EUROPEAN CENTRAL BANK

WORKING PAPER SERIES

NO. 450 / MARCH 2005

EUROSYSTEM INFLATION PERSISTENCE NETWORK

USING MEAN REVERSION AS A MEASURE OF PERSISTENCE

by Daniel Dias

and Carlos Robalo Marques

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USING MEAN REVERSION AS A MEASURE OF PERSISTENCE [1]

by Daniel Dias [2]

and Carlos Robalo Marques [3]

In 2005 all ECB publications will feature a motif taken from the €50 banknote.

1 We especially thank for useful discussions, Stephen Cecchetti, Matteo Ciccarelli, Jordi Gali, Vitor Gaspar, Andrew Levin, James Stock and participants at the ECB's "Inflation Persistence Network" Conference, held at Frankfurt, December 2004. Helpful suggestions from José Maria Brandão de Brito, José Ferreira Machado and João Santos Silva are also acknowledged. The usual disclaimer applies.

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The Eurosystem Inflation Persistence Network

This paper reflects research conducted within the Inflation Persistence Network (IPN), a team of Eurosystem economists undertaking joint research on inflation persistence in the euro area and in its member countries. The research of the IPN combines theoretical and empirical analyses using three data sources: individual consumer and producer prices; surveys on firms' price-setting practices; aggregated sectoral, national and area-wide price indices. Patterns, causes and policy implications of inflation persistence are addressed.

The IPN is chaired by Ignazio Angeloni; Stephen Cecchetti (Brandeis University), Jordi Galí (CREI, Universitat Pompeu Fabra) and Andrew Levin (Board of Governors of the Federal Reserve System) act as external consultants and Michael Ehrmann as Secretary.

The refereeing process is co-ordinated by a team composed of Vítor Gaspar (Chairman), Stephen Cecchetti, Silvia Fabiani, Jordi Galí, Andrew Levin, and Philip Vermeulen. The paper is released in order to make the results of IPN research generally available, in preliminary form, to encourage comments and suggestions prior to final publication. The views expressed in the paper are the author's own and do not necessarily reflect those of the Eurosystem.

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ISSN 1561-0810 (print)
ISSN 1725-2806 (online)

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Abstract

This paper elaborates on the alternative measure of persistence recently suggested in Marques (2004), which is based on the idea of mean reversion. A formal distinction between the "unconditional probability of a given process not crossing its mean in period t" and its estimator, is made clear and the relationship between this new measure and the widely used "sum of the autoregressive coefficients", as alternative measures of persistence, is investigated. Using the law of large numbers and the central limit theorem, properties for the estimator of the new measure of persistence are established, which allow tests of hypotheses to be performed, under very general conditions. Finally, some Monte Carlo experiments are conducted in order to compare the finite sample properties of the estimator for the "unconditional probability of a given process not crossing its mean in period t" and the OLS estimator for the "sum of the autoregressive coefficients".

JEL Classification: E31, C22, E52;

Keywords: Inflation persistence; mean reversion, non-parametric estimator;

NON-TECHNICAL SUMMARY

This paper is a contribution to a recently growing literature trying to measure persistence of inflation. Recently Marques (2004) suggested a non-parametric measure of persistence, which relies on the idea that there is a relationship between persistence and mean reversion. This paper goes a step further in the direction of establishing the properties of the estimator of such alternative measure of persistence, which has the advantage of not requiring the specification and estimation of a model for the series under investigation. The distinction between the theoretical concept of mean reversion, denoted in the paper by γ and its estimator, $\hat{\gamma}$, is now made clear. Some important theoretical properties of the estimator $\hat{\gamma}$, including unbiasedness, consistency and its asymptotic distribution are discussed, under very general conditions. The relationship between the different measures of persistence, with special focus on γ and ρ, the widely used "sum of the autoregressive coefficients" is investigated. Finally, Monte Carlo simulations are used to investigate the finite sample properties of $\hat{\gamma}$ including its robustness against outliers and potential model misspecifications, vis-à-vis the OLS estimator of ρ.

Several results emerge from the paper. First, the process of obtaining the theoretical value of γ for a general data generating process is derived, and the values of γ for a set of AR(1) and AR(2) models are specifically presented, where γ is the unconditional probability of a given process not crossing its mean in period t.

Second, the distributional properties of $\hat{\gamma}$ are investigated for a very broad class of stationary processes. In particular, it is shown that when the mean of the time series process is known, $\hat{\gamma}$ is an unbiased estimator of γ and that, when the mean is unknown, $\hat{\gamma}$ is a consistent estimator of γ. Finally, using the central limit theorem, it is demonstrated that $\hat{\gamma}$ has an asymptotic normal distribution, so that tests of hypotheses on γ can be performed. The implementation of such tests requires an estimate of the spectral density at frequency zero, which may be obtained using, for instance, kernel-based methods.

Third, using Monte Carlo simulations for the AR(1) and AR(2) processes with normal innovations, it is confirmed that i) $\hat{\gamma}$ is an unbiased estimator of γ when the mean of the process is known and slightly downward biased when the mean of the process is unknown; ii) as expected, the OLS estimator of ρ, $\hat{\rho}$, is slightly downward biased when the mean is known and significantly downward biased when the mean is unknown; iii) both $\hat{\gamma}$ and $\hat{\rho}$ are consistent estimators, but the biases of $\hat{\gamma}$ are always smaller than those of $\hat{\rho}$ for the same sample size; iv) the finite sample distribution of $\hat{\gamma}$ is well approximated by the normal distribution while that of $\hat{\rho}$ is not; v) $\hat{\rho}$ may be subject to significant biases coming from the presence of additive outliers in the data or from model misspecifications, but $\hat{\gamma}$ is almost not affected by the first potential source of bias and is, by definition, totally immune to the second.

Finally, it is shown that there is a monotonic relationship between ρ and γ (and some other measures of persistence) when the data are generated by an AR(1) process, but such a monotonic relationship ceases to exist once higher order autoregressive processes are considered. Assuming an AR(2) process with a fixed ρ it is shown that just by choosing alternative combinations of the two coefficients it is possible to make an whole set of alternative measures of persistence (including γ, the half-life, the largest autoregressive root or the number of periods required for 99 percent of the total adjustment to take place) to go through a wide range of variation and in opposite directions. From this evidence it is argued that one should not stick to a single scalar measure to evaluate persistence in higher order processes and, in particular, given their different information content and the robustness of $\hat{\gamma}$, that ρ and γ should be computed as companion measures of persistence, on a regular basis, in empirical applications.

1. INTRODUCTION

Quantifying the response of inflation to shocks hitting the economy is crucial for the implementation of monetary policy. In particular, quantifying the sluggish response of inflation to changes in monetary conditions appears as a fundamental prerequisite if central banks aim at implementing a pre-emptive monetary policy strategy. By definition, however, measuring the speed of the response of inflation to shocks implies evaluating persistence of inflation.

This paper is a contribution to a recently growing literature trying to measure persistence of inflation. Recently Marques (2004) suggested a non-parametric measure of persistence, which relies on the idea that there is a relationship between persistence and mean reversion. This paper goes a step further in the direction of establishing the properties of the estimator of such alternative measure of persistence, which has the advantage of not requiring the specification and estimation of a model for the series under investigation. The distinction between the theoretical concept of mean reversion, denoted in the paper by γ and its estimator, $\hat{\gamma}$, is now made clear. Some important theoretical properties of the estimator $\hat{\gamma}$, including unbiasedness, consistency and its asymptotic distribution are discussed, under very general conditions. The relationship between the different measures of persistence, with special focus on γ and ρ, the widely used "sum of the autoregressive coefficients" is investigated. Finally, Monte Carlo simulations are used to investigate the finite sample properties of $\hat{\gamma}$ including its robustness against outliers and potential model misspecifications, vis-à-vis the OLS estimator of ρ.

A set of relevant results emerges from the paper. First, the process of obtaining the theoretical value of γ for a general data generating process (DGP) is derived, and the values of γ for a set of AR(1) and AR(2) models are specifically presented, where γ is the unconditional probability of a given process not crossing its mean in period t.

Second, we analyse the distributional properties of $\hat{\gamma}$ for a very broad class of stationary processes. In particular, when the mean of the time series process is known, we show that $\hat{\gamma}$ is an unbiased estimator of γ. When the mean is unknown, we use the law of large numbers to show that $\hat{\gamma}$ is a consistent estimator of γ. Finally, using the central limit theorem, we demonstrate that $\hat{\gamma}$ has an asymptotic normal distribution, so that tests of hypotheses on γ can be performed. The implementation of such tests requires an estimate of the spectral density at frequency zero, which may be obtained using, for instance, kernel-based methods.

Third, using Monte Carlo simulations for the AR(1) and AR(2) processes with normal innovations, it is confirmed that i) $\hat{\gamma}$ is an unbiased estimator of γ when the mean of the process is known and slightly downward biased when the mean of the series is unknown; ii) as expected, the OLS estimator of ρ, $\hat{\rho}$, is slightly downward biased when the mean is known and significantly downward biased when the mean is unknown; iii) both $\hat{\gamma}$ and $\hat{\rho}$ are consistent estimators, but the biases of $\hat{\gamma}$ are always smaller than those of $\hat{\rho}$ for the same sample size; iv) the finite sample distribution of $\hat{\gamma}$ is well approximated by the normal distribution while that of $\hat{\rho}$ is not; v) $\hat{\rho}$ may be subject to significant biases coming from the presence of additive outliers in the data or from model misspecifications, but $\hat{\gamma}$ is almost not affected by the first potential source of bias and is, by definition, totally immune to the second.

Finally, it is shown that there is a monotonic relationship between ρ and γ (and some other measures of persistence) when the data are generated by an AR(1) process, but such a monotonic relationship ceases to exist once higher order autoregressive processes are considered. Assuming an AR(2) process with a fixed ρ it is shown that just by choosing alternative combinations of the two coefficients it is possible to make an whole set of alternative measures of persistence (including γ, the half-life, the largest autoregressive root or the number of periods required for 99 percent of the total adjustment to take place) to go through a wide range of variation and in opposite directions. From this evidence it is argued that one should not stick to a single scalar measure to evaluate persistence in higher order processes and, in particular, given their different information content and the robustness of $\hat{\gamma}$, that ρ and γ should be computed as companion measures of persistence, on a regular basis, in empirical applications.

The rest of the paper is organized as follows. Section 2 briefly reviews the measures of persistence of common use in the literature. Section 3 explains the meaning of γ and its estimator $\hat{\gamma}$ and derives the properties of $\hat{\gamma}$ including its asymptotic distribution. Section 4 discusses the relationship among alternative measures of persistence in the context of the AR(1) and AR(2) models. Section 5, using Monte Carlo simulations, investigates the finite sample properties of $\hat{\gamma}$ and $\hat{\rho}$ including the robustness of these two estimators against outliers and model misspecifications. Section 6 concludes.

2. EXISTING MEASURES OF PERSISTENCE

This section presents a formal definition of persistence and discusses the pros and cons of existing scalar measures of persistence[1]. In general terms, persistence of inflation may be defined as the tendency of inflation to revert slowly to its equilibrium or long run level after a shock. Usually, in order to get an estimate for the degree of persistence, i.e., the speed with which inflation converges to its long run level after a shock, an econometric model is specified and estimated. For instance, under the so-called univariate approach, persistence is investigated by looking at the univariate time series representation of inflation. For that purpose it is usually assumed that inflation follows a stationary autoregressive process of order p (AR(p)), which may be written as

$$y_t = \alpha + \sum_{j=1}^{p} \beta_j y_{t-j} + \varepsilon_t \tag{2.1}$$

and reparameterised as:

$$\Delta y_t = \alpha + \sum_{j=1}^{p-1} \delta_j \Delta y_{t-j} + (\rho - 1) y_{t-1} + \varepsilon_t \tag{2.2}$$

where

$$\rho = \sum_{j=1}^{p} \beta_j \tag{2.3}$$

$$\delta_j = -\sum_{i=1+j}^{p} \beta_i \tag{2.4}$$

In the context of model (2.1) inflation is said to be (highly) persistent if, following a shock to the disturbance term, inflation converges slowly to its mean (which in the context of such model is seen as representing the equilibrium level of inflation). Thus,

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[1] This section draws heavily on Marques (2004),

in the context of this parametric representation of inflation, the concept of persistence appears as intimately linked to the impulse response function (IRF) of the AR(p) process. However, as the impulse response function is an infinite-length vector it is not a useful measure of persistence. To overcome such difficulty, several scalar measures of persistence have been proposed in the literature. These include the "sum of the autoregressive coefficients", the "spectrum at zero frequency", the "largest autoregressive root" and the "half-life".

Andrews and Chen (1994) argue that the cumulative impulse response (CIR) is generally a good way of summarizing the information contained in the impulse response function (IRF) and as such a good scalar measure of persistence. In a simple AR(p) process, the cumulative impulse response is simply given by $CIR = \frac{1}{1-\rho}$ where ρ is the "sum of the autoregressive coefficients", as defined in (2.3). As there is a monotonic relation between the CIR and ρ it follows that, under the above assumption, one can simply rely on the "sum of the autoregressive coefficients" as a measure of persistence.

Using the CIR=1/(1-ρ) or simply ρ as a measure of persistence amounts to measuring persistence as the sum of the disequilibria (deviations from equilibrium) generated during the whole convergence period (which is infinite in the AR(p) model). The larger the ρ, the larger the cumulative impact of the shock will be (for the full convergence period). In the context of a simple AR(1) model, with the exception of the half-life, as we shall see below, the different scalar measures of persistence basically deliver the same message in relative terms. This is due to the fact that the speed of convergence as measured in the IRF by ρ^k as $k \to \infty$ is constant throughout the whole convergence period (or, in other words, the disequilibrium in period k, ρ^k, is a fixed proportion, ρ, of the disequilibrium in period $k-1$, ρ^{k-1}).

However, using ρ as a measure of persistence when the series follows an AR(p) process with p>1 may, in some circumstances, be very misleading. Andrews and Chen (1994) discuss several situations in which the CIR and thus also ρ might not be sufficient to fully capture the existence of different shapes in the impulse response function. In particular, ρ, as a measure of persistence, will not be able to distinguish between two series in which one exhibits a large initial increase and then a subsequent quick decrease in the IRF while the other exhibits a relatively small initial increase followed by a subsequent slow decrease in the IRF. We shall show below that ρ is also not able to distinguish between two series in which one exhibits a cyclical behaviour while the other does not. Unfortunately, as we shall illustrate in the context of the AR(2) process, those situations rather than being pathological are very frequent in the context of higher order processes. For such processes the speed of convergence may vary significantly during the convergence period and as this impacts differently on the alternative measures of persistence the monotonic relation among such measures which stands for the AR(1) model ceases to exist for higher order process.

The "spectrum at zero frequency", is a well-known measure of the low-frequency autocovariance of the series and, for the AR(p) process it is given by $h(0) = \frac{\sigma_{\varepsilon}^2}{(1-\rho)^2}$ where σ_{ε}^2 stands for the variance of ε_t. Again, for a fixed σ_{ε}^2, there is a simple correspondence between this concept, the CIR and ρ, and so they can be seen as equivalent measures of persistence. However the two measures can deliver different results if one wants to test for changes in persistence over time. In such a situation the use of the "spectrum at zero frequency" may become problematic because changes in persistence will be brought about not only by changes in ρ but also by changes in σ_{ε}^2. An additional advantage of ρ over $h(0)$ as a measure of persistence is that it is more intuitive and has a small and clearly defined range of potential variation (for a stationary process it varies between –1 and 1), which is not the case of the "spectrum at frequency zero".

The "largest autoregressive root" of model (2.1) has also been used in the literature as a measure of persistence (see, for instance Stock, 2001). The use of this statistic as a measure of persistence is criticised both in Andrews and Chen (1994) and in Pivetta and Reis (2001). The main point against this statistic is that it is a very poor summary measure of the IRF because the shape of this function depends also on the other roots and not only on the largest one. On the positive side, an important argument favouring the use of the largest autoregressive root as a measure of persistence is the fact that an asymptotic theory has been developed and appropriate software is available so that it becomes ease to compute asymptotically valid confidence intervals for the corresponding estimates (see, Stock, 1991 and 2001).

Finally, the "half-life" is defined as the number of periods for which the effect of a unit shock to inflation remains above 0.5. In the case of an AR(1) process given by $y_t = \rho y_{t-1} + \varepsilon_t$ it is easy to show that the half-life may be computed as $h = \frac{\ln(1/2)}{\ln(\rho)}$.

The use of the "half-life" has been criticised on several grounds (see, for instance Pivetta and Reis, 2001). First, if the IRF is oscillating the half-life can understate the persistence of the process. Second, even for monotonically decaying processes this measure will not be adequate to compare two different series if one exhibits a faster initial decrease and then a subsequent slower decrease in the IRF than the other. Third, it may also be argued that for highly persistent processes the half-life is always very large and thus makes it difficult to distinguish changes in persistence over time. On the positive side, the half-life has the attractive feature that persistence is measured in units of time, which is not the case of any of the other three above mentioned measures of inflation persistence, and thus may be preferable for communication purposes. This probably explains why, despite the above criticisms, it still remains the most popular measure in the literature that investigates the persistence of deviations from the "purchasing power parity equilibrium".

For the AR(p) process the exact computation of the "half-life" is more complex and for this reason, the simple expression above is usually used as an approximation to the true half-life. However Murray and Papell (2002) argue that this expression might not be a good approximation to the true "half-life" if the effect of the shock does not converge to zero monotonically. For that reason below we choose to compute the half-life directly from the IRF.

Below, we shall also use as an alternative measure of persistence "the number of time periods required for a given proportion of the total disequilibria to accumulate", which appears as especially suited to evaluate how fast the series "approaches" the equilibrium. Thus, such a measure allows discriminating between two series with the same ρ but with different patterns of shock absorption. Specifically we shall compute the number of time periods required for fifty, ninety five and ninety nine percent of the total disequilibria to accumulate, denoted m_{50}, m_{95}, and m_{99}, respectively[2].

[2] The statistics m_{50}, m_{95} and m_{99} are often used in the empirical econometric literature to measure the speed of adjustment of a given endogenous variable to shocks in the exogenous variables of the model.

The next section discusses the alternative measure of persistence recently suggested in Marques (2004) and establishes its main properties.

3. AN ALTERNATIVE MEASURE OF PERSISTENCE

Marques (2004) has recently suggested an alternative measure of persistence which explores the relationship between persistence and the degree of mean reversion. In particular, Marques (2004) suggested using the statistic

$$\hat{\gamma} = 1 - \frac{n}{T} \tag{3.1}$$

to measure the absence of mean reversion of a given series, where n stands for the number of times the series crosses the mean during a time interval with T+1 observations.

Marques (2004) does not distinguish between the theoretical concept of mean reversion and the corresponding estimator. However, as we shall see below this distinction is important. Thus, we now define γ as the unconditional probability of a given process not crossing its mean in period t, or equivalently as 1 minus the probability of mean reversion of the process. In this context $\hat{\gamma}$ as given in (3.1), whose properties are examined in this paper, appears as a natural estimator of γ.

Even though Marques (2004) has motivated γ in the context of the AR(p) process, after noting the relationship between ρ and the degree of mean reversion in a stationary process, the fact is that the use of γ as a measure of persistence can be motivated in a more general framework, independently of ρ, i.e., without the need of any assumption about the data generating process.

In fact, the use of γ as a measure of persistence follows quite naturally from the very definition of persistence. If a persistent series is the one which converges slowly to its equilibrium level (i.e., the mean) after a shock, then such a series, by definition, must exhibit a low level of mean reversion, i.e., must cross its mean only infrequently. Similarly a non-persistent series must revert to the mean very frequently. And γ simply measures how frequently a given time series reverts to its mean. From an economic and political point of view it is obviously important for the central bank to know how frequently inflation reverts to the mean, i.e., the inflation target.

We note that γ, by definition, and $\hat{\gamma}$, by construction, are always between zero and one. Moreover, in Marques (2004) it is shown that for a symmetric zero mean white noise process $E[\hat{\gamma}] = \gamma = 0.5$, so that values of γ close to 0.5 signal the absence of any significant persistence (white noise behaviour) while figures significantly above 0.5 signal significant persistence. On the other hand, figures below 0.5 signal negative long-run autocorrelation.

It is also shown in Marques (2004) that under the assumption of a symmetric white noise process for inflation (zero persistence) the following result holds:

$$\frac{\hat{\gamma} - 0.5}{0.5/\sqrt{T}} \dot{\frown} N(0;1) \tag{3.2}$$

Equation (3.2) allows us to carry out some simple tests on the statistical significance of the estimated persistence (i.e., γ=0.5). However (3.2) is obtained under the assumption of a pure white noise process and thus if the null of γ=0.5 is rejected, we should expect $\hat{\gamma}$ to have a more complicated distribution, which, in particular, may depend on the characteristics of the data generating process.

An additional interesting property of $\hat{\gamma}$ is that there is a simple relation between the estimate of γ for a given period with T+1 observations and the estimated γ's for two non-overlapping consecutive sub-periods with T_1+1 and T_2+1 observations such that T+1= (T_1+1)+(T_2+1). In fact we have

$$1-\hat{\gamma} = \frac{n}{T} \approx \frac{n_1 + n_2}{T_1 + T_2} = \frac{n_1}{T_1}\frac{T_1}{T_1 + T_2} + \frac{n_2}{T_2}\frac{T_2}{T_1 + T_2} = \alpha(1-\hat{\gamma}_1) + (1-\alpha)(1-\hat{\gamma}_2)$$

or simply

$$\hat{\gamma} \approx \alpha\hat{\gamma}_1 + (1-\alpha)\hat{\gamma}_2 \tag{3.3}$$

so that persistence for the whole period is (approximately) a weighted average of the persistence for the two consecutive periods.

In contrast to ρ which requires the data generating process (DGP) to follow a pure autoregressive process, γ is defined independently of the specific underlying DGP. In this sense γ as a measure of persistence is broader in scope than ρ. To see that let us take the simplest case of an Arma(1,1) process:

$$y_t = \rho y_{t-1} + \varepsilon_t + \theta\varepsilon_{t-1} \tag{3.4}$$

In model (3.4) the parameter ρ (the sum of autoregressive coefficients) is no longer the parameter of interest as it ceases to measure persistence of the y_t series[3]. In fact, we have seen that ρ is used as a measure of persistence in the context of the pure autoregressive model because it displays a monotonic relation with the CIR of the model. However, for model (3.4) we have $CIR = (1+\theta)/(1-\rho)$ so that the one-to-one relationship between ρ and the CIR is lost. The solution, in empirical terms, implies using a finite order autoregressive process to approximate the true Arma model, but this is likely to introduce additional biases into the analysis, especially if the approximation is not good. In strong contrast, γ as a measure of persistence, is defined irrespective of the underlying DGP. Moreover, this nice property translates to its estimator, $\hat{\gamma}$. The estimator $\hat{\gamma}$ has also the advantage of not requiring the researcher to specify and estimate a model for the inflation process. For this reason it can be expected to be robust against potential model misspecifications and given its non-parametric nature also against outliers in the data. We shall investigate such a claim below in section 5.

We now investigate the asymptotic distribution of the $\hat{\gamma}$ statistic. In order to introduce the discussion let us assume that we have a sample of data with T+1 observations, denoted as y_0, y_1,... y_T, generated by a stationary and ergodic process with a known mean, μ.

The estimator of γ is computed as $\hat{\gamma} = 1 - n/T$ where n is the number of times the series, y_t, crosses the mean during the sample period. A useful way to proceed is to think of $\hat{\gamma}$ as being equal to one minus $\bar{x}$, where $\bar{x}$ is the sample mean of a series x_t (t=1, 2, ...T) that equals 1 if the series y_t crosses the mean and zero otherwise. From here it follows that all the results available in the literature concerning consistence and asymptotic distribution of the sample mean of x_t apply directly to the $\hat{\gamma}$ statistic. In particular we can invoke the law of large numbers (LLN) and the central limit theorem (CLT).

As a first step, we can start by noticing that x_t can be seen as a binomial random variable, which equals one with probability (1-γ) and zero with probability γ. Of

[3] Note the contrast with the unit root literature where the parameter ρ is the parameter of interest no matter whether the data is generated by a pure autoregressive or by an Arma process.

course, in general, x_t is not independent of $x_{t-1}, x_{t-2}, \ldots$, etc, but the structure of time dependency is determined by the autocorrelation structure of the assumed underlying stationary and ergodic process for y_t. It is possible to show that if y_t is a stationary and ergodic process then x_t is also a stationary and ergodic process because it is a measurable function of the current and past values of y_t[4]. Thus it follows from the law of large numbers that $\hat{\gamma} = 1 - \bar{x}$ is a consistent estimator of γ[5].

But in our case we can still go somewhat deeper and think of the conditions under which $\hat{\gamma}$ is not only a consistent but also an unbiased estimator of γ. In fact, an alternative way of looking at the x_t series is to think of x_t as a covariance stationary process, which, by definition, meets the conditions i) $E(x_t) = 1 - \gamma$, ii) $E[x_t - (1-\gamma)][x_{t-j} - (1-\gamma)] = r_j \quad \forall t$ and iii) $\sum_{j=0}^{\infty} |r_j| < \infty$. In such a case $\hat{\gamma} = 1 - \bar{x}$ is an unbiased estimator of γ[6].

However, unbiasedness of $\hat{\gamma}$, under the above conditions, can only be guaranteed when the mean of y_t is known[7]. When the mean of y_t is unknown (and $\bar{y}$ is used instead of μ) it follows that the true x_t series is also unknown. What we know is the x_t^* series, which differs from x_t to the extent that the use of $\bar{y}$ instead of μ may imply some additional mean crossings in x_t^* which are not present in x_t. However when $T \to \infty$ we know that $\bar{y}_T \to \mu$ so that $x_t^* \to x_t$ and consistency of $\hat{\gamma}$ follows.

Let us now address the central limit theorem. Under the assumption that x_t (or x_t^*) meets the above conditions for a covariance stationary process it follows immediately by the CLT that $\hat{\gamma}$ is asymptotically normal distributed[8], i.e.

$$\sqrt{T}(\hat{\gamma} - \gamma) \mathrel{\dot{\frown}} N[0, \sigma_\infty^2] \tag{3.5}$$

where σ_∞^2 the "asymptotic variance" of $\sqrt{T}(\hat{\gamma} - \gamma)$ is given by

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[4] See White (1984) Theorem 3.35.

[5] See, for instance, White (1984) theorem 3.34;

[6] See, for instance, Hamilton (1994), chap. 7, section 7.2;

[7] Below we discuss the conditions under which the mean of y_t can be assumed as known or otherwise must be assumed as unknown.

[8] See Hamilton (1994) chap. 7, section 7.2.

$$\sigma_\infty^2 = \lim_{T\to\infty} T.E(\hat\gamma - \gamma)^2 = \sum_{j=-\infty}^{\infty} r_j = r_0 + 2\sum_{1}^{\infty} r_j \qquad (3.6)$$

with $r_j = \mathrm{cov}(x_t, x_{t-j})$.

Below we use Monte Carlo techniques to investigate unbiasedness and consistency of $\hat\gamma$ and to evaluate the finite sample performance of the asymptotic normality result obtained under the CLT.

In order to implement (3.5) in practice we need an estimator for σ_∞^2. In theory any autocorrelation consistent estimator can be used as, for instance, the well-known Newey and West estimator or one of the kernel estimators discussed in Andrews (1991) or den Haan and Levin (1997)[9] [10]. An alternative approach could be the use of more recent bootstrapping time series techniques (see, for instance, Politis and White (2004)).

Under some circumstances it may be advisable to test for the presence of a unit root in the data, before embarking in a persistence evaluation exercise. Thus, a final remark on the potential use of the $\hat\gamma$ statistic to test the null hypothesis of a unit root (ρ=1) in model (2.1) is worth making. Under the assumption of a pure random walk process for the series y_t with no drift and initial value equal to zero ($y_0 = 0$), Burridge and Guerre (1996) demonstrated that the statistic

$$K_T^*(0) = \frac{\sqrt{\frac{\sum(\Delta y_t)^2}{T}}}{\frac{\sum|\Delta y_t|}{T}} \frac{n}{\sqrt{T}} = \theta \frac{n}{\sqrt{T}} \qquad (3.7)$$

is such that $K_T^* \xrightarrow{d} |Z|$ where Z is a standard normal N(0,1) and n is the number of sign changes of y_t during a time interval with T observations. It is straightforward to show that $K_T^*(0)$ can be rewritten in terms of the $\hat\gamma$ statistic as

[9] Note that if we estimate $\bar{x}$ by regressing x_t on a constant, most of the available econometric packages would very easily supply an autocorrelation consistent estimate for the variance of $\bar{x}$ and thus of $\hat\gamma$. Moreover, some econometric packages (PcGive 10, for instance) directly supply estimators based on a data-dependent automatic choice of bandwidth/lag truncation parameters, following Andrews (1991).

[10] One potential difficulty that applies to all tests relying on non-parametric kernel procedures is that if the estimated "asymptotic variance" exhibits significant mean-squared error in finite samples, the resulting inferences using (3.6) may be distorted. For a discussion on the sources of bias in the kernel-based spectral estimators see den Haan and Levin (1997).

$K^*_T(0) = \theta \frac{(T+1)}{\sqrt{T}}(1-\hat{\gamma})$. Once $\hat{\gamma}$ converges to 1 as ρ goes to 1, this statistic could in principle be used to test the null of $\gamma = 1$. However, there seems not to be much to be gained in proceeding in such a direction. On the one hand, statistic (3.7) is valid only under the assumption of a random walk without drift and $y_0 = 0$ and, on the other hand, Burridge and Guerre (1996) evaluated the potential use of $K^*_T(0)$ to test for a unit root and concluded that it has lower power compared to the conventional Dickey-Fuller τ-test. Thus, if testing for a unit root in the data is considered an appropriate first step, using the unit root tests available in the literature (ADF, KPSS, etc.) is recommended, before embarking in a persistence evaluation exercise based on scalar measures of persistence such as γ, ρ, the half-life, or the largest autoregressive root.

4. THE RELATIONSHIP BETWEEN γ AND ALTERNATIVE MEASURES OF PERSISTENCE.

For any given stationary autoregressive process it is always possible to derive the corresponding γ, the unconditional probability of the process not reverting to its mean, in each period t. This allows us to compare γ with alternative measures of persistence, specially ρ, in the context of different stationary AR(p) models. The process of derivation of the values of γ for the AR(p) process, under the assumption of normal innovations, is explained in the Appendix.

Column (2) of Table 1 presents the values of γ corresponding to the value of ρ in column (1) assuming that the data are generated by the AR(1) process $y_t = \rho y_{t-1} + \varepsilon_t$ with normal innovations, ε_t. For instance, for the AR(1) process with ρ=0 (white noise process) γ is 0.50 while, for the AR(1) process with ρ=0.60 the corresponding γ is 0.705. Table (1) thus shows that in the context of the AR(1) process there is a one-to-one correspondence between these two measures of persistence.

Table (1) also reports the half-life, h, defined as the number of periods for which the effect of a unit shock remains above 0.5[11] as well as the m_{50}, m_{95} and m_{99} statistics, in columns (4), (5) and (6), which denote the number of periods required for the

[11] For reasons explained in section 2 we compute the half-life directly from the IRF.

cumulated effect of a unit shock to be (at least) equal to 50%, to 90% and to 99% of the total effect, respectively[12]. Note that these measures are obtained directly from the IRF or the CIR assuming the model in column (1).

Table 1

Comparing ρ with γ – AR(1) model

ρ	γ	h	m_{50}	m_{95}	m_{99}
(1)	(2)	(3)	(4)	(5)	(6)
0.00	0.500	1	1	1	1
0.10	0.532	1	1	1	1
0.20	0.564	1	1	1	2
0.30	0.597	1	1	2	3
0.40	0.631	1	1	3	5
0.50	0.667	1	1	4	6
0.55	0.685	1	1	5	7
0.60	0.705	1	1	5	9
0.65	0.725	1	1	5	9
0.70	0.747	1	1	6	10
0.75	0.770	2	2	10	16
0.80	0.795	3	3	13	20
0.85	0.823	4	4	18	28
0.90	0.856	6	6	28	43
0.95	0.899	13	13	58	89

Looking at columns (3) and (4) we realise that the half-life and m_{50} assume exactly the same values for the different values of ρ, as one would expect under an AR(1) process. However it is apparent from Table (1) that these two measures are completely uninformative for values of $\rho \leq 0.70$. And, even for values of ρ between 0.70 and 0.90, the range of variation of these two statistics is so small that they can

[12] In the case of the AR(1) process we have for instance that m_{99} is the (minimum) number of periods required to get $\frac{(1-\rho^{m_{99}})/(1-\rho)}{1/(1-\rho)} \geq 0.99$, which reduces to the condition $m_{99} = \ln(0.01)/\ln(\rho)$. Notice the similarities with the half-life. As this expression is not valid for the higher order processes we computed m_{99} (as well as m_{50} and m_{95}) directly from the IRF and the CIR.

hardly be seen as useful devices to discriminate among different models. This, of course, represents a strong limitation of both measures[13].

As to the remaining two measures of persistence, m_{95} and m_{99} they exhibit a monotonic relationship with ρ and γ (specially so for m_{99}). Thus in the context of the AR(1) model, ρ, γ and m_{99} (leaving aside for the moment potential estimation problems) appear as giving the same message about the degree of persistence. This result stems from the fact that the speed of convergence to equilibrium in the AR(1) model is constant throughout the adjustment period.

In order to investigate whether the above monotonic relationship among the different measures of persistence found for the AR(1) model carries over to higher order autoregressive models we now consider an AR(2) process (with zero mean) given by $y_t = \rho_1 y_{t-1} + \rho_2 y_{t-2} + \varepsilon_t$. Given the wide range of possibilities for the two autoregressive coefficients we set the sum of the autoregressive coefficients equal to 0.80 (i.e., $\rho = \rho_1 + \rho_2 = 0.80$) and consider different combinations of ρ_1 and ρ_2 with the restriction that $\rho_1 \geq 0$ and such that the resulting model is stationary. Thus models (1) to (18) in Table 2 correspond to different combinations of the two AR coefficients that meet the conditions: i) $\rho_1 \geq 0$, ii) $\rho = \rho_1 + \rho_2 = 0.80$ and iii) the model is stationary[14].

In addition to the statistics considered in Table 1, Table 2 also includes the "largest autoregressive root", lar, as an additional measure of persistence.

In Table 2 models are listed according to the value of ρ_1 in descending order. We start by noticing that models (1) to (6) have complex roots. For these models the IRF denoted in column (8) as being of type (1) oscillates around zero with an overshooting effect in the first periods, the overshooting effect being reduced as we move from model (1) to model (6). Moreover for these models not only the IRF but also the partial sum of the IRF displays an oscillating behaviour. The fact that the partial sum

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[13] This conclusion emerges because we are assuming a discrete range of variation for the half-life. However, it is unclear whether adopting a continuous range of variation (which implies using the formula $h = \ln(0.5)/\ln(\rho)$ in the AR(1) case and interpolating the IRF in the general case) really solves the problem because we get a set of values all ranging between 1 and 2 (for ρ varying between 0 and 0.75). Keeping the half-life on a discrete domain seems more in line with the definition as "the number of periods" is a discrete variable.

[14] As it is well known the required (necessary and sufficient) conditions for stationary for the AR(2) process can be written as $\rho_1 + \rho_2 < 1;\ \ \rho_2 - \rho_1 < 1;\ \ -1 < \rho_2 < 1;$

Table 2 --Different AR(2) models with $\rho = \rho_1 + \rho_2 = 0.80$

Model		γ	h	*lar*	m_{50}	m_{95}	m_{99}	Type of IRF
(1)		(2)	(3)	(4)	(5)	(6)	(7)	(8)
(1)	$\rho_1 = 1.7, \rho_2 = -0.9$	0.853	5	--	1	3	3	(1)
(2)	$\rho_1 = 1.6, \rho_2 = -0.8$	0.849	4	--	1	3	3	(1)
(3)	$\rho_1 = 1.5, \rho_2 = -0.7$	0.844	4	--	1	3	3	(1)
(4)	$\rho_1 = 1.4, \rho_2 = -0.6$	0.839	4	--	2	3	4	(1)
(5)	$\rho_1 = 1.3, \rho_2 = -0.5$	0.834	4	--	2	4	4	(1)
(6)	$\rho_1 = 1.2, \rho_2 = -0.4$	0.828	4	--	2	5	6	(1)
(7)	$\rho_1 = 1.1, \rho_2 = -0.3$	0.821	3	0.600	2	7	10	(2)
(8)	$\rho_1 = 1.0, \rho_2 = -0.2$	0.814	3	0.724	2	9	14	(2)
(9)	$\rho_1 = 0.9, \rho_2 = -0.1$	0.805	3	0.770	2	11	17	(2)
(10)	$\rho_1 = 0.8, \rho_2 = 0.0$	0.795	3	0.800	3	13	20	(2)
(11)	$\rho_1 = 0.7, \rho_2 = 0.1$	0.784	2	0.822	3	15	23	(2)
(12)	$\rho_1 = 0.6, \rho_2 = 0.2$	0.770	2	0.839	3	16	25	(2)
(13)	$\rho_1 = 0.5, \rho_2 = 0.3$	0.753	1	0.852	4	18	28	(3)
(14)	$\rho_1 = 0.4, \rho_2 = 0.4$	0.732	1	0.863	4	20	31	(3)
(15)	$\rho_1 = 0.3, \rho_2 = 0.5$	0.705	1	0.873	4	21	33	(3)
(16)	$\rho_1 = 0.2, \rho_2 = 0.6$	0.667	1	0.881	4	23	35	(3)
(17)	$\rho_1 = 0.1, \rho_2 = 0.7$	0.608	1	0.888	5	24	38	(3)
(18)	$\rho_1 = 0.0, \rho_2 = 0.8$	0.500	1	0.894	6	26	40	(3)

Type of IRF

(1)- Oscillatory around zero with an overshooting effect in the first periods. Overshooting effect reduces as we move from model (1) to model (6). (2) – Non-oscillatory, monotonically decaying to zero after the first periods. (3) – Non-monotonically decaying IRF with increasing oscillatory behaviour as we move from model (13) to model (18).

of the IRF is oscillating has the consequence that the number of periods required for a given proportion of the total adjustment to take place is not uniquely defined. In other words the CIR and thus also ρ are no longer acceptable measures of persistence as they cease to convey any useful information about the speed of convergence in the impulse response function. Of course, this criticism also applies to the half-life, m_{50}, m_{95} and m_{99}[15].

For models (7) to (12) the IRF denoted as type (2) in column (8) is non-oscillatory and monotonically decaying to zero after the first periods. However, in the first periods the IRF with the exception of model (10) (which corresponds to the special case of the AR(1) process) exhibits a non-constant speed of convergence. Finally for models (13) to (18) the IRF denoted as type (3) in column (8) is a non-monotonic decaying process with increasing oscillatory behaviour as we move from model (13) to model (18). Despite the non-monotonic behaviour of the IRF function during the first periods the corresponding CIR increases monotonically for models (7) to (18).

The different types of the IRF can be better understood by looking at Graphs No.1 and No.2. Graph No.1 displays the IRF for the first 30 periods for the extreme cases of models (1), (10) and (18) while Graph No.2 displays the IRF for the intermediate cases of models (7) and (12).

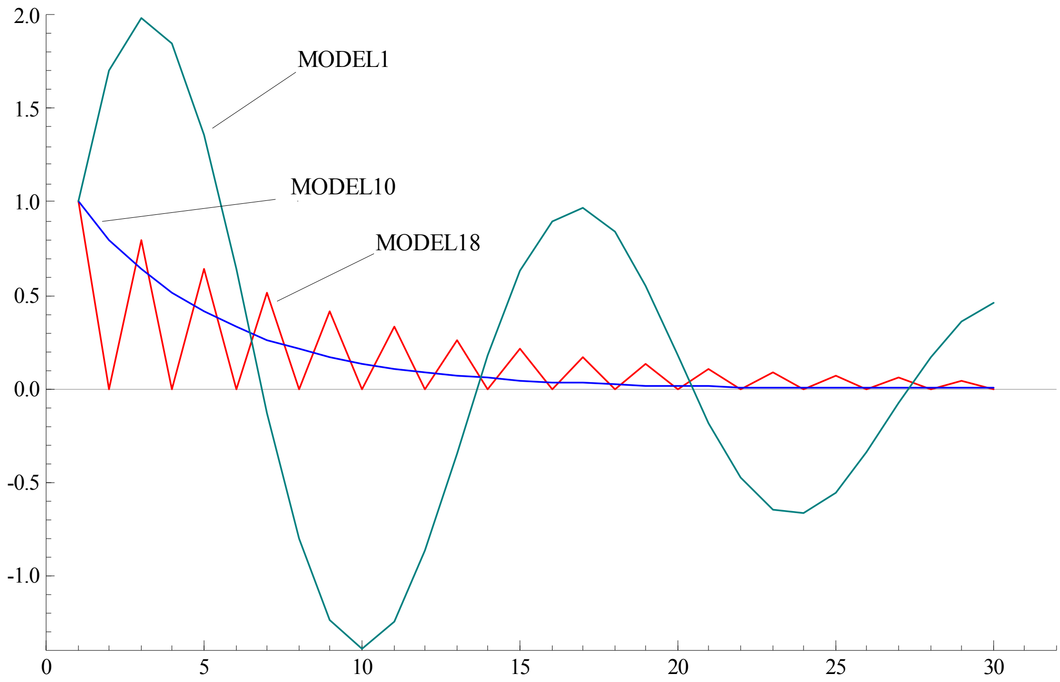
Let us now take a look at Table 2. If we stick to the "sum of the autoregressive coefficients" as the single measure of persistence all the 18 models would be seen as equally persistent with ρ=0.80. However, somewhat surprisingly, as we move from model (1) to model (18) we see that the value of γ and of the half-life, h, decrease monotonically while, in strong contrast, the values of lar, m_{50}, m_{95} and m_{99} increase monotonically. Thus, if instead we stick to one of these alternative six measures the conclusion would be that the 18 models appear as essentially different in terms of persistence. In other words, the 6 alternative measures of persistence behave quite independently from the value of ρ[16].

This is a very important result as it shows that the conclusion obtained above for the AR(1) model about the fact that the alternative measures of persistence exhibit a monotonic relation among them (and with ρ) and thus all convey the same message, does not carry out to more general autoregressive processes.

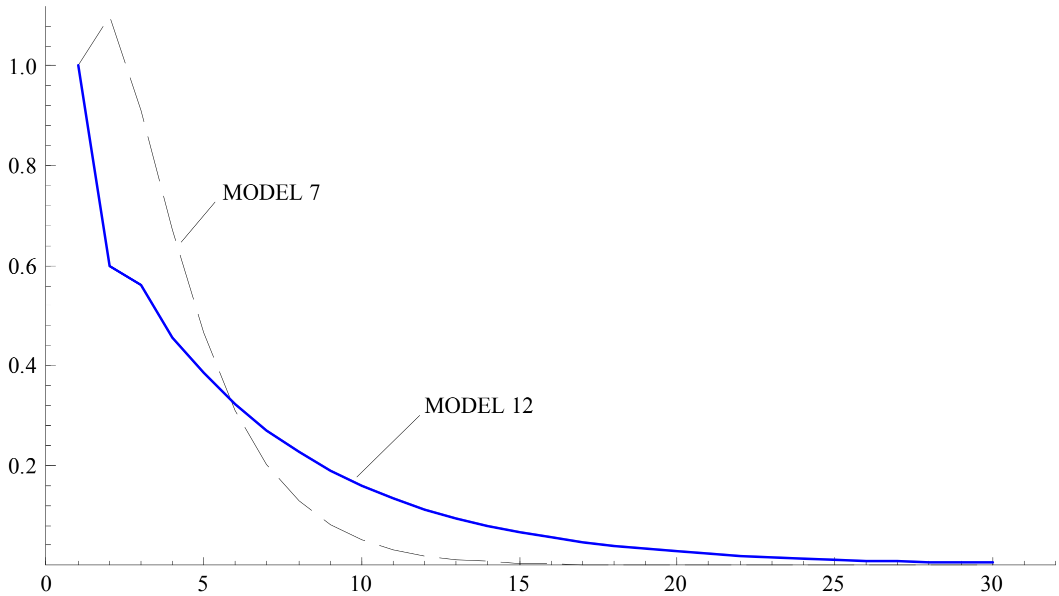
[15] In order to obtain the values of these measures in Table 2 we took the number of periods required for the values of these statistics to be observed by the first time.

[16] For a similar point concerning the absence of a relationship between ρ and the half-life, see Gadzinski and Orlandi (2004).

Graph No.1 – Impulse response functions for the first 30 periods of models (1), (10) and (18)



Graph No.2 – Impulse response functions for the first 30 periods of models (7) and (12)



The idea that the scalar measures of persistence could in some specific situations be very misleading about the true level of persistence is, of course, not new (see Andrews and Chen, 1994, and the discussion in section 2 above). What seems to be new (at least for the authors) is the extension of the problem. More than saying that in some special cases the CIR (and thus all the measures based on the CIR as ρ, m50, m95 or m99) is not a good summary measure of the information contained in the IRF (and thus is not a good measure of persistence) it seems more appropriate to state that with the exception of the very special case of the AR(1) model, the scalar measures of persistence for the general autoregressive model may be very misleading either by suggesting the presence of a strong degree of persistence when it is absent or by suggesting the absence of significant persistence when it is present.

For instance, if we take a look at Graph No.3, which displays a realization of models (1), (10) and (18) with 80 observations it seems difficult to argue that the three models, despite having the same ρ=0.8, should be seen as equally persistent, given the disparate values for the 6 measures of persistence in Table 2[17]. If we think of persistence as the frequency of mean reversion we see that mean reversion in model (10) and model (18) is clearly higher than in model (1) suggesting thus that persistence is lower in those two models. On the other hand, we also see that mean reversion in model (18) is higher than in model (10)[18]. The different degree of mean reversion among the 18 models in Table 2 can be inferred from column (2), which reports the values of γ for each model. We can see that γ starting with model (1) decreases monotonically from a value as high as 0.853 (signalling a very persistent process) to a figure as low as 0.50 which signals a model with zero persistence.

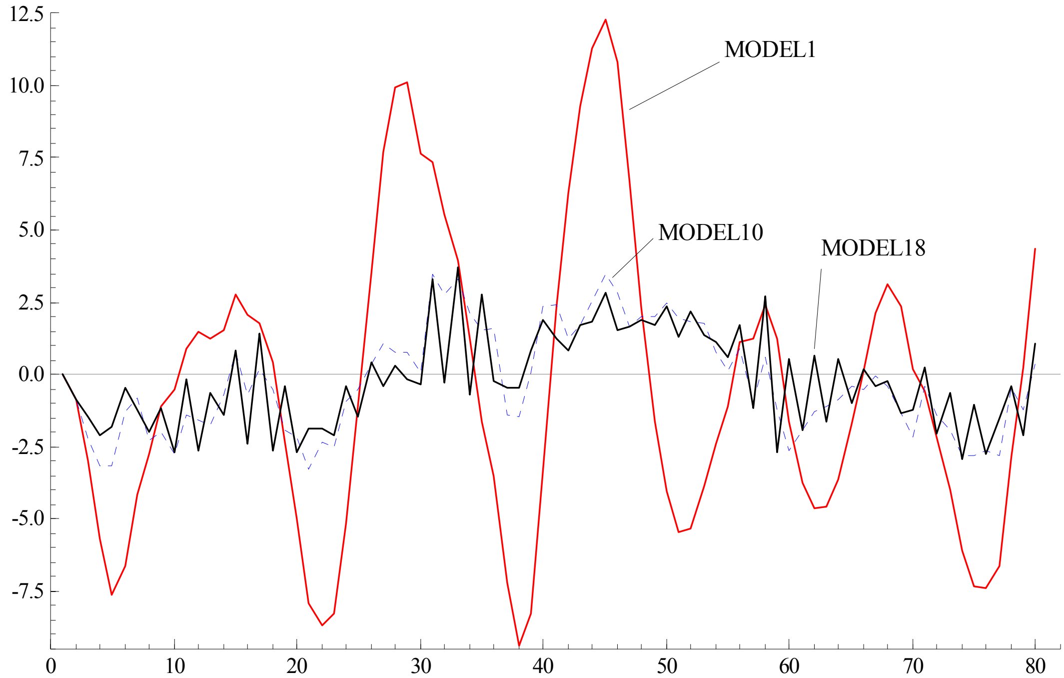
We expect γ to be equal to 0.5 when a white noise process generates the data, but by simply eyeballing the series we see that model (18) in Graph No.3 does not behave like a white noise. Rather it seems to display a kind of cyclical behaviour, which makes the process to cross the mean at irregular intervals, but such that on average it crosses the mean as often as if it were a white noise. This means that γ does not distinguish between a process with a low ρ (close to a white noise behaviour) and a

[17] The three series in graph No.3 were generated using the same series of residuals, which in turn were generated from the N(0,1) distribution.

[18] Notice that the two models behave similarly during most of the sample but mean reversion in model (18) is higher for observations 30-34 and 60-64 suggesting that persistence in model (18) is lower.

process with a cyclical pattern such that during a given time interval the two processes cross the mean on average the same number of times. In other words γ , in contrast to ρ, does not see the cyclical pattern of the process as relevant persistence.

Graph No.3 – Realization of Models (1), (10) and (18)



Of course, the interesting question in such a situation is whether one should think of model (18) as a highly persistent process (with ρ=0.8) or rather as a low persistent one (with γ=0.5). From an economic or even political point of view it is obviously important for the central bank to know how frequently inflation reverts to the inflation target, and that information is given by γ. Thus, in the context of a model with a cyclical pattern, it is γ and not ρ that gives the important information that inflation reverts frequently to the central bank inflation target and so that the situation should not be seen as worrisome in terms of persistence, despite the high estimated ρ[19].

[19] In formal terms choosing between ρ and γ implies discussing whether or not the cyclical pattern of the higher order autoregressive model should be seen as "relevant persistence". We first note that the very concept of persistence as the "speed of convergence of inflation to the equilibrium level" is not well defined when inflation displays a kind of cyclical pattern or more generally oscillates around the mean.

We note, however, that an autoregressive model with a negative ρ also displays an oscillatory pattern around the mean and in such a case no one would argue that such an "oscillatory pattern" should be seen as "relevant persistence". Thus, by analogy, we can argue that the "cyclical pattern" should also not be seen as "relevant persistence" and thus, γ should be preferred to ρ, as a measure of persistence.

By looking at the IFRs of the 18 AR(2) processes we see that it is the non-constant speed of convergence in the IRF which gives rise to the diverging results for the different measures of persistence. In particular, we note that the values of γ for models (8), (9), (11) and (12) which are the ones with an IRF closer to that of the AR(1) process (model (10)) are very close to the value obtained for the AR(1) case. Thus, we can infer that in general what matters for the interpretation of the results is whether or not the IRF closely reproduces the constant speed characteristic of the IRF of the AR(1) model independently of the order of the underlying process. In other words, we can expect the relationship between ρ and γ in Table 1 to hold for higher processes provided the IRF displays a (close to a) constant speed of convergence throughout the adjustment period. In this sense comparing ρ and γ may shed some light on the characteristics of the IRF.

In finalizing this section we stress the idea that relying exclusively on ρ, as the single measure of persistence, as it is current practice in the empirical literature, does not allow uncovering important features of inflation persistence in the context of higher order autoregressive processes. In order to better characterise inflation persistence in such a context it seems advisable to compute at least two alternative scalar measures of persistence capable of delivering complementary information. In this regard using γ and ρ as companion measures of persistence allows obtaining useful information to characterise the degree of persistence that cannot be extracted from γ or ρ in isolation. We have seen that a value of ρ clearly above to what could be expected given the value of γ could be signalling a cyclical pattern in the DGP. We shall see below that an estimate of ρ clearly below the value of $\hat{\gamma}$ may be seen as a signal of significant biases in $\hat{\rho}$ stemming from the presence of additive outliers in the data or from model misspecifications.

5. SOME MONTE CARLO EVIDENCE ON THE FINITE SAMPLE PROPERTIES OF $\hat{\gamma}$ AND $\hat{\rho}$.

In this section we use some Monte Carlo experiments in order to investigate the properties of $\hat{\gamma}$ and $\hat{\rho}$ regarding i) unbiasedness and consistency, ii) finite sample performance of their asymptotic distribution, iii) robustness to outliers and iv) robustness against model misspecifications. These properties are investigated in the context of the AR(1) and AR(2) processes considered in section 4.

From the discussion in section 3 we may expect the information about the mean of the process to be statistically relevant for persistence evaluation. For this reason, below we distinguish the situation in which the mean is known from the situation in which the mean is unknown. Assuming that the mean is known may be realistic for those countries for which an inflation targeting monetary policy was implemented and an explicit inflation target was announced. In this case, the true mean of the series can be computed exogenously to realised inflation and is given by the publicly announced inflation target. However, for most countries the exact (implicit) inflation target used by the central bank when setting monetary policy is unknown. In theses cases the mean must be computed from realised inflation and this implies endogenising the central bank inflation target. As we shall see below this may have noticeable consequences for the process of persistence evaluation if the sample is not very large.

5.1 – Unbiasedness and consistency

Let us start by assuming that true mean of the process is known. Under such circumstances we can extract the mean from realised inflation and assume that the true model does not have an intercept or that the mean of the process is zero[20].

To proceed we start by defining an experiment that constructs the data to follow an AR(1) process (with no intercept) given by $y_t = \rho y_{t-1} + \varepsilon_t$ for ρ ranging between 0 and 1 and where the errors are serially uncorrelated standard normal variables. The data are generated by setting y_{-100}=0 and creating T+100 observations, discarding the first 100 observations to remove the effect of the initial conditions. Samples of size T=50, 75, 100, 150, 250, 500 and 1000 are used in the experiments. Each experiment

[20] Note that under this assumption it does not matter whether we think of the mean as a constant or time varying. Thus conclusions derived below, under the assumption that the mean is known, are valid no matter how the mean is defined (constant, piecewise constant or time varying). However, the conclusions for the case of unknown mean assume that the (unknown) mean is constant throughout the sample period.

is replicated 10,000 times in order to create the sampling distributions of the estimators[21].

It is known that the sampling distribution of the least squares estimator of ρ, $\hat{\rho}$, (namely the expectation and standard deviation of $\hat{\rho}$) depends on the initial value of the process (y_0) (see, for instance, Evans and Savin (1981)). For stationary processes Sawa (1978) argues that there is no noticeable difference whether one assumes that the initial value is a fixed constant or y_0 is a drawing from a stationary distribution. In our case we implicitly assume that the process has started somewhere in the past (y_{-100}=0) so that for estimation purposes our initial value (y_0) may be seen as a random normal variable with $\mathrm{Var}(y_0) = \sigma_\varepsilon^2 / (1-\rho^2)$[22]. Some simulations were carried out in which different values for y_{-100} were assumed. As expected such changes had no noticeable impact on the estimated statistics.

The output of the experiment for T=100 is displayed in Table 3. For values of ρ ranging between 0 and 1, column (2) reports the average value for the Monte Carlo OLS estimates of the ρ parameter ($\overline{\rho}$) and column (3) the median of these estimates ($\hat{\rho}_m$). Similarly, column (6) reports the values of γ (taken from Table 1), column (7) the average value of the Monte Carlo estimates of γ ($\overline{\gamma}$), column (8) the corresponding median ($\hat{\gamma}_m$) and column (9) the standard error of $\hat{\gamma}$[23].

From Table 3 we can see that the OLS estimator of ρ is slightly (mean) downward biased and that the absolute bias increases as ρ increases, as expected. From column (3) we also see that the median of the least squares estimates ($\hat{\rho}_m$) is higher than the mean of the estimates ($\overline{\rho}$) but still lower than ρ. Such evidence is consistent with the result in the literature that the finite sample distribution of $\hat{\rho}$ besides being downward biased is also negatively skewed (see, Sawa, 1978, Phillips, 1977, Evans and Savin, 1981, Andrews, 1993, Andrews and Chen, 1994).

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[21] All the experiments in this paper are replicated 10,000 times with the data generated by setting y_{-100}=0 and creating T+100 observations, discarding the first 100 observations to remove the effect of the initial conditions The replications were carried out in TSP 4.5.

[22] In rigour under the assumption that y_{-100}=0 and that ε_t is i.i.d. N(0, σ_ε^2) we have $\mathrm{var}(y_0) = \sigma_\varepsilon^2 (1-\rho^{2x100})(1-\rho^2)^{-1}$ which reduces to $\mathrm{var}(y_0) = \sigma_\varepsilon^2 / (1-\rho^2)$ as $\rho^{2x100} \approx 0$.

[23] We note that the standard error of $\hat{\gamma}$ and $\hat{\rho}$ is the standard error of the series composed of the 10,000 estimates of γ and ρ, respectively.

Table 3
Monte Carlos simulations – AR(1) model with no intercept (T=100)

True ρ	$\bar{\rho}$	$\hat{\rho}_m$	Mean Bias of $\hat{\rho}$ (%)	St. error of $\hat{\rho}$	True γ	$\bar{\gamma}$	$\hat{\gamma}_m$	St. error of $\hat{\gamma}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
0.00	0.001	0.002	---	0.098	0.500	0.500	0.500	0.049
0.10	0.099	0.101	-1.00	0.098	0.532	0.532	0.530	0.049
0.20	0.197	0.199	-1.50	0.097	0.564	0.564	0.560	0.050
0.30	0.295	0.298	-1.67	0.094	0.597	0.597	0.600	0.050
0.40	0.393	0.397	-1.75	0.091	0.631	0.631	0.630	0.050
0.50	0.491	0.495	-1.80	0.086	0.667	0.667	0.670	0.050
0.55	0.540	0.545	-1.82	0.083	0.685	0.686	0.690	0.051
0.60	0.589	0.594	-1.83	0.080	0.705	0.705	0.710	0.051
0.65	0.638	0.643	-1.85	0.077	0.725	0.725	0.730	0.051
0.70	0.687	0.693	-1.86	0.073	0.747	0.747	0.750	0.051
0.75	0.736	0.742	-1.87	0.068	0.770	0.771	0.770	0.051
0.80	0.785	0.792	-1.88	0.063	0.795	0.796	0.800	0.051
0.85	0.834	0.842	-1.88	0.057	0.823	0.824	0.830	0.050
0.90	0.883	0.891	-1.89	0.049	0.856	0.857	0.860	0.051
0.95	0.933	0.941	-1.79	0.040	0.899	0.899	0.900	0.050

Let us now take a look at the $\hat{\gamma}$ statistic. By comparing the values of γ and $\bar{\gamma}$ in columns (6) and (7) we see that they are not statistically different. The same conclusion is obtained for the simulations with T=50,75,150, 250, 500 and 1000, whose results can be seen in the Appendix (see Table A1)[24]. We thus conclude that $\hat{\gamma}$ is an unbiased estimator of γ, as expected, given the discussion in section 3. Moreover, from columns (7) and (8) we see that the mean ($\bar{\gamma}$) and the median ($\hat{\gamma}_m$) of

[24] Note that the sampling variability allows the estimates of $\bar{\gamma}$ (and of $\bar{\rho}$) to change for a fixed T. As the standard error of $\bar{\gamma}$ may be approximated by $(0.5/\sqrt{T})/\sqrt{10\ 000}$ and $\bar{\gamma}$ has a Normal distribution a 95% confidence interval for $\bar{\gamma}$ is given by $\bar{\gamma} \pm 1.96*(0.5/\sqrt{T})/\sqrt{10\ 000}$, which reduces to $\bar{\gamma} \pm 0.001$ for T=100. From Table A1 and Table A2 we see that, for instance, the values of $\bar{\gamma}$ for T=50 and T=1000 do not differ by more than 0.001, so that the difference can be attributed to sampling variability.

$\hat{\gamma}$ seem to coincide, which suggests that the distribution of $\hat{\gamma}$ is essentially symmetric[25].

Table 4
Monte Carlo simulations – AR(1) model with an intercept (T=100)

True ρ	$\bar{\rho}$	$\hat{\rho}_m$	Mean Bias of $\hat{\rho}$ (%)	True γ	$\bar{\gamma}$	$\hat{\gamma}_m$	Mean Bias of $\hat{\gamma}$ (%)	St. error of $\hat{\gamma}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
0.00	-0.010	-0.008	---	0.500	0.497	0.500	-0.60	0.049
0.10	0.087	0.089	-12.54	0.532	0.528	0.530	-0.75	0.049
0.20	0.184	0.187	-7.78	0.564	0.560	0.560	-0.71	0.049
0.30	0.281	0.285	-6.19	0.597	0.592	0.590	-0.84	0.050
0.40	0.378	0.383	-5.41	0.631	0.626	0.630	-0.80	0.050
0.50	0.475	0.480	-4.95	0.667	0.661	0.660	-0.90	0.051
0.55	0.524	0.528	-4.79	0.685	0.679	0.680	-0.88	0.051
0.60	0.572	0.577	-4.66	0.705	0.698	0.700	-1.00	0.051
0.65	0.620	0.626	-4.55	0.725	0.718	0.720	-0.97	0.051
0.70	0.669	0.675	-4.47	0.747	0.739	0.740	-1.07	0.051
0.75	0.717	0.724	-4.40	0.770	0.762	0.760	-1.04	0.051
0.80	0.765	0.772	-4.36	0.795	0.786	0.790	-1.13	0.051
0.85	0.813	0.821	-4.34	0.823	0.812	0.810	-1.34	0.051
0.90	0.861	0.869	-4.37	0.856	0.842	0.850	-1.64	0.052
0.95	0.907	0.916	-4.52	0.899	0.878	0.880	-2.34	0.052

Let us now assume that the mean of the process is unknown. As discussed above in the process of persistence evaluation this might be a more realistic assumption whenever the central bank inflation target is unknown and thus has to be estimated from the data. The results of the Monte Carlo experiment are in Table 4. The

[25] The differences between $\bar{\gamma}$ and $\hat{\gamma}_m$ stem basically from the fact that the median is a discrete variable. Note in particular that when T=100 (the case in Table 3) the median can only assume the values ....0.630, 0.635, 0.640, 0.645,...etc. What is important is the fact that there are no signs of systematic (positive or negative) bias between $\bar{\gamma}$ and $\hat{\gamma}_m$.

estimates $\hat{\rho}$ were obtained by estimating the AR(1) model with an intercept and those of $\hat{\gamma}$ were computed conditional on the estimated mean[26].

Looking at Table 4 we see that the downward bias of OLS estimator has now significant damaging consequences on the expected estimates of ρ. In fact, the bias has now more than doubled vis-à-vis the situation in Table 3, confirming the claim in Sawa (1978) and Andrews (1993) that the bias of $\hat{\rho}$ is more acute when one assumes that the mean of the process is unknown and has to be estimated from the data using a model with an intercept. In empirical applications this downward bias of $\hat{\rho}$ will naturally translate into all measures of persistence that are computed using an estimate of ρ (the half-life, m_{50}, m_{95} and m_{99} in Table 1). This is the case for which it might be worth using the "approximated median unbiased estimator" suggested in Andrews (1994).[27]

As to the $\hat{\gamma}$ statistic we see that it also appears slightly downward biased as expected given the discussion in section 3. This result is intuitive, as the estimator of the mean (the estimated average) can only increase mean reversion vis-à-vis the situation with the true mean, and thus reduce the estimated γ.

Let us now briefly take a look at the AR(2) process. In our Monte Carlo experiment we consider as our DGP the same 18 models of section 4 for which $\rho = \rho_1 + \rho_2 = 0.80$ with a zero intercept and serially uncorrelated standard normal errors. The output of the experiment is in Table 5. Column (2) reports the average of the OLS estimates of ρ ($\overline{\rho}$) and column (7) the average value for the $\hat{\gamma}$ statistic ($\overline{\gamma}$) obtained under the assumption that the mean of the process is known, i.e., by estimating an AR(2) process without an intercept and computing $\hat{\gamma}$ assuming a zero mean for the process. Column (4) and column (8) report the corresponding values of these statistics under the assumption that the mean is unknown, i.e., by estimating an AR(2) model with an intercept and computing $\hat{\gamma}$ conditional on the estimated mean. To facilitate comparisons, column (6) reports the values of γ taken from Table 2.

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[26] Numbers in the table were obtained by generating the model $y_t = 0.01 + z_t$ with $z_t = \rho z_{t-1} + \varepsilon_t$. We note that the sampling distributions of $\hat{\rho}$ and $\hat{\gamma}$ do not depend on the value of mean, but simply on the fact that mean is unknown and has to be estimated.

[27] However, we shall argue below that far most important biases are brought into the least squares estimator due to potential model misspecifications or to the presence of additive outliers but that such biases may not be overcome by resorting the median unbiased estimator.

Table 5 --Different AR(2) models with $\rho = \rho_1 + \rho_2 = 0.80$ and T=100

Model		Model with no intercept		Model with an intercept		True	Zero mean	Estimated mean	
Model		$\bar{\rho}$	Bias of $\hat{\rho}$ (%)	$\bar{\rho}$	Bias of $\hat{\rho}$ (%)	γ	$\bar{\gamma}$	$\bar{\gamma}$	Bias of $\bar{\gamma}$ (%)
(1)		(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
(1)	$\rho_1 = 1.7, \rho_2 = -0.9$	0.800	0.00	0.797	-0.38	0.853	0.853	0.852	-0.12
(2)	$\rho_1 = 1.6, \rho_2 = -0.8$	0.798	-0.25	0.793	-0.88	0.849	0.848	0.847	-0.24
(3)	$\rho_1 = 1.5, \rho_2 = -0.7$	0.796	-0.50	0.789	-1.38	0.844	0.844	0.842	-0.24
(4)	$\rho_1 = 1.4, \rho_2 = -0.6$	0.794	-0.75	0.785	-1.88	0.839	0.839	0.836	-0.36
(5)	$\rho_1 = 1.3, \rho_2 = -0.5$	0.792	-1.00	0.781	-2.38	0.834	0.834	0.830	-0.48
(6)	$\rho_1 = 1.2, \rho_2 = -0.4$	0.790	-1.25	0.777	-2.88	0.828	0.828	0.823	-0.60
(7)	$\rho_1 = 1.1, \rho_2 = -0.3$	0.788	-1.50	0.773	-3.38	0.821	0.822	0.815	-0.73
(8)	$\rho_1 = 1.0, \rho_2 = -0.2$	0.786	-1.75	0.769	-3.88	0.814	0.814	0.806	-0.98
(9)	$\rho_1 = 0.9, \rho_2 = -0.1$	0.785	-1.88	0.764	-4.50	0.805	0.805	0.797	-0.99
(10)	$\rho_1 = 0.8, \rho_2 = 0.0$	0.783	-2.13	0.760	-5.00	0.795	0.796	0.786	-1.13
(11)	$\rho_1 = 0.7, \rho_2 = 0.1$	0.781	-2.38	0.756	-5.50	0.784	0.784	0.773	-1.40
(12)	$\rho_1 = 0.6, \rho_2 = 0.2$	0.779	-2.63	0.751	-6.13	0.770	0.771	0.758	-1.56
(13)	$\rho_1 = 0.5, \rho_2 = 0.3$	0.777	-2.88	0.747	-6.63	0.753	0.754	0.740	-1.73
(14)	$\rho_1 = 0.4, \rho_2 = 0.4$	0.775	-3.13	0.743	-7.13	0.732	0.733	0.716	-2.19
(15)	$\rho_1 = 0.3, \rho_2 = 0.5$	0.773	-3.38	0.738	-7.75	0.705	0.705	0.686	-2.70
(16)	$\rho_1 = 0.2, \rho_2 = 0.6$	0.772	-3.50	0.734	-8.25	0.667	0.667	0.645	-3.30
(17)	$\rho_1 = 0.1, \rho_2 = 0.7$	0.770	-3.75	0.729	-8.88	0.608	0.608	0.582	-4.28
(18)	$\rho_1 = 0.0, \rho_2 = 0.8$	0.768	-4.00	0.725	-9.38	0.500	0.499	0.471	-5.80

We have just seen, in the context of the AR(1) process, that the biases of $\hat{\rho}$ increase as ρ increases. Now, by looking at Table 5, column (2), we see that the biases of $\hat{\rho}$ also depend on the combination of ρ_1 and ρ_2 and increase monotonically as we move from model (1) to model (18). This is an interesting result because it shows that for higher order processes the bias of $\hat{\rho}$ depends not only on ρ but also on the combinations of ρ_1 and ρ_2 (for a fixed ρ). In strong contrast, by comparing columns (6) and (7) we see that, once again, as expected, $\hat{\gamma}$ behaves as an unbiased estimator.

If we assume that the mean of the process is unknown and estimate a model with an intercept we see that, as expected, the downward biases of the OLS estimator of ρ increase significantly (columns (4) and (5)). From Table 5 we also see that the models which we expect to be more realistic in empirical terms (the ones for which ρ_1<1) are also the ones for which the biases are larger. As regards the $\hat{\gamma}$ statistic we see that, similarly to the AR(1) case, a small downward bias emerges (column (9)) and that the bias increases as we move from model (1) to model (18). However it is always smaller than the bias displayed by $\hat{\rho}$ (column (5)).

Let us now investigate how $\hat{\rho}$ and $\hat{\gamma}$ behave when T increases. Table 6 displays the biases of $\hat{\rho}$ and $\hat{\gamma}$ for the AR(1) and AR(2) processes under the assumption that the mean is unknown. We see that both $\hat{\rho}$ and $\hat{\gamma}$, behave as consistent estimators, as expected. However, the biases of $\hat{\gamma}$ are always significantly smaller than those of $\hat{\rho}$ (for the same T) and become negligible even for relatively small samples (for most models with T=100), which is not the case of $\hat{\rho}$.

Table 6 – Biases in the AR(1) and AR(2) models with an intercept(*)

Biases in % of the AR(1) model with an intercept										
	T=50		T=100		T=250		T=500		T=1000	
True ρ	$\hat{\rho}_b$	$\hat{\gamma}_b$	$\hat{\rho}_b$	$\hat{\gamma}_b$	$\hat{\rho}_b$	$\hat{\gamma}_b$	$\hat{\rho}_b$	$\hat{\gamma}_b$	$\hat{\rho}_b$	$\hat{\gamma}_b$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
0.50	-9.60	-1.65	-5.00	-0.90	-1.80	-0.30	-1.00	-0.30	-0.40	-0.15
0.70	-8.86	-2.14	-4.57	-1.07	-1.71	-0.40	-0.86	-0.27	-0.43	-0.13
0.90	-9.00	-3.50	-4.33	-1.64	-1.67	-0.58	-0.78	-0.23	-0.44	-0.12
Model	Biases in % of the AR(2 model with an intercept									
(1)	-0.88	-0.23	-0.48	-0.12	-0.13	-0.12	-0.00	-0.12	-0.00	-0.00
(4)	-4.00	-0.83	-1.88	-0.36	-0.75	-0.12	-0.38	-0.12	-0.13	-0.00
(7)	-7.13	-1.58	-3.38	-0.73	-1.38	-0.25	-0.63	-0.12	-0.38	-0.00
(10)	-10.4	-2.52	-5.00	-1.13	-1.88	-0.50	-1.00	-0.25	-0.50	-0.13
(13)	-13.8	-4.12	-6.63	-1.73	-2.50	-0.80	-1.25	-0.40	-0.63	-0.13
(16)	-17.1	-7.05	-8.25	-3.30	-3.13	-1.35	-1.63	-0.75	-0.75	-0.45
(18)	-19.4	-11.0	-9.38	-5.80	-3.63	-2.20	-1.75	-1.20	-0.88	-0.60

(*) The entries in the table are equal to $\hat{\rho}_b=(\bar{\rho}-\rho)/\rho*100$ and $\hat{\gamma}_b=(\bar{\gamma}-\gamma)/\gamma*100$, respectively.

Thus, from the preceding analysis, we conclude that in the process of persistence evaluation it may be worth distinguishing between two different possibilities. When the central bank inflation target is known (because it is publicly announced) we may expect to be able to estimate persistence with no (expected) bias if $\hat{\gamma}$ is used or with a small downward bias if $\hat{\rho}$ is used. However, when the central bank inflation target is estimated from the data, which corresponds to the common practice in the literature, we may expect such a fact to introduce an additional downward bias into the conventional measures of persistence. This bias might be particularly significant if OLS estimators are used to get an estimate of ρ. The $\hat{\gamma}$ statistic is very much less affected in such a situation.

5.2 – Finite sample distributions

We know from the literature that $\hat{\rho}$ is asymptotically normally distributed and from section 3 that the same result is valid for $\hat{\gamma}$. However, asymptotic results are of empirical interest only if they can provide a reasonable degree of approximation to the finite sample distributions for the samples usually available. Thus, we now use the output of our Monte Carlo experiments in order to evaluate the finite sample performance of the asymptotic normality distributions for $\hat{\gamma}$ and $\hat{\rho}$.

In order to investigate the sampling distribution of $\hat{\gamma}$ and $\hat{\rho}$ we test for skewness and excess kurtosis in the vectors of the corresponding Monte Carlo estimates obtained for the AR(1) and AR(2) processes as described above. For the AR(1) processes without and with an intercept, Tables A3 and A4 in the Appendix display the p-values for the statistics $(T/6)SK^2$ and $(T/24)KT^2$, where SK and KT denote the estimated skewness and excess kurtosis, respectively[28]. We see that the null hypotheses of zero skewness and zero excess kurtosis for the $\hat{\gamma}$ statistic are never rejected for a 1% test. Only for the single case of T=75 and $\rho = 0.95$, in the no intercept case, is the p-value lower than 5% for the skewness test. In stark contrast, the results for $\hat{\rho}$ suggest that its sampling distribution ceases to be symmetric for values of ρ larger than 0.70 and displays an increasing degree of kurtosis for values of ρ larger than 0.80, confirming the results in the literature that the normal approximation is quite poor in practice

[28] These statistics are both distributed qui-square with 1 degree of freedom.

when ρ is large (Phillips, 1977, Hansen, 1999). Strangely enough there are no signs of normality even in a sample with 1000 observations.

From Table A5 in the Appendix, which displays the p-values for the tests of zero skewness and zero excess kurtosis for the distributions of $\hat{\gamma}$ and $\hat{\rho}$ in the AR(2) case, we see that the null of zero skewness or zero excess kurtosis is never rejected for the $\hat{\gamma}$ statistic independently of whether we assume that the mean is known or unknown. But this is not true for the OLS estimator of ρ. As regards the distribution of $\hat{\rho}$, for the process with no intercept, we see that the two null hypotheses are not rejected for models (1) to (6) (the ones with complex roots), but are clearly rejected for models (7) to (18) (the ones with real roots). The situation looks somewhat better for the process with an intercept, but even in this case the null of zero skewness is rejected for models (9) to (18).

Thus, from the above evidence we conclude that the very often-invoked asymptotic normality of $\hat{\rho}$ is not of much help in finite samples, but that, in strong contrast, the asymptotic normality of $\hat{\gamma}$ may be expected to work fairly well in finite samples, at least for the type of processes investigated in this section.

5.3 – Robustness to outliers

In this section we evaluate the robustness of $\hat{\gamma}$ and $\hat{\rho}$ to the presence of outliers in the data. Given its non-parametric characteristic we expect $\hat{\gamma}$ to emerge as a more robust statistic than $\hat{\rho}$. Usually two types of outliers are dealt with in the literature: the additive outliers and the innovation outliers. The additive outliers are shocks that affect observations in isolation due to some non-repetitive events, which may occur as a result of measurement errors or special events (changes in VAT rates, union strikes, for instance)[29]. In turn, innovation outliers are defined as extreme realisations from the process generating the innovations, ε_t, in the model.

[29] Changes in VAT are, potentially, an important source of additive outliers in the series of inflation for most EU countries. For instance, for Ireland, Italy, France and Belgium the number of revisions on VAT rates during the period 1968-2003 were 28, 19, 18 and 11, respectively (source, Directorate-General Taxation and Customs Union, EU Commission).

More formally, consider the AR(1) process $z_t = \rho z_{t-1} + \varepsilon_t$ where ε_t is a white noise process and suppose that additive outliers of magnitude δ_t may occur with a given probability θ. Hence the time series we observe is

$$y_t = z_t + \delta_t \lambda_t \tag{5.1}$$

where λ_t is a Bernoulli variable taking the value of 1 with probability θ and 0 with probability (1-θ). Note that the additive outlier, δ_t, has a one-shot effect on the observed time series, y_t, and hence will produce a once-and-for-all peak in the series. In the case of innovation outliers the time series we observe can be thought of as being generated by

$$y_t = \rho y_{t-1} + \varepsilon_t + \delta_t \lambda_t \tag{5.2}$$

so that the outlier, δ_t, has an effect on the level of the series y_t, as any other shock ε_t, i.e., it will die out gradually the pace depending on ρ.

For the simple AR(1) model it has been shown (see, Lucas (1995)) that the OLS estimator of ρ converges towards zero when the value of the additive outlier (occurring in the middle of the sample) increases, but that under the same circumstances the consistency of the OLS estimator of ρ is not affected in the presence of an innovation outlier. For that reason in this section we focus on the effect of additive outliers on $\hat{\rho}$ and $\hat{\gamma}$.

With that purpose in mind we define the DGP as corresponding to the AR(1) model without an intercept used in sub-section 5.1 with the addition of 5% of observations drawn from the $N(0,5^2)$ distribution. Thus, in terms of equation (5.1) we have λ_t=1 with probability θ=0.05 (λ_t=0 with probability 1-θ=0.95) and the outliers, δ_t, are realisations from the $N(0,5^2)$ distribution. By constructing the DGP in this way we can, in theory, isolate the effect on $\hat{\rho}$ coming from the usual OLS bias (reported in Table 3) from the effect coming exclusively from the presence of outliers. Table 7 reports the results for 10.000 replications with T=100. Column (4) presents the estimated bias for $\hat{\rho}$ measured as a percentage deviation from the estimated values obtained in the absence of outliers (see column (2) in Table 3). This way we are measuring only the bias due to outliers.

The first important comment is that the presence of additive outliers in the data has a devastating effect on the OLS estimators. For instance, when ρ=0.50 the expected estimated $\hat{\rho}$ ($\overline{\rho}$ in column (2)) is as low as 0.28 (it is equal to 0.49 when no outliers are present) which corresponds to a downward bias of 42.15%. The effect of outliers decreases as ρ increases, but even for values of ρ as large as 0.80 the expected $\hat{\rho}$ is only 0.56 (bias of –28.62%).

As regards the $\hat{\gamma}$ statistic, the estimated bias (measured as a percentage deviation from the estimated values obtained in the absence of outliers in column (7) of Table 3) is reported in column (8). We can see that there is some downward bias as expected (given that some outliers will imply an additional crossing of the mean), but it is quite small. For instance, for the model with ρ=0.50 the expected $\hat{\gamma}$ is now 0.654 while it was 0.667 when no outliers were present. In general the bias of $\hat{\gamma}$ due to outliers increases as ρ increases but it always remains very small.

If instead we take a look at the average standard errors of both $\hat{\rho}$ and $\hat{\gamma}$ (columns (3) and (7)) and compare them to the corresponding standard errors obtained in the absence of outliers (Table 3, columns (5) and (9)), we conclude that the implications are much stronger for the standard errors of $\hat{\rho}$. In fact, while the standard errors of $\hat{\gamma}$ show a small increment (the largest increase is approximately 8% and occurs for the model with ρ=0.95), the standard errors of $\hat{\rho}$ for higher values of ρ more than doubled (for instance for the model with ρ=0.80, the standard error was 0.063 when no outliers were present in the DGP and is now 0.154). The implications for the standard deviations would naturally be reflected in, for instance, the properties of the interquartile range of each estimator. From column (5) we see that (with the exception of models with ρ=0.00 and ρ=0.10) the interquartile range of the OLS estimator does not include the true ρ (nor the estimated ρ when no outliers are present). In contrast, the interquartile range for $\hat{\gamma}$ (column (9)) always includes the true γ (or the estimated γ obtained in Table 3 when no outliers were present in the data).

Results in Table 7 are, of course, specific to the particular way we generated the data, and less extreme outliers are expected to have less damaging consequences for $\hat{\rho}$. However, the exercise carried out shows that in general we can expect $\hat{\gamma}$ to be more robust to the presence of additive outliers in the data than the OLS estimator of ρ.

Table 7 – Robustness to additive outliers
AR(1) process with no intercept (T=100)

True ρ	$\overline{\rho}$	Std. error of $\hat{\rho}$	Bias of $\hat{\rho}$ (%)	Inter Quartile Range	$\overline{\gamma}$	Standard error of $\hat{\gamma}$	Bias of $\hat{\gamma}$ (%)	Inter Quartile Range
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
0.00	0.002	0.094	---	-0.059-0.063	0.500	0.050	0.14	0.47-0.53
0.10	0.053	0.096	-46.32	-0.012-0.115	0.530	0.050	-0.45	0.50-0.56
0.20	0.105	0.102	-46.59	0.036-0.172	0.559	0.050	-0.87	0.53-0.59
0.30	0.160	0.111	-45.81	0.083-0.233	0.590	0.051	-1.25	0.56-0.62
0.40	0.219	0.122	-44.34	0.134-0.302	0.621	0.051	-1.60	0.59-0.66
0.50	0.284	0.134	-42.15	0.191-0.378	0.654	0.051	-1.94	0.62-0.69
0.55	0.320	0.139	-40.74	0.222-0.419	0.671	0.051	-2.12	0.64-0.71
0.60	0.359	0.145	-39.07	0.257-0.463	0.689	0.052	-2.30	0.65-0.72
0.65	0.401	0.149	-37.10	0.297-0.510	0.708	0.052	-2.37	0.67-0.74
0.70	0.448	0.153	-34.76	0.342-0.561	0.729	0.052	-2.50	0.69-0.76
0.75	0.501	0.155	-31.98	0.395-0.616	0.750	0.052	-2.65	0.72-0.79
0.80	0.560	0.154	-28.62	0.459-0.677	0.774	0.053	-2.77	0.74-0.81
0.85	0.630	0.150	-24.50	0.533-0.744	0.801	0.053	-2.83	0.77-0.84
0.90	0.713	0.139	-19.33	0.628-0.818	0.834	0.054	-2.71	0.80-0.87
0.95	0.815	0.117	-12.61	0.755-0.903	0.877	0.054	-2.46	0.84-0.91

5.4 – Robustness to model misspecification

A very nice property of $\hat{\gamma}$ stems from the fact that it is model free, i.e., it does not require the specification and estimation of a model for the series under investigation. In this sense $\hat{\gamma}$ is totally immune to potential model misspecifications, which are likely to affect the performance of alternative estimators, such as the OLS estimator of ρ.

In other words, the analysis carried out in sub-section 5.1, concerning the expected values of $\hat{\gamma}$ and $\hat{\rho}$, may be misleading because it does not compare the real empirical performance of the two estimators. In fact, while in sub-section 5.1 all the uncertainty surrounding $\hat{\gamma}$ was taken into consideration (the single relevant source of potential bias for this statistic comes from the fact that the mean of the process may be unknown and thus has to be estimated) the uncertainty regarding the true model for the y_t series, which is required to get an estimate of ρ, was not taken into account when evaluating the performance of $\hat{\rho}$. Thus, in order to get a real picture on the relative performance of the two estimators we need, in addition, to evaluate the performance of $\hat{\rho}$ to potential model misspecifications.

For that purpose we define a Monte Carlo experiment that exactly matches the one analysed in sub-section 5.1 for the AR(2) model with an intercept and $\rho_1 + \rho_2 = 0.8$. The unique relevant difference is that now we assume that the DGP, including the mean of the series, is unknown, as we want to compare the behaviour of $\hat{\rho}$ and $\hat{\gamma}$ under such circumstances.

Table 8 reports three alternative estimates of ρ, each corresponding to a different estimated model. Column (2) reports the average value of $\hat{\rho}$ when the AR(1) model $y_t = \alpha + \rho y_{t-1} + \varepsilon_t$ is estimated ($\overline{\rho}_{a1}$) while column (5) reports the average value of $\hat{\rho}$ when a searching procedure is carried out starting with the AR(4) model with an intercept and selecting the final model using the Schwarz criterion ($\overline{\rho}_{sw}$). Column (4) reports the proportion of times that the true AR(2) model is selected by the Schwarz criterion. Finally column (7) reports the average estimated $\hat{\rho}$ when an AR(4) is fitted to the data ($\overline{\rho}_{a4}$). Thus, in Table 8 we are considering the effect on the OLS estimators of omitting relevant variables (columns (2) and (3)), introducing irrelevant variables (columns (7) and (8)) or following a searching procedure based on the Schwarz criterion (columns (5) and (6)).

We are interested in comparing the output in Table 8 with the output in Table 5 (columns (4) and (5)), where it was assumed that the DGP was known. The bias reported in Table 8 is the total bias, which includes the pure bias of the OLS estimator (Table 5, column (5)) as well as the bias stemming from model misspecification.

As regards the $\hat{\gamma}$ statistic, the relevant figures are those of Table 5 (columns (8) and (9)) as the single relevant source of potential bias for this statistic comes from the fact that the mean is unknown.

Column (3) in Table 8 reports the omitted variable bias of the OLS estimators when an AR(1) model is estimated. Interestingly enough we see that the sign of the bias is not always negative as in Table 5. The omitted variable bias of the OLS estimators is positive for models (1) to (9) and negative (adding to the OLS pure bias) for models (10) to (18). As expected the omitted variable bias increases as the true model gets farther away from the pure AR(1) model. In the context of the AR(2) process we would expect models (11) (12) and (13) to be the more realistic ones in empirical terms (first coefficient higher than the second and both positive). However, for models (12) and (13) the increased bias stemming from omitted variables is quite significant. For instance for model (13) the expected estimated ρ is as low as 0.655, (bias of –18.13%) while the expected $\hat{\rho}$ when no model misspecification is present is 0.747 (bias of -6.63%, see Table 5).

Table 8 – Robustness to model misspecification
Different AR(2) models with $\rho = \rho_1 + \rho_2 = 0.80$ and T=100

DGP: AR(2) Model		AR(1)		Searching procedure			AR(4)	
		$\bar{\rho}_{a1}$	Bias %(*)	AR(2) %	$\bar{\rho}_{sw}$	Bias %(*)	$\bar{\rho}_{a4}$	Bias %(*)
(1)		(2)	(3)	(4)	(5)	(6)	(7)	(8)
(1)	$\rho_1 = 1.7, \rho_2 = -0.9$	0.893	11.63	96.2	0.796	-0.50	0.790	-1.25
(2)	$\rho_1 = 1.6, \rho_2 = -0.8$	0.885	10.63	96.2	0.793	-0.88	0.787	-1.63
(3)	$\rho_1 = 1.5, \rho_2 = -0.7$	0.876	9.50	96.3	0.789	-1.38	0.783	-2.13
(4)	$\rho_1 = 1.4, \rho_2 = -0.6$	0.866	8.25	96.5	0.785	-1.88	0.779	-2.63
(5)	$\rho_1 = 1.3, \rho_2 = -0.5$	0.854	6.75	96.2	0.781	-2.38	0.774	-3.25
(6)	$\rho_1 = 1.2, \rho_2 = -0.4$	0.841	5.13	94.3	0.777	-2.88	0.770	-3.75
(7)	$\rho_1 = 1.1, \rho_2 = -0.3$	0.826	3.25	81.4	0.777	-2.88	0.766	-4.25
(8)	$\rho_1 = 1.0, \rho_2 = -0.2$	0.809	1.13	48.7	0.781	-2.38	0.761	-4.88
(9)	$\rho_1 = 0.9, \rho_2 = -0.1$	0.789	-1.38	16.1	0.779	-2.63	0.757	-5.38
(10)	$\rho_1 = 0.8, \rho_2 = 0.0$	0.765	-4.38	95.9(+)	0.764	-4.50	0.753	-5.88
(11)	$\rho_1 = 0.7, \rho_2 = 0.1$	0.736	-8.00	8.4	0.741	-7.38	0.748	-6.50
(12)	$\rho_1 = 0.6, \rho_2 = 0.2$	0.700	-12.50	33.6	0.729	-8.88	0.743	-7.13
(13)	$\rho_1 = 0.5, \rho_2 = 0.3$	0.655	-18.13	69.0	0.732	-8.50	0.739	-7.63
(14)	$\rho_1 = 0.4, \rho_2 = 0.4$	0.596	-25.50	90.2	0.738	-7.75	0.735	-8.13
(15)	$\rho_1 = 0.3, \rho_2 = 0.5$	0.517	-35.38	95.4	0.737	-7.88	0.730	-8.75
(16)	$\rho_1 = 0.2, \rho_2 = 0.6$	0.403	-49.63	96.0	0.733	-8.38	0.725	-9.38
(17)	$\rho_1 = 0.1, \rho_2 = 0.7$	0.227	-71.63	95.9	0.728	-9.00	0.721	-9.88
(18)	$\rho_1 = 0.0, \rho_2 = 0.8$	-0.085	-110.6	95.8	0.724	-9.50	0.716	-10.50

(*) Bias = $(\bar{\rho} - 0.8)/0.8 * 100$ where $\bar{\rho}$ is from column (2), (5) or (7) respectively; (+) Percentage of the AR(1) model;

Let us now consider the case of the searching procedure, as this is likely to reflect the best we can do in empirical applications. The first important conclusion is that in general the searching procedure does a very good job. Comparing column (5) of Table

8 with column (5) of Table 5 it is seen that for the bulk of the (18) models, and despite the fact that the true AR(2) is not chosen 100% of the times, the searching procedure ensures that the final bias of $\hat{\rho}$ does not significantly differ from the bias we get in Table 5, i.e., under the assumption that the DGP is known. Curiously enough, the exceptions are models (11), (12) and (13) which, as we have seen, are expected to be the most realistic ones in empirical terms. For these three models the searching procedure does not prevent an additional mean bias of accruing to the original bias, so that the total bias is now between –7.0% and –9.0%. The reason is that the Schwarz criterion picks out the AR(1) model (the wrong model) in a large proportion of times and this implies carrying to column (6) some of the bias found in column (2).

As regards the effect of including irrelevant variables it is seen that there is a small increase in the downward bias of $\hat{\rho}$ when we compare column (8) in Table 8 with column (5) in Table 5, so that in general there seems to be some potential gain in implementing the searching procedure. However, it turns out that for models (11), (12) and (13) just estimating the AR(4) model appears as a better strategy than carrying out the general to specific methodology, based on the Schwarz criterion.

We note that the exercise in this sub-section was designed in order to deliver the best possible results for $\hat{\rho}$. In particular, the fact that the data are generated by an autoregressive process (rather than an Arma model, for example) or that the "searching procedure" or the "general model" approach are designed so that they encompass the true DGP are examples of those favourable designing characteristics of the exercise. In practice, it is important to bear in mind that the DGP may not be an autoregressive model (or not be well approximated by an autoregressive model) or that our "general estimated model" may not encompass the true model if the number of included lags is not large enough. And, in this latter case, the "omitted variable bias" would pop into our estimator through the back door.

On the other hand, it has been argued in the literature (see Schwert (1987)) that there are many good reasons to believe that most economic time series contain moving average components, i.e., are better described by an Arma model rather than by a pure autoregressive process. As we have seen in section 2, assuming that the data are described by an Arma model has the implication that ρ (the sum of autoregressive coefficients) ceases to be the parameter of interest as it no longer measures persistence of the series. The solution, in empirical terms, implies using a finite order autoregressive process to approximate the true Arma model, but this is likely to introduce additional biases into the estimator $\hat{\rho}$. In strong contrast, γ as a measure of persistence, may be defined and its estimator, $\hat{\gamma}$, may be computed directly from the data, irrespective of the underlying DGP.

An interesting question is whether using the "median unbiased estimator" for ρ (see Andrews, 1994 and Hansen, 1999) allows overcoming the limitations of the OLS estimator, $\hat{\rho}$, identified in this section. Resorting to the median unbiased estimator helps alleviating the biases examined in sub-section 5.1, i.e., the biases stemming exclusively from the OLS estimators under the assumption of correct model specification. But, such an estimator does not seem capable of avoiding the biases stemming from the presence of outliers or from model misspecification because it is obtained from Monte Carlo simulations under the assumption of correct model specification and/or distributions of the residuals without outliers.

6. CONCLUSIONS

This paper investigates the use of mean reversion as a measure of persistence. The distinction between the theoretical concept of mean reversion denoted by γ and its estimator, denoted by $\hat{\gamma}$, suggested in Marques (2004), is made clear. The theoretical properties of the estimator $\hat{\gamma}$ are discussed under very general conditions and the relationship between γ and ρ, the "sum of the autoregressive coefficients", is investigated. Using evidence from Monte Carlo simulations, the finite sample properties of $\hat{\gamma}$ are compared to the properties of the OLS estimator of ρ the "sum of the autoregressive coefficients".

Several results emerge from the paper. First, the process of obtaining the theoretical value of γ for a general process is derived where γ is the unconditional probability of a given process not crossing its mean, or equivalently, one minus the probability of mean reversion of the process.

Second, it is shown that, under very general conditions, $\hat{\gamma}$ is an unbiased estimator of γ when the mean of the process is known and a consistent estimator of γ when the mean is unknown. Moreover, it is also shown that, under the same very general conditions, $\hat{\gamma}$ is asymptotically normal distributed, so that tests of hypotheses on γ can be performed very easily.

Third, using Monte Carlo simulations for the AR(1) and AR(2) processes with normal innovations it is shown that while the OLS estimator of ρ, $\hat{\rho}$, is downward biased, $\hat{\gamma}$ is an unbiased estimator when it is assumed that the mean of the process is known. Both estimators are downward biased when the mean of the process is unknown but the biases of $\hat{\gamma}$ are always smaller than those of $\hat{\rho}$. Moreover, for the AR(2) process the biases of $\hat{\rho}$ depend not only on the value of ρ, but also on the specific combination of ρ_1 and ρ_2, for a constant ρ.

Fourth, the same Monte Carlo simulations show that the finite sample distribution of $\hat{\gamma}$ is well approximated by the normal distribution while, as expected, the distribution of $\hat{\rho}$ is not.

Fifth, it is shown that $\hat{\rho}$ may be subject to significant biases coming from the presence of additive outliers in the data or from model misspecifications, but that $\hat{\gamma}$ is almost not affected by the first potential source of bias and is, by definition, totally immune to the second.

Finally, it is shown that there is a monotonic relationship between ρ and γ (and several other measures of persistence) when the data is generated by an AR(1) process, but that such a monotonic relation ceases to exist for higher order process. In particular, it is shown for the AR(2) process with a fixed ρ that the set of measures of persistence considered in the paper (including γ, the half-life, the largest autoregressive root, the number of periods required for 99 percent of the adjustment to take place) behave independently of ρ, with some of them increasing and others decreasing as we change the combination of the autoregressive coefficients ρ_1 and ρ_2, for a constant ρ. We argue in the paper that when γ and ρ give conflicting information about the degree of persistence it is γ and not ρ that delivers the more important message about persistence. Notwithstanding, for higher order processes, using γ and ρ as two companion scalar measures of persistence appears as the best strategy given that the two measures simultaneously may allow uncovering important features of inflation persistence that cannot be extracted from each measure alone. Specifically, values of $\hat{\rho}$ clearly above to what could be expected given the value of $\hat{\gamma}$

can be seen as signalling the presence of a cyclical behaviour in the model of inflation (which the central bank may not find as particularly worrisome as this implies that inflation would cross the inflation target more frequently than could be suggested by the value of $\hat{\rho}$) while estimates of ρ clearly below to the ones we would expect given the estimated γ may be signalling strong downward biases of the OLS (or the median unbiased) estimator of ρ, stemming from the presence of additive outliers in the data or from model misspecification.

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APPENDIX

COMPUTING γ, THE UNCONDITIONAL PROBABILITY OF A PROCESS NOT CROSSING THE MEAN

Let Y_t be a stationary stochastic process such that $E[Y_t]=0\ \forall t$, and define the random variable $X_t = I(Y_t \cdot Y_{t-1}>0)$, where I(.) denotes the indicator function[30]. By noticing that Y_t does not cross the mean (zero) in period t if and only if $Y_t \cdot Y_{t-1}>0$, we may define γ the unconditional probability that the process Y_t does not cross its mean in period t, as:

$$\gamma = E[X_t] = E[I(Y_t \cdot Y_{t-1} > 0)] = P[(Y_t > 0 \wedge Y_{t-1} > 0) \cup (Y_t < 0 \wedge Y_{t-1} < 0)] \quad (A.1)$$

From definition (A.1) it is clear that calculating γ implies knowing the p.d.f. of X_t or, equivalently, the joint p.d.f. of Y_t and Y_{t-1}, which can be very difficult to obtain when Y_t and Y_{t-1} are not independent.

However, γ may very easily be computed if we assume that Y_t follows an AR(p) process with normal innovations:

$$Y_t = \sum_{i=1}^{p} \rho_i \cdot Y_{t-i} + \varepsilon_t, \qquad \varepsilon_t \sim N(0, \sigma_\varepsilon^2) \quad (A.2)$$

As it is well known, in this case, the joint p.d.f. of Y_t and Y_{t-1} is a bivariate normal distribution,

$$(Y_t, Y_{t-1}) \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}; \begin{pmatrix} Var(Y_t) & Cov(Y_t, Y_{t-1}) \\ Cov(Y_t, Y_{t-1}) & Var(Y_t) \end{pmatrix}\right) \quad (A.3)$$

Denoting the cumulative distribution function (c.d.f.) of Y_t and Y_{t-1} by

$$F(y_t, y_{t-1}) = P(Y_t \le y_t \wedge Y_{t-1} \le y_{t-1}) \quad (A.4)$$

and using the fact that the bivariate normal distribution is symmetric relatively to any line that crosses the mean and divides the cartesian plane evenly, calculating γ amounts at calculating

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[30] I(argument) takes value 1 if argument is true and 0 otherwise.

$$\gamma = 2 \times F(0;0) \tag{A.5}$$

which may be easily accomplished resorting to some standard software packages as Mathematica or TSP. As a simple illustration, let us consider an AR(2) process,

$$Y_t = \rho_1 \cdot Y_{t-1} + \rho_2 \cdot Y_{t-2} + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma_\varepsilon^2) \tag{A.6}$$

From (A.6) it follows that Cov(Y_t,Y_{t-1})=$\frac{\rho_1}{1-\rho_2}$ and, without loss of generality, we may assume Var(Y_t)=1, therefore (A.3) reduces to

$$(Y_t, Y_{t-1}) \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}; \begin{pmatrix} 1 & \frac{\rho_1}{1-\rho_2} \\ \frac{\rho_1}{1-\rho_2} & 1 \end{pmatrix} \right) \tag{A.7}$$

For instance, for ρ_1=0.5 and ρ_2=0.2 the corresponding γ is 0.7149, i.e., the unconditional probability of an AR(2) process with ρ_1=0.5 and ρ_2=0.2 not crossing the mean is 71.49%.

Obtaining the value of γ for a general Arma model is straightforward provided normality of the innovations is assumed. Difficulties may arise under a different assumption for the distribution of the innovations. However, given that $\hat{\gamma}$ is an unbiased estimator of γ, when the mean of the process is known, as discussed in section 3, a fairly good approximation to γ may be obtained using Monte Carlo simulations for any assumed distribution of the innovations.

Table A1
Monte Carlo simulations – AR(1) model with no intercept(*)

True ρ	True γ	T=50		T=75		T=150		T=250		T=500		T=1000	
		$\bar{\rho}$	$\bar{\gamma}$	$\bar{\rho}$	$\bar{\gamma}$	$\bar{\rho}$	$\bar{\gamma}$	$\bar{\rho}$	$\bar{\gamma}$	$\bar{\rho}$	$\bar{\gamma}$	$\bar{\rho}$	$\bar{\gamma}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(8)	(9)
0.00	0.500	0.002	0.501	-0.001	0.499	0.001	0.500	0.001	0.500	0.000	0.500	0.000	0.500
0.10	0.532	0.099	0.533	0.097	0.531	0.099	0.532	0.100	0.532	0.100	0.532	0.100	0.532
0.20	0.564	0.195	0.565	0.194	0.563	0.198	0.564	0.199	0.564	0.199	0.564	0.200	0.564
0.30	0.597	0.291	0.598	0.292	0.597	0.297	0.598	0.299	0.597	0.299	0.597	0.299	0.597
0.40	0.631	0.387	0.632	0.389	0.630	0.395	0.632	0.398	0.631	0.399	0.631	0.399	0.631
0.50	0.667	0.483	0.668	0.486	0.666	0.494	0.667	0.497	0.667	0.498	0.667	0.499	0.667
0.55	0.685	0.531	0.686	0.535	0.685	0.543	0.686	0.546	0.686	0.548	0.685	0.549	0.685
0.60	0.705	0.579	0.706	0.584	0.704	0.593	0.705	0.596	0.705	0.598	0.705	0.599	0.705
0.65	0.725	0.627	0.726	0.633	0.724	0.642	0.725	0.645	0.725	0.648	0.725	0.649	0.725
0.70	0.747	0.675	0.747	0.682	0.746	0.691	0.747	0.695	0.747	0.697	0.747	0.699	0.747
0.75	0.770	0.723	0.770	0.730	0.770	0.741	0.771	0.745	0.770	0.747	0.770	0.748	0.770
0.80	0.795	0.771	0.795	0.779	0.795	0.790	0.796	0.794	0.795	0.797	0.795	0.798	0.795
0.85	0.823	0.819	0.823	0.828	0.824	0.840	0.824	0.844	0.823	0.847	0.823	0.848	0.823
0.90	0.856	0.868	0.856	0.878	0.856	0.889	0.857	0.893	0.856	0.897	0.857	0.898	0.856
0.95	0.899	0.919	0.899	0.928	0.900	0.939	0.899	0.943	0.899	0.946	0.899	0.948	0.899

(*) Entries are the average value of Monte Carlo estimates of ρ ($\bar{\rho}$) and of γ ($\bar{\gamma}$).

Table A2
Monte Carlo simulations – AR(1) model with an intercept[(*)]

True ρ	True γ	T=50		T=75		T=150		T=250		T=500		T=1000	
		$\bar{\rho}$	$\bar{\gamma}$	$\bar{\rho}$	$\bar{\gamma}$	$\bar{\rho}$	$\bar{\gamma}$	$\bar{\rho}$	$\bar{\gamma}$	$\bar{\rho}$	$\bar{\gamma}$	$\bar{\rho}$	$\bar{\gamma}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(8)	(9)
0.00	0.500	-0.018	0.495	-0.014	0.494	-0.006	0.498	-0.003	0.499	-0.002	0.499	-0.001	0.500
0.10	0.532	0.077	0.526	0.082	0.526	0.092	0.530	0.096	0.531	0.098	0.531	0.099	0.531
0.20	0.564	0.171	0.557	0.178	0.558	0.190	0.562	0.195	0.563	0.197	0.563	0.198	0.564
0.30	0.597	0.264	0.589	0.274	0.590	0.288	0.595	0.293	0.596	0.296	0.596	0.298	0.597
0.40	0.631	0.358	0.622	0.370	0.624	0.386	0.629	0.392	0.629	0.396	0.630	0.398	0.630
0.50	0.667	0.452	0.656	0.466	0.658	0.484	0.663	0.491	0.665	0.495	0.665	0.498	0.666
0.55	0.685	0.499	0.674	0.514	0.677	0.533	0.682	0.540	0.683	0.545	0.684	0.547	0.685
0.60	0.705	0.545	0.692	0.562	0.696	0.582	0.701	0.589	0.702	0.595	0.703	0.597	0.704
0.65	0.725	0.591	0.711	0.609	0.715	0.631	0.721	0.639	0.722	0.644	0.724	0.647	0.724
0.70	0.747	0.638	0.731	0.657	0.736	0.680	0.742	0.688	0.744	0.694	0.745	0.697	0.746
0.75	0.770	0.684	0.753	0.705	0.758	0.729	0.765	0.737	0.767	0.744	0.768	0.747	0.769
0.80	0.795	0.730	0.775	0.752	0.782	0.777	0.789	0.787	0.791	0.793	0.793	0.797	0.794
0.85	0.823	0.775	0.800	0.800	0.807	0.826	0.816	0.836	0.819	0.843	0.821	0.846	0.822
0.90	0.856	0.819	0.826	0.846	0.837	0.875	0.848	0.885	0.851	0.893	0.854	0.896	0.855
0.95	0.899	0.862	0.854	0.891	0.869	0.922	0.885	0.934	0.891	0.942	0.895	0.946	0.897

[(*)] Entries are the average value of Monte Carlo estimates of ρ ($\bar{\rho}$) and of γ ($\bar{\gamma}$).

Table A3 – Skewness and kurtosis of $\hat{\rho}$ and $\hat{\gamma}$
AR(1) model with no intercept [(*)]

	T=50				T=75				T=100				T=150				T=250			
	$\hat{\rho}$		$\hat{\gamma}$		$\hat{\rho}$		$\hat{\gamma}$		$\hat{\rho}$		$\hat{\gamma}$		$\hat{\rho}$		$\hat{\gamma}$		$\hat{\rho}$		$\hat{\gamma}$	
True ρ	SK	KT	SK	KT	SK	KT	SK	KT	SK	KT	SK	KT	SK	KT	SK	KT	SK	KT	SK	KT
0.00	0.957	0.861	0.934	0.934	0.969	0.806	0.790	0.954	0.998	0.912	0.960	0.945	0.966	0.895	0.985	0.798	0.844	0.876	0.994	0.550
0.10	0.883	0.874	0.972	0.990	0.795	0.823	0.807	0.999	0.820	0.921	0.977	0.985	0.781	0.902	0.928	0.949	0.964	0.920	0.987	0.842
0.20	0.718	0.907	0.954	0.984	0.625	0.865	0.915	0.979	0.645	0.949	0.961	0.953	0.606	0.920	0.907	0.994	0.759	0.961	0.977	0.958
0.30	0.550	0.966	0.855	0.911	0.463	0.941	0.780	0.965	0.482	0.994	0.943	0.996	0.445	0.950	0.820	0.896	0.549	0.990	0.883	0.952
0.40	0.389	0.945	0.880	0.825	0.314	0.940	0.808	0.996	0.338	0.949	0.902	0.989	0.303	0.992	0.666	0.776	0.355	0.998	0.810	0.931
0.50	0.246	0.822	0.791	0.893	0.187	0.769	0.679	0.928	0.217	0.876	0.808	0.982	0.185	0.949	0.700	0.965	0.197	0.987	0.732	0.855
0.55	0.184	0.746	0.735	0.972	0.134	0.664	0.653	0.985	0.164	0.828	0.848	0.988	0.136	0.905	0.760	0.905	0.135	0.965	0.769	0.919
0.60	0.131	0.659	0.687	0.999	0.090	0.549	0.666	0.976	0.118	0.764	0.774	0.990	0.094	0.842	0.769	0.825	0.086	0.926	0.606	0.944
0.65	0.087	0.556	0.615	0.940	0.056	0.426	0.632	0.987	0.077	0.673	0.711	0.960	0.059	0.748	0.611	0.877	0.049	0.864	0.513	0.925
0.70	0.052	0.429	0.589	0.887	0.030	0.301	0.663	0.968	0.044	0.538	0.712	0.971	0.032	0.606	0.613	0.877	0.024	0.764	0.585	0.924
0.75	0.026	0.273	0.509	0.948	0.014	0.181	0.548	0.979	0.019	0.354	0.573	0.956	0.013	0.406	0.453	0.879	0.009	0.603	0.534	0.967
0.80	0.010	0.108	0.413	0.952	0.005	0.081	0.554	0.946	0.006	0.155	0.489	1.000	0.003	0.182	0.500	0.986	0.002	0.360	0.400	0.939
0.85	0.002	0.019	0.343	0.998	0.001	0.019	0.398	0.974	0.001	0.028	0.434	0.958	0.000	0.033	0.418	0.896	0.000	0.098	0.391	0.945
0.90	0.000	0.002	0.226	0.990	0.000	0.001	0.183	0.852	0.000	0.001	0.336	0.837	0.000	0.001	0.209	0.916	0.000	0.002	0.239	0.999
0.95	0.000	0.000	0.062	0.863	0.000	0.000	0.047	0.778	0.000	0.000	0.076	0.993	0.000	0.000	0.07	0.899	0.000	0.000	0.092	0.887

(*) The entries are the P-values of the statistics $(T/6)SK^2$ and $(T/24)KT^2$ where SK and KT denote the estimated skewness and excess kurtosis, respectively.

Table A4 – Skewness and kurtosis of $\hat{\rho}$ and $\hat{\gamma}$
AR(1) model with an intercept (*)

	T=50				T=75				T=100				T=150				T=250			
	$\hat{\rho}$		$\hat{\gamma}$		$\hat{\rho}$		$\hat{\gamma}$		$\hat{\rho}$		$\hat{\gamma}$		$\hat{\rho}$		$\hat{\gamma}$		$\hat{\rho}$		$\hat{\gamma}$	
True ρ	SK	KT	SK	KT	SK	KT	SK	KT	SK	KT	SK	KT	SK	KT	SK	KT	SK	KT	SK	KT
0.00	0.951	0.863	0.981	0.961	0.957	0.845	0.813	0.984	0.988	0.944	0.927	0.979	0.980	0.890	1.000	0.828	0.846	0.822	0.931	0.660
0.10	0.898	0.871	0.987	0.897	0.783	0.867	0.882	0.948	0.808	0.949	0.942	0.940	0.791	0.898	0.912	0.982	0.963	0.859	0.881	0.811
0.20	0.743	0.900	0.948	0.990	0.614	0.913	0.838	0.949	0.637	0.972	0.928	0.964	0.612	0.920	0.916	0.745	0.758	0.894	0.854	0.964
0.30	0.586	0.954	0.931	0.921	0.455	0.990	0.862	0.969	0.478	0.991	0.948	0.883	0.447	0.955	0.944	0.831	0.549	0.921	0.998	0.972
0.40	0.432	0.963	0.826	0.887	0.311	0.894	0.833	0.998	0.342	0.945	0.893	0.937	0.302	0.997	0.786	0.971	0.354	0.932	0.816	0.927
0.50	0.291	0.848	0.716	0.973	0.190	0.732	0.708	0.969	0.226	0.890	0.790	0.984	0.183	0.931	0.759	0.885	0.196	0.922	0.819	0.953
0.55	0.228	0.780	0.662	0.974	0.140	0.637	0.699	0.993	0.176	0.855	0.743	0.989	0.134	0.887	0.711	0.856	0.135	0.906	0.887	0.863
0.60	0.172	0.705	0.692	0.943	0.097	0.535	0.632	0.970	0.132	0.811	0.809	0.966	0.093	0.827	0.761	0.707	0.086	0.877	0.803	0.860
0.65	0.123	0.622	0.684	0.910	0.063	0.431	0.633	0.972	0.092	0.748	0.788	0.997	0.059	0.744	0.723	0.783	0.050	0.831	0.808	0.853
0.70	0.083	0.529	0.658	0.891	0.038	0.327	0.526	0.908	0.058	0.654	0.723	0.914	0.033	0.625	0.600	0.853	0.025	0.756	0.563	0.798
0.75	0.051	0.419	0.581	0.957	0.020	0.222	0.501	0.959	0.031	0.519	0.572	0.893	0.015	0.455	0.525	0.861	0.010	0.633	0.504	0.742
0.80	0.028	0.286	0.553	0.933	0.008	0.117	0.475	0.982	0.013	0.342	0.519	0.989	0.005	0.243	0.434	0.990	0.002	0.437	0.332	0.889
0.85	0.012	0.156	0.440	0.900	0.002	0.031	0.397	0.950	0.003	0.157	0.332	0.908	0.001	0.059	0.408	0.969	0.000	0.178	0.311	0.927
0.90	0.005	0.071	0.294	0.970	0.000	0.001	0.263	1.000	0.000	0.034	0.274	0.987	0.000	0.000	0.239	0.928	0.000	0.000	0.198	0.945
0.95	0.001	0.011	0.144	0.920	0.000	0.000	0.109	0.945	0.000	0.001	0.132	0.958	0.000	0.000	0.090	0.980	0.000	0.000	0.077	0.898

(*) The entries are the P-values of the statistics $(T/6)SK^2$ and $(T/24)KT^2$ where SK and KT denote the estimated skewness and excess kurtosis, respectively.

Table A5 – Skewness and kurtosis of $\hat{\rho}$ and $\hat{\gamma}$
AR(2) process (*) (T=100)

Model		Model with no intercept				Model with an intercep			
		$\hat{\rho}$		$\hat{\gamma}$		$\hat{\rho}$		$\hat{\gamma}$	
		SK	KT	SK	KT	SK	KT	SK	KT
(1)	$\rho_1 = 1.7, \rho_2 = -0.9$	0.757	0.341	0.312	0.126	0.618	0.321	0.266	0.151
(2)	$\rho_1 = 1.6, \rho_2 = -0.8$	0.303	0.499	0.308	0.399	0.265	0.500	0.266	0.422
(3)	$\rho_1 = 1.5, \rho_2 = -0.7$	0.119	0.370	0.230	0.601	0.114	0.423	0.222	0.614
(4)	$\rho_1 = 1.4, \rho_2 = -0.6$	0.051	0.241	0.221	0.668	0.055	0.333	0.172	0.374
(5)	$\rho_1 = 1.3, \rho_2 = -0.5$	0.024	0.153	0.299	0.784	0.029	0.264	0.251	0.647
(6)	$\rho_1 = 1.2, \rho_2 = -0.4$	0.012	0.097	0.267	0.806	0.017	0.214	0.297	0.890
(7)	$\rho_1 = 1.1, \rho_2 = -0.3$	0.006	0.063	0.331	0.909	0.010	0.178	0.299	0.866
(8)	$\rho_1 = 1.0, \rho_2 = -0.2$	0.004	0.042	0.391	0.975	0.007	0.151	0.355	0.950
(9)	$\rho_1 = 0.9, \rho_2 = -0.1$	0.002	0.028	0.408	0.977	0.004	0.128	0.448	0.930
(10)	$\rho_1 = 0.8, \rho_2 = 0.0$	0.001	0.019	0.435	0.916	0.003	0.107	0.441	0.898
(11)	$\rho_1 = 0.7, \rho_2 = 0.1$	0.001	0.012	0.456	0.923	0.002	0.088	0.405	0.905
(12)	$\rho_1 = 0.6, \rho_2 = 0.2$	0.000	0.008	0.542	0.929	0.001	0.071	0.491	0.911
(13)	$\rho_1 = 0.5, \rho_2 = 0.3$	0.000	0.005	0.522	0.946	0.001	0.055	0.530	0.953
(14)	$\rho_1 = 0.4, \rho_2 = 0.4$	0.000	0.003	0.554	0.972	0.001	0.042	0.503	0.959
(15)	$\rho_1 = 0.3, \rho_2 = 0.5$	0.000	0.001	0.607	0.981	0.000	0.032	0.659	0.815
(16)	$\rho_1 = 0.2, \rho_2 = 0.6$	0.000	0.001	0.701	0.997	0.000	0.023	0.731	0.900
(17)	$\rho_1 = 0.1, \rho_2 = 0.7$	0.000	0.000	0.820	0.932	0.000	0.017	0.989	0.879
(18)	$\rho_1 = 0.0, \rho_2 = 0.8$	0.000	0.000	0.988	0.829	0.000	0.012	0.816	0.819

(*) The entries are the P-values of the statistics $(T/6)SK^2$ and $(T/24)KT^2$ where SK and KT denote the estimated skewness and excess kurtosis, respectively.

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