Partial regulation
in vertically differentiated industries[*]

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Abstract

We provide theoretical foundations for quality-adjusted price-cap regulation in industries where a regulated incumbent and an unregulated entrant offer vertically differentiated products competing in price and quality. We show that, whether or not the incumbent anticipates the reaction of the entrant, the optimal weights in the cap depend upon the market served by the entrant, despite the latter not being directly concerned by regulation. We further show that the cap is robust to small errors in the weights. Our findings point to the conclusion that, in partially regulated industries, regulators should use information about the whole sectors rather than on the sole regulated incumbents.

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1 Introduction

In markets where not only price (a monetary dimension) but also quality (a non-monetary dimension) matters, pure price regulation, in general, does not yield overall desirable outcomes (see, for instance, Armstrong and Sappington [3] and Sappington [30]).

Specifically, when firms are compelled to obey a price cap, they are induced to cut costs, which may translate into quality under-provision. This issue regards potentially all network industries in which a price cap is adopted. With reference to telecommunications, Vogelsang [32] observes that concerns about quality deterioration are widespread and that, indeed, such services are subject to price-cap regulation in most OECD countries and in several others. In fact, Rovizzi and Thompson [29] report that after privatization, a noticeable quality reduction was registered in British Telecom's services as soon as the company was submitted to a price cap. According to Crew and Parker [17], among the various quality aspects that might suffer from price ceiling, most penalized seems to be service reliability, which is a crucial part of service value to end users.[1]

De Fraja and Iozzi [19] look for a way to use a price cap in environments with relevant quality aspects, under the motivation that:

> "Price-cap regulation (...) strikes a very good compromise between the theoretically rigorous foundation of the theory of optimal regulation (...) and the practitioner's requirement of the simple, easy-to-understand, easy-to-apply rule."

They integrate quality dimensions into the "standard" price cap, restructuring the latter as a *price-and-quality cap*. Characterizing the ideal composition of this incentive scheme with regard to multi-product monopolies, they find two essential results. The first result is that, in the index that enters the formula, the appropriate weights of the different prices are (proportional to) the optimal quantities of the products sold by the regulated firm. In other words, this cap does not differ from the "standard" price cap, where quality is not an issue[2] (Brennan [12]; Laffont and Tirole [24]). The second result in De Fraja and Iozzi [**19**] is that quality weights in the "extended" cap should be equal to consumer marginal surplus evaluated at the optimal prices and qualities. Billette de Villemeur [8] obtains similar findings with reference to monopolistic airline industries, where relevant dimensions are price and service frequency.[3]

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[1]Service reliability introduces an element of heterogeneity even in electricity, a product that is otherwise perfectly homogeneous. Specifically, in power sectors, reaction lags and supply interruptions are relevant quality dimensions (compare Crampes and Moreaux [14]).

[2]A standard price cap may find specifications according to the context. For instance, Billette de Villemeur, Cremer, Roy and Toledano [9] characterize a price-cap scheme that fits postal sector features.

[3]Under a monopoly (though not necessarily in other frameworks), service frequency is equivalent to a pure quality attribute.

These findings do not need to extend to vertically differentiated oligopolies in which regulation concerns one sole firm. With regard to non-monopolistic sectors, it is known that if a regulated incumbent competes with an unregulated passive fringe, then total market quantities are the optimal weights in the pure price cap only if fringe profits are *not* included in social welfare. By contrast, if fringe profits are taken to contribute to social welfare, then appropriate weights relate to the optimal quantities of the sole regulated firm. These results are found by Brennan [12], who further acknowledges that when competitors are *not* price-takers, a different recipe is required. Yet, the author does not formally provide such a recipe, even just for a pure price cap.

In the present article, we provide theoretical foundations to the price-and-quality cap regulation[4] of oligopolies where a regulated incumbent competes in price and quality with one (or few) strategic rival(s), who operate(s) unregulated. By doing so, we extend the literature about price-and-quality regulation through cap schemes which has focused on monopolistic markets so far, to oligopolistic settings as well.

The framework we consider, namely an oligopoly where the sole incumbent is regulated, closely reflects the most common outcome of the liberalization process recently undergone by many industries that were previously fully regulated. Typically, in those sectors, the former monopolist is subject to regulatory control, whereas the (few) competitors that have entered after liberalization operate unregulated, despite exerting market power in order to gain profits. Our model is meant to stylize concentrated and partially regulated industries of this sort. For example, one may think about competition between regulated telephone companies and unregulated cable voice-over-Internet-Protocol, or wireless cellular companies in voice telephony. One may further consider competition between the regulated and unregulated cable television services. Actually, the model may equally well represent competition across asymmetrically regulated industries. One instance is inter-modal competition between regulated train companies and deregulated air carriers in European transport industries.

To capture the relevance of quality provision and motivate quality regulation, we represent a market where vertically differentiated services are supplied to consumers exhibiting heterogeneous quality valuations. Our choice to model vertical (rather than horizontal) differentiation follows from the observation that in the industrial contexts we refer to, consumers tend to share the same quality ranking, e.g., they perceive the product provided by the incumbent as superior to the product(s) offered by the competitor(s). For instance, in voice telephony, the services provided by telephone companies are generally more reliable than those provided by cable companies. Similarly, most of the time, air transport is considered to be more comfortable and reliable than rail transport. In turn, regulated cable television services typically broadcast higher quality programes and propose less advertising than unregulated competitors.

Importantly, for quality to be regulated, it must be both observable and verifiable. This

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[4]The effects of price ceilings on service quality has already been investigated both theoretically and empirically (see, for instance, Sappington [31] and Weisman [34]).

is actually the case in nearly all network industries. It is indeed possible to observe and collect data about connection interruptions in voice telephony, advertising frequency and program content in cable TV services, travel time and departure/arrival delays in transport services. This is why, in network industries, as the literature has pointed out, service quality is often heavily regulated, whereas infrastructure quality, which is hardly observable and verifiable, in general remains unregulated (see Martimort and Sand-Zantman [27], for instance). As observed by a referee, data collection can be expensive. However, real-world practice seems to suggest that it is worth its cost.[5]

Furthermore, it is frequently the case that the quality of the goods and/or services the utilities provide (though not the inner quality of the network infrastructures) can be adjusted in the short run. To reflect this circumstance, in our model we take quality to be as flexible as price. Yet, we also discuss the consequences of quality being a longer-run decision variable.[6]

We begin by characterizing the optimal price-and-quality pair to be decentralized by the regulator. In the presence of strategic rivals that remain unregulated, the relevant benchmark is given by the equilibrium price-and-quality pair of a market game in which a welfare-maximizing firm competes with one (or more) profit-maximizing rival(s), under the requirement that its profit be non-negative. From this perspective, our work is reminiscent of the mixed oligopoly models in which a public firm competes with one (or more) private operator(s) under the break-even constraint.[7] We characterize the price-and-quality equilibrium for two kinds of market games, namely a Stackelberg game, in which the incumbent/leader chooses price and quality anticipating the reaction of the entrant/follower, and a Nash-Cournot game, in which firms make choices simultaneously taking the rival's as given. In so doing, we account for the circumstance that the regulated firm may or may not enjoy a strategic advantage *vis-à-vis* the unregulated entrant, depending upon how competition, on one side, and regulation, on the other, affect its market position and commitment ability.[8]

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[5]To mention only a few examples, data about TV channels (types of broadcast, audience shares), advertising (spot duration, frequency) and programs (duration, content) are largely available. For instance, in the UK, the Office of Communications (Ofcom) systematically circulates such data through surveys and reviews (see also Carat [13] for information about EU countries). As for transport services, in the U.S., the Bureau of Transportation Statistics of the Research and Innovative Technology Administration provides detailed information about departure and arrival delays for a variety of transportation modes (aviation, maritime, highway, transit, rail). Similarly, in France, the Observatoire des retards du transport aérien, as managed by the Direction générale de l'aviation civile in cooperation with airlines and airports, collects and publishes data on flight punctuality. In Italy, the regulated rail company is currently compelled to disclose information about delays at arrival. Furthermore, delays are being increasingly monitored by consumers' associations and other concerned institutions (see, for instance, the report by Legambiente [26], based on Censis data, about the situation of Italian railways commuter transport).

[6]We thank an Associate Editor for bringing this case to our attention.

[7]Within the domain of literature about mixed oligopolies, Bös [11] reaches the conclusion that a public firm facing the requirement to operate at zero profit should stick to a *modified Ramsey-pricing rule*. In turn, exploring a homogeneous-product Stackelberg game with the public firm in the leader's position, Beato and Mas Colell [5] show that the solution to this game involves average-cost pricing for the public firm. The latter anticipates the competitor's policy choices, setting quantity so as to break even at equilibrium.

[8]On one hand, regulation may reinforce the commitment ability of the firm by limiting its operational flexibility. On the other hand, repeated revisions may progressively inhibit that ability. Subtle is the relationship between the incumbent's strategic behaviour and the regulator's possibility to pursue different

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Once we characterize the Stackelberg equilibrium and the Nash-Cournot equilibrium that constitute the two possible regulatory targets, we demonstrate how each of them can be decentralized by means of a properly structured price-and-quality cap targeted to the sole incumbent. Importantly, it turns out that the optimal weights to be attached to the latter's price and quality, have exactly the same composition, whatever the benchmark. These weights should be set, taking into account not only the market served by the incumbent, but also the market(s) covered by the unregulated competitor(s). It further emerges that, when the Stackelberg target is pursued, the unregulated market(s) should also be considered in order to tighten/relax the global ceiling. These findings implicate that, at the implementation stage, regulatory bodies of liberalized industries should not be restricted to access or use of the information about the solely regulated firms. Rather, they should be allowed to extract and make use of information about the overall industry. This provides a neat argument against the enforcement of norms that prevent regulators from basing their policies on information about unregulated markets and/or activities.

Still concerning implementation, we suggest that both the Stackelberg equilibrium and the Nash-Cournot equilibrium be progressively approached by applying the regulatory scheme iteratively over time, hinging on past data about market activities. We show that when this strategy is indeed followed in either of the relevant frameworks, the quality-adjusted price cap we propose exhibits the desirable property of being robust to small errors in the practical determination of the weights to be attached to the regulated firm's price and quality.

The remainder of the article is organized as follows. In Section 2, we present the framework. In Section 3, we characterize the regulatory benchmarks, i.e., the equilibrium price-and-quality pair of a Stackelberg and of a Nash-Cournot mixed duopoly. In Section 4, we show how either target can be decentralized by means of an appropriate price-and-quality cap. In Section 5, we discuss the generality of our results and provide a few concluding remarks. Most of the mathematical details are relegated to an Appendix.

2   The model

We consider an industry where two firms, namely an incumbent and an entrant, provide vertically differentiated products. The incumbent, denoted as I, is subject to regulation. The competitor, denoted as E, is not.

Firms' strategic variables are price and quality (respectively, p_k and q_k, $k = I, E$). We suppose that both operators choose their own price and quality simultaneously.[9] Further, we take price and quality to be observable and verifiable.

We explore two kinds of market game, namely the Stackelberg and the Nash-Cournot game. In the former, firm I, the leader, anticipates the impact of its own price-and-quality choice on the decision that firm E, the follower, will make. This framework closely

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(more or less ambitious) targets. This opens delicate issues that would deserve specific attention but are beyond the scope of the present work.

[9] See Section 5 for a discussion of the case in which firms choose quality before price.

represents situations in which the incumbent of a previously fully regulated industry enjoys a strategic advantage *vis-à-vis* the newcomer. In the Nash-Cournot game, each provider chooses its own price and quality taking the competitor's as given. This framework mirrors contexts in which, unlike in the previous ones, the incumbent lacks commitment ability *vis-à-vis* the unregulated competitor.

The goods that firms provide are perfect substitutes, except for the difference in quality. Consumers are heterogeneous in their valuation for quality, which is represented by a parameter θ. More precisely, sticking to a quasi-linear framework, we assume that the net surplus a consumer of characteristic θ derives from the consumption of x units of quality q bought at unit price p is written[10]

$$v_\theta\left(x, p, q\right) = u\left(x\right) - \left(p - \theta q\right) x, \tag{1}$$

with $u\left(x\right)$ increasing and concave in the argument. The parameter θ is distributed over the interval $\left[\underline{\theta}, \overline{\theta}\right]$, with $\overline{\theta} > \underline{\theta} \geq 0$, and according to a continuous density function $f\left(\theta\right)$. The associated cumulative distribution function is denoted as $F\left(\theta\right)$. Given her quality valuation, a $\theta-$consumer patronizing firm $k \in \{I, E\}$ faces the so-called generalized price $\widetilde{p}_k\left(\theta\right) \equiv \left(p_k - \theta q_k\right)$, that is the unit price p_k net of the benefits θq_k associated with product quality. A θ-consumer prefers to purchase the good from firm k, rather than from firm $j \neq k \in \{I, E\}$, whenever by doing so he/she bears a smaller generalized price $\left(\widetilde{p}_k\left(\theta\right) < \widetilde{p}_j\left(\theta\right)\right)$. Observe that, by construction, no consumer finds it beneficial to patronize both firms, as is usual in environments with vertical differentiation.

We suppose, without loss of generality, that firm k sells a product of higher quality at a higher price $\left(q_k > q_j, \ p_k > p_j\right)$. The marginal consumer, who is indifferent between the two operators, is characterized by the parameter value

$$\theta_m \equiv \frac{p_k - p_j}{q_k - q_j}. \tag{2}$$

Thus, individuals whose θ exceeds θ_m patronize firm k, whereas individuals whose θ is smaller than θ_m patronize firm j. We remove the possibility that some of the potential consumers abstain from making any purchase, an unlikely case for some "basic" services like telecommunications or daily transport services.

2.1 Consumer valuation of quality, demand and surplus

The demand of a $\theta-$consumer is pinned down by maximizing (1) with respect to x. This yields

$$\frac{\partial u}{\partial x} = \widetilde{p}_k\left(\theta\right), \tag{3}$$

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[10]In a more general formulation, one could allow the marginal valuation of quality to depend on the quality level, namely $\theta\left(q\right)$. However, imposing the restriction that $\theta\left(q\right) = \theta, \ \forall q$, does not affect the very nature of results, as long as variations in the quality level do not yield significant variations in the marginal valuation of quality.

where $\widetilde{p}_k(\theta) = \arg\min\{\widetilde{p}_I(\theta), \widetilde{p}_E(\theta)\}$. Individual consumption $x_k(p_k, q_k; \theta)$ appears to be a function of the sole generalized price, $\widetilde{p}_k(\theta)$, of the consumed commodity. As $\widetilde{p}_k(\theta)$ decreases with θ, and provided $\widetilde{p}_k(\theta_m) = \widetilde{p}_j(\theta_m)$, the ranking of consumers in terms of quality valuation is reflected in their ranking in terms of individual consumption x_θ. Formally, $x_{\theta_1} \leq x_{\theta_2}$ whenever $\theta_1 \leq \theta_2$.

Relying upon (3), it is possible to establish the relationship between the impacts on consumption of marginal changes in price and quality. To see this, observe first that (3) holds for any p_k and q_k. Differentiating both sides with respect to p_k and to q_k and combining the two equations, we obtain

$$\frac{\partial x_k / \partial q_k}{-\partial x_k / \partial p_k} = \theta, \quad \forall k \in \{I, E\}. \tag{4}$$

This evidences that, for the demand of a θ-consumer to remain unchanged as price p_k is increased by one unit, quality q_k should be raised by an amount equal to the individual marginal valuation for quality, namely θ. It also follows that a consumer with a strictly higher quality valuation patronizing the same firm, would consider as strictly beneficial an *increase* in (p_k, q_k) that leaves a θ-consumer indifferent. Conversely, a consumer with a strictly lower valuation would find it detrimental. Opposite appreciations would arise if a *decrease* in (p_k, q_k) that leaves a θ-consumer indifferent were considered.

Firms' aggregate demands are immediately obtained by summing over the relevant ranges of θ. Under the hypothesis that firm k serves high-valuation consumers and firm j serves low-valuation consumers, we formally have

$$X_j(\mathbf{p}, \mathbf{q}) = \int_{\underline{\theta}}^{\theta_m(\mathbf{p}, \mathbf{q})} x_j(p_j, q_j; \theta) f(\theta) \, d\theta, \tag{5a}$$

$$X_k(\mathbf{p}, \mathbf{q}) = \int_{\theta_m(\mathbf{p}, \mathbf{q})}^{\overline{\theta}} x_k(p_k, q_k; \theta) f(\theta) \, d\theta, \tag{5b}$$

where $\mathbf{p}$ and $\mathbf{q}$ denote the vector of prices and qualities respectively. Demands display the rather standard properties that we briefly recall hereafter. For any $k, j \in \{I, E\}$:

1. $(\partial X_k / \partial p_k) < 0$: demand for firm k's product decreases with its own price p_k;

2. $(\partial X_k / \partial q_k) > 0$: demand for firm k's product increases with its own quality q_k;

3. $(\partial X_k / \partial p_j) > 0$: demand for firm k's product increases with the rival price p_j;

4. $(\partial X_k / \partial q_j) < 0$: demand for firm k's product decreases with the rival quality q_j.

It is also straightforward to obtain aggregate consumer surplus as a function of prices and qualities. For this, we plug individual demands, as pinned down by (3), into the surplus function (1) and sum over the relevant ranges of θ. This ultimately returns

$$V(\mathbf{p}, \mathbf{q}) = \int_{\underline{\theta}}^{\theta_m(\mathbf{p}, \mathbf{q})} v_\theta(x_\theta, p_j, q_j) f(\theta) \, d\theta + \int_{\theta_m(\mathbf{p}, \mathbf{q})}^{\overline{\theta}} v_\theta(x_\theta, p_k, q_k) f(\theta) \, d\theta. \tag{6}$$

2.2 Technologies and profits

We denote as $C_k(X_k, q_k)$ the cost function of firm $k \in \{I, E\}$. This function is assumed to be continuous and increasing in both production level and quality. In formal terms $(\partial C_k / \partial X_k) > 0$ and $(\partial C_k / \partial q_k) > 0$, all $k \in \{I, E\}$. We further assume that $\lim_{q_k \to +\infty} C_k(X_k, q_k) = +\infty$. This says that high quality products are so costly to improve that perfect products ($q_k \to +\infty$) are never actually offered on the market. Finally, we assume that the firms never find it profitable to decrease the quality of their products down to zero. Taken together, these hypotheses ensure an interior solution to the determination of quality. Firm k's profit function is written

$$\pi_k(\mathbf{p}, \mathbf{q}) = p_k X_k - C_k(X_k, q_k), \quad \forall k \in \{I, E\}. \tag{7}$$

3 Characterization of the regulatory benchmark

As a first step of the analysis, we need to characterize the (constrained) optimal price-and-quality bundle, that the regulator should take as a target. This is the bundle that would arise at the equilibrium of a mixed duopoly where, under a non-negative profit constraint, a welfare-maximizing (public) firm were to compete with a profit-maximizing (private) firm. We characterize this bundle in the two contexts of our interest, namely the Stackelberg game and the Nash-Cournot game.

3.1 The profit-maximizing $(p_E, q_E)-$pair

We begin by exploring the price-and-quality choice of the profit-maximizing competitor. In either game, firm E takes the price and quality of firm I as given, and optimizes its own price and quality accordingly. Let $\varepsilon_E \equiv (p_E / X_E)(-\partial X_E / \partial p_E)$ be (the absolute value of) the demand elasticity with respect to its own price.

Lemma *The price-and-quality bundle that maximizes $\pi_E(\mathbf{p}, \mathbf{q})$ is characterized by the following pair of conditions:*

$$\frac{p_E - (\partial C_E / \partial X_E)}{p_E} = \frac{1}{\varepsilon_E}, \tag{8}$$

$$p_E \frac{\partial X_E}{\partial q_E} = \frac{\partial C_E}{\partial q_E} + \frac{\partial C_E}{\partial X_E} \frac{\partial X_E}{\partial q_E}. \tag{9}$$

Equation (8) is the standard Lerner formula. It shows that firm E acts as a monopolist *vis-à-vis* the "residual demand," X_E. Moreover, according to Equation (9), quality q_E is chosen so that marginal returns from quality improvements (the left-hand side) equate marginal costs (the right-hand side). The latter are expressed by the sum of the direct costs of quality (the first term) and its indirect costs (as reflected in the second term) that follow from the demand increments induced by quality raise.

Rearranging (9) and combining it with (8) yields

$$\frac{\partial X_E / \partial q_E}{-\partial X_E / \partial p_E} = \frac{1}{X_E} \frac{\partial C_E}{\partial q_E}. \tag{10}$$

Interestingly, by analogy with Equation (4), the ratio in the left-hand side of (10) can be interpreted as the aggregate marginal valuation of quality by firm E's clients. In turn, the right-hand side of (10) represents the average cost of a marginal increase in quality for this same firm. Although firm E is a profit maximizer, no distortion is introduced by the choices it makes in terms of quality. Given consumer valuation, further quality improvement would not appear to be worth its costs.[11]

3.2 The welfare-maximizing (p_I, q_I)-pair

We now move to characterize the regulatory target. Recall that this is the price-and-quality bundle that firm I, the regulated incumbent, should implement so as to pursue social interests without incurring budgetary losses. In formal terms, both in the Stackelberg and in the Nash-Cournot frameworks, the optimal (p_I, q_I)-pair is pinned down by maximizing the social welfare function

$$W (\mathbf{p}, \mathbf{q}) = V (\mathbf{p}, \mathbf{q}) + \pi_I (\mathbf{p}, \mathbf{q}) + \pi_E (\mathbf{p}, \mathbf{q}) \tag{11}$$

subject to the non-negative profit constraint

$$\pi_I (\mathbf{p}, \mathbf{q}) \geq 0, \tag{12}$$

knowing that p_E and q_E obey the rules in (8) and (9).[12] Let λ be the Lagrange multiplier associated with (12). Further denote

$$\begin{aligned}
\widetilde{\theta}_k &\equiv \int_{\theta_m(\mathbf{p},\mathbf{q})}^{\overline{\theta}} \frac{x_k (p_k, q_k; \theta)}{X_k} \theta f (\theta) \, d\theta, \\
\widetilde{\theta}_j &\equiv \int_{\underline{\theta}}^{\theta_m(\mathbf{p},\mathbf{q})} \frac{x_j (p_j, q_j; \theta)}{X_j} \theta f (\theta) \, d\theta,
\end{aligned}$$

as the weighted average of quality valuations by the clients of firm k and j respectively. Taking into account that the incumbent foresees the impact of its decisions on the entrant's decisions in the Stackelberg game, and that it behaves myopically in the Nash-Cournot game, we can state the following proposition:[13]

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[11] This does *not* mean that consumer and firm's objectives are perfectly aligned, even when attention is restricted to the quality dimension. In fact, were the price lower, the demand would be larger. As a result, the average cost of quality would be smaller, calling for a strict improvement in terms of quality.

[12] Because the entrant is not subject to regulatory control, the regulator does not need to be concerned with the financial viability of firm E. Yet, as we assume that the regulated firm does face an unregulated competitor, we implicitly take the entrant's profit to be non-negative at the incumbent's welfare-maximizing price and quality.

[13] See Appendix A.1.1 for mathematical details.

Proposition 1 *Under the Lemma, the incumbent's price-and-quality bundle that maximizes* (11) *subject to* (12) *is characterized by the following pairs of conditions:*

a) *in the Stackelberg game:*

$$\frac{d\pi_I}{dp_I} = \frac{1}{1+\lambda}\left[X_I - X_E\left(\frac{\partial X_E/\partial p_I}{-\partial X_E/\partial p_E}\right)\right. \tag{13}$$
$$\left.+X_E\frac{dp_E}{dp_I} - \widetilde{\theta}_E X_E\frac{dq_E}{dp_I}\right]$$
$$\frac{d\pi_I}{dq_I} = \frac{-1}{1+\lambda}\left[\widetilde{\theta}_I X_I + X_E\left(\frac{\partial X_E/\partial q_I}{-\partial X_E/\partial p_E}\right)\right. \tag{14}$$
$$\left.-X_E\frac{dp_E}{dq_I} + \widetilde{\theta}_E X_E\frac{dq_E}{dq_I}\right];$$

b) *in the Nash-Cournot game:*

$$\frac{\partial\pi_I}{\partial p_I} = \frac{1}{1+\lambda}\left[X_I - X_E\left(\frac{\partial X_E/\partial p_I}{-\partial X_E/\partial p_E}\right)\right] \tag{15}$$
$$\frac{\partial\pi_I}{\partial q_I} = \frac{-1}{1+\lambda}\left[\widetilde{\theta}_I X_I + X_E\left(\frac{\partial X_E/\partial q_I}{-\partial X_E/\partial p_E}\right)\right]. \tag{16}$$

As the incumbent's objective is to attain the highest feasible welfare level, both Equations (13) and (14) and Equations (15) and (16) embody the variations both in net consumer surplus (V) and in the competitor's profit (π_E) that are induced respectively by a raise in the price p_I and in the quality q_I.

Specifically, in (13) and (15), the direct marginal impact on V of a raise in p_I is captured by the term X_I; the direct marginal impact on π_E of a raise in p_I is captured by the term $X_E\left(\frac{\partial X_E/\partial p_I}{-\partial X_E/\partial p_E}\right)$. The latter is the product of the competitor's market size X_E with a fraction equal to the ratio of price changes that would leave unchanged this very same market size:

$$\frac{dp_E}{dp_I} = \frac{\partial X_E/\partial p_I}{-\partial X_E/\partial p_E}.$$

Similarly, in (14) and (16), the direct marginal on V of a raise in q_I is captured by the term $\widetilde{\theta}_I X_I$; the direct marginal on π_E of a raise in q_I is captured by the term $X_E\left(\frac{\partial X_E/\partial q_I}{-\partial X_E/\partial p_E}\right)$. Again, the latter is the product of the competitor's market size X_E with a fraction equal to the ratio of quality changes that would leave unchanged this very same market size:

$$\frac{dq_E}{dq_I} = \frac{\partial X_E/\partial q_I}{-\partial X_E/\partial q_E}.$$

This proves the relevance of cross-price and cross-quality effects ($\partial X_E/\partial p_I$ and $\partial X_E/\partial q_I$) for the determination of the incumbent's optimal price-and-quality pair.

Besides, in the Stackelberg game, strategic interactions across firms are accounted for. In (13) and (14), they are captured by the terms (dp_E/dp_I), (dq_E/dp_I), (dp_E/dq_I) and (dq_E/dq_I). The presence of these terms is due to the circumstance that variations in the

price and quality of the market leader also affect consumer utility through their impact on the rival's price and quality. This involves both a volume and a quality appreciation effect. The former is expressed by the follower's demand, with which the terms under scrutiny are systematically weighed. The latter is measured by the average valuation of quality by the follower's clients, with which the terms are also weighed whenever interactions with quality q_E are concerned. Of course, these terms do not appear in (15) and (16) because, under Nash-Cournot competition, the incumbent does not anticipate the effect of its own decisions on the competitor's decisions.

The apparent complexity of the conditions in Proposition 1, especially as far as the Stackelberg setting is concerned, may induce one to consider the definition of the optimal price and quality as a purely theoretical exercise, with no practical value. If the optimal bundle does not find an explicit expression, then exact implementation is indeed likely to be beyond reach. This makes the results we present hereafter more striking.

4 Decentralization through a quality-adjusted price cap

In Proposition 1, we have identified the benchmark the regulator should effect in the marketplace. It is represented by (13) and (14) when firm I acts as a Stackelberg leader. It is given by (15) and (16) when firm I plays *à la* Nash-Cournot. Decentralization requires that the regulator adopt an appropriate policy device. For either setting, we hereafter propose a quality-adjusted price-cap scheme that allows us to pursue the regulatory objective. To avoid redundancy, we content ourselves with presenting the (somewhat more typical) situation in which, at the concerned target, the incumbent serves high-valuation consumers and the competitor low-valuation consumers (i.e., we take $k = I$ and $j = E$). However, it is noteworthy that arguments carry over, *mutatis mutandis*, in the converse case.[14]

4.1 The ideal quality-adjusted price cap

Suppose first that the regulatory target is given by conditions (13) and (14). Assume that the incumbent is left free to choose both price and quality, provided a *price-and-quality cap* is satisfied. Formally, let firm I pin down the pair (p_I, q_I) that maximizes its profit $\pi_I(\mathbf{p}, \mathbf{q})$ subject to the constraint

$$\alpha p_I - \beta q_I \leq P + \gamma p_E - \delta q_E. \qquad \text{(capS)}$$

This is a modified (single-product) version of the standard quality-adjusted price cap elaborated by De Fraja and Iozzi [19] for (multi-product) monopoly regulation. The novelty is that it is extended to account for the strategic interactions that take place between competitors in a Stackelberg game. As in the standard version, as long as $\alpha > 0$, the regulatory constraint is tightened by an increase in price p_I. With $\beta > 0$, it is relaxed

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by an increase in quality q_I. In addition, the right-hand side of the cap is designed to explicitly embody the competitor's choices. With $\gamma < 0$, a raise in the rival price p_E tightens the constraint; with $\delta < 0$, a raise in the rival quality q_E relaxes it.

When the regulatory target is given by (15) and (16), the extension terms in the right-hand side of (capS) are unnecessary, provided that the incumbent behaves myopically. Thus, in a Nash-Cournot duopoly, $\gamma = \delta = 0$ and, despite the presence of a competitor, the price cap comes back to the more standard quality-adjusted formulation

$$\alpha p_I - \beta q_I \leq P. \tag{capN}$$

Let μ be the Lagrange multiplier associated with either (capS) or (capN). The following proposition summarizes how the regulator should set the weights and the ceiling P for the target to be enforced.[15]

Proposition 2 *Define:*

$$\nu \equiv \left(\frac{\partial X_E^R / \partial p_I}{-\partial X_E^R / \partial p_E} \right) = \left[1 + \frac{q_I^R - q_E^R}{x_m^R f\left(\theta_m^R\right)} \int_{\underline{\theta}}^{\theta_m^R} \left(\frac{-\partial x_E^R}{\partial p_E} \right) f\left(\theta\right) d\theta \right]^{-1}.$$

When the regulatory target is given by (13) and (14), the ideal (capS) weights are set as:

$$\alpha^S = X_I^R - \nu X_E^R \tag{aS}$$
$$\beta^S = \widetilde{\theta}_I^R X_I^R - \theta_m^R \nu X_E^R \tag{bS}$$
$$\gamma^S = -X_E^R \tag{gS}$$
$$\delta^S = -\widetilde{\theta}_E^R X_E^R. \tag{dS}$$

When the regulatory target is given by (15) and (16), the ideal (capN) weights are set as:

$$\alpha^N = X_I^R - \nu X_E^R \tag{aN}$$
$$\beta^N = \widetilde{\theta}_I^R X_I^R - \theta_m^R \nu X_E^R. \tag{bN}$$

P^z *is chosen so that* $\pi_I\left(\mathbf{p}^R, \mathbf{q}^R\right) = 0,\ \forall z \in \{S, N\}.$

Proposition 2 tells that, for (capS) (resp. (capN)) to implement the optimal bundle characterized by (13) and (14) (resp. (15) and (16)), it suffices (i) to set coefficients as in (aS) to (dS) (resp. (aN) and (bN)) and (ii) to decrease P enough to wash out the profit of firm I. The presence of the superscript R indicates that the exact values are those obtained at the optimal price and quality, which are decentralized under the (partial) *regulatory regime*.[16]

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[15]See Appendix A.1.2 and A.2.2 for mathematical details.

[16]To save over notation, here and elsewhere in the text, we append the superscript R to denote the optimal values, whatever the target they refer to. Remark, however, that this is *not* meant to suggest that optimal values are the same in the different cases.

The first, perhaps most striking, point to be made is that the optimal weights to be attached to the incumbent's price and quality are to be determined in the same way in the Stackelberg and in the Nash-Cournot case ((aS) is analogous to (aN) and (bS) to (bN)).

According to (aS) and (aN), the appropriate weight for the incumbent's price is given by the difference between two terms. The first term is the regulated firm's quantity evaluated at $\left(\mathbf{p}^R, \mathbf{q}^R\right)$, namely X_I^R. The second term consists in firm E's quantity evaluated at $\left(\mathbf{p}^R, \mathbf{q}^R\right)$, namely X_E^R, as multiplied by the coefficient ν that reflects product differentiation. Observe that, in general, ν is strictly smaller than one.[17] Thus, firm I's output is given a larger relevance than firm E's output in the composition of the price weight. That is to say, the price weight α is obtained by subtracting from the regulated firm's quantity X_I^R (the "standard" weight in cap formulae), a fraction of the quantity of the (unregulated) competitor, νX_E^R.

Similarly, the quality weight β, as defined by (bS) and (bN), is given by the difference between two terms. The first term, $\widetilde{\theta}_I^R X_I^R$, is an aggregate measure of the quality appreciation by firm I's consumers. The second term is linked to the appreciation of quality by firm E's consumers. As the sole marginal clients of firm E are concerned by changes in q_I, the quality appreciation refers to θ_m^R and *not* to $\widetilde{\theta}_E^R$. Note that, if prices and qualities are observable (as assumed), unlike $\widetilde{\theta}_E^R$, the parameter θ_m^R can be easily computed. Interestingly enough, this marginal quality valuation is to be multiplied by νX_E^R, the exact same part of α that refers to firm E. The coefficient is to be calculated by using the whole demand for firm E's product, X_E^R (which is possibly observable), and not the consumption by firm E's marginal clients (which is not).

The downsizing of X_E^R and $\theta_m^R X_E^R$ in the expressions of α and β respectively, which depends upon the coefficient ν, relates to three elements. First, *ceteris paribus*, the smaller the quality spread between products $\left(q_I^R - q_E^R\right)$, the larger ν (and so the smaller α and β). This suggests that less regulatory pressure needs to be exerted on firm I when products are not very differentiated. Indeed, in that case, the leader is disciplined by fierce competitive pressure. Second, the higher the marginal demand $x_m^R f\left(\theta_m^R\right)$, the weaker the regulation. A similar argument applies here: having a large amount of individuals indifferent between operators signals that, given prices and qualities, products are almost "perfect substitutes," so that competition is again a substitute for regulation. Last, the smaller the term $\int_{\underline{\theta}}^{\theta_m^R}\left(\frac{-\partial x_E^R}{\partial p_E}\right) f\left(\theta\right) d\theta$, the higher ν, meaning that a relatively soft regulation is required when the entrant has market power.

All in all, a clear message can be drawn by looking at the optimal values of α and β characterized in Proposition 2. First of all, neither in the Stackelberg nor in the Nash-Cournot case, can the regulator neglect the presence of the competitor to properly regulate the incumbent. Second, the larger the market share of the unregulated firm, the lower

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[17] As from the definition in Proposition 2, and from Appendix A.1.2, the coefficient ν is the ratio between the marginal variation in X_F^R induced by an increase in p_L and the (absolute value of the) overall (marginal and inframarginal) variation in the same quantity X_F^R as induced by an increase in p_F. In general, the ratio is strictly smaller than 1, because the (cross) effect of price p_L on the entrant's demand is lower than the (own) effect of price p_F. Its specific magnitude depends on the difference between cross and own-price effects. The sole case in which ν equals 1, is the unit demand case, as mentioned later in the text.

the pressure that is necessary to impose on the regulated firm. In fact, if X^R denotes the market size at $\left(\mathbf{p}^R, \mathbf{q}^R\right)$, the optimal weight attached to price p_I in the price-cap formula can be rewritten as $\alpha = \left[X^R - (1 + \nu) X_E^R\right]$. As $\nu > 0$, this also means that competition has a bigger impact on markets than it appears, when considering the sole market share of the unregulated firm $\left(X_E^R\right)$. This further evidences an important feature of our cap: it accounts for (and adapts to) the transition from regulated monopolies to increasingly more competitive markets.

From Proposition 2, it also emerges that when the regulator points to the Stackelberg benchmark, the magnitude of the competitive effect and the relevance of the competitor's product quality for consumers should be considered not only to fix the weights of the regulated price and quality, but also to tighten/relax the overall ceiling. Recall, indeed, from (capS) that in the Stackelberg context, the regulatory constraint is relaxed by a decrease in the entrant's price p_E and by an increase in the entrant's quality q_E. According to (gS), the larger the entrant's demand $\left(X_E^R\right)$, the more a raise in p_E tightens the cap. As prices are strategic complements, an increase in p_I would also trigger an increase in p_E. The larger X_E^R, the greater the negative impact on total welfare. On the other hand, (dS) reveals that the larger the aggregate quality appreciation by firm E's consumers $(\widetilde{\theta}_E^R X_E^R)$, the more a raise in q_E relaxes the cap. As qualities are strategic substitutes, a decrease in q_I would trigger an increase in q_E. The larger $(\widetilde{\theta}_E^R X_E^R)$, the greater the benefits from a raise in q_E against a reduction in q_I.

The predictions of our analysis confirm Brennan [12]'s intuition that, in the presence of competitors endowed with market power, a cap on the incumbent's price cannot be optimally calibrated on the sole incumbent's output. Indeed, unlike in the presence of a passive fringe, the marginal welfare effect of a price variation is affected by the competitors' output. Actually, Proposition 2 formalizes Brennan [12]'s conclusion for price-cap regulation of the incumbent in Stackelberg and Nash-Cournot oligopolies. Besides, it extends that conclusion to environments where not only price but also quality is regulated.

A peculiar case arises when customers allocate a single consumption unit to their preferred operator.[18] The peculiarity is that, ν being exactly equal to 1, the Nash-Cournot target can be pursued by simply adopting a "standard" quality-adjusted price-cap. The latter is a cap of the form $(p_I - \widetilde{\theta}_I^R q_I) \le \overline{p}$, in which the incumbent's price is replaced by its consumers' generalized price. This recipe is equally suitable for decentralization of the Stackelberg target, provided that the incumbent's decisions have no impact on the entrant's quality choice.[19] The striking aspect is that the optimal policy does account for the impact of regulation on the whole industry and, yet, the regulatory target can be

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[18]Examples of unit demand can be found in commuter transport. A commuter allocates his/her consumption unit (i.e., the trip to be made daily to reach the workplace) to the transport mode that makes him/her best off among all available alternatives. Note also that a quality attribute like travel time is both observable and verifiable, hence it can be used for regulatory purposes.

[19]This could be the case in environments where unregulated operators offer some minimum quality level that does not react to variations in the incumbent's price and quality, say, because they obey some given standard or for technological reasons. See Appendix A.1.4, for mathematical details on the unit-demand case in the Stackelberg framework.

enforced by looking at the incumbent only. Despite what this result holds for very specific environments, it may prove important for regulatory practice if, in such environments, relevant markets are difficult to define.

4.2 Implementation and robustness issues

At the implementation stage, the first concern of the regulator is to identify the target that it is possible to pursue, that, in turn, dictates which policy is to be adopted. Actually, this depends upon the incumbent's strategic behaviour. The target is represented by (13) and (14) when firm I acts as a Stackelberg leader, in which case (capS) is to be adopted. The target is given by (15) and (16) when firm I plays *à la* Nash-Cournot, in which case (capN) is the appropriate policy.

From a social perspective, the Nash-Cournot target is less desirable than the Stackelberg target in that it embodies less information about firms' reactions. Yet, it is the target that is feasible whenever the regulated firm is not in a position to move first and/or to commit in the market game. This shows that the incumbent's strategic position affects the regulator's capability to pursue more or less ambitious objectives. Observe, however, that regulation itself may have an impact on the incumbent's strategic position. On one hand, imposing repeated regulatory revisions on the incumbent may progressively remove its strategic advantage. Conversely, by reducing the operational flexibility of the incumbent, regulation may preserve and perpetuate its commitment ability.

Actually, the intertemporal impact of regulation is not a minor aspect because, in the same vein as in Vogelsang and Finsinger [33], De Fraja and Iozzi [19], Billette de Villemeur [8] and Billette de Villemeur and Vinella [10],[20] one can conceive that the targeted pair of vectors $\left(\mathbf{p}^R, \mathbf{q}^R\right)$ be approached through an iterative process. More precisely, as for a "standard" price cap, information on past market performance can be used to update the weights in the constraint at each step. In addition, the firm's profit can be progressively reduced by adjusting P until $\pi_I = 0$. It is worth mentioning that both in the Stackelberg and in the Nash-Cournot cases, our regulatory scheme exhibits the following robustness property.[21]

Proposition 3 *Let* W^S *(resp.* W^N*) be the welfare level that is achieved under* (capS) *(resp.* (capN)*) when* P *is set such that* $\pi_I\left(\mathbf{p}, \mathbf{q}\right) = 0$*. Around the optimal values characterized in Proposition 2, variations in* α *and* β *have zero first-order effects:*

$$\frac{dW^z}{d\alpha} = 0 \ \ and \ \ \frac{dW^z}{d\beta} = 0, \ \ \forall z \in \{S, N\}.$$

According to Proposition 3, variations in the price weight and in the quality weight around their optimal values, have no first-order effect on welfare. This means that the scheme is robust to the possibility that the regulator would be unable to set α and β exactly as dictated in Proposition 2. In other words, the proper functioning of the scheme would not be undermined, should small biases appear in the weight determination. This result should reassure practitioners who might object that it would be problematic to properly set parameters in the cap, especially as far as the quality valuation (namely $\widetilde{\theta}_I^R$ and θ_m^R) and the degree of competition are concerned.

However, this last preoccupation is especially weak in the unit demand cases in which the regulator needs to determine the sole quality weight $\widetilde{\theta}_I^R$. Actually, in these cases, the incentive scheme reduces to a single parameter, to be *exogenously* set. If quality is to be considered by the regulator, then this parameter is the simplest information one can think of. Indeed, it is an *average* marginal valuation of quality by the incumbent's consumers, upon which the regulator may legitimately and more easily collect data.[22] Thus, along the current practice, we suggest that the regulator estimate the social valuation of quality and use this estimate as an attribute in the implementation scheme.[23] With a quality value that is fixed and publicly available, the scheme would be transparent and little prone to manipulations, hence more likely to attract public consensus.

5 Concluding remarks

There are essentially two insights to be drawn from our analysis.

First, in a partially regulated oligopoly, the regulatory agency should be able to hinge upon information for the whole industry. In general, information about the sole regulated firm does not appear to allow for efficient regulation. In the price-and-quality cap we have looked at, appropriate weights depend on the (optimal) quantities provided by both the regulated incumbent and the entrant, despite the fact that the latter is not directly concerned by regulation. These results hold true whether firms compete *à la* Stackelberg or *à la* Nash-Cournot.

Second, under both kinds of competition, price-and-quality cap regulation is robust to small errors in the determination of the weights to be attached to the regulated firm's decision variables. It thus appears reasonable to hinge upon such regulatory mechanisms to account for the quality dimensions of the products sold in vertically differentiated markets, despite the fact that they may rely upon (possibly imperfect) statistical estimates.

We have considered settings in which quality is as flexible as price. However, for some

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[22] For instance, stated preferences can be (and are indeed largely) used to form time value estimates in passenger transport sectors.

[23] This is in contrast to the implementation scheme proposed by De Fraja and Iozzi [19] for monopoly regulation, in which quality valuation is *endogenously* determined by computing at each step consumer marginal surplus $(\partial V/\partial q)$. In the multi-product environment the authors explore, this creates a problem in terms of convergence of the regulatory algorithm to the second-best monopoly prices and qualities. This problem is circumvented by introducing an additional constraint that further limits the regulated firm's choices.

quality dimensions, adjustments can take longer than for price. It is reasonable to think that in liberalized markets in which this occurs, incumbents will still enjoy a strategic advantage in quality setting *vis-à-vis* new competitors. By contrast, this advantage is less likely to survive in price setting. Such situations can be represented by a three-stage game in which a sequential quality choice by the incumbent and then by the entrant is followed by (simultaneous) price competition.[24] In this game, unlike in the Stackelberg context, the entrant anticipates the impact its quality decision will have on both its own and the incumbent's price.[25] For this reason, the regulatory target differs from those we have presented.[26] One might thus expect a different regulatory policy to be required for its decentralization. Perhaps somewhat surprisingly, it turns out that both the structure of the regulatory constraint and the optimal composition of the weights exactly replicate those of the Stackelberg context. Moreover, the scheme maintains the same robustness property at the implementation stage. This points to the conclusion that the regulatory recipe we provide, although far from general, applies to a larger variety of situations than our approach might seem to suggest at first.

Nonetheless, there still is a long way to go for a full understanding of price-and-quality regulation of vertically differentiated oligopolies. Our study represents an initial undertaking in that direction. Because our goal was to pinpoint how competition with a strategic entrant affects the incumbent's regulation, we have found it useful to look at single-product firms as a first step. However, regulated utilities often provide several goods/services. To account for this circumstance, the analysis should be extended to the case of multi-product firms, which has been addressed only for monopolies so far. Moreover, we have taken firms to behave non-cooperatively. In practice, they might have an incentive to collude so as to undo the regulatory policy. That asymmetrically regulated firms can profitably coordinate against the regulator is shown by Aubert and Pouyet [4] in Bayesian environments with adverse selection. It would thus be interesting to study the ideal price-and-quality cap with regard to collusive settings. This is left for further research.

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[24]We are thankful to the associate editor for suggesting to us to consider this setup.

[25]When quality is a longer-run decision variable, as compared to price, it represents the very strategic instrument for firms. This is the situation Grilo [21] and Cremer, De Rycke and Grimaud [15] represent in their mixed oligopoly models. However, these models differ from the three-stage game we refer to in that the public firm is taken to have *no* strategic advantage over the private firm. Competitors play a two-stage game, in which they set qualities anticipating the impact their choices will have on prices. In Grilo [21], it emerges that first best is viable in mixed duopolies, whereas it is not in private regulated duopolies, because public managers are better informed than regulators. First best outcomes also emerge in Cremer, De Rycke and Grimaud [15] as long as the budget constraint of the public firm does not bind. Otherwise, a second-best outcome arises, which is still preferable to the outcome that a private duopoly would yield. By contrast, in the partial regulatory setting we have considered, the first-best outcome is beyond reach even without budget requirements and under "perfect" regulation.

[26]More precisely, although the competitors' pricing rules are the same as in the Cournot framework, the rules that characterize the competitors' qualities differs from both the Cournot and the Stackelberg counterparts. See Appendix B for mathematical details about the three-stage game described in the text.

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A   The Stackelberg and the Nash-Cournot framework

As a first step, we propose the formal analysis for the Stackelberg framework. After characterizing the price-and-quality bundle of firm I (the regulatory target), we derive the optimal weights in the price-and-quality cap, describing both the case in which $p_I > p_E$ and $q_I > q_E$ and that in which $p_I < p_E$ and $q_I < q_E$. This allows us to clarify that analogous conclusions are reached, *mutatis mutandis*, in either case. Thus, to avoid redundancy, we thereafter focus on the former case only. We further show that the scheme is robust to imperfections in the determination of the price and quality weight.

As a second step, we repropose the analysis for the Nash-Cournot framework, taking $p_I > p_E$ and $q_I > q_E$ for the reason previously illustrated.

A.1   The Stackelberg framework

A.1.1   The regulatory benchmark

Firm I maximizes (11) subject to (12). The first-order condition with respect to p_I is given by

$$\frac{d\pi_I}{dp_I} = \left(\frac{-1}{1+\lambda}\right)\left(\frac{dV}{dp_I} + \frac{d\pi_E}{dp_I}\right). \tag{17}$$

The impact on consumer surplus of a change in p_I can be decomposed as

$$\frac{dV}{dp_I} = \frac{\partial V}{\partial p_I} + \frac{\partial V}{\partial p_E}\frac{dp_E}{dp_I} + \frac{\partial V}{\partial q_E}\frac{dq_E}{dp_I}. \tag{18}$$

With firm I serving the market segment $\left[\theta_m, \overline{\theta}\right]$ and firm E the segment $\left[\underline{\theta}, \theta_m\right)$, Roy's identity yields

$$\frac{\partial V}{\partial p_I} = -\int_{\theta_m}^{\overline{\theta}} x_I(p_I, q_I; \theta)\, f(\theta)\, d\theta = -X_I \tag{19a}$$

$$\frac{\partial V}{\partial p_E} = -\int_{\underline{\theta}}^{\theta_m} x_E(p_E, q_E; \theta)\, f(\theta)\, d\theta = -X_E \tag{19b}$$

$$\frac{\partial V}{\partial q_I} = \int_{\theta_m}^{\overline{\theta}} x_I(p_I, q_I; \theta)\,\theta f(\theta)\, d\theta = \widetilde{\theta}_I X_I \tag{19c}$$

$$\frac{\partial V}{\partial q_E} = \int_{\underline{\theta}}^{\theta_m} x_E(p_E, q_E; \theta)\,\theta f(\theta)\, d\theta = \widetilde{\theta}_E X_E. \tag{19d}$$

In the opposite case, denoting the marginal type $\theta_{m,2}$ to avoid confusion, we would get

$$\frac{\partial V}{\partial p_I} = -\int_{\underline{\theta}}^{\theta_{m,2}} x_I(p_I, q_I; \theta)\, f(\theta)\, d\theta = -X_I$$

$$\frac{\partial V}{\partial p_E} = -\int_{\theta_{m,2}}^{\overline{\theta}} x_E(p_E, q_E; \theta)\, f(\theta)\, d\theta = -X_E$$

$$\frac{\partial V}{\partial q_I} = \int_{\underline{\theta}}^{\theta_{m,2}} x_I(p_I, q_I; \theta)\,\theta f(\theta)\, d\theta = \widetilde{\theta}_I X_I$$

$$\frac{\partial V}{\partial q_E} = \int_{\theta_{m,2}}^{\overline{\theta}} x_E(p_E, q_E; \theta)\,\theta f(\theta)\, d\theta = \widetilde{\theta}_E X_E.$$

On the other hand, because $(\partial \pi_E / \partial p_E) = 0$ and $(\partial \pi_E / \partial q_E) = 0$, we can write

$$\frac{d\pi_E}{dp_I} = \frac{\partial \pi_E}{\partial p_I} = X_E \left(\frac{\partial X_E / \partial p_I}{-\partial X_E / \partial p_E} \right). \tag{20}$$

Plugging (19a), (19b) and (19d) into (18) and then (20) and (18) into (17), we obtain (13).

The first-order condition with respect to q_I for a constrained maximum of (11) is given by

$$\frac{d\pi_I}{dq_I} = \left(\frac{-1}{1+\lambda} \right) \left(\frac{dV}{dq_I} + \frac{\partial \pi_E}{\partial q_I} \right). \tag{21}$$

A similar analysis yields the following decomposition of the variation in consumer surplus

$$\frac{dV}{dq_I} = \widetilde{\theta}_I X_I - X_E \frac{dp_E}{dq_I} + \widetilde{\theta}_E X_E \frac{dq_E}{dq_I}. \tag{22}$$

We can also write

$$\frac{\partial \pi_E}{\partial q_I} = X_E \left(\frac{\partial X_E / \partial q_I}{-\partial X_E / \partial p_E} \right). \tag{23}$$

Replacing (22) and (23) into (21), we ultimately obtain (14).

A.1.2   The ideal price-and-quality cap

From (13) and (14), at optimum, α and β are found to be:

$$\begin{aligned} \alpha^S &= X_I^R - X_E^R \left(\frac{\partial X_E^R / \partial p_I}{-\partial X_E^R / \partial p_E} \right), \\ \beta^S &= \widetilde{\theta}_I^R X_I^R + X_E^R \left(\frac{\partial X_E^R / \partial q_I}{-\partial X_E^R / \partial p_E} \right). \end{aligned}$$

The derivatives $\left(\partial X_E^R / \partial p_I \right)$ and $\left(\partial X_E^R / \partial q_I \right)$ reflect only marginal variations, as firm E's inframarginal customers are not concerned by changes in p_I and q_I. The other weights in Proposition 2 follow straightforwardly.

The case of $p_I > p_E$ and $q_I > q_E$ The marginal type is given by $\theta_m \equiv \left(\frac{p_I - p_E}{q_I - q_E} \right)$. Hence, at optimum, we can write

$$\frac{\partial X_E^R}{\partial p_I} = x_m^R f \left(\theta_m^R \right) \frac{\partial \theta_m^R}{\partial p_I} = \frac{x_m^R f \left(\theta_m^R \right)}{q_I^R - q_E^R} \tag{24}$$

together with

$$\begin{aligned} \frac{\partial X_E^R}{\partial q_I} &= x_m^R f \left(\theta_m^R \right) \frac{\partial \theta_m^R}{\partial q_I} \\ &= -\theta_m^R \frac{x_m^R f \left(\theta_m^R \right)}{q_I^R - q_E^R} = -\theta_m^R \frac{\partial X_E^R}{\partial p_I}, \end{aligned} \tag{25}$$

where $x_m^R f\left(\theta_m^R\right)$ measures consumption by marginal clients. We also have

$$
\begin{aligned}
\frac{\partial X_E^R}{\partial p_E} &= \int_{\underline{\theta}}^{\theta_m^R} \frac{\partial x_E^R}{\partial p_E} f\left(\theta\right) d\theta + x_m^R f\left(\theta_m^R\right) \frac{\partial \theta_m^R}{\partial p_E} \\
&= \int_{\underline{\theta}}^{\theta_m^R} \frac{\partial x_E^R}{\partial p_E} f\left(\theta\right) d\theta - \frac{x_m^R f\left(\theta_m^R\right)}{q_I^R - q_E^R}.
\end{aligned} \tag{26}
$$

Thus

$$
\frac{\partial X_E^R / \partial p_I}{-\partial X_E^R / \partial p_E} = \frac{x_m^R f\left(\theta_m^R\right)/(q_I^R - q_E^R)}{\int_{\underline{\theta}}^{\theta_m^R}\left(\frac{-\partial x_E^R}{\partial p_E}\right) f\left(\theta\right) d\theta + \frac{x_m^R f\left(\theta_m^R\right)}{q_I^R - q_E^R}} = \nu, \tag{27a}
$$

$$
\frac{\partial X_E^R / \partial q_I}{-\partial X_E^R / \partial p_E} = -\theta_m^R\left(\frac{\partial X_E^R / \partial p_I}{-\partial X_E^R / \partial p_E}\right) = -\theta_m^R \nu. \tag{27b}
$$

The optimal weights α^S and β^S can thus be rewritten as

$$
\begin{aligned}
\alpha^S &= X_I^R - \nu X_E^R, \\
\beta^S &= \widetilde{\theta}_I^R X_I^R - \theta_m^R \nu X_E^R.
\end{aligned}
$$

Finally, if P^S is chosen so that $\pi^I = 0$, then it must be the case that

$$
\mu^S = \frac{1}{1 + \lambda^R}. \tag{mu-S}
$$

The case of $p_I < p_E$ and $q_I < q_E$ The marginal type is given by $\theta_{m'} = \left(\frac{p_E - p_I}{q_E - q_I}\right)$. Hence, at optimum, we can write

$$
\begin{aligned}
\frac{\partial X_E^R}{\partial p_I} &= \frac{\partial}{\partial p_I}\left(\int_{\theta_{m'}^R}^{\overline{\theta}} x_E^R f\left(\theta\right) d\theta\right) \\
&= -x_{m'}^R f\left(\theta_{m'}^R\right) \frac{\partial \theta_{m'}^R}{\partial p_I} = \frac{x_{m'}^R f\left(\theta_{m'}^R\right)}{q_E^R - q_I^R}
\end{aligned} \tag{28}
$$

together with

$$
\begin{aligned}
\frac{\partial X_E^R}{\partial q_I} &= \frac{\partial}{\partial q_I}\left(\int_{\theta_{m'}^R}^{\overline{\theta}} x_E^R f\left(\theta\right) d\theta\right) \\
&= -x_{m'}^R f\left(\theta_{m'}^R\right) \frac{\partial \theta_{m'}^R}{\partial q_I} \\
&= -\theta_{m'}^R \frac{x_{m'}^R f\left(\theta_{m'}^R\right)}{q_E^R - q_I^R} = -\theta_{m'}^R \frac{\partial X_E^R}{\partial p_I}.
\end{aligned} \tag{29}
$$

We further have

$$\frac{\partial X_E^R}{\partial p_E} = \int_{\theta_{m'}^R}^{\overline{\theta}} \frac{\partial x_E^R}{\partial p_E} f(\theta) \, d\theta - x_{m'}^R f\left(\theta_{m'}^R\right) \frac{\partial \theta_{m'}^R}{\partial p_E}$$

$$= \int_{\theta_{m'}^R}^{\overline{\theta}} \frac{\partial x_E^R}{\partial p_E} f(\theta) \, d\theta - \frac{x_{m'}^R f\left(\theta_{m'}^R\right)}{q_E^R - q_I^R}. \tag{30}$$

Thus

$$\frac{\partial X_E^R/\partial p_I}{-\partial X_E^R/\partial p_E} = \frac{x_{m'}^R f\left(\theta_{m'}^R\right)/(q_E^R - q_I^R)}{\int_{\underline{\theta}}^{\theta_{m'}^R} \left(\frac{-\partial x_E^R}{\partial p_E}\right) f(\theta) \, d\theta + \frac{x_{m'}^R f\left(\theta_{m'}^R\right)}{q_E^R - q_I^R}} = \nu',$$

$$\frac{\partial X_E^R/\partial q_I}{-\partial X_E^R/\partial p_E} = -\theta_{m'}^R \left(\frac{\partial X_E^R/\partial p_I}{-\partial X_E^R/\partial p_E}\right) = -\theta_{m'}^R \nu'.$$

The optimal weights α^S and β^S can thus be rewritten as

$$\alpha^S = X_I^R - \nu' X_E^R,$$
$$\beta^S = \widetilde{\theta}_I^R X_I^R - \theta_{m'}^R \nu' X_E^R,$$

which expression do *not* differ from the one obtained in the case $p_I > p_E$ and $q_I > q_E$.

A.1.3   Robustness of the scheme

Let us focus on the case of $p_I > p_E$ and $q_I > q_E$. As $\pi_I^R = 0$, we can compute

$$\frac{dW^R}{d\alpha} = \frac{d}{d\alpha}\left(V^R + \pi_E^R\right).$$

Omitting superscripts for sake of shortness, we have

$$\frac{dV}{d\alpha} = \left(\frac{\partial V}{\partial p_I} + \frac{\partial V}{\partial p_E}\frac{dp_E}{dp_I} + \frac{\partial V}{\partial q_E}\frac{dq_E}{dp_I}\right)\frac{dp_I}{d\alpha} + \left(\frac{\partial V}{\partial q_I} + \frac{\partial V}{\partial p_E}\frac{dp_E}{dq_I} + \frac{\partial V}{\partial q_E}\frac{dq_E}{dq_I}\right)\frac{dq_I}{d\alpha}$$

$$= \left(-X_I - X_E\frac{dp_E}{dp_I} + \widetilde{\theta}_E X_E\frac{dq_E}{dp_I}\right)\frac{dp_I}{d\alpha} + \left(\widetilde{\theta}_I X_I - X_E\frac{dp_E}{dq_I} + \widetilde{\theta}_E X_E\frac{dq_E}{dq_I}\right)\frac{dq_I}{d\alpha}$$

together with

$$\frac{d\pi_E}{d\alpha} = \frac{\partial \pi_E}{\partial p_I}\frac{dp_I}{d\alpha} + \frac{\partial \pi_E}{\partial q_I}\frac{dq_I}{d\alpha}$$

$$= X_E\left(\frac{\partial X_E/\partial p_I}{-\partial X_E/\partial p_E}\right)\frac{dp_I}{d\alpha} + X_E\left(\frac{\partial X_E/\partial q_I}{-\partial X_E/\partial p_E}\right)\frac{dq_I}{d\alpha}.$$

Hence, we can write

$$\frac{dW}{d\alpha} = \left[-X_I - X_E\frac{dp_E}{dp_I} + \widetilde{\theta}_E X_E\frac{dq_E}{dp_I} + X_E\left(\frac{\partial X_E/\partial p_I}{-\partial X_E/\partial p_E}\right)\right]\frac{dp_I}{d\alpha} \tag{32}$$

$$+ \left[\widetilde{\theta}_I X_I - X_E\frac{dp_E}{dq_I} + \widetilde{\theta}_E X_E\frac{dq_E}{dq_I} + X_E\left(\frac{\partial X_E/\partial q_I}{-\partial X_E/\partial p_E}\right)\right]\frac{dq_I}{d\alpha}.$$

Proceeding similarly with β, we can also write

$$
\begin{aligned}
\frac{dW}{d\beta} & = \frac{d}{d\beta}\left(V + \pi_E\right) \\
& = \left[-X_I - X_E\frac{dp_E}{dp_I} + \widetilde{\theta}_E X_E\frac{dq_E}{dp_I} + X_E\left(\frac{\partial X_E/\partial p_I}{-\partial X_E/\partial p_E}\right)\right]\frac{dp_I}{d\beta} \\
& \quad + \left[\widetilde{\theta}_I X_I - X_E\frac{dp_E}{dq_I} + \widetilde{\theta}_E X_E\frac{dq_E}{dq_I} + X_E\left(\frac{\partial X_E/\partial q_I}{-\partial X_E/\partial p_E}\right)\right]\frac{dq_I}{d\beta}.
\end{aligned}
\tag{33}
$$

Imperfections in α   With $\pi_I^R = 0$, still omitting superscripts, we can write

$$
\frac{d\pi_I}{dp_I}\frac{dp_I}{d\alpha} + \frac{d\pi_I}{dq_I}\frac{dq_I}{d\alpha} = 0,
$$

which yields $\frac{dp_I}{d\alpha} = -\left(\frac{d\pi_I/dq_I}{d\pi_I/dp_I}\right)\frac{dq_I}{d\alpha}$. The firm's first-order conditions with respect to p_I and q_I are written

$$
\frac{d\pi_I}{dp_I} = \mu\left(\alpha - \gamma\frac{dp_E}{dp_I} + \delta\frac{dq_E}{dp_I}\right)
\tag{34}
$$

$$
\frac{d\pi_I}{dq_I} = -\mu\left(\beta + \gamma\frac{dp_E}{dq_I} - \delta\frac{dq_E}{dq_I}\right).
\tag{35}
$$

Hence, we can write

$$
\frac{dp_I}{d\alpha} = \frac{\beta + \gamma\frac{dp_E}{dq_I} - \delta\frac{dq_E}{dq_I}}{\alpha - \gamma\frac{dp_E}{dp_I} + \delta\frac{dq_E}{dp_I}}\frac{dq_I}{d\alpha}.
$$

For the optimal values of the weights, as defined by (aS)-(dS), we have

$$
\frac{dp_I}{d\alpha} = \frac{\widetilde{\theta}_I^R X_I^R - X_E^R\frac{dp_E}{dq_I} + \widetilde{\theta}_E^R X_E^R\frac{dq_E}{dq_I} - \theta_m^R\nu X_E^R}{X_I^R + X_E^R\frac{dp_E}{dp_I} - \widetilde{\theta}_E^R X_E^R\frac{dq_E}{dp_I} - \nu X_E^R}\frac{dq_I}{d\alpha}.
$$

Plug this into (32) and use (27a) and (27b) to obtain $\frac{dW^R}{d\alpha} = 0$.

Imperfections in β   With $\pi_I^R = 0$, omitting superscripts again, we can write

$$
\frac{d\pi_I}{dp_I}\frac{dp_I}{d\beta} + \frac{d\pi_I}{dq_I}\frac{dq_I}{d\beta} = 0,
$$

which yields $\frac{dp_I}{d\beta} = -\left(\frac{d\pi_I/dq_I}{d\pi_I/dp_I}\right)\frac{dq_I}{d\beta}$. Replacing from (34) and (35) returns

$$
\frac{dp_I}{d\beta} = \frac{\beta + \gamma\frac{dp_E}{dq_I} - \delta\frac{dq_E}{dq_I}}{\alpha - \gamma\frac{dp_E}{dp_I} + \delta\frac{dq_E}{dp_I}}\frac{dq_I}{d\beta}
$$

Substituting into (33), for the optimal values of the weights, as defined by (aS)-(dS), we obtain $\frac{dW}{d\beta}^R = 0$.

A.1.4 The case of unit demand

With unit demand, $\nu = 1$ and the price weight becomes $\alpha \equiv \left(X_I^R - X_E^R \right)$ and the quality weight $\beta \equiv (\widetilde{\theta}_I^R X_I^R - \theta_m^R X_E^R)$. Using these results, recalling that $\left(p_I - \theta_m^R q_I \right) = \left(p_E - \theta_m^R q_E \right)$ and noticing that, with $p_I > p_E$ and $q_I > q_E$, we now have $X_E^R = \int_{\underline{\theta}}^{\theta_m^R} f\left(\theta \right) d\theta = F\left(\theta_m^R \right)$ and $\widetilde{\theta}_E^R = \int_{\underline{\theta}}^{\theta_m^R} \frac{\theta f(\theta)}{F(\theta_m^R)} d\theta$, the regulatory constraint becomes

$$\widetilde{p}_I X_I^R \leq P - (\theta_m^R - \widetilde{\theta}_E^R) q_E X_E^R = P - q_E \int_{\underline{\theta}}^{\theta_m^R} F\left(\theta \right) d\theta.$$

When the incumbent's choices have no impact on q_E, this reduces to $(p_I - \widetilde{\theta}_I^R q_I) \leq \overline{p}$.

A.2 The Nash-Cournot framework

A.2.1 The regulatory benchmark

The first-order condition with respect to p_I for a constrained maximum of (11) is written

$$\frac{\partial \pi_I}{\partial p_I} = \left(\frac{1}{1+\lambda} \right) \left[X_I - X_E \left(\frac{\partial X_E / \partial p_I}{-\partial X_E / \partial p_E} \right) \right].$$

The first-order condition with respect to q_I for a constrained maximum of (11) is written

$$\frac{\partial \pi_I}{\partial q_I} = \left(\frac{-1}{1+\lambda} \right) \left[\widetilde{\theta}_I X_I + X_E \left(\frac{\partial X_E / \partial q_I}{-\partial X_E / \partial p_E} \right) \right].$$

A.2.2 The ideal price-and-quality cap

The price-and-quality constraint reduces to

$$\alpha p_I - \beta q_I \leq P. \tag{cap-N}$$

With $p_I > p_E$ and $q_I > q_E$, the optimal price weight is still given by

$$\begin{aligned} \alpha^N &= X_I^R - X_E^R \left(\frac{\partial X_E^R / \partial p_I}{-\partial X_E^R / \partial p_E} \right) \\ &\equiv X_I^R - \nu X_E^R, \end{aligned} \tag{alpha-N}$$

where the superscript N is appended to indicate the *Nash* game. The optimal quality weight is still given by

$$\begin{aligned} \beta^N &= \widetilde{\theta}_I^R X_I^R + X_E^R \left(\frac{\partial X_E^R / \partial q_I}{-\partial X_E^R / \partial p_E} \right) \\ &\equiv \widetilde{\theta}_I^R X_I^R - \theta_m^R \nu X_E^R. \end{aligned} \tag{beta-N}$$

Finally, if P^N is chosen so that $\pi_I = 0$, then it must be the case that

$$\mu^N = \frac{1}{1+\lambda^R}. \tag{mu-N}$$

A.2.3 Robustness of the scheme

Still focusing on the case of $p_I > p_E$ and $q_I > q_E$ and omitting superscripts for sake of shortness, we now have

$$
\begin{aligned}
\frac{dV}{d\alpha} &= \frac{\partial V}{\partial p_I}\frac{dp_I}{d\alpha} + \frac{\partial V}{\partial q_I}\frac{dq_I}{d\alpha} \\
&= -X_I\frac{dp_I}{d\alpha} + \widetilde{\theta}_I X_I \frac{dq_I}{d\alpha}
\end{aligned}
$$

together with

$$
\begin{aligned}
\frac{d\pi_E}{d\alpha} &= \frac{\partial \pi_E}{\partial p_I}\frac{dp_I}{d\alpha} + \frac{\partial \pi_E}{\partial q_I}\frac{dq_I}{d\alpha} \\
&= X_E\left(\frac{\partial X_E/\partial p_I}{-\partial X_E/\partial p_E}\right)\frac{dp_I}{d\alpha} + X_E\left(\frac{\partial X_E/\partial q_I}{-\partial X_E/\partial p_E}\right)\frac{dq_I}{d\alpha}.
\end{aligned}
$$

Hence, we can write

$$
\frac{dW}{d\alpha} = \left[-X_I + X_E\left(\frac{\partial X_E/\partial p_I}{-\partial X_E/\partial p_E}\right)\right]\frac{dp_I}{d\alpha} + \left[\widetilde{\theta}_I X_I + X_E\left(\frac{\partial X_E/\partial q_I}{-\partial X_E/\partial p_E}\right)\right]\frac{dq_I}{d\alpha}.
$$

Similarly,

$$
\frac{dW}{d\beta} = \left[-X_I + X_E\left(\frac{\partial X_E/\partial p_I}{-\partial X_E/\partial p_E}\right)\right]\frac{dp_I}{d\beta} + \left[\widetilde{\theta}_I X_I + X_E\left(\frac{\partial X_E/\partial q_I}{-\partial X_E/\partial p_E}\right)\right]\frac{dq_I}{d\beta}.
$$

Imperfections in α The firm's first-order conditions with respect to p_I and q_I are written

$$
\begin{aligned}
\frac{d\pi_I}{dp_I} &= \mu\alpha^N & \text{(pL-cap-N)} \\
\frac{d\pi_I}{dq_I} &= -\mu\beta^N. & \text{(qL-cap-N)}
\end{aligned}
$$

As $\frac{dp_I}{d\alpha} = -\left(\frac{d\pi_I/dq_I}{d\pi_I/dp_I}\right)\frac{dq_I}{d\alpha}$, we can write $\frac{dp_I}{d\alpha} = \frac{\beta^N}{\alpha^N}\frac{dq_I}{d\alpha}$. For the optimal values (alpha-N) and (beta-N), we have

$$
\frac{dp_I}{d\alpha} = \left(\frac{\widetilde{\theta}_I^R X_I^R - \theta_m^R \nu X_E^R}{X_I^R - \nu X_E^R}\right)\frac{dq_I}{d\alpha}.
$$

Plugging this into the expression for $\frac{dW}{d\alpha}$ at the optimal values, we still have $\frac{dW^R}{d\alpha} = 0$.

Imperfections in β As $\frac{dp_I}{d\beta} = -\left(\frac{d\pi_I/dq_I}{d\pi_I/dp_I}\right)\frac{dq_I}{d\beta}$, we can write $\frac{dp_I}{d\beta} = \frac{\beta^N}{\alpha^N}\frac{dq_I}{d\beta}$. Substituting into the expression for $\frac{dW}{d\beta}$, we still obtain $\frac{dW^R}{d\beta} = 0$.

B   Quality as a longer-run choice variable

We hereafter propose the formal analysis for the situation in which quality is a longer-run choice variable, as compared to price. We first solve a three-stage game in which a sequential quality choice is followed by a simultaneous price choice, in order to characterize the regulatory target. We then proceed as in Appendix A with $p_I > p_E$ and $q_I > q_E$.

B.1   The regulatory benchmark

The game unfolds as follows. At stage 1 firm I chooses q_I. At stage 2 firm E chooses q_E. At stage 3 firm I and firm E choose p_I and p_E respectively. We solve the game backward taking into account that, all over the game, firm I faces the constraint $\pi_I \geq 0$ with multiplier λ.

At stage 3, firm E solves the problem

$$\underset{p_E}{Max} \ \pi_E\left(p_E, p_I; q_E^*, q_I^*\right) = p_E X_E(p_E, p_I; q_E^*, q_I^*) - C_E(X_E(p_E, p_I; q_E^*, q_I^*), q_E^*),$$

where a star is appended to indicate previous choices. The first-order condition with respect to p_E is given by

$$p_E - \frac{\partial C_E}{\partial X_E} = \frac{X_E}{-\partial X_E/\partial p_E}. \tag{36}$$

Simultaneously, firm I chooses p_I facing the objective function

$$W\left(p_I, p_E; q_I^*, q_E^*\right) = V\left(p_I, p_E; q_I^*, q_E^*\right) + \pi_I\left(p_I, p_E; q_I^*, q_E^*\right) + \pi_E\left(p_E, p_I; q_E^*, q_I^*\right).$$

Incorporating (36), the optimal choice is characterized by

$$\frac{\partial \pi_I}{\partial p_I} = \left(\frac{1}{1+\lambda}\right)\left[X_I - X_E\left(\frac{\partial X_E/\partial p_I}{-\partial X_E/\partial p_E}\right)\right]. \tag{37}$$

At stage 2, firm E solves the problem

$$\underset{q_E}{Max} \ \pi_E(p_E, p_I; q_E, q_I^*) = p_E X_E(p_E, p_I; q_E, q_I^*) - C_E(X_E(p_E, p_I; q_E, q_I^*), q_E).$$

Firm E anticipates that the choice of q_E will affect the choice of p_E and p_I at stage 3. The first-order condition with respect to q_E is given by

$$\left(p_E - \frac{\partial C_E}{\partial X_E}\right)\left(\frac{\partial X_E}{\partial q_E} + \frac{\partial X_E}{\partial p_E}\frac{dp_E}{dq_E} + \frac{\partial X_E}{\partial p_I}\frac{dp_I}{dq_E}\right) + X_E\frac{dp_E}{dq_E} - \frac{\partial C_E}{\partial q_E} = 0.$$

Recalling (36), replacing above and manipulating, we get

$$\frac{X_E}{-\partial X_E/\partial p_E}\left(\frac{\partial X_E}{\partial q_E} + \frac{\partial X_E}{\partial p_I}\frac{dp_I}{dq_E}\right) = \frac{\partial C_E}{\partial q_E}. \tag{38}$$

At stage 1, firm I chooses q_I facing the objective function

$$W\left(p_I, p_E; q_I, q_E\right) = V\left(p_I, p_E; q_I, q_E\right) + \pi_I\left(p_I, p_E; q_I, q_E\right) + \pi_E\left(p_E, p_I; q_E, q_I\right).$$

Firm I anticipates that the choice of q_I will have (a) a direct impact on the choice of q_E at stage 2, (b) a direct impact on the choice of p_E and p_I at stage 3, (c) an indirect impact on the choice of p_E and p_I at stage 3 through the impact on the choice of q_E at stage 2.

The choice of q_I is characterized by

$$\frac{dV}{dq_I} + (1+\lambda)\frac{d\pi_I}{dq_I} + \frac{d\pi_E}{dq_I} = 0. \tag{39}$$

We have

$$
\begin{aligned}
\frac{dV}{dq_I} &= \frac{\partial V}{\partial q_I} + \left(\frac{\partial V}{\partial q_E} + \frac{\partial V}{\partial p_I}\frac{dp_I}{dq_E} + \frac{\partial V}{\partial p_E}\frac{dp_E}{dq_E}\right)\frac{dq_E}{dq_I} + \frac{\partial V}{\partial p_I}\frac{dp_I}{dq_I} + \frac{\partial V}{\partial p_E}\frac{dp_E}{dq_I} \\
&= \widetilde{\theta}_I X_I + \left(\widetilde{\theta}_E X_E - X_I\frac{dp_I}{dq_E} - X_E\frac{dp_E}{dq_E}\right)\frac{dq_E}{dq_I} - X_I\frac{dp_I}{dq_I} - X_E\frac{dp_E}{dq_I}. \tag{40}
\end{aligned}
$$

Moreover, we can compute

$$
\begin{aligned}
\frac{d\pi_E}{dq_I} &= \frac{\partial \pi_E}{\partial q_I} + \frac{\partial \pi_E}{\partial q_E}\frac{dq_E}{dq_I} + \frac{\partial \pi_E}{\partial p_E}\frac{dp_E}{dq_I} + \frac{\partial \pi_E}{\partial p_I}\left(\frac{dp_I}{dq_I} + \frac{dp_I}{dq_E}\frac{dq_E}{dq_I}\right) \\
&= \frac{\partial \pi_E}{\partial q_I} + \frac{\partial \pi_E}{\partial p_I}\left(\frac{dp_I}{dq_I} + \frac{dp_I}{dq_E}\frac{dq_E}{dq_I}\right) \\
&= X_E\left[\left(\frac{\partial X_E/\partial q_I}{-\partial X_E/\partial p_E}\right) + \left(\frac{\partial X_E/\partial p_I}{-\partial X_E/\partial p_E}\right)\left(\frac{dp_I}{dq_I} + \frac{dp_I}{dq_E}\frac{dq_E}{dq_I}\right)\right].
\end{aligned}
$$

We can finally rewrite (39) as

$$
\begin{aligned}
\frac{d\pi_I}{dq_I} &= \frac{1}{1+\lambda}\left[X_I - X_E\left(\frac{\partial X_E/\partial p_I}{-\partial X_E/\partial p_E}\right)\right]\frac{dp_I}{dq_I} \\
&\quad -\frac{1}{1+\lambda}\left[\widetilde{\theta}_I X_I + X_E\left(\frac{\partial X_E/\partial q_I}{-\partial X_E/\partial p_E}\right)\right. \\
&\quad +X_E\left(\frac{\partial X_E/\partial p_I}{-\partial X_E/\partial p_E}\right)\frac{dp_I}{dq_E}\frac{dq_E}{dq_I} - X_E\frac{dp_E}{dq_I} \\
&\quad \left.+\left(\widetilde{\theta}_E X_E - X_I\frac{dp_I}{dq_E} - X_E\frac{dp_E}{dq_E}\right)\frac{dq_E}{dq_I}\right].
\end{aligned}
\tag{41}
$$

Overall, the regulatory target is given by the pair of conditions (37) and (41).

B.2   The ideal price-and-quality cap

Let the constraint

$$\alpha p_I - \beta q_I \leq P + \gamma p_E - \delta q_E, \tag{cap-III}$$

Firm I maximizes π_I subject to (cap-III). The Lagrangian is written

$$\ell = \pi_I + \mu\left(P + \gamma p_E - \delta q_E - \alpha p_I + \beta q_I\right),$$

with μ the multiplier associated with (cap-III). We thus have

$$\frac{\partial \pi_I}{\partial p_I} = \mu\alpha \tag{pL-cap-III}$$

$$
\begin{aligned}
\frac{d\pi_I}{dq_I} &= -\mu\left[\gamma\left(\frac{dp_E}{dq_I} + \frac{dp_E}{dq_E}\frac{dq_E}{dq_I}\right) - \delta\frac{dq_E}{dq_I}\right. \\
&\quad \left. -\alpha\left(\frac{dp_I}{dq_I} + \frac{dp_I}{dq_E}\frac{dq_E}{dq_I}\right) + \beta\right].
\end{aligned}
\tag{qL-cap-III}
$$

With $p_I > p_E$ and $q_I > q_E$, the regulator should set

$$
\begin{align}
\alpha^{III} &= X_I^R - \nu X_E^R & \text{(alpha-III)}\\
\beta^{III} &= \widetilde{\theta}_I^R X_I^R - \theta_m^R \nu X_E^R & \text{(beta-III)}\\
\gamma^{III} &= -X_E^R & \text{(gamma-III)}\\
\delta^{III} &= -\widetilde{\theta}_E^R X_E^R & \text{(delta-III)}
\end{align}
$$

and P^{III} such that

$$
\mu^{III} = \frac{1}{1+\lambda^R}, \tag{mu-III}
$$

where the superscript III is appended to indicate the optimal values of the weights and the ceiling in the *three*-stage game.

B.3   Robustness of the scheme

Omitting superscripts for sake of shortness, we now have

$$
\begin{align}
\frac{dV}{d\alpha} &= \frac{\partial V}{\partial p_I}\frac{dp_I}{d\alpha} + \left[\frac{\partial V}{\partial q_I} + \frac{\partial V}{\partial q_E}\frac{dq_E}{dq_I} + \frac{\partial V}{\partial p_E}\left(\frac{dp_E}{dq_I} + \frac{dp_E}{dq_E}\frac{dq_E}{dq_I}\right)\right]\frac{dq_I}{d\alpha}\\
&= -X_I\frac{dp_I}{d\alpha} + \left[\widetilde{\theta}_I X_I + \widetilde{\theta}_E X_E\frac{dq_E}{dq_I} - X_E\left(\frac{dp_E}{dq_I} + \frac{dp_E}{dq_E}\frac{dq_E}{dq_I}\right)\right]\frac{dq_I}{d\alpha}.
\end{align}
$$

together with

$$
\begin{align}
\frac{d\pi_E}{d\alpha} &= \frac{\partial \pi_E}{\partial p_I}\left[\frac{dp_I}{d\alpha} + \left(\frac{dp_I}{dq_I} + \frac{dp_I}{dq_E}\frac{dq_E}{dq_I}\right)\frac{dq_I}{d\alpha}\right] + \frac{\partial \pi_E}{\partial q_I}\frac{dq_I}{d\alpha}\\
&= X_E\left(\frac{\partial X_E/\partial p_I}{-\partial X_E/\partial p_E}\right)\left[\frac{dp_I}{d\alpha} + \left(\frac{dp_I}{dq_I} + \frac{dp_I}{dq_E}\frac{dq_E}{dq_I}\right)\frac{dq_I}{d\alpha}\right]\\
&\quad + X_E\left(\frac{\partial X_E/\partial q_I}{-\partial X_E/\partial p_E}\right)\frac{dq_I}{d\alpha}\\
&= X_E\left[\left(\frac{\partial X_E/\partial p_I}{-\partial X_E/\partial p_E}\right)\left(\frac{dp_I}{dq_I} + \frac{dp_I}{dq_E}\frac{dq_E}{dq_I}\right)\right.\\
&\quad \left. + \left(\frac{\partial X_E/\partial q_I}{-\partial X_E/\partial p_E}\right)\right]\frac{dq_I}{d\alpha} + X_E\left(\frac{\partial X_E/\partial p_I}{-\partial X_E/\partial p_E}\right)\frac{dp_I}{d\alpha}
\end{align}
$$

Hence, we can write

$$
\begin{align}
\frac{dW}{d\alpha} &= \left[X_E\left(\frac{\partial X_E/\partial p_I}{-\partial X_E/\partial p_E}\right) - X_I\right]\frac{dp_I}{d\alpha} + \left\{\widetilde{\theta}_I X_I + \widetilde{\theta}_E X_E\frac{dq_E}{dq_I} - X_E\left(\frac{dp_E}{dq_I} + \frac{dp_E}{dq_E}\frac{dq_E}{dq_I}\right)\right.\\
&\quad \left. + X_E\left[\left(\frac{\partial X_E/\partial p_I}{-\partial X_E/\partial p_E}\right)\left(\frac{dp_I}{dq_I} + \frac{dp_I}{dq_E}\frac{dq_E}{dq_I}\right) + \left(\frac{\partial X_E/\partial q_I}{-\partial X_E/\partial p_E}\right)\right]\right\}\frac{dq_I}{d\alpha}.
\end{align}
$$

Similarly,

$$
\begin{align}
\frac{dW}{d\beta} &= \left[X_E\left(\frac{\partial X_E/\partial p_I}{-\partial X_E/\partial p_E}\right) - X_I\right]\frac{dp_I}{d\beta} + \left\{\widetilde{\theta}_I X_I + \widetilde{\theta}_E X_E\frac{dq_E}{dq_I} - X_E\left(\frac{dp_E}{dq_I} + \frac{dp_E}{dq_E}\frac{dq_E}{dq_I}\right)\right.\\
&\quad \left. + X_E\left[\left(\frac{\partial X_E/\partial p_I}{-\partial X_E/\partial p_E}\right)\left(\frac{dp_I}{dq_I} + \frac{dp_I}{dq_E}\frac{dq_E}{dq_I}\right) + \left(\frac{\partial X_E/\partial q_I}{-\partial X_E/\partial p_E}\right)\right]\right\}\frac{dq_I}{d\beta}.
\end{align}
$$

B.3.1   Imperfections in α

From (pL-cap-III) and (qL-cap-III), we can compute

$$\frac{dp_I}{d\alpha} = \frac{1}{\alpha}\left[\gamma\left(\frac{dp_E}{dq_I} + \frac{dp_E}{dq_E}\frac{dq_E}{dq_I}\right) - \delta\frac{dq_E}{dq_I} \right. $$
$$\left. -\alpha\left(\frac{dp_I}{dq_I} + \frac{dp_I}{dq_E}\frac{dq_E}{dq_I}\right) + \beta\right]\frac{dq_I}{d\alpha}$$

At the optimal values of the weights (alpha-III) - (delta-III), this becomes

$$\frac{dp_I}{d\alpha} = \frac{1}{X_I^R - \nu X_E^R}\left[-X_E^R\left(\frac{dp_E}{dq_I} + \frac{dp_E}{dq_E}\frac{dq_E}{dq_I}\right) + \widetilde{\theta}_E^R X_E^R\frac{dq_E}{dq_I} \right. $$
$$\left. - \left(X_I^R - \nu X_E^R\right)\left(\frac{dp_I}{dq_I} + \frac{dp_I}{dq_E}\frac{dq_E}{dq_I}\right) + \widetilde{\theta}_I^R X_I^R - \theta_m^R \nu X_E^R\right]\frac{dq_I}{d\alpha}.$$

Replacing this, we have $\frac{dW^R}{d\alpha} = 0$.

B.3.2   Imperfections in β

From (pL-cap-III) and (qL-cap-III), we can compute

$$\frac{dp_I}{d\beta} = \frac{1}{\alpha}\left[\gamma\left(\frac{dp_E}{dq_I} + \frac{dp_E}{dq_E}\frac{dq_E}{dq_I}\right) - \delta\frac{dq_E}{dq_I} \right. $$
$$\left. -\alpha\left(\frac{dp_I}{dq_I} + \frac{dp_I}{dq_E}\frac{dq_E}{dq_I}\right) + \beta\right]\frac{dq_I}{d\beta}.$$

At the optimal values of the weights (alpha-III) - (delta-III), this becomes

$$\frac{dp_I}{d\beta} = \frac{1}{X_I^R - \nu X_E^R}\left[-X_E^R\left(\frac{dp_E}{dq_I} + \frac{dp_E}{dq_E}\frac{dq_E}{dq_I}\right) + \widetilde{\theta}_E^R X_E^R\frac{dq_E}{dq_I} \right. $$
$$\left. - \left(X_I^R - \nu X_E^R\right)\left(\frac{dp_I}{dq_I} + \frac{dp_I}{dq_E}\frac{dq_E}{dq_I}\right) + \widetilde{\theta}_I^R X_I^R - \theta_m^R \nu X_E^R\right]\frac{dq_I}{d\beta}.$$

Replacing this, we have $\frac{dW^R}{d\beta} = 0$.