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A Dynamic Duopoly Investment Game under Uncertain Market Growth†

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Abstract

We model investments in capacity in a homogeneous product duopoly facing uncertain demand growth. Capacity building is achieved through adding production units that are durable and lumpy and whose cost is irreversible. There is no exogenous order of moves, no first-mover or second-mover advantage, no commitment, and no finite horizon; while building their capacity over time, firms compete *à la Cournot* in the product market. We investigate Markov Perfect Equilibrium (*MPE*) paths of the investment game, which may include preemption episodes and tacit collusion episodes. However, when firms have not yet invested in capacity, the sole pattern that is *MPE*-compatible is a preemption episode with firms investing at different times, but both have equal value. The first such investment may occur earlier, and therefore be riskier, than socially optimal. When both firms hold capacity, tacit collusion episodes may be *MPE*-compatible with firms investing simultaneously at a postponed time (generating an investment wave in the industry). We show that the emergence of such episodes is favored by higher demand volatility, faster market growth, and lower discount rate (cost of capital).

Key words: Real Options; Dynamic Duopoly; Lumpy Investments; Preemption; Investment Waves; Tacit Collusion.

J.E.L. classification: C73, D43, D92, L13.

1. Introduction

Investment games played by competing firms in oligopolistic markets typically share, not always but often, the following stylized characteristics: (i) the development of the market is uncertain and firms have similar knowledge of the underlying random process; (ii) the firms' production capacities are built over time through new units of significant size; (iii) investments in real assets are substantially irreversible; (iv) firms compete in the product market, given their installed production capacity, while they develop that capacity over time; (v) a firm may invest at any time as the market develops and as its competitors build their own capacity; (vi) at the industry level, new investments sometimes come in waves with firms building new plants simultaneously and sometimes in sequences with firms investing at different dates; (vii) as the market matures, absent drastic innovations, capacity building eventually comes to an end with capacities remaining essentially the same for an indefinite time.

We develop a model with features (i) to (v) above, which generates results (vi) and (vii). Moreover, we are able to address the following questions at some level of generality: What is the link between the level of market or industry development and the level of competition? Do simultaneous investments (investment waves) signal intense competition or tacit collusion? Can investments occur earlier than in perfect competition or in a social optimum? What is the effect of demand volatility, market growth rate, and cost of capital (discount rate) on the intensity of competition?

More specifically, we consider a homogeneous product duopoly industry whose market demand growth is stochastic. Firms compete continuously *à la Cournot*, while building up their capacity through the acquisition of multiple discrete (lumpy) units of capacity at times that are endogenously determined. There is no assumed order of moves in capacity building, no first-mover or second-mover advantage, and no commitment on future investments or strategies. Rather, firms hold investment options that they exercise strategically at optimally chosen dates, which we characterize. We determine the value of those options as well as the value of the firms holding them.[1] This is clearly a complex agenda and the analysis, although simplified as much as possible, remains

[1] Remarkable progresses in the analysis of real option games have been realized in recent years. Among major contributors, Grenadier (1996) uses a game theoretic approach to determine the timing of options exercise in the real estate market; Smets (1995) provides a treatment of the duopoly in a multinational setup, which serves as a basis for the oligopoly discussion in Dixit and Pindyck (1994); Lambrecht and Perraudin (1996), Décamps and Mariotti (2004), and Pawlina and Kort (2006) investigate the impact of asymmetric cost on firms' investment strategies; Baldursson (1998) considers a duopoly model where firms make continuous incremental investments in capacity showing that when firms differ in size initially, substantial time may pass until they are of the same size; Grenadier (2002) provides a general solution approach for deriving the equilibrium investment strategies of symmetric firms facing a sequence of investment opportunities with incremental capacity investments; Weeds (2002), Huisman (2001), Huisman and Kort (2003) study option games in a technology adoption context; Boyer et al. (2004) study a duopoly with multiple investments under Bertrand competition; Smit and Trigeorgis (2004) discuss different strategic competition models in the context of real options,

somewhat intricate.

Methodologically, we consider games involving multiple investments and admitting one or several equilibrium sequences of investments. We assume that investments are irreversible and capacity units do not depreciate. As a result capacity can only increase and it is reasonable to refer to the initial capacities held by the firms at the beginning of the game. A state of the game is characterized by a market development level and installed production capacities. If a *MPE* of the game is such that both firms invest simultaneously *in some state*, we say that there is tacit collusion *in that state*, no matter the sequence of previous or subsequent investments may be. If a *MPE* of the game is such that one firm invests earlier than the other *in some state*, we say that there is preemption *in that state*, no matter the sequence of previous or subsequent investments may be. We characterize the factors and conditions (market demand growth and volatility, discount rate) for which along a given equilibrium sequence one may encounter both preemption investment episodes and collusion-like ones.

As market grows, a *MPE* investment sequence involves several investments as additions to capacity carried out at different times by each firm. Firms may sometimes invest separately, in an episode of preemption, and sometimes simultaneously, in an episode of tacit collusion. Authors focusing on single investment episodes (e.g. Nocke, 2007; Fudenberg and Tirole, 1985) then refer to underinvestment (joint investment, tacit collusion) *MPE* or to diffusion (preemption) *MPE* as such classification raises no ambiguity. But in our case, a *MPE* path may contain both tacit collusion episodes at some times or in some states, and preemption episodes at other times or in other states.

Our main results are as follows. For *any* combination of firm capacities at which further investment is profitable for both firms, there exists a *MPE* starting with an episode of preemption. For *some* capacity combinations, which we characterize, there exists also *MPE* exhibiting an initial episode of tacit collusion. Episodes of tacit collusion never exist in the early stages of market development, that is, when at least one firm holds no capacity.

In preemption episodes, incremental rents are equalized and partly dissipated. In tacit-collusion episodes, firms exercise market power by postponing their respective investments: the next investments by each firm occur simultaneous at a chosen development threshold, thereby generating an investment wave in the industry. When a *MPE* exhibiting an initial episode of tacit collusion exists, there are typically numerous ones, which are all Pareto superior from the firms' viewpoint to *MPE* exhibiting an initial episode of preemption. Furthermore, the market

while Chevalier-Roignant and Trigeorgis (2010) discuss option games. Boyer, Gravel and Lasserre (2010) review that literature.

development level at which joint investment would maximize combined profits is *MPE* compatible only if firms are of equal size; when firms differ in size tacit collusion falls short of maximizing combined profits.

Even though the investment game has no finite horizon, it eventually comes to an end. We characterize the endgame conditions. This allows us to use backward induction to characterize the equilibrium investment sequence. As discussed further below, this result is closely dependent on the way we model market demand growth. If along an equilibrium investment sequence, a point is reached where endgame conditions are close to be met while firms have different capacities, then the smaller firm will be the sole investor for the remainder of the game. Thus, while firms may be of different sizes along the equilibrium path, no size advantage can be maintained forever.

We also show that higher market volatility enlarges the set of conditions under which *MPE* with tacit collusion episodes may exist. Similarly, this set of conditions is enlarged by higher expected market growth as well as by a lower cost of capital. Investment waves (simultaneous investments by both firms) may then signal the collusive exercise of market power in tacit collusion episodes of a *MPE*.

The related literature

The present paper extends the literature on strategic investment, most notably the seminal contributions of Gilbert and Harris (1984), Fudenberg and Tirole (1985), and the more recent contribution of Genc *et al.* (2007), Besanko and Doraszelski (2004) and Besanko *et al.* (2010). These contributions brought forward the analysis of the role of investment competition in shaping the structure of a developing industry, including rent equalization and dissipation in dynamic investment games, tacit-collusion conditions in such games, and the durability of a first-mover advantage. Investigating the role of investment in acquiring market dominance in a very general framework, Athey and Schmutzler (2001) came to the conclusion that, without firms' commitment to future strategic investment plans, there is little hope to obtain definitive predictions outside specific models. In an effort to provide tractable results, modern investment games indeed were often modeled in a way that failed to exhibit some of the stylized facts mentioned above. Such models include models of technology adoption, models of entry, and numerous two-stage models where firms first make and commit to long-term decisions (stage one) before competing in short-term decisions afterwards (stage two). Although the possibility of preemption and collusion equilibria has been widely described within two-stage games,[2] such games cannot account for the

[2]Kreps & Scheinkman (1983), Deneckere & Kovenock (1996), Allen *et al.* (2000), and Mason and Weeds (2005), to name a few.

possible succession of competitive and collusive episodes within a given industry investment path.

Gilbert and Harris recognized early the role of lumpy and strategic investments in dynamic frameworks. Our endgame characterization (no size advantage can be maintained forever) is reminiscent of Gilbert and Harris (1984, Proposition 1) who find an open-loop Cournot-Nash equilibrium where capacities stay within one unit of each other during the whole duopoly development. However, when the commitment assumption that goes with open loop is removed, a drastic change occurs: one firm builds all plants while the other builds none (Proposition 6). This result can be traced back to an *ad hoc* asymmetry in their game formulation to do away with commitment. Fudenberg and Tirole provided the methodological tool for analyzing preemption and joint investment in continuous time without precommitment nor *ad hoc* asymmetry. We use their methodology, adapted to a continuous diffusion process.

Genc *et al.* used the concept of S-adapted open-loop equilibrium of Haurie and Zaccour (2005) to analyze different formulations of dynamic investment games. They showed in particular that more volatility in demand, providing higher profits for firms, could favor entry in the relevant markets. Besanko and Doraszelski developed a computational approach to study discrete dynamic investment games for which both preemption and tacit collusion episodes exist within *MPE*, that is, in their case, a preemption race in the early stages followed by capacity coordination through capital depreciation in later stages. They show that low product differentiation and low sunkness of investments favor such sequences of investments. An important characteristic of these works is the intended and successful application of the models to real industrial cases. Besanko *et al.* wonder why "... some industries experience both preemption races and capacity coordination, (while) others seem to sidestep preemption races altogether." (p. 2). While the comparison with our model is difficult (we have quantity competition and homogenous products while they have price competition and differentiated products; we have infinite Markovian dynamic market uncertainty while they have infinitely repeated independent draws; we have irreversible investment while they have some degree of reversibility), their analysis and ours are first steps in identifying conditions under which episodes of competition may *alternate* with episodes of tacit collusion in an equilibrium sequence of industry investments.

Our model has some similarities with Nocke (2007)'s dynamic game of quality adoption. However, besides differences discussed by Nocke between quality investment games and capacity investment games, the main difference is that we study an industry under continuous stochastic development rather than an industry having reached a given level of development. As a result, investments are not concentrated in equilibrium at any single date or level of market development, but occur in a sequence. Competition intensity may vary as investments proceed along an equilibrium sequence.

As mentioned above, we show that the set of conditions under which tacit collusion episodes may exist within *MPE* is enlarged with higher discount factors (lower discount rates). This is a form of folk theorem which is reminiscent of but different from Dutta's theorem (1995*a*, 1995*b*) stating that, for sufficiently patient players, there exist a tacitly collusive *MPE* equilibrium along which firms maximize their joint value. In our model, joint profit maximization can appear in tacit collusion episodes. However, the investment game eventually comes to an end so that players may not be able to benefit from being sufficiently patient as in Dutta. Consequently other considerations affecting the ability to punish deviations come into play. It turns out that joint value maximization is reachable only if firms are of equal size.

This paper is organized as follows. We present the model, the competition framework, the multiple unit investment game, and we characterize the endgame conditions in Section 2. We begin Section 3 with the explicit analysis of three different situations that essentially cover all relevant ones: no existing capacity; identical positive capacity levels; positive but different capacity levels. We then combine these special cases in such a way that they correspond to alternative states in a single dynamic investment game. We conclude in Section 4. Detailed proofs are provided in the Appendix.

2. The model

2.1 Industry characteristics

We consider the development of an industry where demand is affected by multiplicative random shocks. The inverse demand function at time $t \geq 0$ is given by:

$$P(t, X_t) = Y_t \, D^{-1}(X_t), \tag{1}$$

where $X_t \geq 0$ is aggregate output, $Y_t \geq 0$ is a random shock, and $D : \mathbb{R}_+ \to \mathbb{R}_+$ is the non-stochastic component of demand.

Assumption 1

Demand $D(\cdot)$ is strictly decreasing, continuously differentiable and integrable on $\mathbb{R}_+$, and $D(0) = \lim_{p\downarrow 0} D(p) < \infty$; the mapping $x \mapsto xD^{-1}(x)$ is strictly concave on $(0, D(0))$; aggregate shocks $(Y_t)_{t\geq 0}$ follow a geometric Brownian motion:

$$dY_t = \alpha Y_t dt + \sigma Y_t dZ_t \tag{2}$$

with $Y_0 > 0$, $\alpha > 0$, $\sigma > 0$, and $(Z_t)_{t\geq 0}$ a standard Brownian motion with respect to the complete probability space $(\Omega, \mathcal{F}, P)$.

Multiplicative separability of the inverse demand function is frequently used in dynamic competition models, with D^{-1} referred to as the generic inverse demand. Nocke (2007) uses a unitary elastic version of it in a dynamic game of quality and capacity investment. Bayer (2007) uses another version in a duopoly model of investment timings where the function D is reduced to four possible values implied by two alternative levels of capacity and two states of nature. In contrast, our model involves a continuous volatile market development that proceeds indefinitely. Since there is no variable cost of production, there is a theoretical possibility for the price to become arbitrarily close to zero. We assume $D(0)$ to be finite as no good, even if it is free, is consumed in infinite quantity.

This assumption rules out constant elasticity over the entire price-quantity range, but is compatible with any conventional demand specification such that $D(0)$ is finite, including specifications such as Bayer's. Multiplicative separability in (1) may be interpreted as follow: the number of consumers, actual and potential, in the market remains constant ($D(0)$ is given and finite), but consumers' valuation of the good grows stochastically. Although somewhat restrictive, it is a good representation of markets that grow because actual and potential consumers decide to buy more as they become richer or as their tastes evolve. It also reflects the fact that many if not all markets experience phases of development and decline before reaching maturity. This assumption also turns out to be most useful, especially in the derivation of conditions for the investment game to end.

Firms are risk neutral and discount future revenues at the same rate $r > \alpha$. Investment takes place in a lumpy way. Each capacity unit costs I, which is constant over time, produces at most $Q = 1$ unit of output, does not depreciate, and has no resale value.

2.2 Competition, output, and investment

We consider a duopoly. In Boyer *et al.* (2004) we studied a related preemption model with instantaneous competition in prices (Bertrand) and where each firm may invest at most once. Here, firms make as many investments as they want at dates that they choose. At all dates, they compete in quantities (à la Cournot) subject to capacity constraints. Specifically, within the instant $[t, t+\tau)$, the timing of the game is as follows: (i) first, each firm f chooses how many capacity units ν_t^f to invest in, given the realization of the demand shock Y_t and the existing capital stocks (k_t^f, k_t^{-f}); (ii) next, each firm selects an output level within its capacity, $x_t^f \leq k_t^f + \nu_t^f$; (iii) last, market price is determined according to (1), with $X_t = x_t^f + x_t^{-f}$.

The specification of inverse demand (1) implies that the short-run Cournot quantities are independent of the realization of the current industry-wide shock. We can assume that, in the absence of capacity constraints, this game has a unique equilibrium (x^c, x^c). Let $k^c = \lceil x^c \rceil$ be the minimum capital stock required to produce x^c. It is then easy to check that, with given capacities $k^f \leq k^{-f}$, only three Cournot equilibrium outcomes can occur: (i) both firms are constrained, so that $x^f = k^f$ and $x^{-f} = k^{-f}$; (ii) the smaller firm is constrained, so that $x^f = k^f$, while the bigger firm is not and reacts optimally by choosing x^{-f} on its reaction function; (iii) both firms are unconstrained, so that $x^f = x^{-f} = x^c$. The corresponding instantaneous profit of a firm with capacity k when its competitor holds ℓ capacity units can be conveniently denoted $Y_t \pi_{k\ell}$, where $\pi_{k\ell}$ depends on capacities only. The explicit treatment of Cournot competition not only makes the analysis more transparent; it also gives economic foundation to payoff values that are crucial to the solution of the game and would otherwise appear as *ad hoc* assumptions.

2.3 Markov strategies

A key assumption of our model is that firms cannot (credibly) commit to future investment and output decisions. The game typically generates several investments occurring at endogenous (stochastic) dates. There is no commitment by the firms with respect to their role as first or second mover-investor (the order is endogenous) or to the number of units they will acquire. The natural equilibrium concept here is the Markov perfect equilibrium (*MPE*), in which firms' investment and output decisions at each date depend only on the firms' capital stocks measured in capacity units (k^f, k^{-f}) and the current level of the industry-wide shock y. This rules out implicit collusion between firms when deciding on output: at each date, and given their current capacities, firms play the unique equilibrium of the static Cournot game described above.

In situations where two pure-strategy equilibria exist, where either firm invests first and the other firm second, for identical payoffs, there is a possibility, if firms use pure strategies, that both firms invest simultaneously by mistake. Fudenberg and Tirole (1985)'s concept of mixed strategies for timing games in continuous time avoids this sort of coordination failure. They focus on deterministic environments, while we consider stochastic demand. The basic idea is to construct a continuous-time representation of limits of discrete-time mixed-strategy equilibria by defining a strategy for firm f as a function s^f specifying the *intensity* $s^f_{\nu^f}(k^f, k^{-f}, y) \in [0,1]$ with which firm f invests in ν^f capacity units given the capital stocks (k^f, k^{-f}) and the industry-wide shock $Y_t = y$. A detailed treatment can be found in Boyer *et al.* (2004, Appendix A). In the rest of the paper, we will omit firm and strategy profile indices in the expression of value functions when no ambiguity arises.

2.4 Firm valuation

Since $(Y_t)_{t\geq 0}$ is a time homogeneous Markov process, an outcome may be described as an ordered sequence of investment triggers together with the short-run instantaneous profits of both firms $Y_t\pi_{k\ell}$ and $Y_t\pi_{\ell k}$ between investments. Let y_{ij} (with $y_{ij} = y_{ji}$), where i and j refers to the firms' capacities immediately before Y_t reaches y_{ij} for the first time, denote the value of Y_t that triggers a new investment when total industry capacity is $i + j$. Using the notation y_{ij} for the trigger of the next investment, whether that investment is carried out by the firm of capacity i or by the firm of capacity j, is a convenient way at this stage to avoid discussing which firm is the next investor. Clearly, in general, firms' next investments are triggered by different market development levels and may differ from one *MPE* to another. Since capacity units do not depreciate, higher triggers along a given development path correspond to higher industry capacity levels: $y_{ij} \leq y_{k\ell} \Leftrightarrow i + j \leq k + \ell$. If the game is over, then, by convention, $y_{ij} = \infty$.

Suppose $Y_t = y$ and let us consider, for simplicity, investments of one capacity unit only ($\nu = 1$), as investments in multiple capacity units can be treated as one-unit investments occurring at the same time. Let $L(i, j, y)$ denote the current value of the firm of capacity i if it carries out an investment immediately, while its opponent has capacity j. Let $F(i, j, y)$ be the current value of the firm of capacity i when its competitor with capacity j carries out an investment immediately. Let $S(i, j, y)$ denote the current value of the firm of capacity i, with its competitor holding capacity j, if both firms make a simultaneous investment at some future date, say when Y_t

reaches y_{ij}.

The following lemma gives analytical expressions for the L, F, and S functions. The expressions are divided into a first part corresponding to the current investment and a second part corresponding to the continuation of the game. The latter part is not fully specified at this stage; it will be determined recursively by backward induction, starting from the 'horizon', defined in state space as the first (stochastic) time a situation (a capacity combination) is reached such that no more investment will take place.

Lemma 1 *Let $Y_t = y$. The value of the firm of capacity i, when it invests immediately while the firm of capacity j does not, is given by the following, where $k = i + 1$:*

$$L(i,j,y) = \frac{\pi_{kj}}{r-\alpha} y - I + \left(\frac{y}{y_{kj}}\right)^{\beta} \left[c(k,j,y_{kj}) - \frac{\pi_{kj}}{r-\alpha} y_{kj}\right],$$

where $\beta = \frac{1}{2} - \alpha/\sigma^2 + \sqrt{(\alpha/\sigma^2 - \frac{1}{2})^2 + 2r/\sigma^2} > 1$ and $c(k,j,y)$ is the continuation value of the same firm at the time of the next industry investment, if any (if y_{kj} is finite).

Its value, when it stays put while its competitor of capacity j invests now, is given by the following, where $\ell = j + 1$:

$$F(i,j,y) = \frac{\pi_{i\ell}}{r-\alpha} y + \left(\frac{y}{y_{i\ell}}\right)^{\beta} \left[c(i,\ell,y_{i\ell}) - \frac{\pi_{i\ell}}{r-\alpha} y_{i\ell}\right].$$

Its value, when both firms invest simultaneously at some future trigger value y_{ij}, is given by the following, where $k = i + 1$ and $\ell = j + 1$:

$$S(i,j,y) = \frac{\pi_{ij}}{r-\alpha} y + \left(\frac{y}{y_{ij}}\right)^{\beta} \left(\frac{\pi_{k\ell} - \pi_{ij}}{r-\alpha} y_{ij} - I\right) + \left(\frac{y}{y_{k\ell}}\right)^{\beta} \left[c(k,l,y_{k\ell}) - \frac{\pi_{k\ell}}{r-\alpha} y_{k\ell}\right].$$

Consider the expression for $L(i,j,y)$. The first part $\frac{\pi_{kj}}{r-\alpha} y - I$ gives the expected net present value of the profit flows achieved by increasing capacity from i to $k = i+1$ at a cost of I, assuming that no more investment is made. The second part $\left(\frac{y}{y_{kj}}\right)^{\beta} \left[c(k,j,y_{kj}) - \frac{\pi_{kj}}{r-\alpha} y_{kj}\right]$ adjusts the first one for the effect of subsequent investments, that is for the (equilibrium) exercise by both firms of their investment options. Indeed, $\left(\frac{y}{y_{ij}}\right)^{\beta}$ may be viewed as a discount factor defined in state space rather than in time space and the function $c(k,j,y_{kj})$ is the continuation value function when $Y_t = y_{kj}$. The expressions for $F(i,j,y)$ and $S(i,j,y)$ can be similarly understood.

2.5 Endgame conditions

Although the investment game imposes no restrictions on capacities, we can characterize endgame conditions: the investment game is over if and only if it is known with certainty that no firm will ever invest in additional capacity. The following proposition gives two conditions, one necessary, one sufficient, for the investment game to be over (recall that capacity units do not depreciate).

Proposition 1 *The investment game is over only if (necessity) either condition* A *or condition* B *is satisfied, implying that both firms hold a strictly positive capacity; moreover, the investment game is over if (sufficiency) Condition* A *is satisfied:*

A Neither capacity constraint is binding in the short-run Cournot game, that is,

$k^f \geq k^c = \min\{k \in \mathbb{N} \mid k \geq x^c\},\ f \in \{1, 2\}.$

B Both capacity constraints are binding in the short-run game and would remain binding in case of a unit investment by any one firm.

Proposition 1 indicates (i) that no firm can keep its opponent out of the market in the long-run, and (ii) that a firm cannot use excess capacity in order to maintain a dominant position in the long-run. Condition A falls short of implying equal capacities for both firms. However, it implies that, if capacities are not equal at the end of the game, the number of units used by each firm is the same. If capacities are not equal, some capacity is idle.

Condition A says that the game is over if no firm experiences a capacity constraint in the choice of its Cournot output; since the duopoly game is time independent, this situation will not change as Y_t evolves over time, so that no further investment is called for. Condition A is not necessary, however. But if it is not satisfied at the end of the game, condition B must hold. The latter condition pertains to tacit collusion. It describes a situation where each firm could still profitably increase its capacity if its rival did not react. For such a situation to last forever (game over), it must be the case that firms restrict capacity, hence output, in equilibrium. This implies that any deviation will be adequately punished. Condition B describes a situation where a firm can inflict a punishment on its competitor if the latter deviates. If the former firm were no longer capacity constrained following an investment by its opponent (Condition B not satisfied), then it would not be able to retaliate. Consequently, the (capacity constrained) opponent would invest whenever the level of market development would be adequate, without considering a potential reaction by its competitor.

The mere ability to retaliate is not sufficient to sustain a tacit-collusion equilibrium. We characterize below the conditions under which the retaliatory power is sufficient to offset the gain from deviating. If the firm to be punished is small, it does not lose as much from an increase in the capacity of its opponent as if it were bigger. This implies that retaliation, hence collusion, is likely to be easier between firms of similar size, while this explains also why the investment game cannot be over unless both firms hold strictly positive capacities.

In what follows, firm asymmetry can only take the form of differences in current capacities and may be thought of as inherited from past moves in the industry development game. As discussed above, Lemma 1 provides only a partial characterization of value functions under alternative investment strategies. Completing the characterization requires knowledge of the continuation function $c(\cdot)$ and the appropriate trigger values. These can be determined when the game between the two firms is sufficiently near its end, in the sense of Proposition 1. Once the continuation value function is known in such situations, it is possible to characterize recursively the value function corresponding to the previous steps.

3. Industry development

Assuming multiplicative demand separability and a finite $D(0)$ guarantees that a finite number of capacity units are eventually installed so that the investment game eventually comes to an end. Industry development possibilities are represented in Figure 1 as a tree whose nodes give the number of units held by firms.

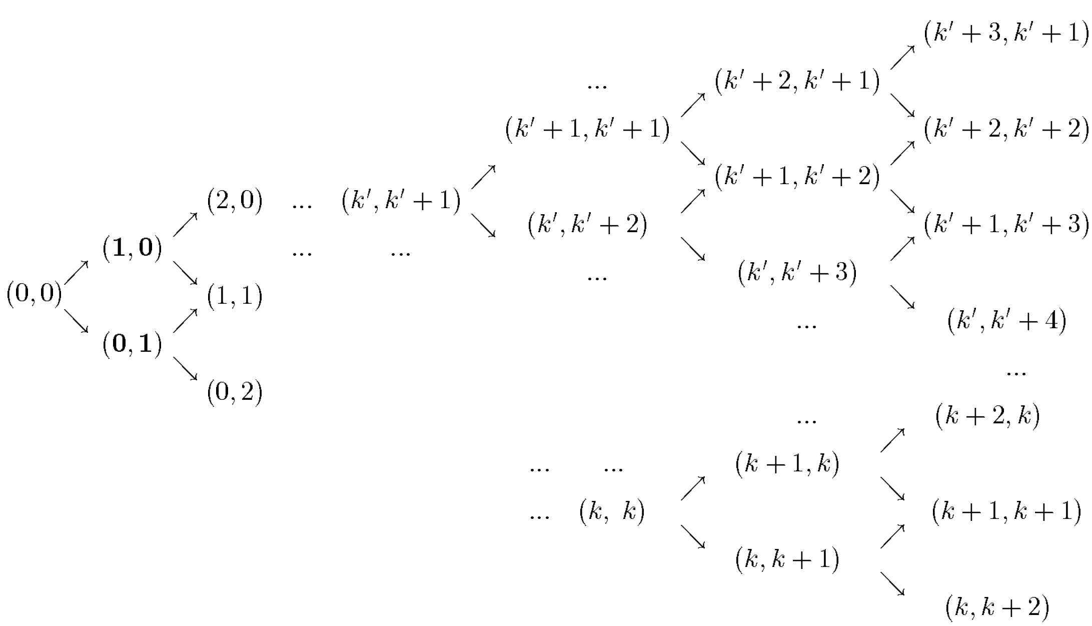


Figure 1: Industry capacity development tree

While the figure indicates possible investment sequences, it does not show any timing, or the market development thresholds at which nodes are reached.

We characterize the capacity acquisition and the competition intensity prevailing at various stages (called episodes of capacity acquisition) of market and industry development: first, in the early stage when firms hold no capacity (Case 1); second, at a later stage, when firms hold similar (Case 2) or different (Case 3) capacities due to the unraveling of their respective investment strategies.

We first consider situations that are "near" the end of the game: from the nodes considered, a limited number of investments will lead to a situation where the investment game is over by Proposition 1. Once these investment developments are characterized, the previous relevant investment episodes can be obtained by backward induction: a limited number of investments will lead to a situation or node from which the (not necessarily unique) unraveling of the investment game has been characterized until endgame conditions are met. Once we have characterized those situations that are "near" the end of the game, we generalize the analysis (in subsection 3.4) to arbitrary nodes in the industry development tree.

3.1 Case 1: No existing capacity

We start with a situation where initial capacities are zero. Let us assume, to simplify the presentation without loss of generality, that the market is such that unconstrained firms would produce at most one unit each in Cournot duopoly (recall subsection 2.2), that is:

Assumption 2 $0 < x^c \leq 1$.

This assumption indicates that, although no firm holds capacity, we are close to the end of the investment game. It allows the monopoly output to exceed unity, so that the acquisition of more than one unit may be considered by any one firm, but it also implies that, if both firms hold one unit or more, the investment game is over by Proposition 1. Assumption 2 also implies that, whatever the (strictly) positive number of capacity units held by its opponent, a firm obtains instantaneous profit $Y_t\pi_{11}$ once it invests in one unit or more; consequently, it will typically not acquire more than one unit.

Therefore, the payoff from not investing immediately is (with $\pi_{1\nu} = \pi_{11}$ by Assumption 2)

$$F(0,0,y) = \left(\frac{y}{y_{0\nu}}\right)^{\beta} \left(\frac{\pi_{11}}{r-\alpha} y_{0\nu} - I\right),$$

where ν is the number of units acquired by the opponent before the firm acquires its first and single unit. The stopping problem faced by the firm is then:

$$F^{*}(0,0,y) = \sup_{y_{0\nu}} \left[\left(\frac{y}{y_{0\nu}}\right)^{\beta} \left(\frac{\pi_{11}}{r-\alpha} y_{0\nu} - I\right)\right] \tag{3}$$

with solution:

$$y_{0\nu}^{*} = y_{01}^{*} = \frac{r-\alpha}{\pi_{11}} I \frac{\beta}{\beta - 1}, \quad \forall \nu \geq 1. \tag{4}$$

Knowing this, the value for the competitor of acquiring at least one unit immediately at $Y_t = y$, and any number of additional units before Y reaches the threshold y_{01}^{*} can be computed explicitly. For example, if it acquires one unit immediately and abstains from any further investment, its value is, according to Lemma 1:[3]

$$L(0,0,y) = \frac{\pi_{10}}{r-\alpha} y - I + \left(\frac{y}{y_{01}^{*}}\right)^{\beta} \frac{\pi_{11} - \pi_{10}}{r-\alpha} y_{01}^{*}, \quad y < y_{01}^{*}, \tag{5}$$

where $\frac{\pi_{11}}{r-\alpha} y_{01}^{*} = c(1,0,y)$, since no more investment is forthcoming beyond y_{01}^{*} by Proposition 1. Similarly, if the investment in the first unit is to take place in the future at $y_{00} > y$, then the value of the firm is:

$$L(0,0,y) = \left(\frac{y}{y_{00}}\right)^{\beta} \left(\frac{\pi_{10}}{r-\alpha} y_{00} - I\right) + \left(\frac{y}{y_{01}^{*}}\right)^{\beta} \left(\frac{\pi_{11} - \pi_{10}}{r-\alpha} y_{01}^{*}\right). \tag{6}$$

Its maximum $L^{*}(0,0,y)$ with respect to y_{00} is reached at:

$$y_{00}^{L} = \frac{r-\alpha}{\pi_{10}} I \frac{\beta}{\beta - 1}. \tag{7}$$

Figure 2 illustrates the functions $L(0,0,y)$, $L^{*}(0,0,y)$ and $F^{*}(0,0,y)$.

[3] We leave to the reader the straightforward task to adapt the formula and the rest of the argument for any number of units acquired by the first investor before the other one invests at y_{01}^{*}. For example, if the firm plans to acquire a second new unit at some y', $y \leq y' < y_{01}^{*}$, the candidate value for $L(0,0,y)$ is $\frac{\pi_{10}}{r-\alpha} y - I + \left(\frac{y}{y'}\right)^{\beta} \left[\frac{\pi_{20} - \pi_{10}}{r-\alpha} y' - I\right] + \left(\frac{y}{y_{01}^{*}}\right)^{\beta} \frac{\pi_{21} - \pi_{20}}{r-\alpha} y_{01}^{*}, \quad y \leq y' < y_{01}^{*}$ where $\pi_{21} = \pi_{11}$ by Assumption 2. If this value is higher than (5), then it gives the correct expression for $L(0,0,y)$; if it is lower, then (5) is the appropriate expression. Note that the number of candidates to try is low as it cannot exceed the monopoly capacity under Assumption 2.

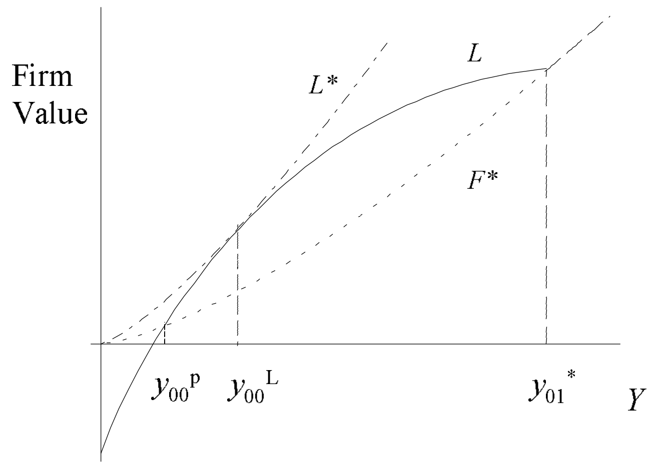


Figure 2: Firm values under alternative strategies

It is straightforward to check from (3)–(6) that, within the interval $(0, y_{01}^*)$, there exists a unique value y_{00}^p, corresponding to the intersection of F^* with L in Figure 2, such that for $y < [>, =]\ y_{00}^p$, $L(0,0,y) < [>, =]\ F^*(0,0,y)$; the corresponding stochastic stopping time is $\tau_{00}^p = \inf\{t \geq 0 \,|\, Y_t \geq y_{00}^p\}$.

We now determine the firms' equilibrium strategies before any firm has invested, that is, in states of the form $(0,0,y)$. If $y < y_{00}^p$, investing is for both firms a strictly dominated strategy while, for $y \geq y_{01}^*$, delaying investment any further is also a strictly dominated strategy. To determine the equilibrium outcome when $y_{00}^p \leq y < y_{01}^*$, it is helpful to consider what would happen if one of the firms were protected from preemption and could thus choose its optimal stand-alone investment date as a monopoly. Given a current industry-wide shock y, the maximal expected payoff that this firm could then achieve by taking the lead is $L^*(0,0,y)$. This is strictly higher than $F^*(0,0,y)$. In a *MPE*, however, such a value gap cannot be sustained. If a firm anticipates that its rival will first invest at y_{00}^L, then it is better-off preempting its rival at $y_{00}^L - dy$. This is true for any y between y_{00}^p and y_{00}^L. When the industry-wide shock Y_t is equal to y_{00}^p, the value of both firms is the same, so each firm is indifferent between investing immediately and letting its rival invest while waiting to invest until Y_t reaches y_{01}^*, at the stochastic time $\tau_{01}^* = \inf\{t \geq 0 \,|\, Y_t \geq y_{01}^*\}$. The following proposition is a transposition of Fudenberg and Tirole (1985, Proposition 2A) in a stochastic context.

Proposition 2 *Under Assumptions 1 and 2, if $Y_0 \leq y_{00}^p$, there exists only one* MPE *outcome of the investment game: one firm invests at τ_{00}^p, while the other firm waits until τ_{01}^* to invest; both times are stochastic. Rents are* *equalized to the value of the second investor given by* (3).

The simple preemption *MPE* episode in this case is characterized by intense competition. The first capacity unit is installed earlier than under protection from preemption since $y_{00}^{p} < y_{00}^{L}$, reflecting a partial dissipation of monopoly rents (Posner, 1975; Fudenberg and Tirole, 1987). A rise in uncertainty increases both F^{*} and L (see Figure 2) so that it may hasten investment (reduce y_{00}^{p}) in a *MPE* preemption episode. The possibility that increased volatility could hasten investment was pointed out by Mason and Weeds (2005).

3.1.1 Socially optimal investment timings

It is more difficult to compare the *MPE* outcome with the social optimum. Specifically, let $k^0 = \lceil D(0) \rceil$ be the minimum capital stock required to produce $D(0)$. The social planner's problem is to find an increasing sequence of stopping times that solves:

$$\sup_{\tau_1 \leq \cdots \leq \tau_{k^0}} \left\{ \mathrm{E}_y \left[\sum_{k=1}^{k^0} \int_{\tau_k}^{\tau_{k+1}} e^{-rt} Y_t \int_0^k D^{-1}(q) dq \right] \right\},$$

where by convention $\tau_{k^0+1} = \infty$. Standard computations imply that it is optimal for the social planner to invest in the first capacity unit when Y_t reaches the investment trigger y^O such that:

$$y^O \int_0^1 D^{-1}(q)\, dq = (r - \alpha) \frac{\beta}{\beta - 1} I. \tag{8}$$

From (7), we obtain $y^O \leq y_{00}^{L}$, with the equality satisfied if $\int_0^1 D^{-1}(q)\, dq = \pi_{10}$. Since $y_{00}^{p} < y_{00}^{L}$ as well, there is no obvious way to compare y^O and y_{00}^{p} in general.

However, let $CS = \int_0^1 D^{-1}(q)\, dq - \pi_{10}$ represent the generic net consumer surplus on the first capacity unit; it only depends on the slope of the demand function on $[0, 1]$. In particular, $CS = 0$ if the inverse demand curve is a step function, $D^{-1}(Q) = D^{-1}(\lceil Q \rceil)$, implying that $y^O = y_{00}^{L} > y_{00}^{p}$. Consequently, for any set of parameters defining y_{00}^{p} and y_{00}^{L}, there exists a set of demand functions $D(.)$ satisfying Assumption 1 such that CS is small enough for y^O to be higher than y_{00}^{p}. This proves the following proposition.

Proposition 3 *For any set of parameters determining y_{00}^{p} and y_{00}^{L}, there exists a set of demand functions $D(.)$ satisfying Assumption 1 such that the first industry investment occurs earlier in the preemption* MPE *than would be socially optimal.*

Leahy (1993) discusses the timing of entry. In a model of industry capacity investments where investment units are of negligible size, he shows that the timing of investments is socially optimal under perfect competition, no matter the demand function. In the duopoly *MPE*, preemption accelerates entry. However here, the indivisibility in capacity units also implies that perfect competition is not socially optimal in general: unlike the social planner, competitive firms do not consider the consumer surplus when they compute profits, so that they underestimate the benefits of entry relative to the social planner. This factor postpones entry relative to the social optimum and may dominate the preemption effect when the net-of-price consumer surplus is big; it is small when the slope of the inverse demand is small and vanishes altogether when the inverse demand is a step function.

The result that the first industry investment may occur earlier under duopoly than is socially optimum does not depend on the market size assumption $k^c = 1$. In particular it applies in the generalized treatment presented in Section 3.4 below.

3.2 Case 2: Symmetric capacities

Let us now investigate the role of existing capacity, starting in this section with situations where firms have identical capacities, as illustrated by the subgame starting at node (k, k) in Figure 1. As in the previous subsection, we will assume that the firms hold a capacity lower than the unconstrained short-run Cournot output, which implies that both firms are initially capacity constrained and that a firm remains constrained if its opponent invests:

Assumption 3 $0 < x^c - k \leq 1$

Assumption 3 is compatible with an unconstrained monopoly output exceeding $k+1$, so that it does not rule out investments exceeding one unit, thereby allowing a firm to get ahead by more than one unit. It does imply that the end of the game is not too far in the sense that, by Proposition 1, the game is over once both firms have acquired at least one more unit. To simplify exposition, we take $k = 1$. Then Assumption 3 implies that $\pi_{21} > \pi_{11}$, $\pi_{22} > \pi_{12}$, and $\pi_{\nu 2} = \pi_{22} = \pi_{2\nu}$, $\forall \nu \geq 2$.

When considering a new investment, firms will now take into account the consequences on the profits they derive from their existing capacity. We will show that, as a result of the cannibalism effect, *MPE* exhibiting tacit collusion at that node $(k, k) = (1, 1)$ may exist besides the *MPE* with preemption, provided that either late joint investment or no more investment dominates preemption over the whole relevant market development range.

3.2.1 Preemption *MPE* episode at (k, k)

A *MPE* characterized by a preemption episode at (k, k) always exists. Indeed, assume that one of the firms has taken the lead by acquiring at least one new unit, bringing its total capacity to $\nu \geq 2$. For its rival, whatever the number of units held by the first investor, it is a dominant strategy by Assumption 3 to acquire one and only one unit at the market development threshold determined by the following stopping-time problem: for $y < y_{1\nu}$,

$$F^*(1, \nu, y) = \sup_{y_{1\nu}} \left[\frac{\pi_{1\nu}}{r-\alpha} y + \left(\frac{y}{y_{1\nu}} \right)^\beta \left(\frac{\pi_{22} - \pi_{1\nu}}{r-\alpha} y_{1\nu} - I \right) \right], \tag{9}$$

that is, at:

$$y_{1\nu}^* = \frac{r-\alpha}{\pi_{22} - \pi_{1\nu}} I \frac{\beta}{\beta - 1}. \tag{10}$$

The situation is similar to the case with no initial capacity, except that the trigger value at which the second investor invests depends on the number ν of units held by the first investor. The higher ν is, the *earlier* the second investor will invest because its profits $\pi_{1\nu}$, while he is waiting, are lower, the higher ν is.

The firm that invests first, whether it acquires one single unit or more units, understands the implications of its investment decision on the behavior of its competitor, so that $L(1, 1, y)$ can be computed explicitly. For example, if the early investor acquires only one unit, its payoff at the current level of $y \leq y_{1\nu}^*$ is, for $\nu = 1$:

$$L(1, 1, y) = \frac{\pi_{21}}{r-\alpha} y - I + \left(\frac{y}{y_{12}^*} \right)^\beta \left(\frac{\pi_{22} - \pi_{21}}{r-\alpha} y_{12}^* \right). \tag{11}$$

As before, if the firm could choose the investment threshold in the absence of any threat of preemption, the maximum $L^*(1, 1, y)$ with respect to y, for $\nu = 1$, would be reached at:[4]

$$y_{11}^L = \frac{r-\alpha}{\pi_{21} - \pi_{11}} I \frac{\beta}{\beta - 1}.$$

But under a preemption threat, it is not possible for the firm to achieve a value of L exceeding F^*; it cannot wait

[4] Again, the reader can adapt the candidate expressions for $L(1, 1, y)$, with $y_{1\nu}^*$ given by (10), for any new capacity purchase exceeding one unit ($\nu > 2$). The highest such candidate gives $L(1, 1, y)$. It is certain to exist because, as shown in the proofs, the candidate for L corresponding to $\nu = 1$ exceeds $F^*(1, 1, y)$ for some range of y values lower than y_{12}^*.

until Y_t reaches $y^L_{1\nu}$; the best it can do is to invest at the trigger level $y^p_{1\nu}$ at which $L(1,1,y) = F^*(1,\nu,y)$, so that rents are equalized. The following result parallels Proposition 2.

Proposition 4 *(Preemption* MPE *episode at* (k,k)*) Under Assumptions 1 and 3, the investment game has a preemption* MPE *episode at node* $(k,k) = (1,1)$ *such that one firm invests when* Y_t *reaches* $y^p_{1\nu}$*, while the other firm invests when* Y_t *reaches* $y^*_{1\nu}$*.*

In this equilibrium, the threat of preemption leads to rent equalization and thus to the complete dissipation of any first-mover advantage. However, with positive capacities, the preemption equilibrium episode may not be the unique equilibrium at node (k,k), as we shall now see.

3.2.2 Tacit-collusion *MPE* episode at (k,k)

As in Fudenberg and Tirole (1985), the fact that the firms hold strictly positive capacities gives rise to the possibility of a different type of *MPE*, the tacit-collusion (or joint investment) *MPE*. The strategies involved consist in coordinating on a random joint investment date or in abstaining from investing forever, thereby increasing firms' values. Note that short-run output decisions are still determined according to Cournot competition. Collusion is achieved through investment, not through production decisions. This implies that the only way firms can sustain a tacit-collusion outcome is by investing simultaneously, rather than at different times, and by doing so at a threshold y^s_{12} exceeding y^*_{12}. Indeed if one of the firms were to invest at some $y < y^*_{12}$, the other firm's unique optimal continuation strategy would be to invest at y^*_{12}. This can be part of an *MPE* only if $y = y^p_{12}$ as shown in the analysis of the preemption *MPE* episode characterized above. Alternatively, if it were to invest at some $y \geq y^*_{12}$, then a dominant strategy for the other firm would be to follow suit immediately. Since simultaneous investments of one unit imply, by Assumption 3, that both firms then hold more capacity than the unconstrained Cournot output, they will not acquire more than one unit. Furthermore, the game is then over by Proposition 1.

Postponing investment or not investing restricts output. In that sense the tacit-collusion equilibrium episode is reminiscent of the early-stopping equilibrium of Fudenberg and Tirole (1983) and of the tacitly collusive underinvestment equilibria of Nocke (2007) described earlier. However, it is important to mention that Fudenberg and Tirole considered a dynamic game with downward sloping reaction functions in continuous investment levels. Such reaction functions imply that capacity is always scarce, leading to a race to invest aimed at inducing the competitor to reduce its investment. We consider a dynamic game in lumpy investment. Once the capacity of the larger firm exceeds a certain level, the smaller firm is able fully to exploit its new capacity irrespective of any reaction by the larger firm; as a result the larger firm cannot indefinitely induce postponement of the smaller firm's investment by acquiring additional capacity. In that sense, even a preemption episode, where one firm invests early while the other firm waits, does not qualify as a race since both firms end up with the same value.

Suppose that the firms could commit to invest simultaneously at some random future date or to abstain from investing forever. Given a current industry-wide shock y, the expected payoff that they could achieve in this way is, according to Lemma 1:

$$S(1,1,y) = \frac{\pi_{11}}{r-\alpha}y + \left(\frac{y}{y_{11}^s}\right)^\beta \left(\frac{\pi_{22}-\pi_{11}}{r-\alpha}y_{11}^s - I\right). \tag{12}$$

If $\pi_{22} > \pi_{11}$, $S(1,1,y)$ has a maximum with respect to y_{11}^s, denoted $S^*(1,1,y)$, at y_{11}^{s*}:

$$y_{11}^{s*} = \frac{r-\alpha}{\pi_{22}-\pi_{11}} I \frac{\beta}{\beta-1} > y_{12}^* = \frac{r-\alpha}{\pi_{22}-\pi_{12}} I \frac{\beta}{\beta-1}, \tag{13}$$

with $\tau_{11}^{s*} = \inf\{t \geq 0 \,|\, Y_t \geq y_{11}^{s*}\}$ as the corresponding investment (stochastic) timing. If $\pi_{22} \leq \pi_{11}$, $S(1,1,y)$ attains a maximum of $\frac{\pi_{11}}{r-\alpha}y$ by letting $y_{11}^{s*} = \infty$ (tacit collusion by inaction), in which case $\tau_{11}^{s*} = \infty$. Clearly if $L(1,1,y)$ exceeds $S^*(1,1,y)$ at any $y \leq y_{11}^{s*}$, tacit collusion cannot occur in equilibrium since each firm then has an incentive to deviate and invest earlier. Hence,

Proposition 5 *(Tacit-collusion* MPE *episode at (k,k)) Under Assumptions 1 and 3, if $Y_0 \leq y_{11}^p$,*

1. *A necessary and sufficient condition for the existence of a* MPE *with a tacit-collusion episode at (k,k) is $L(1,1,y) \leq S^*(1,1,y)$ $\forall y < y_{12}^*$. If this inequality is strict for all such y, there exists a continuum of tacit-collusion* MPE *episode, indexed by their joint investment triggers y_{11}^s in a range $[\underline{y}^s, y_{11}^{s*}]$, where $y_{12}^* \leq \underline{y}^s \leq y_{11}^{s*}$.*

2. *Rents are equalized in each tacit-collusion* MPE *episode and exceed the preemption* MPE *episode rents at the same node; the Pareto optimal tacit-collusion* MPE *episode corresponds to the joint-profit maximizing investment rule under the constraint that firms invest simultaneously if they do.*[5] *In this joint-profit*

[5]In the absence of that constraint, joint-profit maximization would involve sequential investments. Such an investment sequence cannot be sustained as an *MPE* outcome as it would generate a strictly higher expected payoff for the first investor and would, therefore, be subject to preemption.

maximizing tacit collusion MPE *episode, each firm invests in one capacity unit with intensity:*

$$s_1^f(1,1,y) = s_1^{-f}(1,1,y) = \begin{cases} 0 & \text{if } y \in [0, y_{11}^{s*}), \\ \\ 1 & \text{if } y \in [y_{11}^{s*}, \infty). \end{cases}$$

3. *If $\pi_{22} > \pi_{11}$, the Pareto optimal tacit collusion* MPE *episode has both firms investing when Y_t reaches y_{11}^{s*}; otherwise, it is such that neither firm ever invests as $y_{11}^{s*} = \infty$.*

Propositions 2 and 5 highlight the role of existing capacity in the exercise of market power. A firm that holds no capacity has no incentive to restrain output and thus a tacit-collusion *MPE* episode cannot exist if one firm has zero capacity (Proposition 2). In the language of contestability, this says that the level of contestability is stronger when the contesting firm is not yet active. Moreover, the mere existence of an incentive to tacitly collude is not enough to guarantee that tacit collusion is sustainable in equilibrium: firms must also follow investment strategies such that a deviation from the tacit-collusion outcome would trigger a reaction leading to a new equilibrium with a lower value for the deviating firm.

This "punishment" is made difficult because our assumption of a Cournot production equilibrium in any period implies that restraining output can only be achieved by *postponing* capacity investments in the industry. It follows, in particular, that the joint investment trigger in any tacit-collusion equilibrium episode must be higher than both triggers in the preemption *MPE* episode characterized in Proposition 4. Moreover, a firm becomes more vulnerable to a deviation by its competitor once the trigger value for the first investment in the preemption equilibrium has been crossed: once $y > y_{11}^p$ and until y reaches the threshold for the second investment, a deviation yields the defector a higher rent $L(\cdot)$ than the rent $F^*(\cdot)$ obtained by its competitor who would then invest optimally at y_{12}^*. Therefore, the rents $S^*(\cdot)$ under tacit collusion must be attractive enough (Proposition 5(b)) to beat such defection at any level of y preceding y_{12}^*.

Proposition 5 provides a necessary and sufficient condition for *MPE* with tacit collusion at (k,k) to exist. This condition implies restrictions on the components of $L(1,1,y)$ and $S^*(1,1,y)$: first, the four profit values π_{ij} determined by the non-stochastic component of demand $D(\cdot)$ under Cournot competition; second, the parameters underlying real option values, that is, the value of β as determined by the discount rate r as well as the drift α and the volatility σ of the stochastic demand growth process.

Let $\widetilde{\Lambda}(\beta, I) \equiv \{(\pi_{11}, \pi_{12}, \pi_{22}, \pi_{21}) \mid S^*(1,1,y) - L(1,1,y) \geq 0 \, \forall y < y_{12}^*\}$ be the set of π_{ij} quadruples for which tacit-collusion *MPE* episodes exist given β and I. The following proposition states that this set is non-empty, independent of I (that is, $\widetilde{\Lambda}(\beta, I) = \Lambda(\beta)$), and larger in industries with higher volatility, faster growth and lower cost of capital (that is, $\Lambda(\beta') \subset \Lambda(\beta)$ iff $\beta < \beta'$).

Proposition 6 *(Tacit-collusion* MPE *episodes: existence) Under Assumptions 1 and 3,*

1. *There exists a set of market parameters guaranteeing the existence of* MPE *characterized by tacit collusion at (k, k).*
2. *This set is independent of the investment cost I of a capacity unit.*
3. *It is larger, the higher demand volatility, the faster market growth, and/or the smaller the discount rate.*

As we know from the real option literature, increased volatility raises the option value of an irreversible investment under no preemption threat: the firm increases its investment threshold to reduce the probability that the stochastic process reverts to undesirable levels after the firm has invested. The flexibility to do so increases the value of the firm and the more so, the higher the volatility. Such an effect is also present here.

There is another effect of volatility: an increase in volatility raises firm value more under tacit collusion than under preemption, thus favoring the emergence of the former. The reason comes from both timing and discounting. Tacit collusion involves higher investment thresholds (hence longer delays), while an increase in volatility amounts to a lower discount rate (recall that β decreases with volatility σ) because it raises the probability that a given threshold value of y will be reached in a given time span. Although instantaneous profits are always independent of β, the discounted value of the profit flows corresponding to each equilibrium does depend on β: the (state space) discount factors used in (12) and (11) are respectively $\left(\frac{y}{y_{11}^s}\right)^\beta$ and $\left(\frac{y}{y_{12}^*}\right)^\beta$ and, since $y_{11}^s > y_{12}^*$, the former increases more than the latter when β decreases, that is, when volatility increases.

To put it differently, the benefits of restraining output through delaying investments occur in the distant future, that is, in a higher state of market development, while the benefits from deviating occur in the immediate future. Other things equal, higher volatility gives relatively more weight to the distant future, contrary to conventional wisdom whereby increased volatility, because it warrants a risk premium, amounts to an increase in the discount rate.

The intuition for such effects of the (time) discount rate and the market growth rate is similar: a lower discount rate favors future payoffs and a larger expected growth rate raises future prospects relative to immediate ones. Hence, both favor the existence of tacit collusion through a lower β.

3.3 Case 3: Different capacities

While we have shown that existing capacity is a necessary condition for tacit collusion between identical firms, capacity is also often said to play a role as a barrier to entry and thus can be used as a way to acquire and maintain a dominant position or a first-mover advantage.[6]

We consider now situations where firms differ in size. Referring to Figure 1, we now investigate investment subgames starting at nodes (k, k'), $k \neq k'$, and contrast them with those starting at node (k, k) analyzed in the previous section. We showed that, with symmetric strictly positive capacities, there are two possible types of investment episodes: preemption and tacit collusion. The former always exists, is highly competitive and involves rent equalization. The latter exists under some conditions, provides higher rents to both firms, and also involves rent equalization. We will show that some of these characteristics are modified under asymmetric capacities: initial capacity asymmetry prevents rent equalization and makes collusion more difficult in the sense that the maximization of joint profits is no more compatible with a *MPE*.

Without loss of generality, we let one firm hold $k \geq 1$ capacity units and the other $k' = k + 1$ units.

Assumption 4 $0 < x^c - k \leq 2$.

Again, this assumption indicates that we are close to the end of the investment game. Let $k = 1$ to simplify the presentation. The unconstrained Cournot output is then either $1 < x^c \leq 2$ or $2 < x^c \leq 3$, with $\pi_{31} > \pi_{21}$, $\pi_{2\nu} > \pi_{1\nu}$, $\forall \nu$.

Consider first the case where $1 < x^c \leq 2$. The larger firm holding two units may be capacity constrained when the smaller firm holds only one unit, but it will become unconstrained if the smaller firm invests in a second capacity unit. Thus, by Proposition 1, the investment game cannot be over at node $(1, 2)$. If the smaller firm invests, both firms then hold enough capacity to produce x^c and the game is over by Proposition 1. Moreover, the smaller firm benefits more from acquiring one new unit than the bigger firm does and the net benefit from

[6] See for instance Spence (1979), Dixit (1980), Fudenberg and Tirole (1983).

investing is positive at high enough levels of Y_t. Therefore, the smaller firm is the sole investor in equilibrium and the game ends when both firms hold two units of capacity.

Consider now the case where $2 < x^c \leq 3$. Both firms hold a lower capacity than the unconstrained short-run Cournot output so that both are initially constrained and each firm remains constrained if its opponent invests. By Proposition 1 this *may* be the end of the game, although not necessarily so; this possibility will be considered further below. Two alternative candidate preemption equilibrium episodes may be considered: one, where the bigger firm invests first and the smaller firm acts accordingly; another, where the roles are reversed. The corresponding values of the bigger and the smaller firm, acting as first or second investor are respectively $L(2,1,y)$ and $F^*(2,1,y)$ for the bigger firm, and $L(1,2,y)$ and $F^*(1,2,y)$ for the smaller firm.[7]

When the smaller firm invests first, node $(2,2)$ is reached and both firms remain capacity constrained, which is the situation we analyze in subsection 3.2: both firms then hold two units of capacity and assumption 3 holds with $k=2$, so that Propositions 4, 5, and 6 apply. The continuation of the game is then known so that $L(1,2,y)$ and $F^*(2,1,y)$ can be computed. If the bigger firm invests first, then it is a dominant strategy for the smaller firm to invest at some finite future level of Y_t, since $\pi_{13} < \pi_{23} < \pi_{33}$ as the larger firm must accommodate (Cournot equilibrium). It is then straightforward to obtain $L(2,1,y)$ and $F^*(1,2,y)$.

We will show that, unlike the case of symmetric initial capacities, the next investment is undertaken by the smaller firm in any preemption equilibrium episode. In order to prove that result, we need the following lemma.

Lemma 2 *If $L(2,1,y) > F^*(2,1,y)$ for some $y < y_{13}^*$, then there is exactly one value $y_{12}^p \in (0, y_{13}^*)$ such that $L(2,1,y_{12}^p) = F^*(2,1,y_{12}^p)$ and $L(2,1,y) < F^*(2,1,y)$ for $y < y_{12}^p$.*

The lemma indicates that, by investing at $y = y_{12}^p$, the smaller firm leaves the bigger firm indifferent between investing immediately or waiting. Furthermore, we show in the proof of the next proposition that, at $y = y_{12}^p$, the smaller firm strictly prefers to invest. Also, at any other relevant level of y, the gain for the bigger firm from investing first is smaller than the gain for the smaller firm to do so. These results imply that the sole preemption equilibrium is one where the smaller firm catches up. Trivially, if the bigger firm finds it unprofitable to invest, then the smaller firm can invest at its stand-alone date y_{12}^* without worrying about preemption.

[7] Explicit expressions are given in the proof of Lemma 2. As in previous cases, it is tedious but conceptually easy to check whether the first mover acquires one or more new capacity units before its rival invests. To simplify the analysis, we treat the case where the first mover acquires only one extra unit.

Proposition 7 *(*MPE *with preemption episode at (k, k')) Under Assumptions 1 and 4,*

1. *There exists a* MPE *with a preemption episode at (k, k'); in this episode, the smaller firm invests first, when Y_t first reaches* $\min\{y_{12}^p, y_{12}^*\}$.

2. *In this preemption equilibrium episode, the smaller firm enjoys a strictly positive rent from investing first as $L(1, 2, y_{12}^p) - F^*(1, 2, y_{12}^p) > 0$, while the bigger firm is either indifferent between investing immediately and waiting, or prefers waiting as $L(2, 1, y_{12}^p) - F^*(2, 1, y_{12}^p) \leq 0$.*

3. *Once node (2,2) is reached, Proposition 4 applies, mutatis mutandis.*

Unlike the situation with equal capacities, there is not ambiguity as to which firm invests first. The reason is not because the laggard (smaller firm) is in a better position to avoid immediate cannibalism: the drop in price is the same, whichever firm invests. Thus, the source of the first-mover advantage must be found in future decisions rather than current effects. If the bigger firm were investing first, the other firm could plan its own investment at its stand-alone date. Having less to lose from the cannibalism effect, it would invest earlier in the future than a bigger firm would. This reduces, for its bigger opponent, the advantage of taking the lead.

While the same drop in price occurs whichever firm invests first, the laggard (smaller firm) experiences a lower drop in revenues from its existing capacity, simply because it holds fewer units. This, combined with the higher advantage from taking the lead, explains why the smaller firm enjoys a larger gain in value in that preemption episode.

The *MPE* described in Proposition 7 always exists and it is unique in the class of equilibria involving investment by both firms at different dates or investment by one firm only when capacities differ. As with equal capacities, there may exist another class of equilibrium episodes, tacit-collusion equilibrium ones, involving simultaneous investment or inaction by both firms. The next proposition shows that, as with equal capacities, higher volatility, faster growth and lower discount rate make tacit-collusion *MPE* episodes more likely.

Proposition 8 *(Tacit collusion* MPE *episodes at (k, k'))) Under Assumptions 1 and 4,*

1. *If $\pi_{32} - \pi_{22} = 0$: no tacit-collusion equilibrium episode exists.*

2. *If $\pi_{32} - \pi_{22} > 0$: the set of market parameters ensuring the existence of tacit-collusion* MPE *episodes is larger, the larger demand volatility, the faster market growth, and/or the smaller the discount rate.*

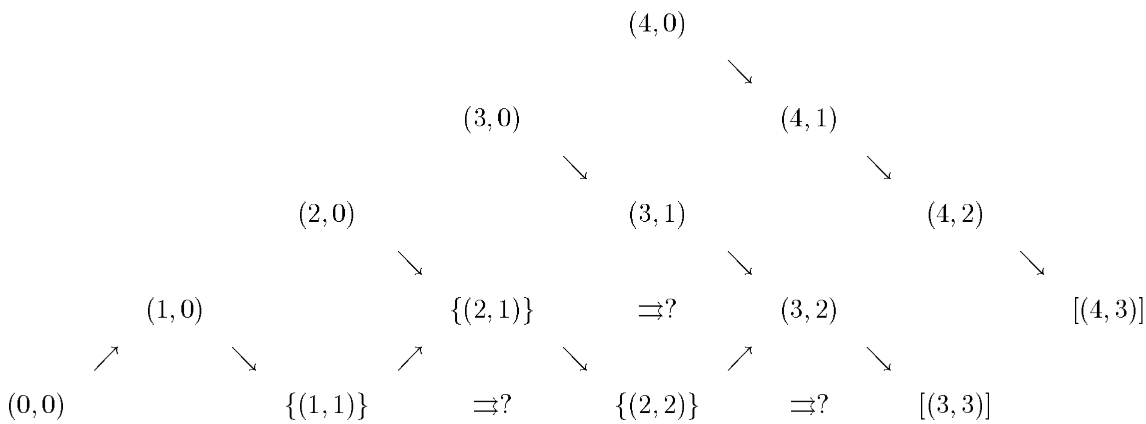
3. *Joint-profits maximization is not compatible with equilibrium (tacit-collusion* MPE *episode).*

As discussed in the case of equal capacities, tacit collusion involves postponing capacity investments in order to restrain output. Benefits from tacit collusion arise in a more distant future than benefits from taking the lead. Consequently, the existence of a tacit-collusion equilibrium rests on conditions under which the future weighs relatively more, either because of significant market growth, or because of high volatility, or because of a low discount rate, as previously. However, tacit collusion is less attractive when firms hold different capacities since joint-profit maximization is not compatible with equilibrium: being different, firms prefer different thresholds for simultaneous investment. The threshold y_{12}^{s*} that maximizes the payoff $S(1,2,y)$ of the smaller firm in case of simultaneous investment is lower than the threshold y_{21}^{s*} for the bigger firm. The joint-profit maximizing threshold is somewhere in between. Since the payoff of the smaller firm decreases in y beyond its maximum, the smaller firm would deviate (invest earlier) from a strategy of joint investment at the joint-profit maximizing threshold.

3.4 Generalization

We have considered explicitly three cases in this section: zero capacity $(0,0)$, equal capacities (k,k), and different capacities (k,k'), each under the assumption that market size was "low", so that any further investments occurred at thresholds higher than y. In each case, a further assumption on market size has limited the number of possible remaining investments (in the sense of Proposition 1): $0 < x^c \leq 1$ for the $(0,0)$ case; $0 < x^c \leq 2$ for the (k,k) case (taking $k=1$); and $0 < x^c \leq 3$ for the $(k,k+1)$ case (taking $k=1$). The three cases cannot be viewed as arising in the same game because these assumptions differ from one another. Let us now replace Assumptions 2, 3, and 4 with $2 < x^c \leq 3$. We now have a single game; it can be solved using the above results, directly or after some adjustment.

This is illustrated in Figure 3 giving all possible capacity combinations if demand is such that $2 < x^c \leq 3$ and $k^m \leq 4$. Since the game is symmetric, we represent only combinations where Firm 1 is at least as big as Firm 2. While a *MPE* is independent of the initial stage of the game, we are interested in industry development, that is, in the characterization of investments occurring in *MPE* equilibrium sequences when initial capacities are $(0,0)$ and y is low. We rule out capacities in excess of the monopoly capacity as the characterization of the *MPE* for such states is trivial.[8] Nodes that are necessarily endgame nodes, according to Proposition $1(A)$, at which no firm holds more than the monopoly capacity, are represented with square brackets in the figure: $[4,3]$ and $[3,3]$. Other possible endgame nodes, in the sense of Proposition $1(B)$, are denoted with curly brackets: $\{2,1\}$, $\{1,1\}$ and $\{2,2\}$; they correspond to tacit-collusion situations, in the sense of Propositions 5, 6, and 8. Equilibrium steps are indicated by single arrows in the cases of single moves (preemption or stand-alone) or double arrows in the cases of simultaneous moves (collusion). A question mark next to an equilibrium step indicates that a *MPE* involving that step may or may not exist in the sense of Propositions 6 and 8. We call the game illustrated in Figure 3 the complete game.



$\{(.,.)\}$: potential endgame; $\rightrightarrows$?: potential tacit-collusion MPE branch;
$[(.,.)]$: endgame (if reached); $\nearrow$: stand-alone or preemption MPE branch.

Figure 3: Complete industry development game when $x^c = 3$

A subset of Figure 3 consisting of any capacity pair (i,j) and all pairs (i',j') such that $i' \geq i$ and $j' \geq j$ may be viewed as a particular stochastic game. With a slight abuse of language, we will call such game *the subgame starting at* (i,j). Although the particular combination (i,j) is not necessarily reached in a *MPE* of the complete game when the initial capacity pair is $(0,0)$, the solution of the subgame starting at (i,j) describes the solution of the complete game for some possible states of that game. For example, although the capacity pair $(4,0)$ is not reached in any *MPE* of the complete game when the initial capacity level is $(0,0)$, the subgame starting at $(4,0)$ has an obvious solution where the bigger firm never invests since it already holds the monopoly capacity, and the smaller firm invests at its stand-alone thresholds until it holds the Cournot capacity $x^c = 3$. This solution describes the investment sequence that applies at capacities $(4,0)$, $(4,1)$, and $(4,2)$ in any *MPE* of the complete

[8] For a linear inverse demand curve $P = \left(1 - \frac{X_1+X_2}{8}\right) Y_t$, $x^c = k^c = 3$ and the maximum monopoly capacity is $k^m = 4$.

game.

The subgame starting at node $(2,2)$ satisfies Assumption 3 for $k=2$. It admits a preemption *MPE* episode described in Proposition 4 leading to an end at $[3,3]$. As described in Propositions 5 and 6 and denoted by the double arrow with a question mark, the subgame starting at $(2,2)$ may also have a tacit-collusion *MPE* episode. In that case, the end is either at $(2,2)$ or at $(3,3)$ and the corresponding firm values are equalized at $(2,2)$, but higher than in the *MPE* with preemption at $(2,2)$.

Trivially, subgames starting at $(3,0)$, $(3,1)$, or $(3,2)$ end up at $[3,3]$ as the bigger firm is then either passive, in which instance the smaller firm invests at its stand-alone thresholds, or is preempted by the smaller firm in *MPE*.[9]

The subgame starting at node $(2,1)$ is studied in Propositions 7 and 8. From $(2,1)$ the preemption equilibrium path leads to $(2,2)$ for a possible end of game at $(2,2)$ or continuation to $(3,3)$, whether directly in a tacit collusion episode or via $(3,2)$ in a preemption episode. There may also exist a tacit collusion episode from $(2,1)$ to $(3,2)$.

The subgame starting at $(1,1)$ has not been studied for $x^c \leq 3$ but only for $x^c \leq 2$. However, now that its two possible continuations, via $(2,1)$ or via $(2,2)$, are known, Propositions 4, 5, and 6 may be adapted accordingly. More precisely, suppose that the equilibrium investment sequence is $(1,1) \rightarrow (2,1)$. When applying Lemma 1 to evaluate $L(1,1,y)$, one must substitute for the continuation value $c(2,1,y)$. If the next equilibrium segment is the preemption segment $(2,1) \rightarrow (2,2)$, this is $F^*(2,1,y)$ as given in the proof of Lemma 2; similarly, the expression for $F^*(1,\nu,y)$, $\nu = 1$, given by (9) for $x^c \leq 2$ must be replaced by the expression applying when $x^c \leq 3$, as provided in the proof of Lemma 2. Alternatively, if the next equilibrium segment is the tacit collusion segment $(2,1) \rightarrow (3,2)$, then $c(2,1,y)$ is equal to $S(2,1,y)$ given in the proof of Proposition 8. The qualitative results are unchanged: the subgame starting at $(1,1)$ always admits a preemption *MPE* episode via $(2,1)$ and $(2,2)$; a collusion *MPE* episode with simultaneous investments leading directly to $(2,2)$ may exist depending on conditions described in Proposition 6. In both cases, the continuation payoffs are known and the game ends at $(2,2)$ or $(3,3)$. If a collusion *MPE* episode exists for the subgame starting at $(2,1)$, Proposition 8 indicates that it does not exhibit rent equalization. Then, by the standard preemption argument used repeatedly in this paper,

[9] For example, consider the possible alternatives from $(3,0)$: either the bigger firm invests first, leading to $(4,0)$ and a continuation with the small firm investing at its stand-alone thresholds until $[4,3]$ is reached; or the small firm invests first, leading to $(3,1)$, $(3,2)$, and $(3,3)$, or to $(3,1)$, $(4,1)$, $(4,2)$ and $(4,3)$. Two conditions are necessary for the first alternative to be an *MPE*: first, the monopoly tenure of the big firm on its fourth unit must be sufficiently long to earn back the investment cost I on the unit; second, the small firm must not invest before the bigger one. Adapting the proof of Proposition 7 where it is shown that the smaller firm invests first in preemption *MPE*, it can be shown that the first condition is violated if the second one is satisfied.

the corresponding preemption *MPE* episode at node $(1,1)$ takes this asymmetry into account: the first investor invests at such a threshold that firm values at node $(1,1)$ are equalized: $L(1,1,y) = F^*(1,1,y)$.

Comparing the subgames starting at $(1,1)$ and at $(2,2)$, we note that a preemption equilibrium episode always exists and collusion equilibrium episodes may exist. However, the subgame starting at $(1,1)$ may also involve a tacit collusion episode from $(2,1)$ to $(3,2)$, unlike the subgame starting at $(2,2)$ where collusion at $(3,2)$ is not possible.

This raises the issue of multiple equilibria. We have shown, however, that firm values are higher under tacit collusion than under preemption in the subgame starting at $(2,2)$. Although we do not provide a formal proof, this is also likely to be the case in the subgame starting at node $(1,1)$ and equilibrium episodes can probably be Pareto ranked. In any case, if tacit collusion from node $(2,1)$ is a possible *MPE* episode, it leads to $(3,2)$. By Proposition 1, this cannot be the end of the game as it is a dominant strategy for the smaller firm to acquire one further unit and for the bigger one to abstain so node $(3,3)$ is reached. By Proposition 1, this is the end of the subgame, with both firms holding equal capacities.

Turning to the subgames starting at $(1,0)$ and $(2,0)$, it can be shown by adapting the proof of Lemma 2 that the small firm invests first from $(1,0)$ or from $(2,0)$. Finally, considering the initial node $(0,0)$, it is now trivial to adapt Proposition 2 replacing the continuation values corresponding to the initial Assumption 2 with the equilibrium values for the subgame starting at $(1,1)$ under the new assumption $x^c \leq 3$. A unique preemption *MPE* prescribing investment from $(0,0)$ to $(1,1)$ via $(1,0)$ exists for each possible continuation payoff at $(1,1)$. As discussed above, several continuation payoff may exist from that node on, all leading to equal size firms at the end of the game, as indicated in Figure 3 and further discussed in the conclusion below.

Thus, under assumption $x^c \leq 3$, the complete game can be solved entirely following the procedure just described. Further generalization to higher-maximum industry sizes would not affect the qualitative properties of the model, which we summarize in the next section.

4. Conclusion

We characterized the development of a stochastically growing market under duopoly where firms build capacity through multiple irreversible lumpy investments in production units without any exogenously given order of moves or any commitment regarding future investments. Firms make optimal strategic use of their flexibility to adapt to the stochastic evolution of the market. The model shares important features of some “real life” investment games and generates stylized characteristics of developing industries, including the occurrence of joint investments (waves) and the eventual reach of maturity.

We found that the early phase of development is characterized by intense competition despite the fact that only one firm may be active at that stage. Competition intensity may cause the first industry investment to occur earlier than would be socially optimal. Indeed equilibrium requires equal payoffs or values for both firms despite the fact that they invest at different market development stages. This leads one firm to enter earlier than would be desirable in order to “destroy” any monopoly advantage it would enjoy over its competitor by being alone in the first phase of the game. This is the sole equilibrium pattern at the beginning of the investment game. The empirical implication of this result is that the first entrant faces riskier returns and a higher probability of bankruptcy than would be the case under perfect competition. Furthermore, uncertainty tends to exacerbate that effect in the sense that higher uncertainty may cause the first industry investment to occur earlier. This was pointed out by Mason and Weeds (2005) when firms are limited to one investment each; we have generalized this result to multiple investments.

The smaller firm eventually catches up and the larger firm cannot indefinitely keep its opponent at bay, even if it holds enough capacity to serve the whole market. Indeed, being bigger makes the threat of early subsequent investment less credible. Consequently, in the initial preemption *MPE* episode of a game starting with firms of different sizes, it is the smaller firm that moves and invests first.

Tacit-collusion episodes may occur in *MPE* only at states of the game where both firms hold positive capacity; they take the form of *postponed* simultaneous investments by both firms. Such equilibria are more likely to exist in highly volatile, low cost of capital, and/or faster growing markets. Suppose that the volatility level is such that no tacit collusion equilibrium episode exists at some stage (k, k', y). Then it is known that a rise in volatility delays investment. However, that effect may be compounded if the rise in volatility further allows a tacit collusion equilibrium episode to come to existence. The firms may then tacitly coordinate and select the higher payoff equilibrium by further delaying their next investments till a common trigger is reached. This effect of volatility appeared in Boyer *et al.* (2001); an anonymous referee pointed out to us that a current version of Mason and Weeds (2005) discusses that issue.

The analysis of multiple investment opportunities has confirmed results that existed with a single opportunity. It has also brought a new perspective on competition at different stages of the development of an industry. Early stages are characterized by intense competition; tacit collusion is only possible at subsequent stages, when firms hold positive capacity; tacit collusion is more likely to exist in industry characterized by high growth, high volatility and/or low cost of capital industry; market power can also be exercised at later stages, whether as tacit collusion implemented by not investing, or as final equilibrium episodes where the smaller firm catches up while investing as a monopoly.

At any industry development stage, however, if tacit collusion is possible in MPE, it is more profitable between firms of equal size than between unequal firms. When firms are of equal size, tacit collusion is compatible with joint-profit maximization, but when firms differ in size, the simultaneous investment threshold that maximizes joint profits is beyond the level that maximizes the expected value of the smaller firm. In that sense, tacit collusion is unable to generate the totality of potential gains from collusion when firms are of unequal sizes. If they can select an equilibrium investment sequence that maintains size equality, it is in the interest of the firms to do so. As extension of the analysis, this also suggests that *explicit* coordination, such as alliances, acquisitions and mergers, may be more attractive, relative to tacit collusion, the more unequal the firm sizes are.

Our results imply that traditional measures of competition intensity may be deceiving: competition is more intense when one single firm is investing, as preemption is then the sole equilibrium action, while tacit collusion is more likely when both firms are active, are of equal size, and the market develops quickly, with much volatility, under a low cost of capital. Indeed faster and more volatile market growth puts remote occurrences within closer reach. As we have shown tacit collusion precisely relies on, and benefits from, exposition to such remote future benefits.

Proof of Lemma 1. Let $Y_t = y$. The value of a firm at date t is the expected present value of its profits over the periods between investments by either firm, minus the present value cost of the investments made by the firm. In the case of a firm of capacity i that invests immediately, at t, while its opponent holds j units and does not make any investment at t,

$$\begin{aligned} L(i,j,y) &= E^y \left\{ \int_t^{\tau_{kj}} e^{-rs} \pi_{kj} Y_s ds + e^{-r\tau_{kj}} \left[c\left(k,j,Y_{\tau_{kj}}\right) \right] \right\} - I \\ &= E^y \left\{ \int_t^{\infty} e^{-rs} \pi_{kj} Y_s ds - \int_{\tau_{kj}}^{\infty} e^{-rs} \pi_{kj} Y_s ds + e^{-r\tau_{kj}} \left[c\left(k,j,Y_{\tau_{kj}}\right) \right] \right\} - I, \end{aligned}$$

where τ_{kj} is the random time, possibly infinite, at which some further investment occurs. The profit flow $\pi_{kj} Y_s$ replaces $\pi_{ij} Y_s$ at t. If it is altered by some new investment by either firm later on, at τ_{kj}, the continuation function $c\left(k,j,Y_{\tau_{kj}}\right)$ accounts for the new state.

The time homogeneity of $(Y_t)_{t \geq 0}$ and the strong Markov property for diffusions imply that, for all $y \geq 0$,

$$L(i,j,y) = \frac{\pi_{kj}}{r-\alpha} y - I + E^y \left\{ e^{-r\tau_{kj}} \left[c\left(k,j,Y_{\tau_{kj}}\right) - \frac{\pi_{kj}}{r-\alpha} Y_{\tau_{kj}} \right] \right\}.$$

We are interested in stopping regions of the form $[y_{kj}, \infty)$. For any $y_{kj} > 0$, let $\tau(y_{kj}) = \inf\{t > 0 \mid Y_t \geq y_{kj}\}$, so that $Y_{\tau(y_{kj})} = y_{kj}$ $P - a.s.$; then $L(i,j,y)$ may be rewritten as:

$$L(i,j,y) = \frac{\pi_{kj}}{r-\alpha} y - I + E^y \left\{ e^{-r\tau(y_{kj})} \right\} \left[c(k,j,y_{kj}) - \frac{\pi_{kj}}{r-\alpha} y_{kj} \right]. \tag{14}$$

Following Harrison (1985, chapter 3), the Laplace transform $E^y \left\{ e^{-r\tau(y_{kj})} \right\}$ is $\left(\frac{y}{y_{kj}} \right)^\beta$ for any $y \in [0, y_{kj})$. Substituting into (14) yields the formula for $L(i,j,y)$ given in the Proposition. The other expressions are obtained in a similar way. ■

Proof of Proposition 1. *A strictly positive capacity is necessary.* Suppose one firm has zero capacity. Then its profit is zero. If it buys one unit, the lowest instantaneous profit it can make at any time after making that investment is $Y_t \pi_{1k}$, where k is the capacity at which its opponent is unconstrained in the short run in response to an output of one: this corresponds to the worst-case scenario where its opponent holds the capacity which leaves the firm the lowest instantaneous profit and the firm does not acquire any further units even if it is profitable for it to do so. The maximized expected discounted present value from buying one capacity unit at some future time τ is, in that worst-case scenario, $V(0,k,y) = \sup_\tau E^y \left\{ \int_\tau^\infty e^{-rt} Y_t \pi_{1k} dt - e^{-r\tau} I \right\}$. Using the approach of Lemma 1 to evaluate V leads to $V(0,k,y) = \sup_{y_{0k}} \left(\frac{y}{y_{0k}} \right)^\beta \left(\frac{\pi_{1k}}{r-\alpha} y_{0k} - I \right)$. The value of y_{0k} that solves the maximization is $y_{0k}^* = \frac{I(r-\alpha)}{\pi_{1k}} \frac{\beta}{\beta-1}$ so that $V(0,k,y) > 0$. Thus, the strategy of never buying in the future is strictly dominated for the firm whose capacity is zero. In consequence, both firms will eventually hold strictly positive capacity.

Either A or B is necessary. Assume that neither A nor B holds, that is: let l and k be the respective capacities; let l be such that the corresponding firm is capacity constrained and let k be such that the firm that holds k units is not constrained if the other firm has a capacity of $l+1$ or more units. If the first firm increases its capacity to $l+1 = n$, its current instantaneous profit increases to $Y_t \pi_{nk} > Y_t \pi_{lk}$ and stays at that level forever since the opponent, not being capacity constrained, has no alternative but to accommodate by reducing output.

The maximized gain in expected discounted present value from bringing capacity to n at some future time τ is $V(l,k,y)=\sup_{y_{lk}}\left(\frac{y}{y_{lk}}\right)^{\beta}\left(\frac{\pi_{nk}-\pi_{lk}}{r-\alpha}y_{lk}-I\right)$. This is positive, implying that a strategy of never investing in a situation where one firm is constrained, while the other is unconstrained or would become unconstrained after a unit investment by its opponent, is strictly dominated.

Condition A is sufficient. If neither capacity constraint is binding, no firm can increase profit by further investing so that the game is necessarily over. ■

Proof of Proposition 2. As shown in the main text, if a firm invests the first time Y_t reaches y from below while the other firm waits, its payoff is $L(0,0,y)$ as given by (5) and the payoff of its opponent is $F^*(0,0,y)$ given by (3). If both firms invest simultaneously at $Y_t=y$, taking $y_{00}^s=y$ in Lemma 1, their payoff is, $\widetilde{S}(0,0,y)=\frac{\pi_{11}}{r-\alpha}y-I$. Let:[10]

$$\begin{aligned}
s_1^f(0,0,y) &= s_1^{-f}(0,0,y)=\begin{cases}0, \text{ if } y\in[0,y_{00}^p)\\ \frac{L(0,0,y)-F^*(0,0,y)}{L(0,0,y)-\widetilde{S}(0,0,y)}, \text{ if } y\in[y_{00}^p,y_{01}^*)\\ 1, \text{ if } y\in[y_{01}^*,\infty)\end{cases}.\\
s_1^f(0,\nu,y) &= s_1^{-f}(0,\nu,y)=\begin{cases}0, \text{ if } \ y\in[0,y_{01}^*),\ \nu\geq 1\\ 1, \text{ if } y\in[y_{01}^*,\infty),\ \nu\geq 1\end{cases}\\
s_1^f(\nu,\nu',y) &= s_1^{-f}(\nu,\nu',y)=0\ \forall y;\ \nu,\nu'\geq 1,
\end{aligned}$$

where $s_1^f(i,j,y)$ is a probability distribution satisfying the detailed definition given in Boyer *et al.* (2004, Appendix A). It can be interpreted as the intensity with which firm f invests in one unit of capacity in state (i,j,y), *i.e.* when it holds i capacity units, its opponent holds j units, and $Y_t=y$. We have shown already that, on $[0,y_{00}^p)$, it is a dominant strategy not to invest, and on (y_{01}^*,∞), it is dominant for a firm with zero capacity to invest if the other holds one unit. The above strategy combination implies that an investment is sure to occur the instant Y_t reaches y_{01}^* because then $s_1^f(0,0,y)$ and $s_1^{-f}(0,0,y)$ start increasing, while no simultaneous investment can occur at y_{00}^p because $s_1^f(0,0,y_{00}^p)$ and $s_1^{-f}(0,0,y_{00}^p)$ are still zero. Once one firm has invested, the other one abstains from investing $\left(s_1^{-f}(0,\nu,y)=0\right)$ until Y_t reaches y_{01}^*.

We now show that the above strategy profile is an *MPE* strategy profile in any subgame starting at $y\in[y_{00}^p,y_{01}^*)$. For $y\in[y_{00}^p,y_{01}^*)$, if firm f deviates by choosing $s'(0,0,y)=0$, the other firm invests at y so that firm f's dominant strategy in the continuation is to invest at y_{01}^* for a continuation payoff of $F^*(0,0,y)$. If it chooses to deviate with intensity $s'(0,0,y)=\lambda\in(0,1]$, its continuation payoff is:

$$\frac{\lambda\left[1-s_1^{-f}(0,0,y)\right]L(0,0,y)+(1-\lambda)\,s_1^{-f}(0,0,y)\,F^*(0,0,y)+\lambda s_1^{-f}(0,0,y)\,\widetilde{S}(0,0,y)}{\lambda-\lambda s_1^{-f}(0,0,y)+s_1^{-f}(0,0,y)}.$$

Substituting for $s_1^{-f}(0,0,y)$, this is equal to $F^*(0,0,y)$. Thus, for any subgame starting at $y\in(y_{00}^p,y_{01}^*)$, both firms are indifferent between all possible choices. At $y=y_{00}^p$, the continuation payoff from the candidate *MPE* strategies is $F^*(0,0,y_{00}^p)=L(0,0,y_{00}^p)$ as for all possible alternatives. Last, the right partial derivative

[10] If the first investor can increase its rent by investing in a second unit, that is if $\Delta L(1,0,y)=\frac{\pi_{20}-\pi_{10}}{r-\alpha}y-I-\left(\frac{y}{y_{01}^*}\right)^{\beta}\frac{\pi_{20}-\pi_{10}}{r-\alpha}y$ is positive on some interval $\left[\underline{y_{10}},\overline{y_{10}}\right]$, then the *MPE* strategy profile must also specify $s_1^f(1,0,y)=\begin{cases}1, \ \underline{y_{10}}\leq y<\overline{y_{10}}\\ 0, \text{ otherwise}\end{cases}$, where $\underline{y_{10}}\leq\overline{y_{10}}<y_{01}^*$. It is tedious, but not difficult, to also work out the corresponding value of y_{00}^p, which is lower since the rent of the first investor would otherwise exceed that of its opponent. We leave it to interested readers to adapt the foregoing proof to such cases where it might be profitable for the first investor to invest more than once before its opponent does.

$\partial_y^+ s_1^f(0,0,y'')$ is strictly positive as required by regularity condition (R_2) in Boyer *et al.* For the proof that there is no other equilibrium outcome, we refer the reader to Fudenberg and Tirole (1985, Appendix 1). ■

Proof of Proposition 3. See the main text. ■

Proof of Proposition 4. For each $y \in (0, y_{12}^*]$, $F^*(1,1,y)$, $L(1,1,y)$, and $\widetilde{S}(1,1,y) = \frac{\pi_{22}}{r-\alpha}y - I$ are respectively the expected payoffs of becoming the first investor, the second investor, and of investing immediately, simultaneously with the other firm. As in the proof of Proposition 1, it can be shown that the strategy profile defined below is an *MPE* strategy profile:

$$
\begin{aligned}
s_1^f(1,1,y) &= s_1^{-f}(1,1,y) = \begin{cases} 0, \ y \in [0, y_{11}^p) \\ \frac{L(1,1,y)-F^*(1,1,y)}{L(1,1,y)-\widetilde{S}(1,1,y)}, \ y \in [y_{11}^p, y_{12}^*) \\ 1, \ y \in [y_{12}^*, \infty) \end{cases} . \\
s_1^f(1,2,y) &= s_1^{-f}(1,2,y) = \begin{cases} 0, \ y \in [0, y_{12}^*) \\ 1, \ y \in [y_{12}^*, \infty) \end{cases} \\
s_1^f(2,2,y) &= s_1^{-f}(2,2,y) = 0 \ \forall y
\end{aligned}
$$
■

Proof of Proposition 5. *1.* Necessity is proven in the main text. To show sufficiency, let $L(1,1,y) \leq S^*(1,1,y)$ $\forall y \in (0, y_{12}^*]$. By the definition of $S^*(1,1,y)$, one also has $L(1,1,y) \leq S^*(1,1,y)$ $\forall y \in (0, y_{11}^{s*}]$ with $y_{11}^{s*} > y_{12}^*$ by (13). We will show that the following (tacit collusion) strategies, whose equilibrium payoff is $S^*(1,1,y)$ for both firms, yield an *MPE*:

$$
\begin{aligned}
s_1^f(1,1,y) &= s_1^{-f}(1,1,y) = \begin{cases} 0, \ y \in [0, y_{11}^{s*}) \\ 1, \ y \in [y_{11}^{s*}, \infty) \end{cases} , \\
s_1^f(1,2,y) &= s_1^{-f}(1,2,y) = \begin{cases} 0, \ y \in [0, y_{12}^*) \\ 1, \ y \in [y_{12}^*, \infty) \end{cases} .
\end{aligned}
$$

For either firm, say f, a deviation from $s_1^f(1,1,y)$ either results in an investment after y_{11}^{s*} is reached, or in an investment before y_{11}^{s*} is reached. In the former instance, since $-f$ has already invested when f invests, the payoff is $F(1,2,y) < F^*(1,2,y) \leq S^*(1,1,y)$, where the last inequality follows from the fact that $y_{11}^s = y_{12}^*$ is admissible in the maximization that defines $S^*(1,1,y)$. If the deviation results in an investment by f before y_{11}^{s*} is reached, then $-f$ applies $s_1^{-f}(1,2,y)$. The payoff to f is $L(1,1,y)$ if the deviation occurs before y_{12}^* is reached and $S(1,1,y)$ if it occurs at or after y_{12}^* (since in that case $-f$ invests immediately). Since $S(1,1,y) \leq S^*(1,1,y)$, the above strategies yield an *MPE* with joint investment at y_{11}^{s*}. This completes the proof that condition $L(1,1,y) \leq S^*(1,1,y)$ $\forall y < y_{12}^*$ is necessary for existence.

With respect to the existence of a *continuum* of tacit-collusion *MPE*, suppose now that $S(1,1,y) > L(2,1,y)$ for each $y < y_{12}^*$, and define $\underline{y}^s$ to be smallest value of $y_{11}^s \in [y_{12}^*, y_{11}^{s*})$ such that:

$$
\frac{\pi_{11}}{r-\alpha}y + \left(\frac{y}{y_{11}^s}\right)^{\beta} \left\{ \frac{\pi_{22}-\pi_{11}}{r-\alpha} y_{11}^s - I \right\} \geq L(2,1,y)
$$

for all $y \in [0, y_{12}^*]$. Then, for any $y_{11}^s \in [\underline{y}^s, y_{11}^{s*}]$, one can, as above, construct an *MPE* such that firms invest jointly at $\tau_{11}^s = \inf\{t \geq 0 \,|\, Y_t \geq y_{11}^s\}$. By definition of y_{11}^{s*}, the expected payoff from jointly investing at τ_{11}^s is an increasing function of the investment trigger y_{11}^{s} over the range $[\underline{y}^{s}, y_{11}^{s*}]$. It follows that these *MPE* are Pareto ranked, and that the Pareto optimal *MPE* corresponds to joint investment at τ_{11}^{s*}.

2. Rents are equal and exceed $F(1,2,y)$ by the definition of S. Since the firms act simultaneously, joint profits equal $2S^*(1,1,y)$ under joint investment at y_{11}^{s*}.

3. As explained in the text, when $\pi_{22} < \pi_{11}$, $y_{11}^{s*} \to \infty$; thus, firms never invest. Otherwise, the above strategy profile implies joint investment at y_{11}^{s*}. ■

Proof of Proposition 6. *3.* Assume that $\pi_{22} - \pi_{11} > 0$. By Proposition 5.1, a tacit-collusion equilibrium exists if and only if $S^*(1,1,y) - L(1,1,y)$ is positive for all $y < y_{12}^*$. Thus, we study the sign of:

$$E(y; I, \beta) \equiv S^*(1,1,y) - L(1,1,y) = -\frac{\pi_{21} - \pi_{11}}{r - \alpha} y + I + K(\beta) y^{\beta}$$

for $y \in [0, y_{12}^*]$ where, after substitution of the expressions for y_{12}^* and y_{11}^{s*},

$$K(\beta) = \left(\frac{\beta - 1}{\beta I}\right)^{\beta - 1} \left(\beta^{-1}\left(\frac{\pi_{22} - \pi_{11}}{(r - \alpha)}\right)^{\beta} + \left(\frac{\pi_{12} - \pi_{22}}{r - \alpha}\right)^{\beta}\left(\frac{\pi_{22} - \pi_{21}}{\pi_{22} - \pi_{12}}\right)\right).$$

The function E is strictly convex, strictly decreasing in a right neighborhood of zero, and $\lim_{y\to\infty} E(y; I, \beta) = \infty$. It follows that E attains its minimum at a unique point $y_E > 0$ that is characterized by the first-order condition:

$$\beta K(\beta) y_E^{\beta - 1} = \frac{\pi_{21} - \pi_{11}}{r - \alpha}. \tag{15}$$

Substituting in the expression for $E(y_E)$, it follows that the minimized value of E is:

$$E^*(\beta) \equiv \min_{y \geq 0} E(y; I, \beta) = (1 - \beta) K(\beta) y_E^{\beta} + I = I - \frac{\beta - 1}{\beta} \frac{\pi_{21} - \pi_{11}}{r - \alpha} y_E. \tag{16}$$

Changes in σ affect the function E only through β; β is a function that is strictly decreasing in σ and α (increasing in r) and that goes to 1 as $\sigma \to \infty$ and as $\alpha \uparrow r$ (as $r \downarrow \alpha$), with $\sigma \geq 0$ and $r > \alpha$. By the envelope theorem, $E^{*\prime}(\beta) < 0$. It follows that if $\beta < \beta'$ and $E^*(\beta') = 0$, then $E^*(\beta) > 0$, so that $E(y; I, \beta) > 0\ \forall y$. Consequently, $\Lambda(\beta') \subset \Lambda(\beta)$. This proves *3.*

2. Using (15) and (16), the condition for $E^*(\beta) \geq 0$ can be written as:

$$\frac{\pi_{21} - \pi_{11}}{\pi_{22} - \pi_{11}} \leq \left[1 + (\beta - 1)\frac{\pi_{22} - \pi_{11}}{\pi_{21} - \pi_{11}}\frac{\pi_{21} - \pi_{22}}{\pi_{22} - \pi_{12}}\left(\frac{\pi_{22} - \pi_{12}}{\pi_{22} - \pi_{11}}\right)^{\beta}\right]^{\frac{1}{\beta - 1}}.$$

This is independent of I.

1. We show existence by constructing an example. Let $\beta = 2$; the condition $E^*(2) \geq 0$ can be written, after some manipulations, as:

$$Q(x) = -x^2 + bx + c \geq 0, \tag{17}$$

where $b = \pi_{22} - \pi_{11}$, $c = (\pi_{21} - \pi_{22})(\pi_{22} - \pi_{12})$, and $x = \pi_{21} - \pi_{11}$. This quadratic expression is subject to features implied by the output competition model, first, under Assumption 1 on demand; and second, under Assumption 3 for the equal-capacity Case 2 under scrutiny. These features are: $\pi_{21} > \pi_{22} > \pi_{11} > \pi_{12} > 0$ and $\pi_{21} - \pi_{11} > \pi_{22} - \pi_{12}$, where the last inequality means that the rise in profit from increasing capacity from one to two units is higher when the opponent holds one unit than when it holds two units. Taking $\pi_{21} - \pi_{22} = 1$ as normalization, the conditions of the Cournot model are equivalent to:

$$x > 1\,; \quad x > c\,; \quad c > b\,; \; b > 0. \tag{18}$$

For values of x, b, and c satisfying conditions (18), $Q(x) \geq 0$ if and only if x is smaller than or equal to the positive root of $Q(x)$, which is equal to $\frac{1}{2}\left(b + \sqrt{b^2 + 4c}\right)$. This is possible if and only if the positive root is greater than both c and 1, or:

$$b \geq \max\{1 - c,\ c - 1\}.$$

Existence of the tacit-collusion *MPE* when $\beta = 2$ is therefore ensured when, in addition to the regular features arising from the Cournot model and under the normalization $\pi_{21} - \pi_{22} = 1$,

$$\pi_{22} - \pi_{11} \geq \max\{1 - (\pi_{22} - \pi_{12}),\ (\pi_{22} - \pi_{12}) - 1\}.$$

For example, if $\pi_{21} = 1$, $\pi_{22} = \frac{5}{6}$, $\pi_{12} = \frac{3}{6}$, $\pi_{11} = \frac{3.5}{6}$, then $\pi_{21} - \pi_{22} = \frac{1}{6}$; normalizing requires multiplying all those values by 6; then $\pi_{22} - \pi_{11} = \frac{9}{6} > \pi_{22} - \pi_{12} - 1 = 1$. ■

Proof of Lemma 2. We need to show that, if the gain $G(2,1,y)$ from investing immediately for the bigger firm is positive at some value of $y < y_{13}^*$, then there exists a value of y, y_{12}^p such that $G(2,1,y) < 0\ \forall\ y < y_{12}^p$. The gain from investing immediately for the bigger firm is

$$G(2,1,y) \equiv L(2,1,y) - F^*(2,1,y), \tag{19}$$

with

$$F^*(2,1,y) = \begin{cases} S^*(2,2,y) \text{ if a collusion episode occurs at } (2,2,y), \\ \frac{\pi_{22}}{r-\alpha} y + \left(\frac{y}{y_{22}^p}\right)^\beta \left(L(2,2,y_{22}^p) - \frac{\pi_{22}}{r-\alpha} y_{22}^p\right) \text{ in case of preemption at } (2,2,y), \end{cases} \tag{20}$$

where $S^*(2,2,y)$ and $L(2,2,y)$ correspond to the tacit-collusion and the preemption equilibria analyzed in subsection 3.2 (for $k' = 2$), respectively given by (12) taken at the joint-profit maximizing trigger (13) and by (11).

In order to compute $L(2,1,y)$, first note that, as an implication of Assumption 4, if the bigger firm invests first, it will then have to accommodate whenever the smaller firm introduces a new unit. Consequently, the smaller firm's dominant policy in that case is to acquire two units successively at its stand-alone trigger values:

$$F^*(1,2,y) = \sup_{y_{13},\, y_{23}} \left[\frac{\pi_{13}}{r-\alpha} y + \left(\frac{y}{y_{13}}\right)^\beta \left(\frac{\pi_{23} - \pi_{13}}{r-\alpha} y_{13} - I\right) + \left(\frac{y}{y_{23}}\right)^\beta \left(\frac{\pi_{33} - \pi_{23}}{r-\alpha} y_{23} - I\right)\right], \tag{21}$$

where $y_{13}^* = \frac{1}{\pi_{23}-\pi_{13}}(r-\alpha) I \frac{\beta}{\beta-1}$, and $y_{23}^* = \frac{1}{\pi_{33}-\pi_{23}}(r-\alpha) I \frac{\beta}{\beta-1}$ are the corresponding investment triggers. Given the dominant policy of the smaller firm when the bigger firm invests first, the value of the latter, if it purchases its third unit at $Y_t = y$ when the small firm holds one unit, is:

$$L(2,1,y) = \frac{\pi_{31}}{r-\alpha} y - I + \left(\frac{y}{y_{13}^*}\right)^\beta \frac{\pi_{32} - \pi_{31}}{r-\alpha} y_{13}^* + \left(\frac{y}{y_{23}^*}\right)^\beta \frac{\pi_{33} - \pi_{32}}{r-\alpha} y_{23}^*. \tag{22}$$

Use (22) and (20) to compute $G(2,1,y)$ from (19) and use (12) and (13) to express $S^*(2,2,y)$ in the resulting expression. Whatever the expression applying in (20), it can be verified that $G(2,1,y)$ is concave and that it is increasing in a right-neighborhood of 0; also, $G(2,1,0) = -I$. Consequently, if $G(2,1,0)$ reaches a strictly positive value for some $y < y_{13}^*$, then there exists at least one value of y in the interval $[0, y_{13}^*]$, such that $G(2,1,y) = 0$. We define y_{12}^p as the smallest root and note that $G(2,1,y_{22}^p)$ is increasing, which proves the last statement in the Lemma. ■

Proof of Proposition 7. The case $0 < x^c - k' \leq 1$ $(k' = 1)$ is discussed in the main text; we focus on $1 < x^c - k' \leq 2$ $(k' = 1)$ in this proof.

1 and 2. The proof relies on comparing the gain $G(1,2,y)$ for the smaller firm to invest immediately with the gain $G(2,1,y)$ for the bigger firm to do so. $G(2,1,y)$ is given by (19) as described in the proof of Lemma **2**; $G(1,2,y)$ will be established further below.

Preliminaries.

Trivially, if $G(2,1,y) \leq 0 \ \forall \ y \leq y_{13}^*$, investing when its opponent holds one unit is a dominated strategy for the bigger firm. Then the result holds, with the smaller firm investing at its stand-alone date, *i.e.* when y reaches y_{12}^* for the first time. We assume that $G(2,1,y) > 0$ for some values of $y \leq y_{13}^*$ in the rest of the proof.

The gain from investing immediately for the smaller firm is

$$G(1,2,y) \equiv L(1,2,y) - F^*(1,2,y) \tag{23}$$

where $F^*(1,2,y)$ is given by (21). $L(1,2,y)$ corresponds to the smaller firm investing first and is given by:

$$L(1,2,y) = \begin{cases} S^*(2,2,y) - I \text{ if a collusion episode occurs at } (2,2,y), \\ \frac{\pi_{22}}{r-\alpha} y + \left(\frac{y}{y_{22}^p}\right)^\beta L(2,2,y_{22}^p) - I \quad \text{in case of preemption at } (2,2,y), \end{cases} \tag{24}$$

where $S^*(2,2,y)$ and $L(2,2,y)$ apply in case of tacit-collusion and preemption at capacities $(2,2)$ respectively, as analyzed in subsection 3.2 (for $k' = 2$). They are respectively given by (12) taken at the joint-profit maximizing trigger (13) and by (11).

Since tacit collusion yields a higher payoff than preemption at $(2,2)$ by Proposition 5, it follows that the gain for the smaller firm to invest immediately, if the alternative is the bigger firm taking the lead, is at least equal to the gain in case the preemption episode occurs:

$$\begin{aligned} G(1,2,y) \quad &\geq \quad -I + \frac{\pi_{22}}{r-\alpha} y + \left(\frac{y}{y_{22}^p}\right)^\beta \left(L(2,2,y_{22}^p) - \frac{\pi_{22}}{r-\alpha} y_{22}^p\right) \\ &- \sup_{y_{13},\, y_{23}} \left[\frac{\pi_{13}}{r-\alpha} y + \left(\frac{y}{y_{13}}\right)^\beta \left(\frac{\pi_{23} - \pi_{13}}{r-\alpha} y_{13} - I\right) + \left(\frac{y}{y_{23}}\right)^\beta \left(\frac{\pi_{33} - \pi_{23}}{r-\alpha} y_{23} - I\right)\right] \end{aligned} \tag{25}$$

Comparing the gains from investing immediately rather than waiting for the smaller versus the bigger firm. An inequality similar to (25) , but in the opposite direction, can be obtained in the case of $G(2,1,y)$ by choosing the lower alternative in (20). Substracting it from (25), we have, for $y \le y_{13}^*$,

$$\begin{aligned}
&G(1,2,y) - G(2,1,y) \ge -I \\
&+ \left\{ \frac{\pi_{22}}{r-\alpha} y + \left(\frac{y}{y_{22}^p} \right)^\beta \left(L(2,2,y_{22}^p) - \frac{\pi_{22}}{r-\alpha} y_{22}^p \right) \right\} \\
&- \frac{\pi_{13}}{r-\alpha} y - \left(\frac{y}{y_{13}^*} \right)^\beta \left(\frac{\pi_{23}-\pi_{13}}{r-\alpha} y_{13}^* - I \right) - \left(\frac{y}{y_{23}^*} \right)^\beta \left(\frac{\pi_{33}-\pi_{23}}{r-\alpha} y_{23}^* - I \right) \\
&- \frac{\pi_{31}}{r-\alpha} y + I - \left(\frac{y}{y_{13}^*} \right)^\beta \left(\frac{\pi_{32}-\pi_{31}}{r-\alpha} y_{13}^* \right) - \left(\frac{y}{y_{23}^*} \right)^\beta \frac{\pi_{33}-\pi_{32}}{r-\alpha} y_{23}^* \\
&+ \left\{ \frac{\pi_{22}}{r-\alpha} y + \left(\frac{y}{y_{22}^p} \right)^\beta \left(L(2,2,y_{22}^p) - \frac{\pi_{22}}{r-\alpha} y_{22}^p \right) \right\}.
\end{aligned}$$

Substituting for $L(2,2,y_{22}^p) = \frac{\pi_{32}}{r-\alpha} y_{22}^p - I + \left(\frac{y}{y_{23}^*} \right)^\beta \frac{\pi_{33}-\pi_{32}}{r-\alpha} y_{23}^*$ and simplifying gives

$$\begin{aligned}
G(1,2,y) - G(2,1,y) \ge{}& 2 \left(\frac{y}{y_{22}^p} \right)^\beta \frac{\pi_{32}-\pi_{22}}{r-\alpha} y_{22}^p \\
& - \left(\frac{y}{y_{13}^*} \right)^\beta \left(\frac{\pi_{23}-\pi_{13}-\pi_{32}+\pi_{31}}{r-\alpha} y_{13}^* - I \right) - \left(\frac{y}{y_{23}^*} \right)^\beta \left(\frac{\pi_{32}-\pi_{23}}{r-\alpha} y_{23}^* - I \right).
\end{aligned}$$

Evaluating the π_{ij} as price times quantity where this helps sign an expression, with p_l defined as the industry price when there are $l = i + j$ capacity units in the industry, this can be written as:

$$\begin{aligned}
&G(1,2,y) - G(2,1,y) \ge \\
&\left(\frac{y}{y_{22}^p} \right)^\beta \left[\frac{6p_5 - 4p_4}{r-\alpha} y_{22}^p - 2I - \left(\frac{y_{22}^p}{y_{13}^*} \right)^\beta \left(\frac{5p_5 - 4p_4}{r-\alpha} y_{13}^* - I \right) - \left(\frac{y_{22}^p}{y_{23}^*} \right)^\beta \left(\frac{p_5}{r-\alpha} y_{23}^* - I \right) \right]
\end{aligned}$$

This shows that the difference in gains is higher than the product of the positive function $\left(\frac{y}{y_{22}^p} \right)^\beta$ by a function δ of three variables evaluated respectively at the trigger values y_{22}^p, y_{13}^*, and y_{23}^*:

$$\begin{aligned}
&\delta(y_{22},\ y_{13}, y_{23}) \\
={}& \frac{6p_5 - 4p_4}{r-\alpha} y_{22} - 2I - \left(\frac{y_{22}}{y_{13}} \right)^\beta \left(\frac{5p_5 - 4p_4}{r-\alpha} y_{13} - I \right) - \left(\frac{y_{22}}{y_{23}} \right)^\beta \left(\frac{p_5}{r-\alpha} y_{23} - I \right)
\end{aligned}$$

We are going to show that $\delta(y_{22}^p,\ y_{13}^*, y_{23}^*)$ is positive. Given $y_{13} = y_{13}^*$, let us treat y_{22} and y_{23} as variables and let us note that $\delta(y_{22},\ y_{13}, y_{23})$ is zero if $y_{13}^* = y_{23} = y_{22}$. First we show that $y_{13}^* < y_{22}^p < y_{23}^*$. Because y_{22}^p is a preemption trigger, it is by definition lower than the corresponding stand-alone trigger $y_{22}^* = \arg\max_{y_{22}} \left\{ \frac{\pi_{22}}{r-\alpha} y + \left(\frac{y}{y_{22}} \right)^\beta \left(\frac{\pi_{32}-\pi_{22}}{r-\alpha} y_{22} - I \right) + \left(\frac{y}{y_{32}} \right)^\beta \frac{\pi_{33}-\pi_{32}}{r-\alpha} y_{32} \right\} = \frac{1}{\pi_{32}-\pi_{22}} (r-\alpha) I \frac{\beta}{\beta-1}$. The inequality $y_{22}^p < y_{23}^*$ follows from the observation that $y_{22}^* < y_{23}^*$. The inequality $y_{13}^* < y_{22}^p$ stems from the following argument. Suppose the investor at y_{22}^p ignored the cannibalism effect, the impact of introducing a new capacity unit on the revenue from its existing unit, effectively treating its existing unit as an independent existing firm. This is feasible and the corresponding payoff is thus reachable. In fact it corresponds to the problem that defines y^*_{13}. Consequently $y^*_{13} \leq y^p_{22}$; equality would occur if the preemption episode at y^p_{22} did not leave any rent to the first investor. However, by Proposition 2, rents are equalized but strictly positive; this requires $y^*_{13} < y^p_{22}$.

Second we note that $\frac{\partial \delta}{\partial y_{22}} > 0$ and we show $\frac{\partial \delta}{\partial y_{23}} > 0$ for $y_{23} \in [y^*_{13}, y^*_{23}]$, which implies, given that $\delta\left(y^*_{13},\ y^*_{13},\ y^*_{13}\right) = 0$ and given $y^*_{13} < y^p_{22} < y^*_{23}$, that $\delta\left(y^p_{22},\ y^*_{13}, y^*_{23}\right) > 0$. Standard computations show that $\frac{\partial \delta\left(y^p_{22},\ y^*_{13}, y^*_{23}\right)}{\partial y_{23}} = (y_{22})^{\beta}\left[(\beta - 1)\frac{p_5}{r-\alpha}\left(y^*_{23}\right)^{-\beta} - \beta\left(y^*_{23}\right)^{-(\beta+1)}\right]$. Substituting y^*_{23}, with $\pi_{23} = 2p_5$ and $\pi_{33} = 3p_6$, it follows that $\frac{\partial \delta\left(y^p_{22},\ y^*_{13}, y^*_{23}\right)}{\partial y_{23}}$ is proportional to $\left(\frac{p_5}{3p_6 - 2p_5} - 1\right)$ which is positive since $p_5 > p_6$. Thus $\frac{\partial \delta}{\partial y_{23}} > 0$ for $y_{23} = y^*_{23}$ and for any lower value of y_{23}. This completes the proof that $G(1,2,y) - G(2,1,y) > 0 \ \ \forall y < y^*_{13}$.

Preemption in MPE. At levels of y such that $G(2,1,y) < 0$, investing is a dominated strategy for the bigger firm, and the best response for the smaller firm is to wait until $G(1,2,y)$ reaches a maximum if that maximum is reached when $G(2,1,y) < 0$. For any y such that $G(2,1,y) \geq 0$, $G(1,2,y) > 0$, so the best response for the small firm to a strategy by the bigger firm of investing at such level of Y_t is to preempt at $y - \varepsilon$. Precisely, by Lemma 2, $G\left(2,1,y^p_{12}\right) = 0$. Since $G(1,2,y) - G(2,1,y) > 0$, it follows that $G\left(1,2,y^p_{12}\right) > 0$. Then, the smaller firm should invest at y^p_{12} which is achieved in equilibrium for the following strategies:

$$s_1(1,2,y) = \begin{cases} 0, & \text{if } y \in [0, y^p_{12}) \\ 1, & \text{if } y \in [y^p_{12}, \infty) \end{cases} \ ; \ s_1(2,1,y) = \begin{cases} 0, & \text{if } y \in [0, y^p_{12}) \\ \frac{L(2,1,y) - F^*(2,1,y)}{L(2,1,y) - S(2,1,y)}, & \text{if } y \in [y^p_{12}, y^*_{12}) \\ 1, & \text{if } y \in [y^*_{12}, \infty) \end{cases} .$$

By Lemma 2, $G\left(2,1,y^p_{12}\right) = 0$. Since $G(1,2,y) - G(2,1,y) > 0$, it follows that $G\left(1,2,y^p_{12}\right) > 0$. Then, the smaller firm should invest at y^p_{12} which is achieved in equilibrium for the following strategies:

$$s_1(1,2,y) = \begin{cases} 0, & \text{if } y \in [0, y^p_{12}) \\ 1, & \text{if } y \in [y^p_{12}, \infty) \end{cases} \ ; \ s_1(2,1,y) = \begin{cases} 0, & \text{if } y \in [0, y^p_{12}) \\ \frac{L(2,1,y) - F^*(2,1,y)}{L(2,1,y) - S(2,1,y)}, & \text{if } y \in [y^p_{12}, y^*_{12}) \\ 1, & \text{if } y \in [y^*_{12}, \infty) \end{cases} .$$

Note that the smaller firm invests first with probability one. Consequently, a preemption episode at (k, k') with the bigger firm as first investor cannot exist in *MPE.*

3 can be readily verified. ■

Proof of Proposition 8. Under Assumption 4 with $k' = 1$:

1. If $\pi_{32} - \pi_{22} = 0$, there exists no value of Y_t at which it is profitable for the bigger firm to invest if the smaller does so; thus, there exists no tacit-collusion MPE with simultaneous investment. Since $\pi_{22} > \pi_{12}$, abstaining from investing is a dominated strategy for the smaller firm; thus, there exists no tacit-collusion MPE by inaction.

2. $\pi_{32} - \pi_{22} > 0$. The sole alternative to the tacit-collusion MPE, if it exists, is the preemption MPE. The proof is similar to that of Proposition 5 so we only introduce the main elements. By Proposition 7, for the bigger firm, the alternative to tacit collusion is to be passive in the preemption MPE; for the smaller firm, the alternative to tacit collusion is to be first investor in the preemption MPE. Consequently, adapting Proposition 5, collusion is an MPE if and only if $S(2,1,y) - F^*(2,1,y) \geq 0$ and $S(1,2,y) - L(1,2,y) \geq 0$ for all $y \leq y_{21}^s$, where y_{21}^s is the threshold at which both firms invest simultaneously. We compute these gains from tacit collusion.

First, we evaluate $S(2,1,y)$ and $S(1,2,y)$. Since $\pi_{32} - \pi_{22} > 0$, $2 < x^c \leq 3$, so that a capacity of three units is necessary to produce the unconstrained Cournot output. In case of simultaneous investment, both firms acquire one unit at some common trigger y_{21}^s to be defined. Then the bigger firm holds three units and must accommodate any increase in production up to x^c by the smaller firm. Thus, it is a dominant strategy for the latter to acquire a third unit at its stand-alone threshold y_{23}^*, if $y_{21}^s \leq y_{23}^*$, or at $y_{23} = y_{21}^s$ if $y_{21}^s > y_{23}^*$. Once both firms hold three units each, the game is over by Proposition 1(A). Thus, the tacit-collusion equilibrium, if it exists, involves simultaneous investment at y_{21}^s, followed, possibly immediately, by an investment by the smaller firm. The corresponding values for the bigger and the smaller firms are respectively:

$$
\begin{aligned}
S(2,1,y) &= \frac{\pi_{21}}{r-\alpha}y + \left(\frac{y}{y_{21}^s}\right)^\beta \left(\frac{\pi_{32}-\pi_{21}}{r-\alpha}y_{21}^s - I\right) + \left(\frac{y}{y_{23}}\right)^\beta \frac{\pi_{33}-\pi_{32}}{r-\alpha}y_{23} \\
S(1,2,y) &= \frac{\pi_{12}}{r-\alpha}y + \left(\frac{y}{y_{21}^s}\right)^\beta \left(\frac{\pi_{23}-\pi_{12}}{r-\alpha}y_{21}^s - I\right) + \left(\frac{y}{y_{23}}\right)^\beta \left(\frac{\pi_{33}-\pi_{23}}{r-\alpha}y_{23} - I\right),
\end{aligned}
$$

where either $y_{23} = y_{23}^* > y_{21}^s$ or $y_{21}^s = y_{23} \geq y_{23}^*$.[11] We now evaluate the gain from colluding for the bigger firm, over its alternative of letting the smaller firm invest first, using (20) for $F^*(2,1,y)$:

$$
\begin{aligned}
GS(2,1,y) &= S(2,1,y) - F^*(2,1,y) \\
&= \frac{\pi_{21}}{r-\alpha}y + \left(\frac{y}{y_{21}^s}\right)^\beta \left(\frac{\pi_{32}-\pi_{21}}{r-\alpha}y_{21}^s - I\right) + \left(\frac{y}{y_{23}}\right)^\beta \frac{\pi_{33}-\pi_{32}}{r-\alpha}y_{23} \\
&\quad - \max\left\{S^*(2,2,y), \frac{\pi_{22}}{r-\alpha}y + \left(\frac{y}{y_{22}^p}\right)^\beta \left(L(2,2,y_{22}^p) - \frac{\pi_{22}}{r-\alpha}y_{22}^p\right)\right\}.
\end{aligned}
$$

Similarly, we evaluate the gain from colluding for the smaller firm, over its alternative of investing first, using (24) for $L(1,2,y)$:

$$
\begin{aligned}
GS(1,2,y) &= S(1,2,y) - L(1,2,y) \\
&= \frac{\pi_{12}}{r-\alpha}y + \left(\frac{y}{y_{21}^s}\right)^\beta \left(\frac{\pi_{23}-\pi_{12}}{r-\alpha}y_{21}^s - I\right) + \left(\frac{y}{y_{23}}\right)^\beta \left(\frac{\pi_{33}-\pi_{23}}{r-\alpha}y_{23} - I\right) \\
&\quad + I - \max\left\{S^*(2,2,y), \frac{\pi_{22}}{r-\alpha}y + \left(\frac{y}{y_{22}^p}\right)^\beta \left(L(2,2,y_{22}^p) - \frac{\pi_{22}}{r-\alpha}y_{22}^p\right)\right\}.
\end{aligned}
$$

Let y_{12}^{s*} and y_{21}^{s*} be the values of y that maximize $S(1,2,y)$ and $S(2,1,y)$ respectively with respect to y_{21}^s. That is, $y_{12}^{s*} = \frac{1}{\pi_{23}-\pi_{12}}(r-\alpha) I \frac{\beta}{\beta-1}$; $y_{21}^{s*} = \frac{1}{\pi_{32}-\pi_{21}}(r-\alpha) I \frac{\beta}{\beta-1}$. Note that $y_{12}^{s*} < y_{21}^{s*}$. Consider y_{12}^{s*} and y_{21}^{s*} as possible triggers in a tacit-collusion equilibrium; since $S(1,2,y)$ is decreasing in y beyond its maximum, it is a dominant strategy for the smaller firm to invest when $y \geq y_{12}^{s*}$. Thus, in *MPE*, $y_{12}^s \leq y_{12}^{s*}$ and simultaneous investment at y_{12}^{s*} yields a higher payoff to both firms than at $y_{12}^s < y_{12}^{s*}$. This equilibrium exists if and only if both $S(1,2,y) - L(1,2,y)$ and $S(2,1,y) - F^*(2,1,y)$ are nonnegative for any $y \leq y_{12}^{s*} = y_{23}$. The rest of the proof of *2*, about parameter conditions, is otherwise similar to that of Proposition 6.

[11] We take the case $y_{23} = y_{21}^s$ corresponding to situations where $y_{23}^* < y_{21}^s$: the second investment of the smaller firm occurs later under the tacit-collusion trigger than under the stand-alone trigger y_{23}^* because the smaller firm delays its first investment beyond y_{23}^* in order to collude. The approach is identical for the alternative case and leads to the same implications.

3. Let us show that the value that maximizes $S(1,2,y) + S(2,1,y)$ is higher than $\min(y_{12}^{s*}, y_{21}^{s*})$. Since y_{12}^{s*} maximizes $S(1,2,y)$, $\frac{\partial S(1,2,y_{12}^{s*})}{\partial y} = 0$ while, since $y_{12}^{s} < y_{12}^{s*}$, $S(2,1,y)$ is rising in y at $y = y_{12}^{s*}$. It follows that $S(1,2,y_{12}^{s*}) + S(2,1,y_{12}^{s*})$ is rising in y, so that its maximum at some value y_{12}^{s**} strictly above y_{12}^{s*}. However, at any $y > y_{12}^{s*}$, it is a dominant strategy for the smaller firm to invest, so that joint investment at y_{12}^{s**} cannot occur in *MPE*. ■

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