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Free Entry in a Cournot Market with Imperfectly Substituting Goods

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Abstract

Two results are shown about the free-entry equilibrium in a Cournot market with asymmetric firms and imperfectly substituting goods. First, only one technology will survive in the production of each good. Second, some good(s) may not be produced. Specifically, we show that in a two-good model only one good is produced if the substitution parameter is higher than a critical value and both goods are produced for smaller substitution parameter values.

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1. Introduction

This paper is concerned with free entry under imperfect competition. We study the free entry equilibrium in a Cournot market with asymmetric firms and imperfectly substituting goods.

Economists have long been interested in the free entry equilibrium in imperfectly competitive markets. The papers by Guesnerie and Hart (1985), Mankiw and Whinston (1986), and Suzumura and Kiyono (1987) all study free entry in a homogeneous good market with imperfect competition. They establish that entry biases exist if firms are not behaving as perfect competitors. For example, Mankiw & Whinston (1986) consider a homogeneous good market with identical firms engaging in Cournot quantity competition. They show that free entry entails excessive entry from a social welfare point of view. Recently, Gotz (2005) and Ohkawa et al. (2005) examine market selection of production technologies under Cournot competition and free entry. They establish that only one type of technologies (or firms) can survive in a free entry homogenous good Cournot market.

The present paper extends the existing literature on free entry under imperfect competition by examining a differentiated goods market. We focus on a market with two imperfectly substituting goods each of which can be produced by multiple technologies (types of firm). Two results are established. First, in the free entry equilibrium, each good is produced by only one type of firm. This generalizes the market selection result by Gotz (2005) and Ohkawa et al. (2005) to a differentiated goods setting. Second, we show that free entry may lead to only one of the goods being produced in equilibrium. This is a new result that implies that free entry in a Cournot market may lead to under-provision of goods.

We use a symmetric linear demand system so as to have a mathematically easily tractable model. Symmetric demands for the goods help highlight that under-provision of goods is not caused by lack of demand but rather due to the effect of entry and the presence of asymmetry in production technology. The results in the paper should hold under more general demand frameworks and cost structures.

2. The Model

Let P_i and Q_i denote the price and total quantity of good i (i = 1, 2). We consider the following linear (inverse) market demand equations:

$$P_1 = \alpha - Q_1 - \gamma Q_2 , \; P_2 = \alpha - \gamma Q_1 - Q_2 ,$$

where $\alpha > 0$ is the maximum price consumers are willing to pay for either good and $\gamma \in [0,1]$ is the substitution parameter. The two goods are independent if $\gamma = 0$ and are perfect substitutes if $\gamma = 1$. Good i can be produced by N_i technologies (types of firms). Type j (j = 1,…, N_i) good i producers have the production technology that has constant marginal cost c_{ij} and fixed cost f_{ij}. We assume that $\alpha > c_{ij}$ for all i and j.

Firms play the Cournot quantity game. We first discuss the Cournot equilibrium when the number of firms is fixed for each type (technology) of either good.[1] Consider a representative type j (j = 1,…, N_1) good 1 producer. This firm's profit is given by

$$\pi_{1j} = P_1 q_{1j} - (c_{1j} q_{1j} + f_{1j}) = (\alpha - Q_1 - \gamma Q_2 - c_{1j}) q_{1j} - f_{1j}, \tag{1}$$

where q_{1j} denotes the firm's output. Taking all other firms' output levels as given, maximizing π_{1j} with respect to q_{1j} yields the first-order condition: $q_{1j} = \alpha - c_{1j} - Q_1 - \gamma Q_2$. Applying this relationship to (1), the firm's equilibrium profit can be written as[2]

$$\pi_{1j}^* = (\alpha - c_{1j} - Q_1^* - \gamma Q_2^*)^2 - f_{1j}.$$

This expression stipulates that in equilibrium each firm's profit depends only on its own cost parameters, the total output by all good 1 producers (Q_1^*), and the total output by all good 2 producers (Q_2^*). It follows immediately that

$$\pi_{1j}^* \geq 0 \text{ if and only if } Q_1^* + \gamma Q_2^* \leq \alpha - d_{1j}, \tag{2}$$

where $d_{1j} = c_{1j} + \sqrt{f_{1j}}$ is type j good 1 firms' average cost of production if they are the only firms in existence in the free-entry equilibrium. Hence, equilibrium profits of type j good 1 producers are positive if total outputs for goods 1 and 2 are such that $Q_1^* + \gamma Q_2^* < \alpha - d_{1j}$, and are negative if $Q_1^* + \gamma Q_2^* > \alpha - d_{1j}$. Note that the condition $Q_1^* + \gamma Q_2^* \leq \alpha - d_{1j}$ is more easily satisfied for smaller values of d_{1j}. This implies that among all good 1 producers those with smaller average costs are more likely to be profitable than those with higher average costs.

Similarly, for a type k (k = 1,…, N_2) good 2 producer,

$$\pi_{2k}^* \geq 0 \text{ if and only if } Q_2^* + \gamma Q_1^* \leq \alpha - d_{2k}, \tag{3}$$

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[1] To save on notation, we do not explicitly specify the number of firms for each type of each good. This does not hinder our presentation since we need only work with the total equilibrium output of each good.

[2] Here and henceforth, superscript * denotes equilibrium value.

where $d_{2k} = c_{2k} + \sqrt{f_{2k}}$ is type k good 2 firms' average cost of production if they are the only firms in existence in the free-entry equilibrium. Hence, equilibrium profit of each type k good 2 producer is positive if $Q_2^* + \gamma Q_1^* < \alpha - d_{2k}$, and is negative if $Q_2^* + \gamma Q_1^* > \alpha - d_{2k}$. It follows that among all good 2 producers those with smaller average costs are more likely to be profitable than those with higher average costs.

3. Free-Entry Equilibrium

We shall neglect the improbable case of ties by assuming that $d_{11} < d_{1j}$ for all j = 2,…, N_1, and $d_{21} < d_{2k}$ for all k = 2,…, N_2. That is, for each good, there is one technology with the smallest average cost.

The free entry equilibrium can be thought of as being obtained via a two stage game. In the first stage, firms of all types decide whether to enter the market. In the second stage, firms that entered in the first stage compete in output a la Cournot.[3] A firm enters if it has the prospect of making a non-negative profit after entry. A firm exits or does not enter if it expects to earn a negative profit.

Since $d_{11} < d_{1j}$ for all j = 2,…, N_1, by the result in (2), if any other type good 1 producer has an incentive to stay or enter more type 1 firms will find it profitable to enter. Hence, entry of type 1 firms will eventually drive out all other existing types of good 1 firms and make entry non-profitable for those types. Mathematically, as the line $Q_1 + \gamma Q_2 = \alpha - d_{11}$ lies higher than lines $Q_1 + \gamma Q_2 = \alpha - d_{1j}$ for $j > 1$ in the first quadrant of the (Q_1, Q_2) space, entry will not stop until the highest line (i.e., $Q_1 + \gamma Q_2 = \alpha - d_{11}$) is reached. Hence, free entry will lead to $Q_1^* + \gamma Q_2^* > \alpha - d_{1j}$ for all $j > 1$, rendering entry non-profitable for good 1 producers of all type $j > 1$.

Similarly, the result in (3) implies that only type 1 good 2 producers may survive in the free-entry equilibrium. We summarize these discussions in the following proposition.

Proposition 1. In the free-entry equilibrium, only one type of firms may exist for each good.

Gotz (2005) and Ohkawa et al. (2005) establish that only one type of firms survive in a free entry homogenous good Cournot market. Proposition 1 generalizes their result to a model

[3] Alternatively, one can think of an entry-exit dynamic process starting from an initial configuration of different types of firms. This process should arrive at the same equilibrium outcome as the two-stage game.

with two substituting goods.[4] Obviously, a similar result should hold for models with more than two goods.

Our next result goes beyond Gotz (2005) and Ohkawa et al. (2005) by showing that it is possible that only one of the two substituting goods will be produced in the free-entry equilibrium. Hence, free entry may lead to under-provision of goods.

We shall now focus on the scenario in which each good is produced by only one type of firms. Denote the cost parameters of good 1 producers as c_1 and f_1, and good 2 producers as c_2 and f_2. Let $d_1 = c_1 + \sqrt{f_1}$ and $d_2 = c_2 + \sqrt{f_2}$ denote the respective average cost measure.

Free entry implies that all surviving firms must make zero profits.[5] Hence, for both goods to be produced in equilibrium, the following two equations must hold:

$$Q_1^* + \gamma Q_2^* = \alpha - d_1, \; Q_2^* + \gamma Q_1^* = \alpha - d_2. \tag{4}$$

Without loss of generality, we shall assume that $d_1 < d_2$. The next proposition states that whether both goods are produced in the free entry equilibrium depends on the value of the substitution parameter.

Proposition 2. In the free-entry equilibrium,
(1) both goods are produced if $0 \le \gamma < (\alpha - d_2)/(\alpha - d_1)$;
(2) only good 1 is produced if $(\alpha - d_2)/(\alpha - d_1) \le \gamma \le 1$.

Proof. It is straightforward to show that if $0 \le \gamma < (\alpha - d_2)/(\alpha - d_1)$ the curves corresponding to the two equations in (4) intersect at an interior point in the first quadrant of the (Q_1, Q_2) space.[6] This intersection point corresponds to the output levels of the two goods produced in the free-entry equilibrium. If $\gamma = (\alpha - d_2)/(\alpha - d_1)$, the two curves intersect at a point on the Q_1 axis. Obviously, only good 1 producers produce at this intersection point. If $(\alpha - d_2)/(\alpha - d_1) < \gamma \le 1$, within the first quadrant, the curve $Q_1 + \gamma Q_2 = \alpha - d_1$ lies entirely outside of the curve $Q_2 + \gamma Q_1 = \alpha - d_2$. In this case, free entry will move (Q_1, Q_2) beyond the curve closer to the origin (i.e., the curve corresponding to $Q_2 + \gamma Q_1 = \alpha - d_2$). Moreover, since good 2 producers are driven out of the market once (Q_1, Q_2) moves beyond $Q_2 + \gamma Q_1 = \alpha - d_2$, free entry will move ($Q_1$, Q_2) to the intersection point of $Q_1 + \gamma Q_2 = \alpha - d_1$ with the Q_1 axis. It

[4] The result in Proposition 1 holds if the two goods are complements ($\gamma < 0$). This is because the arguments presented preceding Proposition 1 hold true for any negative γ.

[5] For simplicity, we ignore the integer problem and assume that the number of firms can be any nonnegative real number. In addition to the integer issue, other factors may also imply non-zero profits in the free-entry equilibrium (e.g., Hurkensa and Vulkan 2003).

[6] Note that the same is true if $\gamma < 0$. Hence, both goods are produced if they are complements.

follows that in equilibrium only the first equation in (4) holds and no good 2 producer is active. Summarizing the last two cases, if $(\alpha - d_2)/(\alpha - d_1) \le \gamma \le 1$ then the free-entry equilibrium corresponds to the intersection point of the first equation in (4) with the Q_1 axis and only good 1 is produced in equilibrium.

The results in this proposition are intuitive. If $\gamma = 0$, the two goods are independent and entry of one type of firms has no effect on the survivability of the other type. Hence, in equilibrium both types enter and both goods are produced. If $\gamma = 1$, the two goods are perfect substitutes. Free entry implies that only the type of firms (good 1 producers) with the cost advantage can survive. With imperfectly substituting goods, good 1 producers as the ones with a cost advantage will always exist. The survivability of good 2 producers depends on how close a substitute good 1 is. For small values of γ, the two goods are distant substitutes and good 2 producers survive. If γ is large, the two goods are close substitutes and good 2 producers cannot survive. The critical value of γ is $(\alpha - d_2)/(\alpha - d_1)$. As is obvious and intuitive, this critical value depends on the firms' average cost parameters.

4. Conclusion

We have shown two results about the free entry equilibrium in a Cournot market with substituting goods. First, each good is produced by only one type of firms. Second, whether both goods are produced depends critically on the level of substitution between the goods. With high substitution levels, only one of the goods is produced, while both are produced with low substitution levels. This result points to the possibility of under-provision of goods under free entry and Cournot competition.

Dating back to Chamberlin (1933), economists have been interested in the issue of the market provision of variety. Spence (1976) and Dixit and Stiglitz (1977) were the first to provide rigorous examinations of the Chamberlinian monopolistic competition model. Their studies were followed by a sizable subsequent literature. Much of this literature focuses on price-taking firms in which each firm's product is a different variety and firms have symmetric cost structures. In such settings, product variety corresponds to the number of active firms, thus entry and exit directly affect the provision of variety. In contrast, the present paper is concerned with non-price taking oligopolistic firms whose entry and exit do not directly affect variety. Hence, our finding that free entry can lead to under-provision of goods (variety) complements the findings in the monopolistic competition literature.

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