TESTING THE EXPECTATIONS THEORY FOR THE PORTUGUESE YIELD CURVE *

*Bernardino Adão***

*Jorge Barros Luís ***

1. INTRODUCTION

According to the expectations theory, long-term interest rates are determined by the present level and by the expected path of short-term interest rates. Thus they can be used to estimate future short-term interest rates. The pure version of the expectations theory sustains that such estimates are unbiased, i.e., that the expected future short-term interest rates implicit in the spot long-term interest rates are on average equal to the forward short-term interest rates. However, long-term interest rates tend to overestimate the future short-term interest rates. This bias can be due to a term premium, i.e., the additional return required by an investor in order for him to choose investments in larger maturities. If this premium is constant, the expectations theory will still hold in a non-pure version, while if the former is variable the expectations theory will not be valid(1).

If the expectations theory is valid, information on the future path of short-term interest rates can be drawn from the yield curve. Since forward short-term interest rates are in fact advanced indicators of important macroeconomics variables, their knowledge is central for economic agents — specially for financial asset portfolio managers. Furthermore, the validity of the expectations theory allows Central Banks to measure the impact of their monetary policies on the term structure of interest rates, and to exclude the possibility of maturity segmentation in the financial markets.

This paper aims at testing the expectations theory for the term structure of Portuguese interest rates. Though no similar research has been developed for Portugal, an extensive set of studies investigate this issue as regards other countries. These researches are almost unanimous in two findings: first, the expectations theory with constant term premium is rejected for the USA; secondly, the expectations theory is not rejected for the remaining countries(2).

This paper utilises the methodology defined by Campbell and Shiller (1991) to analyse if the Portuguese yield curve is in agreement with the expectations theory. A bootstrap-based test was built as to determine the statistical significance of findings.

The paper proceeds as follows: section 2 describes the model; section 3 presents the empirical results; results of the statistical significance tests are shown in the fourth section and section 5 draws the main conclusions.

2. THE MODEL

To analyse the validity of the expectations theory for Portugal, we start by assuming a simple result that relates pairs of interest rates with different maturities; the greatest maturity of both is called the long-term, and the smallest is named the short-term. R_t^n stands for the long-term interest rate observed in moment t (n being the longest

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** Research and Statistics Department.

(1) As indicated in Anderson *et al.* (1996) the premium is due to the investors' risk aversion and to the Jensen's inequality.

(2) Anderson et al. (1996) summarises the findings of the most recent investigations.

maturity) and R_t^m is the short-term interest rate observed in moment t (m being the shortest maturity). Maturities m and n are such that $m<n$ and n/m is an integer. The non-pure linearised version of the expectations theory for the term structure of interest rates says that the theoretical long-term interest rate in moment t ($R_t^{n^*}$) equals a simple average of the present and future values for the short-term interest rates that economic agents in moment t expect for the period running between t and $t+n$($E_t R_{t+mi}^m, i=0,\ldots,(n-m)/m$), plus a constant term premium (c_{nm}):[3].

$$R_t^{n^*} = \frac{m}{n} \sum_{i=0}^{(n-m)/m} E_t R_{t+mi}^m + c_{nm}, \tag{1}$$

where E_t denotes mathematical expectation. The term premium is defined as the additional return (i.e., the yield additional to the expected rate of return of a succession of n/m short-term — m-maturity — investments) demanded by an investor, during the period running from t to $t + n$, which makes the investor indifferent between this investment and a long-term investment with maturity in period $t + n$.

Equation (1) does not specify the process by which expectations are formed. Some tests on the expectations theory are based on adaptive expectations. However, in most tests the expressions $E_t R_{t+mi}^m$ are replaced by their observed values R_{t+mi}^m. Underlying this procedure is the assumption that people do not commit systematic prediction errors, and hence R_{t+mi}^m are taken as unbiased estimates for $E_t R_{t+mi}^m$. Since our data base refers to a very short period (a little longer than 3 years), R_{t+mi}^m cannot be used as an estimate for $E_t R_{t+mi}^m$, since the number of estimates would be small or even nil (e.g., if the maturity is greater than 4 years, no estimate is obtained through such procedure). Furthermore, the assumption of a null prediction error does not imply that those errors are nil at each and every moment.

Campbell and Schiller (1987) proposes a test that overcomes this obstacle. According to this test, expectations regarding the future path of interest rates are assessed by using a vector autoregression (VAR)[4].

Let $S_t^{nm} \equiv R_t^n - R_t^m$ and $S_t^{nm^*} \equiv R_t^{n^*} - R_t^m$ stand respectively for the observed and the theoretical spread between the long-term and short-term interest rates, and $\Delta R_t^m = R_t^m - R_{t-1}^m$ stand for the first difference of short-term interest rates. Rearranging, the following expression for the theoretical spread is obtained:

$$S_t^{nm^*} = \sum_{i=1}^{(n-m)/m} \frac{n-mi}{n} \left(\sum_{j-1}^{m} E_t \Delta_t R_{t+mi+1-j}^m \right) + c_{nm}. \tag{2}$$

Equation (2) indicates that the theoretical spread must predict future changes in the short-term interest rate whenever the expectations theory is verified. As mentioned before, the prediction of the future change in m-maturity rates is obtained with a VAR including S_t^{nm}, ΔR_t^m and lagged values of those variables; these variable are assumed to constitute all the necessary information to forecast future interest rate changes. The first-order VAR can be written as follows:

$$z_t = A z_{t-1} + u_t \tag{3}$$

where $z_t = \left[\Delta R_t^m, \ldots, \Delta R_{t-p+1}^m, S_t^{nm}, \ldots, S_{t-p+1}^{nm}\right]'$, A is the matrix of the VAR coefficients and u_t is a vector of residuals in period t.

The test for each pair of maturities (m, n) consists of: (i) estimating the VAR model given by (3); (ii) calculating the theoretical spread, given by (2); and (iii) testing if the estimated series are similar enough to those observed, by calculating the correlation coefficient between the observed and the theoretical spreads, as well as the ratio between the standard deviations of those spreads. For the null hypothesis not to be rejected, both statistics must be close to one.

Note that the test performed is in fact a compound test of two hypothesis: first, the hypothesis according to which the expectations theory with constant term premium explains the movements exhibited by long-term interest rates; and second,

(3) Campbell (1986) shows that equation (1) is a linear approximation to any one of the non-linear formulas of the expectations theory of the term structure.

(4) However, a test comparing $R_t^{n^*}$ and R_t^n directly as to reject (or not) the expectations theory may become troublesome if the interest rate series are non-stationary. We overcome this difficulty by using interest rates spreads and first differences.

the hypothesis that the best prediction of the future path of the first differences of short-term interest rates is given by a VAR including the past values of the spreads and of the first differences of short-term rates. Consequently, rejecting the null hypothesis could either be an indication that the term premium is not constant over time (i.e., that the expectations theory is not valid) or that economic agents are not using the information encompassed in the VAR (lagged first differences of short-term interest rates and / or lagged interest rate spreads) in their prediction of the future path of the short-term interest rates in first differences.

3. EMPIRICAL RESULTS

Our empirical analysis used daily data. The sample period runs from August 1993 up to October 1996. The database was built by Cassola and Barros Luís (1996) by employing the methods proposed by Svensson (1994) and Nelson and Siegel (1987) to a set of interbank money market and Treasury bond interest rates. The database maturities run from overnight up to ten years.

The stationarity of the involved series was studied(5) and it was observed that between August 1993 and August 1994, series S_t^{nm} exhibit a mean which is slightly different from that of the period beginning in September 1994 up to August 1995, being stationary in both sub-periods. In addition, the short-term interest rates in first differences are stationary in the period as a whole, but are clearly less volatile from September 1995 onwards(6). Given this evidence, our sample was broken-down into three sub-periods with similar sizes: the first from August 1993 to August 1994, the second from September 1994 to August 1995 and the third from September 1995 to October 1996. We also ensured that series S_t^{nm} and ΔR_t^m are stationary in all three sub-periods considered.

One can associate the breakdown of the sample period into three sub-periods to specific domestic market events. Hence, the first sub-period exhibited a negatively sloped yield curve, having occurred disturbances in the money and exchange rate markets. In the second sub-period, the yield curve exhibited a significantly positive slope, and interest rates became less volatile. In the third sub-period, both the yield curve’s slope and the rate's volatility lessened progressively.

The chosen breakdown of the sample period is surely not the only choice possible, but the results do not depend on the size of the sub-periods considered. For example, if any sub-period was made some months longer or some months shorter, our findings would not be altered(7). In addition, our results proved little sensitive to the frequency of observations and to the number of lags considered.

Tables 1, 2 and 3 show the correlations between spreads and the spreads’ standard deviations (in parentheses) for our three sub-periods. These figures correspond to a VAR with 3 lags. The short-term rates considered were the overnight rate and the three-month interbank money market (IMM) rates; the longer-term rates were the 10-year and 5-year Treasury bond spot rates, and the 3-month IMM rates.

The tables show that both the correlation coefficients and the standard deviation ratios are close to one. Furthermore, both statistics are in general closer to one the greatest the difference between short and long maturity. Though the correlation coefficient and the standard deviation ratios’ theoretical distributions are not known, the values we obtain suggest that the null hypothesis is not to be rejected. To validate this conclusion, we searched

(5) The variables were centred as to employ a simplification of (2). Following Campbell and Shiller (1987) equation (2) can be written as follows:

$$S_t^{nm^*} = hA\left[I - \frac{m}{n}\left(I - A^n\right)\left(I - A^m\right)^{-1}\right] \times (I - A)_{zt}^{-1} + C_{nm,}$$

where h stands for the vector selected from the VAR and I is the identity matrix.

The VAR does not include a constant so as to reduce the dimension of matrix A; at the same time, this choice does not exclude matrix A from being non-singular. Therefore variables ΔR_t^m and S_t^{nm} were centred to the mean, so that the estimates for the VAR coefficients were unbiased.

(6) As stressed in Campbell and Shiller (1987), the stationarity of the short-term interest rates in first differences is a sufficient condition to guarantee that the theoretical spread and the observed spread are stationary.

(7) Engsted and Tangaard (1995) studied the prediction abilities of the yield spreads in the Danish bond market. Here, too, was the sample period broken-down into sub-periods, though this procedure was related to the guidance of the Danish monetary policy.

Table 1

CORRELATION COEFFICIENTS AND STANDARD DEVIATION RATIOS BETWEEN S AND S* FOR PORTUGAL

August 93-August 94

n / m	3-month	5-year	10-year
1-day	0.991	0.999	0.999
	(0.986)	(0.951)	1,025
3-month	-	0.998	0.999
	-	(0.785)	(0.947)

Note: The first value in each cell is the correlation coefficient between S and S*, the ratio between the respective standard deviations is in parentheses.

Table 2

CORRELATION COEFFICIENT AND STANDARD DEVIATION RATIOS BETWEEN S AND *S PORTUGAL**

September 94 - August 95

n / m	3-month	5-year	10-year
1-day	0.992	0.999	0.995
	(0.680)	(0.955)	(0.920)
3-month...	-	0.999	0.995
	-	(0.713)	(0.710)

Note: The first value in each cell is the correlation coefficient between *S* and S*; the ratio between the respective standard deviations is in parentheses.

Table 3

CORRELATION COEFFICIENTS AND STANDARD DEVIATION RATIOS BETWEEN S AND *S FOR PORTUGAL**

September 95 - October 96

n / m	3-month	5-year	10-year
1-day.......	0.995	0.999	0.999
	(0.738)	(0.836)	(0.925)
3-month	-	0.997	0.997
	-	(0.606)	(0.785)

Note: The first value in each cell is the correlation coefficient between *S* e S*; the ratio between the respective standard deviations is in parentheses.

for an empirical distribution by using bootstrap procedures. The findings are presented in the following section.

4. STATISTICAL SIGNIFICANCE OF THE ESTIMATES

In this section the bootstrap technique (see Efron (1982)) and the VAR were utilised to generate artificial data so as to obtain a population in line with the expectations theory. The bootstrap seemed more adequate than the Monte Carlo experiments (despite the frequent usage of the latter in literature), since the former allows to generate residuals without specifying any distribution in particular: only the histogram of the residuals obtained initially from the observations for the VAR variables is reproduced. So, this technique allows to determinate the empirical distributions of the correlation coefficient and of the standard deviation ratio between the interest rate spread and the theoretical spread when the null hypothesis is true.

The bootstrap technique uses the residuals obtained in the initial VAR estimation in (3) for each pair of maturities analysed (m, n). This technique is then developed as follows:

(i) a sample of 20 000 observations is generated so that the corresponding histogram must replicate that of the original residuals;

(ii) using the sample generated in (i) as well as the initial values of series $\left\{\Delta R_t^m, S_t^{nm}\right\}_{t=1}^p$ and the initially estimated matrix A, new series are generated for variables ΔR_t^m and S_t^{nm}, with $t = P + 1,..., 20\,000$;

(iii) with the generated series for ΔR_t^m and S^{nm} in step (ii), matrix A is estimated; using (2), the theoretical spread S_t^{nm*} is obtained. Note that the latter was calculated by assuming the test's null hypothesis;

(iv) we then calculate the correlation coefficients and the ratio between the standard deviation of the spreads of the interest rates obeying the expectations theory in the population and the standard deviation of the theoretical spread;

(v) the procedure returns to step (i) and is ran 1000 times.

Proceeding in this way 1000 estimates for the correlation coefficient and for the standard deviation ratio are obtained. Hence, we can estimate the respective empirical distributions, and therefore test the hypothesis in stake.

Table 4, 5 and 6 exhibit the bootstrap results. The earlier conclusions — that the correlation coefficients and the standard deviation ratios calculated with the original series are statistically close to one — is confirmed, and so the null hypothesis

Table 4

STATISTICAL SIGNIFICANCE OF THE CORRELATION COEFFICIENT AND STANDARD DEVIATION RATIOS BETWEEN *S* E *S FOR PORTUGAL**

August 93 - August 94

Probability in 1000 random experiences of a value more distant from one than the estimated value

n / *m*	3-month	5-year	10-year
1-day	0.938	0.393	0.436
	(0.785)	(0.569)	(0.510)
3-month . . .	-	0.999	0.991
	-	(0.977)	(0.996)

Note: The probability of the ratio between the respective standard deviations is in parentheses.

Table 5

STATISTICAL SIGNIFICANCE OF THE CORRELATION COEFFICIENTS AND STANDARD DEVIATION RATIOS *S* E *S FOR PORTUGAL**

September 94 - August 95

Probability in 1000 random experiences of a value more distant from one than the estimated value

n / *m*	3-month	5-year	10-year
1-day	0.701	0.999	0.999
	(0.750)	(0.999)	(0.999)
3-month. . . .	-	0.999	0.999
	-	(0.679)	(0.837)

Note: The probability of the ratio between the respective standard deviations is in parentheses

Table 6

STATISTICAL SIGNIFICANCE OF THE CORRELATION COEFFICIENTS AND STANDARD DEVIATION RATIOS BETWEEN *S* E *S FOR PORTUGAL**

September 95 - October 96

Probability in 1000 random experiences of a value more distant from one than the estimated value

n / *m*	3-month	5-year	10-year
1-day	0.967	0.755	0.849
	(0.728)	(0.733)	(0.871)
3-month. . .	-	0.674	0.579
	-	(0.633)	(0.747)

Note: The probability of the ratio between the respective standard deviations is in parentheses.

is not to be rejected for the tested maturities. In fact, considering all the periods and all the pairs of studied maturities, the smallest number of random experiences with a correlation coefficient between the "observed" (generated) spread and the theoretical spread below that initially estimated is 39%, which occurred for the period between August 1993 and August 1994, for 1-day and 5-year interest rates. As regards the ratio between the standard deviations of those series, that figure stood at 51%, for the same period and for 1-day and 10-year interest rates. Therefore the test's null hypothesis is not rejected at the usual significance levels (5 per cent and 10 per cent)[8].

5. CONCLUSION

The validity of the expectations theory has been analysed by a wide range of international empirical studies. No similar research had been developed as regards the Portuguese case. This paper presents a test to the term structure of Portuguese

(8) These are "one-sided" left-handed tests, i. e., the null hypothesis is rejected whenever the percentage of experiences scoring a value lower than that estimated stands below the significance level of the test. As regards the standard deviation ratio, the null hypothesis is rejected whenever the percentage of experiences scoring a value which is more distant from one that the estimated value stands below the significant level of the test.

interest rates, as to determine if the expectations theory is applicable.

The results obtained by using the method proposed by Campbell and Schiller (1987) point towards the non-rejection of this hypothesis; the bootstrap technique also supported this finding. Hence, the hypothesis according to which the term structure for Portuguese interest rates exhibits a constant risk premium was not rejected for each sub-period considered. This finding allows us to use forward interest rates as the central estimate for the future spot rates (after deducting the respective risk premium) in the sample period. An analysis of the level of the risk premium lies outside the scope of this paper. The available data on the term structure of interest rates suggest interdependence in the determination of short-term and long-term interest rates — that is to say, markets are not segmented.

REFERENCES

Anderson, Nicola, Francis Breedon, Mark Deacon, Andrew Derry e Gareth Murphy (1996), “Estimating and Interpreting the Yield Curve”, John Wiley & Sons, Series in *Financial Economics and Quantitative Analysis.*

Campbell, John (1986), "A Defense of Traditional Hypotheses about the Term Structure of Interest Rates", *Journal of Finance,* 41, pp. 183-193.

Campbell, John Y. and Robert J. Shiller (1991), "Yield Spreads and Interest Rate Movements: A Bird's Eye View", *Review of Economic Studies*, 58, pp. 495-514

Campbell, John Y. And Robert J. Shiller (1987), "-Cointegration and Tests of Present Value Models", *Journal of Political Economy*, 95, pp.1062-1088.

Cassola, Nuno and Jorge Barros Luís (1996), "The Term Structure of Interest Rates: A Comparison of Alternative Estimation Methods with an Application to Portugal", Banco de Portugal, Working Paper No. 17-96.

Efron (1982), “The Jacknife, the Bootstrap and other Resampling Plans”, SIAM, monograph no. 38, CBMS-NSF.

Engsted, Tom and Carsten Tangaard (1995), "The Predictive Power of Yield Spreads for Future Interest Rates: Evidence from the Danish Term Structure", *Scandinavian Journal of Economics*, 97(1), pp. 145-159.

Nelson, Charles R. and Andrew F. Siegel (1987), "P-arsimonious Modelling of Yield Curves", *Journal of Business*, Vol. 60, No.4.

Svensson, Lars E.O. (1994), "Estimating and Interpreting the Forward Interest Rates: Sweden 1992-4", CEPR Discussion Paper Series No. 1051.