Gutiérrez-Romero, Roxana

Conference Paper

The impact of Inequality on Economic Growth: Evidence for Mexico 1895-1994

Proceedings of the German Development Economics Conference, Göttingen 2007 / Verein für Socialpolitik, Research Committee Development Economics, No. 11

Provided in cooperation with:
Verein für Socialpolitik

Suggested citation: Gutiérrez-Romero, Roxana (2007) : The impact of Inequality on Economic Growth: Evidence for Mexico 1895-1994, Proceedings of the German Development Economics Conference, Göttingen 2007 / Verein für Socialpolitik, Research Committee Development Economics, No. 11, http://hdl.handle.net/10419/19867

The impact of Inequality on Economic Growth: Evidence for Mexico 1895-1994*

Roxana Gutiérrez-Romero

Department of Social Policy and Social Work, University of Oxford.

Email: roxana.gutierrezromero@socres.ox.ac.uk

Abstract

The aim of the paper is to explore the influences of initial inequality on the long run distribution of wealth. The paper presents two mathematical models that analyse the occupational choice of individuals in the presence of capital constraints and risk in entrepreneurial activities. The models show that inequality and particularly polarization hinder economic growth. The higher the initial level of polarization is, the lower the long run aggregate wealth of the economy and the higher the long run polarization will be. The models are calibrated using numerical simulations. The implications of the models are assessed empirically using data on economic growth, and income distribution in Mexico, during the period 1895-1994, as well as the "Doing Business" databases of the World Bank. Policy-wise it is found that a more egalitarian wealth distribution and less poverty can be achieved through wealth redistribution policies and by improving the business climate. This can be done by reducing the cost of setting-up firms (technology,

*I am grateful to Professor James Mirrlees, Professor Partha Dasgupta and Professor Tim Besley for their valuable comments. I acknowledge the financial support from CONACYT.

bureaucratic and administrative costs), introducing labour-market reforms encouraging the hiring of those typically excluded such as the poor, improving the access to credit markets by reducing the costs of creating and/or registering collateral and broadening the credit bureau coverage.

1 Introduction

Missing or imperfect credit markets have been suggested as one of the key factors that could lead to poverty and inequality traps. An important contribution within this view, is the theoretical model of Banerjee and Newman (1993). In the Banerjee and Newman model (BN model hereafter), people face an occupational choice problem. Individuals can become entrepreneurs, self-employed, employees or unemployed. However, becoming an entrepreneur requires an up front investment. Only people wealthy enough or those that can borrow will set up a business. In the presence therefore of credit constraints two types of equilibria can be reached, depending on the initial distribution of wealth. If there is a large ratio of very poor to very rich the demand for labour remains low over time, and so does the equilibrium salary. This is because entrepreneurs cannot increase their scale of production over time, due to the assumed technological constraints[1] and that credit-constrained people cannot set up new firms. In contrast, if there is a large ratio of very rich to very poor the demand for labour is high, and the economy achieves a high employment and salary equilibrium. The ratio of credit to non-credit constrained individuals then determines the equilibrium returns to occupations and hence the long run wealth distribution[2].

Although the BN model links the long run wealth distribution to the original distribution, this link depends on the condition that all entrepreneurs set up firms of exactly the same scale, irrespective of differences in their initial wealth levels[3]. Entrepreneurs cannot expand their scales of production, and therefore cannot raise salaries either. As a result growth stops and so does the process of wealth trickling down from entrepreneurs to workers.

The BN model points to the relevance of the initial wealth distribution, credit constraints and production technology in the analysis of growth and inequality. However, in order to draw concrete policy recommendations the determinants of growth and long-run wealth distribution must be analysed under a broader set of conditions, and their importance assessed.

The purpose of this article is to explore the conditions under which initial wealth inequalities determine the long run wealth distribution. Special attention is given to the analysis of the dynamics of inequality and polarisation. Esteban and Ray (1994) and Wolfson (1994)

[1] Entrepreneurs need to acquire a monitoring technology that will enable them to obtain the maximum worker effort. It is assumed that entrepreneurs will all set up the same scale of production, since technology does not allow increases in monitoring capacity.

[2] In this model, the long run distribution of wealth could be one from within a wide range of distributions over the wealth space of two very different stationary distributions, at either low or high aggregate wealth level. However, the randomness of wealth does not affect the long run distribution.

[3] Banerjee and Newman (1993) assumed that there are self-employed individuals that set up businesses at a smaller scale than entrepreneurs. However, there are no differences in scales of production among entrepreneurs nor among the self-employed.

have shown that the trends in inequality and polarisation can diverge. Nonetheless, the theoretical analyses on the dynamics of wealth often focus on the trend of overall inequality or on the welfare of a specific group(s), rather than analysing explicitly the dynamics of overall polarisation.

The article proposes two occupational choice models that consider the effects of imperfect credit markets and random entrepreneurial returns. The first model assumes that all entrepreneurs have firms of exactly the same production scale, and that there is no risk in the economy. The second model allows entrepreneurs to choose the optimum scale of their firms according to their level of wealth, assuming that the returns are not stochastic.

A series of numerical examples is also presented to illustrate the implications of these models. Although this is an artificial and rough approximation, controlling for factors that affect the occupational stratification and the conditions of the economy, it provides an insight into the main forces that drive inequality, polarisation and the process of wealth accumulation[4].

There are three important findings from the models presented here. First, initial wealth distribution plays a decisive role in the long run distribution of wealth, when the earnings that the unemployed people can generate with their own labour do not enable them to accumulate enough wealth to access the credit market. Since the creation of employment relies exclusively on the people who are non-credit constrained, the initial ratio of credit to non-credit constrained individuals determines the long run wealth distribution. Second, when initial wealth determines the long run wealth distribution, the economy can converge to three types of wealth distributions. One with high levels of wealth and perfectly egalitarian, or one with low levels of wealth and polarised or a third one with low levels of wealth and perfectly egalitarian. Third, the larger the proportion of poor people, understood as those that are credit constrained, the more likely that the second type of distribution will be reached. In the extreme case, that all the population is credit constrained, unemployment remains the only option available and the third type of distribution will be reached.

The findings from the second model are that a wealth distribution can converge to a wide range of distributions over time. Given the presence of multiple equilibria and non-linear dynamics, rather than providing a rigorous analytical description of the equilibria, the type of distributions the model predicts, and the conditions that could lead to these distributions are described. These depend on the wealth distribution of the credit and non-credit constrained individuals. The larger and the poorer the bottom of the distribution is, the lower the equilibrium salary will be. The wealthier and the more egalitarian the top of the distribution is the higher the equilibrium salary will be.

Overall the parameters in the two models show that low levels of wealth will be associated to a high cost of setting up a business, a high proportion of credit constrained people, and few and small entrepreneurs. Equilibrium salaries depend on the productivity of labour, but also on the proportion of credit constrained individuals (supply of labour) and how wealthy the proportion of the non-credit constrained individuals is. The wealthier and more numerous the entrepreneurs are, the more likely that they will expand their scales of production and raise salaries.

The implications of these models are assessed using real data from two sources. First, cross-country regressions were estimated to measure the effects of the cost of setting up a business, the cost of creating or registering collateral, the public and private bureau credit coverage, the levels of poverty, and the income share of the top decile on the size of the

[4]For other examples of how numerical simulations can be used to assess the implications of theoretical models with complex dynamics see Champernowne (1952, 1953) and Champernowne and Cowell (1999).

Gross National Income per capita. The data were taken from the 2003 and 2004 "Doing Business" databases for 88 developed and developing countries. Second, a time series for Mexico (1895-1994) was used to measure the impact of poverty, the income share decile of the top 2 deciles and population size on the levels of Gross Domestic Product per capita. The impact of productivity, the size of the labour force and the income share of the top deciles on the level of salaries was also estimated. The data were taken from Alzati (1997).

The issues introduced are analysed in seven sections. Section 2 analyses the first model which assumes that all entrepreneurs have the same scale of production and that their returns are non-stochastic. Section 3 presents a model where entrepreneurs set up the size of their firms according to their wealth level, while their returns are non-stochastic. Section 4 presents the empirical evidence. Section 3.5 presents the concluding remarks. Appendix A describes how the data for poverty were estimated for the Mexican time series 1895-1994.

2 Simple Model with Certainty

This section analyses the dynamics of wealth using a discrete time, deterministic (no randomness) model. Consider an economy there is a constant population, N, of two-period lived individuals belonging to generations of altruistic families. At the beginning of the first period each individual is given an initial wealth w_i and a unit of labour which he can use in one of the three occupations available in the economy: worker, entrepreneur and unemployed[5]. During the second period of life individuals consume, x. The proportion consumed from their wealth, $(1-\beta)$, is assumed to lie between 0 and 1. The utility preferences over consumption and bequest b, $U(x,b)$, are expressed in the following equation.

$$U(x,b) = (1-\beta)\log x + \beta \log b \tag{1}$$

Individuals choose the occupation that maximises their utility subject to their wealth constraint. It is assumed that neither the unemployed nor the workers need any starting wealth in their occupation. The unemployed can produce some fixed amount γ with their labour. A worker earns a salary s, which is endogenously determined in the labour market. To be profitable to become a worker the salary s has to be greater than or equal to γ. It is assumed that both workers and the unemployed save their initial wealth in the first period and in the second period receive the returns on savings $(1+r)w_i$ in addition to the payment for their labour.

The technology available in the economy is a Leontief production function. The entrepreneur invests in machinery at a fixed cost σ and a variable cost $\alpha\sigma$ which depends on the firm's scale of production α. The machinery depreciates, such that the net value of the machinery at time $t+1$ is $c(\sigma+\alpha\sigma)(1+r)$, where c is the proportion left after depreciation, and $(1+r)$ is the discount factor. In addition, a number of α workers is hired in the first period

The project yields a return proportional to the scale of production $\alpha\rho$ and a salary bill αs in the second period. It is assumed that entrepreneurs can only run one firm at a time given managerial and time constraints.

To become an entrepreneur either the initial wealth is equal to or greater than the start-up cost $\sigma + \alpha\sigma$ or the individual requests for credit.

[5] A similar simplification of the Banerjee and Newman (1993) model to the one presented here was developed by Ray (1998) and by Gathak and Jiang (2002). Nonetheless, the model presented here is developed using different credit market and technology specifications.

2.1 Credit Market

Banks offer a loan contract conditional on borrowers providing a collateral, which can only be machinery. Wealth is not collaretisable, given that it is a liquid asset and that entrepreneurs can easily run away with it. Then an entrepreneur will offer as a collateral the property rights of the machinery he can afford to buy with his initial wealth. It is assumed that the interest rate, r is fixed over time. The underlying assumption is that the economy analysed is a small economy subject to international interest rates that remains fixed over time.

Since there are indivisibilities in investment, a Bank will only accept as a collateral the fixed cost σ.

At time t the maximum value of the loan L_i is equal to or smaller than the value of the collateral.

$$L_i \leq \sigma \tag{2}$$

Once the project is set up and makes profits the entrepreneur could try to default on the repayment of the loan $(1+r)L_i$ given that the collateral has depreciated and at $t+1$ is just worth $c\sigma(1+r)$.

To prevent this, banks impose an ex post liability constraint to increase the expected cost of default. It is assumed that banks will seize the total value of the depreciated collateral. An entrepreneur will honour the loan if the cost of the loan is less than or equal to the net value of the collateral at time $t+1$.

$$(1+r)L_i \leq c\sigma(1+r) \tag{3}$$

The entrepreneur with initial wealth w_i and loan L_i will be able to cover the total cost of setting up a business if the following equation is fulfilled.

$$w_i - \sigma + L_i \geq \alpha\sigma \tag{4}$$

Rearranging this equation, the initial wealth required to qualify for a loan is equal to or greater than a wealth threshold denoted by w^E in eq.(5). Note that this wealth threshold does not depend on the level of salaries s nor on the returns of the project ρ.

$$w_i \geq \sigma(1-c) + \alpha\sigma = w^E \tag{5}$$

The earnings of the entrepreneur if he pays back the loan will be equal to the returns proportional to the scale of production, $\alpha\rho$, plus the value of the depreciated machinery $c(\sigma+\alpha\sigma)(1+r)$ and minus the salary bill, αs and the value of the loan $(1+r)(w_i-\sigma-\alpha\sigma)$.

To be profitable to become an entrepreneur, the entrepreneurial earnings have to be greater than or equal to the earnings of a worker or an unemployed individual.

$$\alpha\rho - \alpha s + c(\sigma+\alpha\sigma)(1+r) + (1+r)(w_i-\sigma-\alpha\sigma) \geq w_i(1+r) + s \tag{6}$$

A potential entrepreneur compares the returns that he could obtain as an entrepreneur and as a worker. From eq.(6), it follows that the maximum salary s under which it is more profitable to be an entrepreneur has to be equal to or smaller than a salary threshold denoted by $\overline{s}$ in eq.(7).

$$s \leq \frac{\alpha\rho + (\sigma+\alpha\sigma)(1+r)(c-1)}{(1+\alpha)} = \overline{s} \tag{7}$$

Note that the salary $\overline{s}$ does not depend on the level of wealth of entrepreneurs. Therefore provided that entrepreneurs have wealth $w_i \geq w^E$ all entrepreneurs will be able to pay exactly the same salaries.

To guarantee that people will be willing to become workers at the salary $\overline{s}$ it is assumed that $\overline{s}$ is strictly greater than γ.

Assumption 1 *The returns of unemployed individuals γ are strictly smaller than the maximum salary entrepreneurs are willing to offer.*

$$\gamma < \overline{s} \tag{8}$$

2.2 Labour Market

Assume an initial wealth distribution denoted by $W_t(w)$. The demand for labour D_t is given by the sum of the units of labour demanded, α, by the people that have an initial wealth $w_i \geq w^E$. The proportion of entrepreneurs is denoted by $\alpha \int_{w^E}^{\overline{w}} W_t'(w)dw$ where $W_t'(w)$ is the density function and $\overline{w}$ is the maximum wealth available in the economy.

$$D_t = \left\{ \begin{array}{ll} 0 & \text{if } s_t > \overline{s} \\ \{0, \alpha \int_{w^E}^{\overline{w}} W_t'(w)dw & \text{if } s_t = \overline{s} \\ \alpha \int_{w^E}^{\overline{w}} W_t'(w)dw & \text{if } s_t < \overline{s} \end{array} \right\} \tag{9}$$

The supply of labour Z_t is given by the number of people with initial wealth $w_i < w^E$, if the salary is within the range $\gamma \leq s < \overline{s}$. If the salary s is greater than $\overline{s}$ then all the N individuals of the economy will offer their labour.

$$Z_t = \left\{ \begin{array}{ll} 0 & \text{if } s_t < \gamma \\ \{0, \int_{\underline{w}}^{w^E} W_t'(w)dw & \text{if } s_t = \gamma \\ \{\int_{\underline{w}}^{w^E} W_t'(w)dw, N\} & \text{if } s_t \in (\gamma, \overline{s}) \\ N & \text{if } \quad s_t > \overline{s} \end{array} \right\} \tag{10}$$

Each entrepreneur hires α workers. The salary is determined in a competitive labour market. The following eq. (11) represents the equilibrium salary in different scenarios.

$$\begin{array}{ll} s_t = \overline{s} & \text{if } \alpha \int_{w^E}^{\overline{w}} W_t'(w) > \int_{\underline{w}}^{w^E} W_t'(w)dw \Rightarrow \int_{\underline{w}}^{w^E} W_t'(w)dw < \frac{\alpha}{1+\alpha} \\ s_t = [\gamma, \overline{s}] & \text{if } \alpha \int_{w^E}^{\overline{w}} W_t'(w) = \int_{\underline{w}}^{w^E} W_t'(w)dw \Rightarrow \int_{\underline{w}}^{w^E} W_t'(w)dw = \frac{\alpha}{1+\alpha} \\ s_t = \gamma & \text{if } \alpha \int_{w^E}^{\overline{w}} W_t'(w) < \int_{\underline{w}}^{w^E} W_t'(w)dw \Rightarrow \int_{\underline{w}}^{w^E} W_t'(w)dw > \frac{\alpha}{1+\alpha} \end{array} \tag{11}$$

If the demand for and supply of labour are equal, then the proportion of people that cannot set up business is exactly equal to $\frac{\alpha}{1+\alpha}$. In this case the equilibrium salary can take any value between $[\gamma, \overline{s}]$. The reason for this is that the demand for labour is inelastic to salaries. If there is an excess of demand, the only way to clear this excess is by offering the maximum salary that will make some people indifferent between becoming workers and entrepreneurs. This can be seen from eq. (5) and eq. (7). Entrepreneurs that have access to the credit market, are willing to pay any salary, hiring exactly the same number of workers α, as long as the maximum salary does not exceed $\overline{s}$. The following assumption is made in order to specify what the equilibrium salary would be when the labour demand equals the labour supply.

Assumption 2 *If the demand for and supply of labour are equal,*$\int_{\underline{w}}^{w^E} W_t'(w)dw = \frac{\alpha}{1+\alpha}$ *the equilibrium salary will be $s = \overline{s}$.*

The reason for this assumption is that if $s = \gamma$ then there will be potential workers indifferent between becoming unemployed and becoming workers. Hence, in order to secure market clearing it is assumed the highest salary $\overline{s}$ is offered. If there is excess demand, the salaries will "jump" to $\overline{s}$.

Summarising, the equilibrium salary at time t depends on the proportion of people who can and cannot set up business at time t. If this proportion changes the next period, then so will the salaries.

2.3 Static Equilibrium

The earnings of the unemployed $w_{u,t}$, the workers $w_{w,t}$, and the entrepreneurs $w_{e,t}$, given that $\gamma \leq s_t < \overline{s}_t$, can be expressed in the following three equations.

$$w_u = w_i(1+r) + \gamma \quad (12)$$

$$w_w = w_i(1+r) + s_t \quad (13)$$

$$w_e = w_i(1+r) + \psi_t \quad (14)$$

where $\psi_t = \alpha\rho - \alpha s_t - (1-c)(\sigma + \alpha\sigma)(1+r)$, $\psi_t > 0$

Assumption 3 *The returns of individuals as entrepreneurs are strictly higher than the returns individuals would earn as unemployed* $\psi_t > \gamma$ where $\psi_t > 0$.

The equilibrium salary is equal to $s_t = \gamma$ if $\int_{\underline{w}}^{w^E} W_t'(w)dw > \frac{\alpha}{1+\alpha}$. In this case the proportion $\int_{\underline{w}}^{w^E} W_t'(w)dw - \alpha \int_{w^E}^{\overline{w}} W_t'(w)$ will be unemployed. A proportion $\alpha \int_{\underline{w}}^{w^E} W_t'(w)dw$ will become workers earning $s_t = \gamma$ and a proportion $\alpha \int_{w^E}^{\overline{w}} W_t'(w)$ will become entrepreneurs.

Since the only difference among people is the level of their initial wealth, when there is unemployment those who become workers are selected randomly[6]. If $\int_{\underline{w}}^{w^E} W_t'(w)dw \leq \frac{\alpha}{1+\alpha}$ there is an excess of demand for labour, then salaries jump to the level which equates the returns of workers to entrepreneurs earnings and there is no unemployment. Then all the individuals will earn the same returns, since $s_t = \overline{s}_t$.

In the case that everyone has initial wealth $w_i \geq w^E$ the equilibrium salary will be $s_t = \overline{s}_t$. Therefore, a $\left[\frac{\alpha}{1+\alpha}\right]$ proportion of the economy will become workers and the rest entrepreneurs. If $s_t < \overline{s}_t$ the entire population will become unemployed, since no one will be willing to be hired as a worker.

However, if all the population has wealth $w_i < w^E$ then unemployment becomes the only option available. This situation may seem extreme. However in practice in certain developing regions unemployment rates can reach very high levels. This is due to the lack of job opportunities, either because people are so poor that a market cannot be sustained, or because the few wealthy potential entrepreneurs decide to migrate to more prosperous areas[7].

[6] Potential workers in this model have both the same ability to work and productivity. Hence, people that become involuntarily unemployed are chosen randomnly among all identical individuals. Alternatively, as considered by Mirrlees (1976) and Dasgupta and Ray (1986) it could be assumed that the productivity of workers is related to their wealth. At low levels of wealth, a person will have low nutrition and hence have low productivity.

[7] For instance, South Africa had the highest unemployment rates worldwide, around 28.5% in 2003. According to South African official statistics, in 1994 the total unemployment in the Northern Province reached 47%, while in the Eastern Cape, unemployment reached 45.3%. Information available at: http://www.statssa.gov.za/publications/Report-00-90-02/Report-00-90-021995.pdf

2.4 Dynamics of Wealth Distribution

The dynamics of wealth in this model can be analysed by looking at the number of individuals in each occupation and the evolution of their bequests over time. Each individual leaves a proportion β of his total earnings to his offspring, hence the future starting wealth of the child of an unemployed individual is

$$w_{u,t+1} = \beta\{w_{i,t}(1+r)+\gamma\} \tag{15}$$

The future starting wealth of the child of a worker is

$$w_{w,t+1} = \beta\{w_{i,t}(1+r)+s_t\} \tag{16}$$

The future starting wealth of the child of an entrepreneur is

$$w_{e,t+1} = \beta\{w_{i,t}(1+r)+\psi_t\} \tag{17}$$

$\psi_t = \alpha\rho - \alpha s_t - (1-c)(\sigma+\alpha\sigma)(1+r)$, where $\psi_t > 0$

The difference equations $w_{u,t+1}$, $w_{w,t+1}$,$w_{e,t+1}$ describe the evolution of wealth over time. If one thinks of the relationship between w_t and w_{t+1} as a function $w_{t+1} = f(w_t)$, then the wealth of today equals the wealth of tomorrow when w_{t+1} and w_t are equal. This common value denoted by $\widehat{w}$ is usually called a fixed point. In addition to find when wealth reaches a fixed point, it is important to analyse its stability. It is known that the conditions for the stability of the dynamics of linear recurrence relations with **constant coefficients** can be described by the following *Theorem 1.*

Theorem 1 *If $\widehat{w}$ is a fixed point of the first order recurrence equation $w_{t+1} = f(w_t) = Rw_t + a$, then $\widehat{w}$ is a stable fixed point if $-1 < R < 1$ and an unstable fixed point if $R > 1$.*

Proof. *Let υ_t be the difference between w_t and $\widehat{w}$. Then $\upsilon_t = w_t - \widehat{w}$ and $\upsilon_{t+1} = w_{t+1} - \widehat{w} = f(w_t) - \widehat{w} = f(\widehat{w}+\upsilon_t) - \widehat{w}$.*

By Taylor's theorem it follows that

$\upsilon_{t+1} \approx f(\widehat{w}) + f'(\widehat{w})\upsilon_t - \widehat{w}$.

But $\widehat{w}$ is a fixed point so $\widehat{w} = f(\widehat{w})$ and $f'(\widehat{w}) = R$. Thus $\upsilon_{t+1} \approx R\upsilon_t$.

Since R is a constant, the error υ_t decays to zero if $-1 < R < 1$. However if $R > 1$ the error υ_t continuously increases. ■

Nevertheless, Theorem 1 cannot be applied directly to analyse the dynamics of wealth distribution in the difference equations. Note that the coefficients associated to how wealth changes over time, i.e. $(w_{i,t+1} - w_{i,t})$, are not constant. The difference equations for the workers and entrepreneurs depend on the salaries, which depend on the distribution of wealth. Only the difference equation for the unemployed depends on a constant coefficient γ. However, the existence of unemployment itself depends on the proportion of people that can set up business and this also depends on the distribution of wealth.

Ignoring for a moment how the proportion of people in unemployment changes, the dynamics of wealth for the unemployed can be analysed if further assumptions are introduced. To analyse how wealth changes over time, subtract $w_{u,t}$ from both sides of the difference equation for the unemployed as shown in the following equation.

$$w_{u,t+1} - w_{u,t} = \{\beta(1+r)-1\}w_{u,t} + \beta\gamma \tag{18}$$

This equation shows that wealth accumulates over time, $w_{u,t+1} - w_{u,t} > 0$ if $w_{u,t} > \frac{\beta\gamma}{1-\beta(1+r)}$. No explicit assumptions have been made to determine whether $\frac{\beta\gamma}{1-\beta(1+r)}$ is positive or negative.

The model so far has assumed that the parameters β and γ are positive. If the denominator $1-\beta(1+r)<0$, any unemployed people even with no wealth will over time continuously accumulate wealth to ∞. It does not seem reasonable to assume that the dynasties of unemployed will become rich over time. For this reason the following assumption is being made.

Assumption 4 *A dynasty cannot become rich over time just by saving a fraction of its wealth $\beta(1+r)w_{i,t}$. Therefore it will be assumed that $\beta(1+r)<1$.*

With assumption 4, it is possible to determine the dynamics of wealth for the dynasty of unemployed people. For unemployed people with wealth $w_{u,t}<\frac{\beta\gamma}{1-\beta(1+r)}$, their wealth will increase until it reaches $\frac{\beta\gamma}{1-\beta(1+r)}$, where it will remain constant. For individuals with wealth above $\frac{\beta\gamma}{1-\beta(1+r)}$, their wealth will decrease over time down to the level $\frac{\beta\gamma}{1-\beta(1+r)}$.

The difference equation for the unemployed $w_{u,t+1}=\beta(1+r)w_{u,t}+\beta\gamma$ can be plotted as a line with an intercept equal to $\beta\gamma$ and a slope equal to $\beta(1+r)$. The relationship between $w_{u,t+1}$ and $w_{u,t}$ reaches a steady state when $w_{u,t+1}=w_{u,t}$. Graphically this happens at the intersection between the line $w_{u,t+1}=\beta(1+r)w_{u,t}+\beta\gamma$ and the 45 degree line. To check the stability of the fixed point, take any point on each side of the fixed point. Start at the selected point and draw a vertical line to the function $w_{u,t+1}=\beta(1+r)w_{u,t}+\beta\gamma$. From this point of intersection move horizontally to the 45 degree line, from this point of intersection move vertically to the function $w_{u,t+1}=\beta(1+r)w_t+\beta\gamma$ and so on. Since this path converges to the fixed point the fixed point is said to be stable or attractive.

However, there is an additional aspect that needs to be considered. If wealth converges to $\frac{\beta\gamma}{1-\beta(1+r)}$ and this wealth level is greater than or equal to the wealth threshold w^E, it follows that the dynasties of unemployed will over time accumulate enough wealth to become entrepreneurs. Hence, the dynamics of wealth for the unemployed will in fact be given by the difference equation for entrepreneurs eq.(17). It does not seem realistic though to assume that unemployed people will over time accumulate enough wealth to set up business. Hence, the following assumption is introduced.

Assumption 5 *A dynasty of unemployed people cannot accumulate wealth over time sufficient to set up business. Therefore, it is assumed that $\frac{\beta\gamma}{1-\beta(1+r)}<w^E$.*

Summarising, the wealth of the dynasties of unemployed people will over time converge to a wealth level below the wealth threshold w^E. This is assuming that none of the future generations of the unemployed become workers, which depends on the labour market.

Analysing the dynamics of wealth for entrepreneurs and workers is more complicated than for the unemployed. Their difference equations are not linear, since salaries are determined by the overall wealth distribution, and further, salaries can take two values, either γ or $\overline{s}$. Then to analyse the distribution of wealth over time it is necessary to analyse what happens to salaries over time.

Lemma 1 *The equilibrium salary determined at the initial distribution of wealth remains constant over time.*

Proof. To study the dynamics of wealth, one can analyse at the individual that is located at the population percentile $\left[\frac{\alpha}{1+\alpha}\right]$, i.e. if $\alpha=3$, then we are interested in the person at the $75th$ percentile of the distribution. Define $w_t^p\equiv Max\{w:\int_{\underline{w}}^{w^E}W_t'(w)dw\leq\frac{\alpha}{1+\alpha}\}$. Then if the wealth level of the person at the $\frac{\alpha}{1+\alpha}$ percentile is above the wealth threshold w^E,$w_t^p\geq w^E$ that will mean that there is an excess demand for labour and hence, $\int_{\underline{w}}^{w^E}W_t'(w)dw\leq\frac{\alpha}{1+\alpha}$. This will be reflected in a high equilibrium salary at time t, $s_t=\overline{s}$. Conversely, if $w_t^p<w^E$ there will be an excess supply of labour $\int_{\underline{w}}^{w^E}W_t'(w)dw>\frac{\alpha}{1+\alpha}$, which will imply a low-salary

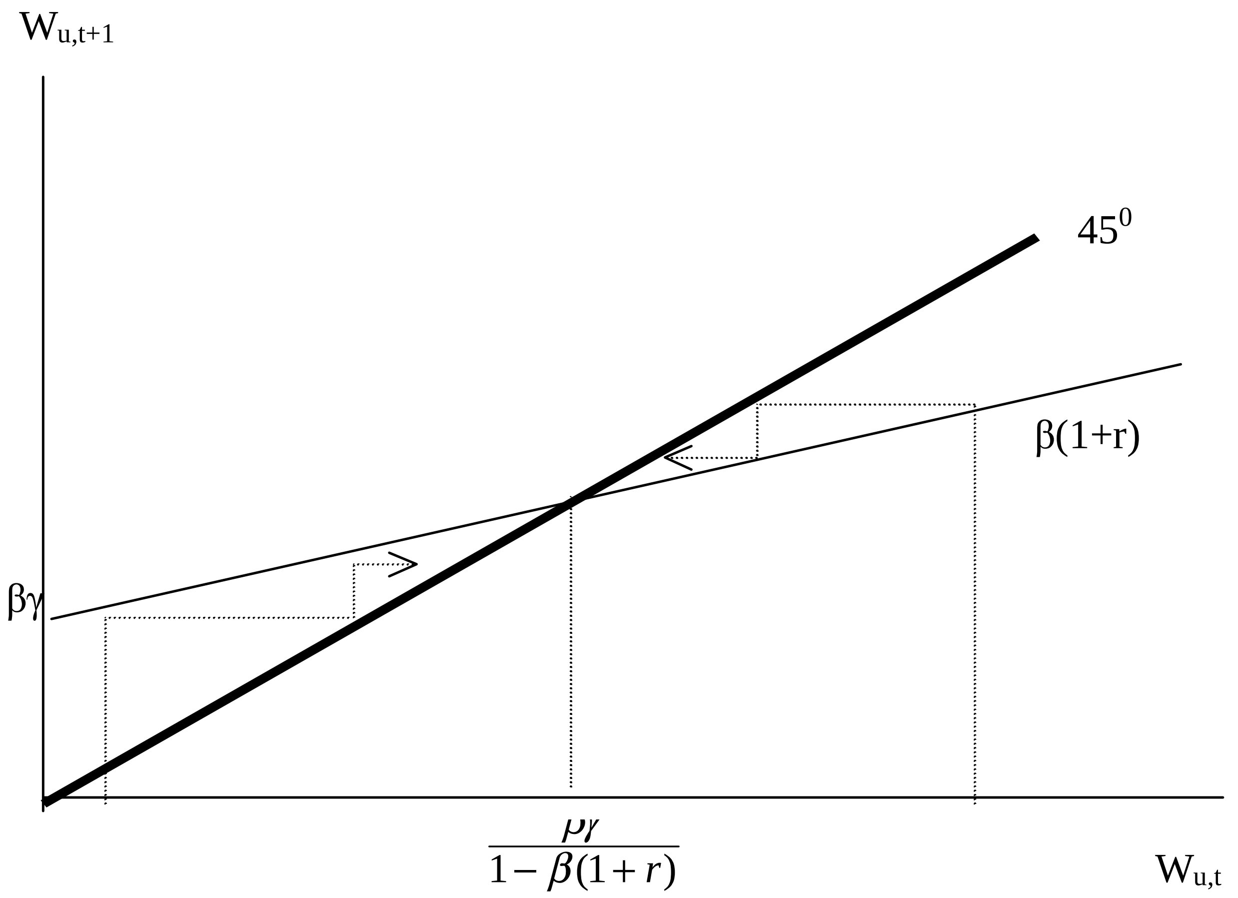


Figure 1: Dynamics of Wealth for the Unemployed

equilibrium $s_t = \gamma$.

If salaries are constant overtime, and at time t the equilibrium salary is equal to γ, then at $t+1$ the equilibrium salary cannot be equal to $\overline{s}$. Note that if $s_{t+1} = \overline{s}$, at time $t+1$ there was either excess demand for labour or the labour market cleared at $\overline{s}$. That implies that the wealth $w^p_{t+1} \geq w^E$. The wealth of the individual at the $\frac{\alpha}{1+\alpha}$ percentile w^p_{t+1} is equal to $w^p_{t+1} = \beta\{w^p_t(1+r) + s_t\}$, and this expression needs to be equal to or greater than w^E. Therefore $\beta s_t \geq w^E - \beta w^p_t(1+r)$. If in the previous period the equilibrium salary was $s_t = \gamma$, then that implies that there was an excess supply of labour and that $w^p_t < w^E$. In the extreme case that $w^p_t = w^E$, by substituting $s_t = \gamma$, it follows that for w^p_{t+1} to be equal to or greater than w^E it needs to be the case that $\frac{\beta\gamma}{1-\beta(1+r)} \geq w^E$. However, by assumption 3, this cannot happen. Hence, if $w^p_{t+1} \geq w^E$ then it can not be the case that $w^p_t < w^E$ because it would imply that $\int_{\underline{w}}^{w^E} W'_t(w)dw > \frac{\alpha}{1+\alpha}$ and $s_t = \gamma$.

Conversely, if the equilibrium salary is equal to $s_t = \overline{s}$ then in the next period salaries cannot decrease $s_{t+1} \neq \gamma$. For instance, if $s_{t+1} = \gamma$ then $w^p_{t+1} < w^E$ and $\beta\{w^p_t(1+r)+s_t\} < w^E$, which implies that $\frac{\beta s_t}{1-\beta(1+r)} < w^E$, hence the equilibrium salary is $s_t = \gamma$ and therefore $s_t \neq \overline{s}$.
■

In conclusion, the equilibrium salary at the initial distribution will remain constant over time. Therefore, the dynamics of all the occupations can be treated as difference equations with constant coefficients. From Theorem 1 and Lemma 1 it is then possible to find the condition under which the earnings in each occupation converge to a fixed point and to determine whether the fixed point is stable or unstable.

The fixed points in each occupation are found by equating the level of wealth $w_{t+1} = w_t$ in the equations (15), (16) and (17). Then the earnings by occupation will converge to $\widehat{w}_u$ for the unemployed, $\widehat{w}_w$ for workers and $\widehat{w}_e$ for entrepreneurs.

$$\widehat{w}_u = \frac{\beta\gamma}{1-\beta(1+r)} \tag{19}$$

$$\widehat{w}_w = \frac{\beta s}{1-\beta(1+r)} \tag{20}$$

$$\widehat{w}_e = \frac{\beta\psi_t}{1-\beta(1+r)} \tag{21}$$

where $\psi_t = \alpha\rho - \alpha s_t - (1-c)(\sigma + \alpha\sigma)(1+r)$, $\psi_t > 0$

The stability of these fixed points is given by the following propositions.

Proposition 1 *The long run earnings of workers, the unemployed and entrepreneurs are positive and stable if* $\beta(1+r) < 1$.

Proof. *The earnings of the unemployed, workers and entrepreneurs at time* $t+1$ *are given by eq.(19), eq.(20) and eq.(21) respectively. Since the numerators of these equations are positive* $\beta\gamma > 0$, $\beta s > 0$ and $\beta\{\alpha\rho - \alpha s + (\sigma + \alpha\sigma)(1+r)(c-1)\} > 0$ *to have positive long run earnings the denominator* $1-\beta(1+r)$ *must also be positive, therefore* $\beta(1+r) < 1$. *Since the recurrence relation between* w_{t+1} *and* w_t *decays to zero only if* $-1 < \beta(1+r) < 1$, *the fixed points are positive and stable if* $\beta(1+r) < 1$. ■

Intuitively proposition 1 implies that one cannot get richer over time by saving the bequest from his parents. It also guarantees that the fixed points are positive and stable. However, further requirements are necessary to make sure that the fixed points for each occupational group are consistent with their derivation, in other words that each individual maximises his utility subject to his wealth constraint.

In the case of the fixed points for workers and the unemployed, these need to be below the level of wealth for which it is more profitable to set up business. Otherwise in the long run they would make more profits by becoming entrepreneurs and not workers or unemployed.

Assumption 6 *The fixed point for workers' earnings lies below the wealth threshold to set up a business* $w^E > \widehat{w}_w$, *if salaries remain low over time* $s = \gamma$.

Note that if $s = \overline{s}$ then entrepreneurs and workers have the same wealth and in the long run both will be able to set up business.

Equally entrepreneurs must maximize their wealth in the long run, therefore the fixed point will be equal to or higher than the wealth threshold necessary for becoming an entrepreneur.

Assumption 7 *The fixed point for entrepreneurs' earnings is equal to or greater than the wealth threshold to set up business,* $w^E \leq \widehat{w}_e$.

Hence $\widehat{w}_e = \frac{\beta\psi_t}{1-\beta(1+r)} \geq w^E$. Since ψ_t depends on salary s_t, solving for s_t it follows that the long run equilibrium salary needs to be equal to or less than $s_t \leq \frac{\alpha\sigma+(1+c)(\frac{1}{\beta}-(1+r))-\overline{s}(1+\alpha)}{\alpha} = \widehat{s}$. The long run salary will be equal to or below the maximum salary that the economy can pay $\widehat{s} \geq \overline{s}$ if $\overline{s} \leq \frac{\alpha\sigma+(1+c)(\frac{1}{\beta}-(1+r))}{(\alpha+2\alpha^2)}$, which depends on the parameters of the economy. If $\widehat{s} < \overline{s}$, that means that although in the short run entrepreneurs are willing to offer salaries equal to

$\overline{s}$, in the long run this salary will not allow entrepreneurs to accumulate enough wealth to stay in business since $\widehat{s} < \overline{s}$ and therefore $\widehat{w}_e < w^E$.

Lemma 1 and assumptions 1-7 imply that there exists a stable stationary fixed point for each occupation. Hence it follows that,

Proposition 2 *Given any initial wealth distribution, there exists a unique stationary wealth distribution to which it converges.*

Lemma 1 and assumption 4 guarantees that the fixed points are stable for all the occupations. Assumption 1 guarantees that people are willing to work if their wealth is $w_i < w^E$ and assumptions 5-7 guarantee that there is no inconsistency between the occupational choice in the long run and individuals' stable fixed points, i.e. a worker with low salaries in the long run cannot become an entrepreneur or an entrepreneur in the long run can stay in business. However, these propositions do not rule out the existence of multiple equilibria.

Proposition 3 *The initial distribution of wealth converges to either of three types of stationary distributions*

Stationary Distribution Type 1: If $\int_{\underline{w}}^{w^E} W_t^{'}(w)dw > \frac{\alpha}{1+\alpha}$ *the long run distribution converges to two fixed and stable points. For individuals with initial wealth* $w_{i,t} < w^E$ *the fixed points converge to* $\widehat{w}_u = \widehat{w}_w = \frac{\beta\gamma}{1-\beta(1+r)}$. *For individuals with initial wealth* $w_{i,t} \geq w^E$ *their wealth converges to* $\widehat{w}_e = \frac{\beta\psi_t}{1-\beta(1+r)}$.

Stationary Distribution Type 2: If $\int_{\underline{w}}^{w^E} W_t^{'}(w)dw \leq \frac{\alpha}{1+\alpha}$ *the long run distribution converges to one fixed and stable point* $\widehat{w} = \frac{\beta\overline{s}}{1-\beta(1+r)}$.

Stationary Distribution Type 3: If $w_i < w^E$ $\forall i$ *the long run distribution converges to one fixed and stable point* $\widehat{w}_u = \frac{\beta\gamma}{1-\beta(1+r)}$ $\forall i$.

The stationary distribution type 1 will be achieved when there is an excess supply of labour, $\int_{\underline{w}}^{w^E} W_t^{'}(w)dw > \frac{\alpha}{1+\alpha}$ which implies a low-equilibrium salary. In this case, the aggregate wealth in the long run will be low and denoted by W_L. It will be given by the sum of the wealth of workers, of the unemployed, and the wealth of entrepreneurs.

$$W_L = \frac{\beta\gamma}{1-\beta(1+r)} \int_{\underline{w}}^{w^E} W_t^{'}(w)dw + \frac{\beta\psi_t}{1-\beta(1+r)} \int_{w^E}^{\overline{w}} W_t^{'}(w)dw \tag{22}$$

The aggregate wealth achieved in the long run in the stationary distribution type 2, is denoted by W_H. It is achieved when there is an excess demand for labour, or when the labour market clears at $\overline{s}$. In this case, all the population, N, accumulates the same level of wealth.

$$W_H = \frac{\beta\overline{s}}{1-\beta(1+r)}N \tag{23}$$

The economy will achieve a poor level of wealth in the long run, W_P, and a stationary distribution type 3, when no one in the economy is able to set up business. In this situation, not even the wealthiest person in the economy can afford to set up a business.

$$W_P = \frac{\beta\gamma}{1-\beta(1+r)}N \tag{24}$$

The dynamics of wealth in the first type of distribution are shown in figure (2). The individuals with wealth greater than w^E will converge to $\widehat{w}_e$, which is higher than the level of wealth for workers $\widehat{w}_w$ and unemployed $\widehat{w}_u$, where $\widehat{w}_w = \widehat{w}_u$.

Proposition 4 *The stationary distribution type 1 is more polarised in the long run than the original wealth distribution.*

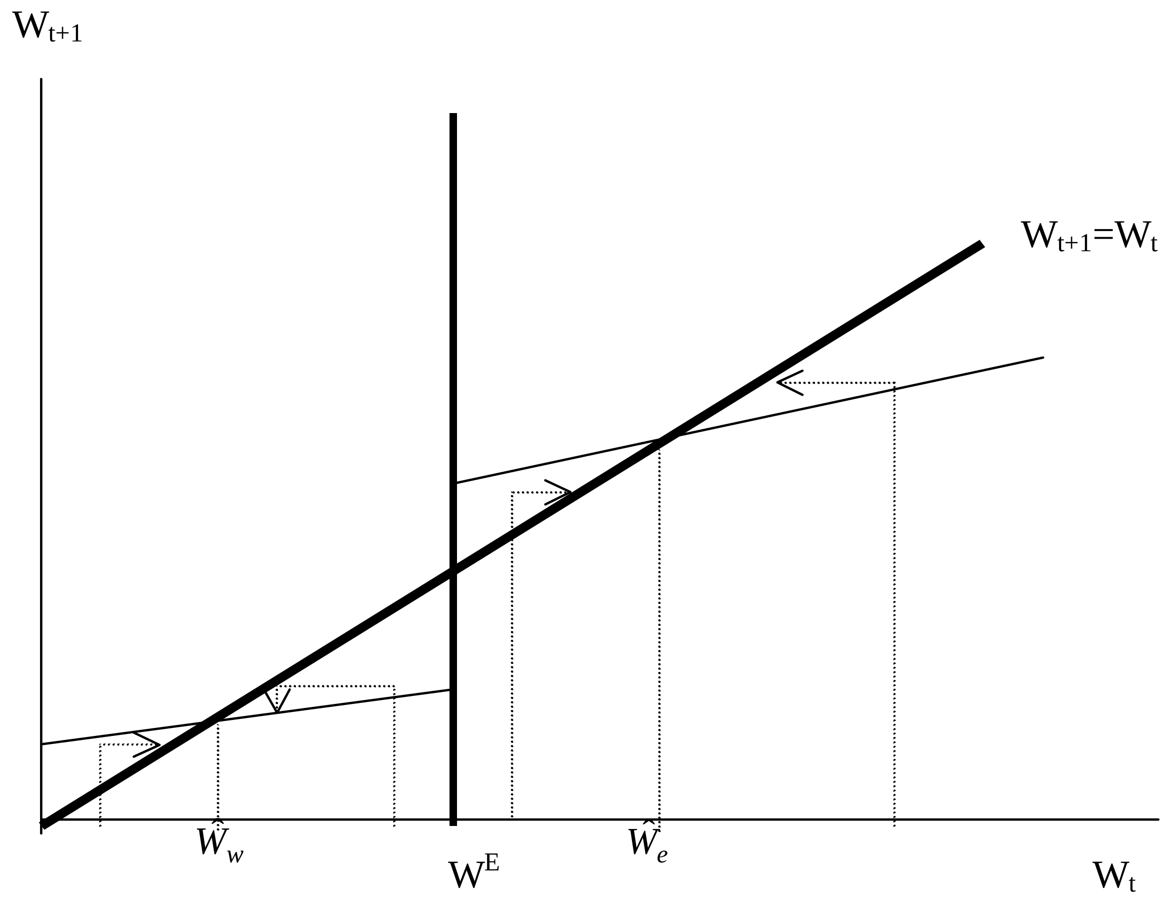


Figure 2: Dynamics of First Type of Wealth Distribution ($s = \gamma$)

Note that in the long run there will be no inequality among workers and the unemployed since their wealth will converge to the same level $\frac{\beta\gamma}{1-\beta(1+r)}$. Similarly, there will not be inequality in wealth among entrepreneurs since their wealth will converge to the same level $\frac{\beta\psi_t}{1-\beta(1+r)}$. Since inequality within these two groups is driven to zero, and the difference in wealth between these two groups increases over time, polarisation increases. Inequality will decline as well if the increase in the between group inequality outweighs the reduction in within group inequality.

The dynamics of wealth in the second type of the wealth distribution will look as shown in figure (3). The individuals with wealth greater than w^E will converge to $\widehat{w}_e$, which is the same as the level of wealth for workers $\widehat{w}_w$. There is no unemployment.

Similarly, the stationary distribution type 3 will converge to one stable fixed point like distribution type 2, with the difference that in the long run everyone is unemployed and the aggregate wealth is lower than in the second distribution.

Since the stationary distributions type 2 and 3 are characterised by no inequality in the long run, the following proposition follows:

Proposition 5 *The stationary distributions type 2 and type 3, will become perfectly egalitarian in the long run but, the stationary distribution type 2 will have a higher aggregate wealth level than the distribution type 3.*

Comparing the long run wealth levels W_L, W_H and W_P it follows that the larger is the proportion of poor people, understood as those unable to set up business, the lower the long

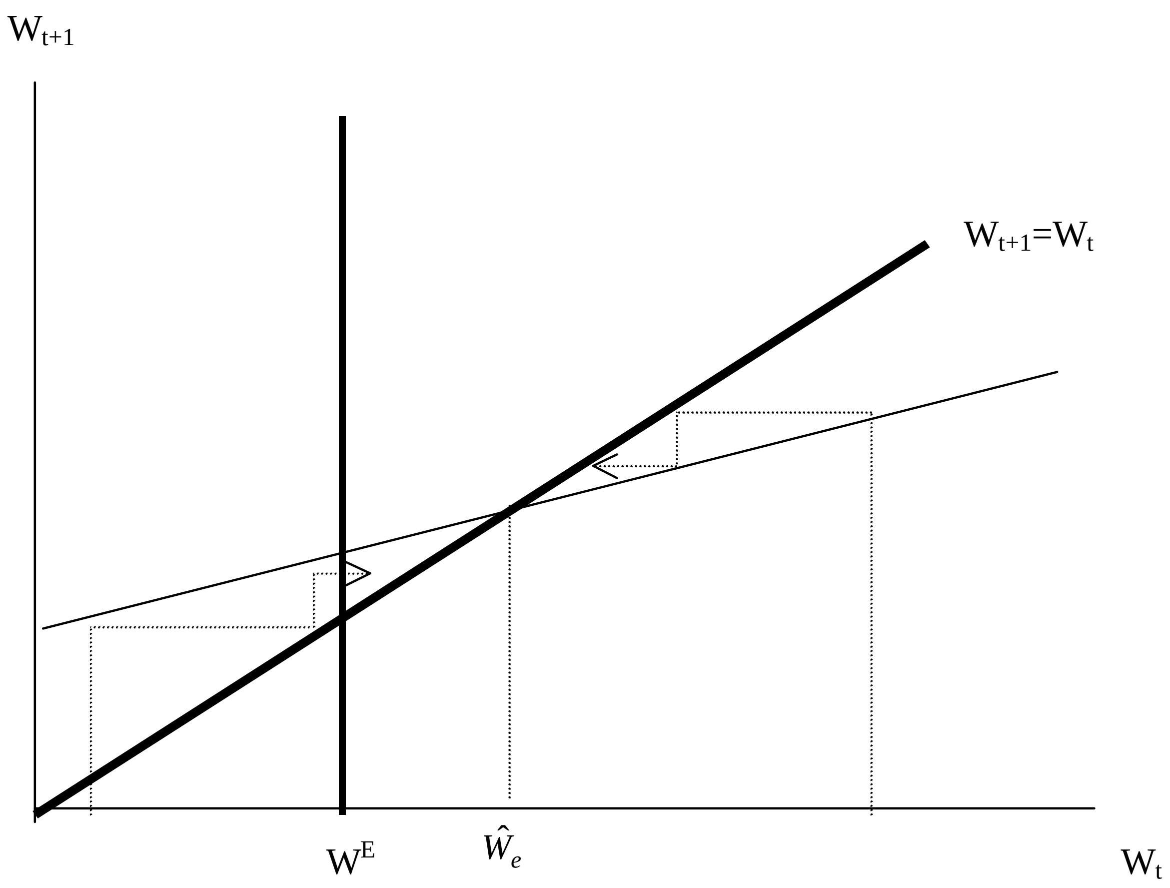


Figure 3: Dynamics of Second Type of Wealth Distribution ($s = \overline{s}$)

run wealth will be. For instance, in the case of the stationary distributions type 1 and 3, the smaller the proportion of credit constrained people, the higher the aggregate wealth level will be in the long run. The distribution type 2 achieves the highest possible aggregate wealth level in the economy, given the smaller ratio of credit to non-credit constrained people.

Proposition 6 *The larger the proportion of poor people the lower the long run wealth level will be.*

2.5 Policy Implications

One can increase the long run wealth level, or even achieve a certain stationary distribution by carrying out a series of transfers. For instance, if the economy has a low long run wealth level, W_L, a one-shot Pigou-Dalton transfer from an entrepreneur to a worker/unemployed will permanently increase the wealth level. This is the case if the entrepreneur can still stay in business, and the recipient of the transfer can afford to become an entrepreneur. Meanwhile, the highest long run wealth level, W_H can be achieved if a number of one-shot Pigou-Dalton transfers is made such that it enables the person in the percentile $\frac{\alpha}{1+\alpha}$ to become an entrepreneur. This will reduce the excess supply of labour to zero and the economy will converge to a high-equilibrium salary.

In the case of the third type of stationary distribution, one way to increase the wealth level in the long run is to carry out some non-Pigou-Dalton transfers. This is by guaranteeing funds to a certain individual to allow him to pursue entrepreneurial activities. These transfers will not be order preserving, since the wealthiest people will need to transfer wealth to the poorest individuals to enable them to set up business. Alternatively, the poorest individuals will need to transfer wealth to the wealthiest individuals to enable them to set up business. There are ways to encourage employment other than by of using these non-order preserving transfers. For instance, another way to secure employment is by allowing individuals to share wealth in order to set up joint-firms. Policy makers could reduce the costs of setting up firms, either by aiding entrepreneurs to gain access to credit, or by subsidising the cost of setting up firms such as the cost of machinery (i.e. technology).

In summary, one-shot transfers can increase the level of long run wealth, if these transfers reduce the excess supply of labour. If the excess supply of labour is driven to zero, the salaries will increase to $\overline{s}$ and an egalitarian distribution in the long run can be achieved. Note that the transfers required are one-shot transfers, that is once in the lifetime of a dynasty rather than continuous over time. Alternatively, long run aggregate wealth can be increased by changing the parameters of technology, peoples' preferences, or the cost of setting up the firm.

Table (1) summarises the effects of changes in the parameters of the model. An increase in the proportion of wealth left to offspring, β, will increase the steady state wealth level of every dynasty. Increases in the interest rate r, will increase the steady state for workers and unemployed. Increases in the interest rate will also increase the steady state for entrepreneurs as long as $\beta\alpha(\rho - s) > (1 - c)(\sigma + \alpha\sigma)(1 + r)$.

An increase in γ, the amount that the unemployed can produce with their own labour, will increase the steady state wealth level of both workers and the unemployed. This is because γ is seen as a reservation salary for workers. If the economy converges to the low salary level, then an increase in γ will reduce the profits of entrepreneurs and hence their steady state level of wealth.

An increase in the return of the firm ρ will not affect the wealth threshold w^E, but it will increase the maximum salary that entrepreneurs are willing to pay. Hence, only if the economy converges to the high salary equilibrium does the steady state wealth level increase for both workers and entrepreneurs will be increased.

An increase in the scale of production α, will require higher demand for machinery and labour, and will raise the wealth threshold necessary for setting up a business. Although this does not affect directly the steady state wealth level of workers, the proportion of people credit constrained individuals will increase, hence it will become more difficult to converge to a high-salary equilibrium. Given that the returns of a firm are proportional to the scale of production α, increases in α will increase the steady state wealth level of entrepreneurs.

The lower the cost σ, the higher the steady state wealth level of entrepreneurs $\widehat{w}_e$ and the lower w^E will be, hence the more likely it is that a high-salary equilibrium will be achieved. Similarly, a lower depreciation rate (i.e. higher c) will increase the size of loans available to entrepreneurs, reduce the wealth threshold w^E and increase the steady state for entrepreneurs $\widehat{w}_e$.

Δ	β	r	γ	ρ	α	σ	c
w^E	-	-	-	-	Δ	Δ	∇
$\widehat{w}_u$ and $\widehat{w}_w$	Δ	Δ	Δ	-	-	-	-
$\widehat{w}_e$	Δ	Δ^*	∇	Δ	Δ	∇	Δ

where Δ denotes an increase, ∇ a decrease, - no change and Δ^* denotes an increase depending on an additional constraint.

Table 1: How Changes in the Parameters Affect the Steady States by Occupations and Wealth Threshold w^E.

It is important to note in this model that the workers' salaries remain constant over time, either at the lowest or highest level. If assumption 5 did not hold, such that the unemployed and workers were able to accumulate sufficient wealth to set up business when salaries are at their lowest level, γ, then everyone would be able to set up business in the future and hence salaries would rise to $\overline{s}$. In this case the salaries would increase from γ to $\overline{s}$ once the person in the percentile $\frac{\alpha}{1+\alpha}$ accumulated enough wealth to set up business, that is $w^p \geq w^E$.

2.6 Numerical Examples

At the end of each of the models presented, a series of numerical examples will be shown to illustrate the dynamics of each of the models. In order to be able to compare the results of these numerical examples across models, only two initial wealth distributions will be used. These two distributions (defined as type I and type II), have the same mean and aggregate wealth, but type II is more unequal. Each example will provide the main statistics for the initial and long run distribution, as well as the levels of inequality and polarisation. Inequality is measured by the Gini coefficient and polarisation is measured by the modified Wolfson index (denoted by Wolfson*) proposed by Gutiérrez-Romero (2001). This modified Wolfson index instead of measuring polarisation as the income deviation from the median as proposed by Wolfson (1994), measures polarisation as the income deviation from the mean income. This index is given by the following equation,

$$\text{Wolfson}^*(p^*) = (F(\mu) - Lorenz[F(\mu)] - \frac{Gini}{2})\frac{\mu}{x}Q^* \tag{25}$$

where μ is the mean income, p^* is the population percentile whose income x takes the closest value to the mean income, $x \leq \mu$, $F(\mu)$ is the frequency density function evaluated at the mean income, $Lorenz[F(\mu)]$ is the Lorenz curve of the density function evaluated at the mean income and Q^* is a normalising factor[8].

The Wolfson* index takes the value of zero when there is perfect equality and takes the value of one when the group with income below the mean income has zero income and the group above the mean accumulates all the aggregate income, assuming that there is no within-group inequality in the two groups. The greater the distance between these two groups and the lower the within-group inequality in each group the higher the Wolfson* index will be.

Initial Wealth Distribution Type I

The characteristics of the initial wealth distribution type I are shown in table (2).

Total Wealth	Mean	Median	Min	Max	Gini	Wolfson*
273	2.73	0.25	0.24	22.63	0.38	0.14

Table 2: Wealth Distribution Type 1

[8]It is equal to the inverse of the value of the non-normalised Wolfson index, assuming that the people in or below the population percentile p^* have no income and those above p^* have distributed all the income in the economy equally among themselves.

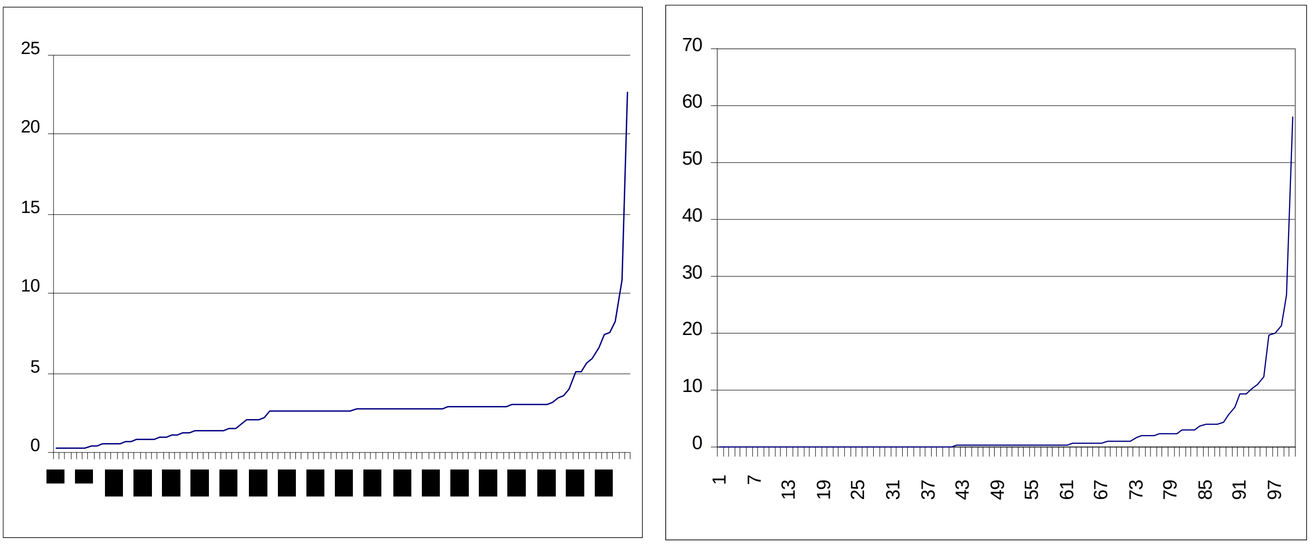
The left panel in figure (4) shows the shape of the initial wealth distribution type I. The horizontal axis measures the number of individuals and the vertical one the level of wealth.

Initial Wealth Distribution Type II

A series of **regressive transfers** (poor to rich) were carried out in the wealth distribution type I such that the total and mean wealth levels did not change. The resulting wealth distribution, denoted as type II, is more unequal. Table (3) shows the main statistics and the right panel in figure (4) shows the shape of the wealth distribution after the transfers.

Total Wealth	Mean	Median	Min	Max	Gini	Wolfson*
273	2.73	0.25	0.00001	58.00	0.83	0.66

Table 3: Initial Wealth Distribution Type II



Type I

Type II

Figure 4: Initial Wealth Distributions

Example I

This first example uses the initial distribution type I. Table (4) shows the assumed parameter values that satisfy assumptions 1-6 of the first model presented.

N	β	c	σ	r	ρ	γ	α
100	0.1	0.4	1.7	0.2	120	.05	1

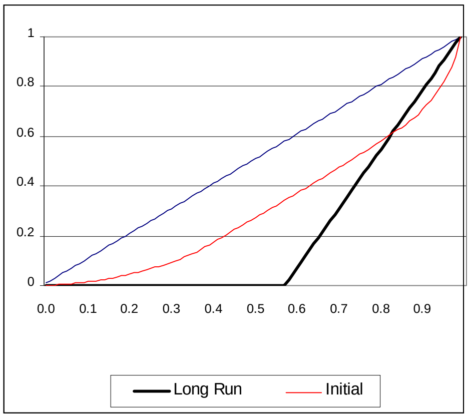
Table 4: Parameters of Example I

Given these parameters and the initial distribution of wealth assumed, the minimum wealth needed to access credit is $w^E = 2.72$. There are 41 entrepreneurs that have wealth above w^E. Since the demand for labour is less than the supply of labour the prevailing salary is equal to γ. The earnings of workers and the unemployed converge to 0.006 and the earnings of entrepreneurs to 13.35. The total wealth increased as a result of the accumulation of wealth among entrepreneurs and workers. The mean wealth increased, although the median decreased.

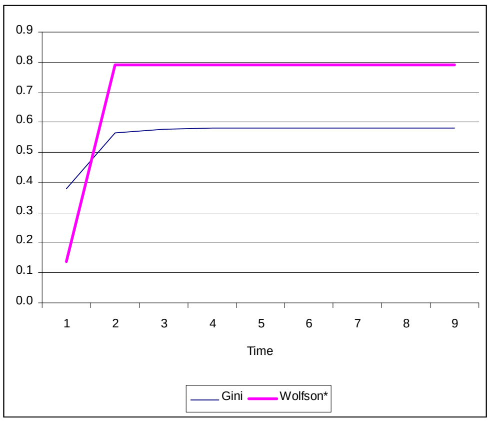
w^E	D	z	s=γ	$\hat{w}_u = \hat{w}_w$	$\hat{w}_e$	Total Wealth	Mean	Median	Min	Max	Gini	Wolfson*
2.72	41	59	0.05	0.01	13.35	560	5.60	0.01	0.01	13.35	0.58	0.82

Table 5: Long Run Wealth Distribution of Example I

Note that both inequality and polarisation increased. This can be seen in the Lorenz curves of the initial and long run wealth distributions in figure (5). The within-group inequality among workers and entrepreneurs was reduced to zero, while the inter-group inequality increased. Hence, polarisation increased. Inequality increased as a result of the increased inter-group inequality. In this example from an initial Gini coefficient of 0.38 the long run inequality converges to 0.58. The Wolfson* increased more than proportional than the Gini coefficient to converge to the level of 0.79.



Lorenz Curves



Gini and Wolfson*

Figure 5: Dynamics of Inequality and Polarisation in Example I

There are two elements to stress in this example. First, note that the maximum proportion of workers required to achieve a higher salary equilibrium is $\frac{\alpha}{1+\alpha} = \frac{1}{2}$. That is, up to half of the population can be workers in order to guarantee high-equilibrium salary. In this example, there are 59 workers out of 100 people. Transfers of wealth are one way to reduce this excess supply of labour.

Another important remark is that the dynamics of inequality depend on the lower and upper boundaries of the salary, that is γ and $\bar{s}$. If one assumes that the level of subsistence γ is higher than assumed above, but still lower than $\bar{s}$, then the long run inequality will be lower.

Example II

A second example is presented assuming the same initial distribution I and the same parameter values as in the first example, with the exception of γ that is equal to 19.

N	β	c	σ	r	ρ	γ	α
100	0.1	0.4	1.7	0.2	120	19	1

Table 6: Parameters of Example II

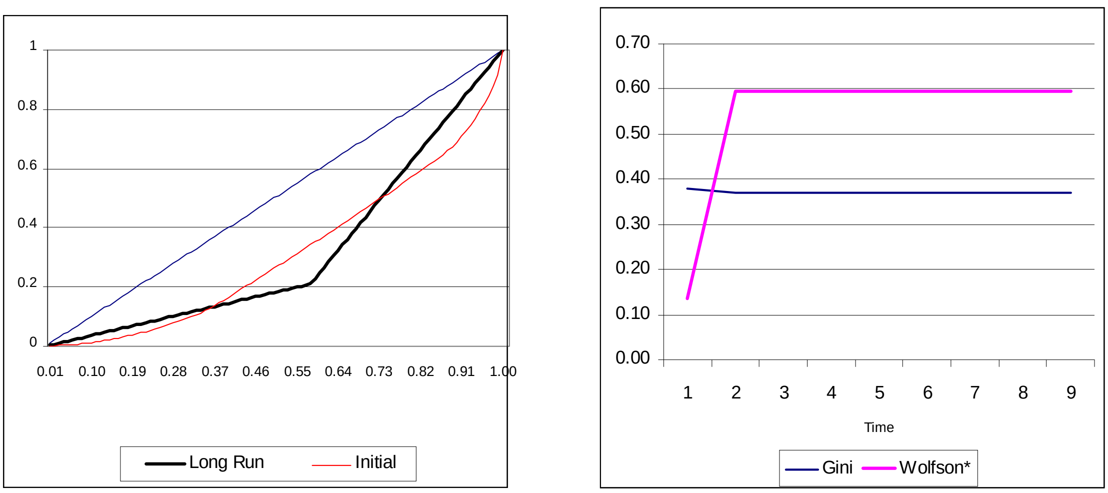
Given these parameters and the initial distribution assumed, the minimum wealth needed to access credit is the same as in the previous example $w^E = 2.72$. There are 41 entrepreneurs that have wealth above w^E. Since there is an excess supply of labour the equilibrium salary is equal to γ. The earnings of workers and the unemployed will converge to a higher level than in the previous example 2.16. The earnings of entrepreneurs converge to 11.20, a lower value than the one reached in example I. This is because in example II entrepreneurs are paying higher salaries, hence getting lower returns.

w^E	D	Z	s=γ	$\hat{w}_u = \hat{w}_w$	$\hat{W}_e$	Total Wealth	Mean	Median	Min	Max	Gini	Wolfson*
2.72	41	59	19	2.16	11.20	594	5.94	2.16	2.16	11.20	0.37	0.60

Table 7: Long Run Wealth Distribution of Example II

The long run aggregate wealth, mean and median wealth levels increase. In addition, the wealth distribution converges to a lower level of inequality and polarisation than in example I.

In contrast to the previous example, inequality decreases over time and the Lorenz curve presents a lower inequality and polarisation than in example I. Note that despite the increase in γ in example II, polarisation between the group of workers and entrepreneurs remains present (see figure (6)).



Lorenz Curves

Gini and Wolfson*

Figure 6: Dynamics of Inequality and Polarisation in Example II

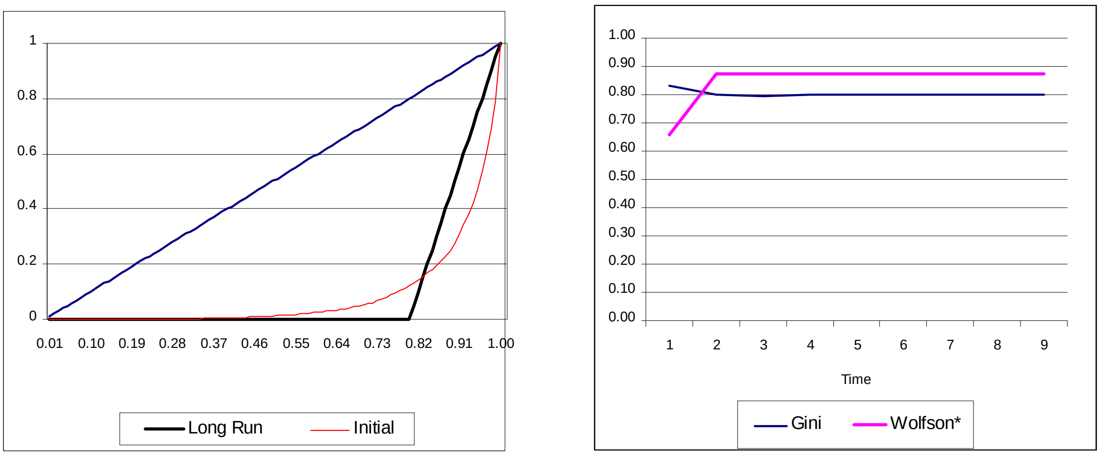
Example III

A third example is presented, which assumes the same parameter values as example I, but uses the initial distribution type II. Using this more unequal distribution, only 20 individuals are able to set up business. Since there is an excess supply of labour the equilibrium salary is equal to the subsistence level. Given that a higher proportion of individuals is credit constrained the aggregate long run wealth level is below the one in example I. The long run distribution converges to a more unequal and polarised wealth distribution.

w^E	D	z	s=γ	$\hat{w}_u = \hat{w}_w$	$\hat{w}_e$	Total Wealth	Mean	Median	Min	Max	Gini	Wolfson*
2.72	20	80	0.05	0.006	13.35	267	2.67	0.01	0.01	13.32	0.80	0.70

Table 8: Long Run Wealth Distribution of Example III

The Lorenz curve for the long run distribution of wealth shows that the inequality within workers and entrepreneurs was reduced to zero. Given that there is a large proportion of workers/unemployed this reduced the overall inequality compared to the original distribution. However, polarisation increased compared to the initial distribution. Although inequality decreased marginally, polarisation increased (see figure (7)).



Lorenz Curves

Gini and Wolfson*

Figure 7: Dynamics of Inequality and Polarisation in Example III

An egalitarian distribution can be achieved in example III either through a redistribution of wealth, or by allowing more individuals to become entrepreneurs. The next example illustrates this alternative.

Example IV

Using the initial distribution type II and the parameter values shown in table (9). The fixed cost to start up a business is reduced to $\sigma = .15$ and $\gamma = 19$.

N	β	c	σ	r	ρ	γ	α
100	0.1	0.4	0.15	0.2	120	19	1

Table 9: Parameters of Example IV

In this example, since the cost of starting up a business is reduced considerably, more people can set up a business and there is an excess demand for labour such that the equilibrium salary equals $\overline{s} = 59.9$. Then the earnings of entrepreneurs equal the earnings of workers. The long run distribution of wealth is perfectly egalitarian. In addition, the aggregate, mean and median wealth levels increase.

w^E	D	Z	s=γ	$\hat{w}_u = \hat{w}_w$	$\hat{w}_e$	Total Wealth	Mean	Median	Min	Max	Gini	Wolfson*
0.24	50	50	59.9	6.8	6.8	680	6.8	6.8	6.8	6.8	0.00	0.00

Table 10: Long Run Wealth Distribution of Example IV

Figure (**??**) shows that both polarisation and inequality are reduced to zero.

In summary, this model addresses in a simple way the dynamics of wealth, including the levels of inequality and polarisation. There are several ways in which this simplistic model could be improved to work in a more realistic scenario. One way to do this is by taking into account how risk affects the distribution of wealth.

3 Model with Firm Size Variance

The two previous models have assumed that all entrepreneurs will set up firms of exactly the same scale. This ignores the possibility that poorer entrepreneurs will set up smaller firms, and that wealthier entrepreneurs can afford to set up larger firms.

The model could be made more realistic by allowing individuals to choose different scales of production according to their wealth. The question then is whether this would affect the dynamics of wealth.

In this section it is assumed that entrepreneurs can set up firms of different scales depending on their wealth. If entrepreneurs use exclusively their wealth, they will be able to set up a scale of production $\widehat{\alpha}_i$. As in the previous two models, it is assumed that the technology available is a Leontief production function. Entrepreneurs invest in machinery at a fixed cost σ and incur a variable cost $\widehat{\alpha}_i\sigma$, which depends on the scale of production of the firm $\widehat{\alpha}_i$. A number of $\widehat{\alpha}_i$ workers are hired in the first period. The machinery depreciates, such that the net value of the machinery at time $t+1$ is $c(\sigma + \widehat{\alpha}_i\sigma)(1+r)$.

The project yields a return proportional to the scale of production $\widehat{\alpha}_i\rho$ and a salary bill $\widehat{\alpha}_i s$ in the second period.

Therefore, an entrepreneur using exclusively his initial wealth equal to w_i can set up a scale of production $\widehat{\alpha}_i$ if he covers the start up cost,

$$w_i = \sigma + \widehat{\alpha}_i\sigma \tag{26}$$

Given the scale of production $\widehat{\alpha}_i$, the net return $\widehat{\eta}_i$ of the project at time $t+1$ is the return $\widehat{\alpha}_i\rho$ plus the value of the depreciated machinery $c(\sigma + \widehat{\alpha}_i\sigma)(1+r)$, less the salary bill $\widehat{\alpha}_i s$.

$$\widehat{\eta}_i = \widehat{\alpha}_i\rho + c(\sigma + \widehat{\alpha}_i\sigma)(1+r) - \widehat{\alpha}_i s \tag{27}$$

The scale of production $\widehat{\alpha}_i$ can be increased further if the entrepreneur gets a loan.

3.1 Credit Market

Banks offer a loan contract conditional on borrowers providing collateral. This collateral is equal to the net value of the machinery, which depends on the scale of production, $\widehat{\alpha}_i$.

An entrepreneur will honour the loan if the cost of the loan is less than or equal to the net value of the collateral at time $t+1$.

$$(1+r)L_i \leq c(\sigma + \widehat{\alpha}_i\sigma)(1+r) \tag{28}$$

From eq.(26) $\sigma + \widehat{\alpha}_i\sigma$ is equal to w_i, hence the loan L_i is proportional to the wealth level of the entrepreneur,

$$L_i = cw_i \tag{29}$$

An entrepreneur with initial wealth, w_i and loan L_i will run the project at scale α_i,which is greater than $\widehat{\alpha}_i$ the scale of the project the entrepreneur would have run without the loan.

$$w_i + L_i = \sigma + \alpha_i\sigma \tag{30}$$

Solving for the scale α_i, the optimum scale of the project is given by:

$$\alpha_i = \frac{w_i + L_i - \sigma}{\sigma} \tag{31}$$

Substituting L_i, the optimal scale is,

$$\alpha_i = \frac{w_i(1+c) - \sigma}{\sigma} \tag{32}$$

The net returns η_i of entrepreneurs with loan L_i are determined by the net returns of the scale of production $\alpha_i\rho$, plus the value of the depreciated machinery $c(\sigma+\alpha_i\sigma)(1+r)$, minus the salary bill $\alpha_i s$ and minus the repayment of the loan $(1+r)L_i$.

$$\eta_i = \alpha_i\rho + c(\sigma + \alpha_i\sigma)(1+r) - \alpha_i s - (1+r)L_i \tag{33}$$

Entrepreneurs require loans only if the project yields higher returns than the net return earned from being a worker.

$$\eta_i \geq w_i(1+r) + s \tag{34}$$

Substituting η_i eq.(33), α_i eq.(32) and L_i eq.(29) into eq.(34), the initial wealth w_i, has to be equal to or greater than a wealth threshold w^A (see eq.(35)) in order for entrepreneurs to enjoy returns η_i higher than the workers' return,

$$w_i \geq \frac{\rho}{\frac{1+c}{\sigma}\rho + (1+r)c^2 - (1+r) - (\frac{1+c}{\sigma})s} = w^A \tag{35}$$

Another interpretation of eq.(34) can be given by solving for the maximum salary that the entrepreneur i can afford could still earn pay to have a higher return than workers.

$$s_i \leq \frac{w_i\{\frac{1+c}{\sigma}\rho + (1+r)c^2 - (1+r)\} - \rho}{w_i(\frac{1+c}{\sigma})} = \overline{s}_i \tag{36}$$

Note that the wealth threshold w^A varies with the level of the equilibrium salary, while the maximum salary that entrepreneurs are willing to pay $\overline{s}_i$ depends on the individuals' wealth.

At very low levels of wealth, although individuals may be able to set up a small business, individuals cannot afford to pay positive salaries. Both the wealth threshold w^A and s_i increase with respect to wealth. However, the wealth threshold w^A reaches an upper limit above which further salary increases will make the returns to entrepreneurial activities negative. This can be seen in eq. (35). When the salary is higher than or equal to the expression below, the wealth threshold w^A becomes negative and entrepreneurs make negative profits.

$$s \geq \rho + \frac{(1+r)(c^2-1)(1+c)}{\sigma} \tag{37}$$

Hence, there is a maximum level of salaries in the economy, above which further salary increases will make profits negative even for the wealthiest entrepreneur.

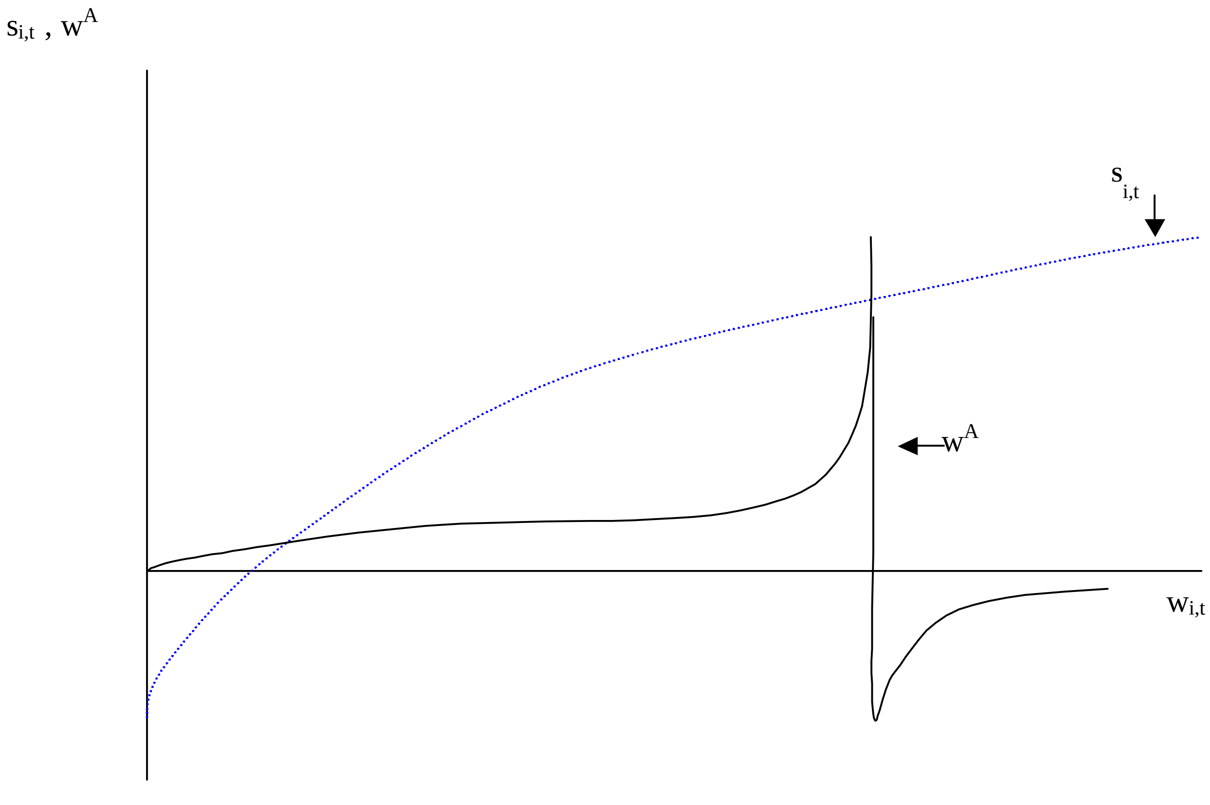


Figure 8: Salary $\overline{s}_i$ and Wealth Threshold w^A

3.2 Labour Market

Note that every entrepreneur is willing to pay different salaries, and this depends on the scale of production α_i and hence on their level of wealth. The higher the scale of production, the higher the salaries individuals are willing to pay. This is because returns are proportional to the scale of production. Hence, the larger the scale, the higher the returns and the higher the salary bill that entrepreneurs can afford to pay. The equilibrium salary in period t is obtained from labour demand, D_t and labour supply Z_t.

In order to find the equilibrium salary, define w^p as the wealth level of the individual that is willing to pay the salary s_i^* which clears the labour market, where $s_i^* \geq \gamma$. This equilibrium salary s_i^* also determines the wealth threshold w^A.

The demand for labour D_t is given by the number of entrepreneurs that have initial wealth $w_i \geq w^A$.

$$D_t = \left\{ \begin{array}{ll} 0 & \text{if } s_t > \overline{s}_j \\ \int_{w^A}^{\overline{w}} \alpha_i(w) W_t'(w) dw & \text{if } s_t \leq s_i^* \end{array} \right\} \tag{38}$$

where $\overline{s}_j$ indicates the maximum salary that the wealthiest entrepreneur is willing to pay.

The supply of labour Z_t is given by the number of people with initial wealth $w_i < w^A$, if the salary $s_i^* \leq s < \overline{s}_j$. If the salary is $s > \overline{s}_j$ all the N individuals of the economy will offer their labour.

$$Z_t = \left\{ \begin{array}{ll} 0 & \text{if } s_t < \gamma \\ \int_{\underline{w}}^{w^A} W_t'(w) dw & \text{if } s_t \geq s_i^* \\ N & \text{if } s_t \geq \overline{s}_j \end{array} \right\} \tag{39}$$

Every individual with wealth $w_i > \frac{\sigma}{(1+c)}$ will be able to set up a business with a positive scale of production, independently of the salaries offered in the labour market. However, depending on the equilibrium salary, individuals will decide whether to become workers or entrepreneurs.

If at the lowest possible salary γ, there is an excess supply of labour, then all the individuals that are able to pay a salary greater than or equal to γ will set up a business.

$$s_t = \gamma \quad if \int_{w^A}^{\overline{w}} \alpha_i(w) W_t'(w) dw < \int_{\underline{w}}^{w^A} W_t'(w), (w^A | s_t = \gamma) \tag{40}$$

If at the lowest possible salary γ, there is an excess demand for labour, then the salary will rise to the level that it will clear the labour market. This will be given by the salary s_i^* that makes the individual with wealth w^p indifferent between becoming a worker and an entrepreneur. $s_t = s_i^* > \gamma$.

$$s_t = s_i^* \quad if \int_{w^A}^{\overline{w}} \alpha_i(w) W_t'(w) dw > \int_{\underline{w}}^{w^A} W_t'(w), (w^A | s_t = \gamma) \tag{41}$$

The equilibrium salary will be equal to $\overline{s}_j$ (the salary that the wealthiest individuals in the economy are willing to pay) if the labour market clears, or if there is an excess demand for labour at $\overline{s}_j$.

Assumption 1 *As in the previous two models, the returns of unemployed individuals γ are strictly smaller than the maximum salary entrepreneurs are willing to offer.* In this model, then $\gamma < \overline{s}_j$, where $\overline{s}_j$ is the salary that the wealthies individuals in the economy are willing to pay.

Note that if only the wealthiest person in the economy is willing to set up a business, the equilibrium salary will be given by the salary that the second wealthiest person in the economy would be willing to pay. This is because the labour market is competitive, and in order to clear the labour market entrepreneurs will raise salaries up to the level that leaves the person with wealth equal to w_i^p indifferent between becoming an entrepreneur and a worker.

$$s_t = \overline{s}_j \quad if \int_{w^A}^{\overline{w}} \alpha_i(w) W_t^{'}(w)dw \geq \int_{\underline{w}}^{w^A} W_t^{'}(w), (w^A | s_t = \overline{s}_j) \tag{42}$$

Assumption 2 *As in the previous two models, if the demand for and supply of labour are equal the salaries offered will make individuals indifferent between becoming workers and entrepreneurs.*

However in this model the equilibrium salary can take any value between $[\gamma, \overline{s}_j]$ depending on the wealth distribution. The reason for this is that the demand for labour is not inelastic to salaries as in the first model. If there is excess demand for labour, the only way to clear this excess is by offering a higher salary that will make some individuals indifferent between becoming workers and entrepreneurs, such that the labour market clears. Although changes in salaries do not change the scale of production people can set up, the profits that entrepreneurs will make change. Hence, as long as the salary does not exceed the individual limit $\overline{s}_i$ people will remain in business.

3.3 Static equilibrium

The next three equations summarize the earnings of the unemployed, w_u, the workers w_w, and the entrepreneurs, w_e, given that $\gamma \leq s < \overline{s}_j$.

$$w_u = w_i(1+r) + \gamma \tag{43}$$
$$w_w = w_i(1+r) + s_t \tag{44}$$
$$w_e = w_i \mu_t - \delta_t \tag{45}$$

where $\mu_t = (\frac{1+c}{\sigma})(\rho - s_t) + (1+r)c^2$ and $\delta_t = \rho - s_t$. The term $\mu_t > 0$ and $\delta_t > 0$, since $s_t < \rho$ given that the salaries are endogenously determined to have a positive return and that $c > 0$.

Assumption 3 *The returns of individuals as entrepreneurs are strictly higher than the returns individuals would obtain as unemployed* $\delta_t > \gamma$, $\delta_t > 0$ and $\mu_t > (1+r)$.

The equilibrium salary at time t will determine the earnings of individuals in each occupation. As in the previous two models, there will be no employment if no one in the economy has wealth equal to or greater than the wealth threshold necessary to set up a business, $w_i < w^A$, not even when the salary is at the lowest level, γ.

3.4 Dynamics of Wealth Distribution

The dynamics of wealth distribution depend on the equilibrium salary, and this can take any value between $[\gamma, \overline{s}_j]$, which depends on the distribution of wealth. This indicates the existence of more equilibria than in the first two models.

To examine the dynamics of wealth for each dynasty, first, the fixed points are obtained and second their stability is analysed.

The future wealth of the child of an unemployed individual is,

$$w_{u,t+1} = \beta\{w_{i,t}(1+r) + \gamma\} \tag{46}$$

The future wealth of the child of a worker is,

$$w_{w,t+1} = \beta\{w_{i,t}(1+r) + s_t\} \tag{47}$$

The future wealth of the child of an entrepreneur with high earnings is,

$$w_{e,t+1} = \beta\{\alpha_{i,t}(\rho - s_t) + c(\sigma + \alpha_{i,t}\sigma)(1+r) - (1+r)L_{i,t}\} \tag{48}$$

Substituting the values of $\alpha_{i,t}, L_{i,t}$,

$$w_{e,t+1} = \beta\{w_t\mu_t - \delta_t\} \tag{49}$$

where $\mu_t = (\frac{1+c}{\sigma})(\rho - s_t) + (1+r)c^2$ and $\delta_t = \rho - s_t$. The term $\mu_t > 0$ and $\delta_t > 0$, since $s_t < \rho$ given that the salaries are endogenously determined to have a positive return and that $c > 0$.

Then the fixed points by occupation are given by $\widehat{w}_u$ for the unemployed, $\widehat{w}_w$ for workers and $\widehat{w}_e$ for entrepreneurs.

$$\widehat{w}_u = \frac{\beta\gamma}{1-\beta(1+r)} \tag{50}$$

$$\widehat{w}_w = \frac{\beta s}{1-\beta(1+r)} \tag{51}$$

$$\widehat{w}_e = \frac{-\beta\delta_t}{1-\beta\mu_t} \tag{52}$$

The stability of these fixed points can be analysed if further assumptions are made. Since the fixed points for the workers and the unemployed are not different than the ones in the first two models, the following assumption is made.

Assumption 4 *As in the previous two models, a dynasty cannot become rich over time just by saving a fraction of its wealth $\beta(1+r)w_{i,t}$. Therefore it will be assumed that $\beta(1+r) < 1$.*

Assumption 5 *As in the previous two models, a dynasty of unemployed people cannot accumulate wealth over time sufficient to set up business. Therefore, it is assumed that $\frac{\beta\gamma}{1-\beta(1+r)} < w^A$.*

Assumption 6 *As in the previous two models, the fixed point for workers' earnings lies below the wealth threshold to set up a business $w^E > \widehat{w}_w$, if salaries remain low over time $s_t = \gamma$.*

Assumption 7 *As in the previous two models, the fixed point for entrepreneurs' earnings is equal to or greater than the wealth threshold to set up business, $w^A \leq \widehat{w}_e$.*

The main difference with the two previous models is given by the fixed point for entrepreneurs. This can be negative for certain parameter values. To guarantee a positive wealth level for entrepreneurs the following proposition is made.

Proposition 1.1 *The long run earnings of workers and the unemployed are positive and stable if $\beta(1+r) < 1$.*Note that in contrast to proposition 1 made in the previous two models, here the long run earnings of the entrepreneurs are not guaranteed to be positive and stable if $\beta(1+r) < 1$.

Proposition 1.2 *The long run earnings of entrepreneurs have a positive fixed point if* $\beta\mu_t > 1$.

This is because the long run earnings of entrepreneurs are given by eq.(33). The numerator of this equation is negative since $-\beta\delta_t < 0$, $\beta > 0$ and $\delta_t > 0$. To secure a positive fixed point the denominator must also be negative, which implies $\beta\mu_t > 1$.

If $\beta(1 + r) < 1$ and $\beta\mu_t > 1$ and assuming that salaries are constant over time at the lowest possible value, γ, the dynamics of wealth will be as depicted in figure (9).

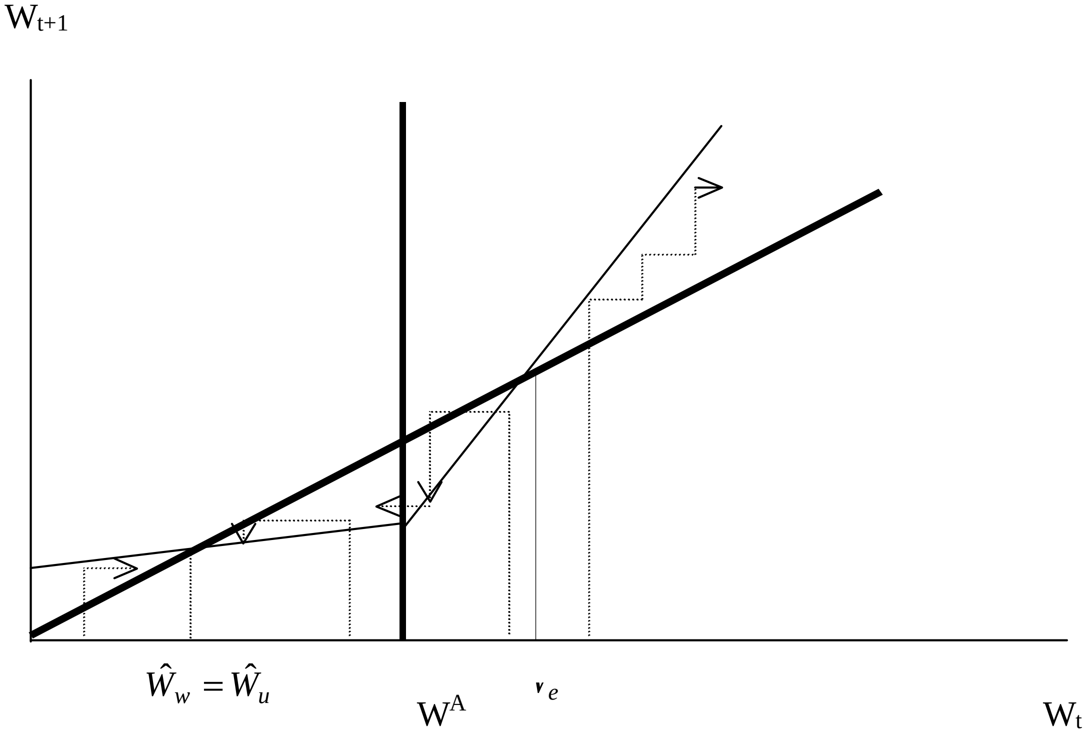


Figure 9: Dynamics of Wealth Distribution (having $s_t = \gamma$ constant)

Individuals with wealth below w^A will become workers or unemployed, while individuals with wealth above w^A will become entrepreneurs. However, the main difference with respect to the first two models is that small entrepreneurs with wealth below $\widehat{w}_e$ will "decapitalise" over time even if the salaries are kept constant at γ. For entrepreneurs with wealth above $\widehat{w}_e$ their wealth will expand towards ∞. Nonetheless, if some entrepreneurs keep expanding their firms, and others keep dropping out of business it is unlikely that salaries will remain constant.

In the case where there is a larger proportion of entrepreneurs with expanding rather than contracting wealth, salaries will increase. Figure (10) shows how an increase in salaries changes the dynamics of wealth for individuals in each occupation. The fixed wealth point for workers will increase as a result of having higher salaries. In other words the line that denotes the dynamics of wealth for workers will shift upwards from line WB towards the line WB'. Since labour is now more expensive this will reduce the rate of accumulation of wealth by entrepreneurs. As a result of the increase in labour payments the line representing the dynamics of wealth for entrepreneurs shifts from BA rightwards towards $B'A'$. The wealth threshold for becoming an entrepreneur will move from W^A to $W^{A\prime}$ and only the wealthiest entrepreneurs will remain in business, since they are the only ones that can afford to continue expanding their scale of production after the increase in salaries.

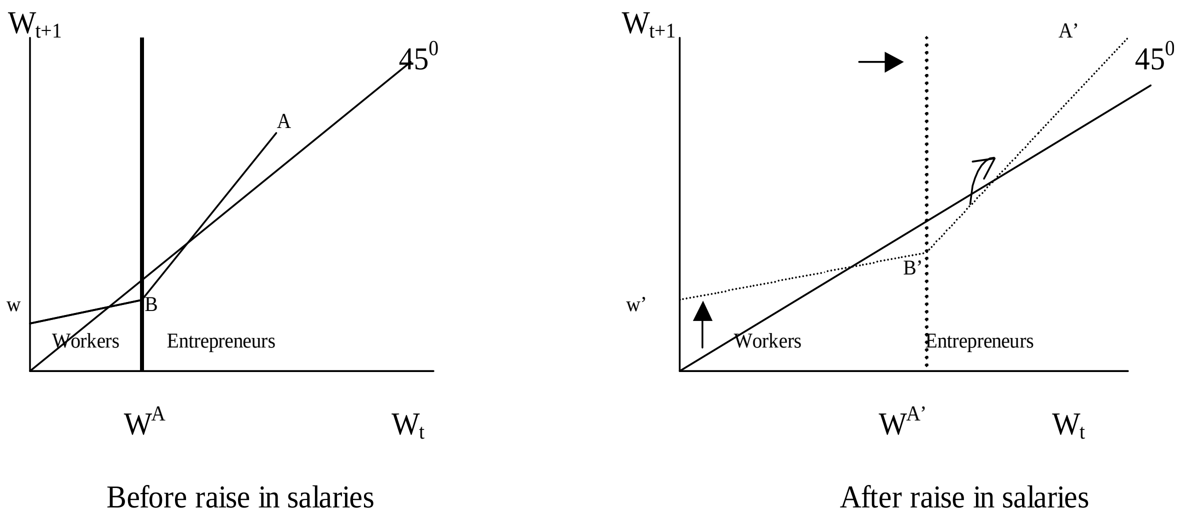


Figure 10: Dynamics of Wealth Distribution when Salaries Increase

Therefore, the dynamics of wealth will not follow a linear path, but the one presented in figure (11). The wealth of some entrepreneurs will decrease towards $-\infty$. The scale of production of these entrepreneurs' firms is too small to keep financing the cost of replacing the depreciated machinery. Therefore at some point they will be forced to shut down their business and become either workers or unemployed. In contrast, the wealth of some entrepreneurs will continuously expand towards ∞. However, the growth of their wealth will be constrained by the labour capacity of the economy. Hence, their wealth will tend towards an upper threshold denoted by the dot in the curve that intersects the 45^o line.

Since there is an upper limit to wealth expansion, the wealth threshold w^A also tends to stabilize at an upper level defined by $\overline{w}^A$.

Although the wealth of some people keeps increasing, it is not certain what happens to the dynamics of salaries. This depends on the distribution of wealth, how wealthy are the richest in the economy and the inequality among entrepreneurs.

For instance, if at the initial distribution there are already very wealthy entrepreneurs hiring almost the entire population, they will not expand their scale of production much further, nor will they raise salaries significantly. If there is no inequality among the wealthiest entrepreneurs all of them will remain in business since their willingness to pay higher salaries is the same. Nonetheless, if among the entrepreneurs there is one that is much richer, he could raise salaries to keep potential entrepreneurs out of business, in order to secure the expansion of his scale of production.

The figures (12, 13 and 14) illustrate how salaries and the wealth threshold w^A change over time. Let us order the wealth levels from the **highest to the lowest level**. Figure (12)

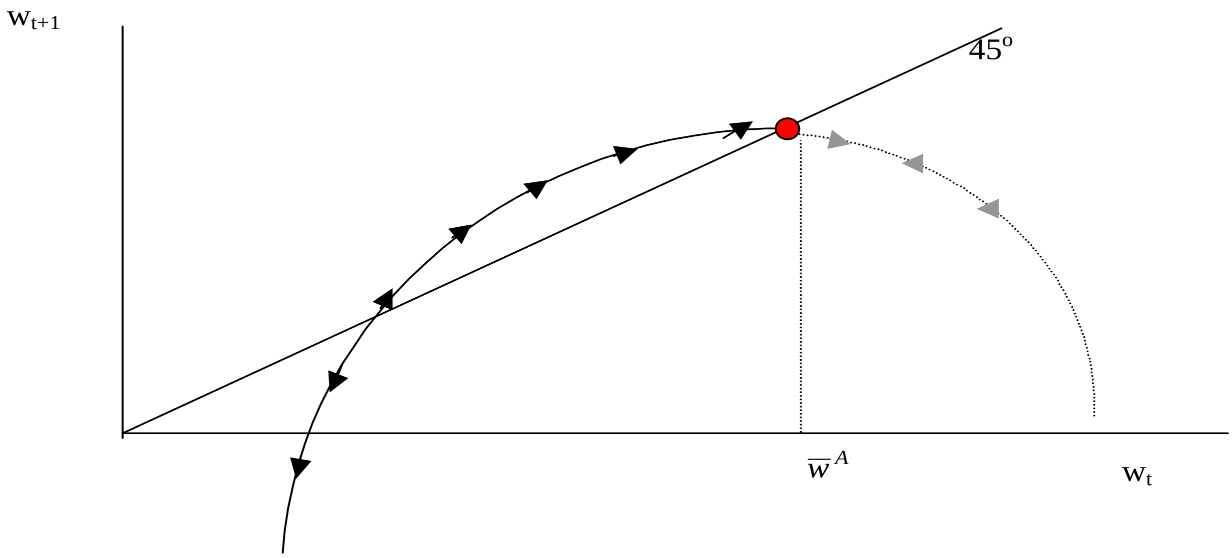


Figure 11: Long Run Wealth Threshold w^A and $\overline{w}^A$

shows the scale of production that each individual can set up according to his level of wealth at time t.

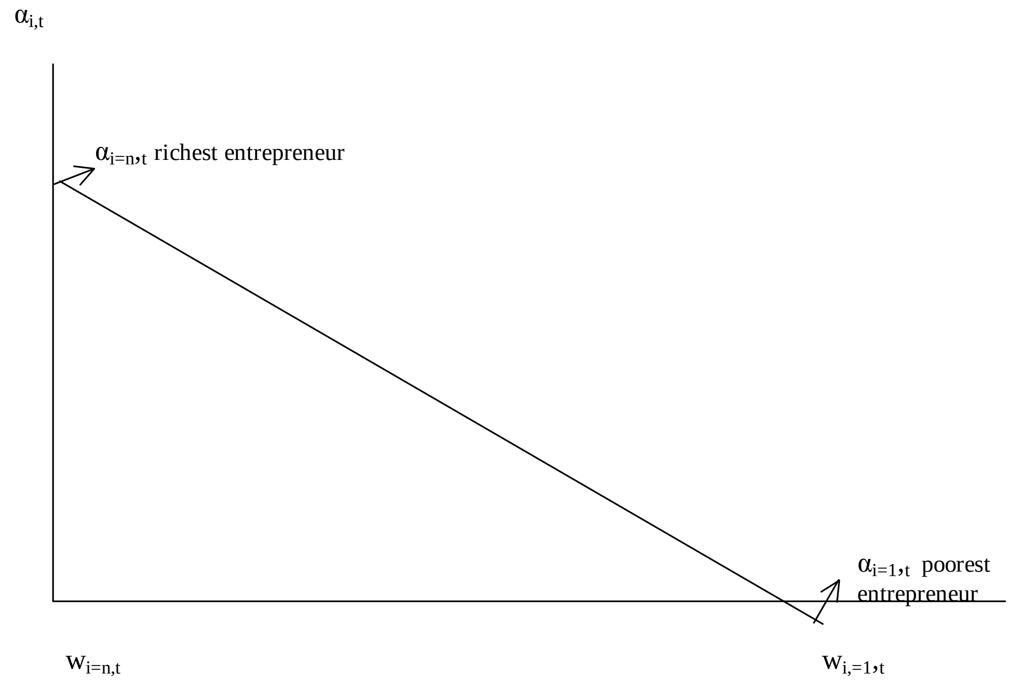
Note that the scale of production depends only on the level of wealth, $\alpha_i = \frac{w_i(1+c)-\sigma}{\sigma}$ and not on the level of salaries. The largest scale of production can only be set up by the wealthiest in the economy, while people with zero levels of wealth will be unable to set up any positive scale of production.

The potential demand for labour is obtained by adding up the different scales of production across firms. The left figure in (13), shows the accumulated demand for labour in the economy, starting from the largest scale. The right figure in (13) shows the potential supply of labour. For instance, if only the wealthiest entrepreneur sets up a business, then the supply of labour will be given by $N-1$ number of people in the economy.

Subtracting the potential supply of labour from the demand for labour, the excess of either demand or supply in the labour market is obtained. This is shown in the top figure in (14). The labour market clears when the labour demand equals the supply of labour. That will happen at the specific level of wealth $w^*_{i,t}$. If at this level of wealth an entrepreneur can afford to set up a positive scale of production, then the equilibrium salary will be given by the maximum salary that this person is able to pay. Once the salary level is established, the wealth threshold w^A can be obtained. This is shown in the bottom figure in (14).

Lemma 3 *The equilibrium salary could be characterised by either of the following 4 types of dynamics:*

Type 1: Salaries remain constant over time, at level γ. This will happen if at the initial distribution of wealth the labour market clears at the salary γ, either because there is only one entrepreneur hiring all the population, or because there are various entrepreneurs that hire the entire population and no entrepreneur can expand further his scale of production by increasing salaries. Under these two circumstances salaries will not increase over time, given that the demand for labour cannot expand any further.



Note: Wealth is ordered starting from the wealthiest to the poorest individual.

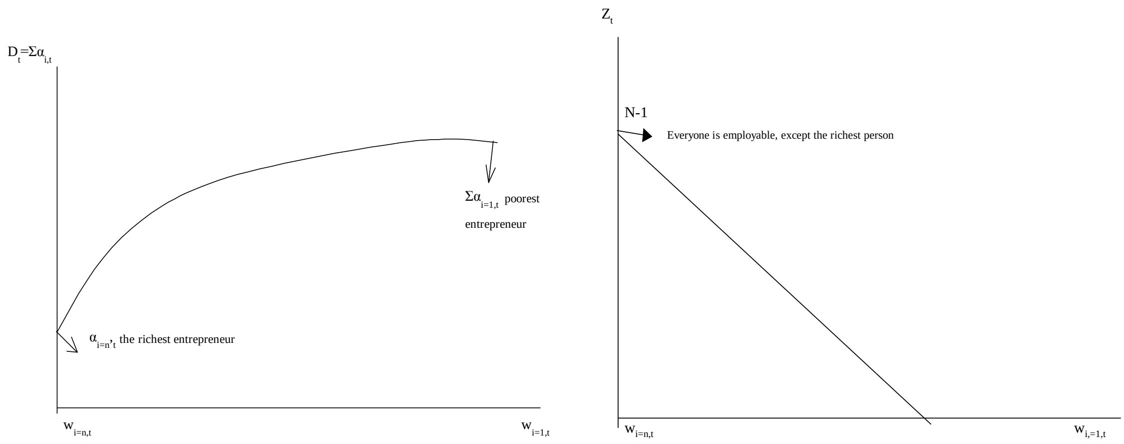
Figure 12: Labour Demand

Type 2: Salaries present fluctuation and then stabilize at any point in $[\gamma, \overline{s}_j]$. This will happen if at any equilibrium salary there is a proportion of people that drops out of business next period, while others keep expanding their wealth. The changes in salaries therefore are either positive or negative depending on whether the proportion that is expanding business is greater than, equal to or smaller than the proportion of people dropping out of business. In the long run, when the remaining entrepreneurs reach full employment, the labour demand will remain constant at a point in $[\gamma, \overline{s}_j]$.

Type 3: Salaries increase over time to then stabilize at any point in $(\gamma, \overline{s}_j]$. This will happen if the proportion of entrepreneurs expanding business is always greater than the proportion of people dropping out of business. In the long run, when entrepreneurs reach full employment, the labour demand will remain stable at a point in $(\gamma, \overline{s}_j]$.

Type 4: Salaries increase to reach a peak and then decrease over time to γ. This will happen if there are various entrepreneurs expanding business over time, but there is only one entrepreneur that keeps increasing salaries such that he hires all the population. Notice that although the richest entrepreneur behaves competitively, the equilibrium salaries are equal to the highest salary that the second wealthiest person would be willing to pay. Since the second wealthiest person is a worker, his wealth is declining over time to the fixed point $\widehat{w}_w$. Therefore, the salary that the second wealthiest individual can pay keeps falling over time, and so do the equilibrium salaries over time.

The aggregate wealth level and the distribution of wealth in the long run depend on the dynamics of salaries. The type of long run wealth distribution that can be achieved is described in proposition 2.



Note: Wealth is ordered starting from the wealthiest to the poorest individual.

Figure 13: Demand and Supply of Labour

Proposition 2 *As in the previous two models, given any initial wealth distribution, there exists a unique stationary wealth distribution to which it converges.*

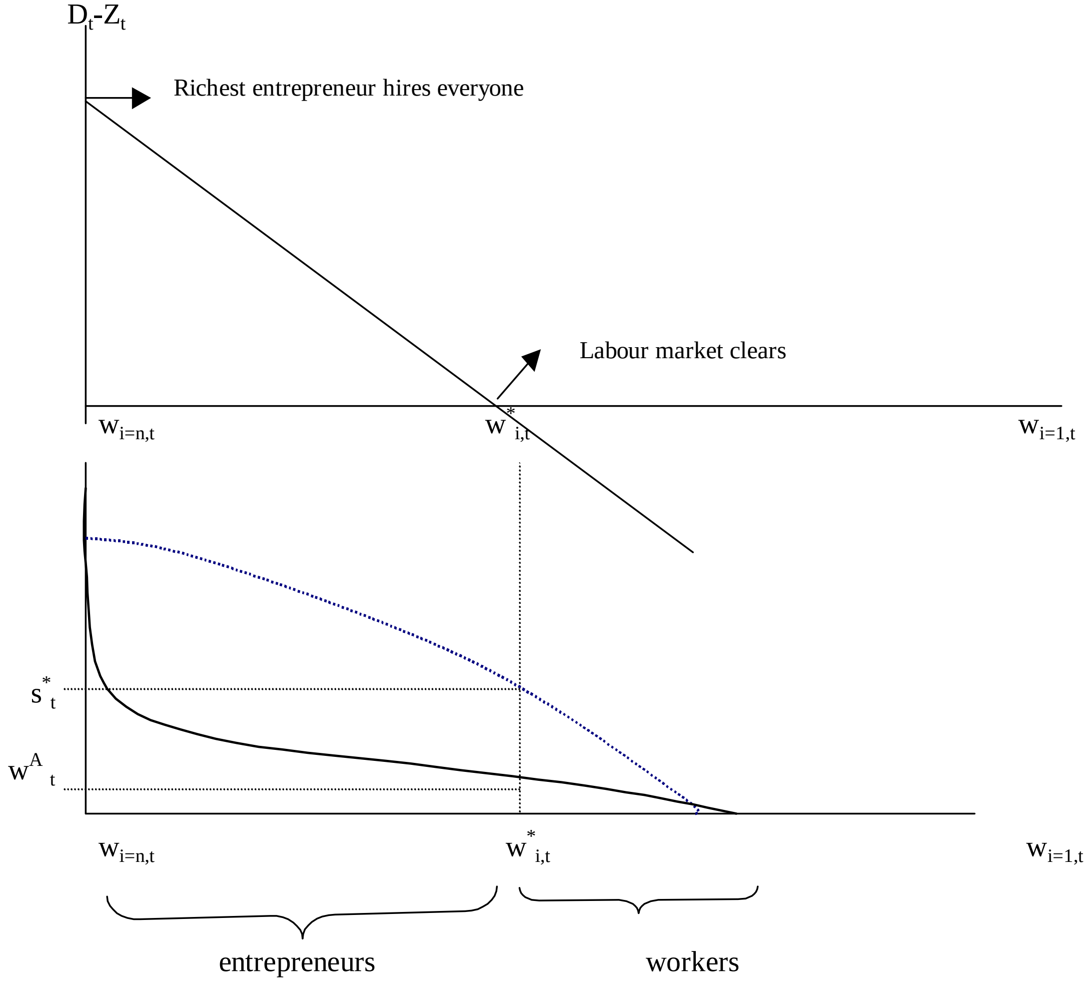
Proposition 3 *As in the previous two models, the initial distribution of wealth converges to either of three types of stationary distributions.*

Stationary Distribution Type 1: The long run distribution converges to two fixed and stable points. For individuals with initial wealth below the wealth threshold required for setting up business at the equilibrium salary $w_{i,t} < \overline{w}^A$ *the fixed points converge to* $\widehat{w}_u = \widehat{w}_w = \frac{\beta s}{1-\beta(1+r)}$. *For individuals with initial wealth equal to or greater than the wealth threshold to set up business* $w_{i,t} \geq \overline{w}^A$ *their wealth converges to* $\widehat{w}_e = \frac{\beta \psi_t}{1-\beta(1+r)}$, at which there is full employment and the scales of production cannot be expanded further even if salaries increase.

Stationary Distribution Type 2: The long run distribution converges to one fixed and stable point $\widehat{w} = \frac{\beta \overline{s}_j}{1-\beta(1+r)}$, *where both entrepreneurs and workers get the same wealth. In this case there will be full employment and the equilibrium salary will equalise the returns of entrepreneurs and workers.*

Stationary Distribution Type 3: The long run distribution converges to one fixed and stable point $\widehat{w}_u = \frac{\beta \gamma}{1-\beta(1+r)}$ $\forall i$. *This will happen if the initial wealth of all individuals is below the wealth threshold necessary for setting up business* $w_{i,t} < \overline{w}^A$ *at the lowest possible salary* γ. *Then the only option available is unemployment. Alternatively, this stationary distribution can also be reached, if at the lowest possible salary* γ, *the scale of production of all the firms is too small to keep financing the cost of replacing machinery. Therefore, in the long run all firms will shut down and unemployment will be the only option available. This will happen when the wealth of all individuals is below the fixed point* $\widehat{w}_e$ *at* $s_t = \gamma$.

In the stationary distribution type 1 the returns of entrepreneurs will be different to the returns of workers. In this case, the aggregate wealth in the long run will be low and denoted by $\overline{W}_L$. It will be given by the sum of the wealth of workers and the unemployed,



Note: Wealth is ordered starting from the wealthiest to the poorest individual.

Figure 14: Equilibrium Salary

a $W(\overline{w}^A)$ proportion of the population and by the wealth of entrepreneurs, a proportion of the population $\int_{\overline{w}^A}^{\overline{w}} W_t'(w)dw$.

$$\overline{W}_L = \widehat{w}_w \int_{\underline{w}}^{\overline{w}^A} W_t'(w)dw + \widehat{w}_e \int_{\overline{w}^A}^{\overline{w}} W_t'(w)dw \tag{53}$$

The long run aggregate wealth achieved by the stationary distribution type 2, is denoted by $\overline{W}_H$. It is achieved when there is an excess demand for labour in the long run, or when the labour market clears at $\overline{s}_j$. In this case, all the population, N, enjoys the same level of wealth.

$$\overline{W}_H = \frac{\beta \overline{s}_j}{1-\beta(1+r)} N \tag{54}$$

The economy will achieve the stationary distribution type 3 $\overline{W}_P$, when no one in the economy can afford to set up a business. In this situation, not even the wealthiest person in the economy can set up a business.

$$\overline{W}_P = \frac{\beta\gamma}{1-\beta(1+r)}N \tag{55}$$

In the stationary distribution types 2 and 3 there is no inequality in the long run.

Comparing the long run wealth levels of $\overline{W}_L$,$\overline{W}_H$ and $\overline{W}_P$ it follows that the larger is the proportion of wealthy people, understood as those who can set up a business or remain in business, the higher the salaries and the higher the long run wealth level will be. This is due to by the fact that the wealthiest entrepreneurs will be able to expand their scales of production and raise salaries at the same time. Salaries will increase faster, the less poor the non-entrepreneurs are. This is because the reservation salary is not given only by γ, but actually by the maximum salary that every individual is willing to pay, $\overline{s}_i$. The higher this is the higher the equilibrium salary will be.

3.5 Policy Implications

In contrast to the first two models presented, a one-shot Pigou-Dalton wealth transfer will not permanently increase the wealth level, even if this transfer enabled some people to set up business. The reason for this is that even if salaries remain constant over time, for small entrepreneurs wealth decreases over time, forcing them to shutdown their firm. This is because their scale of production is too small to keep financing the cost of replacing machinery. Therefore, even if some workers receive transfers to enable them to set up a business, their dynasties might not afford to keep financing the required investment in machinery.

Furthermore, given that some wealthier entrepreneurs keep expanding their scales of production salaries will rise and hence so will the salary cost for small firms. Therefore wealth transfers will increase the long run wealth only if the transfers are large enough to enable small entrepreneurs to keep increasing their scale of production, despite the increase in salaries.

In summary, a one-shot transfer of wealth will not guarantee an expansion of business over time, or a higher salary over time. To ensure the creation of firms over time, policy makers could alternatively reduce the costs of setting up firms, but similarly this reduction in costs needs to be continuous over time, to guarantee both that entrepreneurs with financial difficulties can afford to stay in business and to encourage the creation of new firms.

3.6 Numerical Examples

A series of numerical exercises based on this model is now presented. Assume the initial wealth distribution type I (shown in table (2)) and the following parameter values:

Example V

N	β	c	σ	r	ρ	γ
100	0.1	0.4	3	0.2	120	.05

Table 11: Parameters in Example VIII

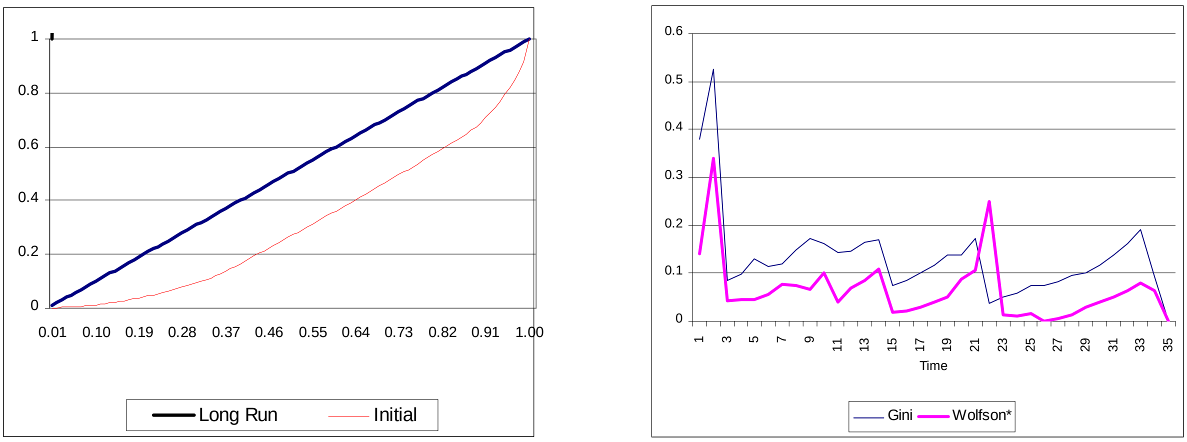
Given these parameters and the initial distribution assumed, at time $t = 0$ there is excess demand for labour at the salary level equal to γ. At that salary there are 63 individuals willing to set up business. Therefore, the salary rises to 21 and clears the labour market. Then the minimum wealth level required for access to credit is $w^A = 5.62$ and the number of entrepreneurs that have wealth above that level is $n = 54$. Given that entrepreneurs keep increasing their demand for labour, salaries also increase. In the long run there is full employment and the earnings of workers and entrepreneurs converge to the same level 10.67. In the long run the aggregate, mean and median wealth levels increase, achieving an egalitarian distribution.

$D = \Sigma\alpha_i$	z	$\hat{s}$	$\hat{w}_w$	$\hat{w}_e$	Total Wealth	Mean	Median	Min	Max	Gini	Wolfson*
$\Sigma\alpha_i$= 40, n=54 at t=0 $\Sigma\alpha_i$= 84, n=16 long run	46 at t=0 84 long run	94	10.67	10.67	1067	10.67	10.67	10.67	10.7	0	0

Table 12: Long Run Wealth Distribution of Example V

As figure (??) shows, there was an increase in salaries at the beginning in order to clear the excess demand for labour. Subsequently, the salaries remain high given that entrepreneurs keep increasing their scales of production. At time $t = 23$ there is an excess demand for labour, the salaries increase and only 16 entrepreneurs remain in business. Nevertheless, these entrepreneurs are big enough to hire the entire labour force in the long run, keeping salaries high. Eventually, the returns of entrepreneurs are equated to the earnings of workers.

With respect to inequality, at the beginning there is an increase in inequality given that 54 individuals become entrepreneurs earning more than workers. As time goes by salaries keep rising, this reduces the trend in inequality and polarisation until a completely egalitarian distribution is reached (see figure (15)).



Lorenz Curves Gini and Wolfson*

Figure 15: Dynamics of Inequality and Polarisation in Example V

Example VI

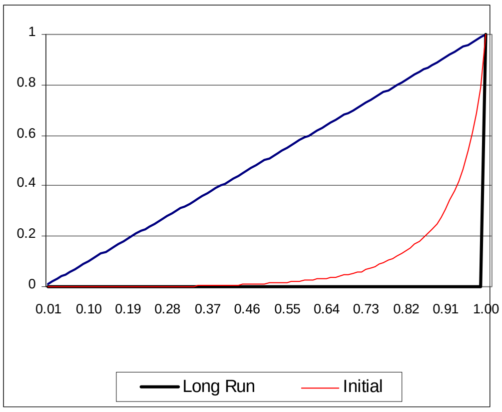
Another example is presented using the initial wealth distribution type II (shown in table (3)), and the parameter values of example VIII. With this more unequal initial wealth distribution, in the long run there is only one entrepreneur that remains in the economy, hiring all the labour force and offering the lowest possible salary.

$D=\Sigma\alpha_i$	z	$\hat{s}$	$\hat{w}_u = \hat{w}_w$	$\hat{w}_e$	Total Wealth	Mean	Median	Min	Max	Gini	Wolfson*
$\Sigma\alpha_i$= 86, n=12 at t=0 $\Sigma\alpha_i$= 99, n=1 long run	88 at t=0 99 long run	0.05	0.006	1324	1325	13.25	0.01	0.01	1324.89	0.99	0.99

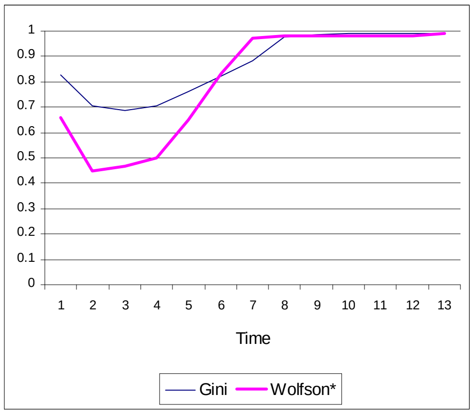
Table 13: Long Run Wealth Distribution in Example VI

In this example at time $t = 0$ there is an excess demand for labour and salaries increase to 28. In the following period, salaries increase once again, but only one entrepreneur remains in the economy hiring the entire labour force. As time goes by the only entrepreneur in the labour market reduces salaries given that he does not face any competition and that the wealth of potential entrepreneurs keeps falling over time. In the long run the salaries equal the subsistence level γ.

Inequality at the beginning decreases given the increase in salaries. However, once the only entrepreneur reduces salaries inequality increases. Aggregate wealth increases due to the increase in the wealth of the remaining entrepreneur. The wealth distribution worsens over time in terms of inequality and polarisation. The long run distribution is characterised by almost the maximum level of inequality possible, where one individual holds all the wealth in the economy (see figure (16)).



Lorenz Curves



Gini and Wolfson*

Figure 16: Dynamics of Inequality and Polarisation in Example VI

In summary, this third model addresses the dynamics of wealth assuming that entrepreneurs can vary their scales of production over time. It has been shown that even in the absence of risk in entrepreneurial returns and even if salaries remain low and constant small firms will shut down business. The next section investigates how risk affects the dynamics of wealth in this model.

3.7 Summary of the Two Models

In the first model of this article, the distribution of wealth can converge to three different types of equilibria. The initial wealth distribution will determine the long run distribution, if in the long run the earnings that the unemployed get using their own labour is below the wealth threshold necessary to set up a business, $\widehat{w}_u < w^E$. A low wealth level and bi-polar distribution will be achieved in the long run if at the starting point there is an excess supply of labour. A high wealth level and an egalitarian distribution will be achieved in the long run if at the starting point the demand equals or exceeds the supply of labour. An egalitarian long run distribution but with low levels of wealth will be achieved if at the starting point there is no one in the economy that can afford to set up a business.

In the second model, the distribution of wealth can converge to three types of "family" distributions, which depend on the initial overall wealth distribution. An egalitarian long run distribution with high levels of wealth will be achieved if many entrepreneurs can keep expanding their firms, such that the equilibrium salary equates the returns of workers and entrepreneurs. An egalitarian long run distribution, but with low levels of wealth, will be achieved if either at the starting point there is no one in the economy that can set up a business, or if all the entrepreneurs in the economy are unable to stay in business even if the salaries remain at the lowest possible level and constant. The wider the initial wealth differences between workers and entrepreneurs, the more likely that the salaries will remain low over time.

In the first model, one could increase the aggregate wealth level or achieve a specific type of distribution by making one-shot transfers, once in the life of a dynasty. In contrast, in the second model a series of one-shot transfers will not necessarily increase the aggregate wealth level, nor will it change the long run distribution. To improve the long run wealth distribution one would need to carry out a series of continuous transfers either to small entrepreneurs to enable then to stay in business, or to credit constrained people to enable them to set up business.

In the fist model, the initial number of firms and their size will remain constant over time. As a consequence, the initial equilibrium salary will remain constant over time at either a low (γ) or a high level ($\overline{s}$). In the second model, the number and size of firms will vary over time. Small entrepreneurs will drop out of business because the proportion of their returns they consume or the lost value of machinery due to depreciation means that they cannot afford the investment needed to stay in business. In contrast, medium and large firms keep expanding their scale of production. The equilibrium salary will vary depending on changes in the number and size of the firms remaining in business.

In summary, the main implications of the models analysed concern eleven issues:

1) Cross-country differences in wealth levels are determined by different historical starting points. The higher the proportion of credit-constrained people is, the lower the aggregate wealth will be.

2) Dynamics in salaries are driven by the proportion and levels of wealth of credit and non-credit constrained people. The larger the entrepreneurial group, the more likely that salaries will rise.

3) Salaries are driven by the productivity of workers, and hence by how much labour supply increases will increase returns.

4) The dynamics of inequality and polarisation can differ. In particular, the wider are the initial wealth differences between workers and entrepreneurs, the more likely that polarisation will increase more than inequality. Within-group inequalities will decline over time, but between-group inequality depends on whether equilibrium salaries reach a high level.

5) The higher the cost of setting up a business the lower the proportion of entrepreneurs will be. Then it is also more likely that a low equilibrium salary and low levels of aggregate wealth will be reached.

6) Similarly, the more difficult the access to credit markets is, the lower the proportion of entrepreneurs will be. Credit constraints are caused by high costs in obtaining an asset that can be used as a collateral, such as machinery.

The implications from the second model are:

7) Salary levels depend on how wealthy entrepreneurs are, the more wealthy they are the higher the salaries they can afford to pay, but this also depends on the proportion and wealth of workers. The poorer the workers, the lower the salaries that entrepreneurs need to offer them to clear the labour market.

8) Small or young firms are the most likely to drop out of business. This is because their scale of production is too small to afford any increase in salaries or to replace depreciated machinery.

9) Bigger or older firms will have larger scales of production and higher profits than small firms.

10) The long run size of firms depends on the entrepreneurs' initial distribution of wealth. The more unequal their initial wealth is, the larger the variance in scales of production will be.

11) Bigger firms have access to more credit, based on the collateral they can offer.

The following section uses three data sources to assess empirically some of the implications of the models.

4 Empirical Evidence

The aim of this section is not to fully validate or calibrate empirically the models presented, but to assess whether there is any empirical support for their predictions. This will be shown in terms of both the relevance of the assumptions made and the results obtained. Three data sources are used for this purpose: the "Doing Business" databases of the World Bank (2003 and 2004), historical data for Mexico from 1894 to 1994 and a micro-business survey for Mexico 1992 to 1998.

The predictions of the models that will be discussed are that the higher the levels of inequality and poverty a country has, the lower will be the long run wealth and the higher the long run levels of polarisation and inequality. In addition, "business-friendly environments" could help countries boost their economies, through low costs for setting up business and easy access to credit markets. Then the differences in wealth across-countries could be partly attributed to their different starting points and business environments.

The "Doing Business" dataset is an annual report that investigates the scope and manner of the regulations that enhance business activity and those that constrain it. The indicators included focus on the regulation for starting a small/medium-sized firm, credit market information, employment regulations, court efficiency and regulation for closing a business (bankruptcy). With regards to the regulation of entry, it includes the estimated costs of obtaining all necessary permits, licenses, verifications and notifications that will allow an entrepreneur to start a business. In addition, it includes the minimum capital requirement for starting a business. These estimated costs and minimum capital can be thought as the main costs of entry and therefore can be used to approximate the wealth thresholds necesary for running a business as discussed in the theoretical models presented. The dataset also contains information on credit registries, which are institutions that gather and disseminate information on credit histories, and on the estimated cost of creating or registering a collateral, that the lender can seize and sell. These data can be used to describe the credit market conditions and to assess the implications of the models presented.

Combining the "Doing Business" dataset with other income distribution statistics (Gross National Income (GNI), Gini coefficient, head count poverty ratio, income share of the top decile) , table (23) shows that the countries/regions with higher Gross National Income (GNI) per capita have lower levels of poverty, lower cost of setting up business and more developed credit markets. Sub-Saharan Africa, the region with the lowest GNI per capita, has the highest cost of setting up business, and the worse credit market conditions. South Asia, with the second lowest levels of GNI per capita, has a better business environment than Sub-Saharan Africa, but if compared with Europe and Central Asia has higher costs for setting up business and worse credit market conditions, in terms of the private and public bureau coverage. There are large differences in business environments between developed and developing countries. In developing countries the cost of setting up a firm (as % of GNI per capita) is 6.7 times higher than in developed countries, while the minimum capital required to set up a business is 1.5 times higher in developing countries. There is also a wide difference in the cost of creating or registering collateral, in the private and public credit bureau coverage and in the public credit registry index. This index measures how well the public credit registry rules are designed to support credit transactions. It is an average of four sub-indices (collection, distribution, access and quality) and its values can range from 0 to 100, where the higher values indicates a more extensive registry. Therefore one can see that developing countries have a considerably lower extensive registry than in developed countries.

It is important to note the limitations of this cross-country comparison. The results for each country are derived from national surveys using different definitions and measures of poverty and this limits the comparability of the indicators. Although most inequality and poverty statistics were taken from the World Bank, the definitions of poverty vary considerably among nations. For example, rich nations generally employ more generous standards of poverty than poor nations, while some countries measure poverty in terms of income rather than consumption.

Economy	GNI per Capita (2002)	Gini	Poverty	Income Share Top Decile	Cost (% of GNI per capita)	Min. Capital (% of GNI per capita)	Cost to Create Collateral (% of income per capita) 2003	Private Bureau Coverage (borrowers per 1000 capita)	Public Credit Registry (PCR) Coverage (borrowers/1000 capita)	Public Credit Registry Index
East Asia & Pacific	6,780	41.7	27.1	32.6	56.8	68.0	2	107.8	12.9	63
Europe & Central Asia	2,511	32.5	34.9	25.8	21.7	123.9	7.6	38.6	2	49
Latin America & Caribbean	2,829	50.9	43.8	39.5	70.1	85.6	19.4	196.6	53.2	50
Middle East & North Africa	4,937	36.1	20.7	29.5	54.5	410.2	18.6	14.9	3.8	43
OECD: High income	23,135	31.1	15.4	24.3	10.2	61.2	5.2	443.5	43.2	58
South Asia	464	34.7	31.9	29.5	76.3	86.1	8	1.8	0.4	46
Sub-Saharan Africa	559	45.9	53.5	36.2	255.5	237.6	41.8	36.3	0.8	39
Developed Countries	14,843	34.5	20.3	27.0	18.5	114.3	6.4	310.7	33.4	22.7
Developing Countries	909	42.6	44.5	34.0	121.4	174.6	25.0	33.0	7.9	24.9

Source: 2003 Doing Business Databases, World Bank. Cost to create Collateral: Doing Business 2004, World Bank. Gini and Income Share Decile 10: World Development Indicators 2002. World Bank. Poverty: CIA World Fact book, 2003, Poverty for Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, the Netherlands, Portugal, Spain, Sweden and the UK are taken from EUROMOD [version 27A] for 2001.

Note: Public Credit Registry Index measures how well the public credit registry rules are designed to support credit transactions. This index is an average of four sub-indices (collection, distribution, access, and quality) and its values can range from 0 to 100, where higher values indicate a more extensive registry.

Table 14: Business Environment and Income Distribution

Using the data mentioned above, a series of cross-country regressions was run to measure the impact of the cost of starting a business, size of population, levels of poverty, the income share of the top decile, and the accessibility to credit markets on the size of the Gross National Income. Although the aim of the regressions is not per se to validate directly all the implications of the models presented, they could shed light on five specific issues. First, whether countries with business-friendly environments have higher levels of wealth (GNI). Second, whether the size of the population, a rough proxy for the total labour force available in the models presented, constrains the capacity to expand wealth. Third, whether high levels of poverty and inequality deter growth. Fourth, whether the wealthier the richest are in the economy, a proxy for the wealth of the wealthiest entrepreneurs, the higher the aggregate levels of wealth will be. Fifth, whether the easier it is to access to credit markets (measured by the cost of creating or registering collateral and the credit bureau coverage) the higher the levels of aggregate wealth will be.

The regressions were estimated using the log-log specification, where all variables are measured in logarithms and the coefficients represent the elasticities of the dependent variable with respect to the explanatory variable. In the first regression, the dependent variable is the level of the GNI per capita and the explanatory variables are the log of the cost of opening a business (cost), population size (pop), poverty head count ratio (pov), and the income share of the top decile (rich). The error term is measured by ε, and k denotes countries. The standard errors used in the computation of the statistics presented in the regression tables of this article are based on traditional variance estimation methods and not robust variance estimates.

$$\log \mathrm{GNI}_k = \beta_0 + \beta_1 \mathrm{cost}_k + \beta_2 \mathrm{pop}_k + \beta_3 \mathrm{pov}_k + \beta_4 \mathrm{rich}_k + \varepsilon_k \tag{56}$$

The first regression run for 57 developing countries (table (24)), explains 55% of the differences in GNI across these countries. The coefficients of this regression show a significant elasticity between the levels of GNI and all the regressors. Additional F-tests show that all the coefficients estimated are significantly different from zero.

The coefficients show that the higher the income share of the top decile, the higher the GNI will be. The higher the cost of starting a business, the larger the population of the country and the higher the levels of poverty, the lower the GNI will be. These results coincide with the predictions of the models presented.

Number of obs	57
Dependent variable log GNI	OLS
Constant	8.06 (4.66)
log cost	-0.46 (-5.99)
log population	-0.13 (-2.14)
log poverty	-0.44 (-2.37)
log rich	1.17 (2.87)
R-squared	0.55
F-Tests of Restriction:	
log cost=0	35.93
log population=0	4.6
log poverty=0	5.62
log rich=0	8.23
Previous 4 constraints together=0	15.68

Note: T-statistics are in parentheses.

Table 15: Regression Results for Cost of Business Entry and GNI levels in Developing Countries

The Ramsey test was also performed. This test is a regression specification error test for omitted variables. The null hypothesis of the test is that the model is not misspecified (Gujarati, 1995, p.464-466). The F-statistic equal to 1.08 is not statistically significant at the 10 percent level of significance and therefore the hypothesis that the model is correctly specified cannot be rejected.

An additional regression was estimated to include the variable of public credit index (see table (25)). Not all the countries have information on this variable, so the sample consists of 30 countries only. Although the coefficient for the public credit index is positive, it is not statistically significant. Additional regressions were estimated using either the private bureau coverage or the cost to create collateral variables. Both variables had a positive coefficient (.30 and .012 respectively), but were nonetheless not significant.

Number of obs	30
Dependent variable log GNI	OLS
Constant	7.47 (2.61)
log cost	-0.50 (-2.84)
log population	-0.21 (-2.29)
log poverty	-0.36 (-1.33)
log rich	1.36 (2.28)
log pcr index	0.31 (0.74)
R-squared	0.57

Note: T-statistics are in parentheses.

Table 16: Second Regression Results for Cost of Business Entry, Credit Market, and GNI levels in Developing Countries

The same regressions were run for developed countries. With a sample of 24 countries, it was found that a higher cost of starting a business reduces the GNI (see table (26)). In addition, it was found that the size of the population and the levels of poverty are not statistically significant. Hence, these two variables were dropped from the regression. The reason for the non-significance of these two variables could be inferred from the theoretical models presented in this article. These models indicate that having a low proportion of poor people does not affect growth, as long as there is a high proportion of people able to set up business. Hence, the levels of poverty are not likely to be significant for developed countries. However, it is notorious that in contrast to developing countries, the regression showed that a higher income share of the top decile lowers the level of the GNI. A similar result was shown with the numerical examples in the case of the third model that was presented this article. The higher the proportion of rich people is the higher the demand for labour will be and hence the higher the salaries will be. However, if there is a very high concentration of wealth among the richest entrepreneurs, then increases in salaries will force small and medium-sized firms to shut down. In the long run therefore, few entrepreneurs with big firms will remain, but paying low salaries. In this particular case, a high concentration of wealth among the richest entrepreneurs will affect the wealth of aggregate level as the results of the regression seem to suggest here.

Number of obs	24	34
Dependent variable log GNI	OLS	OLS
Constant	15.48 (8.02)	15.45 9.24
log cost	-0.33 (-4.11)	-0.35 -4.05
log population	0.05 (0.55)	
log poverty	0.07 (0.35)	
log rich	-1.92 (-4.07)	-1.62 -3.13
R-squared	0.73	0.52
F-Tests of Restriction:		
log cost=0		35.93
log population=0		4.6
log poverty=0		5.62
log rich=0		8.23
Previous 4 constraints together=0		15.68

Note: T-statistics are in parentheses.

Table 17: Regression Results for Cost of Business Entry and GNI levels in Developed Countries

Two additional regressions were estimated, one including the variable of public credit index and the other the private bureau coverage. Both variables seem to be positively associated to the levels of GNI, but only the private bureau coverage is statistically significant. With a coefficient of .19, and a probability of .092. The cost to create or register collateral was also estimated. It had a coefficient of .012, and was not significant (t-statistic of .13).

Then, the regressions were estimated for both developing and developed countries. With 81 observations, the cost of starting a business and the levels of poverty were negatively related to the levels of GNI. Although the income share of the top decile was positively related to the levels of GNI, this relation was not significant. The variable of public credit index was positively related to the levels of GNI, however its coefficient was not significant. The cost to create collateral was also estimated, but its coefficient of .079 was not significant (t-statistic .92).

Number of obs	81	42
Dependent variable log GNI	OLS	OLS
Constant	11.98 (7.38)	9.64 (2.6)
log cost	-0.58 (-6.49)	-0.57 (-3.01)
log poverty	-0.86 (-4.14)	-0.65 (-2.19)
log rich	0.09 (0.17)	0.09 (0.12)
log pcr index		0.41 (0.73)
R-squared	0.65	0.60

Note: T-statistics are in parentheses.

Table 18: Regression Results for Business Environment and GNI levels in Developed and Developing Countries

Additional regressions were run to measure the effect of the minimum cost of setting up business, poverty, income share of the top decile and the cost to create collateral on the GNI per capita. With a sample of 35 developing countries, the cost to create collateral and the income share of top decile were not found to be statistically significant.

The same regression was run for developed countries. The sample consisted only of 13 countries. The only statistically significant variable was the minimum cost of setting up a firm.

Considering 44 developed and developing countries together, the cost to create collateral and the income share of the top decile were again not significant.

Summarising, the cross-country evidence presented coincides with the predictions of the theoretical models presented. The business environment has an important explanatory power on the differences in wealth across-countries. The effect of the distribution of wealth (in terms of the income share of the top decile and poverty) on the level of wealth depends on the stage of development countries are in.

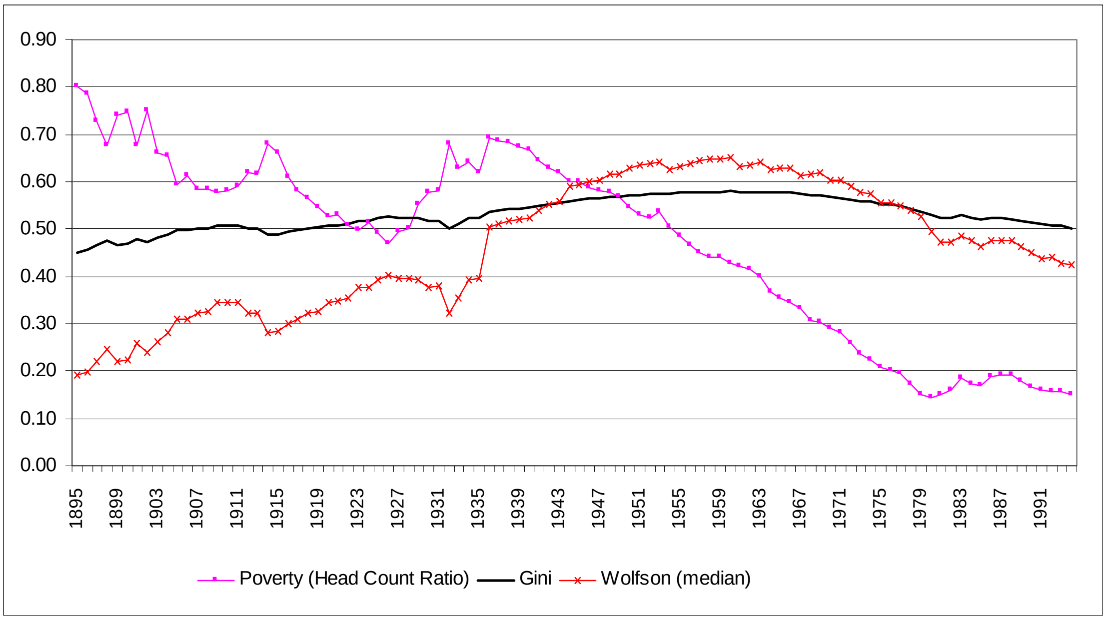
Although the data shed light on the effects of the income distribution and cost of starting business on the changes in GNI across-countries, it is not possible using these cross section data to assess the historic relationship between these variables or how the causal relationships (if any) may differ across countries.

In the next sub-section a time series for Mexico for the period 1895-1995 is used to examine the long run effects of high initial levels of inequality and poverty on wealth. Although, this time series does not include information on the cost of setting up business or access to the credit market, it sheds light on how the income distribution in Mexico might have affected growth over time.

4.1 Time Series for Mexico

The models presented suggested that high levels of initial inequality and polarisation deter growth over time. This section estimates the relationship between growth, and income distribution in Mexico, during the period 1895-1994, and can be used therefore to assess the empirical relevance of the models' predictions. The data used are the historical annual data for Mexico obtained by Alzati (1997). The dataset consists of over twenty macro-variables for Mexico. Using the income shares by decile estimated by Alzati (1997), this article estimated the poverty head count ratio, the Wolfson index proposed by Wolfson (1994) and the modified Wolfson index proposed by Gutiérrez-Romero (2001)[9].

Figure (17) shows the trends for poverty, inequality and polarisation. During this period, extreme poverty decreased from 80 percent in 1895 to 16 percent in 1994. Although inequality, measured by the Gini coefficient, increased from 0.45 in 1895 to 0.58 during the early 60s, it then decreased to reach .50 in 1994. Polarisation has been more variable than inequality. In 1895 the Wolfson index was 0.20, it reached its maximum level of .65 during the early 60s and thereafter decreased to reach 0.52 in 1994.



Source: Own estimates using the income share by deciles estimated by Alzati (1997)

Figure 17: Poverty, Inequality, Polarisation for Mexico 1895-1994

Despite the growth in GDP per capita over the period analysed, it is surprising that the levels of inequality (measured by the Gini coefficient) did not change to a great extent. In contrast, the Wolfson indices showed greater variability over the entire period.

One of the most important implications of the theoretical models presented in this article is that the changes in the wealth distribution over time are driven by the changes in the equilibrium salary. The time series analysed for Mexico shows a close relationship between the changes in the income distribution and the trend in salaries for Mexico over the period analysed (1895-1994). Specifically, the trend in poverty was inversely related to the changes in the levels of salaries for unskilled workers (increases in salaries were followed by lower

[9] The Wolfson index measures polarisation as the income deviation from the median. The modified Wolfson index measures polarisation as the income deviation from different population percentiles in the income distribution. These population percentiles are specified by the researcher and polarisation can be estimated for instance from different deciles points. The modified Wolfson index reaches its maximum level when polarisation is measured as the income deviations from the population percentile that holds the mean income.

poverty levels and vice versa)[10]. Moreover, the decline in the income share held by the top two deciles (the richest 20% of the population) was also inversely related to the changes in the levels of salaries (see figure (18)).

The dataset of Alzati (1997) also provides data on the real value of Gross Domestic Product (GDP), GDP per capita, the ratio of capital to labour, the labour force and the output per employee (labour productivity) (see figure (18)). These data provide valuable information for the implications and assumptions of the models presented. For instance, the models presented assumed a Leontief production function, which implies that there is a fixed ratio of capital to labour over time. As shown in figure (18) this ratio has changed considerably over the period considered. Note that the ratio of capital to labour increased despite the increase in the size of the labour force and labour productivity. The data for Mexico shows that entrepreneurs were unlikely to have been constrained by size of the size of the labour force. For instance, despite the increase in the labour force, both the real GDP and the GDP per capita also experienced an upward trend.

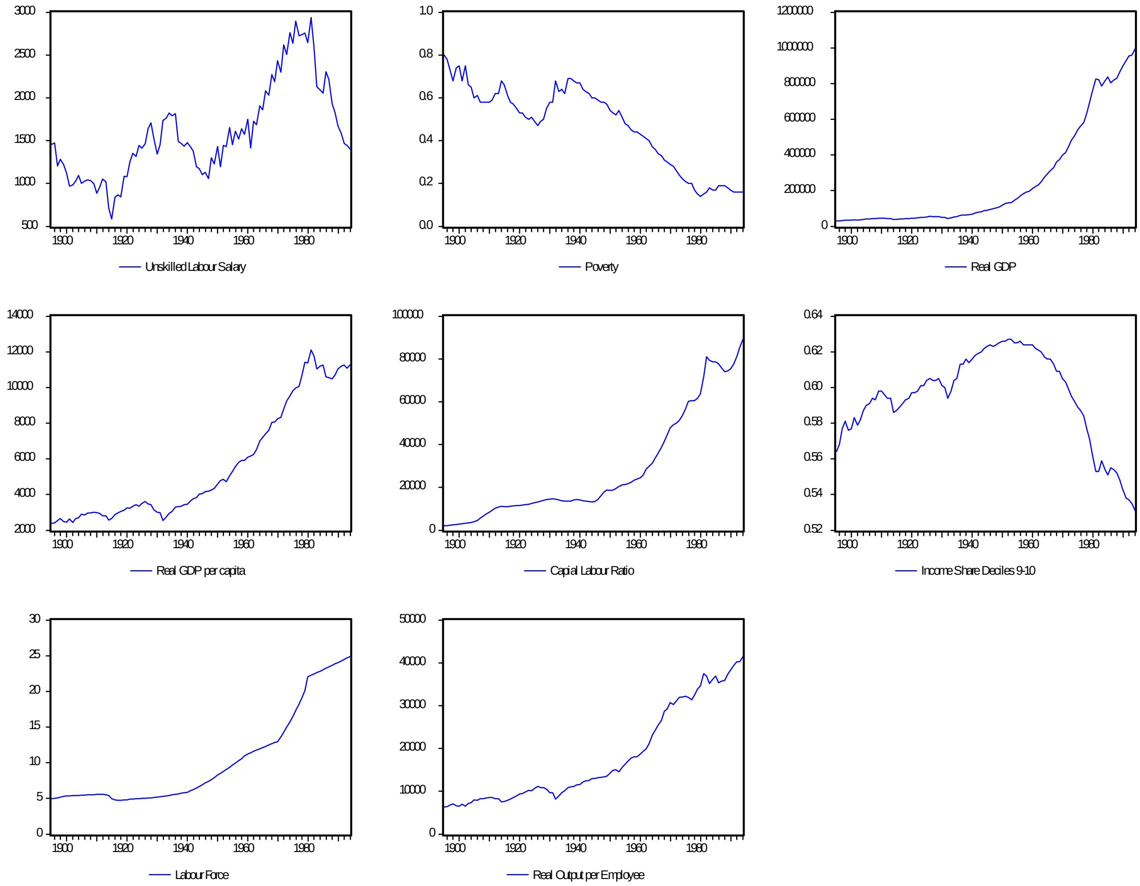
A number of time series regressions was run to assess the statistical significance of the relations between the income distribution, growth and the equilibrium salaries implied by the theoretical models. The aim of this econometric analysis is to assess two of the main implications of the models presented. First, whether the levels of poverty and the wealth of the rich in the population affect growth. Second, whether the level of salaries depends on the size of the labour force, the productivity of labour, the levels of poverty and the wealth of the rich in the population.

The first time series regression was run to measure the impact of the levels of poverty, and the income share of the deciles 9-10 on the size of the GDP per capita. All the variables were measured in logarithms. The results show that increases in the levels of extreme poverty deterred the growth of the GDP per capita, while the income share of the deciles 9-10 was associated to increments in the levels of GDP per capita. The Durbin-Watson statistic shows that the residuals are correlated with their own lagged values. Therefore a first-order autoregressive specification was added to the regression model.

Number of obs	99
Dependent variable log Real GDP	OLS
Constant	8.19 (8.64)
LOG Extreme Poverty	-0.60 (13.36)
LOG Income Share Decile 9-10	1.92 (4.15)
AR(1)	1.01 (104.06)
R-squared	0.99
Durbin-Watson stat	1.98

Note: T-statistics are in parentheses.

[10]The data also show that between 1895 and 1920 salaries showed an overall decreasing trend, despite the constant size of the labour force. During this period Mexico, under the dictatorship of Porfirio Díaz (1895-1910) experienced labour turmoil in the form of two major labour strikes (Cananea strike in 1906 and Rio Blanco strike in 1907), and the Mexican Revolution (1910-1920). These events could account for the downward trend in salaries.



Source: Alzati (1997). Data on Poverty own estimates using the income shares by deciles estimated by Alzati (1997)

Figure 18: Main Statistics for Mexico 1895-1994

Table 19: Regression Results for Income Distribution and Growth in Mexico

Figure (19) shows the residuals and how the regression model fits the log of GDP per capita. The trend of the GDP per capita indicates that there was a structural change during the 40s and 80s. For instance, the trend in GDP between 1895 and 1940 was relatively constant, but it boomed between 1940 to 1980. From the 1980s onwards the GDP per capita remained constant due to the recession in the 80s.

A series of test were performed to assess whether there was a strucutral change. The Chow-Breakpoint test was performed. This test confirmed that there were two breakpoints in the time series, one in 1940 and another in 1980.

Chow Breakpoint Test: 1940 1980			
F-statistic	8.206038	Probability	0
Log likelihood ratio	55.66062	Probability	0

Table 20: Chow Test

Two additional regressions were run for the periods 1895-1940 and 1940-1994. In these periods extreme poverty deterred growth in GDP per capita. During 1895-1940 the income

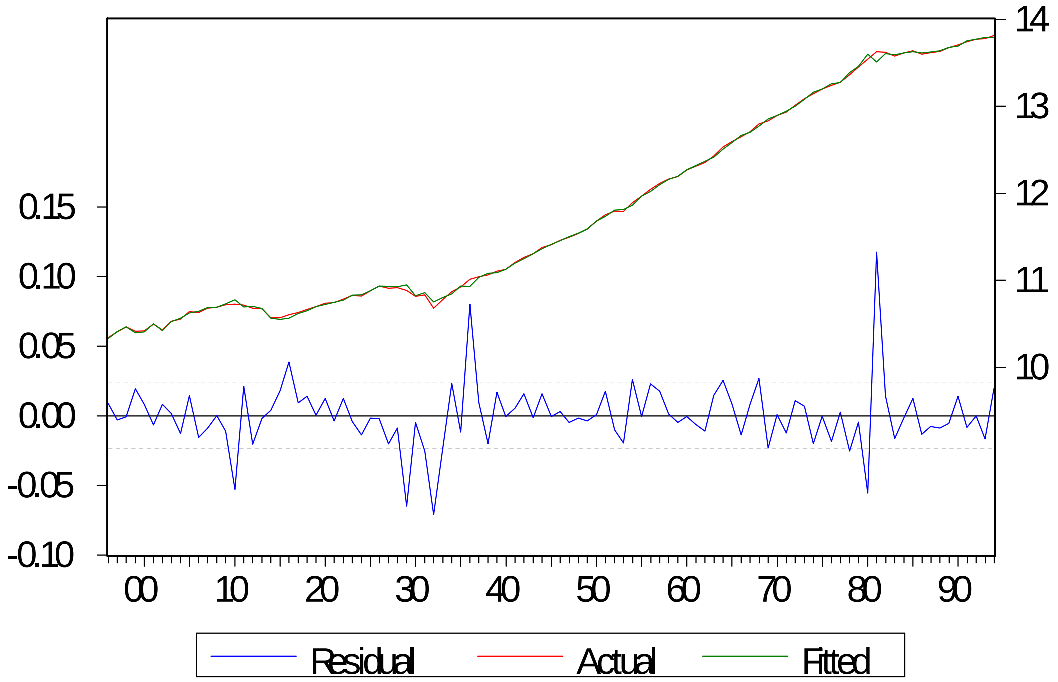


Figure 19: Regression Fit

share of the deciles 9-10 contributed to the increase in GDP per capita, but this relationship was reversed during 1940-1994 and was no longer (statistically) significant.

Period	1895-1940	1940-1994
Dependent variable log Real GDP	OLS	OLS
Constant	10.28 (35.28)	8.09 (23.58)
LOG Extreme Poverty	-0.47 (-8.09)	-0.37 (-5.59)
LOG Income Share Decile 9-10	4.88 (8.97)	-1.00 (-1.66)
AR(1)	0.77 (12.92)	0.95 (50.84)
R-squared	0.97	0.99
Durbin-Watson stat	1.93	2.29

Note: T-statistics are in parentheses.

Table 21: Regressions for Polarisation and Growth in Mexico

The Wald test for structural change was also performed for the regressions in table 3.30. The Wald test statistic has a value of 12.69. The 5 percent critical value from the F statistic for 95 degrees of freedom is 5.63. Therefore, on the basis of the Wald test, the hypothesis that the same coefficient vector applies to the two sub-periods 1895-1940 and 1940-1994 is rejected. The Hansen's test of model stability also rejects the hypothesis of stability in the model. The Hansen's test is based on the cumulative sum of the least squares residuals. The Hansen test statistic for the regression result in table 3.30 has a value of 3.70. The Hansen test statistic for the 95 percent significance level is 3.54. Therefore the tests suggest that there was a structural change between 1895-1940 and 1940-1994.

To confirm whether the contribution of the rich in the economy becomes (statistically) non-significant in the period 1940-1994 the same regressions were re-run, including the polarisation Wolfson index instead of the income share of the top 2 deciles as a regressor. A measure of polarisation was used instead of a conventional inequality measure such as the Gini coefficient, because inequality changed very little over the period 1895-1994, while polarisation changed noticeably. Given that the Wolfson index measures the distance between the groups separated by the median income, it does not measure whether the group in the bottom of the distribution is poor or not. Therefore, the poverty head count ratio was kept in the regressions. The results show that polarisation overall had a positive effect on growth in GDP per capita over the period 1895-1994. Two extra regressions were estimated for the period 1895-1940 and for 1940-1994. The Wolfson index contributed to the increase in GDP per capita during the period 1895-1940, but this relationship was reversed during 1940-1994 and was no longer significant. This result coincides with the one obtained when the income share of the top 2 deciles was used as an explanatory variable in the regression.

The (statistical) non-significance of polarisation or the income share of the top decile in the period 1940-1994 could be explained by the predictions of the third theoretical model presented. In that model the contribution of the rich entrepreneurs to growth does not necessarily remain positive over time. If there is a wide gap between the rich and the poor, without a middle class that could set up small or middle firms, then the rich entrepreneurs do not need to keep increasing salaries to increase their scales of production. In fact, entrepreneurs can reduce salaries if they face no potential competition and the labour force keeps expanding as was the case in Mexico.

Period	1895-1994	1895-1940	1940-1994
Dependent variable log Real GDP	OLS	OLS	OLS
Constant	7.74 (10.62)	8.22 (141.65)	8.61 (23.58)
LOG Extreme Poverty	-0.51 (-12.31)	-0.44 (-8.28)	-0.43 (-7.06)
LOG Wolfson Index (median)	0.32 (7.41)	0.42 (0.39)	-0.01 (-0.08)
AR(1)	1.01 (104.57)	0.83 (14.87)	0.96 (56.90)
R-squared	0.99	0.98	0.99
Durbin-Watson stat	1.89	1.87	2.41

Note: T-statistics are in parentheses.

Table 22: Regression for Real GDP and Polarisation in Mexico

A regression was run to assess whether the level of salaries was related to the wealth of entrepreneurs and the levels of poverty. Specifically, the regression measured the impact of the levels of extreme poverty, income share of deciles 9-10, the size of the labour force and the real output per employee on the size of the real unskilled labour salary. The levels of extreme poverty were not significant in explaining the changes in the unskilled labour salary, hence this variable was dropped from the regression. Then the results indicate that the size of the labour force is negatively associated to the level of salaries. Hence, increases in labour supply will reduce the real salaries. Both the income share of deciles 9-10 and the output per employee are positively related to increases in the unskilled labour salary. Hence, the more productive workers are, and the wealthier the top deciles are, a proxy for the wealth of entrepreneurs, the higher salaries will be.

Number of observations	99	99
Dependent variable log Real Unskilled Labour Salary	OLS	OLS
LOG Extreme Poverty	-0.34 (-1.36)	
LOG Income Share Decile 9-10	2.51 (2.38)	2.02 (2.01)
LOG Labour Force	-1.10 (-4.13)	-0.76 (-7.73)
LOG Real Output per Employee	1.10 (13.57)	1.02 (16.45)
AR(1)	0.13 (2.31)	0.13 (2.42)
R-squared	0.37	0.36
Durbin-Watson stat	1.17	1.17

Note: T-statistics are in parentheses.

Table 23: Regression for Salaries in Mexico

The same regression was run replacing the income share of deciles 9-10 with their of the 10th decile. The coefficient of the income share variable increase to 4.6 and was significant with a t-statistic of 2.5. Two additional regressions were run to measure the impact of polarisation instead of the income share of the top deciles on the level of salaries. Although the coefficient of the Wolfson index was positive, it was not statistically significant.

Number of observations	99	99
Dependent variable log Real Unskilled Labour Salary	OLS	OLS
LOG Extreme Poverty	-0.26 (-1.03)	
LOG Wolfson Index (median)	0.26 (1.64)	0.21 (1.40)
LOG Labour Force	-1.10 (-3.76)	-0.82 (-6.7)
LOG Real Output per Employee	0.99 (18.19)	0.95 (26.11)
AR(1)	0.14 (2.60)	0.14 (2.65)
R-squared	0.35	0.34
Durbin-Watson stat	1.17	1.17

Note: T-statistics are in parentheses.

Table 24: Regression for Salaries and Polarisation in Mexico

In summary, in Mexico during 1895-1994 there was a substantial decrease in the levels of extreme poverty. There were distributional changes, which were reflected in the rising levels of polarisation. As the theoretical model in this article predicts, changes in GDP per capita were associated to the proportion of poor people in the population and the income share of the rich people. In the case of Mexico, it seems that the extent of extreme poverty reduced the levels of aggregate GDP, while the income share of the top two deciles contributed to the increases in GDP per capita.

It must be acknowledged that the direction of causality in the regressions presented for Mexico might be problematic in that polarisation could be affected by growth. A series of Granger causality tests was carried out and this suggests that the direction of causality runs from polarisation to growth, as assumed in the regressions.

With respect to the changes in the salary, these were related to both the productivity of workers and also to the income share of the richest in the economy.

Although the time series used shed light on various issues that the theoretical models suggest, it included no data on the accessibility to credit, or the cost of setting up a firm, or on the number and size of firms over time. However, there is an additional Mexican Survey that one could explore for these purposes.

5 Concluding Remarks

The aim of the paper was to explore the influences of initial inequality on the long run distribution of wealth.This article presented two models that assess how wealth inequality, credit market imperfections and the overall business-environment affect the dynamics of wealth.

The first model presented are simplified versions of other models presented elsewhere in the literature (Banerjee and Newman, 1993; Ray, 1998; Gathak and Jiang, 2002), and are presented as a basis for the extensions made in the second model. In the second model when entrepreneurs are allowed to change their scales of production over time, the equilibria depend not just on the proportion of credit to non-credit constrained individuals, but also on the wealth distribution within these two groups. This was suggested in Banerjee and Newman (1993), but they did not show or describe how the long run distribution of wealth is affected by the wealth distribution within the credit and non-credit constrained groups. As the models in this article show, it is not always the case that a low proportion of non-credit constrained individuals will lead to a higher aggregate wealth level, higher salaries and lower inequality levels as previous models of the Banerjee and Newman type have suggested. This is especially evident in the numerical examples accompanying the second model. When there is a large difference in wealth between the rich and the middle-scaled entrepreneurs, the large entrepreneurs can drive out of the market the middle- and smaller-scaled entrepreneurs by increasing salaries in the short run. In the long run, once the middle- and smaller-scaled entrepreneurs are out of the market the large entrepreneurs can end up paying low salaries, given the reduced level of competition in the market. Previous models have not established such a link between the distribution of wealth within the credit and non-credit constrained groups and the long run distribution of wealth. Therefore, this article has shed light on the determinants of the long run distribution of wealth.

The main conclusion from the first first model, where entrepreneurs have firms of the same size is that initial conditions do matter. If at the beginning of the period there are not enough entrepreneurs to increase the demand for labour and hence salaries, the long run distribution will converge to a unique bipolar distribution of wealth. Entrepreneurs will consistently have higher levels of wealth than workers. However, the long run distribution could converge to a unique perfectly egalitarian distribution of wealth if at the beginning of the period there is excess demand for labour, which increases salaries.

In the second model where entrepreneurs are allowed to have different scales of production,without random returns, the long run distribution of wealth can converge to a wide range of distributions over time. These depend on the wealth distribution of the credit and non-credit constrained individuals. The larger and the poorer the bottom of the distribution is, the lower the equilibrium salary will be. The wealthier and the more egalitarian the top of the distribution is the higher the equilibrium salary will be.

In summary, the two models presented show that a more egalitarian distribution of wealth can be achieved in two ways. First by targeting wealth transfers to those at the bottom of the distribution of wealth, and second by facilitating access to credit and the means for diversifying risk. The former would raise workers incomes up to the minimum required for enabling occupational mobility. The latter would prevent possible decapitalisation of firms and decrease the importance of the randomness in entrepreneurial activities.

The empirical evidence presented in the article supports the main implications of these models. The cross-country regressions show that low levels of GNI are caused by the high cost of setting up firms, a high proportion of people living in poverty, and the difficulty in accessing credit markets. In developing countries, the income share of the top decile is associated to higher levels of GNI. The time series for Mexico 1895-1994 shows that the level of GDP per capita was reduced by the levels of poverty, and increased with the income share of the top decile. Changes in salaries were caused by changes in labour productivity, and in the income share of the top two deciles.

References

[1] Aghion Philippe and Patrick Bolton. (1997). "A Theory of Trickle-Down and Development." Review of Economic Studies 64, 151-72.

[2] Banerjee, A, Andrew Newman, (1993) "Occupational Choice and the Process of Development" Journal of Political Economy, 101,274-298.

[3] Benabou, R. (1996). "Inequality and Growth" in Ben Bernanke and Julio Rotemberg (eds), National Bureau of Economic Research Macroeconomics Annual, Cambridge: MIT Press, 11-74.

[4] Boltvinik Kalinka, Julio. (1994). Pobreza y Estratificación Social en México, INEGI, Mexico.

[5] CIA World Factbook, December 2003

[6] COPLAMAR. (1986), Macroeconomía de las Necesidades Esenciales en México: Situación Actual y Perspectivas al año 2000, vol. I, Alimentación, Siglo XXI, Mexico.

[7] Cummingham, W. and Malon W. Maloney. (1998) "Heterogenity among Mexico's Micro-Enterprises, An aplication of Factor and Cluster Analysis" Policy Research Working Paper WPS1999, The World Bank.

[8] Champernowne, D.G. (1952) "The Graduation of income distributions", Econometrica, 20, 591-615.

[9] Champernowne, D.G. (1953) "A Model of Income Distribution", Economic Journal, 63, 318-351.

[10] Champernowne, D.G and F. A. Cowell (1998), Economic inequality and income distribution, Cambridge: Cambridge University Press.

[11] Datt, Gaurav and Martin Ravallion. (1992). "Growth and Redistribution Components of Changes in Poverty Measures: A Decomposition with Applications to Brazil and India in the 1980s." Journal of Development Economics 38, 275-95.

[12] Dasgupta, P. and D. Ray. (1986). "Inequality as a Determinant of Malnutrition and Unemployment: Theory", the Economic Journal, 96, 1011-1034.

[13] Esteban, Joan and Debraj Ray. (1994). "On the Measurement of Polarisation.", Econometrica 62, 819-51.

[14] EUROMOD statistics on Distribution and Decomposition of Disposable Income, accessed at www.econ.cam.ac.uk/dae/mu/emodstats/DecompStats.pdf on [September 2004] using EUROMOD version no. [27A].

[15] Foster, J., J. Greer, and E. Thorbecke. (1984). "A Class of Decomposable Poverty Measures." Econometrica 52, 761-66.

[16] Galor, Oded and Joseph Zeira. (1993). "Income Distribution and Macroeconomics", The Review of Economic Studies, 60, 35-52.

[17] Gathak, M. Neville. Nien-Huei Jiang. (2002) "A simple model of inequality, occupational choice and development", Journal of Development Economics 69, 205-226.

[18] Heino, H. and J. Pagán (2000), "Assesing the Need for Microenterprises in Mexico to Borrow Start-up Capital", Journal of Microfinance, 3, 131-144.

[19] INEGI. (1992,1994,1996,1998) *Encuesta Nacional de Micronegocios (ENAMIN)*, Aguascalientes, Ags: Instituto Nacional de Estadística, Geografía e Informática, Mexico.

[20] Mirrlees, J.A. (1975). "A Pure Theory of Underdeveloped Economies", In Agriculture in Development Theory (ed. L. Reynolds). New Haven, Connecticut: Yale University Press.

[21] Persky, Joseph. (1992). Retrospectives:Pareto's Law, The Journal of Economic Perspectives, Vol. 6, pp. 181-92.

[22] Djankov, S, et al (2002) "The Regulation of Entry", Quarterly Journal of Economics, 117, 1-37.

[23] Steven Stern, Simulation-Based Estimation, Journal of Economic Literature, XXXV, 2006-2039.

[24] Villaseñor, J and B.C. Arnold. (1989) "Elliptical Lorenz curves." Journal of Econometrics 40, 327-38.

[25] Wolfson, Michel (1994). "When Inequalities Diverge.", American Economic Review 84, 353-58.

[26] World Bank. 2002. World Development Indicators 2002. CD-ROM. Washington, D.C.

5.0.1 Appendix A

There is information on Mexico's income shares by deciles based on household income-expenditure surveys for the years 1950, 1956, 1958, 1963, 1968, 1975, 1977, 1983 and 1994. Alzati (1997) used these data to estimate an annual series of income shares by deciles from 1895 to 1994. To do so, Alzati (1997) first extrapolated an annual series of income shares from 1950-1994. Then using this series, he extrapolated the income shares to 1895. The extrapolated series were regressed against the series of Mexico's Real GDP. Using these data, then Alzati (1997) obtained the Gini coefficient using the formula

$$G = 1 - (1/n) \sum_{i=j}^{n} Min(q_i, q_j) \tag{57}$$

where q_i and q_j are the income proportions of individuals i and j respectively.

Using these series of income shares, article three measured poverty and polarisation for Mexico. The polarisation measure computed was the index proposed by Wolfson (1994).

To estimate poverty, first, the Lorenz curve for each of the one hundred years was estimated using the method proposed by Villaseñor and Arnold (1989). They show that the Lorenz curve is approximated by the quadratic form

$$ax^2 + bxy + cx + dy^2 + ey + f = 0 \tag{58}$$

where x and y express the cumulative proportions of population and income. By manipulating the general equation, and defining $t = y(1-y)$, $u = (x^2 - y)$, $v = y(x-1)$ andw=x-y,theLorenzcurvecanbeexpressedas

$$t = au + bv + cw \tag{59}$$

and can be estimated by regressing t on u, v, and w by ordinary least squares.

The second step was to apply the Datt and Ravallion (1992) formula that allows estimation of the poverty measures from the Lorenz curve.

$$H = -[n + r(b + 2z/\mu)\{(b + 2z/\mu)^2 - m\}^{-1/2}/2 \tag{60}$$

$m = b^2 - 4a$, $n = 2be - 4c$, $r = (n^2 - 4me^2)^{1/2}$ and μ is the mean income and z is the poverty line.

Since there is no information on the poverty lines for the period analysed, the poverty lines were estimated in an indirect way. There is information on the levels of extreme/chronic poverty that prevailed in Mexico during various years in the period 1950-1995 (COPLAMAR (1986) and Boltvinik (1994)). Then four poverty lines were obtained to give the same poverty levels that previous research has found for the above period.