Federal Reserve Bank of San Francisco

Spring

1987

Economic Review

Number

2

The Federal Reserve Bank of San Francisco's Economic Review is published quarterly by the Bank's Research and Public Information Department under the supervision of Jack H. Beebe, Senior Vice President and Director of Research. The publication is edited by Gregory J. Tong, with the assistance of Karen Rusk (editorial) and William Rosenthal (graphics).

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Bank Capital Regulation and Asset Risk

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This paper examines theoretically the effects of more stringent capital regulation on a bank's incentive to increase asset risk and on the expected liability of the deposit insurance system. Our analysis shows that regulatory increases in capital standards will not require greater regulatory efforts to restrain asset risk because a bank's incentive to increase asset risk declines as its capital increases. Thus, as long as regulatory efforts to contain asset risk, such as bank examinations, are not reduced, more stringent capital regulation will reduce the expected liability of the deposit insurance system.

Over the past several years, bank regulators have placed greater emphasis on the regulation of bank capital. For example, the three federal bank regulatory agencies have raised capital requirements for banks and bank holding companies and established more uniform standards among themselves.[1] Most recently, the federal bank regulatory agencies have put forth proposals for risk-based capital requirements that would be coordinated with the Bank of England.[2]

These regulatory measures, in part, are reactions to deteriorating capital positions, particularly among the larger banking organizations. For example, among the twenty largest bank holding companies, the average ratio of the book value of common equity to assets was over 6 percent in 1968 but only about 4 percent in 1980.[3] The increase in the number of bank failures and the correspondingly sharp rise in the Federal Deposit Insurance Corporation's (FDIC's) expenses in recent years also have intensified interest in capital regulation. Total expenses of the FDIC, which fluctuated between about $50 million and $200 million per year in the 1970s, rose to about $2 billion per year in 1985 and 1986. Such increases in expenses for the deposit insurance system have focused attention on increasing the stringency of bank capital regulation to limit the FDIC's exposure to losses and to blunt the incentives for "excessive" risk-taking by federally insured banks.

The move to more stringent capital standards in banking, however, has met with considerable controversy as well as some skepticism. Some argue that higher capital requirements will cause banks simply to invest in more risky assets, and thereby offset, or even more than offset, the desired effects of higher capital. This view often is echoed in the financial press. In a New York Times article about a Federal Reserve proposal to require banks to hold capital in connection with agreements involving interest rate and currency swaps, William McDonough, vice chairman of First National Bank of Chicago, is quoted as saying that ". . . the proposal could lead banks to take on riskier business to compensate for the lower returns they would almost assuredly get by having to maintain more capital."[4]

The effectiveness of capital regulation also has come under question in the academic literature. A study by Koehn and Santomero (1980), which assumes that banks maximize utility in a mean-variance framework,[5] is representative of the literature on the theoretical relationship between capital

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requirements and bank asset risk. They conclude that ". . . a case could be argued that the opposite result can be expected to that which is desired when higher capital requirements are imposed."[6]

In this paper, we evaluate the popular view of capital regulation and the conclusions of earlier theoretical studies on the effectiveness of capital regulation. In contrast to the popular view and the earlier academic work, we find that more stringent capital standards alone would not give a bank more of an incentive to increase the riskiness of its assets. In fact, the incentive to increase asset risk falls as a bank's capital increases. This implies that, as long as regulatory and supervisory efforts to limit asset risk in banking, such as bank examinations, are not relaxed, increasing a bank's capital will lower that bank's chance of failure and reduce the expected liability of the deposit insurance system.

We also show in the Appendix that the conclusions reached by earlier theoretical studies using the mean-variance framework were derived from internally inconsistent assumptions. In essence, these studies implicitly (but unintentionally) assume that bank failure is not possible by assuming that borrowing costs are unrelated to bank risk. Yet, they seek to analyze the effects of capital regulation on the probability of bank failure. Moreover, these studies fail to incorporate the effect of the deposit insurance guarantee on risk-taking. Although the results of these studies regarding the effects of capital regulation on the incentive to increase asset risk are technically correct when bank failure is not possible, such findings are of little policy relevance since capital regulation and concern over risk-taking are relevant only when banks can fail.

The Appendix also contains an example to show that the results of these earlier studies do not generally hold when subsidized deposit insurance and the possibility of bankruptcy are taken into account. Specifically, we show that when the asset return distribution is binomial, the incentive to increase asset risk does not increase as the stringency of capital regulation increases.

The analytic framework used in the body of this paper is the state-preference model rather than the mean-variance model used in the older literature on the topic. One reason for this choice is that the state-preference model, unlike the mean-variance model, can easily accommodate the possibility of bankruptcy and an analysis of the effects of mispriced deposit insurance on a bank's choice of leverage and asset risk.[7] Moreover, with the state-preference model, the effects of changes in capital requirements on both banks' gains from increasing asset risk and the expected liability of the deposit insurance system can be evaluated directly.

Another advantage of the state-preference framework is that it can be applied to the analysis of both value-maximizing and utility-maximizing banks. Utility maximization might be appropriate for certain smaller, closely held banks where the owners' risk preferences affect the riskiness of the banks' portfolios, whereas value maximization is more suitable for most other banks, particularly the large publicly held banks whose stockholders can hold diversified portfolios. Value-maximizing banks would seek to maximize the current market value of their equity, which is independent of the risk preferences of the owners.[8]

In the next section, we start with a discussion of the nature of bank capital and the issues that higher capital requirements raise for bank regulators. In Section II we introduce the state-preference model and use it to analyze the effects of capital regulation on asset risk and the liability of the federal deposit insurance system, under the assumption that banks choose to maximize the value of stockholders' wealth. Section III contains a similar analysis, applying the state-preference model to utility-maximizing banks. The conclusions and policy implications are presented in the final section.

I. Issues in Capital Regulation

A bank's financial capital — that is, its equity — is the difference between the value of the institution's assets and liabilities. Banks use both capital and liabilities to finance loans and investments.[9] The two sources of funding are distinguished in that variations in earnings on assets are borne first by capital holders. The larger the proportion of assets funded by capital, the greater the range of returns on assets that will be borne solely by equity holders and the more likely the promised obligation to liability holders will be met. Thus, if banks were not insured, both equity and liability holders, including depositors, would have an interest in the level of a bank's capital. As with other firms, the stockholders and liability holders (depositors) of unregulated banks would be expected to "decide" on a satisfactory combination of capital financing and promised return on bank debt.

The regulation of bank capital, then, must be predicated on the assumption that a market determination of the level of capital would not be satisfactory from a public policy perspective. While capital regulation predates federal deposit insurance, partly because of the externalities argued to be associated with bank failures, the provision of the federal deposit guarantee commonly is cited as the main reason that the level of bank capital must be a regulatory concern.

The federal deposit insurance system, by guaranteeing deposits, in essence takes on the role of a bank liability holder and has an interest in bank capital similar to that of private liability holders in an uninsured firm. Indeed, some have argued that the deposit insurance system has taken on the role of covering virtually all bank liability holders in the event of an insolvency. If so, the insurance system would be the only liability holder with an interest in bank capital.

From a regulatory perspective, a bank with more capital relative to assets will be less likely to fail, and, if it does fail, will impose smaller losses on the insurance fund, all other things equal. However, the probability of failure and the contingent liability of the insurance system also depend on the variability of the return on assets.[10] The higher the variability of the return on assets for a given amount of capital, the greater the chance of bank failure.[11]

Consequently, a central issue in capital regulation is whether banks would respond to higher regulatory capital requirements by choosing riskier assets to offset or even more than offset the effects of higher capital on the exposure of the deposit insurance system to bank risk. Below, we consider this issue and examine under what conditions, if any, regulation-induced increases in bank capital would lower the expected losses of the deposit insurance system.

II. Value Maximization

A value-maximizing bank chooses its portfolio solely to maximize the current market value of equity. Such a bank's portfolio decisions are independent of the risk preferences of its individual owners because the owners are fully able to adjust the composition of their personal portfolios to attain any level of risk they desire. Therefore, even though actual returns on the bank's portfolio are uncertain (risky), a value-maximizing bank does not consider the risk preferences of the owners.

Some of the implications of bank capital regulation for value-maximizing banks within the state-preference framework are discussed in Dothan and Williams (1980), Sharpe (1978), and Kareken and Wallace (1978).[12] All of these studies provide theoretical support for restricting leverage in banking when there is subsidized deposit insurance. They do not, however, deal with the issue of how the asset investment strategies of such insured banks might be altered by capital regulation. Nor do they consider how the behavior of a utility-maximizing bank in the state-preference framework might differ from that of a value-maximizing bank.

This portion of the paper addresses the first of these two issues by extending the examination of bank capital regulation within a state-preference framework. For the reader who is not familiar with this framework, a brief description of the state-preference model is presented in Box 1. Below, we first show why leverage constraints are necessary for insured, value-maximizing banks. Then, we assess the likely effects of changes in capital requirements on the asset risk of such banks and on the liability of the deposit insurance system. The discussion in Section III turns to the implications of deposit insurance and capital regulation for utility-maximizing banks within a state-preference model.

Value-Maximizing Banks

Although the state-preference model can be applied to an individual investor's decisions, it also can be used to analyze the portfolio and leverage decisions of an insured bank that maximizes its current value (the market value of its equity). Since the current value of such a bank is independent of the risk preferences of the owners, we can put aside any consideration of utility functions and focus instead on how an insured bank's investment opportunity frontier itself is affected by leverage and capital regulation.

The effects of leverage and the role of capital regulation can be seen most easily through a numerical example with two states and two securities. In this example, security A represents a promise to pay $4 if state 1 occurs and $6 if state 2 occurs, and is summarized as A(4, 6). The second security, security B, is summarized as B(1, 1). Security A is a risky investment (a different payout in each state) and security B is a riskless security (the same payout in each state). For expositional purposes, we assume that the current market price of a dollar payment in state 1 is $.35 and the price of a dollar payment in state 2 is $.60. The current price of a share of security A then is $5. ($\$.35 \times 4 + \$.6 \times 6$) and the current price of security B is $.95 ($\$.35 \times 1 + \$.6 \times 1$).[13]

The bank is assumed to invest only in the risky security. The bank's purchases of that security are funded with a combination of capital and the proceeds from issuing shares of security B. Shares of security B can be thought of as deposits that are "insured" at a fixed-premium rate by the federal government. In the example, the premium is set at zero, but the analysis and conclusions would hold even with a positive, fixed-rate premium.[14] Initial capital is set at $500 by assumption. With no deposits, the bank would have 100 shares of security A, and leverage would be one.

The calculations in Table 1 demonstrate what happens to the total net worth (current value) of the bank's equity as leverage increases. In line 2, the bank increases leverage to 2 by issuing deposits with a current value of $500 and purchasing an additional 100 shares of security A. In both states 1 and 2 the bank promises to repay depositors $526.32 ($500/.95). The net claims (future wealth) of the bank in each state after paying off deposits are shown in column 6. The current value of the bank to the owners, column 8, is derived by multiplying the net claims by the price of a dollar claim in the appropriate state. The addition to the value of the bank from the free deposit insurance, presented in the last column, is derived by subtracting the initial capital, $500, from the total value of net worth.

As Table 1 shows, initially the bank's value (net worth) is not affected by issuing deposits. At the lower levels of leverage, the bank would be indifferent to the amount of borrowing because its value (column 8) would be unaffected.[15] Although the risk of the bank increases with leverage, as reflected in the growing disparity between the claims in the two states (column 6), bankruptcy could not occur and the deposit insurance fund would not be at risk with leverage of 4 or less. Moreover, depositors would be indifferent to the risk of the bank whether or not their funds were insured as long as leverage was less than or equal to 4.

It is easy to see why the insurance fund as well as depositors are not at risk at low levels of leverage. Up to a point, the bank is able to meet its promised payments to depositors in both states 1 and 2. The bank would not fail in either state since its liabilities would not exceed its assets. Therefore, while risk increases with leverage, as long as the bank's capital is sufficient to ensure payment, the added risk is borne only by the bank.[16]

As leverage continues to increase, the bank eventually will be unable to meet its promised obligations to depositors in the first state. Without a third party guarantee such as deposit insurance, depositors would not be willing to lend to a bank in return for a promise to pay only $1 (per share) in each state. With leverage equal to 5, for example, the bank would issue a deposit with a current value of $.95 per share but would have to promise to pay about $1.03 in each state instead of $1 (the actual pay-

Box 1
The State-Preference Model

The state-preference model can be used to analyze investors' decisions that affect their future consumption. In this model, an investor chooses among combinations of claims to wealth in all possible future states of the world.*

In its simplest form, the state-preference approach includes only two time periods: now and the future. The future is uncertain because only one of a number of possible states of the world actually will occur. The possible states of the world, for example, might be represented by inflation, deflation, and unchanged prices. An investor's claim to future wealth (measured in terms of real purchasing power) if state i were to occur can be expressed as W_i.

To secure future claims to wealth, the investor in the state-preference model can purchase (or issue) securities. The securities are characterized by the payments made in the various possible states. Consider, for example, a world in which there are two possible future states, 1 and 2, and two securities, A and B. One share of security A pays a_1 real dollars in state 1 and a_2 in state 2. We can summarize these characteristics by $A(a_1, a_2)$. One share of security B can be summarized as $B(b_1, b_2)$.

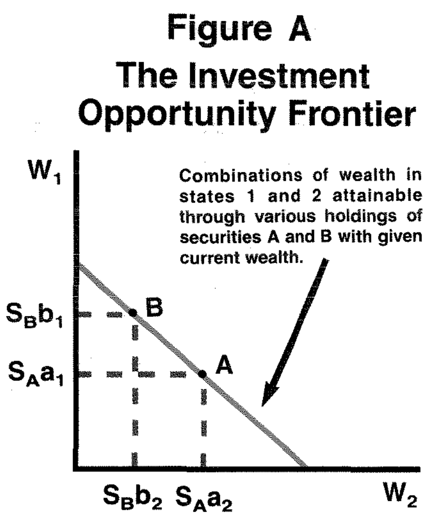
The current value or price of a share of either security can be expressed in terms of two current market prices: p_1, the current price of a future payment of one dollar if state 1 occurs, and, p_2, for a one dollar claim in state 2. (These prices are taken as given by the investor and are unaffected by his decisions.)**

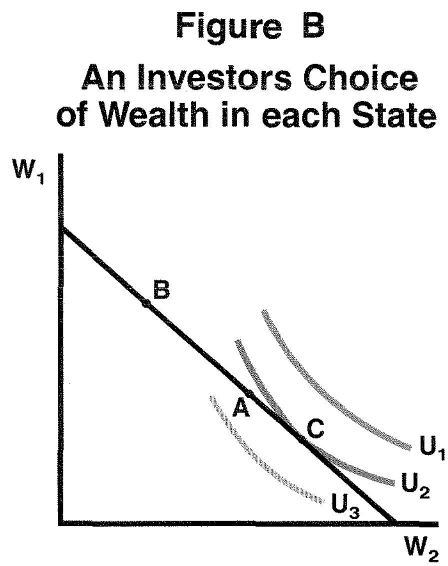
The current price of security A then is: $p_A = p_1a_1 + p_2a_2$. And the price of security B is: $p_B = p_1b_1 + p_2b_2$. The key insight of the state-preference model is that all future uncertain claims have a certainty-equivalent current value as determined by the prices of a dollar claim in each state.

The number of shares of securities that an investor can purchase is determined as follows. The amount of "capital," K, that an investor decides to allocate to purchase or secure claims on future wealth is taken as given. If only security A were purchased, the investor's future claims would be the number of shares purchased times the per share payout in each state: (S_Aa_1, S_Aa_2), where S_A is the number of shares of security A and $S_A = K/p_A$. If only security B were purchased, the claims would be (S_Bb_1, S_Bb_2), where S_B is the number of shares of security B and $S_B = K/p_B$.

In Figure A, we have plotted the various combinations of wealth in state 1 and state 2, denoted as W_1 and W_2, that can be attained through various holdings of security A and security B. These combinations are represented by the negatively sloped straight line in the figure called the investment opportunity frontier. Point A represents the wealth outcomes (S_Aa_1, S_Aa_2) if only security A were purchased, and point B, the outcomes (S_Bb_1, S_Bb_2) if only security B were purchased. The points between A and B represent combinations where K is divided between holdings of both securities.

For points on the opportunity frontier below point A, the investor issues security B (borrows) and uses the funds to purchase additional shares of security A. Similarly, for the points on the frontier above point B, the investor issues security A and purchases additional shares of B. Thus, through various combinations of holding and issuing securities A and B, an investor can attain any combination of wealth in the two states along the frontier. Borrowing, however, does not affect the current value of the investor's net future claims (claims after the payments on the issued security have been made), which is equal to K.***





Given the constraint on current wealth, the actual combination of assets chosen is determined by the investor's preferences concerning the trade-off between wealth in the different future states. Those preferences can be represented generally as the utility function $U(W_1, W_2)$. A number of factors could affect the nature of utility functions, including the investors' assessments of the probabilities of the states occurring. As is generally the case in economic models, in the state-preference model an investor's utility function is assumed to be convex. In the state-preference model, the convexity of utility functions indicates that investors are risk-averse.

Examples of convex utility functions are shown in Figure B, where utility rises such that $U_1 > U_2 > U_3$. An investor will choose a combination of securities — a point on opportunity frontier (the line on which wealth equals K) — that results in the highest utility. The point of highest utility would be the one and only point at which a utility curve is tangent to the opportunity frontier. In the figure, that point is represented by C. Point C is attainable by issuing shares of security B and investing in shares of security A.

* For a complete discussion of the state-preference model see Sharpe (1970).

**Arrow (1964) and Debreu (1959) have shown that a competitive market equilibrium determines these prices.

***This is always true in the absence of any subsidized third party guarantee of such borrowing. However, as is shown in the body of the paper, subsidized deposit insurance does affect current wealth.

ments would be about $1.03 in state 2 and about .95 in state 1). The current value of the depositor's claims would be unaffected because the higher payment in state 2 would compensate for the lower payment in state 1. The deposit guarantee, however, would allow a bank with leverage greater than 4 to continue promising $1 to depositors in both states because the deposit insurance fund would cover the shortfall in state 1.

As seen in column 8 of Table 1, once leverage is extended to a point at which bank failure becomes possible, the current net worth of the bank begins to increase with leverage. The addition to net worth represents the current value of the deposit insurance guarantee (column 9). A bank gains from increasing leverage and simultaneously investing additional deposits in the risky security because the net claims of the owners in state 1 can never be less than zero, no matter how large the "promised" payments, while the potential claims in state 2 are unlimited. The state-preference model therefore predicts that a value-maximizing bank with an insurance premium less than the current value of the insurance payout would limit its leverage only if forced to do so by regulation.

Table 1

Effects of Leverage with 100% Insurance of Deposits

Security A ***A(4,6)*** $p_1 = \$.35$ ***Initial Bank Capital = $500***
Security B (Deposit) ***B(1,1)*** $p_2 = \$.6$

	(1)	(2)	(3)		(4)	(5)	(6)	(7)	(8)	(9)
					Future			Current		
	Initial Leverage	Shares of Sec.A	Current Value of Depsoits (Sec. B)	State	Gross Claims	Deposit Payments	Net Claims (4-5)	Price of $1 Claim	Value of Bank (6x7)	Value of Deposit Guarantee (Col. 8-$500)
(1)	1	100	$ 0	1	$ 400.00	$ 0.00	$ 400.00	.35	$140.00	
				2	600.00	0.00	600.00	.6	360.00	
									$500.00	$ 0.00
(2)	2	200	500	1	800.00	526.32	273.68	.35	95.79	
				2	1200.00	526.32	673.68	.6	404.21	
									500.00	0.00
(3)	3	300	1000	1	1200.00	1052.63	147.37	.35	51.58	
				2	1800.00	1052.63	747.37	.6	448.42	
									500.00	0.00
(4)	4	400	1500	1	1600.00	1578.95	21.05	.35	7.37	
				2	2400.00	1578.95	821.05	.6	492.63	
									500.00	0.00
(5)	5	500	2000	1	2000.00	2105.26	0.00[1]	.35	0.00	
				2	3000.00	2105.26	894.74	.6	536.84	
									536.84	36.84
(6)	6	600	2500	1	2400.00	2631.58	0.00[2]	.35	0.00	
				2	3600.00	2631.58	968.42	.6	581.05	
									581.05	81.05
(7)	7	700	3000	1	2800.00	3157.89	0.00[3]	.35	0.00	
				2	4200.00	3157.89	1042.11	.6	625.27	
									625.27	125.27
(8)	8	800	3500	1	3200.00	3684.21	0.00[4]	.35	0.00	
				2	4800.00	3684.21	1115.79	.6	669.47	
									669.47	169.47

1. Actual net claim in state 1 is - $105.26.
2. Actual net claim in state 1 is - $231.58.
3. Actual net claim in state 1 is - $357.89.
4. Actual net claim in state 1 is - $484.21.

Capital Requirements and Risk

This brings us to the main question facing regulators: will regulatory efforts to force banks to hold more capital be partially or even totally offset by banks that then acquire riskier assets?

To answer this question, another risky asset has to be introduced. In addition to security A(4, 6), we assume that the bank also can hold the more risky security, security D(0, 8.33), where the numbers in parentheses are the dollar claims per share of the securities in the two possible states. The price of security D is \$5 ($\$.35 \times 0 + \$.6 \times 8.33$). A bank with a given degree of leverage can alter its net claims in future states by investing available funds in different combinations of these two risky assets.

Table 2 demonstrates how the value of a bank with an initial leverage of 3, initial capital of \$500, and underpriced deposit insurance is affected by shifting from holding only security A to holding greater proportions of its assets in security D. Parallel to the case of increased leverage with asset risk held constant (Table 1), a bank with a given level of leverage benefits from increasing its asset risk with underpriced deposit insurance only when bankruptcy is possible (that is when deposit claims exceed the bank's gross claims in state 1). Once bankruptcy is possible, the value of the bank increases with asset risk (that is, with higher proportions of security D). Therefore, even if leverage were limited through regulation, a value-maximizing

TABLE 2

Effects of Asset Risk with 100% Insurance of Deposits

Security A — ***A(4,6)*** — ***$p_1 = \$.35$*** — ***Initial Bank Capital = \$500***
Security B — ***B(1,1)*** — ***$p_2 = \$.6$*** — ***Initial Assets = \$1,500***
Security D — ***D(0,8.33)*** — ***Initial Leverage = 3***

	(1)	(2)	(3)		(4)	(5)	(6)	(7)	(8)	(9)
					Future			Current		
	Shares of Sec. A	Shares of Sec. D	Percent Shares in Sec. D	State	Gross Claims	Deposit Payments	Net Claims (4-5)	Price of \$1 Claim	Value of Bank (6x7)	Value of Deposit Guarantee (Col. 8-\$500)
(1)	300	0	0	1	\$1200.00	\$1052.63	\$ 147.37	.35	51.58	
				2	1800.00	1052.63	747.37	.6	448.42	
									\$500.00	\$ 0.00
(2)	240	60	20	1	960.00	1052.63	0.00[1]	.35	0.00	
				2	1940.00	1052.63	887.37	.6	532.42	
									532.42	32.42
(3)	180	120	40	1	720.00	1052.63	0.00[2]	.35	0.00	
				2	2080.00	1052.63	1027.37	.6	616.42	
									616.42	116.42
(4)	120	180	60	1	480.00	1052.63	0.00[3]	.35	0.00	
				2	2220.00	1052.63	1167.37	.6	700.42	
									700.42	200.42
(5)	60	240	80	1	240.00	1052.63	0.00[4]	.35	0.00	
				2	2360.00	1052.63	1307.37	.6	784.42	
									784.42	284.42
(6)	0	300	100	1	0.00	1052.63	0.00[5]	.35	0.00	
				2	2500.00	1052.63	1447.37	.6	868.42	
									868.42	368.42

1. Actual net claim in state 1 is - \$92.63
2. Actual net claim in state 1 is - \$332.63
3. Actual net claim in state 1 is - \$578.63
4. Actual net claim in state 1 is - \$812.63
5. Actual net claim in state 1 is - \$1057.63

bank with deposit insurance would want to hold the risky security that maximized the value of the deposit guarantee. (In the example, this would be a portfolio that includes only security D.)

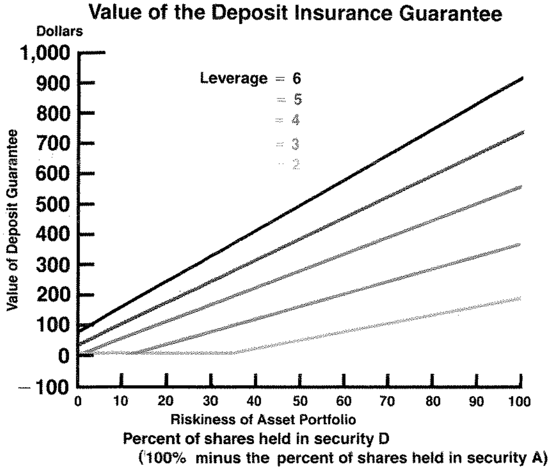
Figure 1 shows how the gains from increasing asset risk are affected by leverage. Each of the lines in the figure tracks the current value of the deposit guarantee to the bank that invests greater proportions of funds in security D (and correspondingly smaller proportions in security A), for a given degree of leverage. The marginal value to a bank from increasing asset risk (holding greater proportions of its asset in security D) is represented by the slope of a line.

With low levels of leverage and asset risk, the marginal value to increasing asset risk is zero (the lines are horizontal). However, for higher levels of leverage, the slopes of the lines increase as leverage increases, indicating that the marginal value of increasing asset risk increases with leverage. Put another way, as the capital of an insured bank increases, the marginal value to that bank of shifting to a riskier composition of assets falls. This means that more stringent capital requirements would *not* give banks a greater incentive to invest in riskier assets, and would reduce the liability of the deposit insurance system.[17]

With regulatory constraints on leverage, a bank still would want to hold the risky asset, security D.

Figure 1

Effects of Leverage and Risk on the Value of the Deposit Insurance Guarantee



Conclusions similar to those derived from the state-preference model regarding the implications of capital regulation for risk-taking (and the resulting contingent liability of the deposit insurance fund) can be derived by modeling the deposit insurance guarantee as a put option. In such an options approach, the bank is viewed as "purchasing" an option from the insurance fund to sell (put) its assets to the fund at an exercise price equal to the value of the bank's insured deposits (which we assume represent all bank liabilities). The bank would exercise this option only if it were insolvent, that is, when the assets were worth less than the liabilities.

Following Merton (1977), the Black-Scholes formula for a European option (one that can be exercised only at maturity) can be adapted to apply to the deposit insurance guarantee. Assuming all earnings are retained, the value of the insurance option can be expressed as

$$V_t = v(A, D, s, t),$$

where V_t is the current value of the contingent insurance liability, A is the current value of assets (excluding any insurance subsidy), D is the current value of insured deposits, s measures risk and is the standard deviation of the rate of return on assets, and t is the interval between examinations.*

Merton modified the put option model to apply to deposit insurance rather than equity securities. In his formulation, the value of assets of the bank replaces the stock price, the value of deposits represents the exercise or strike price,** the standard deviation of the rate of return on assets replaces the standard deviation of the return on the stock, and the examination interval corresponds to the time to maturity.

This approach generates the result that the value of the insurance guarantee increases with leverage.

Box 2
Capital Regulation and Asset Risk: An Options Approach

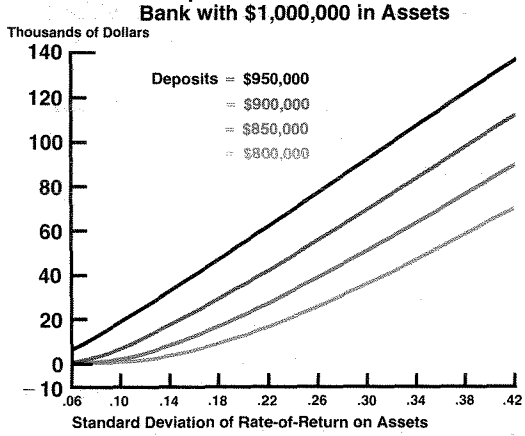
Greater leverage (that is, more deposits) in place of capital results in an increase in the exercise price and makes it more likely that the option would be exercised. Also, holding riskier assets, which would be reflected in a higher variation in the rate-of-return, would make it more likely that the value of assets would fall below the value of deposits, thus raising the value of the option. If a bank were not required to pay the full value of the option, then the bank could add to its owners' wealth by increasing leverage and/or asset risk.

The effects of leverage and asset risk on the value of the insurance guarantee from the options formula are illustrated graphically in the figure in this Box. For purposes of the figure, examinations are assumed to take place at the same time each year ($t = 1$) and the initial assets of the bank (excluding the value of the deposit guarantee) are set at $1 million.

In the figure, holding leverage constant (moving along a line from left to right), the contingent liability of the insurance fund increases with asset risk. Of importance to this paper is the observation that the marginal value of (and hence, incentive for) increasing asset risk increases with leverage. When the standard deviation of the rate of return on assets is held constant, the slopes of the lines in the figure increase as leverage increases. That is, at higher leverage (less capital), the change in the value of the insurance option for a given change in asset risk is greater.

As in the example using the state-preference model, then, raising capital standards (lowering leverage) reduces the marginal value of increasing asset risk for an insured bank. Unless other regulatory constraints are relaxed, the options approach also implies that regulating capital should not lead to higher asset risk and should reduce the contingent liability of the deposit insurance system.

Value of Deposit Insurance Guarantee for a Bank with $1,000,000 in Assets



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* The options formula for deposit insurance is

$$V_t = DF(X + s\sqrt{t}\,) - AF(X)$$

where

$$X = (\log (D/A) - (\frac{s^2}{2})t)/s\sqrt{t}$$

and F(·) is the standard normal cumulative density function evaluated from $-\infty$ to (·).

**The exercise price itself is $X = De^{rt}$, the value of deposits at time the bank would be examined. In the put options formula only D appears because the exercise price is multiplied by e^{-rt}, and $Xe^{-rt} = D$.

As a result, regulators also might put controls on asset risk to reduce the liability of the deposit insurance system. For example, regulation might limit a bank to holding less than 30 percent of its assets in security D. However, if regulatory limits on the composition of bank assets and monitoring (examinations) of banks were sufficient to constrain the asset risk of a bank for a given level of leverage, they would be sufficient for any lower level of leverage because banks would have even less of an incentive to evade them. Consequently, as long as regulators did not react to lower leverage (higher capital) by relaxing their efforts to limit asset risk, a bank would not increase its asset risk, and the liability of the insurance fund would decline.[18]

Summary

Not surprisingly, capital regulation is necessary with subsidized deposit insurance to limit the liability of the insurance fund. However, more stringent capital standards for banks do not confound regulatory efforts to limit the riskiness of bank assets because higher capital does not increase the incentives of a value-maximizing bank to hold riskier assets. In fact, the marginal value from increasing asset risk for an insured bank declines as leverage is lowered. This conclusion does not depend solely on the state-preference framework. As discussed in Box 2, a positive relation between leverage and the gains from risk-taking also can be derived from an options approach to evaluating the gains from risk and leverage in banking.

III. Utility Maximization

In this section we incorporate utility maximization into the state-preference model. With utility maximization, the state-preference model implies that capital regulation is either irrelevant because risk-averse owner-managers will hold sufficiently conservative portfolios to make bankruptcy impossible or capital regulation will limit the liability of the deposit insurance system in the same way that it can for value-maximizing banks.

Utility-Maximizing Banks

Utility maximization has been rationalized as being more applicable than value maximization to smaller, owner-managed banks because the investment opportunity set for such banks and their owners may be one and the same. The assumption behind this rationalization is that the owner-manager cannot attract capital in addition to his own and that most of his portfolio is invested in the bank. Consequently, unlike a bank that is maximizing its current market value, the owner's preference toward risk would influence the bank's portfolio decisions.[19]

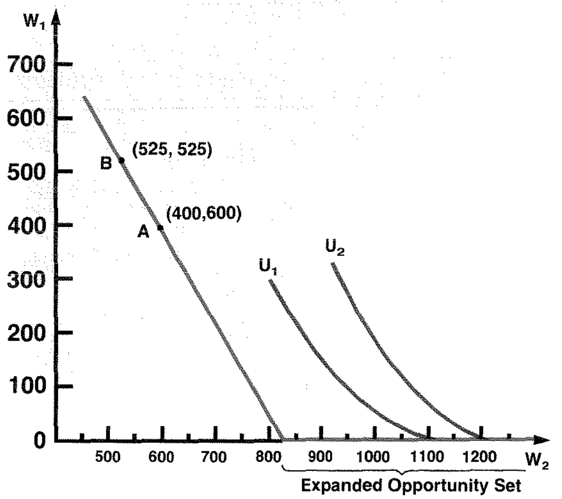
It is assumed that the owner-managers are risk-averse. As pointed out in Box 1, in a simple two-state world, risk aversion means that utility functions are convex with respect to the origin. That is, future wealth has diminishing marginal utility in each state of the world. As is shown in the figure in Box 1, in a world without deposit insurance, an investor will allocate his capital between the available securities to maximize utility along the investment opportunity frontier.

The introduction of underpriced deposit insurance expands the opportunity frontier. The opportunity frontier with free deposit insurance can be derived from the types of calculations presented in Table 1. (Recall there is only one risky security A(4,6), riskless borrowing by issuing security B(1,1), free deposit insurance, and initial bank capital of \$500.) Figure 2 shows the various combinations of wealth (W_1 and W_2) that can be attained by increasing leverage and investing in the risky security (security A). Point A in Figure 2 indicates the combination attainable with no leverage, and the shaded segment includes points attainable by increasing leverage.

The key difference between the choice set with deposit insurance and the set without deposit insurance is that there is no limit to the amount of wealth that would be attained in state 2 once bankruptcy would occur in state 1. That is, various wealth outcomes (W_1, W_2), such as (0,895), (0,968), (0,1042), can be attained only with free deposit insurance. With free (or underpriced) deposit insurance and no capital regulation, there is no limit to the amount of wealth that could be attained in state 2 by increasing leverage.

Whether the effects of deposit insurance on the opportunity set will influence the owner-manager's

Figure 2
Subsidized Deposit Insurance Expands the Opportunity Set



leverage decisions depends on the owner-manager's preferences. To benefit from deposit insurance, the owner-manager must be willing to risk bankruptcy — that is, there must be some amount of wealth in state 2 that will compensate for zero wealth in state 1. If there were no such level of wealth in state 2, utility functions would approach the axes asymptotically and interior solutions (some wealth in both states) would be obtained (that is, a point on the frontier to the left of point C). Without the possibility of bankruptcy, the deposit insurance fund would not be at risk, and no capital regulations would be required. Moreover, even if capital regulations were imposed, portfolios that make bankruptcy possible would not be held.

Alternatively, if there were some level of wealth in state 2 that could compensate the bank owner for zero wealth in state 1, the utility functions would intersect the axis. Such a set of preferences is depicted in Figure 2 as indifference curves U_1 and U_2. In Figure 2, raising leverage would increase utility ($U_2 > U_1$) because wealth in state 2 would increase while wealth in state 1 would remain zero. As shown in the previous section, this is the same reason that the current value of the bank increases with leverage. Consequently, maximizing the utility of future wealth in state 2 for the type of preferences depicted in Figure 2 (that is, indifference curves that intersect the axis) is equivalent to maximizing the current value of the bank. Like a value-maximizing bank, a utility-maximizing bank that is willing to accept bankruptcy will maximize leverage.

Capital Requirements and Risk

The predictions of the state-preference model regarding the effect of capital regulations on asset risk for utility-maximizing banks (that will accept bankruptcy) also are similar to those for value-maximizing banks. As just stated, maximizing utility is equivalent to maximizing wealth in state 2 (the nonbankruptcy state) for a bank that has underpriced deposit insurance and will accept a nonzero risk of bankruptcy. Table 2 shows that if leverage were restricted by regulation to 3, wealth in state 2 could be increased by increasing asset risk (holding a larger proportion of assets in security D). Therefore, it could be necessary to regulate even utility-maximizing banks' asset portfolios to prevent such banks from increasing asset risk.[20]

Table 2 also indicates that wealth in state 2 is directly proportional to the current value of the bank when bankruptcy is possible in state 1. (With bankruptcy possible in state 1, wealth in state 2 is equal to the current value of the bank divided by .6.) This means that the marginal effect on wealth in state 2 for a given increase in asset risk declines as leverage declines, just as does the marginal effect on the current value of the bank (see Figure 1).[21] Therefore, as long as the regulatory efforts to prevent a bank from increasing asset risk are not lessened, imposing a lower leverage position would not increase the incentives for a utility-maximizing bank to increase asset risk.

Summary

In sum, incorporating utility maximization into the state-preference model does not affect our conclusion that more stringent leverage requirements will reduce payouts from the deposit insurance fund as long as the stringency of portfolio regulation remains unchanged. Some owner-managers might be so risk-averse that they would be unwilling to risk bankruptcy even with deposit insurance. However, the owner-manager that will risk bankruptcy in one state for a sufficiently high claim in the other state would seek to maximize wealth in the non-bankruptcy state. For such persons, utility maximization and value maximization are comparable and all of the results of the earlier section apply.

IV. Summary and Conclusions

This paper analyzes the theoretical relationships among capital regulation, bank asset risk, and the liability of the federal deposit insurance system. We demonstrated that a bank can benefit from underpriced deposit insurance by increasing leverage and/or asset risk. As a result, some degree of capital regulation is needed to limit the liability of the deposit insurance fund.

More importantly, the analysis suggests that regulatory increases in capital standards will *not* require greater efforts to restrain asset risk. On the contrary, the marginal value of increasing asset risk declines as leverage falls — that is, less leverage (more capital) reduces the gain from risk-taking. In other words, banks with the least capital have the most incentive to increase asset risk.

We have shown under the assumption of value maximization that more stringent capital regulation lowers the contingent liability of the deposit insurance system as long as the stringency of asset portfolio restraints is not reduced. This result follows for value-maximizing banks in both the state-preference and options models. Moreover, incorporating utility maximization into the state-preference model does not change this conclusion.

The key policy implication that stems from our analysis is that regulatory efforts to raise capital standards in banking would not by themselves lead to more risky asset portfolios.

FOOTNOTES

1. Actions taken in 1981, 1983 and 1985 raised capital requirements for banks and bank holding companies, and made the federal bank regulatory agencies' definitions of capital more uniform.

2. The proposal for risk-based capital standards was made public in January 1987.

3. The measure of common equity used in these ratios is not the current regulatory definition of equity capital that includes loan loss reserves and preferred stock.

4. See, "Fed Urgues Swap Plan for Banks," New York Times, March 5, 1987.

5. Other studies that consider the effects of capital regulation on bank asset risk within the mean-variance framework are Kahane (1977), Blair and Heggestad (1978), and Hanweck (1984).

6. Koehn and Santomero (1980), p. 1244.

7. For purposes of this paper, failure and bankruptcy occur when the market value of a bank's liabilities exceeds that of its assets.

8. We recognize that the utility maximization model also might be rationalized by appealing to the notion of the separation of ownership and control so that the firm's operating decisions depend on the manager's risk preferences.

9. Some articles appear to confuse financial capital with physical capital. For example, Santomero and Watson (1977) view financial capital as a physical investment that could have been made in other sectors of the economy. Financial capital, however, is not directly related to physical investment, and higher bank capital does not limit the amount of physical investment in other sectors of the economy. The amount of capital relative to liabilities is simply a reflection of the way a bank finances its assets. Bank capital as well as liabilities are available to be invested in nonbank physical investment through bank loans, for example, as well as in bank facilities.

10. From an economic standpoint, a bank (or any other firm) fails when the value of its capital falls below zero. Mathematically, the probability of failure is the probability that the value of end-of-period assets is less than that of end-of-period liabilities:

$$\text{Prob}[\text{Failure}] = \text{Prob}[(1 + \tilde{p})A < (1 + r)L], \quad (1)$$

where: $\tilde{p}$ = rate of return on assets (which is assumed to be random),

A = initial assets,

r = promised rate on liabilities,

L = initial value of liabilities, and

Prob[ ] = denotes the probability of [ ].

Without information on the type of probability distribution governing $\tilde{p}A$, the probability of failure can be bounded by using the Tchebichef inequality (see Koehn and Santomero, 1980). However, by assuming that the normal distribution approximates the distribution of $\tilde{p}A$ (i.e., $\tilde{p}A \sim N[E(\tilde{p}A), \sigma^2(\tilde{p}A)]$), we can solve for the probability of failure:

$$\text{Prob}[\text{Failure}] = F\left[\frac{L - A + rL - E(\tilde{p}A)}{\sigma(\tilde{p}A)}\right] \quad (2)$$

where: F = the standard ($\mu = 0$, $\sigma^2 = 1$) unit normal cumulative distribution function.

11. Equation 2 in footnote 10 indicates that the probability of failure increases as the riskiness of the asset portfolio, $\sigma(\tilde{p}A)$, increases, and as leverage (as reflected in the quantity of liabilities relative to assets) increases. To prove this, the equation can be differentiated with respect to the applicable parameter as follows:

$$\frac{\delta \text{Prob}[\text{Failure}]}{\delta\sigma} = \frac{-f(\ )[L - A + rL - E(\tilde{p}A)]}{\sigma^2} > 0, \quad (1)$$

$$\frac{\delta \text{Prob}[\text{Failure}]}{\delta L} = \frac{f(\ )(1 + r)}{\sigma^2} > 0 \quad (2)$$

where f( ) is the standard normal density function evaluated at the initial position. The first inequality holds because the term in brackets is negative. Also note, in the second inequality, assets are held constant and, thus, an increase in liabilities reflects a corresponding decrease in capital.

12. Other studies such as Merton (1977) and Pyle (1984) provide useful insights into the regulation of bank leverage and asset risk by using options models to analyze the value of the federal deposit guarantee. In Box 2, we analyze the effects of capital standards on asset risk in banking using an options model.

13. This implies a risk-free real interest rate of 5.26 percent, [($1.00/$.95) – 1] x 100%.

14. The analysis would be essentially the same if the premium rates were variable as long as the premiums paid were less than the value of the deposit guarantee.

15. The indifference of a low leverage bank in our example to the degree of leverage parallels Proposition I (the market value of a firm is independent of its capital structure) in Modigliani and Miller (1958). While Modigliani and Miller do not use a state-preference model, Hirshleifer (1966) uses the state-preference approach to show that Proposition I still holds in that framework. In fact, the state-preference model can be used to show that the Modigliani-Miller theorem holds with or without bankruptcy, when there is no subsidized deposit insurance.

16. One policy implication here is that the distortions of deposit insurance could be eliminated if risk in banking were borne only by the banks. Along this line, it has been suggested that risk would not be shifted to the insurance fund if there were timely closures of banks. With continuous (and costless) monitoring of banks, this would correspond to closing a bank before its market net worth reached zero (Furlong and Keeley, 1985). With periodic examinations of banks, the state-preference approach indicates that losses to the insurance fund could be avoided only if banks had enough capital to ensure their solvency in all possible states.

17. In this two-state model, the probability of failure actually would decline only if leverage and risk were restricted in such a way that the bank could meet obligations in each state. In a model with more than two states, the probability of failure would decline with decreased leverage as the number of states in which the bank was able to meet promised payments increased.

18. In this paper, we have not dealt directly with bankruptcy costs. As shown in Dothan and Williams (1980), such costs can lead to a determinate degree of leverage for a value-maximizing *uninsured* bank. However, bankruptcy costs are not sufficient to limit leverage if banks have access to a subsidized deposit insurance. Although bankruptcy costs are not incurred in all future states, they nonetheless can be evaluated in terms of their effects on the current value in the state-preference framework. It can be shown that, with free deposit insurance, bankruptcy costs that are fixed or proportional to assets generally will be insufficient to limit leverage. This result holds whether bankruptcy costs fall on the bank or on the depositors.

19. Despite legitimate questions as to whether this assumption would apply to any real-world banks since owners of small, privately held banks can diversify their portfolios, we hold to it.

20. It is possible that, with leverage held sufficiently low, the wealth attainable in state 2 from investing in the riskier security would not be adequate to compensate a utility maximizing bank owner for risking zero wealth in state 1, even if the utility curves crossed the axis. In such a case, the bank would choose a portfolio along the AC portion of the opportunity frontier in Chart 2, and no other portfolio restraints would be required. However, at some higher level of leverage the same bank would begin to take advantage of the opportunity to increase wealth in state 2 through investing in the riskier asset, security D.

Similar results hold in a model with more than two states. With very low leverage, a bank may not be able to realize sufficient compensation in the nonbankruptcy states to justify risking bankruptcy in even one possible future state. It would not be necessary to regulate the composition of such a bank's assets. At higher levels of leverage, the bank ultimately would increase asset risk and allow for bankruptcy in at least some states.

21. In a multi-state world it also is the case that the marginal effect on wealth in each of the nonbankruptcy states with positive payouts would increase with leverage.

APPENDIX

Capital Regulation and Asset Risk in a Utility-Maximization, Mean-Variance Framework

Introduction

A number of studies have attempted to analyze the effects of bank capital regulation on asset risk and the probability of bankruptcy while assuming that banks maximize utility in a mean-variance framework. This literature is best typified by articles by Kahane (1977), and Koehn and Santomero (1980).

We show below that the conclusions reached by these studies were derived using internally inconsistent assumptions. Both studies assume that a bank's borrowing cost would be unrelated to bank risk. That is, a bank's borrowing cost (per dollar of liabilities) is assumed to be constant regardless of its asset risk or leverage. Thus, these studies implicitly, but unintentionally, assume that bank failure cannot occur. Yet, they seek to analyze the effects of capital regulation on the probability of bank failure. Moreover, these studies fail to take into account the effect of underpriced deposit insurance on the incentive to take on excessive risk.

A possible explanation for why these studies overlook the effects of bankruptcy on the bank's borrowing cost is that the basic mean-variance framework used is adapted from the finance literature on investment, which does not allow for bankruptcy since borrowing and lending are assumed to take place at the risk-free interest rate. While this simplifying assumption may be appropriate for certain investment decisions, it is not appropriate for the analysis of banking with underpriced deposit insurance. The reason is that concern over the exposure of the deposit insurance system to risk in banking arises only when bankruptcy is possible.

In this Appendix, we first construct a prototype of the utility-maximization, mean-variance model used in past studies to analyze the effects of bank capital regulation on asset risk. We show that when bankruptcy is not possible, and, thus, when there is no deposit insurance subsidy, the results from our prototypical model are identical to those of the previous studies. Specifically, the effect on asset risk of moving from one binding capital constraint to a more stringent one depends on the nature of the preferences of the bank's owner-manager. Restricting such an analysis to situations where bankruptcy is not possible, however, makes these conclusions irrelevant for policy purposes since capital regulation is needed only when bank failures and deposit insurance payouts are possible.

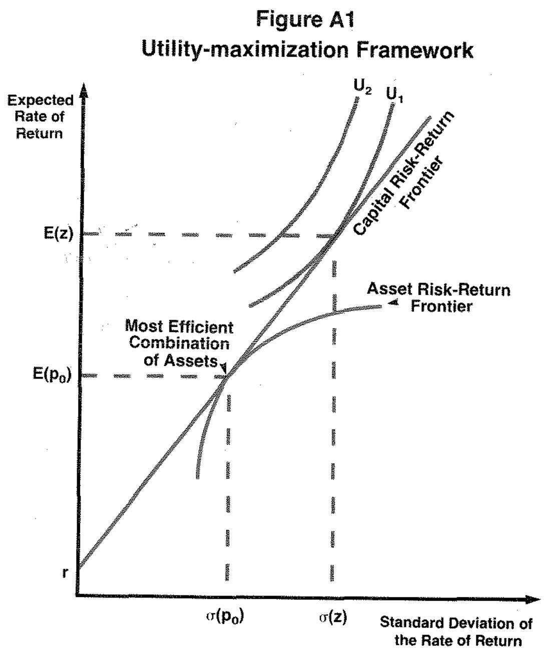
In the next section of the Appendix, we add the possibility of bankruptcy and subsidized deposit insurance to the model. Doing so changes markedly the bank's investment opportunity set. In addition, we present a specific numerical example to illustrate that, when the asset return distribution is binomial, the incentive to increase asset risk does not increase as the stringency of capital regulation increases regardless of the nature of the bank owner-manager's preferences.

Background

The utility-maximization framework assumes that the bank owner-manager's preferences toward risk can be characterized by the expected rate of return on capital, $E(\tilde{z})$, and the standard deviation of the rate of return $\sigma(\tilde{z})$. Moreover, assuming risk-aversion, these preferences can be depicted by a set of concave, upward sloping iso-utility functions depicting the tradeoff between the expected rate-of-return and risk.

Such iso-utility functions, U_1 and U_2 ($U_i = U[E(\tilde{z}), \sigma(\tilde{z})]$), are depicted in Figure A1. The vertical axis represents the expected rate of return and the horizontal axis represents risk as measured by the standard deviation of the rate of return. Along indifference curve U_1, the investor is indifferent among the various combinations of expected return and risk. However, the investor prefers the points on U_2 to those on U_1 because, for any given level of risk (standard deviation) the expected rate of return on U_2 is larger.

The ideas behind this characterization of preferences are that initial wealth is given and that the



investor is concerned about the expected value of future wealth along with its standard deviation. Future wealth is equal to one plus the rate of return times current wealth. Consequently, with current wealth (capital) assumed fixed, the mean and variance of future wealth are mathematically equivalent to the mean and variance of the rate of return on capital, respectively. Thus, similar to the state-preference model, utility maximization in a mean-variance framework also focuses on the probability distribution of future wealth.

Utility-Maximization Without Bankruptcy

A bank must decide on how risky an asset portfolio to hold and by how much to leverage that portfolio. Given the owner's preferences toward risk, utility will be maximized subject to a constraint that relates the expected return on capital, $E(\tilde{z})$, to the standard deviation of the rate of return $\sigma(\tilde{z})$. To derive this constraint, note that if the bank chooses sufficiently low leverage and asset risks to make *bankruptcy impossible* (that is, promised obligations to depositors are always met regardless of the asset return that is realized, the rate of return on capital, $\tilde{z}$, is given by the gross returns on assets, $A\tilde{p}$, minus promised (which equals the actual) payments to liability holders, Lr, divided by initial capital, K, or

$$\tilde{z} = [A\tilde{p} - Lr]/K \qquad \text{(A1)}$$

where

A = initial assets
L = initial liabilities
K = initial capital
$\tilde{p}$ = rate of return on assets, assumed to be random
$\tilde{z}$ = rate of return on capital, which is random
r = promised (which equals actual) rate of return paid on (and cost of) liabilities.

(As discussed more fully later, if bankruptcy were possible, the cost of liabilities would no longer be fixed since actual payments to depositors would sometimes be less than promised payments. This implies that the cost of deposits to the bank would be a random variable, which depends on the rate of return on assets realized and leverage chosen. Consequently, equation A1 would not apply to realizations of $\tilde{p}$ when bankruptcy occurred.)

Equation A1 may be rewritten by noting that $L = A - K$ to give

$$\tilde{z} = [A/K]\,[\tilde{p} - r] + r. \qquad \text{(A2)}$$

The expected rate of return on capital, $E(\tilde{z})$ may be found by taking expected values of both sides of equation A2. This gives:

$$E(\tilde{z}) = [A/K]\,[E(\tilde{p}) - r] + r, \qquad \text{(A3)}$$

as long as r is fixed and not random, which it would be as long as bankruptcy were not possible.[A1] Thus, increasing leverage, as measured by the asset-to-capital ratio increases the owner's expected rate of return on capital linearly as long as default is not possible.[A2] (We later show this result changes when bankruptcy is possible).

Similarly, the standard deviation of the return on capital, $\sigma(\tilde{z})$, may be derived from equation A2 by noting that when bankruptcy is not possible, the covariance of r and $\tilde{p}$ is zero. In this case,

$$\sigma(\tilde{z}) = [A/K]\sigma(\tilde{p}). \qquad \text{(A4)}$$

(This equation is not valid when bankruptcy is possible since the covariance of $\tilde{p}$ and the cost of deposits is not zero.)

Equations A3 and A4 may be jointly solved to eliminate the [A/K] term to give

$$E(\tilde{z}) = [\sigma(\tilde{z})/\sigma(\tilde{p})][E(\tilde{p}) - r] + r. \quad (A5)$$

In other words, expected return on capital varies linearly with the standard deviation of return on capital for a given expected asset return and standard deviation of the return.

This linear relationship, equation A5, is plotted as the straight line intersecting the vertical axis at r in Figure A1. It is assumed that the particular asset portfolio with expected return $E(\tilde{p}_0)$ and standard deviation $\sigma(\tilde{p}_0)$ is being leveraged. With no leverage $(A = K)$, the expected rate of return and standard deviation of return on capital are equal to the expected rate of return and standard deviation of return on that particular asset portfolio — $E(\tilde{p}_0)$ and $\sigma(\tilde{p}_0)$, respectively. As leverage increases, the expected rate of return and standard deviation of return both increase linearly.

In general, it is assumed that a bank faces a variety of different asset risk-return combinations as determined by the availability of investment alternatives in its market (known as the asset risk-return frontier). As shown in Figure A1, asset portfolios with more risk are assumed to yield larger expected returns. Also, it is assumed that the banking sector is small enough that the asset risk-return frontier is unaffected by banks' behavior. Thus, that frontier is taken as given by banks in their optimizing decisions.

In this framework, the most efficient asset portfolio is the one where a line from the constant borrowing rate, r, is tangent to the asset risk-return frontier. This is depicted as point $E(\tilde{p}_0)$, $\sigma(\tilde{p}_0)$ in Figure A1. By leveraging this asset portfolio, the bank can obtain the highest expected return on its capital for any degree of risk. Since this tangency point does not depend on the bank owner's preferences, the asset portfolio (that is, the particular combination of assets) chosen depends only on the risk-free interest rate and the asset risk-return frontier. The degree of leverage chosen, however, is determined by the tangency of the owner's iso-utility function with the risk-return frontier for capital (the straight line in Figure A1). However, the assumption here is that the unconstrained bank would choose a degree of leverage for which bankruptcy is not possible.

Capital Requirements and Risk

In Figure A1, we showed how a particular asset portfolio may be leveraged (assuming no bankruptcy) to obtain the capital risk-return frontier. Of course, any asset portfolio may be leveraged although there would be no reason for a bank owner to leverage any asset portfolio other than the most efficient one in a world without capital or asset portfolio regulation. When capital constraints are imposed, however, the bank owner generally will be able to increase utility by leveraging asset portfolios other than the one characterized by the parameters $E(\tilde{p}_0)$, $\sigma(\tilde{p}_0)$. For example, suppose that the maximum asset-to-capital ratio allowed were 5. Then the standard deviation of return on capital would be 5 times the standard deviation of return on the asset portfolio chosen, and the expected return on capital also would be five times greater.

Figure A2
Imposing a Binding Capital Constraint Causes Asset Risk to Rise

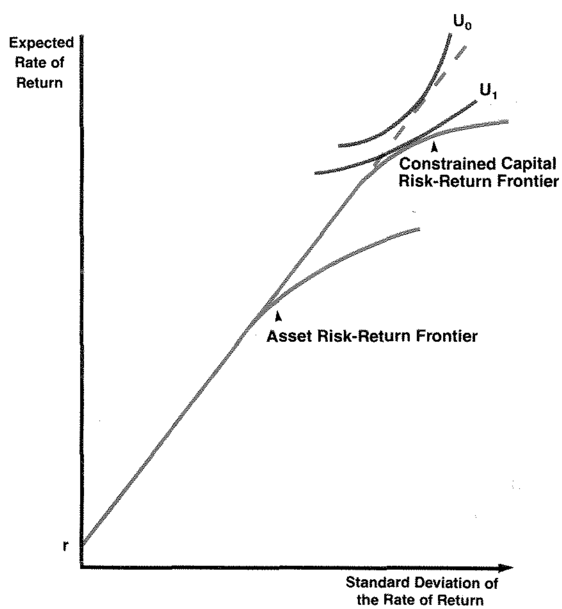
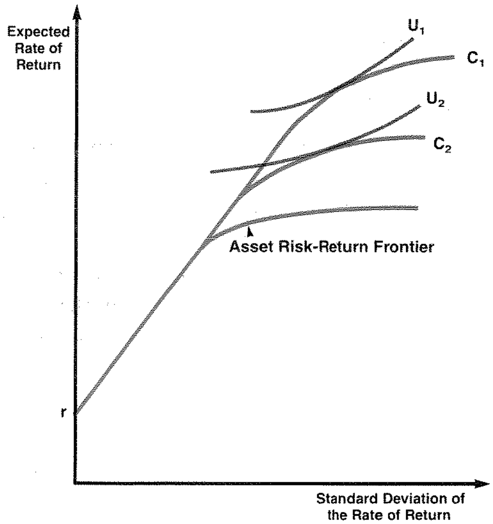


Figure A3
Increasing the Stringency of Capital Regulation



In geometric terms, the risk and return on capital for a given asset portfolio and leverage can be found by extending a ray from the constant borrowing rate through the asset portfolio chosen up to the maximum leverage allowed. As Figure A2 shows, when leverage is limited by regulation, the capital-risk return frontier becomes convex once the leverage constraint becomes binding. As a result, if a binding capital constraint were imposed on a previously unconstrained bank, the bank would choose a more risky asset portfolio. This is shown as a move from U_0 to U_1 in Figure A2.

In Figure A3, two binding capital constraints, C1 and C2, and a particular set of preferences are depicted. However, as one moves from one binding capital constraint, C1, to a more stringent one, C2, the effects on asset risk depend on the owner's risk preferences — asset risk can either increase, decrease, or remain the same, which is the basic conclusion reached by the traditional literature.

Introducing Bankruptcy and Deposit Insurance

The analysis above was derived under the assumption that an unconstrained bank would always make asset and leverage choices such that bankruptcy could not occur. Such a bank could attract deposits at the risk-free rate because it would always make the payments promised regardless of the return on assets realized. Consequently, the cost of deposits to such a bank would be fixed at the risk-free rate r, and not be a random variable.

With free deposit insurance, a bank could issue deposits at a fixed risk-free promised rate even if bankruptcy were possible. However, the cost of deposits to the bank would no longer necessarily equal the risk-free rate. When bankruptcy occurs, the bank effectively would pay less than the promised rate on deposits, r. Only when bankruptcy does not occur does the cost of deposits to the bank equal the risk-free rate. Put another way, the excess of contractual debt obligations over assets when bankruptcy occurs corresponds to the option value of deposit insurance (see Box 2). Thus, the effective deposit cost to the bank is a random variable related to the rate of return on assets and leverage.

As a result, the expected cost of deposits to the bank would decline with increasing leverage and would be less than the risk-free rate. This means that neither the expected rate-of-return equation, A3, nor the standard-deviation of the rate-of-return equation, A4, would hold. Instead, the rate of return on capital, $\tilde{z}$, to a bank with free deposit insurance is given by:

$$\tilde{z} = \begin{cases} [A\tilde{p} - Lr]/K & \text{if bankruptcy does } \textit{not} \text{ occur, that is, if } A(1+\tilde{p}) \geqslant L(1+r). \\ -1 & \text{if bankruptcy does occur, that is, if } A(1+\tilde{p}) < L(1+r). \end{cases} \quad \text{(A6)}$$

The expected rate of return on capital, $E(\tilde{z})$, found by taking the expected value of equation A6 is:

$$E(\tilde{z}) = -1 \text{ Prob[Failure]} + E[(A\tilde{p} - rL)/K \mid \tilde{p} > p^*](1 - \text{Prob[Failure]}) \quad \text{(A7)}$$

where

$$p^* = -\frac{K(1+r)}{A} + r \quad \text{(A8)}$$

is the level of $\tilde{p}$ above which bankruptcy does not occur and

$$\text{Prob[Failure]} \equiv \text{Prob}[\tilde{p} < p^*] \equiv \text{Prob}[\tilde{p} < -\frac{K(1+r)}{A} + r]. \qquad (A9)$$

Equations A6 and A7 indicate that the simple linear relationship between $E(\tilde{z})$ and $\sigma(\tilde{z})$ presented in equation A5, generally would not be valid.[A3]

Moreover, Equation A7 indicates that the bank owner would never lose more than his or her initial capital (that is, the minimum $\tilde{z}$ would be -1, even though $\tilde{z}$ would be less than minus 1 if the promised obligation to depositors were met in the event of a bankruptcy). Also, equation A9 indicates that, depending on the asset rate-of-return distribution, the probability of failure can increase up to a point as leverage increases. However, in the limit as leverage increases (and K/A goes to zero), the probability of failure approaches the probability that the rate of return on assets, $\tilde{p}$, is less than the promised rate on deposits, r.

Consequently, by increasing leverage, the owner can increase without limit the expected rate of return on capital as long as at least some part of the asset rate-of-return distribution exceeds the promised deposit rate. Thus, even if the expected rate of return on assets were less than the promised rate on deposits, a bank with underpriced deposit insurance would gain from leverage as long as this condition held. This conclusion contrasts with the results obtained when bankruptcy is not possible. In that

TABLE A1

Expected Return and Standard Deviation of Return on an Initial $100 Investment of Capital as Leverage Increases

(1) Leverage	(2) Assets	(3) Deposits	State	(4) End-of-Period Assets	(5) Actual Deposit Payments	(6) End-of-Period Capital (4 - 5)	(7) Deposit Insurance Payments	(8) Expected End-of-Period Capital	(9) Standard Deviation of End-of-Period Capital
			1	110	0	110	0		
1	100	0	2	100	0	100	0	105	5
			1	220	104	116	0		
2	200	100	2	200	104	96	0	106	10
			1	330	208	122	0		
3	300	200	2	300	208	92	0	107	15
			1	440	312	128	0		
4	400	300	2	400	312	88	0	108	20
⋮									
			1	2,860	2,600	260	0		
26	2,600	2,500	2	2,600	2,600	0	0	130	130
			1	2,970	2,704	266	0		
27	2,700	2,600	2	2,700	2,700	0	4	133	133
			1	3,080	2,808	272	0		
28	2,800	2,700	2	2,800	2,800	0	8	136	136
⋮									
			1	11,000	10,296	704	0		
100	10,000	9,900	2	10,000	10,000	0	296	352	352

Asset return = .5 probability of .10 (state 1)
.5 probability of .00 (state 2)

Deposit rate = .04
Initial Capital = 100

case, leverage can increase expected return only if the expected return on assets exceeds the promised rate.

A Numerical Example

Below, a simple numerical example is presented that allows for both subsidized deposit insurance and the possibility of bankruptcy. This example shows that a bank can gain from subsidized deposit insurance only by assuming a nonzero risk of bankruptcy. The example also shows that when asset returns are distributed binomially, more stringent capital regulation will not increase the incentive to increase asset risk. Thus, this example demonstrates that the results in the cited mean-variance literature do not hold generally.

The calculations presented in Table A1 demonstrate what happens to the expected return and standard deviation of return on an initial $100 investment of capital in a bank (with deposit insurance provided to it at no cost by the government) as leverage increases. To simplify the calculations, we assume that the rate of return on the asset being leveraged is drawn from a binomial probability distribution with a .5 probability of a 10 percent rate of return and a .5 probability of a 0 rate of return (for an expected rate of return of 5 percent). It is assumed that the bank is able to attract any quantity of deposits it wants at a promised 4 percent rate of return because deposit insurance (which is provided free to the bank) covers any shortfalls when bankruptcy occurs.

The calculations in the Table demonstrate what happens to expected end-of-period capital and its standard deviation as leverage increases. (The rate of return on capital, in percentage terms, is just end-of-period capital minus 100). In line 2, the bank increases leverage to 2 by issuing $100 of deposits and promising to return $104 to depositors at the end of the period. The end-of-period capital for each of the two possible asset returns after paying the deposit claims is shown in column 6. The expected end-of-period capital and standard deviation of end-of-period capital are shown in columns 8 and 9. The payment from the deposit insurance fund is shown in column 7.

As the Table indicates, initially, as leverage increases up to 26, the expected return and standard deviation of return both increase linearly and there is no bankruptcy. After this point (when leverage exceeds 26), there is a 50 percent chance of realizing the low asset return (denoted as outcome 2) and going bankrupt. However, there is also a 50 percent chance of drawing the high asset return and profiting from leverage. As leverage increases, end-of-period capital increases without limit as long as bankruptcy does not occur.[A4]

Rationality implies that a person will prefer a lottery that pays $100 with a 50 percent chance and $0 with a 50 percent chance to one that pays $10 with a 50 percent chance and $0 with a 50 percent chance.[A5] Thus, this example illustrates that even a risk averse bank owner that is willing to risk bankruptcy (that is, one who is willing to participate in the type of lottery just described) in return for a sufficiently high payoff when the higher asset return is realized would prefer unlimited leverage. Consequently, in this example, maximizing utility is equivalent to maximizing value as long as the bank owner is willing to risk bankruptcy.

In this simple model, a similar result would apply to asset risk under leverage constraints. As long as some non-zero probability of bankruptcy were acceptable, a bank owner would maximize asset risk since that would maximize end-of-period capital if bankruptcy did not occur and would not affect end-of-period capital if bankruptcy did occur. These implications are in sharp contrast to those from the cited mean-variance literature which claims that risk aversion would limit leverage and asset risk.

Moreover, as in the state preference model presented in this paper, it can be shown that the gain from increasing asset risk is positively related to leverage. Thus, in the case of binomially distributed asset returns, more stringent capital regulation does not increase the incentive to increase asset risk.

Summary

The results of previous studies using the mean-variance framework regarding the effect of capital regulation on asset risk can be replicated assuming that bankruptcy is not possible. However, when bankruptcy is possible and underpriced deposit insurance is provided to banks, the results of these studies no longer hold generally.

APPENDIX ENDNOTES

A1. An uninsured bank could attract deposits by paying a fixed rate of interest, independent of its leverage or asset risk as long as its asset risk were low enough relative to capital that the probability of bankruptcy were zero. As long as bankruptcy were not possible, bank liability holders would not be at risk of loss due to default and would accept bank liabilities as riskless. At some point, however, as leverage increased (for a given nonzero asset risk), bankruptcy would become possible and the bank would have to pay a higher deposit rate to compensate depositors for the risk of default.

The utility-maximization literature cited assumes a constant borrowing rate environment, but does not explicitly acknowledge that this would be consistent only with a zero probability of bankruptcy. Kahane does allow for a stochastic deposit rate but assumes the promised rate equals the rate the bank expects to pay. Moreover, he assumes the expected cost of deposits and the promised rate are independent of leverage and asset risk. These assumptions would hold only if bankruptcy were not possible.

A2. Thus, it is crucial to distinguish the asset from the capital risk-return frontier. Blair and Heggestad (1978) fail to do so.

A3. However, we do not mean to imply that equations A6 and A7 necessarily can be used to derive the appropriate risk-return constraint for utility maximization in a mean-variance framework. One reason is that variance no longer adequately characterizes risk when bankruptcy is possible.

A4. After the point where bankruptcy becomes possible, the relationship between the expected rate of return on capital (column 8 minus 100) and its standard deviation changes (the expected rate of return rises more rapidly and the standard deviation rises less rapidly with leverage.)

A5. This is true even though the standard deviation of the first alternative is larger.

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