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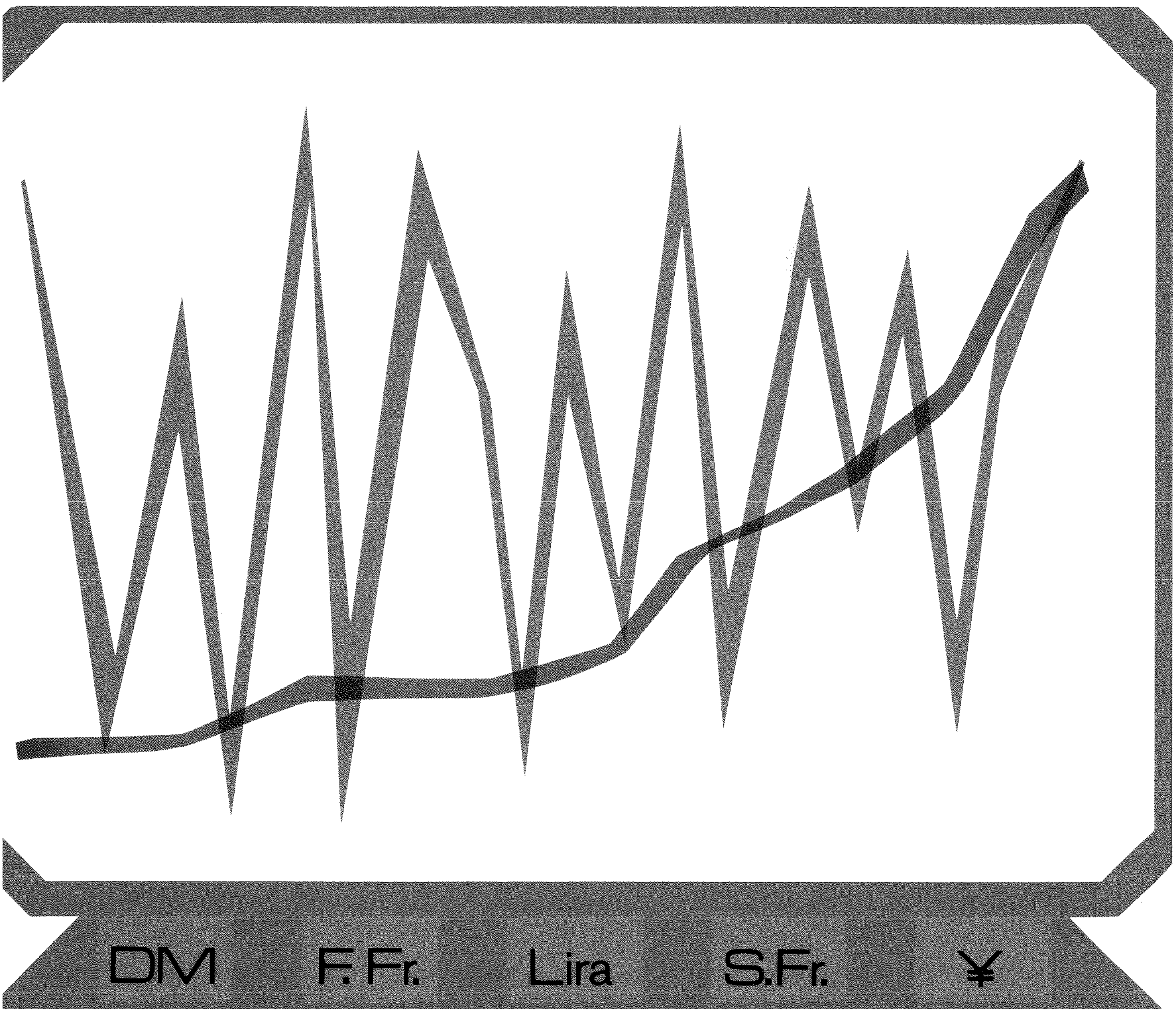

MONEY, PRICES, AND EXCHANGE RATES

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Estimating The Underlying Inflation Rate

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It is widely recognized that every wiggle in the consumer price index or in the GNP implicit price deflator does not signify a change in what we typically mean by inflation. Inflation is usually defined as an on-going, systematic rise in prices, while many of the influences which operate to produce month-to-month or even quarter-to-quarter changes in prices—like strikes, crop failures, temporary dislocations due to inclement weather and the like—do not persist. Indeed, their effects are unsystematic and ephemeral.

Only the systematic changes in prices are of any use in forecasting future prices; by definition, the unsystematic, transitory changes contain no information about the future course of prices. The persistence of relatively high and variable rates of inflation in recent years has measurably increased the marketplace's stake in efficiently forecasting prices. One would expect therefore that the marketplace makes some attempt to discriminate between the systematic forces operating on prices—the things that determine the *underlying inflation rate*—and the short run, transitory and unsystematic part of price changes.[1]

This paper presents a model of how individuals might rationally extract information about the underlying inflation rate from observed price changes, and how they might use that information to forecast future prices. The model is then estimated by assuming that people use these forecasts, among other things, to determine how much to spend on consumption.

Traditionally, economists have assumed that economic agents form their expectations about future events *adaptively*, i.e., the forecast for next period is formed by adjusting this period's forecast by some fraction of this period's forecast error. Price expectations are commonly modelled this way, although the adaptive model is in fact ill-suited for this purpose because it leads to chronic underprediction of prices if prices are growing. The reason is fairly obvious. The adaptive model implies that forecast prices are a weighted average of current and past prices, which will always be less than the current level when prices are growing. The forecasting model developed in this paper represents a generalization of the adaptive model that allows for systematic growth in prices and therefore avoids the problem of chronic underprediction. The model has the added attraction of being derived from optimizing behavior, rather than adduced on an ad hoc basis as is typically done.

Information about the market's perception of the underlying inflation rate is valuable to the policy maker for at least two reasons. In the first place, such information should provide relatively efficient estimates of ingrained inflation, which presumably is what policy makers are interested in. Almost by definition it *is* the problem—the inflation that won't go away. Certainly the agonizing that goes on in Washington every month over what the price indices are telling us suggests that the chief preoccupation of policy makers is with the underlying inflation rate. This is understandable, of course, because that underlying rate is probably the appropriate target for the conventional macroeconomic remedies for inflation: tight money and stringent government budgets. These traditional policy tools are too cumbersome, inflexible or blunt in their impact to be used to counteract every vagary of the price indices.

The second reason why the policy maker should be interested in how the market estimates the underlying inflation rate has to do with the putative trade-off between employment and inflation summarized in the now-familiar Phillips Curve. One popular explanation for the trade-off is that it is caused by temporary diver-

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gences between the perceived or anticipated rate of inflation on the one hand, and the actual rate of inflation on the other. According to this view, a decline in the actual rate of inflation, for example, produces a (temporary) increase in unemployment and corresponding decline in output as perceptions about the course of inflation lag behind events. Obviously the longer it takes perceptions about inflation to adjust, the longer will be the adjustment period during which employment and output are below their full-employment levels. It follows, therefore, that the costs in terms of lost output and employment of a successful anti-inflation policy depend, among other things, on the speed with which perceptions adjust. Knowledge about how the market estimates the underlying inflation rate—which presumably comes close to the theoretical notion of the perceived rate of inflation—can provide the policy maker with one estimate of this critical parameter.

The estimates of the underlying inflation rate yielded by the model suggest several important conclusions. First, perceptions of the ingrained inflation are currently quite high (about seven percent) and have been so during all of the current expansion. Second, these perceptions appear to respond very sluggishly to changes in the actual inflation rate, which suggests that a successful assault on inflation will entail a protracted adjustment period (and possibly one that involves significant losses in output and employment). Finally, this sluggishness in perceptions may be attributable to a high variance in the unsystematic part of price changes, which makes it difficult for individuals to distinguish changes in underlying inflation from random movement in the indices. A certain amount of evidence suggests that this problem has become worse since 1970.

Section I of our paper develops our basic theory—a standard model of consumption behavior and a sketch of how people might rationally forecast prices. Section II expands on the latter point with a technical discussion of the theory of optimal prediction. The reader who is not interested in the details may skip this section and proceed directly to Section III, which discusses the empirical results and presents estimates of the underlying inflation rate. Section IV concludes the paper with a summary and touches briefly on one important policy implication of the empirical findings.

I. Basic Theory

Prices and Consumption

Almost all modern theories of consumption start from two fundamental propositions. First, people are free from any significant money illusion, i.e., what matters is the amount of goods and services that the dollars allocated to consumption will buy. The second proposition is that the decision about the amount to spend on consumption today is part of a broader plan which encompasses decisions about how much to spend over a significant and indefinite period in the future. The first proposition is typically incorporated in empirical work by measuring consumption in real terms, i.e., as consumption spending deflated by some appropriate price index. The second proposition is handled by making consumption a function not of current income alone, but rather of people's longer-term income position as measured by their wealth or permanent income. A familiar and widely-accepted hypothesis about consumption behavior—the permanent-income hypothesis—embodies these two points in the following simple formulation:

$$\frac{C}{P_t}t = B_o y_t^p \tag{1.1}$$

Here C is nominal, or current-dollar consumption; P is some price index; y^p is permanent real income; and B_o is the marginal (and average) propensity to consume.[2] Note that (1.1) assumes that all relations are contemporaneous—that today's (time *t*'s) consumption depends on today's prices and permanent income. If the time period used as the unit of observation is long enough, this assumption of strict contemporaneity is probably not too far-fetched. A year, for example, is probably enough time for people to make consumption plans and to adjust those plans as they receive new information about prices and income. However, the assumption is doubtless strained for quarterly data such as we use, and for that reason quarterly consumption models typically assume that consumption adjusts with a lag to changes in prices and income. As is well known, such models are indistinguishable from specifications which make consumption a function of expected, or forecast, prices and income where the forecasts of a particular variable are based on its past values. Hence we can turn (1.1) into a quarterly model by replacing actual prices and permanent income with their forecast values. We assume that consumption plans are revised each quarter, and the relevant forecasts therefore are one-period-ahead forecasts, i.e., forecasts for next quarter. Thus consumption plans for the next quarter (time t+1) are made today on the basis of today's forecasts (denoted by bars over the variables) of next quarter's permanent income and prices:

$$C_{t+1}/\overline{P}_{t+1} = B_0\overline{y}^{P}_{t+1} \tag{1.2}$$

Equation (1.2) implies that nominal consumption deflated by expected prices should be more stable than nominal consumption deflated by actual prices, which is the usual measure of real consumption. Or to put the point in a slightly different way, part of the observed variation in the conventional measure of real consumption is spurious in the sense that it reflects the unintended effect of errors in forecasting prices. To see this, let $c_{t+1} = C_{t+1}/P_{t+1}$ be the conventional measure of real consumption. Then we have

$$\begin{aligned} c_{t+1} &= (C_{t+1}/\overline{P}_{t+1})\,(\overline{P}_{t+1}/P_{t+1}) \\ &= B_0 y^{P}_{t+1}\ \ (\overline{P}_{t+1}/P_{t+1}) \end{aligned} \tag{1.3}$$

As equation (1.3) makes clear, real consumption depends not only on forecast permanent income, but also inversely on the relative error in forecasting prices, $(\overline{P}_{t+1}/P_t)$.

To complete the specification of the determinants of real consumption, we need to recognize that there are accidental, unforeseen influences which cause consumption to deviate temporarily from its planned levels—things like illness, sudden trips, unannounced sales, discoveries of new products and new places to shop, and so on. These unpredictable influences on consumption we model as an additive random-error term in the logarithms of the variables.[3] Thus we complete (1.3), after writing it in logarithms, as

$$\begin{aligned} \ln c_{t+1} = \ln B_0 + \ln \overline{y}^{P}_{t+1} - (\ln P_{t+1} - \ln \overline{P}_{t+1}) \\ + \ln u_{t+1}, \end{aligned} \tag{1.4}$$

where $\ln u_{t+1}$ is a random variable which has mean zero and which is uncorrelated with the other right-hand variables.

We shall derive estimates of forecast prices by estimating equation (1.4) on quarterly, U.S. postwar data. To do so, however, we must be able to distinguish the consumption effects of the forecast errors in prices from all of the unpredictable influences captured in $\ln u_{t+1}$. To do that, we next turn to a discussion of how prices are forecast.

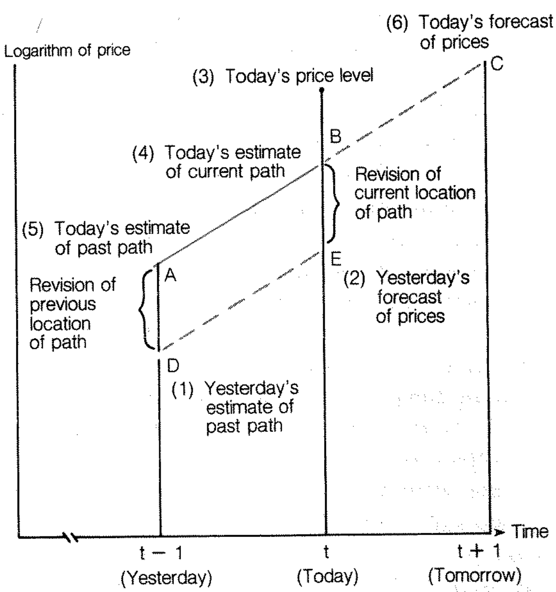
Forecasting Prices

Again, the problem in forecasting prices is to separate the systematic, sustained rise in prices from the random and transient. We can visualize this distinction by thinking of the systematic influences as operating to push prices along a path, while the unsystematic forces temporarily displace prices away from that path. By definition, only the systematic part of the price change is predictable, and the problem of forecasting prices therefore comes down to one of extrapolating the systematic, underlying path. Two types of uncertainty intrude to make this a difficult problem. First, the underlying inflationary process is not fully understood, so that the systematic path of prices cannot be precisely inferred from one's model of inflation. For example, suppose for the sake of argument that monetary growth is the main cause of inflation. Our understanding of the links between money and prices is still too imprecise to permit complete certainty about how prices will behave given the behavior of money. For this reason, we should look at the current behavior of prices themselves as another indicator of the underlying rate of inflation. However, that introduces the second source of uncertainty: the prices we actually observe can deviate in an unpredictable way from the underlying inflation path. These random deviations act like measurement error—they cause observed prices to differ from the underlying prices which we are interested in.

The next section develops an explicit model of how consumers would rationally forecast prices in the context of these two types of uncertainty. Because the non-technical reader may wish to skip that section, we may summarize the main points here and in Chart 1. The essence of the optimal forecasting scheme is that forecasts are revised each period as new information about prices is received. This new information is used in two ways: (1) to locate the current position of the underlying inflation path (point B) and (2) to determine its slope (line $\overline{AB}$), i.e., to determine how fast prices are growing along the underlying path. This latter variable, of course, is what we mean by the underlying inflation rate. The two variables then are used to extrapolate the underlying path, and that extrapolation is used as the forecast of next period's prices (C).

It is clear from these remarks that the forecast of prices is an estimate of where the underlying path will be tomorrow. When tomorrow comes, however, actual prices in general will differ from this estimate, and the question then is how much of the forecast error to attribute to a mistake in estimating underlying prices, and how much to ascribe to the random deviations of observed prices from the underlying path. The former of course should be used to revise one's estimate of where the underlying path (B) is; the latter is merely "noise" and should be disregarded. The theory of optimal prediction provides the following solution to this problem: add to last period's forecast (E) a fraction (EB) of the forecast error, and use that result as the best estimate of the current position of the underlying path. This fraction, which we denote by K, is a number between 0 and 1. Its value is determined by the amount of random variation found in observed prices. If this measurement error is negligible, so that observed prices stay close to the underlying path, K will be 1, because the estimates of underlying prices should always be adjusted to equal observed prices. At the opposite extreme, where observed prices contain no information about the underlying path, one should disregard the entire forecast error and hence K will be 0.

Chart 1
Price Paths Over Time



The new information about prices allows us not only to estimate the current position of the underlying path, but also to re-estimate its position last period. The idea involved here is a familiar one in navigation: a navigator's current readings allow him both to estimate his current position and to revise his estimate of where he was previously. This approach provides an up-to-date estimate of a second point on the underlying path, which means that the slope of the path can be estimated and hence an estimate of the underlying rate can be calculated. The theory of optimal prediction indicates that the revision in the estimate of last period's position (DA) should be proportional to the revision in the estimate of the current position (EB). The factor of proportionality, which we denote by D, must lie between 0 and a number less than 1. Its particular value depends upon the amount of knowledge market participants have about the inflationary process. Where knowledge is fairly complete—where one can be reasonably confident about his estimate of the underlying inflation rate—D should be close to 1, so that the revisions in the estimates of today's and last period's positions leave the slope of the path unchanged. By the same token, where one has only a vague idea about what causes inflation and therefore must rely heavily on observed price changes as an indicator, D should be close to 0.

This will mean that any forecast error leads to a relatively large revision in the estimate of the current location of the path, to relatively little revision in the estimate of where the path was yesterday, and consequently to a relatively large revision in the rate of growth between the two points.

As noted earlier, Chart 1 illustrates the sequence of steps involved in forecasting prices. Logarithms of prices are used here because the empirical results in Section III are expressed in those terms. This representation also has the advantage that slopes of straight-line segments can be interpreted as rates of change—as rates of inflation, in other words.

Clearly, the estimate of the underlying inflation rate—the slope of the line segment AB—is a function of how much estimates of the current and previous locations of the underlying path are revised, given the forecast errors. Thus the estimate of underlying inflation depends on K and D. It is also clear that the forecast for period $t+1$ depends on the same factors. These two observations suggest the possibility of obtaining estimates of the underlying inflation rate by using data on price forecasts to infer the values of D and K. Of course, we do not have direct observations on forecast prices. But we do have indirect evidence because real consumption is a function, among other things, of the price-forecast error. However, to deduce the forecast error from observed movements in real consumption, we must be able to isolate its effects from all of the other influences on consumption. In order to do that, we need to introduce the final result from Section II—that the forecast error depends on the sequence of current and past accelerations in prices, i.e., on how fast the rate of inflation has been changing. Hence our methodology consists of substituting a distributed lag in price accelerations for the forecast error in the consumption function (equation 1.4), estimating the distributed-lag co-efficients, calculating estimates of K and D from these distributed-lag estimates, and, finally, using the estimates of K and D to calculate estimates of the market's perception of the underlying inflation rate.

II. Optimal Prediction

The problem of forecasting prices can be formally characterized as one of forecasting a variable with incomplete knowledge of the causes of its movements and with errors involved in its observations. The model sketched here is summarized by equations (2.1a) and (2.1b). The first describes the path of prices generated by the underlying inflationary process; the asterisks are used to distinguish these prices—which are not directly observed—from actual or observed prices, P. The variable ϕ summarizes all of the available information about how fast prices are growing along the underlying path. Thus ϕ is what we mean by the underlying inflation rate.

$$P^*_{t+1} = (1 + \phi_{t+1})\, P^*_t + w_{t+1}. \qquad (2.1a)$$

$$P_t = P^*_t + v_t. \qquad (2.1b)$$

Note that this representation of the inflationary process is completely general. It can as easily accommodate a pure monetary explanation of inflation as a cost-push one. The question of what causes inflation is essentially a question about the determinants of ϕ. This, of course, is an important issue, but one which we need not address here.

Uncertainty about the inflationary process is represented by the random variable, w. Since by definition this uncertainty provides no information about prices, we require that it have zero mean and be uncorrelated with its past (and therefore with past P*s). A common name for random variables with these properties is *white noise*. Equation (2.1b) expresses the point that prices are measured with error. Thus observed prices (P) differ from underlying prices (P*) by a random term, v. Again, since v is uninformative about inflation, we require that it be white noise, and also that it be uncorrelated with w.

Consider now the problem of forecasting prices in the context of equations (2.1a) and (2.1b). Before proceeding, we should note that while the following discussion provides only a heuristic justification for our final forecasting equations, it is easy enough to show that these equations generate minimum mean-square error forecasts and therefore are optimal in that sense.[4] As we noted in the previous section, the problem of forecasting is viewed as a problem in extrapolating the underlying inflationary path. Formally this can be divided into two parts: (1) determining the current position of the underlying path, i.e., determining what P^*_t is, to serve as a starting point; and (2) determining the rate of change of P^* so that the path can be extrapolated. Let the estimate of the current location of the path be $\hat{P}^*_t$ and the estimated rate of change, $\bar{\phi}_{t+1}$. Then equation (2.1a) suggests that our best forecast of tomorrow's prices, $\bar{P}_{t+1}$, is given by

$$\bar{P}_{t+1} = (1 + \bar{\phi}_{t+1})\hat{P}^*_t \ . \tag{2.2}$$

The estimate $\hat{P}^*_t$ is based on two sources of information: all prior information which is incorporated in last period's forecast, $\bar{P}_t$, and new information received in the form of today's prices. However, the latter is not fully informative about inflation, which suggests that only a fraction of the new information should be incorporated in estimating P^*_t:

$$\hat{P}^*_t = \bar{P}_t + K(P_t - \bar{P}_t). \tag{2.3}$$

The factor K is essentially the ratio of the uncertainty about underlying prices to uncertainty about the amount of error in observed prices. The latter is measured by the variance of v, while the uncertainty in underlying prices is a function both of this uncertainty and uncertainty about the underlying inflationary process as measured by the variance of w. If we let σ_v^2 be the variance of v, σ_w^2 the variance of w, and σ^{*2} the uncertainty in underlying prices, we have

$$K = \sigma^{*2}_t \ / \ \sigma_v^2 \ , \tag{2.4a}$$

$$\sigma^{*2}_t = \cfrac{1}{\cfrac{1}{(1+\phi_t)^2 \ \sigma^{*2}_{t-1} + \sigma_w^2} + \cfrac{1}{\sigma_v^2}} \ . \tag{2.4b}$$

Clearly K lies on the closed interval [0,1]. Relatively low measurement uncertainty or high process uncertainty (low σ_v^2 or high σ_w^2) corresponds to Ks close to 1, while the opposite ranking of uncertainties produces Ks close to 0.[5]

We assume that people identify the underlying inflation rate with the speed at which P^* is currently changing. In order to determine that velocity, it is necessary to know not only what P^* currently is, but also what it was last period. Let $P^*_{t-1/t}$ denote the latter. The t−1 subscript denotes that this is an estimate of where the underlying path was yesterday; the t subscript indicates that is a retrospective estimate, i.e., one made today. In general, people's perceptions today of where the underlying path was yesterday will differ from where they thought it was at the time. The latter is obviously last period's analogue of P^*_t, which we denote by P^*_{t-1}. The theory of optimal prediction indicates that people revise their estimate of the last period's position by a fraction, D,[6] of the revision in their estimate of the current position:

$$\hat{P}^*_{t-1/t} = \hat{P}^*_{t-1} + D(\hat{P}^*_t - \bar{P}_t). \tag{2.3a}$$

First-order approximations to equations (2.2) and (2.3) yield the following relationship in the logarithms of forecast prices:

$$\ln \bar{P}_{t+1} - \ln P_t = \bar{\phi}_{t+1} - (1-K)(\ln P_t - P_t) \tag{2.5}$$

It is clear from this expression that our forecasting scheme is a mixed extrapolative-regressive one of the sort first proposed by deLeeuw (1965) and subsequently used by Modigliani and Sutch (1966), among others, in their work on forecasting interest rates. The extrapolative element is ϕ_{t+1}—the rate at which prices are forecast to grow in the future. The regressive element in the forecast is represented by the second term on the right-hand side of the expression. It indicates that, ceteris paribus, prices are forecast to revert partially to their present level. The smaller is K, the larger is the influence of this regressive element. The estimate of the underlying inflation rate is given by

$$\bar{\phi}_{t+1} \equiv (\hat{P}^*_t - \hat{P}^*_{t-1/t}) / \hat{P}^*_{t-1/t}, \tag{2.6a}$$

which to a first-order approximation is

$$\bar{\phi}_{t+1} \cong (1-K)(\ln \bar{P}_t - \ln \bar{P}_{t-1}) + K(\ln P_t - \ln P_{t-1}) - DK(\ln P_t - \ln \bar{P}_t) \quad (2.6b)$$

Equations (2.5) and (2.6b) together yield the following relationship in the logarithms of forecast prices:

$$\ln \bar{P}_{t+1} - \ln \bar{P}_t = (1-K)(\ln \bar{P}_t - \ln \bar{P}_{t-1}) + K(\ln P_t - \ln P_{t-1}) + (1-D)K(\ln P_t - \ln \bar{P}_t) \quad (2.7)$$

If the last term were missing, (2.1) would imply that the growth rate of forecast prices is an exponentially declining weighted average of current and past rates of price change—the familiar adaptive-expectations result. For forecasting the *level* of prices, this is clearly suboptimal if a change occurs in the average rate of growth of prices. Consider, for example, what would happen if the inflation rate permanently increased. The growth rate of forecast prices would follow with a lag, and approach as a limit the new, higher inflation rate. But it would never exceed the actual inflation rate, and consequently the level of forecast prices would always fall short of the level of actual prices. For this reason, (2.7) has a term in the forecast error, $\ln P_t - \ln \bar{P}_t$, which is designed to adjust the growth of forecast prices to remove any systematic discrepancy between actual and forecast prices.

Finally, (2.7) is easily recast in terms of forecast errors to produce

$$(\ln P_{t+1} - \ln \bar{P}_{t+1}) = [2(1-K)+DK](\ln P_t - \ln \bar{P}_t) - [1-K](\ln P_{t-1} - \ln \bar{P}_{t-1}) + \Delta^2 \ln P_{t+1}, \quad (2.8)$$

where $\Delta^2 \ln P_{t+1}$, the second difference in the logarithm of prices, measures price accelerations, i.e., changes in the rate of growth of prices. Repeated lagging of (2.8) and substitution back into itself yields a solution for the forecast error as a distributed lag in current and past accelerations in prices:

$$\ln P_{t+1} - \bar{P}_{t+1} = \sum_{j=0}^{\infty} a_j \, \Delta^2 \ln P_{t+1-j} \quad (2.9)$$

Since (2.9) is a particular solution of (2.8), the distributed-lag coefficients, a_j, must be functions of K and D. In particular, we must have

$$a_0 = 1$$

$$a_1 = 2(1-K) + DK$$

$$\cdots$$

$$a_{j+1} = [2(1-K) + DK]a_j - (1-K)a_{j-1}, \; j \geq 1$$

$$\lim_{j \to \infty} a_{j+1} = 0 \quad (2.10)$$

It is clear from equation (2.9) that a constant inflation rate, i.e., $\Delta^2 \ln P_{t+j} = 0$ for current and all past periods, produces a zero forecast error. In other words, when prices are growing at a steady rate, the actual and forecast levels of prices are the same. A permanent change in the inflation rate, on the other hand, produces a transitory (though by no means short-lived) divergence of actual from forecast prices. The distributed-lag coefficients trace out the path of the forecast error during the transition. Thus the requirement that $a_0=1$ indicates that a one-percentage-point increase in the rate of inflation initially raises actual prices above forecast prices by exactly the same amount. Thereafter, the gap between actual and forecast prices may continue to widen for awhile, or may begin to close; the particular path followed depends on the values of D and K, which determine the speed with which forecasts are revised. Ultimately, however, as the last condition on the a_j indicates, the gap must close and in the limit go to zero. Thus in the new steady-state equilibrium, forecast and actual prices again grow along the same path.

III. Empirical Results

Estimating the Consumption Function

Our consumption function, after substituting a distributed lag in price accelerations (denoted by $\Delta^2 \ln P_{t+1-j}$) for the forecast error, is

$$\ln c_{t+1} = \ln B_0 + \ln(\bar{y}^p_{t+1}) - \sum_{j=0} a_j \Delta^2 \ln P_{t+1-j} + \ln u_{t+1} \quad (3.1)$$

For the purpose of estimation, consumption is defined to exclude expenditure on new consumer durables, which is more properly treated as a form of savings.[7] Forecast permanent income, $\bar{y}^p_{t+1}$, is computed recursively from the formula

$$\bar{y}^p_{t+1} = (1 + .0048)\,(0.1y_t + 0.9\bar{y}^p_t),$$

where y is measured per capita real income, .0048 is the quarterly trend rate of growth of y for the period 1947:1–1977:4, and the weights 0.1 and 0.9 are taken from Darby (1972).

Measured income is defined as the sum of disposable personal income plus undistributed corporate profits. On theoretical grounds alone the latter should be included, since permanent income is viewed as the flow of income generated by a broadly defined concept of wealth that includes corporate wealth. Moreover, empirical evidence suggests that households treat changes in the value of their equity holdings as part of their income. (See, for example, David and Scadding [1975].) The implicit price deflator for GNP, rather than the consumer-price index or consumption-spending deflator, is used to measure P. This is done because a "true" cost-of-living index—i.e., one that corresponds to the notion of permanent income—should include the prices of both current and *future* consumption. No existing index approaches this ideal, of course, but a broad-based index like the GNP deflator presumably comes closest, because it implicitly includes the prices of future consumption through its inclusion of producers' goods prices.

Two restrictions are imposed in estimating (3.1): (1) the forecast errors are assumed to average out to zero over the sample period; and (2) the forecast errors and permanent income are assumed to be uncorrelated. Both are imposed on the grounds that people make efficient forecasts, i.e., that roughly speaking, they use all available information. Consider the first restriction. If, for example, the forecast error were systematically positive, people would ultimately recognize their chronic underforecasting and would adjust their forecasts upwards to remove the discrepancy. This recognition might take some time, but not to the extent that errors would systematically cumulate over our entire sample period of 24 years. Next, consider the second restriction. Recall that the permanent-income variable in (3.1) is *forecast* permanent income. If this variable were correlated with the forecast error in prices, people could use this association to improve their forecasts of permanent income. It would pay them to do so until the association disappeared, i.e., until permanent income and the forecast error in prices became uncorrelated.

The two restrictions are easily imposed by estimating (3.1) in two stages. First, real consumption is regressed on a constant and permanent income. The residuals from this estimation are then regressed on the distributed lag in accelerations in prices to obtain estimates of the a_j. The latter will be unbiased provided the restrictions are true.

Equation (3.2) reports the results of the first-stage regression. The sample period is 1953:3-1977:4, and both consumption and permanent income are in per capita terms. The adjusted multiple R^2, standard error of estimate, Durbin Watson statistic and estimated first-order serial correlation in the error term (ρ) are shown below. The standard errors of the estimated coefficients are shown in parentheses beneath their respective estimates.

$$\underset{}{\ln c_{t+1}} = \underset{(.0752)}{-.2820} + \underset{(.0573)}{1.0015}\ \ln(\bar{y}^p_{t+1}) \qquad (3.2)$$

$\bar{R}^2 = .9986$ D.W. $= 1.7458$
S.E. $= .0057$ $\hat{\rho} = .9434$

The appropriateness of the restrictions imposed in estimating (3.1) can be roughly gauged by comparing the coefficient estimates in (3.2) with comparable estimates from other consumption studies. Such a comparison indicates no significant bias in the estimates, which suggests that the restrictions may not be unreasonable. Thus the point estimate of the coefficient on $\bar{y}^p_{t+1}$, which measures the permanent-income elasticity of consumption, is effectively unity. This agrees completely with the permanent-income specification of the consumption function, and it is supported by a large body of other evidence.[9] The estimated constant in (3.2) implies a marginal propensity to consume of approximately .75.

This is somewhat low—most estimates cluster around .80—but given its relatively high standard error, it is surely compatible with other estimates.

Estimates of Forecasting Parameters

The results of estimating the second-stage regression, in which the residuals from estimating the consumption function (3.2) are regressed on the distributed lag in price accelerations, are summarized in Chart 2 and Table 1. Chart 2 graphs the estimated lag coefficients on current and past accelerations in prices, while Table 1 reports the implied estimates of K and D and the summary statistics of the regressions.[10] Separate results are given for the whole sample period, 1954:1–1977:4, and for two subperiods, 1954:1–1970:4 and 1971:1–1977:4.

The familiar Almon polynomial distributed-lag estimator (with the Cochrane-Orcutt correction for serial correlation) was used to estimate the coefficients. Experiments with different lag shapes and lengths led to the choice of a third-degree polynomial with a 20-quarter lag. In all cases, the far end of the distributed lag was constrained to be zero in order to incorporate the requirement that the steady-state (long-run) forecast error be zero.

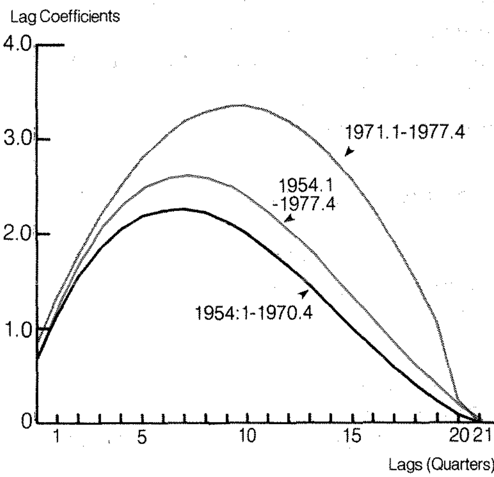
In several instances the results square remarkably with our theory. All of the estimates of the coefficient on the contemporaneous price acceleration are within two standard errors of their a priori value, 1. Similarly, all of the point estimates of K lie within the a priori bounds, [0,1]. The point estimates of D are ostensibly an exception—they are greater than 1 while our theory predicts just the opposite. Nevertheless, the difference is probably not statistically significant. The estimate of D is calculated from a ratio of distributed-lag estimates, and such ratio estimators typically have large standard errors. The numerical differences from unity of about 12 to 16 percent are probably well within two standard errors of estimate.

Table 1
Estimates of K and D

Sample Period	Estimates of K and D		Summary Statistics R^2	S.E.	Rho	D.W.
1954.1-1977.4	.1471	1.1248	.9302	.0044	.9528	1.8939
1954.1-1970.4	.1528	1.1591	.9367	.0044	.9607	2.1394
1971.1-1977.4	.0881	1.1177	.9128	.0040	.6194	1.4414

The equation estimated is $\hat{z}_{t+1} = \sum_{j=0}^{21} a_j \Delta^2 \ln P_{t+1-j}$, where $\hat{z}$ is the (raw) residual from the regression $\ln c_{t+1} = \ln B_o + \ln \bar{y}^P_{t+1}$. The a_js were constrained to lie along a third-degree polynominal with $a_{21} = 0$.

Chart 2
Distributed Lag Estimates



As noted earlier, people are assumed to revise their estimate of the underlying inflation path when prices turn out differently from what was forecast. Roughly speaking, the revision in the estimated *level* of the path varies directly with the size of K, while the revision in the estimated *slope* of the path varies inversely with the size of D. The relatively low values of K and relatively high values of D indicate that people's perceptions of underlying prices and the underlying inflation rate are slow to respond to changes in the actual inflation rate. Consequently, forecast prices can deviate substantially from actual prices, and the discrepancy can persist for a long period of time. The distributed-lag coefficients (Chart 2) can be interpreted as tracing out the sequence of forecast errors after a permanent one-percentage-point increase in the inflation rate. They indicate that forecast prices can differ by as much as three percent or more from actual prices for every one-percentage-point increase in inflation, and that it takes about five years for the difference to disappear altogether. On the face of it, a lag of five years between actual and forecast prices may seem rather long, but it is in fact relatively short by comparison with typical results obtained from studies of the relationship between prices and interest rates. Some observers have rejected these long lags as being implausible, given our knowledge of how prices are formed.[11] However, once errors of measurement are allowed, they may not be so implausible: where one is unsure about the amount of information contained in price movements, it is not irrational to ignore them unless they continue for a long time.

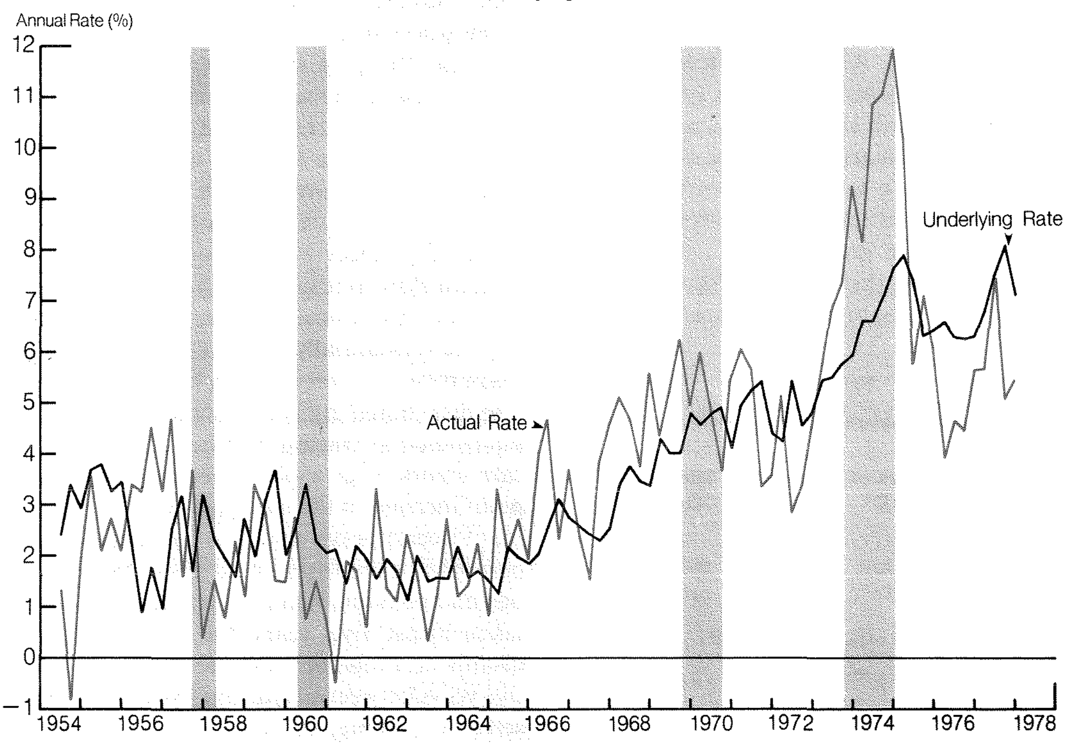
The low values of K and high values of D also suggest that most of the uncertainty in forecasting prices stems from measurement error in prices, i.e., from the fact that a significant part of the observed variation in prices represents random shocks which are unrelated to systematic inflation. The decline in the value of K for the later subperiod suggests as well that prices have become more unpredictable since 1970. This point has been made elsewhere on the basis of different evidence,[12] and agrees with one's casual impression that the price level in the Seventies has been subject to more frequent and severe shocks than was the case in prior decades.

Estimates of Underlying Inflation Rate

Estimates of the underlying inflation rate, $\overline{\phi}_{t+1}$, along with the actual quarterly inflation rates, are shown in Chart 3. Clearly, the estimates of the underlying inflation rate have the sort of properties one would expect of such a series: a much greater quarter-to-quarter stability than the actual inflation rate, and an ability to track faithfully the longer-run movements in the actual inflation rate. However, the underlying inflation rate can differ from the actual rate for substantial periods of time, reflecting the long adjustment lags.

It is also clear that successively higher levels of inflation have become embedded in the economy since 1960. Thus the underlying inflation rate fluctuated around 1.7 percent until the late Sixties, averaged about 4.8 percent from 1971 to 1973, and in the current expansion has hovered around 7 percent. Apparently, neither the 1969–70 nor the 1973–75 recession made a sizable dent in the underlying rate; at most, they seemed capable only of stabilizing the inflation

Chart 3
Actual and Underlying Inflation Rates



rate until some new disturbance carried it off to a higher plateau.

There is no evidence that the 1971 price and wage controls had any noticeable effect on people's perceptions about the underlying inflation rate. The decline in the underlying inflation rate after the second quarter of 1971 was negligible compared to the fall in the actual rate, and it did not last as long. Some numbers make this point more forcefully. In the four quarters ending in 1971:2, the inflation rate, measured by the growth in the GNP implicit price deflator, averaged 5.2 percent. In the four subsequent quarters, inflation declined by nearly 1½ percentage points to 3.8 percent. By comparison, the underlying inflation rate was 4.8 percent in the first period, and 4.9 percent in the second—effectively unchanged, in other words.

Much of the spectacular run-up in inflation rates in late 1973 and in 1974 appears to have been treated by economic participants as transitory, and thus was not viewed as symptomatic of a deterioration in the underlying rate (though that did happen). This perception was borne out by the subsequent sharp decline in inflation rates after 1974. By the same token, the underlying rate did not follow the actual rate down as the latter fell from its 1974 highs. Again, this perception appears to have been borne out by the bounce-back in inflation rates after mid-1976.

IV. Summary and Conclusions

Obviously, the estimates of the underlying inflation rate calculated here should not be accepted uncritically. Nevertheless, the congruence of our estimation results with the predictions of theory, and their conformance with historical experience, are too striking to be ignored. This congruence lends our estimates a high degree of plausibility.

Two points seem worth repeating because they bear on the important question of what a successful assault on inflation is likely to cost in terms of lost output and employment. First, the ingrained rate of inflation currently perceived by the market is dismally high by historical standards—around 7 percent at an annual rate—and has stubbornly remained at this level throughout the current expansion. This persistence of a high perceived underlying inflation rate doubtless has given inflation an important momentum of its own, because market participants, in an effort to protect themselves against future inflation, have built this perception into their wage and price demands.

Secondly, even if aggregate-demand growth could be moderated, pressure for price and wage increases would continue to emanate from the cost side for a considerable time. The implication of this for output and employment is not reassuring. If pressures from inflationary expectations do not abate after growth in aggregate demand slows down, the difference presumably has to come out of real income growth. This is essentially the modern explanation for the observed trade-off between unemployment and inflation described by the Phillips Curve. This explanation of course stresses the *temporary* nature of the trade-off. Once expectations of inflation have fully caught up with the actual rate, output and employment are assumed to return to their normal levels. But our finding about the length of the adjustment period—about five years—suggests that temporary can still be a long time. Hence, output and employment may have to remain below normal levels for a fairly protracted time if any significant progress is to be made against inflation.

FOOTNOTES

1. For some evidence that the market discounts spurious evidence of inflation in the consumer price index see, E. Fama, "Interest Rates and Inflation: The Message in the Entrails," **American Economic Review,** 67 (June 1977), pp. 487-96.

2. Some controversy surrounds the proposition implicit in (1.1) that consumption is strictly proportional to permanent income. This restriction was not imposed in the estimation, but the empirical results were so consistent with it that I have written (1.1) in the traditional form.

3. The variable u, which can be interpreted to be the ratio of actual to planned consumption, has a lognormal distribution if we assume, in the usual way, that 1n u is normally distributed. Hence the assumption that the mean of 1n u is zero corresponds to assuming that the

median ratio of actual to planned consumption is unity, and it is in this sense that actual and desired consumption are "on average" the same.

4. A good account of the theory involved can be found in A. Bryson and Y. Ho, **Applied Optimal Control** (Waltham, Mass.: Blaisdell, 1969)

5. K has a steady-state solution for constant ϕ. Although obviously we do not want to assume the latter, we shall assume that the relative variation in ϕ is so small that K is approximately constant. There is ample precedent in the literature for doing so, presumably because without such a simplification the forecasting problem has no closed-form solution.

6. The expression for D is

$$D = \frac{\sigma^{*2}_t (1+\phi_t)}{\sigma^{*2}_t (1+\phi_t)^2 + \sigma^2_w}$$

where, as before, σ^{*2} measures the uncertainty in underlying prices and σ^2_w stands for the uncertainty about the underlying inflationary process. Clearly D is bounded from above by a number less than 1, while it cannot be less than zero.

7. To be totally consistent, we should add to consumption the imputed service flow from the existing stock of consumer durables. We did not do this simply because quarterly estimates are not readily available; it is doubtful that the omission has any practical significance.

8. For a more thorough discussion of this point, see A. Alchian and B. Klein, "On a Correct Measure of Inflation," **Journal of Money, Credit and Banking** 5 (February 1973, Part II), pp. 173-91.

9. For an up-to-date survey of evidence on the consumption function, see R. Ferber,"Consumer Economics: A Survey," **Journal of Economic Literature** 11 (December 1973), esp. pp. 1307-08. Estimating (3.1) in two stages does not appear to have affected the estimates except trivially. When (3.1) is estimated in one step, the estimate of the marginal propensity to consume is .78 rather than .75, while the estimated income elasticity is .98 rather than 1—differences which are without statistical or economic significance. The distributed-lag estimates are even less affected: they are virtually indistinguishable from the estimates graphed in Chart 2.

10. The estimates of K and D are obtained by substituting the estimated a_j into the restrictions $a_{j+1} = (2(1-K) + DK)\ a_j - (1-K)\ a_{j-1} = 0$ and solving for K and D. The choice of which a_j to use is arbitrary: any four consecutive ones will do, and I chose a_2 through a_5. See G. Box and G. Jenkins, **Time Series Analysis** (San Francisco: Holden Day, 1970), page 383.

11. See for example, T. Sargent, "Interest Rates and Prices in the Long Run," **Journal of Money, Credit and Banking** 5(February 1973, Part II), pp. 384-449.

12. B. Klein, "Our New Monetary Standard: The Measurement and Effects of Price Uncertainty,1880-1973," **Economic Inquiry** 13 (December 1975), pp. 462-84, argues that the shift from a monetary constitution based on the gold standard to a managed fiduciary standard increased uncertainty about future prices. He places the watershed in the mid-Sixties, at the latest. However, my experiments with different subperiods produced clear evidence for a break around 1970. My conjecture is that it took the monetary laxity of the late Sixties to convince the public that monetary arrangements had fundamentally changed—a perception that was soon borne out by the collapse of the Bretton Woods System.

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