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REMINDER

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A Reexamination of Mean-Variance Analysis of Bank Capital Regulation

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The mean-variance framework has been used to analyze the effects of bank capital regulation on the asset and bankruptcy risk of insured, utility-maximizing banks. This literature claims that more stringent capital regulation will increase asset risk and can increase bankruptcy risk. These conclusions are notable because they are opposite to those obtained for insured, value-maximizing banks. In this paper, we show that the utility-maximization literature does not support its conclusions regarding the effects of bank capital regulation because it has mischaracterized the bank's investment opportunity set by neglecting the option value of deposit insurance.

In recent years, federal bank regulatory agencies have increased reliance on bank capital regulation, in part, because of heightened concerns over the risk exposure of the deposit insurance system. Indeed, the primary rationale for existing capital regulations, as well as proposals for more stringent capital regulation, is to reduce the insurance system's risk exposure by reducing leverage.

The idea that capital and other restrictions might be needed by liability holders in general to protect themselves from equity holders has been discussed extensively in the theoretical corporate finance literature. For example, Jensen and Meckling (1976), modeling the equity of a firm as a call option on its assets, show that equity holders have an incentive to increase the non-systematic risk of assets once debt has been issued or to issue additional debt. The reason is that increasing asset risk or issuing new debt increases the value of their option on the firm's assets and hence decreases the value of outstanding debt. As a result, bondholders often impose covenants constraining such things as future debt issues, dividend payments, and leverage.

In banking, the interests of the deposit insurance system parallel those of a private bondholder because the deposit insurance system, not the insured depositors, stands to lose in the event of a bank failure. In this vein, regulatory capital requirements and other portfolio restrictions could be viewed as similar to private bond covenants.[1]

It is within this context that a number of articles have analyzed the need for bank regulation. One strand of the literature has shown that when deposit insurance underprices risk, banks seeking to maximize the value of their stockholders' equity will attempt to maximize the value of the insurance subsidy by increasing asset risk and leverage (see Sharpe 1978, Kareken and Wallace 1978, and Dothan and Williams 1980). The reason is that the option value of deposit insurance increases as leverage or asset risk increases (see Merton 1977). As a result, with fixed-rate deposit insurance both capital and asset portfolio regulation are needed to limit the liability of the deposit insurance fund.

Moreover, as we have shown elsewhere (Furlong and Keeley 1989), the marginal value of the deposit insurance option with respect to increasing asset risk declines as leverage declines. Consequently, value-maximizing banks would have less of an incentive to increase asset risk as a result of more stringent capital regulation. Thus, more stringent capital regulation will reduce the risk exposure of the insurance system as long as the stringency of the regulation of asset portfolio risk remains unchanged. (That is, as long as the resources devoted to enforcing, and the penalties for evading, asset regulations remain unchanged, more stringent capital regulation will cause the probability of bank failure to decline.)

In contrast, another strand of the literature focusing on utility-maximizing banks questions the effectiveness of capital regulation. The original contributions to this literature perhaps are best typified by Kahane (1977) and Koehn and Santomero (1980), hereafter referred to as KKS. Moreover, the basic framework developed by KKS continues to be used, as in the work of Kim and Santomero (1988) and others.[2] KKS claim to show that, in the context of a Markowitz two-parameter portfolio model, more stringent bank capital regulation will cause a utility-maximizing bank owner-manager to increase asset risk and may, as a result, increase the risk of bank failure (and thus implicitly increase the expected liability of the deposit insurance fund).[3] These results are notable in large part because they run counter to the general finance literature and suggest that capital regulation may be counterproductive.[4]

In this paper, we show that KKS's use of the Markowitz two-parameter portfolio model to analyze the effects of bank capital regulation on bankruptcy risk is inappropriate because of the model's assumption of constant borrowing rates and costs independent of portfolio (default) risk.[5] While this assumption is appropriate for certain investment decisions where the probability of bankruptcy (default on debt) is zero or can be ignored, it is logically inconsistent to use it to analyze the effects of bank capital regulation on bankruptcy risk.

First, in a world without deposit insurance when the probability of bankruptcy is nonzero, the promised deposit rate demanded by uninsured depositors will depend on the risk of the bank's portfolio, which in turn depends on leverage and asset risk. Also, if default is possible, the cost of deposits will be a random variable. Moreover, if depositors are risk-averse, the expected cost of deposits (per dollar) will rise with risk. Thus, the models of KKS, which assume constant borrowing rates and costs, are not applicable to uninsured banks.

Second, and more importantly, while it might appear that the Markowitz assumption of constant borrowing costs employed by KKS is applicable to insured banks since insured depositors will supply funds at a constant risk-free promised rate, we show below that it is not.[6] The reason is that the expected net marginal cost (expected interest cost plus an assumed fixed-rate premium) of deposits (per dollar) to the bank declines as the quantity of deposits increases, because the option value of the deposit guarantee increases as leverage increases. In effect, KKS confuse the expected cost of deposits with the promised return under situations where the probability of default is nonzero.

By assuming that changes in the probability of bank failure do not affect deposit rates or costs, KKS mischaracterize the risk-return tradeoff even for a bank with fixed-rate deposit insurance by neglecting changes in the value of the insurance subsidy that occur when leverage or asset risk changes and by using an inappropriate measure of risk when bankruptcy is possible. These oversights are crucial since limiting the deposit insurance subsidy is the main reason for capital requirements in the first place.[7]

In Section I we first construct a prototype of the Markowitz portfolio model used by KKS to analyze the effect of bank capital regulation on asset risk. We show that when bankruptcy is not possible, and, thus, when there is no deposit insurance subsidy, the results from our prototypical model parallel those of KKS regarding the effects of capital regulation on increasing asset risk. However, increases in asset risk due to more stringent capital regulation cannot increase the probability of bankruptcy under the assumptions that KKS use to derive the model since these assumptions imply that the probability of bankruptcy must be zero.

In Section II, we demonstrate that accounting for deposit insurance and the possibility of bankruptcy markedly changes the bank's opportunity set. Moreover, the variance of return no longer adequately characterizes risk. As a result, KKS mischaracterize the risk-return tradeoff absent capital regulation as well as the effect of capital regulation on the risk-return tradeoff when bankruptcy is possible or when deposit insurance is subsidized. Because of this, KKS's model cannot be used to support their results.

Section III presents our summary and conclusions.

I. A Prototypical Model of an Uninsured Bank's Portfolio Decisionmaking

KKS analyze bank risk-taking as a portfolio management problem for a risk-averse bank owner-manager whose entire net worth is invested in the bank. The owner-manager's equity risk depends on the bank's asset portfolio risk and on leverage. We assume that, in the absence of regulation, banks will leverage only efficient asset portfolios (those with maximum expected return for any given level of risk).[8] Given the owner's preferences towards risk, expected utility will be maximized subject to a constraint that relates the gross expected return (one plus the expected rate of return) on capital, E(Z), to the standard deviation of that return, $\sigma(Z)$.

To derive this risk-return constraint, we assume that the bank's deposits are not insured but that the bank can attract deposits at a fixed promised deposit rate unrelated to the bank's risk. This implies that the bank has to choose a combination of leverage and asset risk to make bankruptcy impossible (that is, the realized return on assets will be such that the promised obligations to depositors always will be met). With bankruptcy not possible, the gross return on capital, Z, is given by the gross return on assets, A_0P, minus the promised (which equals the actual) obligation to liability holders, L_0R, divided by initial capital, K_0, or

$$(1) \qquad Z = \frac{A_0P - L_0R}{K_0},$$

where

- A_0 = initial assets,
- L_0 = initial liabilities (deposits),
- K_0 = initial capital,
- P = gross return on the bank's portfolio of assets, assumed to be random, which equals one plus the rate of return,
- Z = gross return on capital, which is random, which equals one plus the rate of return,
- R = promised (which equals actual) gross certain return paid on (and per dollar cost of) liabilities, which equals one plus the rate of return.

(1) may be rewritten by noting that $L_0 = A_0 - K_0$ to give

$$(2) \qquad Z = [\frac{A_0}{K_0}][P - R] + R.$$

The expected gross return on capital, E(Z), may be found by taking expected values of both sides of (2). As long as R is fixed and not random, which it would be as long as bankruptcy were not possible, this gives:

$$(3) \qquad E(Z) = [\frac{A_0}{K_0}][E(P) - R] + R.$$

Thus, increasing leverage, as measured by the asset-to-capital ratio, increases the owner's expected return on capital linearly when default is not possible for any given asset portfolio.

Similarly, the standard deviation of the return on capital, $\sigma(Z)$, may be derived from (2). When bankruptcy is not possible, the covariance of R and P is zero and

$$(4) \qquad \sigma(Z) = [\frac{A_0}{K_0}]\sigma(P).$$

so that the standard deviation of return on capital also varies linearly with leverage for a given asset portfolio.

(3) and (4) may be jointly solved to eliminate the $[A_0/K_0]$ term to give

$$(5) \qquad E(Z) = [\frac{\sigma(Z)}{\sigma(P)}][E(P) - R] + R$$

In other words, the expected (gross) return on capital varies linearly with the standard deviation of return on capital for a given expected asset return and asset standard deviation. This is a standard result in the CAPM models of the finance literature on investment (see Sharpe 1970).

In general, it is assumed that a bank faces a variety of different asset portfolio risk-return combinations (the asset risk-return frontier). Asset portfolios with more risk are assumed to yield larger expected returns and thus the asset risk-return frontier is convex (see Figure 1). Moreover, it is assumed that the banking sector is small enough that the asset risk-return frontier is unaffected by banks' behavior. Thus, the frontier is taken as given by banks in their optimizing decisions.

An unconstrained bank's efficient investment frontier consists of linear combinations of a particular asset portfolio and the single risk-free liability. As is well known (see Hirshleifer 1970, chap. 10, or Fama and Miller 1972, chap. 7), the most efficient asset portfolio is the one where a line from the constant (gross) borrowing rate, R, is tangent to the asset risk-return frontier. (This is depicted as point E(P_0), $\sigma(P_0)$ in Figure 1.) By leveraging this asset portfolio the bank can obtain the highest expected return on its capital for any degree of risk. Since this tangency at point E(P_0), $\sigma(P_0)$ does not depend on the bank owner's risk preferences, the asset portfolio chosen depends only on the risk-free interest rate and the asset risk-return frontier.

An unconstrained bank's optimal position on this linear investment (or capital) risk-return frontier is the point at which the marginal rate of substitution between risk and expected return, $d\sigma^2(Z)/dE(Z)\,|_{U=\bar{U}}$, is equated with the tradeoff between risk and expected return along the efficient investment portfolio frontier. Following Koehn and Santomero, assuming the distribution of portfolio returns (Z) is symmetric (as it would be for a diversified portfolio in which bankruptcy was not possible), and taking a second order Taylor-series expansion of the utility function, U, around the initial capital, K_0, of the bank, and then taking expected values gives:

$$(6) \qquad E(U) = U(K_0) + U'(K_0)\{E(Z) - b[(E(Z))^2 + \sigma^2(Z)^2]\}$$

where

$$b = -\frac{U''(K_0)K_0}{2U'(K_0)}$$

is the coefficient of relative risk aversion of the underlying utility function and U' and U'' are the first and second derivatives of U. For this utility function the marginal rate of substitution is:

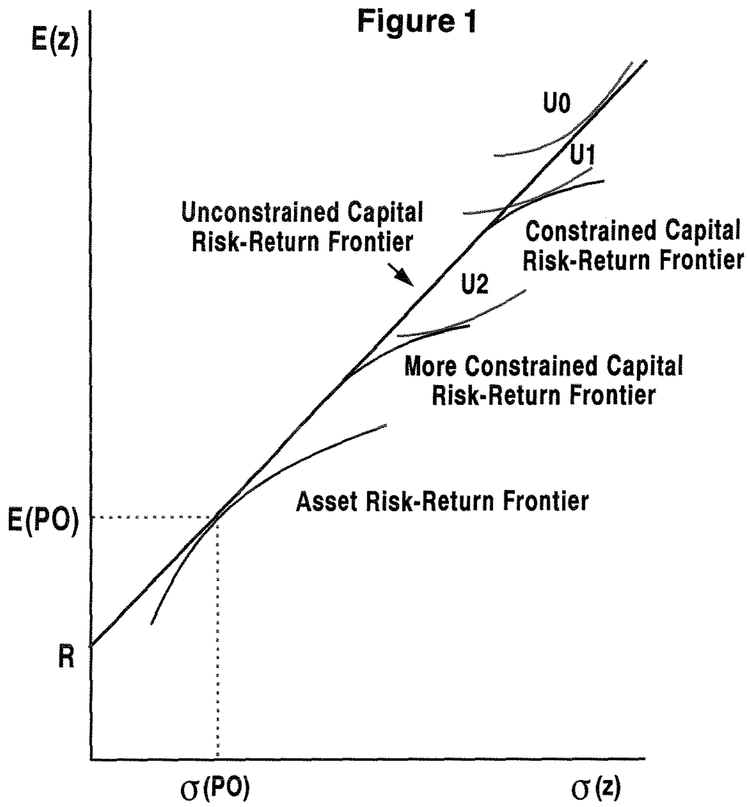
$$(7) \qquad \frac{d(\sigma^2(Z))}{d(E(Z))}\Big|_{U=\bar{U}} = \frac{1}{b} - 2E(Z) = MRS.$$

Thus, the optimal portfolio requires that $MRS = \lambda$, where λ is the tradeoff between variance and expected return on the efficient investment frontier. Thus, the degree of leverage chosen is determined by the owner's risk preferences, although the assumption is that the unconstrained bank would choose a degree of leverage for which bankruptcy is not possible.

When capital constraints are imposed, the bank owner generally will be able to increase utility by leveraging asset portfolios with more risk than the one characterized by the parameters $E(P_0)$ and $\sigma(P_0)$. The reason is that a binding capital constraint changes the shape and location of the capital risk-return frontier, making it convex once the constraint becomes binding. The capital risk-return frontier under binding capital regulation is convex because it represents a linear mapping of the asset risk-return frontier which is assumed convex.[9]

In Figure 1, the effect of such a binding capital constraint on the capital risk-return frontier is depicted. As Koehn and Santomero point out, a reduction in permissible leverage reduces expected return on capital and investment risk over the entire constrained frontier for any given asset portfolio. Moreover, λ is larger on the constrained frontier. Thus, if a binding capital constraint were imposed on a previously unconstrained bank (at $U0$), the bank would choose a more risky asset portfolio (and move to $U1$). (The bank could have chosen a less or equally risky asset portfolio when not constrained, but did not, which precludes a new equilibrium on the old capital frontier.) It

Figure 1



should be emphasized that this result of an unambiguous increase in asset risk depends critically on the assumption that capital regulation alters the bank's risk-return frontier from a linear to a convex constraint, thereby increasing λ and reducing expected return on capital. However, as we demonstrate in the next section, in the absence of capital regulation the capital risk-return frontier of a bank that can fail is not linear, nor does leverage have a linear effect on risk and return.[10]

The effects of further reducing leverage through regulation are ambiguous, however, and depend on the shape of the bank's (owner's) utility function. The case depicted in Figure 1 is one with constant relative risk aversion, where increasing the stringency of capital regulation (lowering the permissible degree of leverage) leads the bank to choose a more risky and higher-return asset portfolio (by moving to $U2$).[11] This result too, however, depends critically on the assumption that leverage affects the risk-return frontier linearly—a result that does not hold for banks that can fail.

This basic result of increased asset risk caused by more stringent capital regulation is one result emphasized by KKS. More important is their claim that the increased asset risk caused by more stringent capital regulation could increase the probability of bank failure and thus could be counterproductive. However, under the constant borrowing cost assumption they use to derive their model, an increase in asset risk due to more stringent capital regulation cannot affect the probability of failure, which remains zero.

The reason is that the probability of failure must be zero in order for borrowing costs to be constant and for the effect of leverage on risk and return to be linear. As we show below, neither insured nor uninsured banks that can fail have constant borrowing costs, and thus there is not a linear effect of leverage on risk and return as KKS suppose. Moreover, with underpriced deposit insurance, the capital frontier may be nonconvex. Thus, KKS's model is not applicable to assessing the effects of capital regulation on the probability of bankruptcy. In the next section we show how the constraint changes and why KKS's analysis of capital regulation is inappropriate when a nonzero probability of bankruptcy and deposit insurance are introduced.

II. Introducing Bankruptcy and Deposit Insurance

The analysis above, which is consistent with KKS, assumes that a bank always would make asset and leverage choices such that bankruptcy could not occur. Such a bank could attract deposits at the risk-free rate because it always would make the payments promised regardless of the return on assets realized. Consequently, the cost of deposits (per dollar) to such a bank would equal the promised risk-free deposit rate and would not be a random variable so that the capital risk-return frontier in the absence of capital regulation would be linear.

In the absence of deposit insurance a bank's expected borrowing cost would rise as leverage (and thereby the probability of bankruptcy) increases if depositors are risk-averse. Thus, leverage does not have a linear effect on risk and return for uninsured banks that can fail.

More importantly, even with fixed-rate deposit insurance (under which a bank could attract deposits at a promised risk-free rate even though bankruptcy is possible), the per dollar deposit cost is random and the expected per dollar cost of deposits to the bank would vary with default risk and would no longer be equal to the promised risk-free deposit rate plus the deposit insurance premium rate.[12] As a result, leverage would not have linear effects on risk and return. The reason is that a fixed-rate deposit insurance guarantee represents an option to the bank to put the bank's assets to the insuring agency at a striking price equal to the promised maturity value of its liabilities. The value of the option (per dollar of deposits) increases as leverage (deposits) increases, but its price (per dollar of deposits) is fixed. This increase in the option's net value, in effect, lowers the expected marginal cost of deposits. As a result, the expected cost of deposits to the bank is less than the risk-free rate plus the deposit insurance premium. Moreover, the expected cost of deposits is not independent of the bank's asset portfolio risk—in fact, the expected cost of deposits also declines as asset portfolio risk increases because the net value of the deposit insurance option also increases as asset risk increases.

To demonstrate how the deposit insurance guarantee affects the risk-return tradeoff, it is assumed that a minimal form of capital regulation is in place (a bank owner must invest his or her entire net worth in the bank), and that the deposit insurance premium is zero.[13,14] The expected gross return on capital, then, is given by:

$$\text{(8)} \qquad E(Z) = \int_{P^*}^{\infty} \left[\frac{A_0 P - L_0 R}{K_0}\right] f(P)\, dP,$$

where

R = the promised gross rate on deposits,
$f(P)$ = the probability density function of P,
P^* = $[L_0/A_0]R$, which is the lowest asset return for which depositors are repaid in full, that is, when bank capital is exhausted.

(8) indicates that the expected gross return on capital is the expected value of gross asset returns minus liability obligations, conditional on nonbankruptcy. (If $P < P^*$, bankruptcy occurs and the gross return on capital is zero.) (8) can be rewritten by adding and subtracting the same term to give:

$$(9) \qquad E(Z) = \int_{-\infty}^{\infty} [\frac{A_0 P - L_0 R}{K_0}] f(P) dP - \int_{-\infty}^{P^*} [\frac{A_0 P - L_0 R}{K_0}] f(P) dP.$$

Noting that $L_0 = A_0 - K_0$ and taking the integral of the first term of (9) and rearranging terms in the second integral gives:

$$(10) \qquad E(Z) = \{[\frac{A_0}{K_0}][E(P) - R] + R\} + \int_{-\infty}^{P^*} [\frac{L_0 R - A_0 P}{K_0}] f(P) dP.$$

Note that the first term of (10) in braces is identical to the right-hand side of (3), the formula for the expected gross returns on capital of a bank that cannot go bankrupt. However, the second term of (10) represents an integration over bankruptcy states of the obligations to depositors in excess of returns on assets, which, by definition, are positive in each bankruptcy state (since if $P < P^*, L_0 R - A_0 P > 0$). The value of this integral, however, depends positively on leverage. (That is, the derivative of the integral with respect to liabilities holding constant equity is

$$\int_{-\infty}^{P^*} \frac{[R - P]}{K_0} f(P) dP$$

which is positive since $R > P$ for $P < P^*$.) This means that the cost of an additional dollar of deposits holding equity constant (which increases leverage), is not R, but is R minus the increase in the value of the integral.

This second term of (10), the expected value (conditional on bankruptcy) of the obligations to depositors in excess of returns on assets per dollar of invested capital, corresponds to the option value of deposit insurance as described by Merton (1977). This is the term that is neglected by both Kahane and Koehn and Santomero. By neglecting the option value of deposit insurance per dollar of invested capital, the linear relationship between expected return and leverage assumed by KKS no longer holds, nor does the linear relationship between risk and leverage.[15] In effect, the expected return on capital with deposit insurance is the sum of the expected return posited by KKS plus the expected return of the option. Below, the implications of these changed relationships for the effects of bank capital regulation on the relationship between leverage and expected return are explored.

Following Merton (1977), under the stochastic assumptions employed by Black and Scholes (1973), the value of the integral neglected by KKS—the option value of deposit insurance per dollar of capital invested—is:

$$(11) \qquad I_0 = \frac{L_0 F(\sigma\sqrt{t} - x) - A_0 F(-x)}{K_0},$$

where

I_0 = the value of the option per dollar of capital invested, which equals

$$\int_{\infty}^{P^*} [\frac{L_0 R - A_0 P}{K_0}] f(P) dP,$$

L_0 = the current value of insured deposits, which earn the risk-free interest rate and are assumed to constitute all deposits,

A_0 = the current value of assets (excluding the value of the insurance option),

σ = the standard deviation of the rate of return on assets, which is the measure of risk,

t = the interval to the next examination,

$$x = \frac{\log(\frac{A_0}{L_0}) + (\frac{\sigma^2 t}{2})}{\sigma\sqrt{t}}, \text{ and}$$

$F(\,)$ = the standard normal cumulative density function.

First, consider how the value of the option varies with leverage, holding initial capital constant. (We chose this method of varying leverage since it corresponds to KKS's assumption that the bank owner's capital is fixed.) Using the results in Jarrow and Rudd (1983),

$$(12) \qquad \frac{dI_0}{dL_0}\Big|_{K_0} = [\frac{\partial I_0}{\partial L_0} + \frac{\partial I_0}{\partial L_0}][\frac{1}{K_0}]$$

or

$$(13) \qquad \frac{dI_0}{dL_0}\Big|_{K_0} = [F(\sigma\sqrt{t} - x) - F(-x)][\frac{1}{K_0}] > 0.$$

That is, increasing deposits, holding capital constant, increases the option value of deposit insurance. Moreover, the second derivate of I_0 with respect to L_0 is positive.

Thus, the overall expected return increases more rapidly and nonlinearly with leverage than the linear relationship posited by KKS, thereby making the relation between leverage and expected return nonconvex. Because of this, risk-aversion would no longer necessarily constrain bank risk-taking. In fact, as we have shown elsewhere (Furlong and Keeley 1987), for a binomial asset return distribution, as long as the bank owner is willing to risk bankruptcy, absent regulation, optimal leverage is infinite even though the bank owner is risk-averse.[16]

Moreover, consider how the expected return varies with increased asset portfolio risk. The value of the option varies with asset risk as

$$(14) \qquad \frac{dI_0}{d\sigma} = \frac{A_0 \sqrt{t}\, F'(x)}{K_0} > 0.$$

Thus, independent of the positive market relationship between asset risk and return presumed by KKS, the value of the option also increases with asset risk, thereby changing the shape of the capital risk-return frontier holding leverage constant. That is, the capital risk-return frontier is no longer a linear mapping of the asset risk-return frontier.

Finally, as we have pointed out elsewhere (Furlong and Keeley, 1989), the gain from increased risk-taking (in terms of increased option value) increases as leverage increases because:

$$(15) \qquad \frac{d^2 I_0}{d\sigma dL_0} \Big|_{K_0} = [A_0 \sqrt{t}\, F''(x) \frac{dx}{dL_0} + F'(x)\sqrt{t}\,][\frac{1}{K_0}] > 0$$

(which is positive because $F''(x) < 0$, $dx/dL_0 < 0$ and $F'(x)>0$). (15) implies that the gain from increased risk-taking is not independent of leverage as KKS assume.[17]

As the above results demonstrate, the relationship between expected return, leverage and asset risk is straightforward, but the relationship between true capital risk and return is not. Although the variance of Z under subsidized deposit insurance is easily calculated, $\sigma(Z)$ alone no longer adequately characterizes risk for the bank owner when bankruptcy is possible. Specifically, it is well known that variance alone is an unreliable measure of risk for truncated, skewed distributions such as that of Z when bankruptcy is possible. Since the equity of the bank is a call option on its assets at a striking price equal to the promised maturity value of the deposits, the return on equity will be positively skewed. As Cox and Rubinstein (1985, pp. 317-342) show, for utility functions with constant proportional risk-aversion, expected utility depends on the skewness as well as the mean and variance of the return. By neglecting the skewness of the return distribution, KKS mischaracterize the shape of the capital risk-return tradeoff absent capital regulation and how that shape is affected by leverage and capital regulation.

Thus, KKS's analysis does not demonstrate that more stringent capital regulation would lead a utility-maximizing bank with fixed-rate deposit insurance to take on more asset risk. Moreover, from the analysis above, risk-aversion alone will not necessarily be sufficient to limit leverage and asset risk as is concluded by KKS. As a result, KKS's analysis cannot support their claim that more stringent capital regulation will be counterproductive for bank owners with certain preference structures. For example, Furlong and Keeley (1987) demonstrate that for a binomial asset return distribution, the probability of bankruptcy declines as the stringency of capital regulation is increased as long as the stringency of asset portfolio risk regulation remains unchanged regardless of the bank owner's preference structure.

III. Summary and Conclusions

Two inconsistent strands of the finance literature come to opposite conclusions regarding the effects of capital regulation on bank risk-taking. On the one hand, the options literature suggests that for risk-neutral or value-maximizing banks capital regulation will reduce the risk exposure of the deposit insurance system under a given stringency of asset regulation. On the other hand, the utility-maximization literature utilizing the Markowitz two-parameter portfolio model, as typified by KKS, claims that for risk-averse banks more stringent capital regulation may increase the probability of bank failure (and hence implicitly the risk exposure of the insurance system) and thus be counterproductive.

In this paper we show that the KKS model does not support its claimed results. KKS apply the Markowitz two-parameter portfolio model to analyze bank risk-taking under a nonzero probability of bankruptcy inappropriately. Specifically they neglect the option value of the deposit insurance subsidy and use an inappropriate measure of risk, thereby mischaracterizing both the risk-return frontier absent capital regulation and the shift in the risk-return frontier due to capital regulation. Because of these oversights, the models used by KKS are not applicable to analyzing the effects of bank capital regulation on asset risk and cannot be used to support their results.

NOTES

1. See Black, Miller, and Posner (1978) for a discussion of why bank regulation is analogous to the contractual enforcement of private lending agreements by private debtholders. Black, Miller, and Posner also argue, but do not formally prove, that the less capital the bank has, the greater its incentives for risk-taking. As a result, they call for more stringent capital requirements to protect the insurance fund.

2. Also, see articles by Wall (1985), Lam and Chen (1985) and Hanweck (1985).

3. As Koehn and Santomero (1980, p. 1244) put it, "In fact, a case could be argued that the opposite result can be expected to that which is desired when higher capital requirements are imposed."

4. Although Kahane and Koehn and Santomero conclude that both capital and asset regulation are necessary, as does the value-maximizing literature typified by Sharpe, Kareken and Wallace, and Dothan and Williams, it is important to recognize that KKS's models cannot support this conclusion. (It is true that, under the assumption of value maximization, both capital and asset regulation are needed to limit the liability of the deposit insurance system.) KKS claim that both capital and asset regulation are needed because more stringent capital regulation leads to greater asset portfolio risk, which in turn can increase the probability of failure. However, as we show later, KKS's models cannot be used to show that more stringent regulation will lead to an increased probability of failure.

5. Koehn and Santomero explicitly assume a constant deposit rate and corresponding constant borrowing cost per dollar of liabilities. Similarly, Kahane, following Hart and Jaffee (1974), assumes that the deposit rate is stochastic but unrelated to the bank's portfolio risk. For example, Kahane (1977, p. 209) states that ". . . the distributions of the random variables (the returns on assets and liabilities) must be exogenously given and independent of the value of the vector x (the portfolio allocation). . . " (parenthetical statements added).

6. In private correspondence Anthony Santomero indicated that the Koehn and Santomero model was properly interpreted as applying to insured banks. Moreover, the more recent Kim and Santomero (1988) model specifically is claimed to apply to insured banks.

7. It should be noted that the analysis of KKS is not applicable to uninsured banks either. To analyze uninsured banks, one would have to account for the increase in expected borrowing costs as bank risk increased and the variance of deposit returns increased, which would result from the behavior of risk-averse depositors.

8. See Merton (1972) for a discussion of how quadratic programming can be used to solve for the efficient asset portfolio.

9. To derive the shape of the capital risk-return frontier, the risk-return combination resulting from leveraging each asset portfolio to the maximum degree allowed may be traced out. As (3) and (4) show, both expected return, E(Z), and risk, R_i, are linear functions of leverage. Thus, the risk and return on capital for a given asset portfolio and leverage can be found geometrically by extending a ray from the constant borrowing rate through the particular asset portfolio up to the maximum leverage allowed. The locus of such points is the constrained capital risk-return frontier. Also, note that a particular point on a capital risk-return frontier has greater asset risk than another point on the same or on a different frontier if the slope of a line from R through the point is smaller.

10. Although the shape of the risk-return frontier depends on whether the bank's deposits are insured at a fixed rate, neither insured nor uninsured frontiers are linear.

11. For a formal proof see Koehn and Santomero (1980).

12. An anonymous referee noted that once it is realized that the deposit rate (from the bank's perspective) is random, the nonlinearity of the investment frontier is self-evident since it is simply a combination of positive and negative risky assets. While this is true, our point is that a fixed-rate deposit insurance guarantee alters the risk-return frontier in a specific way so as to subsidize risk-taking.

13. The assumption implicit in the utility maximization framework is that a (potential) bank owner has an exogenously given initial capital (wealth) K_0, all of which must be invested in the bank in order to obtain deposit insurance. However, this assumption implies some form of capital regulation since a risk-averse bank owner generally would prefer to segregate his capital (and make a relatively safe investment) and start a bank with no capital, thereby acquiring an option of potentially unlimited value. This suggests that the utility-maximization model may not be applicable to many actual banks since even owners of small banks have limited liability, and, absent capital regulation, would not have to risk all of their own funds. It also suggests that this limited form of capital regulation must reduce the probability of bank failure—a result opposite to KKS.

14. Since we are interested in fixed-rate deposit insurance systems, the analysis of a zero-premium rate system is essentially the same as a positive fixed-rate system.

15. With fixed-rate deposit insurance, a more risky asset portfolio, even if it has the same expected return, increases the expected return on capital because the option value of deposit insurance increases. Similarly, by increasing leverage, the owner can increase without limit the expected return on capital as long as some part of the asset distribution exceeds the promised rate. Under these circumstances, even if the expected rate on assets were less than the promised rate on deposits, a bank with underpriced deposit insurance would gain from leverage. This is in sharp contrast to the results without deposit insurance when bankruptcy is not possible. In that case, leverage can increase expected return only if the expected return on assets exceeds the expected cost of

deposits (see (3)). Thus, the provision of underpriced deposit insurance can cause risk-averse bank owners to assume more risky portfolios.

16. Rationality implies that a lottery that costs $1 and pays $100 with a 50 percent chance and $0 with a 50 percent chance will be preferred to one that also costs $1 but pays $10 with a 50 percent chance and $0 with a 50 percent chance, even though the variance of the first lottery's outcomes is higher. Thus, for some asset distributions, such as the binomial, a bank owner's true risk is limited (for sufficiently high leverage there is a constant probability, which is invariant with greater leverage and is less than one, that he will lose his capital), but his (certain) return if failure does not occur (and thus his expected return) is unlimited as leverage increases. (That is, as leverage increases, end-of-period capital increases without limit as long as bankruptcy does not occur and is zero if bankruptcy does occur. See Furlong and Keeley (1987, p. 39) for a more detailed discussion.)

Thus, even a risk-averse bank owner (willing to risk bankruptcy in return for a sufficiently high payoff) might prefer unlimited leverage. A similar result applies to asset risk. In contrast, KKS argue that risk-aversion necessarily limits bank risk-taking. While this may be the case in the absence of subsidized deposit insurance, it need not be so with subsidized deposit insurance. It may be the case that even a risk-averse bank owner-manager will try to maximize the option value of deposit insurance. Thus, in general it is not meaningful to analyze the effects of capital regulation absent asset regulation as do KKS.

17. (15) also implies that, for value-maximizing banks, more stringent capital regulation will reduce the risk exposure of the deposit insurance system as long as the stringency of asset portfolio regulation is not reduced. See Furlong and Keeley (1989) for a more complete discussion.

REFERENCES

Black, Fischer, Merton Miller, and Richard Posner. 1978. "An Approach to the Regulation of Bank Holding Companies." *Journal of Business* 51, pp. 379-413.

_________, and Myron Scholes. 1973. "The Pricing of Options and Corporate Liabilities." *Journal of Political Economy* 81, pp. 632-659.

Cox, John C., and Mark Rubinstein. 1985. *Options Markets*. Englewood Cliffs, NJ: Prentice Hall.

Dothan, Uri, and Joseph Williams. 1980. "Banks, Bankruptcy and Public Regulation." *Journal of Banking and Finance* 4, pp. 65-88.

Fama, Eugene F., and Merton H. Miller. 1972. *The Theory of Finance*. New York: Holt, Rinehart and Winston.

Furlong, Frederick T., and Michael C. Keeley. 1987. "Bank Capital Regulation and Asset Risk." Federal Reserve Bank of San Francisco *Economic Review* (Spring) pp. 20-40.

_________, and _________. 1989. "Capital Regulation and Bank Risk-taking: A Note." *Journal of Banking and Finance* 13, pp. 883-891.

Hanweck, Gerald A. 1985. "A Theoretical Comparison of Bank Capital Adequacy Requirements and Risk-related Deposit Insurance Premia." Research Papers in Banking and Finance, no. 72. Board of Governors of the Federal Reserve System.

Hart, Oliver D., and Dwight M. Jaffee. 1974. "On the Application of Portfolio Theory to Depository Financial Institutions." *Review of Economic Studies* 41, pp. 129-147.

Hirshleifer, J. 1970. *Investment, Interest, and Capital*. Englewood Cliffs, NJ: Prentice Hall, Inc.

Jarrow, Robert A., and Andrew Rudd. 1983. *Option Pricing*. Homewood, IL: Richard D. Irwin, Inc.

Jensen, Michael C., and William H. Meckling. 1976. "Theory of the Firm: Managerial Behavior, Agency Costs and Ownership Structure." *Journal of Financial Economics* 3, pp. 305-360.

Kahane, Yehuda. 1977. "Capital Adequacy and the Regulation of Financial Intermediaries." *Journal of Banking and Finance* 2, pp. 207-217.

Kareken, John H., and Neil Wallace. 1978. "Deposit Insurance and Bank Regulation: A Partial Equilibrium Exposition." *Journal of Business* 51, pp. 413-438.

Kim, Daesik, and Anthony M. Santomero. 1988. "Risk in Banking and Capital Regulation." *Journal of Finance* 43 (December) pp. 1219-1233.

Koehn, Michael, and Anthony M. Santomero. 1980. "Regulation of Bank Capital and Portfolio Risk." *Journal of Finance* 35 (December) pp. 1235-1250.

Lam, Chun H., and Andrew H. Chen. 1985. "Joint Effects of Interest Rate Deregulation and Capital Requirements on Optimal Bank Portfolio Adjustments." *Journal of Finance* 40 (June) pp. 563- 576.

Merton, Robert C. 1972. "An Analytic Derivation of the Efficient Portfolio Frontier." *Journal of Financial and Quantitative Analysis* (December) pp. 1851-1872.

_________. 1977. "An Analytic Derivation of the Cost of Deposit Insurance and Loan Guarantees." *Journal of Banking and Finance* pp. 3-11.

Sharpe, William F. 1970. *Portfolio Theory and Capital Markets*. New York, McGraw-Hill.

_________. 1978. "Bank Capital Adequacy, Deposit Insurance and Security Values." *Journal of Financial and Quantitative Analysis* (November) pp. 701-718.

Wall, Larry D. 1985. "Regulation of Banks' Equity Capital." Federal Reserve Bank of Atlanta *Economic Review* (November) pp. 4-18.