon$ -agent's problem. If there is trade between the  $\varepsilon$ -agent and the tenant, the  $\varepsilon$ -agent can store the output that he buys and sell it later at price  $p$ .

**Assumption A3:** (Symmetric storage capacities) The  $\varepsilon$ -agent has the same storage capacity  $\bar{Q}$  as the landlord.

Due to symmetric capacities,<sup>7</sup> like the landlord, the revenue of  $\varepsilon$ -agent from the product market at output  $Q$  is given by  $\Psi^p(Q)$  in (5).

Suppose the landlord has offered to buy the tenant's output at a price  $\gamma \in [1, p]$ . Then the tenant will not trade with the  $\varepsilon$ -agent if the latter offers him any price less than  $\gamma$ . Therefore, if in any SPBE there is trade between the tenant and the  $\varepsilon$ -agent, the price offered by the  $\varepsilon$ -agent must exactly equal  $\gamma$  that makes the tenant indifferent between trading with him and the landlord.

## 5.2 The stage game of $G_1(\varepsilon)$ following the landlord's contract offer

Now consider the stage game of  $G_1(\varepsilon)$  that follows the landlord's contract  $(\alpha, \beta, \gamma)$ . Following this contract, the tenant sells his output at price  $\gamma$  regardless of who the buyer is. So his problem stays the same as in the unperturbed game  $G_1$  and it depends only on  $\theta = \gamma\alpha$ . The output produced is  $F(\theta)$  and the tenant obtains  $\Phi(\theta) - \beta$  (Lemma 4). As before, the landlord sets the rent  $\bar{\beta}(\theta) = \Phi(\theta) - \Phi(\alpha_0)$  that binds the tenant's participation constraint. So it is sufficient to consider contracts  $(\alpha, \bar{\beta}(\theta), \gamma)$  for  $\theta = \gamma\alpha \in [0, p]$ ,  $\alpha \in [\theta/p, 1]$  and  $\gamma \in [\theta, p]$ .<sup>8</sup>

**Lemma 5** *Let  $p > 1$ . Suppose the landlord offers the contract  $(\alpha, \bar{\beta}(\theta), \gamma)$  to the tenant, where  $\theta = \gamma\alpha \in (0, p]$ ,  $\alpha \in [\theta/p, 1]$  and  $\gamma \in [\theta, p]$ . In any SPBE of  $G_1(\varepsilon)$  after this offer the following hold.*

(i) *The output produced is  $F(\theta)$  and after paying the landlord's share and rent, the output at the tenant's disposal is  $\alpha F(\theta)$ .*

(ii) *If the  $\varepsilon$ -agent trades with the tenant, then he pays the tenant  $\theta F(\theta)$  and obtains*

$$\pi_A^p(\theta, \alpha) = \Psi^p(\alpha F(\theta)) - \theta F(\theta) \quad (19)$$

*There is trade between the tenant and the  $\varepsilon$ -agent if and only if  $\pi_A^p(\theta, \alpha) \geq 0$ .*

(iii)  *$\pi_A^p(\theta, \alpha)$  is strictly increasing in  $\alpha$ ,  $\pi_A^p(\theta, 1) = \Psi^p(F(\theta)) - \theta F(\theta)$  and  $\pi^p(\theta, \theta/p) \leq 0$ .*

(iv) *Trading between the  $\varepsilon$ -agent and the tenant depends on  $\theta$  and  $\alpha$  as follows.*

<sup>7</sup>As the  $\varepsilon$ -agent represents a small trader in the village, it is more realistic to assume that his storage capacity is lower compared to the landlord. We assume symmetric storage capacities for analytic convenience and our qualitative conclusions will not be altered if the  $\varepsilon$ -agent has a lower capacity than the landlord, as long as it is not too low. If the capacity of the  $\varepsilon$ -agent is too low, then he does not pose any serious competition to the landlord in the product market.

<sup>8</sup>As  $\gamma \leq p$ , we have  $\theta = \gamma\alpha \leq \alpha p$  implying  $\alpha \geq \theta/p$ . As  $\alpha \leq 1$ , we have  $\theta = \gamma\alpha \leq \gamma$ , so  $\gamma \geq \theta$ .

(a) If  $\pi_A^p(\theta, 1) < 0$ , the  $\varepsilon$ -agent does not trade with the tenant and if  $\pi_A^p(\theta, 1) = 0$ , he trades with the tenant if and only if  $\alpha = 1$ .

(b) If  $\pi_A^p(\theta, 1) > 0$ , then  $\exists \alpha_A^p(\theta) \in [\theta/p, 1)$  such that for  $\alpha \in [\theta/p, 1]$ ,  $\pi_A^p(\theta, \alpha) \begin{matrix} \leq \\ \geq \end{matrix} 0 \Leftrightarrow \alpha \begin{matrix} \leq \\ \geq \end{matrix} \alpha_A^p(\theta)$ . Consequently the  $\varepsilon$ -agent trades with the tenant if and only if  $\alpha \geq \alpha_A^p(\theta)$ .

**Proof** See the Appendix. ■

### 5.3 The problem of the landlord

Now we move to the first stage of  $G_1(\varepsilon)$  and solve the landlord's problem. As we know, for the landlord it is sufficient to consider contracts  $(\alpha, \bar{\beta}(\theta), \gamma)$  for  $\theta = \gamma\alpha \in [0, p]$ ,  $\alpha \in [\theta/p, 1]$  and  $\gamma \in [\theta, p]$ . Then the output produced is  $F(\theta)$ . To determine the expected payoff of the landlord in the perturbed game  $G_1(\varepsilon)$ , observe that:

- (a) Regardless of the emergence or trading nature of the  $\varepsilon$ -agent, the landlord obtains the rental transfer  $\bar{\beta}(\theta) = \Phi(\theta) - \Phi(\alpha_0)$  from the tenant.
- (b) If the  $\varepsilon$ -agent does not trade with the tenant, the landlord has the total output  $F(\theta)$  (his own share  $1 - \alpha$  plus the tenant's share  $\alpha$ ), which yields the revenue  $\Psi^p(F(\theta))$  from the product market. Since the landlord pays  $\gamma\alpha F(\theta) = \theta F(\theta)$  to the tenant, his net revenue from the product market is  $\Psi^p(F(\theta)) - \theta F(\theta)$ .
- (c) If the  $\varepsilon$ -agent trades with the tenant: (i) the landlord does not have to make any payment for the tenant's share and (ii) he has only his share  $(1 - \alpha)F(\theta)$ . So his revenue from the product market is simply the revenue from output  $(1 - \alpha)F(\theta)$ , given by  $\Psi^p((1 - \alpha)F(\theta))$ .

By Lemma 5(iv), whether the  $\varepsilon$ -agent trades with the tenant or not depends on  $\theta$  and  $\alpha$ . Let  $\lambda(\theta, \alpha)$  be the indicator variable where

$$\lambda(\theta, \alpha) = \begin{cases} 1 & \text{if the } \varepsilon\text{-agent trades with the tenant,} \\ 0 & \text{otherwise} \end{cases}$$

As the  $\varepsilon$ -agent emerges with probability  $\varepsilon$ , there is trade between  $\varepsilon$ -agent and the tenant with probability  $\varepsilon\lambda(\theta, \alpha)$  and no trade between them with probability  $1 - \varepsilon\lambda(\theta, \alpha)$ . So by (a)-(c), the landlord's expected payoff is

$$\Pi_\varepsilon^p(\theta, \alpha) = \bar{\beta}(\theta) + \varepsilon\lambda(\theta, \alpha) \underbrace{\Psi^p((1 - \alpha)F(\theta))}_{\text{revenue in (c)}} + [1 - \varepsilon\lambda(\theta, \alpha)] \underbrace{[\Psi^p(F(\theta)) - \theta F(\theta)]}_{\text{revenue in (b)}} \quad (20)$$

When the  $\varepsilon$ -agent does not trade with the tenant, the landlord's payoff is the same as in the unperturbed game  $G_1$ . It depends only on  $\theta$ , given by

$$\Pi^p(\theta) = \Psi^p(F(\theta)) - \theta F(\theta) + \bar{\beta}(\theta)$$

When the  $\varepsilon$ -agent trades with the tenant, the payoff changes from  $\Pi^p(\theta)$  (the change could be positive, negative or zero). This change is the difference between the revenues of (c) and (b):

$$\Omega^p(\theta, \alpha) = \Psi^p((1 - \alpha)F(\theta)) - [\Psi^p(F(\theta)) - \theta F(\theta)] \quad (21)$$

By (20) and (21), the landlord's expected payoff in  $G_1(\varepsilon)$  is

$$\Pi_\varepsilon^p(\theta, \alpha) = \Pi^p(\theta) + \varepsilon\lambda(\theta, \alpha)\Omega^p(\theta, \alpha) \quad (22)$$

Let us define

$$\mathbb{J}^p = \{(\theta, \alpha) | \alpha \in [\theta/p, 1], \theta \in [0, p]\}$$

The landlord's problem in the first stage of  $G_1(\varepsilon)$  is to choose  $(\theta, \alpha) \in \mathbb{J}^p$  to maximize  $\Pi_\varepsilon^p(\theta, \alpha)$ . As the functions in (22) are bounded for  $(\theta, \alpha) \in \mathbb{J}^p$ , the maximization problem has a solution, i.e., the game  $G_1(\varepsilon)$  has a Subgame Perfect Bayesian Equilibrium (SPBE) for any  $\varepsilon \in (0, 1)$ .

Note from (22) that when  $\varepsilon = 0$ , the game coincides with the unperturbed game  $G_1$ . The landlord's problem there is to choose  $\theta \in [0, p]$  to maximize  $\Pi^p(\theta)$  and it is optimal to choose  $\theta = 1$  (Prop 3). The next lemma summarizes the properties of the function  $\Omega^p(\theta, \alpha)$  when  $\theta$  is close to 1.

**Lemma 6** *There is  $\bar{\delta} > 0$  satisfying  $(1 - \bar{\delta}, 1 + \bar{\delta}) \subset [0, p]$  such that the following hold for all  $\theta \in (1 - \bar{\delta}, 1 + \bar{\delta})$  where  $\pi_A^p(\theta, \alpha) = \Psi^p(\alpha F(\theta)) - \theta F(\theta)$  (the  $\varepsilon$ -agent's payoff if it trades with the tenant).*

(a)  $\exists \alpha_A^p(\theta) \in [\theta/p, 1)$ , satisfying  $\pi_A^p(\alpha_A^p(\theta), \theta) = 0$  (i.e., at  $\alpha = \alpha_A^p(\theta)$ , the  $\varepsilon$ -agent just breaks even in his trade with the tenant), such that

$$\Pi_\varepsilon^p(\theta, \alpha) = \begin{cases} \Pi^p(\theta) & \text{if } \alpha \in [\theta/p, \alpha_A^p(\theta)), \\ \Pi^p(\theta) + \varepsilon\Omega^p(\theta, \alpha) & \text{if } \alpha \in [\alpha_A^p(\theta), 1]. \end{cases} \quad (23)$$

(b)  $\Omega^p(\theta, \alpha)$  is strictly decreasing in  $\alpha$  and  $\exists \alpha_L^p(\theta) \in (\alpha_A^p(\theta), 1)$  such that for  $\alpha \in [\alpha_A^p(\theta), 1]$ ,  $\Omega^p(\theta, \alpha) \geq 0 \Leftrightarrow \alpha \leq \alpha_L^p(\theta)$ .

(c) (*Pareto improving region*) If  $\alpha \in [\alpha_A^p(\theta), \alpha_L^p(\theta)]$ , then  $\pi_A(\theta, \alpha) \geq 0$  (the  $\varepsilon$ -agent obtains a non-negative payoff by trading with the tenant) and  $\Omega(\theta, \alpha) \geq 0$  (change in the landlord's payoff due to trading between the  $\varepsilon$ -agent and the tenant is non-negative).

(d)  $\Pi_\varepsilon^p(\theta, \alpha_A^p(\theta)) > \Pi^p(\theta)$  and the unique maximum of  $\Pi_\varepsilon^p(\theta, \alpha)$  over  $\alpha \in [\theta/p, 1]$  is attained at  $\alpha = \alpha_A^p(\theta)$ , where the  $\varepsilon$ -agent trades with the tenant and just breaks even.

**Proof** See the Appendix. ■

In Figure 1,  $\Pi_\varepsilon^p(\theta, \alpha)$  is depicted as function of  $\alpha$  for fixed  $\theta \in (1 - \bar{\delta}, 1 + \bar{\delta})$ . For  $\alpha \in [\theta/p, \alpha_A^p(\theta))$ , the  $\varepsilon$ -agent does not trade with the tenant and the landlord obtains  $\Pi^p(\theta)$  (the line  $AB_0$ ). For  $\alpha \in [\alpha_A^p(\theta), 1]$ , the  $\varepsilon$ -agent trades with the tenant. As the trading pattern changes at  $\alpha = \alpha_A^p(\theta)$ , the payoff function has a jump there. In Figure 1, the payoff for  $\alpha > \alpha_A^p(\theta)$  has been drawn for two different values of  $\varepsilon$ :  $\varepsilon_1 < \varepsilon_2$ , presented by the curves  $B_{\varepsilon_1}C_{\varepsilon_1}$  and  $B_{\varepsilon_2}C_{\varepsilon_2}$ . As  $\varepsilon$  becomes close to zero, these curves converge to the line  $B_0C_0$ .

As the  $\varepsilon$ -agent trades with the tenant for  $\alpha > \alpha_A^p(\theta)$ , the landlord is left with his own share  $(1 - \alpha)F(\theta)$ . His revenue falls with  $\alpha$ , so the drawback of a fixed rental contract is immediate. At  $\alpha = 1$ , his revenue from the product market drops from  $\Psi^p(F(\theta)) - \theta F(\theta) > 0$  to zero [ $\Omega^p(\theta, 1) < 0$ ]. The function  $\Omega^p(\theta, \alpha)$  continues to stay negative until  $\alpha$  falls to  $\alpha_L^p(\theta)$ . The set  $[\alpha_A^p(\theta), \alpha_L^p(\theta)]$  of share contracts presents the *Pareto improving* (PI) region: trading

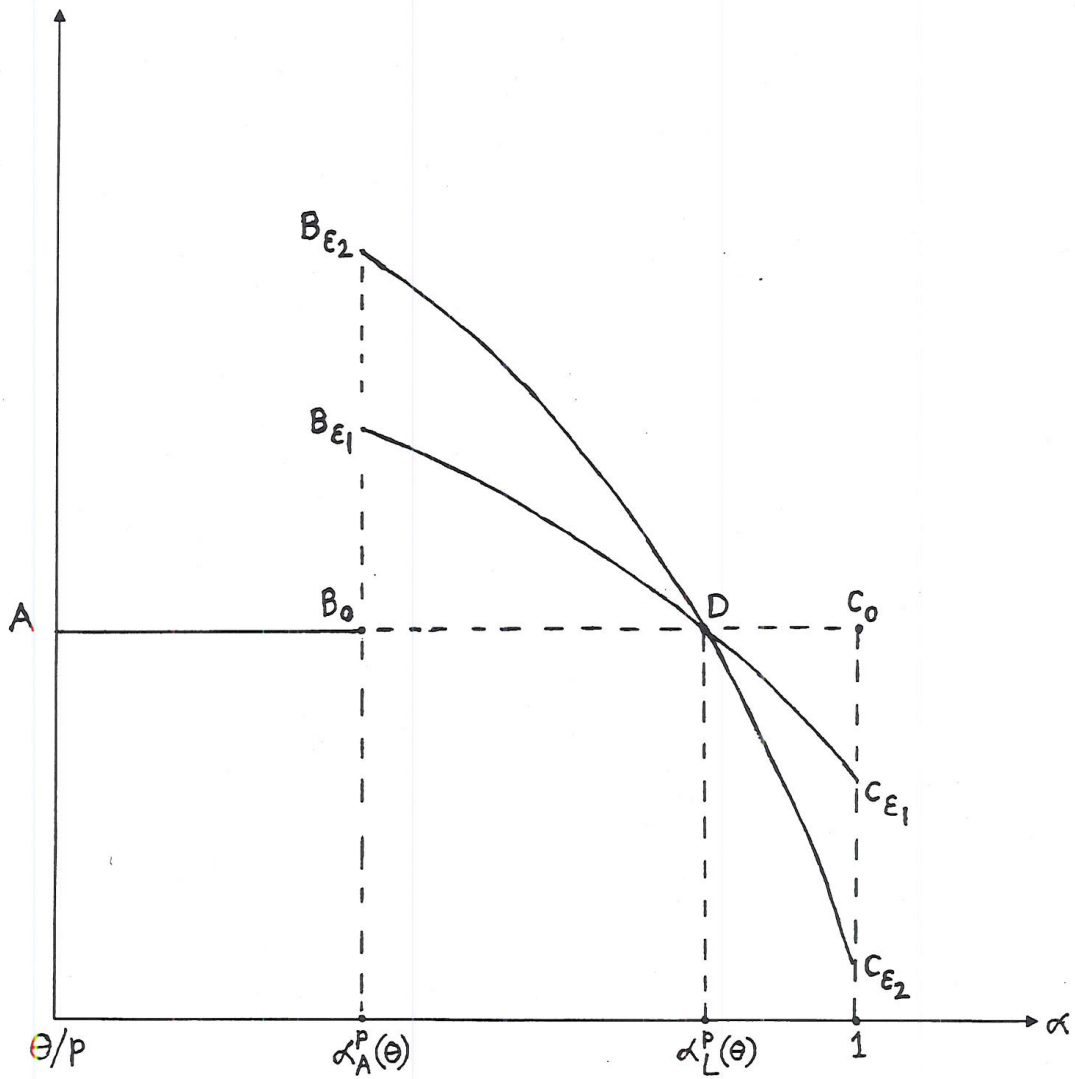


Figure 1:  $\Pi_\epsilon^p(\theta, \alpha)$  as function of  $\alpha$  for fixed  $\theta \in (1 - \bar{\delta}, 1 + \bar{\delta})$

between the  $\epsilon$ -agent and the tenant improves the payoffs of both the landlord and the  $\epsilon$ -agent, keeping the tenant indifferent. However, the gains of the landlord and the  $\epsilon$ -agent move in

opposite directions: each prefers to have a high output at his disposal, so the landlord prefers  $\alpha$  to be low while the  $\varepsilon$ -agent prefers it to be high. The landlord, having the advantage of choosing the contract, sets  $\alpha = \alpha_A^p(\theta)$ , which is the lowest possible  $\alpha$  in the PI region, and the  $\varepsilon$ -agent just breaks even.

Why is there a non-empty PI region, i.e., why it is the case that  $\alpha_A^p(\theta) < \alpha_L^p(\theta)$ ? When there is trade between the  $\varepsilon$ -agent and the tenant, the tenant's payoff does not change, the landlord's change in payoff is  $\Omega^p(\theta, \alpha) = \Psi^p((1 - \alpha)F(\theta)) - [\Psi^p(F(\theta)) - \theta F(\theta)]$  and the  $\varepsilon$ -agent obtains  $\pi_A^p(\theta, \alpha) = \Psi^p(\alpha F(\theta)) - \theta F(\theta)$ . The total trading surplus is

$$\Omega^p(\theta, \alpha) + \pi_A^p(\theta, \alpha) = \Psi^p(\alpha F(\theta)) + \Psi^p((1 - \alpha)F(\theta)) - \Psi^p(F(\theta)).$$

Due to decreasing returns to scale property of the revenue function  $\Psi^p(Q)$  [Lemma 3(vi)], the total trading surplus is positive. Hence if the surplus of one party is zero, the other party must have a positive surplus. As  $\pi_A^p(\theta, \alpha) = 0$  at  $\alpha = \alpha_A^p(\theta)$ ,  $\Omega^p(\theta, \alpha_A^p(\theta))$  must be positive. As  $\Omega^p(\theta, \alpha)$  is decreasing in  $\alpha$  and  $\Omega^p(\theta, \alpha_L^p(\theta)) = 0$ , it follows that  $\alpha_A^p(\theta) < \alpha_L^p(\theta)$ , resulting in a non-empty PI region.

## 5.4 Unique robust contract is a share contract

Now we state the main result of this section, where it is shown that the unique contract that is robust to the emergence of the  $\varepsilon$ -agent results in sharecropping.

**Proposition 4** (I) *Let  $p > 1$ . For any  $\varepsilon \in (0, 1)$  the perturbed game  $G_1(\varepsilon)$  has an SPBE. Let  $(\alpha_\varepsilon, \beta_\varepsilon, \gamma_\varepsilon) \in \mathbb{I}$  be the contract offered by the landlord in an SPBE of  $G_1(\varepsilon)$  and let  $\theta_\varepsilon = \gamma_\varepsilon \alpha_\varepsilon$ . There exists  $\bar{\varepsilon} \in (0, 1)$  such that for  $\varepsilon \in (0, \bar{\varepsilon})$ :*

- (i)  $\alpha_\varepsilon = \alpha_A^p(\theta_\varepsilon)$  and  $\beta_\varepsilon = \Phi(\theta_\varepsilon) - \Phi(\alpha_0)$ .
- (ii) *The tenant obtains his reservation payoff  $\Phi(\alpha_0)$ . If the  $\varepsilon$ -agent emerges, he buys the tenant's share of output  $\alpha_A^p(\theta_\varepsilon)F(\theta_\varepsilon)$  by paying the tenant  $\theta_\varepsilon F(\theta_\varepsilon)$ . If the  $\varepsilon$ -agent does not emerge, the landlord buys the tenant's share by making the same payment.*
- (iii) *By trading with the tenant the  $\varepsilon$ -agent just breaks even. He obtains  $\pi_A^p(\theta_\varepsilon, \alpha_A^p(\theta_\varepsilon)) = 0$ .*
- (iv) *The landlord obtains the expected payoff  $\Pi^p(\theta_\varepsilon) + \varepsilon \Omega^p(\theta_\varepsilon, \alpha_A^p(\theta_\varepsilon)) > \Pi^p(\theta_\varepsilon)$ .*

(II) *Let  $p > 1$ . Consider the set  $\mathbb{I}_p^*$  of all optimal interlinked contracts for the landlord in the unperturbed game  $G_1$ . There is a unique  $(\alpha, \beta, \gamma) \in \mathbb{I}_p^*$  that is robust to the emergence of the  $\varepsilon$ -agent. This contract is a share contract (i.e.  $0 < \alpha < 1$ ) where the landlord offers to buy the tenant's share of output at a price higher than 1 (i.e.  $\gamma > 1$ ). Specifically, it has  $\alpha = \alpha_A^p(1)$ ,  $\beta = \Phi(1) - \Phi(\alpha_0)$  and  $\gamma = 1/\alpha_A^p(1)$ .*

**Proof** See the Appendix. ■

Proposition 4 shows that the unique contract that is robust to the emergence of the  $\varepsilon$ -agent results in a share contract. Competition in the product market generates a subset of Pareto improving share contracts out of the multiple optimal contracts of the unperturbed game  $G_1$ . It is optimal for the landlord to choose that specific contract in this subset where his own share is maximum. As a result, the  $\varepsilon$ -agent just breaks even in trading with the tenant.



## 6 Concluding remarks

In this paper we have proposed a theory of sharecropping on the basis of price behavior in agriculture and imperfectly competitive nature of rural product markets. Considering a contractual setting where the landlord can take advantage of seasonal variation of price but the tenant-farmer cannot, first we have shown the optimality of sharecropping in the class of tenancy contracts. Then considering interlinked contracts, we have shown that there are multiple optimal contracts. Finally proposing an equilibrium refinement that incorporates imperfect competition in the rural product market, we have shown that the unique contract that is robust to this refinement results in a share contract. In our model, the price differential between the contracting parties is the main driving force behind sharecropping. It is further reinforced by the emergence of a small trader who seeks to gain from arbitrage. When the price differential goes down, this rationale for sharecropping will gradually disappear. For example, if there are entities (e.g., the government or an outside firm that has no stake at small village-level competition) that can credibly assure the tenant of a high price, then fixed rental contracts would be gradually more preferable for the landlord.

In proposing a theory of tenancy contracts based on price fluctuations, this paper relates to two general themes of development economics: (i) volatilities of different kinds have important effects on rural economies of poor countries and (ii) institutions and contractual forms can often be endogenous to these volatilities. Our model is consistent with some of the stylized facts of tenancy contracts (e.g., agent-specific and crop-specific variation of contractual forms, incidence of cost-sharing with share contracts, interlinkage of tenancy and marketing). However, further empirical work is necessary to see the extent to which price variation plays a role in explaining these facts in specific contexts.

It is well recognized that there cannot be a single explanation of the sharecropping institution. As Singh (1989: 34) points out:

“Sharecropping has existed in various times and places in various forms. It has disappeared over time and reappeared. Sometimes the output share equals the cost share; sometimes it does not. Sometimes the tenant’s share is one-half; sometimes it is not. Sometimes productivity is higher on sharecropped land than on other types of tenancy or with self-cultivation; sometimes it is not. Sometimes sharecroppers are poor; sometimes they are prosperous. Sometimes sharecroppers produce risky cash crops; sometimes they produce for subsistence. I do not think a single theory can capture all of these aspects of sharecropping”

In this spirit, it can be said that this paper complements the existing theories of the literature.

## Appendix

**Proof of Lemma 2** Parts (i) and (ii) are direct from (5).

(iii) If  $Q_1 + Q_2 \leq \bar{Q}$ , then both  $Q_1$  and  $Q_2$  are less than  $\bar{Q}$ , and (5) yields the result.

Now let  $Q_1 + Q_2 > \bar{Q}$ . Then by (5),  $\Psi^p(Q_1 + Q_2) = Q_1 + Q_2 + (p - 1)\bar{Q}$ .

If both  $Q_1, Q_2$  do not exceed  $\bar{Q}$ , then  $\Psi^p(Q_1) + \Psi^p(Q_2) = pQ_1 + pQ_2$  and  $\Psi^p(Q_1) + \Psi^p(Q_2) - \Psi^p(Q_1 + Q_2) = (p - 1)(Q_1 + Q_2 - \bar{Q}) > 0$ .

If  $Q_i > \bar{Q}$  for at least one  $i$ , without loss of generality, let  $Q_1 > \bar{Q}$ . Then  $\Psi^p(Q_1) = Q_1 + (p-1)\bar{Q}$  and by part (ii),  $\Psi^p(Q_2) > Q_2$ . Hence  $\Psi^p(Q_1) + \Psi^p(Q_2) > Q_1 + Q_2 + (p-1)\bar{Q} = \Psi^p(Q_1 + Q_2)$ . ■

**Proof of Proposition 2** (I) Let  $p > 1$ . By (13) and (14), under the set of contracts  $\mathbb{K}$ , the landlord's problem is to choose  $\alpha \in [0, 1]$  to maximize  $\Pi_K^p(\alpha) = \Psi^p(s(\alpha) - \Phi(\alpha_0))$ . By the monotonicity of  $\Psi^p(\cdot)$ , the problem reduces to maximizing  $s(\alpha)$  over  $\alpha \in [0, 1]$ . By (15), it follows that the unique optimal contract has  $\alpha = 1$  (fixed rent) with rent in kind  $\bar{\beta}(1) = \Phi(1) - \Phi(\alpha_0)$ . The payoffs are immediate.

(ii) Let  $p > 1$ . By (11), under the set of contracts  $\mathbb{C}$ , the landlord's problem is to choose  $\alpha \in [0, 1]$  to maximize  $\Pi_C^p(\alpha) = \Psi^p(H(\alpha)) + \Phi(\alpha) - \Phi(\alpha_0)$ . Since  $H(\alpha) \leq H(\tilde{\alpha})$  (Lemma 3) and  $H(\tilde{\alpha}) < \bar{Q}$  (Assumption A2), we have  $H(\alpha) < \bar{Q}$ . Then by (5),  $\Psi^p(H(\alpha)) = pH(\alpha)$  for all  $\alpha \in [0, 1]$  and hence

$$\Pi_C^p(\alpha) = pH(\alpha) + \Phi(\alpha) - \Phi(\alpha_0) \quad (24)$$

Since  $\Phi'(\alpha) = F(\alpha)$  (Lemma 1) and  $H'(\alpha) = (1 - \alpha)F'(\alpha) - F(\alpha)$ , we have

$$\Pi_C'^p(\alpha) = pH'(\alpha) + F(\alpha) = p(1 - \alpha)F'(\alpha) - (p - 1)F(\alpha) \quad \text{and} \quad (25)$$

$$\Pi_C''^p(\alpha) = p(1 - \alpha)F''(\alpha) - (2p - 1)F'(\alpha) \quad (26)$$

Since  $F''(\alpha) \leq 0$  (Assumption A1) and  $F'(\alpha) > 0$ , it follows from (26) that  $\Pi_C''^p(\alpha) < 0$  for  $\alpha > 0$ , so  $\Pi_C^p(\alpha)$  is strictly concave in  $\alpha$ . As  $H'(\tilde{\alpha}) = 0$  (Lemma 3) and  $p > 1$ , from (25) we have  $\Pi_C'^p(\tilde{\alpha}) = F(\tilde{\alpha}) > 0$  and  $\Pi_C'^p(1) = -(p - 1)F(1) < 0$ . Hence  $\exists$  a unique  $\alpha_p^* \in (\tilde{\alpha}, 1)$  such that  $\Pi_C'^p(\alpha_p^*) = 0$  and  $\alpha_p^*$  is the unique maximizer of  $\Pi_C^p(\alpha)$  over  $\alpha \in [0, 1]$ . Taking  $\alpha = \alpha_p^*$  in (6) gives the cash rent  $\bar{\beta}(\alpha_p^*)$ . Now we prove properties (a)-(b).

Property (a) is immediate from (24), so consider (b). We have already shown that  $\alpha_p^* > \tilde{\alpha}$ . To prove the monotonicity of  $\alpha_p^*$ , let  $1 < p_1 < p_2$ . Since  $\Pi_C'^{p_1}(\alpha_p^*) = 0$  for any  $p > 1$ , from (25),  $\Pi_C'^{p_2}(\alpha_p^*) = (p_2 - p_1)H'(\alpha_p^*)$ . Since  $H'(\alpha) < 0$  for  $\alpha > \tilde{\alpha}$  (Lemma 3) and  $\alpha_p^* > \tilde{\alpha}$ , we have  $\Pi_C'^{p_2}(\alpha_p^*) < 0 = \Pi_C'^{p_2}(\alpha_{p_2}^*)$ . The strict concavity of  $\Pi_C^p(\alpha)$  then yields  $\alpha_{p_2}^* < \alpha_p^*$ .

To prove the limiting properties of  $\alpha_p^*$ , first note that when  $p = 1$ , the optimal contract is the fixed rental contract  $(1, \bar{\beta}(1))$ , i.e.,  $\alpha_1^* = 1$  (part(i)), hence  $\lim_{p \downarrow 1} \alpha_p^* = 1$ . Next consider any small  $\delta > 0$  and let  $\alpha \in [\tilde{\alpha} + \delta, 1]$ . Since  $H'(\alpha) < 0$  for  $\alpha > \tilde{\alpha}$  and  $H'(\cdot)$  and  $F(\cdot)$  are bounded, from (25) it follows that  $\exists P(\delta) > 1$  such that for any  $p > P(\delta)$ ,  $\Pi_C'^p(\alpha) < 0$  for all  $\alpha \in [\tilde{\alpha} + \delta, 1]$ . Hence  $\alpha_p^* \in (\tilde{\alpha}, \tilde{\alpha} + \delta)$  for  $p > P(\delta)$ , proving that  $\lim_{p \rightarrow \infty} \alpha_p^* = \tilde{\alpha}$ .

(III) Suppose

$$H(\tilde{\alpha}) > \bar{\beta}(1) = \Phi(1) - \Phi(\alpha_0) \quad (27)$$

Note by (15) that  $s(1) = \Phi(1) > s(\tilde{\alpha}) = H(\tilde{\alpha}) + \Phi(\tilde{\alpha})$ , so that  $\Phi(\alpha_0) - \Phi(\tilde{\alpha}) - [H(\tilde{\alpha}) - \bar{\beta}(1)] = s(1) - s(\tilde{\alpha}) > 0$ . Hence, if (27) holds, then

$$\Phi(\alpha_0) - \Phi(\tilde{\alpha}) > H(\tilde{\alpha}) - \bar{\beta}(1) > 0 \quad (28)$$

Define

$$\bar{p} := [\Phi(\alpha_0) - \Phi(\tilde{\alpha})]/[H(\tilde{\alpha}) - \bar{\beta}(1)] > 1 \quad (29)$$

Consider the contract  $(\tilde{\alpha}, \bar{\beta}(\tilde{\alpha})) \in \mathbb{C}$  [i.e. the share contract with the tenant's share  $\tilde{\alpha} \in (0, 1)$  and cash rent  $\bar{\beta}(\tilde{\alpha}) = \Phi(\tilde{\alpha}) - \Phi(\alpha_0)$ ]. Taking  $\alpha = \tilde{\alpha}$  in (24), the landlord's payoff under this contract is

$$\Pi_C^p(\tilde{\alpha}) = pH(\tilde{\alpha}) + \Phi(\tilde{\alpha}) - \Phi(\alpha_0) \quad (30)$$

Note from part (I) that under the fixed rental contract in kind  $(1, \bar{\beta}(1))$ , the landlord obtains  $\Pi_K^p(1) = \Psi^p(\bar{\beta}(1))$ . Since  $H(\tilde{\alpha}) > \bar{\beta}(1)$  [by (27)] and  $H(\tilde{\alpha}) < \bar{Q}$  (Assumption A2), we have  $\bar{\beta}(1) < \bar{Q}$  and by (5),  $\Psi^p(\bar{\beta}(1)) = p\bar{\beta}(1)$ . Hence  $\Pi_K^p(1) = \Psi^p(\bar{\beta}(1)) = p\bar{\beta}(1)$  and by (30),

$$\Pi_C^p(\tilde{\alpha}) - \Pi_K^p(1) = p[H(\tilde{\alpha}) - \bar{\beta}(1)] - [\Phi(\alpha_0) - \Phi(\tilde{\alpha})] = [H(\tilde{\alpha}) - \bar{\beta}(1)](p - \bar{p})$$

where  $\bar{p}$  is given in (29). Hence we conclude that if (27) holds, then  $\Pi_C^p(\tilde{\alpha}) > \Pi_K^p(1)$  for all  $p > \bar{p}$ . This completes the proof of (III). ■

**Proof of Proposition 3** Note that the landlord's problem is to choose  $\theta \in [0, p]$  to maximize (17) where  $\theta = \alpha\gamma$ . Since  $F(1) > \bar{Q}$  (Assumption A2),  $F(0) = 0$  and  $F$  is monotonic (Lemma 3),  $\exists \bar{\theta} \in (0, 1)$  such that  $F(\theta) \geq \bar{Q} \Leftrightarrow \theta \geq \bar{\theta}$ .

**Case 1**  $\theta \in [0, \bar{\theta}]$ . For this case  $F(\theta) \leq \bar{Q}$  and by (5),  $\Psi^p(F(\theta)) = pF(\theta)$ . Then by (17),

$$\Pi^p(\theta) = pF(\theta) - \theta F(\theta) + \Phi(\theta) - \Phi(\alpha_0) \quad (31)$$

As  $\Phi'(\theta) = F(\theta)$ , for  $\theta \in [0, \bar{\theta}]$  we have  $\Pi'^p(\theta) = (p - \theta)F'(\theta)$ . As  $F' > 0$  and  $\theta < \bar{\theta} < p$ , it follows that  $\Pi'^p(\theta) > 0$ , so  $\Pi^p(\theta)$  is strictly increasing for  $\theta \in [0, \bar{\theta}]$ .

**Case 2**  $\theta \in (\bar{\theta}, p]$ . Then  $F(\theta) > \bar{Q}$  and by (5),  $\Psi^p(F(\theta)) = F(\theta) + (p - 1)\bar{Q}$ . So by (17),

$$\Pi^p(\theta) = F(\theta) + (p - 1)\bar{Q} - \theta F(\theta) + \Phi(\theta) - \Phi(\alpha_0) \quad (32)$$

As  $\Phi'(\theta) = F(\theta)$ , for  $\theta \in (\bar{\theta}, 1]$ , we have  $\Pi'^p(\theta) = (1 - \theta)F'(\theta) \geq \bar{Q} \Leftrightarrow \theta \leq 1$ . Since  $\bar{\theta} < 1 < p$ , from Cases 1 and 2 it follows that the unique maximum of  $\Pi^p(\theta)$  over  $\theta \in [0, p]$  is attained at  $\theta = 1$ .

Taking  $\theta = 1$  in (16), the rent in any optimal contract is  $\bar{\beta}(1) = \Phi(1) - \Phi(\alpha_0)$  that binds the tenant's participation constraint. Since  $\theta = \gamma\alpha$ , any contract  $(\alpha, \bar{\beta}(1), \gamma)$  with  $\gamma\alpha = 1$  is an optimal contract, proving that the set of optimal contracts is the set  $\mathbb{I}_p^*$  given in (18).<sup>9</sup> The landlord's payoff is obtained by taking  $\theta = 1$  in (32). ■

**Proof of Lemma 5** Part (i) is direct.

(ii) If the  $\varepsilon$ -agent trades with the tenant, he offers the price  $\gamma$  (the same price offered by the landlord). As the tenant has output  $\alpha F(\theta)$ , the  $\varepsilon$ -agent pays  $\gamma\alpha F(\theta) = \theta F(\theta)$  to the tenant to buy this output. Since the  $\varepsilon$ -agent's revenue from the product market at output  $\alpha F(\theta)$  is  $\Psi^p(\alpha F(\theta))$ , his payoff is given by (19). The last statement is (ii) is immediate.

(iii) As  $F(\theta) > 0$  for  $\theta > 0$ , from (19), the monotonicity of  $\pi_A^p(\theta, \alpha)$  with respect to  $\alpha$  follows by the monotonicity of  $\Psi^p(\cdot)$ . The second statement is direct from (19). To prove the third statement, note from Lemma 2(ii) that  $\Psi^p(Q) \leq pQ$ . Taking  $(\theta/p)F(\theta) = Q$ , we have  $\theta F(\theta) = pQ$  and  $\pi^p(\theta, \theta/p) = \Psi^p(Q) - pQ \leq 0$ .

Part (iv) follows directly by part (iii). ■

<sup>9</sup>As  $\gamma \leq p$ , we have  $\theta = \gamma\alpha \leq \alpha p$  implying  $\alpha \geq \theta/p$ . So, when  $\theta = \gamma\alpha = 1$ , we have  $\alpha \geq 1/p$ , i.e.,  $\alpha \in [1/p, 1]$ .

**Proof of Lemma 6** Denote  $(1 - \bar{\delta}, 1 + \bar{\delta}) \equiv N(\bar{\delta})$ . Note that  $\pi_A^p(\theta, 1) = \Psi^p(F(\theta)) - \theta F(\theta)$ . As  $F(1) > \bar{Q}$  (Assumption A2), by (5),  $\Psi^p(F(1)) = F(1) + (p-1)\bar{Q}$  and  $\pi_A^p(\theta = 1, 1) = (p-1)\bar{Q} > 0$ . Hence  $\exists \bar{\delta} > 0$  satisfying  $N(\bar{\delta}) \subset [0, p]$  such that for all  $\theta \in N(\bar{\delta})$ , we have (i)  $\pi_A^p(\theta, 1) > 0$  and (ii)  $F(\theta) > \bar{Q}$ . We prove that (a)-(c) hold for all  $\theta \in N(\bar{\delta})$ .

(a) Since  $\pi_A^p(\theta, 1) > 0$  for  $\theta \in N(\bar{\delta})$ , by Lemma 5(iv)(c) it follows that  $\exists \alpha_A^p(\theta) \in [\theta/p, 1)$  such that  $\lambda(\theta, \alpha) = 1$  (i.e. the  $\varepsilon$ -agent trades with the tenant) if and only if  $\alpha \geq \alpha_A^p(\theta)$ . Using this, (23) follows from (22).

(b) Since  $F(\theta) > \bar{Q} > 0$  for  $\theta \in N(\bar{\delta})$ , by the monotonicity of  $\Psi^p(\cdot)$  it follows from (21) that  $\Omega(\theta, \alpha)$  is strictly decreasing in  $\alpha$ . The last part is proved by showing that (i)  $\Omega(\theta, 1) < 0$  and (ii)  $\Omega(\theta, \alpha_A^p(\theta)) > 0$ .

(i) Since  $\pi_A^p(\theta, 1) > 0$  for  $\theta \in N(\bar{\delta})$ , by (21), we have  $\Omega(\theta, 1) = -\pi_A^p(\theta, 1) < 0$ .

(ii) As  $\pi_A^p(\theta, \alpha_A^p(\theta)) = \Psi^p(\alpha_A^p(\theta)F(\theta)) - \theta F(\theta) = 0$ , from (21), we have

$$\Omega(\theta, \alpha_A^p(\theta)) = \Psi^p((1 - \alpha_A^p(\theta))F(\theta)) + \Psi^p(\alpha_A^p(\theta)F(\theta)) - \Psi^p(F(\theta))$$

Since  $F(\theta) > 0$  and  $0 < \alpha_A^p(\theta) < 1$ , taking  $Q_1 = (1 - \alpha_A^p(\theta))F(\theta) > 0$  and  $Q_2 = \alpha_A^p(\theta)F(\theta) > 0$ , we have  $Q_1 + Q_2 = F(\theta) > \bar{Q}$  and Lemma 2(iii) yields  $\Omega(\theta, \alpha_A^p(\theta)) = \Psi^p(Q_1) + \Psi^p(Q_2) - \Psi^p(Q_1 + Q_2) < 0$ .

(c) Follows from part (b).

(d) As  $\Omega(\theta, \alpha_A^p(\theta)) > 0$  [by (b)], (23) yields  $\Pi_\varepsilon^p(\theta, \alpha_A^p(\theta)) > \Pi^p(\theta)$ . Since  $\Omega^p(\theta, \alpha)$  is strictly decreasing in  $\alpha$ , by (23),  $\Pi_\varepsilon^p(\theta, \alpha_A^p(\theta)) > \Pi_\varepsilon^p(\theta, \alpha)$  for  $\alpha \in (\alpha_A^p(\theta), 1]$  which proves (d). ■

**Proof of Proposition 4** (I) Let  $(\alpha_\varepsilon, \beta_\varepsilon, \gamma_\varepsilon)$  be the contract offered by the landlord in an SPBE of  $G_1(\varepsilon)$  and  $\theta_\varepsilon = \gamma_\varepsilon \alpha_\varepsilon$ . Then  $\beta_\varepsilon$  equals  $\bar{\beta}(\theta_\varepsilon) = \Phi(\theta_\varepsilon) - \Phi(\alpha_0)$  that binds the tenant's participation constraint and  $(\theta_\varepsilon, \alpha_\varepsilon)$  is a maximizer of  $\Pi_\varepsilon^p(\theta, \alpha)$  over  $\mathbb{J}^p = \{(\theta, \alpha) | \alpha \in [\theta/p, 1], \theta \in [0, p]\}$  where

$$\Pi_\varepsilon^p(\theta, \alpha) = \Pi^p(\theta) + \varepsilon \lambda(\theta, \alpha) \Omega^p(\theta, \alpha)$$

(I) When  $\varepsilon = 0$ , the landlord's problem there is to choose  $\theta \in [0, p]$  to maximize  $\Pi^p(\theta)$  and it is optimal to choose  $\theta = 1$  (Prop 3). By continuity, it follows that  $\lim_{\varepsilon \downarrow 0} \theta_\varepsilon = 1$ . Now consider the constant  $\bar{\delta} > 0$  of Lemma 5. As  $\lim_{\varepsilon \downarrow 0} \theta_\varepsilon = 1$ ,  $\exists \bar{\varepsilon} \in (0, 1)$  such that for all  $\varepsilon \in (0, \bar{\varepsilon})$ ,  $\theta_\varepsilon \in N(\bar{\delta}) \equiv (1 - \bar{\delta}, 1 + \bar{\delta})$ . Then by Lemma 5(d), we have  $\alpha_\varepsilon = \alpha_A^p(\theta_\varepsilon)$  which proves part (i). Part (ii) is immediate. Parts (iii) and (iv) follow from Lemma 5(d).

(II) By part (I), for any  $\varepsilon \in (0, \bar{\varepsilon})$ , we have  $\alpha_\varepsilon = \alpha_A^p(\theta_\varepsilon)$  and hence  $\lim_{\varepsilon \downarrow 0} \alpha_\varepsilon = \lim_{\varepsilon \downarrow 0} \alpha_A^p(\theta_\varepsilon)$ . Since  $\lim_{\varepsilon \downarrow 0} \theta_\varepsilon = 1$ , by continuity, it follows that  $\lim_{\varepsilon \downarrow 0} \alpha_\varepsilon = \alpha_A^p(1)$ . Hence the unique robust contract is given by  $(\alpha, \beta, \gamma)$  where  $\alpha = \alpha_A^p(1)$ ,  $\beta = \Phi(1) - \Phi(\alpha_0)$  and  $\gamma = 1/\alpha_A^p(1)$ . Since  $\alpha_A^p(1) \in [1/p, 1)$  (Lemma 4), we have  $0 < \alpha_A^p(1) < 1$ , proving that the unique robust contract is a share contract. ■

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