DEPARTMENT OF ECONOMICS AND FINANCE

COLLEGE OF BUSINESS AND ECONOMICS

UNIVERSITY OF CANTERBURY

CHRISTCHURCH, NEW ZEALAND

ESTIMATING STANDARD ERRORS FOR THE PARKS MODEL: CAN JACKKNIFING HELP?

W. Robert Reed and Rachel S. Webb

WORKING PAPER

No. 18/2009

Department of Economics and Finance
College of Business and Economics
University of Canterbury
Private Bag 4800, Christchurch
New Zealand

WORKING PAPER No. 18/2009

ESTIMATING STANDARD ERRORS FOR THE PARKS MODEL: CAN JACKKNIFING HELP?

W. Robert Reed [1] and Rachel S. Webb [2*]

November 15, 2009

Abstract: Non-spherical errors, namely heteroscedasticity, serial correlation and cross-sectional correlation are commonly present within panel data sets. These can cause significant problems for econometric analyses. The FGLS(Parks) estimator has been demonstrated to produce considerable efficiency gains in these settings. However, it suffers from underestimation of coefficient standard errors, oftentimes severe. Potentially, jackknifing the FGLS(Parks) estimator could allow one to maintain the efficiency advantages of FGLS(Parks) while producing more reliable estimates of coefficient standard errors. Accordingly, this study investigates the performance of the jackknife estimator of FGLS(Parks) using Monte Carlo experimentation. We find that jackknifing can -- in narrowly defined situations -- substantially improve the estimation of coefficient standard errors. However, its overall performance is not sufficient to make it a viable alternative to other panel data estimators.

Keywords: Panel Data estimation, Parks model, cross-sectional correlation, jackknife, Monte Carlo.

JEL Classifications: C23, C15

[1] Department of Economics & Finance, University of Canterbury, Private Bag 4800, Christchurch 8020, New Zealand
[2] Department of Economics & Finance, University of Canterbury, Private Bag 4800, Christchurch 8020, New Zealand

*Corresponding Author: Rachel S. Webb, Department of Economics & Finance, University of Canterbury, Private Bag 4800, Christchurch 8020, New Zealand; email: rachel309@gmail.com.

WORKING PAPER No. 18/2009

ESTIMATING STANDARD ERRORS FOR THE PARKS MODEL: CAN JACKKNIFING HELP?

I. INTRODUCTION

Panel data commonly suffer from a variety of nonspherical error behaviours, including heteroscedasticity, serial correlation, and cross-sectional correlation. As is well known, the simultaneous occurrence of serial and cross-sectional correlation bedevils existing estimation procedures. The Parks model (Parks, 1967) remains the most commonly used estimation procedure for simultaneously handling cross-sectional and serial correlation.[1] However, while FGLS(Parks) is consistent and asymptotically efficient, it can produce notoriously bad estimates of coefficient standard errors in finite samples.

To address this problem, Beck and Katz (1995) proposed a two-step estimator that they claim produces reliable standard error estimates at no cost to estimator efficiency when compared to FGLS(Parks). In a recent paper, Chen, Lin and Reed (2009) show that the latter claim does not generally hold. Specifically, the PCSE estimator compares poorly with FGLS(Parks) on efficiency grounds when data are characterized by both serial and cross-sectional correlation. There remains, therefore, a demand for an estimation procedure that produces both relatively efficient coefficient estimates and reliable standard errors.

This paper uses Monte Carlo experiments to study whether jackknifing the FGLS(Parks) estimator provides a solution to this problem. On the face of it, jackknifing would appear to be a promising avenue. As a result of increased computer processing

[1] For example, the options available with the Stata command “xtgls” are all variations of the Parks model.

speeds, jackknifing has become increasingly feasible (Breunig, 2002; Sunil, 2002). Further, it has been shown to reliably estimate coefficient standard errors in a variety of settings (Schucany, 1989; Jennrich, 2008). Potentially, jackknifing would allow one to maintain the efficiency advantages of FGLS(Parks) while producing more reliable estimates of coefficient standard errors. And unlike bootstrapping, jackknifing is easily incorporated within existing estimation procedures, as FGLS(Parks) is easily estimated when panels are unbalanced.

Unfortunately, our Monte Carlo simulations find that while jackknifing can improve estimation of coefficient standard errors, its overall performance is not sufficient to make it a viable alternative to other panel data estimators.

II. THE PARKS ERROR STRUCTURE AND THE PROBLEM WITH ESTIMATING STANDARD ERRORS

The data generating process. This paper analyzes the following panel data problem. Let the DGP be represented as follows:

(1) $$\boldsymbol{y} = \begin{bmatrix} \boldsymbol{i} & \boldsymbol{x} \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_x \end{bmatrix} + \boldsymbol{\varepsilon} = \boldsymbol{X\beta} + \boldsymbol{\varepsilon},$$

where N and T are the number of cross-sectional units and time periods; β_0 and β_x are scalars; and $\boldsymbol{y}$, $\boldsymbol{i}$, $\boldsymbol{x}$, and $\boldsymbol{\varepsilon}$ are, respectively, $NT \times 1$ vectors of observations of the dependent variable, a constant term, observations of the exogenous explanatory variable, and unobserved errors, where $\boldsymbol{\varepsilon} \sim N(\boldsymbol{0}, \boldsymbol{\Omega}_{NT})$.

The $NT \times NT$ error variance-covariance matrix, $\boldsymbol{\Omega}_{NT}$, is structured according to the Parks model (Parks, 1967). It assumes (i) groupwise heteroscedasticity; (ii) first-order serial correlation; and (iii) time-invariant cross-sectional correlation.[2] This implies the following specification for $\boldsymbol{\Omega}_{NT}$:

(2) $\boldsymbol{\Omega}_{NT} = \boldsymbol{\Sigma} \otimes \boldsymbol{\Pi}$,

where $\boldsymbol{\Sigma} = \begin{bmatrix} \sigma_{\varepsilon,11} & \sigma_{\varepsilon,12} & \cdots & \sigma_{\varepsilon,1N} \\ \sigma_{\varepsilon,21} & \sigma_{\varepsilon,22} & \cdots & \sigma_{\varepsilon,2N} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{\varepsilon,N1} & \sigma_{\varepsilon,N2} & \cdots & \sigma_{\varepsilon,NN} \end{bmatrix}$, $\boldsymbol{\Pi} = \begin{bmatrix} 1 & \rho & \rho^2 & \cdots & \rho^{T-1} \\ \rho & 1 & \rho & \cdots & \rho^{T-2} \\ \rho^2 & \rho & 1 & \cdots & \rho^{T-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho^{T-1} & \rho^{T-2} & \rho^{T-3} & \cdots & 1 \end{bmatrix}$.

The GLS estimators for $\boldsymbol{\beta}$ and var($\hat{\boldsymbol{\beta}}$) are given by the usual formulae: $\hat{\boldsymbol{\beta}} = \left(\boldsymbol{X}'\boldsymbol{\Omega}_{NT}^{-1}\boldsymbol{X}\right)^{-1}\boldsymbol{X}'\boldsymbol{\Omega}_{NT}^{-1}\boldsymbol{y}$ and $Var\left(\hat{\boldsymbol{\beta}}\right) = \left(\boldsymbol{X}'\boldsymbol{\Omega}_{NT}^{-1}\boldsymbol{X}\right)^{-1}$. In the case of Feasible Generalized Least Squares (FGLS), $\boldsymbol{\Omega}_{NT}$ is replaced with $\hat{\boldsymbol{\Omega}} = \hat{\boldsymbol{\Sigma}} \otimes \hat{\boldsymbol{\Pi}}$, so that $\boldsymbol{Var}\left(\hat{\boldsymbol{\beta}}_{\boldsymbol{FGLS}}\right) = \left(\boldsymbol{X}'\hat{\boldsymbol{\Omega}}^{-1}\boldsymbol{X}\right)^{-1}$. In other words, FGLS does not adjust coefficient standard errors for the additional uncertainty that arises from the fact that the elements of $\boldsymbol{\Omega}_{NT}$ are unknown and must be estimated. This causes FGLS to underestimate coefficient standard errors. As there are a total of $\frac{N(N+1)}{2}+1$ unique elements in $\boldsymbol{\Omega}_{NT}$, the degree of underestimation may be quite substantial.

III. JACKKNIFING THE FGLS(PARKS) ESTIMATOR

Let $\hat{\boldsymbol{\beta}}$ be the FGLS(Parks) estimator given NT data points. Define $\hat{\boldsymbol{\beta}}_i$ as the FGLS(Parks) estimate derived from dropping the i^{th} observation,

[2] In its most general form, the Parks model assumes groupwise, first-order serial correlation. In contrast, our experiments model the DGP with a common AR(1) parameter, ρ, that is the same across groups. We do this to facilitate comparison with previous Monte Carlo studies of this problem that have also assumed a common AR(1) parameter (cf., Chen et al. 2009)

$\hat{\boldsymbol{\beta}}_i = \left(\boldsymbol{X}'\hat{\boldsymbol{\Omega}}_{NT-1}^{\ -1}\boldsymbol{X}\right)^{-1}\boldsymbol{X}'\hat{\boldsymbol{\Omega}}_{NT-1}^{\ -1}\boldsymbol{y}$, where $\boldsymbol{X}$ and $\boldsymbol{y}$ are the data observations corresponding to the *NT-1* observations, and $\hat{\boldsymbol{\Omega}}_{NT-1}$ is the estimate of the corresponding error variance-covariance matrix.

The *ith* "pseudovalue" is defined by $\hat{\boldsymbol{\beta}}_i{*} = (NT)\hat{\boldsymbol{\beta}} - (NT-1)\hat{\boldsymbol{\beta}}_i$. The jackknife estimate of $\boldsymbol{\beta}$ is given by $\hat{\boldsymbol{\beta}}{*} = \frac{1}{NT}\sum_{i=1}^{NT}\hat{\boldsymbol{\beta}}_i{*}$, and the corresponding standard error for each of the elements of $\hat{\boldsymbol{\beta}}{*}$ is given by $s.e.(\hat{\beta}{*}) = \sqrt{\frac{\sum_{i=1}^{NT}\left(\hat{\beta}_i{*} - \hat{\beta}{*}\right)^2}{NT(NT-1)}}$.

A complication arises when constructing $\hat{\boldsymbol{\Omega}}_{NT-1}$. Not only must the values of ρ and the $\sigma_{\varepsilon,ij}$s be re-estimated with the deletion of an observation, but $\hat{\boldsymbol{\Omega}}$ now has dimensions $(NT-1)\times(NT-1)$. Let the deleted observation be indexed by *it*. For the i^{th} group, $\boldsymbol{\Pi}$ must be modified to account for the deleted t^{th} observation. To illustrate, if *T=5* and *t=3*, $\boldsymbol{\Pi}_i$ becomes

$$\boldsymbol{\Pi}_i = \begin{vmatrix} 1 & \rho & \rho^3 & \rho^4 \\ \rho & 1 & \rho^2 & \rho^3 \\ \rho^3 & \rho^2 & 1 & \rho \\ \rho^4 & \rho^3 & \rho & 1 \end{vmatrix} .$$

IV. DESCRIPTION OF THE MONTE CARLO EXPERIMENTS

One of the challenges of Monte Carlo analysis of panel data estimators given complicated error structures is how to set the population parameters associated with $\boldsymbol{\Omega}_{NT}$. As noted above, the Parks model has $\frac{N(N+1)}{2}+1$ unique elements in $\boldsymbol{\Omega}_{NT}$. This study employs the methodology described in Reed and Ye (2009). The idea is to produce simulated data sets that are similar to "real world" data sets.

Two "families" of data sets are constructed: one patterned after cross-country, real, per capita GDP data; the other after real, per capita personal income (PCPI) data from U.S. states. In each case, for given N and T values, a large number of OLS regressions are estimated using the basic specification $y_i = \beta_0 + \beta_x x_i + Country/State\ Dummies + Time\ Dummies + \varepsilon_i$, $i=1,2,\ldots,NT,$ where the dependent variable is either the level or growth of real per capita GDP, or the level or growth of real per capita PCPI; and the explanatory variable is either governmental share of GDP or the state's effective tax rate. Specification (1) includes only Country/State Dummy variables. Specification (2) also includes Time Dummy variables.

The residuals from these regressions are then used to construct representative parameter values for $\boldsymbol{\Omega}_{NT}$. Thus, for any given N and T values, eight different population, error variance-covariance structures are created, encompassing a large range of values of cross-sectional and serial correlations similar to those found in "real world" data sets. These population error variance-covariance structures are then used to generate the simulated data sets used in our experiments.

An "experiment" is defined by the original data set (International GDP Data/U.S. State PCPI Data, Level/Growth), the size of the data set (N,T), and the regression specification (1/2) from which the values of $\boldsymbol{\Omega}_{NT}$ are set. We follow Beck and Katz (1995) by setting N and T values that correspond to commonly chosen values in the literature. *N* is restricted to be less than *T*, otherwise the Parks model is not estimable. A total of 1000 replications are conducted for each experiment. Further details are provided in Reed and Ye (2009).

V. RESULTS AND DISCUSSION

The focus of our study is the "coverage rates" produced by the FGLS(Parks) and jackknife estimators, where the respective coverage rates are defined as the percent of 95% confidence intervals that contain the true population value of β_x. Coverage rates should be close to 95%.

Our main findings are:

1. The jackknife estimator can produce substantial improvements in coverage rates over FGLS(Parks).

2. Coverage rates for the jackknife estimator are unsatisfactory, except when N=T, and then only for some types of data.

TABLE 1 demonstrates the improvement that can come from jackknifing FGLS(Parks) estimates.

The numbers in the table represent the difference in coverage rates between FGLS(Parks) and the jackknife estimator. For example, using a population error variance-covariance matrix patterned after International GDP data (Level, Specification 1) and data sets of size $N=5$ and $T=5$, we find that FGLS(Parks) and the jackknife estimator produce coverage rates of 45.4 and 84.5 percent, respectively. Thus, the jackknife estimator has coverage rates that are 39.1 percentage points higher than the FGLS(Parks) estimator. It is the latter number that is reported in the table.

In general, the performance advantage of the jackknife estimator diminishes, and is sometimes reversed, as $\frac{T}{N}$ increases. This is primarily due to the better performance of FGLS(Parks). The last row of TABLE 1 averages the difference in coverage rates for values of N and T across the different population data sets. This generally confirms the observation that jackknifing results in greatest performance improvements when N=T.

To be a viable estimator, jackknifing should not only produce more reliable estimates of coefficient standard errors, but it should also have satisfactory coverage rates of its own. Unfortunately, TABLE 2 makes clear that this is not the case. Coverage rates are rarely close to 95 percent and are frequently less than 50 percent. When N=T, the jackknife estimator does slightly better. Overall, the coverage rates of the jackknife estimator compare poorly with alternative panel data estimators, such as the PCSE estimator (Beck and Katz, 1995).

One disadvantage of our experimental methodology is that we do not directly control the values of cross-sectional and serial correlation. This is outweighed by the advantage of being able to measure estimator performance in simulated data environments patterned after the “real world.” The fact that the jackknife estimator performs poorly under these conditions eliminates it as a viable alternative to existing panel data estimators. Until a better approach is developed, the recommendation of Reed and Ye (2009) remains valid: Researchers should use FGLS(Parks) if the goal is estimator efficiency, and another estimator (e.g. the PCSE) if the concern is reliable hypothesis testing.

REFERENCES

Beck, N., & Katz, J. N. (1995). What To Do (And What Not To Do) With Time-Series Cross-Secton Data. *American Political Science Review* Vol. 89, No. 3, 634-647.

Breunig, R. (2002). Bias Correction for Inequality Measures: An Application to China and Kenya. *Applied Economics Letters* Vol. 9, No. 12, 783-786.

Chen, X., Lin, S., & Reed, W.R. (2009). A Monte Carlo Evaluation of the PCSE Estimator. *Applied Economics Letters,* forthcoming.

Jennrich, R. I. (2008). Nonparametric Estimation of Standard Errors in Covariance Analysis Using the Infinitesimal Jackknife. *Psychometrika* Vol. 73, No. 4, 579-594.

Parks, R. (1967). Efficient Estimation of a System of Regression Equations When Disturbances Are Both Serially and Contemporaneously Correlated. *Journal of the American Statistical Association* Vol. 62, 500-509.

Reed, W.R. and Ye, H. (2009). Which Panel Data Estimator Should I Use? *Applied Economics,* forthcoming.

Schucany, W. & Sheather, S.J. (1989). Jackknifing R-estimators. *Biometrika* Vol. 76, No. 2, 393-398.

Sunil. S. (2002). A Jackknife Maximum Likelihood Estimator for the Probit Model. *Applied Economics Letters* Vol. 9, No. 2, 73-74.

TABLE 1
Difference in Coverage Rates for FGLS (Parks) and Jackknife Estimators

Spec.[a]	*Experimental Data Patterned After…*[a]	*N=5*					*N=10*				*N=20*	
		T=5	*T=10*	*T=15*	*T=20*	*T=25*	*T=10*	*T=15*	*T=20*	*T=25*	*T=20*	*T=25*
1	*International GDP Data (Level)*	39.1	-8.6	-34.7	-45.6	-54.1	60.2	25.6	-3.4	-29.6	57	48.7
1	*International GDP Data (Growth)*	32.6	-18.5	-30.5	-42.3	-44.6	50.6	-9.1	-35.3	-44.2	70.6	42.6
1	*U.S. State PCPI Data (Level)*	42.1	22.7	3.7	5	-2.1	52.9	37.6	32.5	33.6	44.5	62.3
1	*U.S. State PCPI Data (Growth)*	39.7	9.4	-2.7	-14.8	-18.4	51.1	27.4	10.7	-9.5	53.2	57.7
2	*International GDP Data (Level)*	45.9	4.6	-23.4	7.1	-16.3	64.3	34.5	70.7	36.6	61.9	81.5
2	*International GDP Data (Growth)*	45.6	-13.2	-38.4	1.8	-67.4	58.9	20.6	61	49.5	69.5	84
2	*U.S. State PCPI Data (Level)*	39.1	-9.4	34.8	0.5	-25.5	69.5	45.1	82.4	13.6	64.1	88.5
2	*U.S. State PCPI Data (Growth)*	35.9	-21.3	5.5	-32.3	-21.9	65.6	24.9	54.4	17.8	67.9	89.7
AVERAGE		**40**	**-4.3**	**-10.7**	**-15.1**	**-31.2**	**59.1**	**25.8**	**34.1**	**8.5**	**61.1**	**69.4**

[a] See text for an explanation of "Specification" 1 and 2 and the methodology used to produce simulated data sets patterned after the respective data.

TABLE 2
Coverage Rates for FGLS (Parks) with Jackknifed Standard Errors

RGF	*Model Data*	*N=5*					*N=10*				*N=20*	
		T=5	*T=10*	*T=15*	*T=20*	*T=25*	*T=10*	*T=15*	*T=20*	*T=25*	*T=20*	*T=25*
1	*International GDP Data (Level)*	84.5	57.6	35.7	28.7	21.4	81.3	71.4	50.9	33.4	66	73
1	*International GDP Data (Growth)*	81.7	59.7	52.1	44.4	43.3	83.7	53.8	37.9	34.5	85.5	79
1	*U.S. State PCPI Data (Level)*	89.7	87	74.3	76.5	69.6	82.3	91	89.5	87.5	53.1	74.5
1	*U.S. State PCPI Data (Growth)*	89.5	83.5	79	70.5	67.4	86.3	90.4	82.9	67.5	70.2	93.2
2	*International GDP Data (Level)*	52.4	42.5	38.4	80	62.4	64.4	40.6	81.7	53.2	61.9	81.5
2	*International GDP Data (Growth)*	54.1	34.7	28.1	84.5	21	59.2	27.4	79.4	79.3	69.5	84
2	*U.S. State PCPI Data (Level)*	45.8	24.9	87.6	64.9	47.1	69.7	49.7	89.9	34.2	64.1	88.5
2	*U.S. State PCPI Data (Growth)*	44.1	18.8	70.7	45	66.6	65.9	31.4	70.6	47.6	67.9	89.7