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The Taylor Rule and Interest Rate Uncertainty in the U.S. 1970-2006

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Abstract

This paper shows how to estimate forecast uncertainty about future short-term interest rates by combining a time-varying Taylor rule with an unobserved components model of economic fundamentals. Using this model I separate interest rate uncertainty into economically meaningful components that represent uncertainty about future economic conditions and uncertainty about future monetary policy. Results from estimating the model on U.S. data suggest important changes in uncertainty about future short-term interest rates over time and highlight the relative importance of the different elements which underlie interest rate uncertainty for the U.S.

Keywords: Monetary policy, reaction functions, state-space models, output-gap forecasts, inflation forecasts
JEL Classification: E52, C32, C53

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1. INTRODUCTION

This paper presents an empirical analysis of time-variation in uncertainty about future short-term interest rates in the U.S. Its main contribution is the decomposition of this uncertainty in various components that can be interpreted economically as uncertainty about future behavior of the central bank, uncertainty about the future state of the economy and a residual component.

Estimates of interest rate uncertainty are important for a wide range of financial market applications such as portfolio allocation, derivative pricing, risk management etc. Uncertainty about short-term interest rates is also important for evaluating monetary policy as the most important factors for the determination of money-market interest rates are interest rates set by the central bank. For example, the extent of uncertainty about future money market rates is an indicator of the credibility and predictability of the central bank's monetary policy (e.g. Caporale and Cippolini 2002, Swanson 2006). To keep this uncertainty low an important goal of central banks' communication policy is to "guide" expectations about its future policy moves (e.g. Reinhart 2003, European Central Bank 2008). Rising uncertainty about how interest rates will be set by the central bank in the future can have negative effects on economic stability (e.g. Poole 2005). For example, an increase in the volatility of money market rates can be transmitted through the yield curve (Ayuso, Haldane, and Restoy 1997) causing the volatility of longer-term interest rates to rise as well which can have a negative effect on growth and investment (e.g. Muellbauer and Nunziata 2004, Byrne and Davis 2005).

The empirical importance of time-variation in uncertainty about short-term interest rates has been documented in many studies. However, measures of interest rate uncertainty have usually been constructed from the time series of historical interest rate changes and, therefore, are difficult to interpret economically. These measures have been derived either by estimating ARCH/GARCH models (e.g. Chuderewicz 2002, Lanne and Saikkonen 2003), stochastic volatility models (e.g. Caporale and Cipollini 2002) or regime switching models of volatility (e.g. Sun 2005). A third approach uses derivative prices to estimate interest rate uncertainty (e.g. Fornari (2005).

To derive economically interpretable measures of interest rate uncertainty the starting point of this paper is the fact that monetary policy is the most important determinant of short-term interest rates. Hence, for analyzing interest rate forecast uncertainty, I suggest an approach based on the way financial markets perceive monetary policy to be made in response to changing economic conditions. (See Swanson (2006) for a similar argument.) By combining a widely used interest rate rule with a structural economic model, various elements of interest rate uncertainty can be separated and attributed to their economic sources.

The basis of the analysis is the famous Taylor rule (Taylor 1993) which is widely accepted as an approximate description of how central banks set short-term interest rates in response to (expected) economic conditions. Even though central banks certainly do not exactly follow the Taylor rule when adjusting interest rates, financial market participants often use Taylor-type rules as tools for forecasting future interest rates set by the central bank. Predicting interest rates using such a rule requires forecasts of the economic conditions the central bank will have to react to in the future. Hence, uncertainty concerning the forecasts of the information the central bank is expected to act upon, is the first source of uncertainty about future interest rates (fundamental uncertainty).
The second element of uncertainty is related to imperfect knowledge about the central bank's reaction to given future economic conditions. The reaction coefficients in simple interest rate rules such as the Taylor rule have been shown to change over time (e.g. Cogley and Sargent 2003, Boivin 2006, Kim and Nelson 2006). One reason for this is that the coefficients in optimally derived monetary policy reaction functions depend on both, the central bank's preferences about output stabilization, inflation and possibly other goals as well as on structural parameters of the central bank's model of the economy (e.g. Svensson 1997). Changes in preferences and changes in the structure of the economy will both affect the parameters in the monetary policy reaction function. Furthermore, simple interest rate rules in general are only crude approximations to an optimal monetary policy reaction function. Central banks base their policy decisions on a much richer data set than, for example, (forecasts of) the output gap and inflation accounted for in the Taylor rule. Consequently, situations with identical (forecast) values of the output gap and inflation can be significantly different economically if judged by the much larger optimal information set. Thus, the central bank does not necessarily have to react to (apparently) identical economic situations in the same way and this will lead to changes in the parameters of simple, i.e. not completely specified monetary policy reaction functions. Finally, changes in the reaction coefficients can also result from fitting a linear reaction function when the true reaction function is in fact non-linear. Forecasting interest rates based on a simple monetary policy reaction function implies that changing coefficients in this reaction function are a second source of uncertainty about future interest rates.
The third element of interest rate uncertainty is due to the fact that the estimated reaction function is an approximation. It is embodied in the error term in the empirically estimated in-

terest rate rule and represents the approximation error of the rule relative to actual monetary policy.

This paper presents an empirical model of U.S. monetary policy that identifies these different components of uncertainty about future interest rates. The model consists of a Taylor rule with time-varying coefficients combined with an unobserved components model of output, inflation, and the output gap which is used to generate forecasts of the variables entering the Taylor rule.

The model opens a new field of applications for the growing empirical literature on time-varying monetary policy rules: the study of uncertainty about future monetary policy. Previous analyses have focused on ex-post descriptions of Federal Reserve policy: For example, Clarida, Galì and Gertler (2000) provide evidence of pronounced changes in Taylor-type interest rate rules for the U.S. using split-sample regression analysis. They show a strong shift in the Fed's reaction function related to the appointment of Fed Chairman Volcker in 1979. More recently Boivin (2006) and Kim and Nelson (2006) estimate forward-looking Taylor rules with time-varying parameters and report sizeable but more gradual changes in the coefficients. Trecroci and Vassali (2006) show that time-varying monetary policy reaction functions for the U.S., the U.K., Germany, France and Italy perform superior to constant parameter rules in accounting for observed changes in interest rates. Time-varying Taylor rules have also been estimated for the Deutsche Bundesbank by Kuzin (2005) and using a regime-switching model by Assenmacher-Wesche (2008).

The paper is structured as follows: Section 2 outlines the empirical models for the monetary policy reaction function and for the economic fundamentals that enter into it. Section 3 discusses the results from estimating the models and Section 4 presents the results for the uncertainty about future interest rates.

2. A MODEL OF POLICY AND ECONOMIC FUNDAMENTALS

2.1 The Taylor rule

The standard approach to model the setting of the short-term interest rate by the central bank is the specification of an interest rate rule that relates the monetary policy instrument, i.e. the short term interest rate, to a limited number of economic variables. The most widely used interest rate rule is the Taylor rule which assumes that the central bank's target value for the monetary policy instrument responds to current or expected values of inflation and the output gap

$$i_t^* = a_{0,t} + a_{\pi,t} E_t \pi_{t+j} + a_{z,t} E_t z_{t+k}, \tag{1}$$

where i_t^* is the target short-term interest rate. π_{t+j} is the inflation rate j periods in the future and z_{t+k} is the output gap j periods ahead. E_t denotes expectations formed conditional on information at time t. Equation (1) allows for time variation in the coefficients $a_{0,t}$, $a_{\pi,t}$ and $a_{z,t}$.

The actual interest rate is adjusted gradually towards the target interest rate given by (1), i.e.

$$i_t = (1 - \rho_t) i_t^* + \rho_t i_{t-1} + \epsilon_t, \qquad 0 < \rho_t < 1, \tag{2}$$

where ϵ_t is a random disturbance term which captures the non-systematic component of monetary policy and the approximation error of the Taylor rule relative to the actually observed interest rate.

Assuming the time-varying Taylor rule coefficients to follow random walks and imposing the restriction $0 < \rho_t < 1$ leads to the following time-varying policy rule

$$\begin{aligned} i_t &= (1 - \rho_t) a_{0,t} + a_{\pi,t} E_t \pi_{t+j} + a_{z,t} E_t z_{t+k}) + \rho_t i_{t-1} + \epsilon_t \\ &= \beta_{0,t} + \beta_{\pi,t} E_t \pi_{t+j} + \beta_{z,t} E_t z_{t+k} + \rho_t i_{t-1} + \epsilon_t & (3) \\ \rho_t &= \frac{1}{1 + \exp(-\beta_{\rho,t})} & (4) \\ \beta_{t+1} &= \beta_t + w_{t+1}, \quad w_t \sim \text{i.i.d} N(0, \Sigma_w), & (5) \end{aligned}$$

with $\beta_{0,t} = (1 - \rho_t) a_{0,t}$, $\beta_{\pi,t} = (1 - \rho_t) a_{\pi,t}$, $\beta_{z,t} = (1 - \rho) a_{z,t}$,
$\beta_t = [\beta_{0,t} \quad \beta_{\pi,t} \quad \beta_{z,t} \quad \beta_{\rho t}]'$, $w_t = [w_{0,t} \quad w_{\pi,t} \quad w_{z,t} \quad w_{\rho t}]'$ and Σ_w as a diagonal matrix.

Various assumptions about j and k have been used in the literature. Here, I assume $j = k = 2$. Due to the high degree of autocorrelation of the forecasts the choice of the forecast horizon has only modest effects on the results (e.g. Boivin 2006). Several studies have documented drastic changes in the variance of the interest-rate shock ϵ (e.g. Stock and Watson 2002, Cogley and Sargent 2003). Ignoring these changes may lead to incorrect estimates of the time-varying parameters in the Taylor rule. To account for time variation in the variance of the interest rate shock, the variance of the disturbance term ϵ_t is approximated by a GARCH(1,1) process

$$\epsilon_t|\Psi_{t-1} \sim N(0, \sigma^2_{\epsilon,t}) \tag{6}$$

$$\sigma^2_{\epsilon,t} = \kappa_0 + \kappa_1 \epsilon^2_{t-1} + \kappa_2 \sigma^2_{\epsilon,t-1}, \tag{7}$$

where Ψ_{t-1} is the period $t-1$ information set.

2.2 Output gap and inflation forecasts

The output gap which enters the Taylor rule (3) is an unobservable variable and can only be inferred indirectly from observed output dynamics. Various empirical decompositions of actual output into a long-run trend component (potential output) and a short-run cyclical component (output gap) have been proposed in the literature. These include the Hodrick-Prescott filter, linear or quadratic detrending as well as decompositions suggested by Watson (1986) and Clark (1989).

The output gap is also likely to be linked to the inflation rate by a Phillips curve-type relationship. To exploit both sources of information on the output gap, it is preferable to jointly model the dynamics of inflation and of the output gap as suggested in Kuttner (1994): The output equation is based on Watson (1986) and decomposes the log of real GDP (y) into a random walk and a stationary AR(2) component

$$y_t = n_t + z_t \tag{8}$$

$$z_t = \phi_1 z_{t-1} + \phi_2 z_{t-2} + e^z_t \tag{9}$$

$$n_t = \mu_y + n_{t-1} + e^n_t. \tag{10}$$

n is the trend component and follows a random walk with drift μ_y while z is the (log) deviation of real GDP from potential output, i.e. the output gap.

Inflation dynamics are modelled as an ARIMA process in which the change in the rate of inflation depends on the lagged output gap. (Preliminary unit-root tests strongly reject the hypothesis of a stationary inflation rate and suggest a model in first differences.)

$$\alpha(L)\Delta\pi_t = \mu_\pi + \gamma(L)z_{t-1} + \delta(L)\nu_t, \tag{11}$$

where $\alpha(L)$, $\gamma(L)$, and $\delta(L)$ are polynomials in the lag operator of order p, r, and q. μ_π is a constant and ν is a normally i.i.d error term.

The model (8 - 11) can be written in state-space form leading to the observation equation (see Appendix A)

$$Y_t = \mu + H\tilde{x}_t + A(L)Y_{t-1} + e_t, \tag{12}$$

and the transition equation for the state variables

$$\tilde{x}_{t+1} = F\tilde{x}_t + \zeta_{t+1}. \tag{13}$$

From this model I generated inflation and output gap forecasts to estimate the Taylor rule. Since output and inflation cannot be observed within the current period the forecasts are based on information up to period $t-1$. Hence, the predicted output gap for period $t+2$ is

$$z_{t+2|t-1} = 1_z' FFF \cdot \tilde{x}_{t|t-1}, \tag{14}$$

where 1_z is a unit vector for the first element of $\tilde{x}$. A forecast of inflation in $t+2$ based on data up to $t-1$ is

$$\pi_{t+2|t-1} = \pi_{t-1} + 1_\pi' \left[3\mu + H(I + F + FF)\tilde{x}_{t|t-1} + \sum_{i=0}^{2} A(L)Y_{t+i-1|t-1} \right]. \tag{15}$$

These forecasts enter the Taylor rule equation (3) by replacing $E_t\pi_{t+2}$ and $E_t z_{t+2}$ with $\pi_{t+2|t-1}$ and $z_{z+2|t-1}$.

Kim and Nelson (2006) used actual values of inflation and the output gap derived from the Congressional Budget Office estimates of potential output to estimate the time-varying Taylor rule. While this allows them to estimate the time-varying coefficients, constructing the interest rate forecasts needed to compute the measures for interest rate uncertainty requires a forecasting model for the economic fundamentals in the Taylor rule. It is feasible to estimate the time-varying Taylor rule coefficients as in Kim and Nelson (2006) in combination with some other forecasting model for the output gap and the inflation rate. However, from the point of view of financial market participants who try to predict future monetary policy based on forecasts about the future state of the economy it is more consistent to use the same forecast both for estimating the Taylor rule parameters and in deriving the interest rate uncertainty estimates. A possible drawback of this approach is that measurement errors in the forecasts relative to the forecasts actually used by the Fed might lead to biased estimates of the Taylor-rule coefficients. However, I corrected my estimates for this by using the Heckman-type (1976) procedure suggested in Kim and Nelson (2006, 1951-1954). (A similar two-step approach to estimate an interest rate rule, however without a bias correction, is used in Cogley and Sargent (2003) who construct forecasts from a vector autoregression.)

3. ESTIMATION AND RESULTS

The parameters in (12) and (13) were estimated by maximum likelihood using the Kalman filter. The data consists of quarterly observations on real GDP and the GDP deflator in the U.S. from 1960Q1 to 2006Q4 obtained from FRED II. The inflation rate is defined as 100 times the first log difference of the GDP deflator.

The inflation equation (11) was specified as an ARIMA(|4|,1,3)-model with two lags of the output gap and with μ_π restricted to zero. (To check the robustness of the results I repeated the complete analysis for different ARIMA-models with very similar results.)

Table 1 shows the coefficient estimates for the output gap/inflation model. Tables 2 and 3 present the results of a number of diagnostic tests. There is no evidence of missing inflation and output terms and autocorrelation in the residuals. (I also estimated a specification which allows for a direct effect of output growth on the change in the inflation rate. In contrast to Kuttner (1994), the relevant coefficient always turned out to be insignificant.)

« insert Table 1»

« insert Table 2»

« insert Table 3»

Using the estimates from the output gap/inflation model to estimate the reaction function parameters requires the forecasts of inflation and the output gap to be uncorrelated with the error term ϵ_t. This assumption is violated if the forecasts used to estimate the Taylor rule differ from the forecasts actually used by the Fed. To arrive at bias corrected estimates of the

Taylor rule coefficients I applied the Heckman-type two-step estimation procedure presented in Kim and Nelson (2006). As instruments I used four lags of the following variables: the output gap as measured by the Congressional Budget Office, the inflation rate, oil price inflation, and the growth rate of M2. (The oil price variable is the spot price of West Texas Intermediate. All variables were obtained from FRED II except for the output gap.)

Figure 1 displays the estimated output gap along with bounds of 1.96 standard deviations. The output gap is the one-sided estimate from the Kalman-Filter algorithm and only based on observations up to period t. The error bounds are constructed using the Monte Carlo approach from Hamilton (1994) and reflect both the Kalman filter uncertainty and the uncertainty about the model parameter estimates.

« insert Figure 1 »

The two-period inflation forecast $\pi_{t+2|t-1}$ is shown in Figure 2 together with the actual inflation rate.

« insert Figure 2 »

From the time series for $z_{t+2|t-1}$, the inflation forecasts $\pi_{t+2|t-1}$ and observations on the Federal Funds Rate I estimated the parameters of the time-varying Taylor rule. Since the Federal Funds Rate cannot be used as the principal indicator for the Fed's monetary policy before 1966 the estimation starts in 1966Q1 (e.g. Lansing 2002). (The log-likelihood function was evaluated over the period 1970Q1 to 2006Q4. The first 20 observations from 1966Q1 on were used to obtain initial values for the time-varying parameters.) Table 4 presents the estimated parameters of the time-varying Taylor rule. The bias correction coefficients ϑ_1 and ϑ_2 are relatively small indicating that there is no substantial bias caused by the generated regressors.

« insert Table 4»

4. INTEREST-RATE UNCERTAINTY

4.1 The one-period case

Uncertainty about the Federal Funds Rate in the next quarter can be defined as

$$\mathbf{E}_t\left[(i_{t+1}-\hat{i}_{t+1|t})^2|\Omega_t\right]. \tag{16}$$

Define $b_t = (\beta_{0,t} \quad \beta_{\pi,t} \quad \beta_{z,t} \quad \rho_t)'$ and $x_t = (1 \quad \pi_{t+2|t-1} \quad z_{t+2|t-1} \quad i_{t-1})$, then

$$\hat{i}_{t+1|t} = \mathbf{E}_t\left[i_{t+1}|\Omega_t\right] = \mathbf{E}_t\left[x'_{t+1}b_{t+1}|\Omega_t\right]. \tag{17}$$

Ω_t represents the information available to market participants immediately after the interest rate is set at time t. The information set contains the estimated coefficients in Tables 1 and 4, all past values of y and π but not their current values y_t and π_t which cannot be observed contemporaneously. It also contains past and current values of i and the current values of the central bank's estimates of the output gap $z_{t|t-1}$ and of the inflation rate $\pi_{t|t-1}$.

Since b and x are uncorrelated the one-step-ahead forecast of the Federal Funds Rate is,

$$\hat{i}_{t+1|t} = \mathbf{E}_t\left[x'_{t+1}|\Omega_t\right]\mathbf{E}_t\left[b_{t+1}|\Omega_t\right] = \hat{x}'_{t+1|t}b_{t+1|t}. \tag{18}$$

Note that since $x_t = (1 \quad \pi_{t+2|t-1} \quad z_{t+2|t-1} \quad i_{t-1})$, the forecast of x_{t+1} based on Ω_t, is $\hat{x}_{t+1|t} = (1 \quad \pi_{t+3|t-1} \quad z_{t+3|t-1} \quad i_t)$. However, the forecast of b_{t+1} based on Ω_t is $b_{t+1|t}$, as i_t is part of the information set. The forecast (18) is shown in Figure 3

« insert Figure 3»

Combining (16), (17) and (18) leads to (see Appendix B)

$$\begin{aligned}
\mathbf{E}_t\left[(i_{t+1}-\hat{i}_{t+1|t})^2|\Omega_t\right] &= \mathbf{E}_t\left[(x'_{t+1}b_{t+1}-\hat{x}'_{t+1|t}b_{t+1|t})^2|\Omega_t\right] \\
&= \hat{x}'_{t+1|t}P_{b,t+1|t}\hat{x}'_{t+1|t} + b'_{t+1|t}P_{x,t+1|t}b_{t+1|t} + \sigma^2_\epsilon.
\end{aligned} \tag{19}$$

$P_{b,t+1|t} = \mathbf{E}_t\left[(b_{t+1}-b_{t+1|t})(b_{t+1}-b_{t+1|t})'\right]$ is obtained from the Kalman filter as the forecast variance of the unobserved coefficients in $t+1$ based on period-t information. The first term in (19) is the component of the overall interest rate forecast uncertainty due to possible changes in the way the Fed responds to the fundamental variables in its reaction function.

$P_{x,t+1|t} = \mathbf{E}_t\left[(x_{t+1}-x_{t+1|t})(x_{t+1}-x_{t+1|t})'|\Omega_t\right]$ represents the uncertainty about the forecast of the economic variables the interest rate responds to. A detailed derivation of this expression can be found in Appendix B. The last term in equation (19) is uncertainty due to the Taylor rule residual ϵ.

The results from (19) are presented in Figure 4 which shows the three elements of interest rate uncertainty.

« insert Figure 4 »

The Figure shows some pronounced peaks in uncertainty about the future coefficients in the Fed's reaction function in the mid 1970s, the early, the mid and late 1980s. Except for some small increases in the mid 1990s and in 2001 uncertainty about the Fed's reactions in the next quarter was much lower since 1990 compared to the 1970s and 80s. Relative to uncertainty about the reaction coefficients uncertainty about future inflation and output gap is very moderate through all of the sample period. It rises above uncertainty about the reaction coefficients only in the late 1990s. Uncertainty caused by the approximation error in the Taylor rule (residual uncertainty) contributed strongly to Federal Funds Rate forecast uncertainty in the mid 1970s, early 1980s and in 2004 but was very modest over the rest of the time period. Extreme values for this residual uncertainty are a result of the deteriorating fit of the Taylor rule in these episodes which also leads to the strong increases in uncertainty about the policy rule coefficients.

While these empirical results do not provide any indication of a systematic decrease in forecast uncertainty about the output gap and the inflation rate after 1985 uncertainty about the Fed's reaction function coefficients is markedly lower than before. Similar results about increased predictability of the Fed's monetary policy have also been documented in other studies (e.g. Sellon 2008) and have been cited as one possible explanation for the recent decline in economic volatility (e.g. Stock and Watson 2002, Gordon 2005).

4.2 The two-period case

Forecast uncertainty about the short-term interest rate that will be set two periods in the future is

$$\mathbf{E}_t\left[(i_{t+2} - \hat{i}_{t+2|t})^2|\Omega_t\right], \tag{20}$$

where

$$\begin{aligned}\hat{i}_{t+2|t} &= \mathbf{E}_t\left[(i_{t+2}|\Omega_t\right] = \mathbf{E}_t\left[x'_{t+2}b_{t+2}|\Omega_t\right] \\ &= \mathbf{E}_t\left[x'_{t+2}|\Omega_t\right]\mathbf{E}_t\left[b_{t+2}|\Omega_t\right] = \hat{x}'_{t+2|t}b_{t+2|t},\end{aligned} \tag{21}$$

Expanding (21) yields

$$\begin{aligned}\mathbf{E}_t\left[(i_{t+2}-\hat{i}_{t+2|t})^2|\Omega_t\right] &= \mathbf{E}_t\left[(x'_{t+2}b_{t+2}-\hat{x}'_{t+2|t}b_{t+2|t})^2|\Omega_t\right] \\ &= \hat{x}'_{t+2|t}P_{b,t+2|t}\hat{x}'_{t+2|t}+b'_{t+2|t}P_{x,t+2|t}b'_{t+2|t}+\sigma^2_\epsilon.\end{aligned} \tag{22}$$

$P_{b,t+2|t} = \mathbf{E}_t\left[(b_{t+2}-b_{t+2|t})(b_{t+2}-b_{t+2|t})'|\Omega_t\right]$ can be computed as $P_{b,t+2|t} = P_{b,t+1|t} + \Sigma_w$. (For the derivation of $P_{x,t+2|t}$ refer to Appendix B.)

Figure 5 shows the various elements of the two-period forecast uncertainty about the Federal Funds Rate. The time series of uncertainty about the reaction function coefficients is very similar to the one shown in Figure 4. Uncertainty about the output gap and inflation in the future is markedly higher than for the one-period forecasts. It peaks in the mid 1970s, the early 1980s, the early 1990s and again in the late 1990s. In 2005 and 2006 it rises again. Throughout the 1990s this uncertainty about the future state of the economy is much higher than uncertainty about the reaction function coefficients. Uncertainty about the approximation error is always small compared to the two other sources of uncertainty. The low impact of the approximation error is caused by the low estimate of κ_2 which implies a quick reduction of the conditional variance in the GARCH process.

« insert Figure 5 »

5. SUMMARY AND CONCLUSIONS

Using a simple model based on a Taylor rule I estimated the forecast uncertainty about the future Federal Funds Rate and separated it into uncertainty about the state of the economy in the future, uncertainty about how the Fed will react to future economic conditions, and uncertainty about the quality of the Taylor rule as an approximation of the Fed's monetary policy.

The results showed considerable variation in uncertainty about the Federal Funds Rate in the next quarter with marked peaks in the mid 1970s and the early, mid and late 1980s. In some of these episodes the quality of the approximation of actually observed monetary policy by the Taylor rule deteriorated strongly. For a forecast horizon of one quarter uncertainty about the coefficients in the Fed's reaction function dominated uncertainty about the future state of the economy. For a longer forecast horizon however, uncertainty about future economic fundamentals was higher than uncertainty about reaction coefficients throughout the 1990s.

APPENDIX A: THE STATE-SPACE MODEL OF OUTPUT GAP AND INFLATION

The observation equation (9) is

$$Y_t = \mu + H\tilde{x}_t + A(L)Y_{t-1} + e_t, \tag{A1}$$

with

$$\begin{aligned}
Y_t &= \begin{bmatrix} \Delta y_t \\ \Delta\pi_t \end{bmatrix}, \mu = \begin{bmatrix} \mu_y \\ \mu_\pi \end{bmatrix}, e_t = \begin{bmatrix} e_t^n \\ 0 \end{bmatrix} \\
A_4 &= \begin{bmatrix} 0 & 0 \\ 0 & \alpha_4 \end{bmatrix} \\
H &= \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \gamma_1 & \gamma_2 & 1 & \delta_1 & \delta_2 & \delta_3 \end{bmatrix} \\
\mathbf{E}e_t e_t' = \Sigma_Y &= \begin{bmatrix} \sigma_{e,n}^2 & 0 \\ 0 & 0 \end{bmatrix},
\end{aligned}$$

where the matrices A_1 to A_3 of the autoregressive coefficients have been set equal to zero as in the preferred model specification in the paper. The transition equation for the state variables can be written as

$$\tilde{x}_{t+1} = F\tilde{x}_t + \zeta_{t+1}, \tag{A2}$$

with

$$\begin{aligned}
\tilde{x}_t &= \begin{bmatrix} z_t & z_{t-1} & z_{t-2} & \nu_t & \nu_{t-1} & \nu_{t-2} & \nu_{t-3} \end{bmatrix}', \\
\zeta_t &= \begin{bmatrix} e_t^z & 0 & 0 & e_t^\nu & 0 & 0 & 0 \end{bmatrix}',
\end{aligned}$$

$$F = \begin{bmatrix} \phi_1 & \phi_2 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\mathbf{E}\zeta_t\zeta_t' = \Sigma_\zeta = \begin{bmatrix} \sigma^2_{e,z} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma^2_{e,\nu} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

The shocks e^ν, e^n and e^z are assumed to be serially and mutually uncorrelated.

APPENDIX B: UNCERTAINTY MEAUSURES

B.1 Uncertainty in the one-period case

Derivation of (19):

$$\begin{aligned} \mathbf{E}_t\left[(i_{t+1} - \hat{i}_{t+1|t})^2|\Omega_t\right] &= \mathbf{E}_t\left[(x'_{t+1}b_{t+1} - \hat{x}'_{t+1|t}b_{t+1|t})^2|\Omega_t\right] \\ &= \mathbf{E}_t\left[b'_{t+1}x_{t+1}x'_{t+1}b_{t+1}|\Omega_t\right] - b'_{t+1|t}\hat{x}_{t+1|t}\hat{x}'_{t+1|t}b_{t+1|t} \\ &\quad +\sigma^2_{\epsilon,t+1|t} \qquad \text{(B1)} \end{aligned}$$

I use a Taylor-Approximation to write

$$\begin{aligned} \mathbf{E}\left[b'_{t+1}x_{t+1}x'_{t+1}b_{t+1}|\Omega_t\right] &\approx b'_{t+1|t}\hat{x}_{t+1|t}\hat{x}'_{t+1|t}b_{t+1|t} \qquad \text{(B2)} \\ &\quad +\hat{x}'_{t+1|t}\mathbf{E}\left[(b_{t+1} - b_{t+1|t})(b_{t+1} - b_{t+1|t})'|\Omega_t\right]\hat{x}_{t+1|t} \\ &\quad +b'_{t+1|t}\mathbf{E}\left[(x_{t+1} - \hat{x}_{t+1|t})(x_{t+1} - \hat{x}_{t+1|t})'|\Omega_t\right]b_{t+1|t}. \end{aligned}$$

Substituting this expression into (B1) yields (19)

$$\begin{aligned}\mathbf{E}_t\left[(i_{t+1}-\hat{i}_{t+1|t})^2|\Omega_t\right] &= \hat{x}'_{t+1|t}\mathbf{E}_t\left[(b_{t+1}-b_{t+1|t})(b_{t+1}-b_{t+1|t})'|\Omega_t\right]\hat{x}_{t+1|t} \\ &\quad +b'_{t+1|t}\mathbf{E}_t\left[(x_{t+1}-\hat{x}_{t+1|t})(x_{t+1}-\hat{x}_{t+1|t})'|\Omega_t\right]b_{t+1|t} \\ &\quad +\sigma^2_{\epsilon,t+1|t} \\ &= \hat{x}'_{t+1|t}P_{b,t+1|t}\hat{x}'_{t+1|t}+b'_{t+1|t}P_{x,t+1|t}b_{t+1|t}+\sigma^2_{\epsilon,t+1|t}, \qquad \text{(B3)}\end{aligned}$$

with $P_{b,t+1|t}=\mathbf{E}_t\left[(b_{t+1}-b_{t+1|t})(b_{t+1}-b_{t+1|t})'\right]$ and
$P_{x,t+1|t}=\mathbf{E}_t\left[(x_{t+1}-x_{t+1|t})(x_{t+1}-x_{t+1|t})'|\Omega_t\right]$.

Since $x_{t+1}=(1 \quad \pi_{t+3|t} \quad z_{t+3|t} \quad i_t)$ and $\hat{x}_{t+1}=(1 \quad \pi_{t+3|t-1} \quad z_{t+3|t-1} \quad i_t)$ I can write

$$\begin{aligned}P_{x,t+1|t} &= \mathbf{E}_t\left[(x_{t+1}-\hat{x}_{t+1|t})(x_{t+1}-\hat{x}_{t+1|t})'|\Omega_t\right] \\ &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & p_{\pi,\pi,t+1} & p_{\pi,z,t+1} & 0 \\ 0 & p_{\pi,z,t+1} & p_{z,z,t+1} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \qquad \text{(B4)}\end{aligned}$$

where $p_{\pi,\pi,t+1}=\mathbf{E}\left[(\pi_{t+3|t}-\pi_{t+3|t-1})^2|\Omega_t\right]$, $p_{z,z,t+1}=\mathbf{E}\left[(z_{t+3|t}-z_{t+3|t-1})^2|\Omega_t\right]$,
and $p_{\pi,z,t+1}=\mathbf{E}\left[(\pi_{t+3|t}-\pi_{t+3|t-1})(z_{t+3|t}-z_{t+3|t-1})|\Omega_t\right]$.

The individual elements can be derived as follows: The inflation forecast the central bank will react to in the next period is $\pi_{t+3|t}=\pi_{t-1}+\Delta\pi_t+\sum_{i=1}^{3}\Delta\pi_{t+i|t}$ while the forecast of this variable based on information dated $t-1$ is $\pi_{t+3|t-1}=\pi_{t-1}+\sum_{i=0}^{3}\Delta\pi_{t+i|t-1}$. Hence, using (A1) the forecast error is

$$\begin{aligned}\pi_{t+3|t}-\pi_{t+3|t-1} &= =(\Delta\pi_t-\Delta\pi_{t|t-1})+\sum_{i=1}^{3}(\Delta\pi_{t+i|t}-\Delta\pi_{t+i|t-1}) \\ &= \mathbf{1}'_2\left[(Y_t-Y_{t|t-1})+\sum_{i=1}^{3}(Y_{t+i|t}-Y_{t+i|t-1})\right] \\ &= \mathbf{1}'_2\left[H(\tilde{x}_t-\tilde{x}_{t|t-1})+e_t+\sum_{i=1}^{3}H(\tilde{x}_{t+i|t}-\tilde{x}_{t+i|t-1})\right] \qquad \text{(B5)} \\ &= \mathbf{1}'_2\left[H(\tilde{x}_t-\tilde{x}_{t|t-1})+e_t+H(F+FF+FFF)(\tilde{x}_{t|t}-\tilde{x}_{t|t-1})\right],\end{aligned}$$

with $\mathbf{1}_2=(0 \quad 1)'$.

At the time the policy rate in period t is announced, uncertainty about $\pi_{t+3|t}$, the estimate of inflation the central bank will react to in the next period stems from two sources: First, $(\Delta\pi_t - \Delta\pi_{t|t-1})$ is the error made in estimating the change in the inflation rate from the previous to the current period. Second, $\sum_{i=1}^{3}(\Delta\pi_{t+i|t} - \Delta\pi_{t+i|t-1})$ is the difference between the changes in inflation from period $t+1$ to $t+3$ forecast by the central bank at the time it has to set i_{t+1} – and thus formed with knowledge of π_t – and the forecast of the changes in inflation made by the public in $t-1$ without knowing π_t.

Use of the Kalman-filter updating equations results in

$$\begin{aligned}\pi_{t+3|t} - \pi_{t+3|t-1} &= \mathbf{1}_2'\big[H(\tilde{x}_t - \tilde{x}_{t|t-1}) + e_t + H(F + FF + FFF)K_{t|t-1}(H(\tilde{x}_t - \tilde{x}_{t|t-1}) \\ &\quad + e_t)\big] \\ &= \mathbf{1}_2'\big[H(I + (F + FF + FFF)K_{t|t-1}H)(\tilde{x}_t - \tilde{x}_{t|t-1}) \\ &\quad + (I + H(F + FF + FFF)K_{t|t-1})e_t\big],\end{aligned} \tag{B6}$$

with $K_{t|t-1} = P_{\tilde{x},t|t-1}H'\big[HP_{\tilde{x},t|t-1}H' + \Sigma_Y\big]^{-1}$. This can be written as

$$\pi_{t+3|t} - \pi_{t+3|t-1} = \mathbf{1}_2'\Big[D_{1,\tilde{x}}(\tilde{x}_t - \tilde{x}_{t|t-1}) + D_{1,e_t}e_t\Big]. \tag{B7}$$

Using this expression the result for $p_{\pi,\pi,t+1}$ is

$$\begin{aligned}p_{\pi,\pi,t+1} &= \mathbf{E}\left[(\pi_{t+3|t} - \pi_{t+3|t-1})^2|\Omega_t\right] \\ &= \mathbf{1}_2'\Big[D_{1,\tilde{x}}P_{\tilde{x},t|t-1}D_{1,\tilde{x}}' + D_{1,e_t}\Sigma_Y D_{1,e_t}'\Big]\mathbf{1}_2.\end{aligned} \tag{B8}$$

$z_{t+3|t}$ is the (1,1) element of $\tilde{x}_{t+3|t} = FFF\tilde{x}_{t|t}$, while $z_{t+3|t-1}$ is the (1,1) element of $\tilde{x}_{t+3|t-1} = FFF\tilde{x}_{t|t-1}$. Hence,

$$\begin{aligned}z_{t+3|t} - z_{t+3|t-1} &= \mathbf{1}_1'FFF(\tilde{x}_{t|t} - \tilde{x}_{t|t-1}) \\ &= \mathbf{1}_1'FFFK_{t|t-1}(H(\tilde{x}_t - \tilde{x}_{t|t-1}) + e_t),\end{aligned} \tag{B9}$$

where the last step makes use of the Kalman filter updating equation for $\tilde{x}$.

Defining

$$z_{t+3|t} - z_{t+3|t-1} = \mathbf{1}_1' \Big[B_{1,\tilde{x}}(\tilde{x}_t - \tilde{x}_{t|t-1}) + B_{1,e_t} e_t \Big], \tag{B10}$$

with the respective coefficients shown in (B9) leads to

$$\begin{aligned} p_{z,z,t+1} &= \mathbf{E}\left[(z_{t+3|t} - z_{t+3|t-1})^2 | \Omega_t \right] \\ &= \mathbf{1}_1' \mathbf{E}\left[(\tilde{x}_{t+1|t} - \tilde{x}_{t+1|t-1})(\tilde{x}_{t+1|t} - \tilde{x}_{t+1|t-1})' | \Omega_t \right] \mathbf{1}_1 \\ &= \mathbf{1}_1' \left[B_{1,\tilde{x}} P_{\tilde{x},t|t-1} B_{1,\tilde{x}}' + B_{1,e_t} \Sigma_Y B_{1,e_t}' \right] \mathbf{1}_1, \end{aligned} \tag{B11}$$

with $\mathbf{1}_1 = (1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0)'$. Uncertainty about the central bank's forecast for the output gap is due to the fact that when policy is set next period additional information in form of observations of π_t and y_t will be available.

Finally, combining (B7) with (B10) yields

$$\begin{aligned} p_{\pi,z,t+1} &= \mathbf{E}\left[(\pi_{t+3|t} - \pi_{t+3|t-1})(z_{t+3|t} - z_{t+3|t-1}) | \Omega_t \right] \\ &= \mathbf{1}_2' \Big[D_{1,\tilde{x}} P_{\tilde{x},t|t-1} B_{1,\tilde{x}}' + D_{1,e_t} \Sigma_Y B_{1,e_t}' \Big] \mathbf{1}_1. \end{aligned} \tag{B12}$$

All these expressions can be evaluated using the parameter estimates from section 3 and the results from the Kalman filter algorithm applied to model from Appendix A.

B.2 Uncertainty in the two-period case

(22) in the paper is derived analogous to (19). To evaluate (22) the following expression is required

$$\begin{aligned} P_{x,t+2|t} &= \mathbf{E}_t\left[(x_{t+2} - \hat{x}_{t+2|t})(x_{t+2} - \hat{x}_{t+2|t})' | \Omega_t \right] \\ &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & p_{\pi,\pi,t+2} & p_{\pi,z,t+2} & p_{\pi,i,t+2} \\ 0 & p_{\pi,z,t+2} & p_{z,z,t+2} & p_{i,z,t+2} \\ 0 & p_{\pi,i,t+2} & p_{i,z,t+2} & p_{i,i,t+2} \end{bmatrix}, \end{aligned} \tag{B13}$$

where

$$
\begin{aligned}
p_{\pi,\pi,t+2} &= \mathbf{E}\left[(\pi_{t+4|t+1} - \pi_{t+4|t-1})^2|\Omega_t\right], \\
p_{z,z,t+2} &= \mathbf{E}\left[(z_{t+4|t+1} - z_{t+4|t})^2|\Omega_t\right], \\
p_{\pi,z,t+2} &= \mathbf{E}\left[(\pi_{t+4|t+1} - \pi_{t+4|t-1})(z_{t+4|t+1} - z_{t+4|t})|\Omega_t\right], \\
p_{i,i,t+2} &= \mathbf{E}\left[(i_{t+1} - \hat{i}_{t+1|t})^2|\Omega_t\right], \\
p_{\pi,i,t+2} &= \mathbf{E}\left[(\pi_{t+4|t+1} - \pi_{t+4|t-1})(i_{t+1} - \hat{i}_{t+1|t})|\Omega_t\right], \\
p_{i,z,t+2} &= \mathbf{E}\left[(i_{t+1} - \hat{i}_{t+1|t})(z_{t+4|t+1} - z_{t+4|t})|\Omega_t\right].
\end{aligned}
$$

The inflation forecast the central bank will react to two periods in the future is
$\pi_{t+4|t+1} = \pi_{t-1} + \Delta\pi_t + \Delta\pi_{t+1} + \sum_{i=2}^{4} \Delta\pi_{t+i|t+1}$,
while $\pi_{t+4|t-1} = \pi_{t-1} + \Delta\pi_{t|t-1} + \Delta\pi_{t+1|t-1} + \sum_{i=2}^{4} \Delta\pi_{t+i|t-1}$. Thus,

$$
\begin{aligned}
\pi_{t+4|t+1} - \pi_{t+4|t-1} &= (\Delta\pi_t - \Delta\pi_{t|t-1}) + (\Delta\pi_{t+1} - \Delta\pi_{t+1|t-1}) \\
&\quad + \sum_{i=2}^{4}(\Delta\pi_{t+i|t+1} - \Delta\pi_{t+i|t-1}) \\
&= \mathbf{1}_2'\left[(Y_t - Y_{t|t-1}) + (Y_{t+1} - Y_{t+1|t-1}) + \sum_{i=2}^{4}(Y_{t+2|t+1} - Y_{t+2|t-1})\right] \\
&= \mathbf{1}_2'\left[H(\tilde{x}_t - \tilde{x}_{t|t-1}) + e_t + H(\tilde{x}_{t+1} - \tilde{x}_{t+1|t-1}) + e_{t+1}\right. \\
&\quad \left. + \sum_{i=2}^{4} H(\tilde{x}_{t+i|t+1} - \tilde{x}_{t+i|t-1}) + A_4(Y_t - Y_{t|t-1})\right] \\
&= \mathbf{1}_2'\left[(A_4 + I)H(\tilde{x}_t - \tilde{x}_{t|t-1}) + (A_4 + I)e_t + H(\tilde{x}_{t+1} - \tilde{x}_{t+1|t-1})\right. \\
&\quad \left. + e_{t+1} + H(F + FF + FFF)(\tilde{x}_{t+1|t+1} - \tilde{x}_{t+1|t-1})\right] \\
&= \mathbf{1}_2'\left[(A_4 + I)H(\tilde{x}_t - \tilde{x}_{t|t-1}) + (A_4 + I)e_t + H(\tilde{x}_{t+1} - \tilde{x}_{t+1|t-1}) + e_{t+1}\right. \\
&\quad \left. + H(F + FF + FFF)(\tilde{x}_{t+1|t+1} - \tilde{x}_{t+1|t} + F(\tilde{x}_{t|t} - \tilde{x}_{t|t-1}))\right], \qquad \text{(B14)}
\end{aligned}
$$

with $\mathbf{1}_2 = (0 \quad 1)'$ and using the fact that $(Y_t - Y_{t|t-1}) = H(\tilde{x}_t - \tilde{x}_{t|t-1}) + e_t$ Using (12) and (13) and the updating equations from the Kalman-filter algorithm leads to

$$
\begin{aligned}
\pi_{t+4|t+1} - \pi_{t+4|t-1} &= \mathbf{1}_2'\left[(A_4 + I)H(\tilde{x}_t - \tilde{x}_{t|t-1}) + (A_4 + I)e_t + H(F(\tilde{x}_t - \tilde{x}_{t|t-1}) + \zeta_{t+1})\right. \\
&\quad + e_{t+1} \\
&\quad + H(F + FF + FFF)[K_{t+1|t}(H(\tilde{x}_{t+1} - \tilde{x}_{t+1|t}) + e_{t+1}) \\
&\quad \left. + FK_{t|t-1}(H(\tilde{x}_t - \tilde{x}_{t|t-1}) + e_t))\right]
\end{aligned}
$$

$$
\begin{aligned}
\pi_{t+4|t+1} - \pi_{t+4|t-1} \;=\;& \mathbf{1}_2' \Big[\big(A_4 H + H(I + F + (F + FF + FFF)[K_{t+1|t}(HF(I - K_{t|t-1}H)) \\
& + FK_{t|t-1}H]) \big) (\tilde{x}_t - \tilde{x}_{t|t-1}) \\
& + (A_4 + I + H(F + FF + FFF)(FK_{t|t-1} - K_{t+1|t}HFK_{t|t-1}))e_t \\
& + (I + H(F + FF + FFF)K_{t+1|t})e_{t+1} \qquad \text{(B15)} \\
& + H(I + (F + FF + FFF)K_{t+1|t}H)\zeta_{t+1} \Big],
\end{aligned}
$$

Define

$$
\begin{aligned}
\pi_{t+4|t+1} - \pi_{t+4|t-1} \;=\;& \mathbf{1}_2' \Big[D_{2,\tilde{x}}(\tilde{x}_t - \tilde{x}_{t|t-1}) + D_{2,e_t} e_t + D_{2,e_{t+1}} e_{t+1} \qquad \text{(B16)} \\
& + D_{2,\zeta}\zeta_{t+1} \Big],
\end{aligned}
$$

where the respective coefficients are shown in (B15). This leads to

$$
\begin{aligned}
p_{\pi,\pi,t+2} &= \mathbf{E}\left[(\pi_{t+4|t+1} - \pi_{t+4|t-1})^2 | \Omega_t \right] \qquad \text{(B17)} \\
&= \mathbf{1}_2' \Big[D_{2,\tilde{x}} P_{\tilde{x},t|t-1} D_{2,\tilde{x}}' + D_{2,e_t} \Sigma_Y D_{2,e_t}' + D_{2,e_{t+1}} \Sigma_Y D_{2,e_{t+1}}' + D_{2,\zeta} \Sigma_\zeta D_{2,\zeta}' \Big] \mathbf{1}_2.
\end{aligned}
$$

$z_{t+4|t+1}$ is the (1,1) element of $\tilde{x}_{t+4|t+1} = FFF\tilde{x}_{t+1|t+1}$, while $z_{t+4|t-1}$ is the (1,1) element of $\tilde{x}_{t+4|t-1} = FFF\tilde{x}_{t+1|t-1}$. Hence,

$$
\begin{aligned}
z_{t+4|t+1} - z_{t+4|t-1} &= \mathbf{1}_1' FFF(\tilde{x}_{t+1|t+1} - \tilde{x}_{t+1|t-1}) \\
&= \mathbf{1}_1' FFF(\tilde{x}_{t+1|t+1} - \tilde{x}_{t+1|t} + \tilde{x}_{t+1|t} - \tilde{x}_{t+1|t-1}) \\
&= \mathbf{1}_1' \Big[FFF[K_{t+1|t}HF(I - K_{t|t-1}H) + FK_{t|t-1}H](\tilde{x}_t - \tilde{x}_{t|t-1}) \\
&\quad + FFF[FK_{t|t-1} - K_{t+1|t}HFK_{t|t-1}]e_t \\
&\quad + FFFK_{t+1|t}e_{t+1} + FFFK_{t+1|t}H\zeta_{z+1} \Big], \qquad \text{(B18)}
\end{aligned}
$$

Define

$$
\begin{aligned}
z_{t+4|t+1} - z_{t+4|t-1} \;=\;& \mathbf{1}_1' \Big[B_{2,\tilde{x}}(\tilde{x}_t - \tilde{x}_{t|t-1}) + B_{2,e_t} e_t + B_{2,e_{t+1}} e_{t+1} \qquad \text{(B19)} \\
& + B_{2,\zeta}\zeta_{t+1} \Big],
\end{aligned}
$$

with the respective coefficients shown in (B18). Hence

$$\begin{aligned} p_{z,z,t+2} &= \mathbf{E}\left[(z_{t+4|t+1} - z_{t+4|t-1})^2|\Omega_t\right] \qquad \text{(B20)} \\ &= \mathbf{1}_1'\mathbf{E}\left[B_{2,\tilde{x}}P_{\tilde{x},t|t-1}B_{2,\tilde{x}}' + B_{2,e_t}\Sigma_Y B_{2,e_t}' + B_{2,e_{t+1}}\Sigma_Y B_{2,e_{t+1}}' + B_{2,\zeta}\Sigma_\zeta B_{2,\zeta}'\right]\mathbf{1}_1. \end{aligned}$$

From (B16) and (B19) it follows that

$$\begin{aligned} p_{\pi,z,t+2} &= \mathbf{E}\left[(\pi_{t+4|t+1} - \pi_{t+4|t-1})(z_{t+4|t+1} - z_{t+4|t-1})|\Omega_t\right] \qquad \text{(B21)} \\ &= \mathbf{1}_2'\left[D_{2,\tilde{x}}P_{\tilde{x},t|t-1}B_{2,\tilde{x}}' + D_{2,e_t}\Sigma_Y B_{2,e_t}' + D_{2,e_{t+1}}\Sigma_Y B_{2,e_{t+1}}' + D_{2,\zeta}\Sigma_\zeta B_{2,\zeta}'\right]\mathbf{1}_1. \end{aligned}$$

Next are the correlations of the forecast errors for the output gap and inflation with the forecast error for the interest rate. The latter one is

$$\begin{aligned} i_{t+1} - \hat{i}_{t+1|t} &= x_{t+1}'b_{t+1} - \hat{x}_{t+1|t}'b_{t+1|t} + \epsilon_{t+1} \\ &= x_{t+1}'(b_t + v_{t+1}) - \hat{x}_{t+1|t}'b_{t|t} + \epsilon_{t+1} \\ &= (x_{t+1} - \hat{x}_{t+1|t})'b_{t|t} + x_{t+1}'(b_t + v_{t+1} - b_{t|t}) + \epsilon_{t+1}, \qquad \text{(B22)} \end{aligned}$$

with v_t being the vector of innovations to the Taylor rule coefficients. The first three elements of v are the first three innovations in w_t in (5) with the fourth being the innovation to ρ_t.

Since $x_{t+1}' = (1 \quad \pi_{t+3|t} \quad z_{t+3|t} \quad i_t)$ and $\hat{x}_{t+3|t}' = (1 \quad \pi_{t+3|t-1} \quad z_{t+3|t-1} \quad i_t)$ the above expression can be expanded to

$$\begin{aligned} i_{t+1} - \hat{i}_{t+1|t} &= (\pi_{t+3|t} - \pi_{t+3|t-1})\beta_{\pi,t|t} + (z_{t+3|t} - z_{t+3|t-1})\beta_{z,t|t} \\ &\quad +(\beta_{0,t} - \beta_{0,t|t}) + \pi_{t+3|t}(\beta_{\pi,t} - \beta_{\pi,t|t}) \\ &\quad +z_{t+3|t}(\beta_{z,t} - \beta_{z,t|t}) + i_t(\rho_t - \rho_{t|t}) \\ &\quad +x_{t+1}'v_{t+1} + \epsilon_{t+1}. \qquad \text{(B23)} \end{aligned}$$

The inflation forecast made in period $t+1$ is

$$
\begin{aligned}
\pi_{t+3|t} &= \pi_{t-1} + \Delta\pi_t + \sum_{i=1}^{3} \Delta\pi_{t+i|t} \\
&= \pi_{t-1} + \mathbf{1}_2' \Big[4\mu + H(I - K_{t|t-1}H + (I + F + FF + FFF)K_{t|t-1}H) \\
&\quad (\tilde{x}_t - \tilde{x}_{t|t-1}) + (I - HK_{t|t-1} + H(I + F + FF + FFF)K_{t|t-1})e_t \\
&\quad + H(I + F + FF + FFF)\tilde{x}_{t|t-1} + A_4 \sum_{i=0}^{3} Y_{t+i-4} \Big], \qquad \text{(B24)}
\end{aligned}
$$

and $(\pi_{t+3|t} - \pi_{t+3|t-1})$ is shown in (B7).

$$
\begin{aligned}
z_{t+3|t} &= \mathbf{1}_1' \tilde{x}_{t+3|t} \\
&= \mathbf{1}_1' FFF \tilde{x}_{t|t} \\
&= \mathbf{1}_1' FFF(\tilde{x}_{t|t-1} + K_{t|t-1}(H(\tilde{x}_t - \tilde{x}_{t|t-1}) + e_t)) \\
&= \mathbf{1}_1' \Big[FFFK_{t|t-1}H(\tilde{x}_t - \tilde{x}_{t|t-1}) + FFFK_{t|t-1}e_t + FFF\tilde{x}_{t|t-1} \Big], \qquad \text{(B25)}
\end{aligned}
$$

and $(z_{t+3|t} - z_{t+3|t-1})$ is shown in (B9).

Hence, using (B7), (B10), and (B23)-(B25)

$$
\begin{aligned}
p_{\pi,i,t+2} &= \mathbf{E}\left[(\pi_{t+4|t+1} - \pi_{t+4|t-1})(i_{t+1} - i_{t+1|t}) | \Omega_t \right] \\
&= \mathbf{1}_2' \left[D_{2,\tilde{x}} P_{\tilde{x},t|t-1} D_{1,\tilde{x}}' \beta_{\pi_{t|t}} + D_{2,e_t} \Sigma_Y D_{1,e_t}' \beta_{\pi_{t|t}} \right] \mathbf{1}_2, \qquad \text{(B26)} \\
&\quad + \mathbf{1}_2' \left[D_{2,\tilde{x}} P_{\tilde{x},t|t-1} B_{1,\tilde{x}}' \beta_{z_{t|t}} + D_{2,e_t} \Sigma_Y B_{1,e_t}' \beta_{z_{t|t}} \right] \mathbf{1}_1.
\end{aligned}
$$

and

$$
\begin{aligned}
p_{i,z,t+2} &= \mathbf{E}\left[(z_{t+4|t+1} - z_{t+4|t-1})(i_{t+1} - i_{t+1|t}) | \Omega_t \right] \\
&= \mathbf{1}_1' \left[B_{2,\tilde{x}} P_{\tilde{x},t|t-1} D_{1,\tilde{x}}' \beta_{\pi_{t|t}} + B_{2,e_t} \Sigma_Y D_{1,e_t}' \beta_{\pi_{t|t}} \right] \mathbf{1}_2, \qquad \text{(B27)} \\
&\quad + \mathbf{1}_1' \left[B_{2,\tilde{x}} P_{\tilde{x},t|t-1} B_{1,\tilde{x}}' \beta_{z_{t|t}} + B_{2,e_t} \Sigma_Y B_{1,e_t}' \beta_{z_{t|t}} \right] \mathbf{1}_1.
\end{aligned}
$$

Finally, $p_{i,i} = \mathbf{E}\left[(i_{t+1|t} - \hat{i}_{t+1|t-1})^2 | \Omega_t \right]$ is known from the one-step-ahead forecast uncertainty.

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Table 1: Parameter estimates for the output gap - inflation model

output equation			inflation equation			gap equation		
μ_y	0.82	(0.05)	γ_1	0.36	(0.12)	ϕ_1	1.55	(0.09)
			γ_2	-0.31	(0.11)	ϕ_2	-0.60	(0.07)
			δ_1	-0.93	(0.11)			
			δ_2	0.29	(0.07)			
			δ_3	-0.03	(0.01)			
			α_4	0.20	(0.07)			
$\sigma_{e,n}$	0.63	(0.06)	$\sigma_{e,\nu}$	0.82	(0.08)	$\sigma_{e,z}$	0.43	(0.08)
SE	0.49		SE	0.63				

NOTE: Standard errors in parentheses. Estimation from 168 quarterly observations from 1960:1 to 2006:4. Log-likelihood value: -322.23.

Table 2: Diagnostic tests for output equation

residual auto-correlation (order)				missing $\Delta\pi$-terms	
1	2	3	4	$\Delta\pi_{t-1}$	$\Delta\pi_{t-2}$
1.99	1.71	1.30	0.73	0.44	1.53
(0.37)	(0.30)	(0.43)	(0.33)	(0.51)	(0.22)

NOTE: The LM-statistics are distributed as χ^2 with degrees of freedom equal to the number of restrictions. p-values in parentheses.

Table 3: Diagnostic tests for inflation equation

residual auto-correlation (order)				missing Δy and $\Delta\pi$-terms			
1	2	3	4	Δy_{t-1}	Δy_{t-2}	$\Delta\pi_{t-1}$	$\Delta\pi_{t-2}$
0.60	0.60	0.39	1.68	0.19	0.20	0.37	0.48
(0.74)	(0.74)	(0.82)	(0.43)	(0.67)	(0.91)	(0.54)	(0.79)

NOTE: The LM-statistics are distributed as χ^2 with degrees of freedom equal to the number of restrictions. p-values in parentheses.

Table 4: Parameter estimates for monetary policy reaction function

A. innovations			
$\sigma_{w,0}$	$\sigma_{w,\pi}$	$\sigma_{w,z}$	$\sigma_{w,i}$
0.11	0.16	0.33	0.04
(0.15)	(0.06)	(0.17)	(0.03)
B. GARCH parameters			
κ_0	κ_1	κ_2	
0.01	1.06	0.13	
(0.01)	(0.20)	(0.11)	
C. correction parameters			
ϑ_1	ϑ_2		
-0.03	-0.01		
(0.03)	(0.01)		

NOTE: Standard errors in parentheses. Standard error of estimate is 1.02. Estimation on quarterly observations from 1970Q1 to 2006Q4. Log-likelihood: -136.27

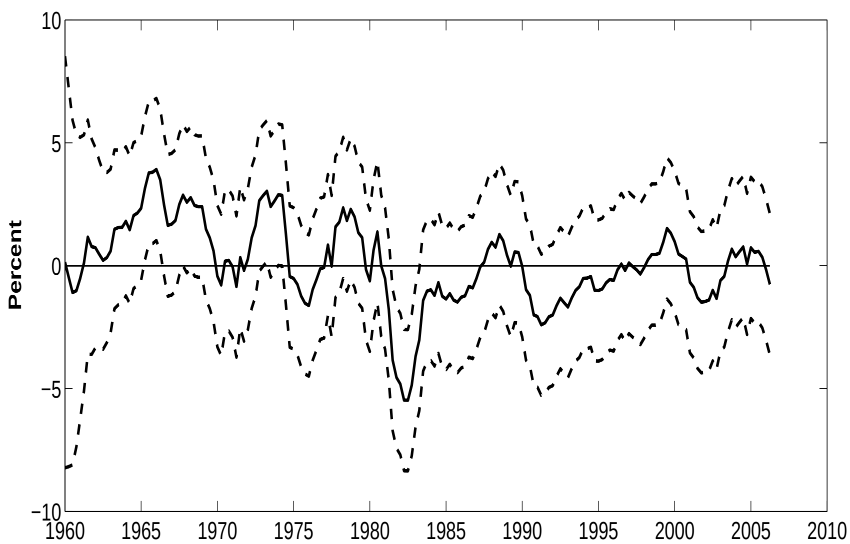

Figure 1: Estimated output gap with 1.96 standard deviation bands.

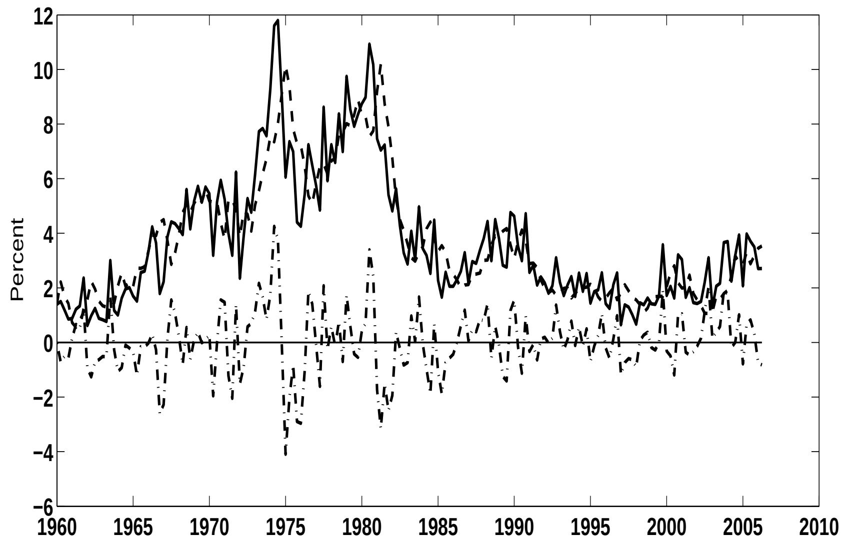

Figure 2: Inflation forecast and forecast errors. Solid: inflation rate. Dashed: inflation forecast $t+2|t-1$. Dotted: forecast error.

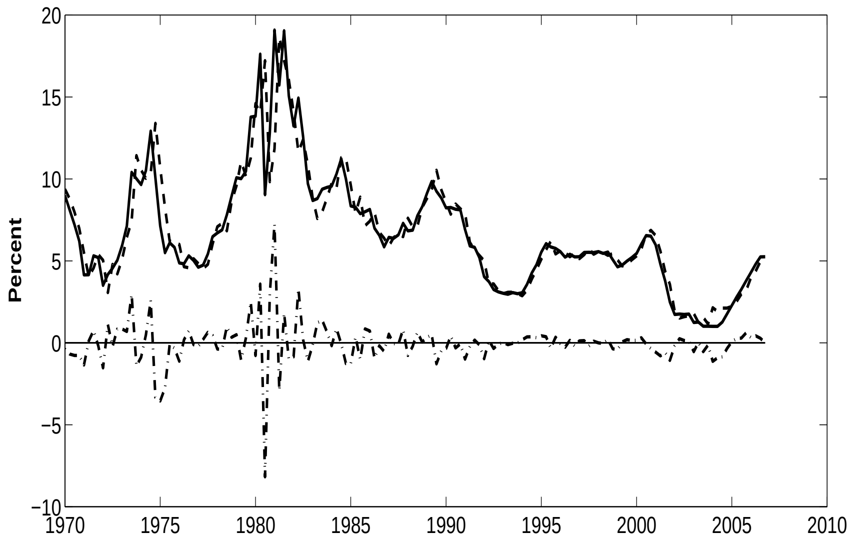

Figure 3: Federal Funds Rate (FFR) forecasts and forecast errors. Solid: Federal Funds Rate. Dashed: Federal Funds Rate forecast $t+2|t-1$. Dotted: forecast error.

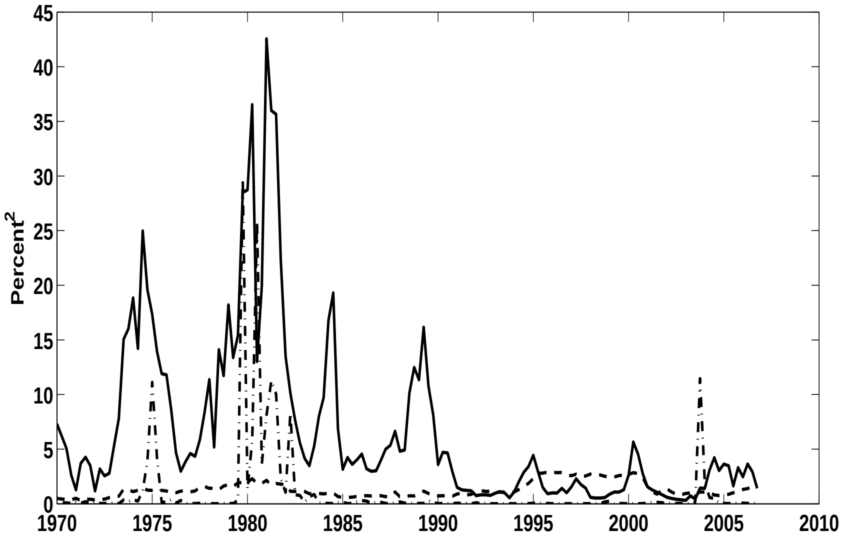

Figure 4: Uncertainty about one-quarter ahead Federal Funds Rate. Solid: uncertainty about future central bank reaction coefficients. Dashed: uncertainty about future fundamentals. Dotted: uncertainty about approximation error.

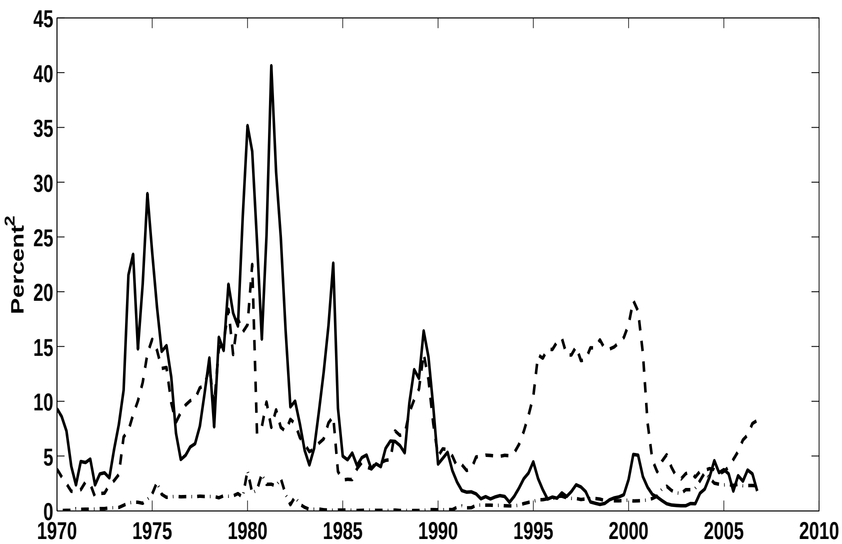

Figure 5: Uncertainty about two-quarter ahead Federal Funds Rate. Solid: uncertainty about future central bank reaction coefficients. Dashed: uncertainty about future fundamentals. Dotted: uncertainty about approximation error.