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*Can Labor Market Imperfections Cause  
Overprovision of Public Inputs?*

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JEL Classification numbers: H41, J51

Keywords: Public Inputs, Trade Unions.



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# Can Labor Market Imperfections Cause Overprovision of Public Inputs?\*

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## Abstract

In a model where trade unions dominate the labor market, a relationship is derived between the rate of unemployment and the provision of a public input in the production. This relationship implies that for conventional rates of unemployment, the public input will be overprovided compared to the first-best level.

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# 1 Introduction

An important question in public economics is whether the provision of a public good in a second-best economy will exceed, or fall short of, the quantity provided in a first-best setting? In the literature, the main focus has been to analyze how the use of distortionary taxes,<sup>1</sup> or the presence of labor market distortions,<sup>2</sup> may influence the provision of a public good.

These issues have received far less attention in the context of public inputs in the production.<sup>3</sup> In particular, no previous study has made an attempt to relate the quantity provided of a public input to the rate of unemployment. This is somewhat surprising since the provision of a public input will be influenced by the presence of unemployment.<sup>4</sup> The argument is that if the public input is complementary with labor in the production, then the government will have an incentive to provide more of the public input. The question is then whether this incentive will be sufficiently strong to cause overprovision compared to the first-best quantity?

The purpose of this paper is to analyze this question. Since we want to focus on the connection between labor market imperfections and public input provision, we do not include other types of imperfections, such as distortionary taxes, in the model. The labor market imperfection is assumed to arise because of trade union wage setting and the main result is that when agents are risk-neutral and the production is of Cobb-Douglas type, then the public input will be overprovided in relation the first-best quantity as long as the rate of unemployment does not exceed approximately 60%.

In section 2, we present the basic model while the main result is derived in Section 3. The paper is concluded in Section 4.

## 2 The Basic Model

Consider an economy made up of firms, consumers, trade unions and a government. The firms are identical and their number is normalized to one. The production function is of Cobb-Douglas

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<sup>1</sup>See, for example, Pigou (1947), Wilson (1991), Chang (2000), Gaube (2000, 2007).

<sup>2</sup>See, for example, Bovenberg and van der Ploeg (1996), and Aronsson and Sjögren (2003).

<sup>3</sup>Exceptions are Martinez and Sanchez (2009a,b) who analyze the effects of distortionary taxes on the provision of a public input.

<sup>4</sup>See Aronsson and Wehke (2008) who characterize the optimal provision of public inputs in an economy where the labor market is dominated by trade unions, and Aronsson and Koskela (2008) who characterize public input provision in the presence of unemployment and outsourcing.

type

$$F(N, K, G) = N^\alpha \cdot K^{1-\alpha} \cdot G^\beta \quad (1)$$

where  $\alpha, \beta \in (0, 1)$ ,  $N$  is labor,  $K$  a fixed factor and  $G$  a public input. This formulation of the technology means that the public input is factor augmenting. In the following, the fixed factor will be normalized to one, in which case we can write the profit (i.e. the return to the fixed factor) as

$$\pi = F(N, G) - w \cdot N \quad (2)$$

where  $w$  is the wage rate. The first-order condition for profit maximization implies  $w = F_N(N, G)$ , which implicitly defines the following labor demand function

$$N(w, G) = \alpha^{\frac{1}{1-\alpha}} \cdot G^{\frac{\beta}{1-\alpha}} \cdot w^{-\frac{1}{1-\alpha}} \quad (3)$$

Substituting the labor demand function into equation (2) defines the profit function  $\pi(w, G)$ .

Turning to the consumption side of the economy, all consumers have identical preferences for consumption,  $c$ , and these preferences are described by a utility function  $u(c)$  which is increasing in  $c$ . There are three types of consumers: a firm-owner, employed workers and unemployed workers, and they are distinguished by the superindices "f", "e" and "u", respectively. Beginning with the firm-owner, he is endowed with the fixed factor. The firm-owner does not work and he receives the profit income in return for providing the fixed factor to the firm. The firm-owner's budget constraint is given by  $c^f = \pi - T^f$  where  $T^f$  is a lump-sum tax. Turning to the labor force, it is made up of  $M$  workers out of whom  $N$  are employed and  $M - N$  are unemployed. An unemployed worker receives a net of tax unemployment benefit,  $b$ , from the government meaning that his utility is given by  $u^u = u(b)$ . As for an employed worker, he faces the budget constraint  $c^e = w - T^e$ , where  $T^e$  is a lump-sum tax.

All workers are assumed to belong to a trade union which dominates the labor market.<sup>5</sup> The trade union has a utilitarian objective function

$$V = N \cdot u(w - T^e) + [M - N] \cdot u(b) \quad (4)$$

where  $M - N$  is the number of unemployed workers. The trade union chooses  $w$  so as to maximize equation (4) subject to the restriction  $N = N(w, G)$ . The first-order condition becomes

$$N_w(w, G) \cdot [u(w - T^e) - u(b)] + N(w, G) \cdot u'(w - T^e) = 0 \quad (5)$$

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<sup>5</sup>Since the number of firms is normalized to one, so is also the number of trade unions.

Equation (5) implicitly determines the wage as a function<sup>6</sup>  $w = w(b, T^e)$ . If this wage exceeds the market-clearing wage, there will be unemployment in the equilibrium.

### 3 Optimal Policy

The government maximizes a utilitarian welfare function

$$W = N \cdot u^e + [M - N] \cdot u^u + u^f \quad (6)$$

and the budget constraint is given by

$$N \cdot T^e + T^f - G - [M - N] \cdot b = 0 \quad (7)$$

The policy instruments are  $b$ ,  $G$ ,  $T^e$  and  $T^f$ , and the Lagrangian corresponding to the government's problem can be written as

$$\begin{aligned} \mathcal{L} = & N \cdot u(w - T^e) + [M - N] \cdot u(b) + u(\pi - T^f) \\ & + \gamma \cdot [N \cdot T^e + T^f - G - (M - N) \cdot b] \end{aligned} \quad (8)$$

where  $\gamma$  is a Lagrange multiplier.

Let us first consider the optimal policy chosen in a first-best setting. Within the context of this model, this corresponds to the policy chosen when there is no unemployment. Substituting  $N = M$  into equation (8) and maximizing w.r.t. to the policy instruments, it is straightforward to show that the solution to this problem produces the standard condition for the optimal provision of the public input

$$F_G(M, G^*) = 1 \quad (9)$$

where  $G^*$  denotes the first-best quantity.

Next, we turn to the outcome when there is unemployment in equilibrium. Differentiating the Lagrangian w.r.t.  $b$ ,  $G$ ,  $T^e$  and  $T^f$ , and combining the first-order conditions for  $T^f$  and  $G$  while using that  $w = F_N$ , it is straightforward to derive the following condition for the optimal provision of the public input

$$\begin{aligned} 0 = & F_G(N(w, G^o), G^o) - 1 \\ & + N_G(w, G^o) \cdot \left[ \frac{u(w - T^e) - u(b)}{\gamma} + (T^e + b) \right] \end{aligned} \quad (10)$$

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<sup>6</sup>Note that the functional form of  $N(w, G)$  in equation (3) implies that  $w$  will be independent of  $G$ .

Let  $G^\circ$  denote the quantity of the public input chosen in the presence of unemployment. Equation (10) shows that two incentives influence the provision of the public input in this case; a 'pure' motive captured by the term  $F_G - 1$  and an employment motive reflected by the terms in the second row of equation (10). Beginning with the 'pure' motive, observe first that since  $N$  and  $G$  are complements in production, it follows that  $F_G(N, G) < F_G(M, G)$  for any level of employment satisfying  $N < M$ . This means that the 'pure' motive for providing the public input is reduced in the presence of unemployment. The employment motive, on the other hand, will induce the government to provide more of the public input as long as  $N_G > 0$ . Each worker who goes from the state of unemployment to employment will give rise to two welfare effects. First, there will be a direct and positive utility effect equal to  $u^e - u^u > 0$  and second, each worker who leaves unemployment will give rise to a net tax revenue improvement equal to  $T^e + b > 0$ .

The discussion above indicates that the presence of unemployment will give rise to two conflicting effects regarding the provision of the public input. On one hand, the 'pure' motive for providing  $G$  will be weakened whereas the employment motive will provide an incentive to increase the provision of  $G$ . The question is then if the employment motive is sufficiently strong to cause overprovision of the public input in the sense that  $G^\circ > G^*$ ? Without making further assumptions we cannot answer this question. Let us, therefore, assume that the individuals are risk-neutral, meaning that we can write the utility function  $u(c) = c$ . In this situation, equation (10) can be simplified to<sup>7</sup>

$$F_G(N, G^\circ) - 1 + N_G(w, G^\circ) \cdot w = 0 \quad (11)$$

where  $N$  is determined by equation (3). To be able to compare  $G^\circ$  with  $G^*$ , let us add and subtract  $F_G(M, G^*)$  in equation (11) and use that  $F_G(M, G^*) - 1 = 0$ . This produces

$$F_G(M, G^*) = F_G(N, G^\circ) + N_G(w, G^\circ) \cdot w \quad (12)$$

Next, substituting

$$F_G(N, G) = \beta \cdot N^\alpha \cdot G^{\beta-1} \quad (13)$$

$$N_G(w, G) = \frac{\beta}{1-\alpha} \cdot \frac{N}{G} \quad (14)$$

$$w = F_N(N, G) = \alpha \cdot N^{\alpha-1} \cdot G^\beta \quad (15)$$

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<sup>7</sup>Here we use that risk neutrality will imply  $\gamma = 1$ .

into equation (12), we can derive the following relationship

$$\left(\frac{G^o}{G^*}\right)^{1-\beta} = \frac{1}{1-\alpha} \cdot \left(\frac{N}{M}\right)^\alpha \quad (16)$$

If there is overprovision of the public input in the presence of unemployment, then  $G^o/G^* > 1$ . As such, equation (16) implies that employment rates associated with overprovision will satisfy the following inequality

$$\frac{N}{M} > h(\alpha) \quad (17)$$

where

$$h(\alpha) = (1-\alpha)^{\frac{1}{\alpha}} \quad (18)$$

The function  $h(\alpha)$  is decreasing in  $\alpha$  and the start- and the endpoints of this function are given by

$$\lim_{\alpha \rightarrow 0} h(\alpha) = e^{-1} \approx 0.3679 \quad (19)$$

$$\lim_{\alpha \rightarrow 1} h(\alpha) = 0 \quad (20)$$

Three observations can be made from this analysis. First, since  $h(0) \approx 0.3679$  and since  $h(\alpha)$  is decreasing in  $\alpha$ , the public input will always be overprovided as long as the rate of employment exceeds 36.79% (or equivalently, as long as the rate of unemployment does not exceed approximately 63%). Second, the larger  $\alpha$  is, the smaller will be the rates of employment associated with underprovision of the public input. Third, if  $\alpha = 1$  then the public input will always be overprovided in the presence of unemployment.

The analysis above can be summarized in the following proposition;

**Proposition:** *Consider an economy where monopoly union wage setting creates unemployment, individuals are risk-neutral and the production technology is of Cobb-Douglas type. If the level of employment in this setting satisfies the inequality  $N > M \cdot (1-\alpha)^{\frac{1}{\alpha}}$ , then the public input will be overprovided compared to the first-best quantity.*

## 4 Concluding Remarks

In this paper, we show that in the presence of unemployment, the government has an employment motive to overprovide a public input in the production. An avenue for future research would be to analyze to what degree this result will hold in the presence of distortionary taxes. It would also be interesting to study whether the result derived in the paper is reflected in empirical data.

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