School of Economics

HENLEY
UNIVERSITY OF READING

Growth and Public Infrastructure

by
Nigar Hashimzade and Gareth D. Myles

2009
076

School of Economics
Henley Business School
University of Reading
Whiteknights
Reading
RG6 6AA
United Kingdom

www.henley.reading.ac.uk

Growth and Public Infrastructure

Nigar Hashimzade
University of Reading

Gareth D. Myles
University of Exeter and Institute for Fiscal Studies

11 January 2009

Abstract: The paper analyzes a multi-country extension of the Barro model of productive public expenditure. In the presence of infrastructural externalities between countries the provision of infrastructure will be inefficiently low if countries do not coordinate. This provides a role for a supra-national body, such as the EU, to coordinate the policies of the individual governments. It is shown how the supra-national body can ensure the efficient level of infrastructure provision and, as a result, obtain an increased rate of growth. The results of the paper also show how capital flows between countries act to equalize growth rates. This can help explain why there is limited empirical evidence for tax rates causing a difference in growth rates between countries. This is not the same as saying taxation does not affect growth: if production requires public infrastructure then taxation is needed for growth. The flow of capital acts to distribute the benefit of this across countries.

Contact details: Nigar Hashimzade: School of Economics, University of Reading, Reading, RG6 6AA, UK, n.hashimzade@henley.reading.ac.uk

Gareth Myles: Department of Economics, University of Exeter, Exeter, EX4 4PU, UK, gdmyles@ex.ac.uk

Acknowledgements: Thanks are due to Sebastian Kessing, seminar participants in Reading, Dublin, and participants in the DG ECFIN workshop on the Quality of Public Finances.

1. Introduction

One factor promoting endogenous growth is the supply of public infrastructure that complements the investments of the private sector. The importance of infrastructure is widely recognized, not least by the EU which pursues an active programme to support the investment activities of member states. The policy problem facing the EU is to ensure that member states undertake an efficient level of infrastructural expenditure that ensures the maximum rate of growth. The determination of the level has to take into account the full consequences of an infrastructure project for the EU, not just the direct benefits for the member state undertaking the investment. There are three significant issues that confront this policy programme. First, infrastructural investment has significant spill-overs across member states. Second, mobility of the tax base results in tax externalities between the member states, and between the member states and the EU. Third, the EU is faced with a decision on how to allocate support for infrastructural expenditure across the different member states. This interacts with the process of revenue-raising, and with the extent to which the projects are financed jointly by the EU and member states.

The economic modelling of the impact of infrastructure on economic growth has focussed on the Barro (1990) model of public expenditure as a public input and its extensions (Chen *et al*. 2005, Turnovsky, 1999). This literature has identified the concept of an optimal level of expenditure, and has highlighted the deleterious effects of both inadequate and excessive expenditure. These are important insights, but do not address the spill-over issues that confront the EU. Infrastructural spill-overs between member states can be positive, which occurs when improvements in infrastructure in one member state raise productivity in another, or they can be negative if they induce relocation of capital between member states. In either case, it is important that the consequences of spill-overs are addressed in order that the role of productive public expenditure can be fully understood. Ignoring either form of spill-over will result in an inefficient level and allocation of expenditure.

The financing of infrastructure in the Barro model is through a simple tax on output levied at the national level. The position in the EU is much more complex. Each member state levies national taxes. Part of these taxes are retained by the member states, the remainder is remitted to, and redistributed by, the EU. In economic terms, if there is mobility of the tax base then there are horizontal tax externalities between member states, and a vertical tax externality between member states and the EU. These tax externalities have a key role in determining the growth-maximizing level of expenditure.

In this paper we construct multi-country extensions of the Barro model of productive public infrastructure. In addition, the benefits of infrastructure spill-over between countries. The spill-over between countries is a form of positive externality which results in inefficient investment in infrastructure if countries act independently. If there are infrastructural externalities between countries then the provision of infrastructure will be inefficiently low when countries do not coordinate policies. This gives a role to a supra-national body, such as the EU, to act as a coordinator of the policies of individual governments. By ensuring the efficient level of infrastructural investment it is possible for the supra-national body to counter the externality and obtain a higher rate of growth.

The results of the paper also show how capital flows between countries act to equalize growth rates. The observation that capital flows reduce growth differentials between countries has been made previously by Razin and Yuen (1997). They argued that labour mobility equalized incomes across countries when there were human capital externalities. We demonstrate that the mobility of physical capital equalizes the growth rate across countries. Similar issues have also been addressed by Bianconi and Turnovsky (1997), but in a model that does not have public infrastructure. A substantial empirical literature has failed to find a convincing link between the rate of tax and the rate of economic growth. Our model provides a possible explanation for this: In a cross-section of countries the infrastructural externality and the flow of capital equalize the growth rate across countries regardless of the tax policy that each country operates. This is not to say that taxation does not matter. Actually, the opposite is correct: Taxation is even more important than in a world without spill-overs since additional public infrastructure in one country can raise the growth rate in all. This holds if all countries are operating with less than the optimum level of infrastructure, as they will be in an equilibrium without policy coordination.

Section 2 of the paper provides a brief review and discussion of evidence on the link between taxation and economic growth. A basic version of the endogenous growth model with a productive public input is analyzed in Section 3. Section 4 studies the role of a supra-national body in coordinating the choices of individual countries when there is an infrastructural externality. The analysis is extended to incorporate capital mobility in Section 5. Conclusions are given in Section 6. An appendix provides the calculations that support the results reported in the main text.

2. Empirical Evidence

There is a considerable body of published work that investigates the link between the level of taxation and the growth rate of gross domestic product (GDP). Much of this literature is summarized in Myles (2007). The conclusions that can be drawn from that literature are open to debate and interpretation, but the essential issues can be identified from considering some standard data.

Consider the data presented in Figures 1 and 2. Figure 1 plots the growth rate of US GDP and federal government tax revenue as a percentage of GDP since 1930. Trend lines have been fitted to the time series using ordinary least squares. The two trend lines show a steady rise in taxation (the upper line) and a very slight decline in the growth rate (the lower line). Although the variance of the growth rate is lower after 1940, statistical tests on US data have found no significant between the average rate of growth prior to 1942 and after 1942. The data for the UK in Figure 2 tell a very similar story. The trend lines show an increase in taxation but, in contrast to the US, an increase in the rate of growth. Over the long periods illustrated in these figures no clear relationship between the average tax rate and the growth rate of GDP is apparent.

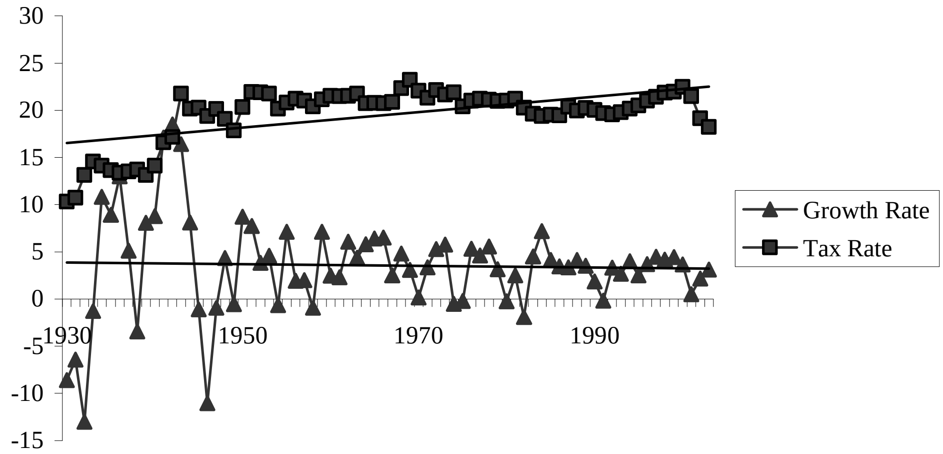

Figure 1: US Tax and Growth Rates.

Source: US Department of Commerce: www.bea.doc.gov]

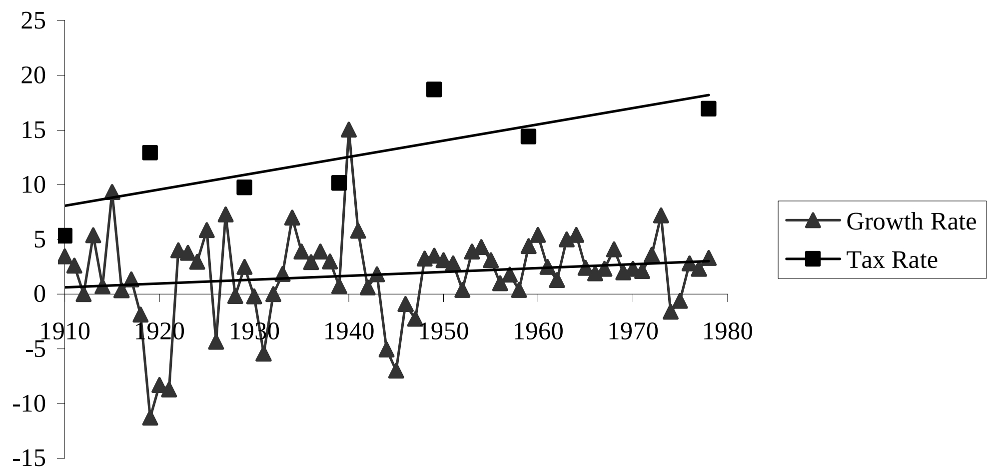

Figure 2: UK Tax and Growth Rates.

Source: Feinstein (1972), UK Revenue Statistics, Economic Trends

The interpretation of these figures must be considered carefully. There are two reasons for this. First, a contrast between the tax rate and the growth rate across time cannot answer the counter-factual question "if taxes had been lower, would growth have been higher?" An answer to this question requires, at least, a study involving a range of countries with different regimes. Second, there are substantive issues that have to be resolved about the definition of the tax rate that should enter into any such comparison. In particular, economic theory focuses on the marginal rate of tax as the determinant of behaviour but the figures employ the average rate of tax.

One route through which the first issue can be addressed is to consider the same data for a cross-section of countries. This approach was pioneered by Plosser (1993) who calculated the correlation between the rate of growth of per capita GDP and a range of variables for the OECD countries. The share of income and profit taxes in GDP was found to have a correlation of -0.52 with the growth rate of GDP. However, Plosser warns against taking the correlation as evidence of causality and presents several potential explanations for the lack of robustness in regression equations: most policies operate through investment; policies are complex and not easily represented by variables in regressions; and policies are highly correlated. Even so, the work of Plosser is often cited as evidence that an increase in taxation leads to a fall in the growth rate.

The left-hand panel of Figure 3 displays an updated version of Chart 6 in Plosser (1993) that extends the sample period through to 2004. The data points are found by averaging the growth rate and the tax rate over this period for each country. A straight line fitted by least squares shows the negative relationship between the growth rate and the average tax rate. There are three countries that are unusual in this data set: Korea (the only Asian Tiger in this sample), the Czech Republic and the Slovak Republic (both newly liberalized). These countries represent the three outliers in the data set. The right-hand panel shows the effect of removing these outliers: the negative relationship virtually vanishes (formally, it is statistically insignificant). This observation supports the claim that the negative relationship cannot be accepted until it has been shown to survive the consideration of all relevant covariates in a regression analysis.

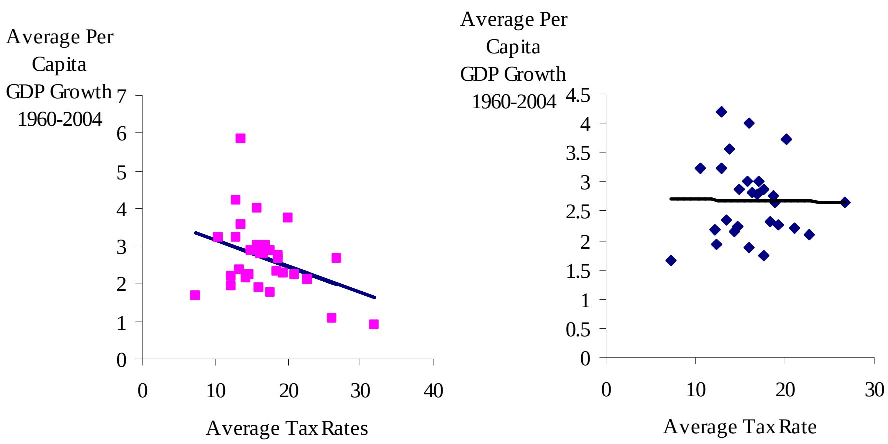

Figure 3: Real income growth and tax rates in OECD countries 1960 -- 2004
Source: Penn World Table Version 6.2

The second issue is the definition of the appropriate tax rate. The figures above use a measure of the average rate but economic theory argues that it is the marginal tax rate that matters for the degree of distortion introduced into choices. Using an average rate of tax to explain growth does not capture this important feature of taxation. This issue was addressed by Koester and Kormendi (1989) who used IMF data on 63 countries to construct measures of the average tax rate and the marginal tax rate. Their results are reported in Table 1. There is little evidence of an effect of either the average or the marginal tax rate upon the growth rate, but the marginal tax rate is claimed to have an effect on the level of activity. The tax rates are significant when used as the sole regressor but become insignificant when the level of initial GDP is included in the regression.

Variable	*A*	*B*	*C*	*D*
Constant	0.060	0.053	0.058	0.060
Average tax	-0.074 (-2.18)		-0.005 (-0.11)	
Marginal tax		-0.25 (-1.87)		-0.011 (-087)
Initial GDP			-0.052 (-2.65)	-0.048 (-3.03)
R^2	0.072	0.05	0.17	0.18

Table 1: Regressions on marginal and average tax rates
Source: Koester and Kormendi (1989)

This methodology is also open to criticism since it assumes a constant marginal rate of tax despite significant changes in the tax systems in several of the countries over the period of the data set. There is also an issue concerning aggregation bias since the industrialized and non-industrialized countries may have very different responses of growth to taxation. Easterly and Rebelo (1993) employ several different measures of the marginal rate of tax plus a range of other potential determinants of growth (initial income, school enrolments, assassinations, revolutions and war casualties). They conclude "The evidence that tax rates matter for economic growth is disturbingly fragile".

The reasons for why no strong relationship is evident in the data are explored in Slemrod (1995). Figure 4 provides an updated view of the data used by Slemrod. The figure plots growth in per capita GDP against government expenditure as a proportion of GDP for 78 countries in 2004 using data from the Penn World Tables. As Slemrod observed there is no discernible pattern in this data. If there were a strong link between government and growth it should be evident in the figure.

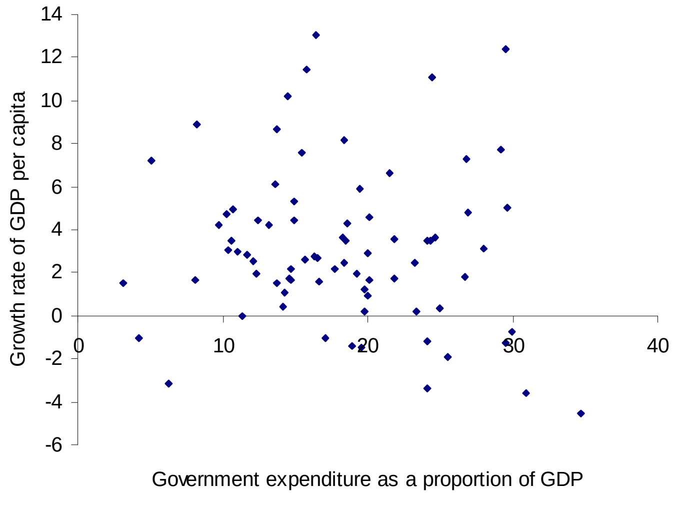

Figure 4: Growth and government expenditure 2004.

Source: Penn World Tables Version 6.2

The main argument of Slemrod (1995) is that the data is generated by the interaction of two structural relationships. On the one hand, an increase in government expenditure results in higher taxes implying further distortions that reduce the growth of GDP. On the other hand, growth in GDP affects the demand for government expenditure (for example, demand will increase if Wagner's law applies). The estimation methodology has not adequately resolved the simultaneity between these two relationships and therefore the estimated coefficients do not represent the underlying structural equations. Moreover, if the level of government expenditure is chosen to maximize the rate of growth then the data should exhibit little variation: if the countries are similar the observations represent points clustered around the maximum of the relationship. If there are any differences in the relationship between expenditure and growth across countries then, combined with an underlying optimization process, this will make for an even less meaningful relationship in the data.

There is much evidence in favour of the argument that the data reviewed above demonstrates no significant effect of the tax rate upon the rate of growth. The remainder of the paper is devoted to exploring the extent to which the observation of no link between taxation and growth in cross-country data can be consistent with a world in which taxation does affect economic growth. There are two components to the ideas we explore. First, we adopt the idea from endogenous growth theory that public sector expenditures are productive. In particular, we consider expenditure to be made on infrastructure that increases economic output. Therefore, taxation is not just a cause of distortion but supports expenditure that contribute to output. It can be expected that governments will exercise rationality in the choice of the tax rate. This is a formalization of the argument that the data should represent countries clustered around an optimum. Second, we explore the existence of a mechanism through which growth is endogenously equalized between countries, or at least the differences are reduced. In our modelling this occurs in two ways: spill-overs of the benefits of public infrastructure between countries and the mobility of capital. The consequence of these mechanisms is that an increase in taxation in one country can (in certain circumstances) raise growth in all countries; hence, cross-country comparisons taken at one point in time will reveal no systematic relationship. These comments agree with a general perspective that what should be explained is not the differences in the growth rates between countries but instead why growth rates have been so similar over such large time spans.

3. Public Infrastructure

Endogenous growth can occur when capital and labour are augmented by additional inputs in a production function that otherwise has non-increasing returns to scale. One interesting case for understanding the link between government policy and growth is when the additional input is a public good or public infrastructure financed by taxation. The need for public infrastructure to support private capital in production provides a positive role for public expenditure and a direct mechanism through which policy can affect growth. Introducing infrastructure permits an analysis of the optimal level of public expenditure in an endogenous growth model.

This section first reviews the Barro (1990) model of productive public expenditure. In this model public expenditure is financed by a tax on output. We then introduce the approach we adopt to analyze externalities by re-phrasing the analysis as a comparison across balanced growth paths with a tax upon the private capital input.

Public infrastructure can be introduced by assuming that the production function for the representative firm at time t takes the form

$$Y_t = AL_t^{1-\alpha} K_t^{\alpha} G_t^{1-\alpha}, \tag{1}$$

where A is a positive constant and G_t is the quantity of public infrastructure. The form of this production function ensures that there are constant returns to scale in labour, L_t, and private capital, K_t, for the firm given a fixed level of public infrastructure. Although returns are decreasing to private capital as the level of capital is increased for fixed levels of labour and public input, there are constant returns to scale in public input and private capital together. For a fixed level of L_t, this property of constant returns to scale in the other two inputs permits endogenous growth to occur.

The analysis of Barro (1990) assumes that public infrastructure is financed by a tax upon output. Assuming that capital does not depreciate, the profit level of the firm is

$$\pi_t = [1-\tau] AL_t^{1-\alpha} K_t^{\alpha} G_t^{1-\alpha} - r_t K_t - w_t L_t, \tag{2}$$

where r_t is the interest rate, w_t the wage rate, and τ the tax rate. The government budget constraint requires that tax revenue finances the public infrastructure, so

$$G_t = \tau Y_t. \tag{3}$$

Now assume that labour supply is constant at $L_t = 1$ for all t and that the economy's representative consumer has preferences described by the utility function

$$U = \sum_{t=1}^{\infty} \beta^t \frac{C_t^{1-\sigma} - 1}{1-\sigma}. \tag{4}$$

This specific form of utility is adopted to permit an explicit solution for the growth path. The optimality condition for intertemporal choice is

$$\frac{\partial U / \partial C_t}{\partial U / \partial C_{t+1}} = \frac{C_t^{-\sigma}}{\beta C_{t+1}^{-\sigma}} = 1 + r_{t+1}, \tag{5}$$

which can be combined with the input choices of the firm to show that the growth rate of consumption is related to the tax rate by

$$\frac{C_{t+1} - C_t}{C_t} = \beta^{1/\sigma} \left[1 + [1-\tau] \alpha A^{1/\alpha} \tau^{[1-\alpha]/\alpha} \right]^{1/\sigma} - 1. \tag{6}$$

The result in (6) demonstrates the two channels through which the tax rate affects the growth rate of consumption. Taxation reduces the growth rate of consumption through the term $1-\tau$ which represents the effect on the marginal return of capital reducing the amount of capital used. The tax rate increases growth through the term $\tau^{[1-\alpha]/\alpha}$ which represents the gains through the provision of the public input.

Further insight into these effects can be obtained by plotting the relationship between the tax rate and consumption growth. This is shown in Figure 5 under the assumption that $A = 1$, $\alpha = 0.5$, $\beta = 0.95$ and $\sigma = 0.5$. The figure displays two notable features. First, for low levels of the public input the rate of growth is negative, so a positive tax rate is required for there to be consumption growth. Second, the relationship between growth and the tax rate is non-monotonic: there is a tax rate which maximizes the growth rate of consumption.

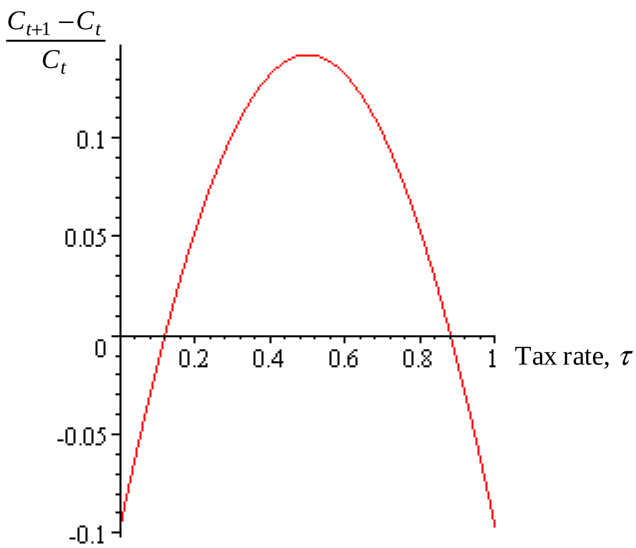

Figure 5: Tax Rate and Consumption Growth

Differentiation of (6) determines the tax rate that maximizes consumption growth as

$$\tau = 1 - \alpha . \quad (7)$$

For the values in the figure, this optimal tax rate is τ = 0.5. To see what this tax rate implies, observe that

$$\frac{\partial Y_t}{\partial G_t} = [1 - \alpha]\frac{Y_t}{G_t} = 1 , \quad (8)$$

using $G_t = \tau Y_t$ and $\tau = 1 - \alpha$. Hence, the tax rate that maximizes consumption growth ensures that the marginal product of the public input is equal to 1 which is also its marginal cost.

The analysis of growth described above works successfully for this particular form of the model. However, it is difficult to generalize the approach to more complex settings in a way that permits explicit results to be derived. As a consequence we adopt a different approach in the modelling that follows. The basis of this approach is that instead of looking at the growth path from an arbitrary starting point we instead focus on balanced growth paths. Along a balanced growth path all real variables grow at the same rate, so it can be interpreted as describing the process of growth in the long-run.

We model the consumer as choosing a balanced growth path given the path of tax rates announced by the government. The government then chooses the path of tax rates to maximize consumer welfare. If the tax is distortionary the resulting growth rate will not be first-best optimal. We see this analysis as the dynamic equivalent of maximizing welfare in a standard static Diamond-Mirrlees type framework. In characterizing the equilibrium we exploit two equivalences. The first equivalence is that between the market equilibrium and the outcome when the consumer chooses the path of capital directly. This is a standard result that has been widely exploited to simplify the derivation of the path of capital accumulation in growth models. The second is that, in the long-run, the outcome with the consumer choosing the path for capital is equivalent to the consumer directly choosing the rate of growth of the capital stock on the balanced growth path. The equivalence holds provided the economy always tends to a balanced growth path, a property that we assume is applies to the economies we study.

We now make two changes to the specification of the Barro model to demonstrate this methodology. First, we assume that government spending is funded from a tax levied on the private capital input. Second, we assume that public infrastructure and private capital depreciate in use at rates $\delta_G \geq 0$ and $\delta_K \geq 0$ respectively. Under these assumptions the budget constraint of the government, or the law of motion for public infrastructure, at time *t* is

$$G_t = [1-\delta_G]G_{t-1} + \tau_t K_t . \tag{9}$$

The firm belongs to a representative infinitely-lived household whose preferences are from this point described by an instantaneous utility function, $U_t = \ln C_t$. The household maximizes the infinite discounted stream of utility

$$\max U = \sum_{t=0}^{\infty} \beta^t \ln C_t , \tag{10}$$

subject to the sequence of intertemporal budget constraints,

$$Y_t = C_t + K_{t+1} - [1-\delta_K - \tau_t]K_t , \tag{11}$$

and with the sequence of taxes and government infrastructure taken as given.

We focus on balanced growth path equilibria, along which the tax rate is constant and all real variables grow at the same constant rate. The first step is to show that when K_t grows at a constant rate γ and taxes are constant the law of motion for G_t also converges to growth at the same constant rate. Assume that the private capital stock grows at rate γ from time 0 with capital stock K_0. Recursive substitution into (9) gives

$$\begin{aligned} G_{t+1} &= [1-\delta_G]G_t + \tau K_{t+1} \\ &= [1-\delta_G]^{t+1} G_0 + \sum_{i=0}^{t} [1-\delta_G]^i \tau K_{t+1-i} . \end{aligned} \tag{12}$$

From the relation

$$K_{t+1-i} = [1+\gamma]^{t+1-i} K_0 , \tag{13}$$

it follows that

$$\begin{aligned} \sum_{i=0}^{t} [1-\delta_G]^i \tau K_{t+1-i} &= \tau K_0 [1+\gamma]^{t+1} \sum_{i=0}^{t} \left[\frac{1-\delta_G}{1+\gamma}\right] \\ &= \tau K_0 \frac{1+\gamma}{\gamma+\delta_G} \left[[1+\gamma]^{t+1} - [1-\delta_G]^{t+1} \right]. \end{aligned} \tag{14}$$

Hence,

$$G_{t+1} = [1-\delta_G]^{t+1} G_0 + \tau K_0 \frac{1+\gamma}{\gamma+\delta_G}\left[[1+\gamma]^{t+1} - [1-\delta_G]^{t+1}\right]$$

$$= [1-\delta_G]^{t+1}\left[G_0 - \tau K_0 \frac{1+\gamma}{\gamma+\delta_G}\right] + \tau K_{t+1}\frac{1+\gamma}{\gamma+\delta_G}. \quad (15)$$

From (15) it can be seen that the effect of the initial levels disappears with time, and for t large enough

$$\frac{G_t}{K_t} \cong \tau \frac{1+\gamma}{\gamma+\delta_G}. \quad (16)$$

In particular, this result is consistent with the static balanced budget constraint $G_t = \tau K_t$ if $\delta_G = 1$. When the economy is on the balanced growth path at time 0 it must be the case that

$$G_0 = \tau K_0 \frac{1+\gamma}{\gamma+\delta_G}. \quad (17)$$

The level of consumption at time t if the balanced growth path is achieved with capital K_0 is

$$C_t = [1+\gamma]^t \left[A K_0^{\alpha} G_0^{1-\alpha} - K_0[\gamma+\delta_K+\tau]\right]. \quad (18)$$

This gives the objective of the household as

$$\max_{\{\gamma\}} \sum_{t=0}^{\infty} \beta^t \ln\left([1+\gamma]^t \left[A K_0^{\alpha} G_0^{1-\alpha} - K_0[\gamma+\delta_K+\tau]\right]\right). \quad (19)$$

The household takes government actions as given when optimizing so the values of τ and G_0 are treated as fixed in the choice of the balanced growth path. The objective function can be rearranged as

$$\max_{\{\gamma\}} \sum_{t=0}^{\infty} \beta^t \ln\left([1+\gamma]^t\right) + \sum_{t=0}^{\infty} \beta^t \ln\left(A K_0^{\alpha} G_0^{1-\alpha} - K_0[\gamma+\delta_K+\tau]\right)$$

$$= \frac{\beta}{[1-\beta]^2}\ln(1+\gamma) + \frac{1}{[1-\beta]}\ln\left(A K_0^{\alpha} G_0^{1-\alpha} - K_0[\gamma+\delta_K+\tau]\right). \quad (20)$$

Assuming an interior solution exists, the necessary condition for the choice of γ is

$$\frac{\beta}{[1+\gamma][1-\beta]} - \frac{K_0}{A K_0^{\alpha} G_0^{1-\alpha} - K_0[\gamma+\delta_K+\tau]} = 0. \quad (21)$$

Solving this gives

$$1+\gamma = \beta\left[A\left[\frac{G_0}{K_0}\right]^{1-\alpha} + 1 - \delta_K - \tau\right]. \quad (22)$$

The expression in (22) determines the balanced growth path chosen by the household in response to the tax rate and level of public infrastructure selected by the government. This equation summarizes the behaviour of the private sector in the model.

There are two alternatives ways of modelling the choice problem of the government. Either the government can choose τ to maximize γ, or it can choose τ to maximize U taking into account the effect of τ on γ. In both cases the chosen value of τ and the level of government infrastructure are related by (22). It is now shown that these two options are, in fact, equivalent. Rearrangement of the objective function with further substitution from (22) gives

$$U = \frac{\beta}{[1-\beta]^2}\ln(1+\gamma)+\frac{1}{[1-\beta]}\left[\ln(K_0)+\ln\left(A\left[\frac{G_0}{K_0}\right]^{1-\alpha}-\gamma-\delta_K-\tau\right)\right]$$

$$= \frac{1}{[1-\beta]^2}\ln(1+\gamma)+\frac{1}{[1-\beta]}\left[\ln(K_0)+\ln\left(\frac{1-\beta}{\beta}\right)\right]. \quad (23)$$

Since only the first term depends upon policy instruments and is itself an increasing function of γ, it follows that maximizing γ is equivalent to maximizing U.

Using (22) and (17) the growth-maximizing tax rate solves

$$A[1-\alpha]\left[\beta\left[A\frac{\alpha}{1-\alpha}\tau+1-\delta_K\right]\right]^{1-\alpha} = \tau^{\alpha}\left[\beta\left[A\frac{\alpha}{1-\alpha}\tau+1-\delta_K\right]-1+\delta_G\right]^{1-\alpha}. \quad (24)$$

One special case can be explicitly solved. With full depreciation of private and public capital,

$$\tau = [A[1-\alpha]]^{1/\alpha}, \quad (25)$$

so the optimal tax rate increases as the elasticity of output with respect to public infrastructure increases. For the general result it can be seen that the optimal tax rate is decreasing in δ_G, increasing in δ_K, and decreasing in β. Hence, the more patient is the household the lower should be the optimal tax rate supporting public infrastructure.

This model of public infrastructure illustrates a sense in which there can be an optimal level of government expenditure in an endogenous growth model. The analysis shows how a study of the optimal tax rate can be undertaken by considering choice over different balanced growth paths. We now develop this technique in the context of a world economy with infrastructural externalities between countries.

4. Infrastructural Spill-Over

This section analyzes a model that incorporates infrastructural spill-overs between countries. The results show that uncoordinated optimization by countries will lead to under-investment in infrastructure. This provides a role for a supra-national body to coordinate the decisions of individual countries so as to secure an increase in the growth rate. We develop these results by retaining the focus upon the comparison of balanced growth paths.

The model of the previous section is extended to a multi-country setting in which production benefits from positive externalities created by global infrastructure. For country i at time t the level of output is given by

$$Y_{it} = AK_{it}^{\alpha}\left[G_{it}^{1-\rho}\Gamma_t^{\rho}\right]^{1-\alpha}. \tag{26}$$

The measure of global infrastructure at time t, Γ_t , is defined as the total public investment in infrastructure, $\Gamma_t = G_{it} + \overline{G}_{it}$, where $\overline{G}_{it}$ is the public investment in infrastructure in all countries other than i. The infrastructural externality is generated by the term $\overline{G}_{it}$ appearing here. The interpretation is that both infrastructure within a country (the term involving G_{it} in (26)) and the total level of infrastructure (the term involving Γ_t) are relevant.

We focus on balanced growth paths along which all real variables in all countries grow at the same rate and the tax rates are constant over time. The equality of the growth rates across countries here is imposed, since the law of motion of the public capital in one country only ensures that the growth rates of the stock of public and private capital are equal in that country, but there is no reason of why the growth rates should be equal across countries. If we did not impose this assumption then the output of one country would eventually become arbitrarily small relative to the output of the other. An extension to the model that ensures the endogenous equalization of the growth rates is presented in the next section.

When the growth rates in the countries are equal, along the balanced growth path Γ_t grows at the same constant rate as all other real variables, so that at any time t

$$G_{it}^{1-\rho}\Gamma_t^{\rho} = G_{it}\left[\frac{\Gamma_t}{G_{it}}\right]^{\rho} = G_0[1+\gamma]^t\left[\frac{\Gamma_0}{G_0}\right]^{\rho}. \tag{27}$$

The level of consumption at time t if the balanced growth path is reached with capital stock K_0 is

$$C_{it} = [1+\gamma]^t\left[AK_0^{\alpha}\left[G_0\left[\frac{\Gamma_0}{G_0}\right]^{\rho}\right]^{1-\alpha} - K_0[\gamma+\delta_K+\tau]\right]. \tag{28}$$

This gives the objective of the household as

$$\max_{\{\gamma\}} \sum_{t=0}^{\infty}\beta^t \ln\left([1+\gamma]^t\left[AK_0^{\alpha}\left[G_0\left[\frac{\Gamma_0}{G_0}\right]^{\rho}\right]^{1-\alpha} - K_0[\gamma+\delta_K+\tau]\right]\right), \tag{29}$$

which is the same as in the closed economy case, with total factor productivity now augmented by the factor $[\Gamma_0/G_0]^{\rho[1-\alpha]}$. Therefore, we immediately obtain the expression for the chosen growth rate in the open economy with externalities,

$$1+\gamma=\beta\left[A\left[\frac{\Gamma_0}{G_0}\right]^{\rho[1-\alpha]}\left[\frac{G_0}{K_0}\right]^{[1-\alpha]}+1-\delta_K-\tau\right], \tag{30}$$

where G_0 is related to K_0 through (17), and $\Gamma_0 = G_0 + \overline{G}_0$. Using this expression we obtain for the lifetime utility function

$$U=\sum_{t=0}^{\infty}\beta^t \ln\left([1+\gamma]^t\left[AK_0^{\alpha}\left[G_0\left[\frac{\Gamma_0}{G_0}\right]^{\rho}\right]^{[1-\alpha]}-K_0[\gamma+\delta_K+\tau]\right]\right)$$

$$=\frac{1}{[1-\beta]^2}\ln(1+\gamma)+\frac{1}{[1-\beta]}\left[\ln(K_0)+\ln\left(\frac{1-\beta}{\beta}\right)\right], \tag{31}$$

as in the closed economy case. Thus, for the government maximizing the welfare of the representative household is equivalent to maximizing the growth rate.

4.1 Independent Choice

A government that does not internalize the externality chooses τ to maximize γ, taking K_0 and $\overline{G}_0$ as given. The growth-maximizing tax rate is determined implicitly by the solution to the pair of equations

$$\tau=[1-\alpha]\left[\frac{1+\gamma}{\beta}-1+\delta_K+\tau\right]\left[1-\rho\frac{\overline{G}_0/K_0}{\varphi(\gamma,\tau)+\overline{G}_0/K_0}\right], \tag{32}$$

$$\gamma=\beta\left[A\left[1+\frac{\overline{G}_0/K_0}{\varphi(\gamma,\tau)}\right]^{\rho[1-\alpha]}\varphi(\gamma,\tau)^{1-\alpha}+1-\delta_K-\tau\right]-1, \tag{33}$$

where $\varphi(\gamma,\tau)=\tau\dfrac{1+\gamma}{\gamma+\delta_G}$.

From this point we assume that the world consists of two countries. Along the balanced growth path all real variables in both countries grow at the same constant rate, γ. The governments do not coordinate their choice of taxes, so each government maximizes only the welfare of its representative household. We assume that the discount rates and depreciation rates are equal across the countries. Furthermore, we assume that the technologies and the endowments in two countries are such that both countries have the same rate of growth. The latter assumption only holds if the technologies and the endowments in two countries satisfy a certain relationship. This relationship is met if we assume symmetry between the countries

In Figure 6 a symmetric equilibrium without coordination between countries is illustrated. Denote the technology levels in the two countries by A and $\overline{A}$, and the initial capital stocks when the balanced growth path is achieved by K_0 and $\overline{K}_0$. The values of the parameters used for the simulation are: β = 0.9, ρ = 0.5, α = 0.5, $\delta_K=\delta_G=0.2$, $A=\overline{A}=0.5$, $K_0=\overline{K}_0=2$. The solid curve depicts the utility-maximizing growth rate (the optimal choice of the private sector) given the tax rate and level of public infrastructure and is, therefore, determined by the fundamentals of the economy -- endowments, preferences, and the production technology. The dashed curve describes the optimal choice of the tax rate by the government. The equilibrium occurs at the intersection of these two curves. One can see that the equilibrium tax rate is too low: with a higher tax rate higher growth, and, hence, higher welfare can be achieved. This is a consequence of the externality created by the infrastructural spill-over.

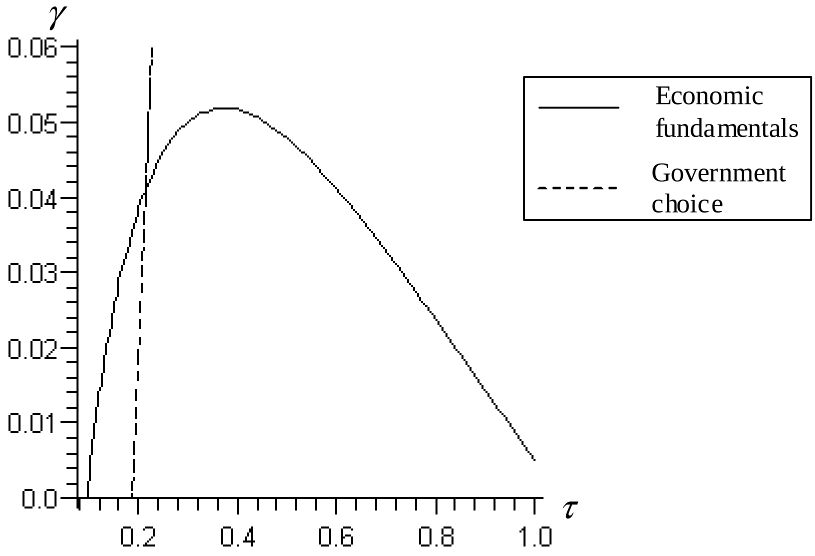

Figure 6: Optimal tax rate without coordination
$\tau = \bar{\tau} = 0.21,\ \gamma = \bar{\gamma} = 0.042$.

4.2 Coordination between Two Countries

We showed in the previous section that without coordination the equilibrium growth rate is below the efficient level: when choosing the tax rate, and hence when choosing the level of public investment, each government ignores the positive externality of the investment in its own country upon the productivity and growth in the other country. In this section we show that the efficient rate of growth can be achieved via coordination.

Assume that the two governments coordinate their policies by choosing simultaneously their tax rates to maximize the sum of the welfare of the representative households. This optimization can be written as the following:

$$\max_{\{\tau,\bar{\tau}\}} U(\gamma) + U(\bar{\gamma}) \text{ s.t. } \gamma = \bar{\gamma}. \tag{34}$$

where γ and $\bar{\gamma}$ are implicitly defined by

$$\gamma = \beta\left[A\left[1 + \frac{\bar{\tau}\bar{K}_0}{\tau K_0}\right]^{\rho[1-\alpha]}\left[\tau\frac{1+\gamma}{\gamma+\delta_G}\right]^{[1-\alpha]} + 1 - \delta_K - \tau\right] - 1, \tag{35}$$

$$\bar{\gamma} = \beta\left[\bar{A}\left[1+\frac{\tau K_0}{\bar{\tau}\bar{K}_0}\right]^{\rho[1-\alpha]}\left[\bar{\tau}\frac{1+\bar{\gamma}}{\bar{\gamma}+\delta_G}\right]^{[1-\alpha]}+1-\delta_K-\bar{\tau}\right]-1. \quad (36)$$

The optimization can be stated equivalently as $\max_{\{\tau,\bar{\tau}\}}\gamma$, with $\gamma=\bar{\gamma}$. When the two countries are identical the necessary condition for τ simplifies to

$$\tau = A2^{\rho[1-\alpha]}[1-\alpha]\left[\tau\frac{1+\gamma}{\gamma+\delta_G}\right]^{[1-\alpha]}. \quad (37)$$

Plotting (37) along with the equation for γ produces Figure 7, which uses the same values for the model parameters as for Figure 6. It can be seen that the coordinated tax rate is higher than in the case without coordination, and the growth rate is the highest that can be achieved in the economy with the given fundamentals. As expected, coordination achieves the efficient outcome.

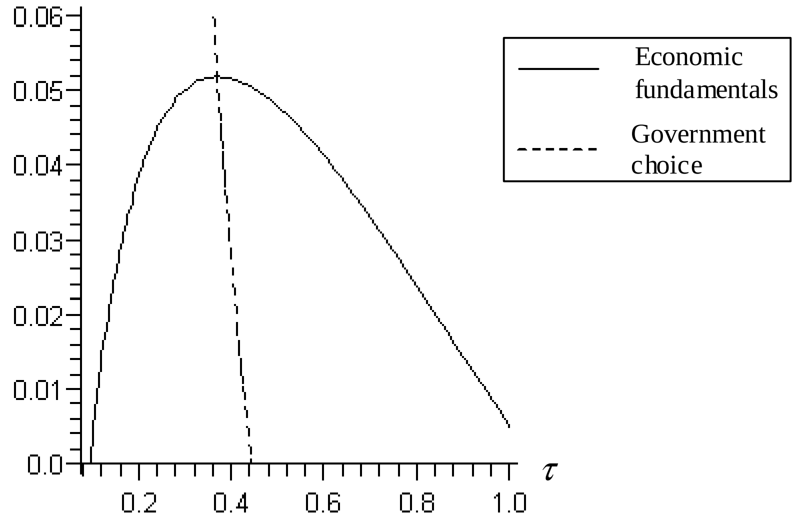

Figure 7: Coordinated choice of optimal tax rate
$\tau=\bar{\tau}=0.37$, $\gamma=\bar{\gamma}=0.052$.

4.3 Redistribution

In this section we consider the possibility of intervention by a supra-national body that collects and redistributes tax revenues between the countries. The interaction between the central body and the national government is modelled as the following multi-stage game. At the first stage the supra-national body announces what share of the tax revenues will be collected from each national government for a centralized fund. At the second stage the national governments choose optimal tax rates. At the third stage the supra-national body announces how the centralized fund will be divided between the two countries. Finally, the investments are made and production takes place. There is no coordination between the two national governments at any stage.

We assume that a fraction θ of the tax revenue is collected by the supra-national body from the first country (and a fraction $\bar{\theta}$ from the second country), and a fraction μ (1 – μ for the second country) of the total amount collected is returned to the national government. Thus, the law of motion of the public capital is

$$G_{t+1} = [1-\delta_G]G_t + [1-\theta]\tau_{t+1}K_{t+1} + \mu\Omega_{t+1}, \tag{38}$$

where $\Omega_{t+1} = \theta\tau_{t+1}K_{t+1} + \bar{\theta}\bar{\tau}_{t+1}\bar{K}_{t+1}$. Thus, if the balanced growth path is achieved from $t = 0$

$$G_0 = \frac{1+\gamma}{\gamma+\delta_G}\left[[1-\theta+\theta\mu]\tau K_0 + \bar{\theta}\mu\bar{\tau}\bar{K}_0\right]. \tag{39}$$

Since the households take the public capital investment into production as given, the optimization problem for each household is the same as in the case without redistribution solved in the previous section. Thus, the welfare-maximizing growth rate is determined by

$$\gamma = \beta\left[A\left[\frac{\Gamma_0}{G_0}\right]^{\rho[1-\alpha]}\left[\frac{G_0}{K_0}\right]^{[1-\alpha]} + 1 - \delta_K - \tau\right] - 1, \tag{40}$$

where now G_0 is defined by (39), and

$$\Gamma_0 = G_0 + \bar{G}_0 = \tau K_0 + \bar{\tau}\bar{K}_0\,. \tag{41}$$

We assume that the redistribution is costless, so that the total public investment is not changed.

Maximization of welfare by the government leads to the implicit solution for the optimal tax rate,

$$\tau = [1-\alpha]\left[\frac{1+\gamma}{\beta} - 1 + \delta_K + \tau\right]\left[1 - \rho\frac{\bar{\tau}\bar{K}_0}{\tau K_0 + \bar{\tau}\bar{K}_0}\right] - \bar{\tau}\frac{\bar{\theta}\mu}{1-\theta+\theta\mu}\frac{\bar{K}_0}{K_0}\,. \tag{42}$$

Similarly, the second country's optimal tax rate is

$$\bar{\tau} = [1-\alpha]\left[\frac{1+\gamma}{\beta} - 1 + \delta_K + \tau\right]\left[1 - \rho\frac{\tau K_0}{\tau K_0 + \bar{\tau}\bar{K}_0}\right] - \tau\frac{\theta[1-\mu]}{1-\bar{\theta}+\bar{\theta}[1-\mu]}\frac{K_0}{\bar{K}_0}\,. \tag{43}$$

It can be seen from (42) and (43) that by correctly choosing $(\theta, \bar{\theta}, \mu)$ it is possible to induce the efficient levels of the national tax rates when there is no cooperation between the governments.

This is illustrated in Figure 8. A comparison with the no coordination case shows that the redistribution results in a shift of the curve describing the optimal choice of the government, and this shift can be adjusted so that the resulting equilibrium is the one with the highest growth rate, *i.e.* the same outcome as with coordination. Note that to achieve this θ and $\bar{\theta}$ must be negative: the central body announces subsidies to public investment in infrastructure. Such a policy is typical for correction of inefficiency in the presence of positive externalities.

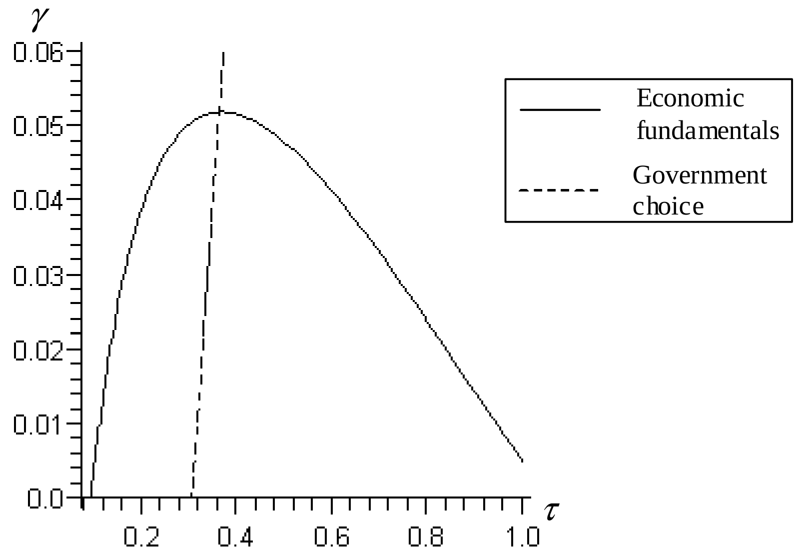

Figure 8: Equilibrium with redistribution
$\theta = \bar{\theta} = -0.65$, $\mu = 0.5$.

5. Capital Mobility

A limitation of the analysis in the previous section was the assumption that both countries would be on the same balanced growth path. This assumption was required because the model lacked a mechanism that would guarantee the equalization of growth rates. Any asymmetry between the countries would imply that the GDP per capita of one country would eventually become insignificant relative to the other. This is unsatisfactory since growth paths are similar in practice so the model needs an endogenous mechanism that ensures the equality of the growth rates.

The process we use to ensure the equality of balanced growth paths is to allow capital to be mobile between countries. As the investors relocate capital to seek the highest after-tax return the outcome will ensure that the growth rates in the two countries are the same. If the growth rates are equalized then the countries impose an additional dynamic externality on each: if one country raises its tax rate it will affect the growth rates of all countries.

In this section the model is extended to allow the consumers in the two countries to relocate capital costlessly. Let k_t ($\bar{k}_t$) denote the stock of capital owned by the "domestic" ("foreign") consumer. With perfect capital mobility each consumer will choose to invest in the country where the after-tax return on investment is higher. We assume that the tax on capital is collected at the destination, and the output produced in each country is divided between the two investors proportionally to their capital investments.

Let $\lambda_t \in [0,1]$ denote the fraction of k_t invested in the home country and $1-\bar{\lambda}_t \in [0,1]$ denote the fraction of $\bar{k}_t$ invested there. Then

$$K_t = \lambda_t k_t + \left[1-\overline{\lambda}_t\right]\overline{k}_t . \tag{44}$$

Similarly, for the foreign country

$$\overline{K}_t = \left[1-\lambda_t\right]k_t + \overline{\lambda}_t \overline{k}_t . \tag{45}$$

Razin and Yuen (1997) hypothesized that capital mobility across countries ensures equalization of the growth rates along the balanced growth path, and illustrated this result in a two-country framework of endogenous growth with a human capital input. In what follows we show that a similar result holds in our model. The law of motion of the public capital in the foreign country is

$$\overline{G}_{t+1} = \left[1-\delta_G\right]\overline{G}_t + \overline{\tau}\left[\left[1-\lambda_t\right]k_{t+1} + \overline{\lambda}_t \overline{k}_{t+1}\right]. \tag{46}$$

Let the capital stock owned by domestic investors, k_t, grow at a constant rate γ, and that owned by foreign investors, $\overline{k}_t$, grows at rate $\overline{\gamma}$. Assuming the tax rate and the share of domestic investment are constant, and iterating in (46), we obtain the following:

$$\begin{aligned}\overline{G}_{t+1} &= \left[1-\delta_G\right]\left[\left[1-\delta_G\right]\overline{G}_{t-1} + \overline{\tau}\left[\left[1-\lambda\right]k_t + \overline{\lambda}\overline{k}_t\right]\right] + \overline{\tau}\left[\left[1-\lambda\right]k_{t+1} + \overline{\lambda}\overline{k}_{t+1}\right] \\ &= \left[1-\delta_G\right]^{t+1}\left[\overline{G}_0 - \overline{\tau}\left[\frac{1+\gamma}{\gamma+\delta_G}\left[1-\lambda\right]k_0 + \frac{1+\overline{\gamma}}{\overline{\gamma}+\delta_G}\overline{\lambda}\overline{k}_0\right]\right] \\ &+ \overline{\tau}\left[\frac{1+\gamma}{\gamma+\delta_G}\left[1-\lambda\right]k_0\left[1+\gamma\right]^{t+1} + \frac{1+\overline{\gamma}}{\overline{\gamma}+\delta_G}\overline{\lambda}\overline{k}_0\left[1+\overline{\gamma}\right]^{t+1}\right].\end{aligned} \tag{47}$$

Dividing both sides by $\overline{k}_{t+1}$,

$$\begin{aligned}\frac{\overline{G}_{t+1}}{\overline{k}_{t+1}} &= \left[\frac{1-\delta_G}{1+\overline{\gamma}}\right]^{t+1}\frac{1}{\overline{k}_0}\left[G_0 - \overline{\tau}\left[\frac{1+\gamma}{\gamma+\delta_G}\left[1-\lambda\right]k_0 - \frac{1+\overline{\gamma}}{\overline{\gamma}+\delta_G}\overline{\lambda}\overline{k}_0\right]\right] \\ &\quad + \overline{\tau}\left[\left[1-\lambda\right]\frac{1+\gamma}{\gamma+\delta_G}\frac{k_0}{\overline{k}_0}\left[\frac{1+\gamma}{1+\overline{\gamma}}\right]^{t+1} + \frac{1+\overline{\gamma}}{\overline{\gamma}+\delta_G}\overline{\lambda}\right].\end{aligned} \tag{48}$$

It can be seen that in the long run the first term converges to zero and the second term is constant if and only if $\gamma = \overline{\gamma}$. Thus, in the balanced growth path equilibrium the long-run growth rates for the two countries must be equal. Also, if the balanced growth path is achieved at $t = 0$ then

$$\overline{G}_0 = \overline{\tau}\frac{1+\gamma}{\gamma+\delta_G}\left[\left[1-\lambda\right]k_0 + \overline{\lambda}\overline{k}_0\right]. \tag{49}$$

Similarly, for the home country it must be the case that

$$G_0 = \tau\frac{1+\gamma}{\gamma+\delta_G}\left[\lambda k_0 + \left[1-\overline{\lambda}\right]\overline{k}_0\right]. \tag{50}$$

The allocation of capital across the countries is chosen by each consumer to maximize their utility. An interior solution for λ corresponds to equalized net returns, so

$$\frac{\alpha\lambda k_0 + [1-\bar{\lambda}]\bar{k}_0}{\lambda k_0 + [1-\bar{\lambda}]\bar{k}_0}\frac{Y_0}{K_0} - \tau = \frac{\bar{\lambda}\bar{k}_0 + \alpha[1-\lambda]k_0}{\bar{\lambda}\bar{k}_0 + \alpha[1-\lambda]k_0}\frac{\bar{Y}_0}{\bar{K}_0} - \bar{\tau}\,. \tag{51}$$

If the tax rates and the fundamentals are such that the left-hand side of (51) is greater than the right-hand side, *i.e.* the domestic net return to domestically-owned capital is higher that the foreign net return, then $\lambda = 1$, the home consumer invests all his capital at home. Conversely, if this left-hand side is less, *i.e.* the foreign net returns are higher than the domestic ones then $\lambda = 0$, and the home consumer invests all his capital abroad. The solution for the foreign consumer's optimization problem is similar. Note that in every case we consider each consumer chooses the allocation of capital between countries taking the decision of other consumer, as well as the tax rates and public capital inputs in both countries, as given.

5.1 Non-cooperative equilibrium

When the two governments do not cooperate they simultaneously choose the tax rates to maximize the welfare of their own consumers, or, equivalently, the growth rates of their economies. Consider the decision of the domestic government, assuming that the optimal allocation of capital by the domestic investor is in the interior. The government does not internalize the externality and chooses τ to maximize γ, taking k_0, $\bar{k}_0$, $\bar{\tau}$, and $\bar{G}_0$ as given. However, each government realizes that its choice of tax will affect the capital allocation decisions of investors in both countries.

Thus, the home government's objective is to maximize γ implicitly defined by

$$\gamma = -1 + \beta\Bigg[\lambda A\left[1 + \frac{G_0}{\varphi(\gamma,\tau)[\lambda k_0 + [1-\bar{\lambda}]\bar{k}_0]}\right]^{\rho[1-\alpha]}\varphi(\gamma,\tau)^{1-\alpha}$$
$$+ [1-\delta_K] - \tau\lambda - \bar{\tau}[1-\lambda]$$
$$+ [1-\lambda]\bar{A}\left[1 + \frac{\varphi(\gamma,\tau)[\lambda k_0 + [1-\bar{\lambda}]\bar{k}_0]}{\bar{G}_0}\right]^{\rho[1-\alpha]}\left[\frac{\bar{G}_0}{\bar{\lambda}\bar{k}_0 + [1-\lambda]k_0}\right]^{1-\alpha}\Bigg], \tag{52}$$

where $\varphi(\gamma,\tau) \equiv \tau\dfrac{1+\gamma}{\gamma+\delta_G}$, and λ and $\bar{\lambda}$ solve (51) and the counterpart of the latter for the foreign investor.

There is no need to impose the equality of the growth rates: in equilibrium this is ensured by the optimal choice of capital allocation by the consumers. However, the equilibrium growth rate, in general, is not efficient because the externalities are not internalized even with mobile capital.

In Figure 9 the completely symmetric equilibrium with mobile capital when the governments do not coordinate their tax choice is illustrated for the same values of the model parameters as in the previous numerical examples. One can see that the equilibrium tax rates are even lower than in the no-coordination case with immobile capital, because of the tax competition effect: each government cuts their tax rate in order to attract capital from abroad. Thus, capital mobility in the absence of coordination exacerbates the inefficiency caused by the externality.

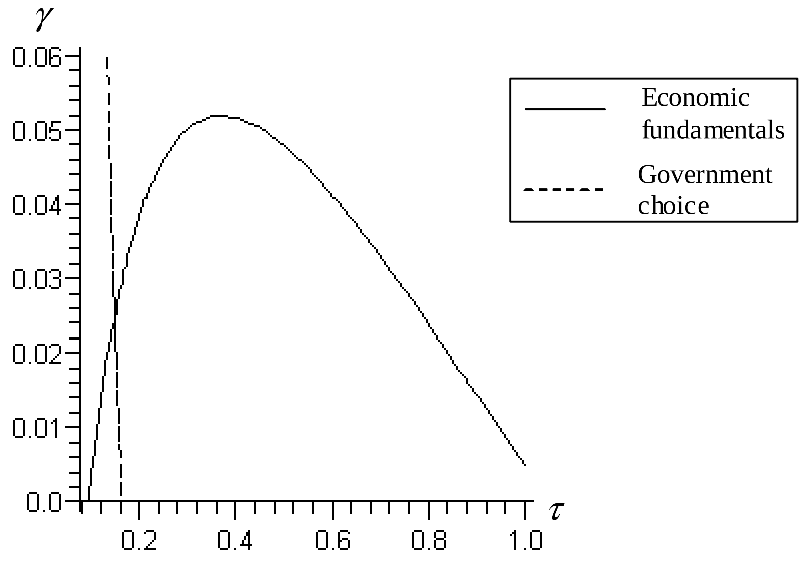

Figure 9: Optimal tax rate without coordination, with mobile capital
$\tau = \bar{\tau} = 0.1487$, $\gamma = \bar{\gamma} = 0.0259$.

5.2 Cooperative equilibrium

If the two governments cooperate in choosing taxes the equilibrium outcome can be improved. The efficient equilibrium growth rate is achieved when the externalities are internalized, *i.e.* the two governments choose their tax rates simultaneously to maximize the sum of the two welfare functions.

This is equivalent to solving

$$\max_{\{\tau,\bar{\tau}\}} \gamma, \tag{53}$$

where γ is implicitly defined by

$$\begin{aligned}\gamma = \beta\Bigg[& \lambda A\left[1+\frac{\bar{\tau}\left[\bar{\lambda}\bar{k}_0+[1-\lambda]k_0\right]}{\tau\left[\lambda k_0+\left[1-\bar{\lambda}\right]\bar{k}_0\right]}\right]^{\rho[1-\alpha]}\left[\tau\frac{1+\gamma}{\gamma+\delta_G}\right]^{1-\alpha} \\ & +[1-\delta_K]-\tau\lambda-\bar{\tau}[1-\lambda] \\ & +[1-\lambda]\bar{A}\left[1+\frac{\tau\left[\lambda k_0+\left[1-\bar{\lambda}\right]\bar{k}_0\right]}{\bar{\tau}\left[\bar{\lambda}\bar{k}_0+[1-\lambda]k_0\right]}\right]^{\rho[1-\alpha]}\left[\bar{\tau}\frac{1+\gamma}{\gamma+\delta_G}\right]^{1-\alpha}\Bigg]+1,\end{aligned} \tag{54}$$

5.3 Non-cooperative equilibrium with redistribution

As in the case of immobile capital considered in the previous section, a policy of intervention by a supra-national government can achieve the first-best outcome through correction of the externality. The intervention is again modelled as a multi-stage game. Since the households take the public capital investment into production as given, the optimization problem for the household is the same as in the case without redistribution. We assume that the central body runs a balanced budget.

Under the redistribution scheme along the balanced growth path

$$G_0 = \frac{1+\gamma}{\gamma+\delta_G}\left[\left[1-\theta+\theta\mu\right]\tau K_0 + \bar{\theta}\mu\bar{\tau}\bar{K}_0\right], \tag{55}$$

$$\bar{G}_0 = \frac{1+\gamma}{\gamma+\delta_G}\left[\left[1-\bar{\theta}+\theta\left[1-\mu\right]\right]\tau K_0 + \theta\left[1-\mu\right]\bar{\tau}\bar{K}_0\right], \tag{56}$$

where

$$K_0 = \lambda k_0 + \left[1-\bar{\lambda}\right]\bar{k}_0, \tag{57}$$

and

$$\bar{K}_0 = \bar{\lambda}\bar{k}_0 + \left[1-\lambda\right]k_0. \tag{58}$$

The domestic government chooses τ to maximize γ implicitly defined by

$$\gamma = -1+\beta\left[\lambda A\left[1+\frac{\bar{G}_0}{\tilde{\varphi}(\gamma,\tau)\left[\lambda k_0+\left[1-\bar{\lambda}\right]\bar{k}_0\right]}\right]^{\rho[1-\alpha]}\tilde{\varphi}(\gamma,\tau)^{1-\alpha}\right.$$
$$+\left[1-\delta_K\right]-\tau\lambda-\bar{\tau}\left[1-\lambda\right]$$
$$\left.+\left[1-\lambda\right]\bar{A}\left[1+\frac{\tilde{\varphi}(\gamma,\tau)\left[\lambda k_0+\left[1-\bar{\lambda}\right]\bar{k}_0\right]}{\bar{G}_0}\right]^{\rho[1-\alpha]}\left[\frac{\bar{G}_0}{\bar{\lambda}\bar{k}_0+\left[1-\lambda\right]k_0}\right]^{1-\alpha}\right]. \tag{59}$$

where

$$\tilde{\varphi}(\gamma,\tau) = \frac{1+\gamma}{\gamma+\delta_G}\left[\left[1-\theta+\theta\mu\right]\tau+\bar{\theta}\mu\bar{\tau}\frac{\bar{K}_0}{K_0}\right]. \tag{60}$$

The foreign government faces a similar optimization.

By choosing $\{\theta,\bar{\theta},\mu\}$ the supra-national body can shift the reaction functions of the two governments so that the equilibrium choice of taxes coincides with the first-best outcome. There is no closed-form solution, but we can characterize it qualitatively. Since the externality from public investment is positive, in the non-cooperative equilibrium the taxes will be set too low. In addition, capital mobility will cause tax competition between the governments, which will result in even lower taxes. To achieve efficiency the supra-national body needs to encourage higher taxes, which requires θ and $\bar{\theta}$ to be negative. Once the supra-national body announces that it will subsidize public investment in each country, the governments set higher taxes. After that the subsidies are claimed back. Figure 10 demonstrates that the maximum growth rate can be achieved by this process.

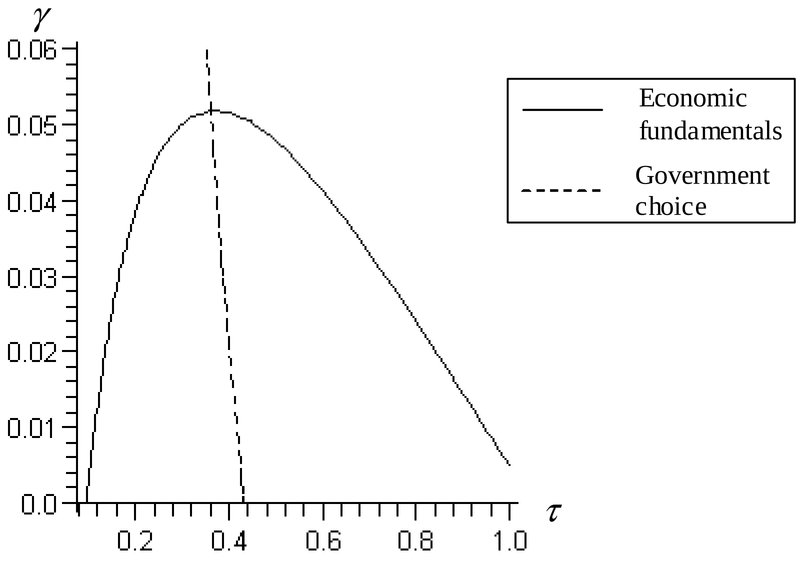

Figure 10: Effect of central body, with mobile capital
$\tau = \bar{\tau} = 0.361$, $\theta = \bar{\theta} = -1.27$.

6. Conclusions

There is no convincing empirical evidence of a relationship between taxation and economic growth in cross-country data. We have argued that the lack of a relationship is not inconsistent with the growth rate being increased by additional public sector expenditure. The explanation offered to resolve this apparent contradiction is that public sector expenditure is productive, that there are spill-overs of the benefits of public infrastructure between countries, and that capital is internationally mobile. These mechanisms make it possible for an increase in taxation in one country to raise the growth rate in all countries. This effect will not be apparent in cross-country comparisons taken at one point.

This argument is reflected in the summary of results presented in Table 2. This reports the growth rate for the two models of immobile capital and mobile capital for a range of values of the fraction θ of the tax revenue collected by the supra-national body. The first observation is that with no intervention (θ = 0) the growth rate is below the maximum level. With immobile capital this reflects the inefficiency resulting from the spill-over of infrastructure. The growth rate with no intervention is even lower when capital is mobile because this creates an additional externality. The rate of growth is increased when θ becomes negative since this provides an incentive for the countries to increase their tax rates.

θ	-1.4	-1.2	-1.0	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4
Immobile	0.039	0.045	0.049	0.051	0.052	0.050	0.047	0.042	0.034	0.025
Mobile	0.052	0.052	0.051	0.048	0.045	0.042	0.034	0.026	0.017	0.006

Table 2: Growth rate and intervention

The policy implications of our analysis are that although public expenditure can assist growth there is no guarantee that the optimal level of growth will be achieved. The design of public expenditure has to take into account the infrastructure externalities and the capital flows. If the choices of individual countries are not coordinated then the outcome will be inefficient and growth will not be welfare-maximizing. A coordinating body, such as the European Commission, has a role to play in attaining an efficient level of expenditure on public infrastructure. This role involves supporting the expenditure decisions of individual countries to raise the overall level of expenditure.

Appendix

The Appendix provides the derivation of the results used in the main text.

A4.1 Independent Choice

A government that does not internalize the externality chooses τ to maximize γ, taking K_0 and $\overline{G}_0$ as given. The resulting value of γ is given by

$$\gamma = \beta\left[A\left[1+\frac{\overline{G}_0 / K_0}{\varphi(\gamma,\tau)}\right]^{\rho[1-\alpha]}\left[\varphi(\gamma,\tau)^{1-\alpha}+1-\delta_K-\tau\right]\right]-1, \quad \text{(A1)}$$

where $\varphi(\gamma,\tau)=\tau\frac{1+\gamma}{\gamma+\delta_G}$. Total differentiation gives

$$\frac{1}{\beta}d\gamma = A\left[1+\frac{\overline{G}_0 / K_0}{\varphi(\gamma,\tau)}\right]^{\rho[1-\alpha]}\varphi(\gamma,\tau)^{1-\alpha}$$

$$\times\left[-\frac{\rho[1-\alpha]}{1+\frac{\overline{G}_0 / K_0}{\varphi(\gamma,\tau)}}\frac{\overline{G}_0}{K_0}\frac{d\varphi}{\varphi^2}+[1-\alpha]\frac{d\varphi}{\varphi}\right]-d\tau . \quad \text{(A2)}$$

Using (A1) this can be re-written as the following:

$$\left[\frac{1}{\beta}+\frac{[1-\delta_G][1-\alpha]\left[\frac{1+\gamma}{\beta}-1+\delta_K+\tau\right]}{[1+\gamma][\gamma+\delta_G]}\left[1-\rho\frac{\overline{G}_0 / K_0}{\varphi(\gamma,\tau)+\overline{G}_0 / K_0}\right]\right]d\gamma$$

$$=[1-\alpha]\left[\left[\frac{1+\gamma}{\beta}-1+\delta_K+\tau\right]\left[1-\rho\frac{\overline{G}_0 / K_0}{\varphi(\gamma,\tau)+\overline{G}_0 / K_0}\right]-\tau\right]\frac{d\tau}{\tau}. \quad \text{(A3)}$$

Thus, the growth-maximizing tax rate is determined implicitly by the solution to the pair of equations

$$\tau=[1-\alpha]\left[\frac{1+\gamma}{\beta}-1+\delta_K+\tau\right]\left[1-\rho\frac{\overline{G}_0 / K_0}{\varphi(\gamma,\tau)+\overline{G}_0 / K_0}\right], \quad \text{(A4)}$$

$$\gamma=\beta\left[A\left[1+\frac{\overline{G}_0/K_0}{\varphi(\gamma,\tau)}\right]^{\rho[1-\alpha]}\varphi(\gamma,\tau)^{1-\alpha}+1-\delta_K-\tau\right]-1. \qquad \text{(A5)}$$

If the world consists of two countries the equilibrium tax rates and the growth rate solve the following system of equations (assuming the discount rates and depreciation rates are equal across the countries):

$$\tau=[1-\alpha]\left[\frac{1+\gamma}{\beta}-1+\delta_K+\tau\right]\left[1-\rho\frac{\overline{\tau}\overline{K}_0}{\tau K_0+\overline{\tau}\overline{K}_0}\right], \qquad \text{(A6)}$$

$$\overline{\tau}=[1-\alpha]\left[\frac{1+\gamma}{\beta}-1+\delta_K+\overline{\tau}\right]\left[1-\rho\frac{\tau K_0}{\tau K_0+\overline{\tau}\overline{K}_0}\right], \qquad \text{(A7)}$$

$$\gamma=\beta\left[A\left[1+\frac{\overline{\tau}\overline{K}_0}{\tau K_0}\right]^{\rho[1-\alpha]}\left[\tau\frac{1+\gamma}{\gamma+\delta_G}\right]+1-\delta_K-\tau\right]-1, \qquad \text{(A8)}$$

and

$$A\left[1+\frac{\overline{\tau}\overline{K}_0}{\tau K_0}\right]^{\rho[1-\alpha]}\left[\tau\frac{1+\gamma}{\gamma+\delta_G}\right]^{[1-\alpha]}-\tau=\overline{A}\left[1+\frac{\tau K_0}{\overline{\tau}\overline{K}_0}\right]^{\rho[1-\alpha]}\left[\overline{\tau}\frac{1+\gamma}{\gamma+\delta_G}\right]^{[1-\alpha]}-\overline{\tau}. \qquad \text{(A9)}$$

The last equality is obtained from the equality of the two growth rates. This implies an additional restriction on the fundamentals, $\{A,\overline{A},K_0,\overline{K}_0\}$. In other words, to achieve the balanced growth path a certain relationship between the technologies and the endowments in two countries must hold. These conditions are met if we assume symmetry between the countries in the sense that $A=\overline{A}$, and $K_0=\overline{K}_0$.

A4.2 Coordination between Two Countries

Assuming symmetry with $A=\overline{A}$, $\tau=\overline{\tau}$ and $K_0=\overline{K}_0$, the optimization simplifies to

$$\max_{\{\tau\}}=\beta\left[A2^{\rho[1-\alpha]}\left[\tau\frac{1+\gamma}{\gamma+\delta_G}\right]^{[1-\alpha]}+1-\delta_K-\tau\right]-1. \qquad \text{(A10)}$$

This gives the necessary condition for τ as

$$\tau=A2^{\rho[1-\alpha]}[1-\alpha]\left[\tau\frac{1+\gamma}{\gamma+\delta_G}\right]^{[1-\alpha]}. \qquad \text{(A11)}$$

A4.3 Redistribution

The law of motion of the public capital is

$$G_{t+1}=[1-\delta_G]G_t+[1-\theta]\tau_{t+1}K_{t+1}+\mu\Omega_{t+1}, \qquad \text{(A12)}$$

where $\Omega_{t+1}=\theta\tau_{t+1}K_{t+1}+\overline{\theta}\overline{\tau}_{t+1}\overline{K}_{t+1}$. Iterating with respect to t yields

$$G_{t+1}=[1-\delta_G]^{t+1}G_0+\sum_{i=0}^{t}[1-\delta_G]^i\left[[1-\theta]\tau_{t+1}K_{t+1-i}+\mu\Omega_{t+1-i}\right], \qquad \text{(A13)}$$

and if the capital stock grows at a constant rate γ in both countries, whereas the tax rates and redistribution rates are constant,

$$G_{t+1} = [1-\delta_G]^{t+1} G_0 + \sum_{i=0}^{t} [1-\delta_G]^i [[1-\theta]\tau K_0 + \mu\Omega_0][1+\gamma]^{t+1-i}$$

$$= [1-\delta_G]^{t+1}\left[G_0 - \frac{1+\gamma}{\gamma+\delta_G}[[1-\theta+\theta\mu]\tau K_0 + \theta[1-\mu]\overline{\tau}\overline{K}_0]\right]$$

$$+[1+\gamma]^{t+1}\frac{1+\gamma}{\gamma+\delta_G}[[1-\theta+\theta\mu]\tau K_0 + \overline{\theta}\mu\overline{\tau}\overline{K}_0]. \qquad \text{(A14)}$$

Thus, if the balanced growth path is achieved from $t = 0$

$$G_0 = \frac{1+\gamma}{\gamma+\delta_G}[[1-\theta+\theta\mu]\tau K_0 + \overline{\theta}\mu\overline{\tau}\overline{K}_0]. \qquad \text{(A15)}$$

Since the households take the public capital investment into production as given, the optimization problem for the household is the same as in the case without redistribution. Thus, the welfare-maximizing growth rate is determined by

$$\gamma = \beta\left[A\left[\frac{\Gamma_0}{G_0}\right]^{\rho[1-\alpha]}\left[\frac{G_0}{K_0}\right]^{[1-\alpha]} + 1 - \delta_K - \tau\right] - 1, \qquad \text{(A16)}$$

where

$$\Gamma_0 = G_0 + \overline{G}_0 = \tau K_0 + \overline{\tau}\overline{K}_0. \qquad \text{(A17)}$$

A government that does not internalize the externality chooses τ to maximize γ, taking K_0 and $\overline{G}_0$ as given. Now

$$\gamma = \beta\left[A\left[1+\frac{\overline{G}_0/K_0}{\tilde{\varphi}(\gamma,\tau)}\right]^{\rho[1-\alpha]}\varphi(\gamma,\tau)^{1-\alpha} + 1 - \delta_K - \tau\right] - 1, \qquad \text{(A18)}$$

where

$$\tilde{\varphi}(\gamma,\tau) = \frac{G_0}{K_0} = \frac{1+\gamma}{\gamma+\delta_G}\left[[1-\theta+\theta\mu]\tau + \overline{\theta}\mu\overline{\tau}\frac{\overline{K}_0}{K_0}\right]. \qquad \text{(A19)}$$

Total differentiation gives

$$\frac{1}{\beta}d\gamma = A\left[1+\frac{\overline{G}_0/K_0}{\tilde{\varphi}(\gamma,\tau)}\right]^{\rho[1-\alpha]}\tilde{\varphi}(\gamma,\tau)^{1-\alpha}\left[-\frac{\rho[1-\alpha]}{1+\frac{\overline{G}_0/K_0}{\tilde{\varphi}(\gamma,\tau)}}\frac{\overline{G}_0}{K_0}\frac{d\tilde{\varphi}}{\tilde{\varphi}^2} + [1-\alpha]\frac{d\tilde{\varphi}}{\tilde{\varphi}}\right] - d\tau$$

$$= [1-\alpha]A\left[1+\frac{\overline{G}_0/K_0}{\tilde{\varphi}(\gamma,\tau)}\right]^{\rho[1-\alpha]}\tilde{\varphi}(\gamma,\tau)^{1-\alpha}\left[1-\rho\frac{\overline{G}_0/K_0}{\tilde{\varphi}(\gamma,\tau)+\overline{G}_0/K_0}\right]$$

$$\times\left[d\tau\frac{1-\theta+\theta\mu}{[1-\theta+\theta\mu]\tau+\bar{\theta}\mu\bar{\tau}\frac{\bar{K}_0}{K_0}}-d\lambda\frac{1-\delta_G}{[1+\gamma][\gamma+\delta_G]}\right]-d\tau\,. \tag{A20}$$

We therefore obtain the implicit solution for the optimal tax rate,

$$\tau=[1-\alpha]\left[\frac{1+\gamma}{\beta}-1+\delta_K+\tau\right]\left[1-\rho\frac{\bar{\tau}\bar{K}_0}{\tau K_0+\bar{\tau}\bar{K}_0}\right]-\bar{\tau}\frac{\bar{\theta}\mu}{1-\theta+\theta\mu}\frac{\bar{K}_0}{K_0}\,. \tag{A21}$$

Similarly, the second country's optimal tax rate is

$$\bar{\tau}=[1-\alpha]\left[\frac{1+\gamma}{\beta}-1+\delta_K+\tau\right]\left[1-\rho\frac{\tau K_0}{\tau K_0+\bar{\tau}\bar{K}_0}\right]-\tau\frac{\theta[1-\mu]}{1-\bar{\theta}+\bar{\theta}[1-\mu]}\frac{K_0}{\bar{K}_0}\,. \tag{A22}$$

A5. Capital Mobility

The law of motion of the public capital in the foreign country is

$$\bar{G}_{t+1}=[1-\delta_G]\bar{G}_t+\bar{\tau}\left[[1-\lambda_t]k_{t+1}+\bar{\lambda}_t\bar{k}_{t+1}\right]. \tag{A23}$$

Let the capital stock owned by domestic investors, k_t, grow at a constant rate γ, and that owned by foreign investors, $\bar{k}_t$, growth at rate $\bar{\gamma}$. Assuming the tax rate and the share of domestic investment are constant, and iterating in (A23), we obtain the following:

$$\bar{G}_{t+1}=[1-\delta_G]\left[[1-\delta_G]\bar{G}_{t-1}+\bar{\tau}\left[[1-\lambda]k_t+\bar{\lambda}\bar{k}_t\right]\right]+\bar{\tau}\left[[1-\lambda]k_{t+1}+\bar{\lambda}\bar{k}_{t+1}\right]$$

$$=[1-\delta_G]^{t+1}\left[\bar{G}_0-\bar{\tau}\left[\frac{1+\gamma}{\gamma+\delta_G}[1-\lambda]k_0+\frac{1+\bar{\gamma}}{\bar{\gamma}+\delta_G}\bar{\lambda}\bar{k}_0\right]\right]$$

$$+\bar{\tau}\left[\frac{1+\gamma}{\gamma+\delta_G}[1-\lambda]k_0[1+\gamma]^{t+1}+\frac{1+\bar{\gamma}}{\bar{\gamma}+\delta_G}\bar{\lambda}\bar{k}_0[1+\bar{\gamma}]^{t+1}\right]. \tag{A24}$$

Dividing both sides by $\bar{k}_{t+1}$,

$$\frac{\bar{G}_{t+1}}{\bar{k}_{t+1}}=\left[\frac{1-\delta_G}{1+\bar{\gamma}}\right]^{t+1}\frac{1}{\bar{k}_0}\left[G_0-\bar{\tau}\left[\frac{1+\gamma}{\gamma+\delta_G}[1-\lambda]k_0-\frac{1+\bar{\gamma}}{\bar{\gamma}+\delta_G}\bar{\lambda}\bar{k}_0\right]\right]$$

$$+\bar{\tau}\left[[1-\lambda]\frac{1+\gamma}{\gamma+\delta_G}\frac{k_0}{\bar{k}_0}\left[\frac{1+\gamma}{1+\bar{\gamma}}\right]^{t+1}+\frac{1+\bar{\gamma}}{\bar{\gamma}+\delta_G}\bar{\lambda}\right]. \tag{A25}$$

The first term converges to zero and the second term is constant if and only if $\gamma=\bar{\gamma}$. If the balanced growth path is achieved at $t = 0$

$$\bar{G}_0=\bar{\tau}\frac{1+\gamma}{\gamma+\delta_G}\left[[1-\lambda]k_0+\bar{\lambda}\bar{k}_0\right]. \tag{A26}$$

Similarly, for the home country it must be the case that

$$G_0 = \tau \frac{1+\gamma}{\gamma+\delta_G}\left[\lambda k_0 + [1-\bar{\lambda}]\bar{k}_0\right]. \tag{A27}$$

With capital flows the budget constraint of the domestic consumer becomes

$$C_t = \frac{\lambda k_t}{K_t} Y_t + \frac{[1-\lambda]k_t}{\bar{K}_t}\bar{Y}_t + [1-\delta_K]k_t - \tau\lambda k_t - \bar{\tau}[1-\lambda]k_t - k_{t+1}, \tag{A28}$$

and along the balanced growth path

$$C_t = [1+\gamma]^t \left[\frac{\lambda k_0}{K_0} Y_0 + \frac{[1-\lambda]k_0}{\bar{K}_0}\bar{Y}_0 + [1-\delta_K]k_0 - \tau\lambda k_0 - \bar{\tau}[1-\lambda]k_0 - [1+\gamma]k_0\right]$$

$$= [1+\gamma]^t C_0, \tag{A29}$$

where

$$C_0 = \left[\lambda \frac{Y_0}{K_0} + [1-\lambda]\frac{\bar{Y}_0}{\bar{K}_0} + [1-\delta_K] - \tau\lambda - \bar{\tau}[1-\lambda] - [1+\gamma]\right]k_0, \tag{A30}$$

and

$$\frac{Y_0}{K_0} = \left[\frac{G_0}{\lambda k_0 + [1-\bar{\lambda}]\bar{k}_0}\left[\frac{\Gamma_0}{G_0}\right]^{\rho}\right]^{1-\alpha}, \tag{A31}$$

$$\frac{\bar{Y}_0}{\bar{K}_0} = \left[\frac{\bar{G}_0}{\bar{\lambda}\bar{k}_0 + [1-\lambda]k_0}\left[\frac{\Gamma_0}{\bar{G}_0}\right]^{\rho}\right]^{1-\alpha}. \tag{A32}$$

The lifetime utility of the home consumer is

$$U = \sum_{t=0}^{\infty}\beta^t \ln(C_t) = \frac{1}{1-\beta}\ln(C_0) + \frac{\beta}{[1-\beta]^2}\ln(1+\gamma). \tag{A33}$$

Assume that two consumers simultaneously choose the allocation of capital and after that simultaneously choose the optimal growth rate. Solving backwards we first solve the first-order condition for γ. For the home consumer we have

$$\frac{\partial U}{\partial \gamma} = \frac{1}{1-\beta}\frac{1}{C_0}\frac{\partial C_0}{\partial \gamma} + \frac{\beta}{[1-\beta]^2}\frac{1}{1+\gamma},$$

$$C_0 = -\frac{\partial C_0}{\partial \gamma}\frac{1-\beta}{\beta}[1+\gamma] = \frac{1-\beta}{\beta}[1+\gamma]k_0, \tag{A34}$$

and, substituting this back into the utility function,

$$U(\gamma(\lambda)) = \frac{1}{1-\beta}\ln\left(\frac{1-\beta}{\beta}\right) + \frac{1}{1-\beta}\ln(k_0) + \frac{1}{[1-\beta]^2}\ln(1+\gamma(\lambda)). \tag{A35}$$

Now differentiation with respect to λ gives

$$\frac{dU}{d\lambda} = \frac{1}{[1-\beta]^2}\frac{1}{1+\gamma}\frac{d\gamma}{d\lambda}. \tag{A36}$$

For the interior solution this is equal to zero. From (A34)

$$\frac{1-\beta}{\beta}[1+\gamma]=\frac{C_0}{k_0}$$

$$=\lambda\frac{Y_0}{K_0}+[1-\lambda]\frac{\bar{Y}_0}{\bar{K}_0}+[1-\delta_K]-\tau\lambda-\bar{\tau}[1-\lambda]-[1+\gamma], \tag{A37}$$

so that

$$\frac{1}{\beta}[1+\gamma]=\lambda\frac{Y_0}{K_0}+[1-\lambda]\frac{\bar{Y}_0}{\bar{K}_0}+[1-\delta_K]-\tau\lambda-\bar{\tau}[1-\lambda]. \tag{A38}$$

Differentiation gives

$$\frac{1}{\beta}\frac{d\gamma}{d\lambda}=\frac{Y_0}{K_0}-\frac{\bar{Y}_0}{\bar{K}_0}-\tau+\bar{\tau}-\lambda\frac{Y_0}{K_0}\frac{[1-\alpha]k_0}{K_0}+[1-\lambda]\frac{\bar{Y}_0}{\bar{K}_0}\frac{[1-\alpha]k_0}{K_0}$$

$$=\frac{\alpha\lambda k_0+[1-\bar{\lambda}]\bar{k}_0}{\lambda k_0+[1-\bar{\lambda}]\bar{k}_0}\frac{Y_0}{K_0}-\tau-\left[\frac{\bar{\lambda}\bar{k}_0+\alpha[1-\lambda]k_0}{\bar{\lambda}\bar{k}_0+\alpha[1-\lambda]k_0}\frac{\bar{Y}_0}{\bar{K}_0}-\bar{\tau}\right]. \tag{A39}$$

This equation is expressed as the difference in net returns. Thus, an interior solution for λ corresponds to the equalized net returns. If the tax rates and the fundamentals are such that this difference is positive, *i.e*. domestic net returns to domestically-owned capital then λ = 1, the home consumer invests all his capital at home. Conversely, if this difference is negative, *i.e*. the foreign net returns are higher than the domestic ones then λ = 0, and the home consumer invests all his capital abroad.

The solution for the foreign consumer is similar:

$$\frac{1}{\beta}\frac{d\gamma}{d\bar{\lambda}}=\frac{\alpha\bar{\lambda}\bar{k}_0+[1-\lambda]k_0}{\bar{\lambda}\bar{k}_0+[1-\lambda]k_0}\frac{\bar{Y}_0}{\bar{K}_0}-\bar{\tau}-\left[\frac{\lambda k_0+\alpha[1-\bar{\lambda}]\bar{k}_0}{\lambda k_0+[1-\bar{\lambda}]\bar{k}_0}\frac{Y_0}{K_0}-\tau\right], \tag{A40}$$

so that $\bar{\lambda}$ = 1 when the difference is positive, $\bar{\lambda}$ = 0 when the difference is negative, and $\bar{\lambda}$ is between zero and one when the two net returns are equalized. Note that each consumer chooses his capital allocation taking the decision of other consumer, as well as the tax rates and public capital inputs in both countries as given. Thus, in (A39) and (A40)

$$\frac{Y_0}{K_0}=A\left[\frac{G_0}{\lambda k_0+\alpha[1-\bar{\lambda}]\bar{k}_0}\left[\frac{\Gamma_0}{G_0}\right]^{\rho}\right]^{1-\alpha}, \tag{A41}$$

$$\frac{\bar{Y}_0}{\bar{K}_0}=\bar{A}\left[\frac{\bar{G}_0}{\bar{\lambda}\bar{k}_0+[1-\lambda]k_0}\left[\frac{\Gamma_0}{\bar{G}_0}\right]^{\rho}\right]^{1-\alpha}. \tag{A42}$$

Suppose, λ is in the interior. Then one can see from (A41) and (A42) that $\bar{\lambda}$ is also in the interior i.f.f. $Y_0/K_0=\bar{Y}_0/\bar{K}_0$. Therefore, interior solutions for both λ and $\bar{\lambda}$ may exist for a particular combination of endowments and productivities. If the optimal choice of $\bar{\lambda}$ is zero, then it must be the case that in equilibrium $Y_0/K_0<\bar{Y}_0/\bar{K}_0$.

Conversely, if the optimal choice of $\bar{\lambda}$ is one it must be the case that in equilibrium $Y_0 / K_0 > \bar{Y}_0 / \bar{K}_0$.

A5.1 Non-cooperative equilibrium

The domestic government's objective is to maximize γ implicitly defined by

$$\gamma = -1 + \beta \Bigg[\lambda A \left[1 + \frac{\bar{G}_0}{\varphi(\gamma,\tau)\left[\lambda k_0 + \left[1-\bar{\lambda}\right]\bar{k}_0\right]} \right]^{\rho[1-\alpha]} \varphi(\gamma,\tau)^{1-\alpha}$$

$$+ \left[1-\delta_K\right] - \tau\lambda - \bar{\tau}\left[1-\lambda\right]$$

$$+ \left[1-\lambda\right]\bar{A}\left[1 + \frac{\varphi(\gamma,\tau)\left[\lambda k_0 + \left[1-\bar{\lambda}\right]\bar{k}_0\right]}{\bar{G}_0}\right]^{\rho[1-\alpha]} \left[\frac{\bar{G}_0}{\bar{\lambda}\bar{k}_0 + \left[1-\lambda\right]k_0}\right]^{1-\alpha} \Bigg], \qquad \text{(A43)}$$

where $\varphi(\gamma,\tau) \equiv \tau \frac{1+\gamma}{\gamma + \delta_G}$, and λ and $\bar{\lambda}$ solve

$$\frac{\alpha\lambda k_0 + \left[1-\bar{\lambda}\right]\bar{k}_0}{\lambda k_0 + \left[1-\bar{\lambda}\right]\bar{k}_0} \frac{Y_0}{K_0} - \tau = \frac{\bar{\lambda}\bar{k}_0 + \alpha\left[1-\lambda\right]k_0}{\bar{\lambda}\bar{k}_0 + \alpha\left[1-\lambda\right]k_0} \frac{\bar{Y}_0}{\bar{K}_0} - \bar{\tau}, \qquad \text{(A44)}$$

and

$$\left\{ \frac{\alpha\lambda k_0 + \left[1-\bar{\lambda}\right]\bar{k}_0}{\lambda k_0 + \left[1-\bar{\lambda}\right]\bar{k}_0} \frac{Y_0}{K_0} - \tau = \frac{\bar{\lambda}\bar{k}_0 + \alpha\left[1-\lambda\right]k_0}{\bar{\lambda}\bar{k}_0 + \alpha\left[1-\lambda\right]k_0} \frac{\bar{Y}_0}{\bar{K}_0} - \bar{\tau} \vee \frac{Y_0}{K_0} = \frac{\bar{Y}_0}{\bar{K}_0} \right\}, \qquad \text{(A45)}$$

or

$$\left\{ \bar{\lambda} = 0 \vee \frac{Y_0}{K_0} > \frac{\bar{Y}_0}{\bar{K}_0} \right\}, \text{ or } \left\{ \bar{\lambda} = 1 \vee \frac{Y_0}{K_0} < \frac{\bar{Y}_0}{\bar{K}_0} \right\}. \qquad \text{(A46)}$$

Here, from the viewpoint of the domestic government,

$$\frac{Y_0}{K_0} = A\left[\varphi(\gamma,\tau)\left[1 + \frac{\bar{G}_0}{\varphi(\gamma,\tau)\left[\lambda k_0 + \left[1-\bar{\lambda}\right]\bar{k}_0\right]} \right]^{\rho} \right]^{1-\alpha}, \qquad \text{(A47)}$$

$$\frac{\bar{Y}_0}{\bar{K}_0} = \bar{A}\left[\frac{\bar{G}_0}{\bar{\lambda}\bar{k}_0 + \left[1-\lambda\right]k_0} \left[1 + \frac{\varphi(\gamma,\tau)\left[\lambda k_0 + \left[1-\bar{\lambda}\right]\bar{k}_0\right]}{\bar{G}_0} \right]^{\rho} \right]^{1-\alpha}. \qquad \text{(A48)}$$

The foreign government chooses τ to maximize γ defined implicitly by

$$\gamma = -1 + \beta \Bigg[\bar{\lambda}\bar{A} \left[1 + \frac{G_0}{\varphi(\gamma,\bar{\tau})\left[\bar{\lambda}\bar{k}_0 + \left[1-\lambda\right]k_0\right]} \right]^{\rho[1-\alpha]} \varphi(\gamma,\bar{\tau})^{1-\alpha}$$

$$+ \left[1-\delta_K\right] - \bar{\tau}\bar{\lambda} - \tau\left[1-\bar{\lambda}\right]$$

$$+\left[1-\bar{\lambda}\right]A\left[1+\frac{\varphi(\gamma,\tau)\left[\bar{\lambda}\bar{k}_0+[1-\lambda]k_0\right]}{G_0}\right]^{\rho[1-\alpha]}\left[\frac{G_0}{\lambda k_0+\left[1-\bar{\lambda}\right]\bar{k}_0}\right]^{1-\alpha}\Bigg], \tag{A49}$$

but, since it takes G_0 as given, from the viewpoint of the foreign government

$$\frac{Y_0}{K_0}=A\left[\frac{G_0}{\lambda k_0+\left[1-\bar{\lambda}\right]\bar{k}_0}\left[1+\frac{\varphi(\gamma,\bar{\tau})\left[\bar{\lambda}\bar{k}_0+[1-\lambda]k_0\right]}{G_0}\right]^{\rho}\right]^{1-\alpha}, \tag{A50}$$

$$\frac{\bar{Y}_0}{\bar{K}_0}=\bar{A}\left[\varphi(\gamma,\bar{\tau})\left[1+\frac{G_0}{\varphi(\gamma,\bar{\tau})\left[\bar{\lambda}\bar{k}_0+[1-\lambda]k_0\right]}\right]^{\rho}\right]^{1-\alpha}. \tag{A51}$$

There is no need to impose the equality of the growth rates: in equilibrium this is ensured by the optimal choice of capital allocation by the consumers. However, the equilibrium growth rate, in general, is not efficient because the externalities are not internalized even with mobile capital.

A5.1.1 *Symmetric case: identical endowments and productivities*

Consider the case where $A=\bar{A}$ and $k_0=\bar{k}_0$. The choice of the domestic and the foreign investors, λ and $\bar{\lambda}$, solve

$$\frac{\alpha\lambda+\left[1-\bar{\lambda}\right]}{\lambda+\left[1-\bar{\lambda}\right]}\frac{Y_0}{K_0}-\tau=\frac{\bar{\lambda}+\alpha[1-\lambda]}{\bar{\lambda}+[1-\lambda]}\frac{\bar{Y}_0}{\bar{K}_0}-\bar{\tau}, \tag{A52}$$

and

$$\left\{\frac{\alpha\lambda+\left[1-\bar{\lambda}\right]}{\lambda+\left[1-\bar{\lambda}\right]}\frac{Y_0}{K_0}-\tau=\frac{\bar{\lambda}+\alpha[1-\lambda]}{\bar{\lambda}+\alpha[1-\lambda]}\frac{\bar{Y}_0}{\bar{K}_0}-\bar{\tau}\vee\frac{Y_0}{K_0}=\frac{\bar{Y}_0}{\bar{K}_0}\right\}, \tag{A53}$$

or

$$\left\{\bar{\lambda}=0\vee\frac{Y_0}{K_0}>\frac{\bar{Y}_0}{\bar{K}_0}\right\},\text{ or }\left\{\bar{\lambda}=1\vee\frac{Y_0}{K_0}<\frac{\bar{Y}_0}{\bar{K}_0}\right\}. \tag{A54}$$

Here

$$\frac{Y_0}{K_0}=\frac{A}{k_0^{1-\alpha}}\left[\frac{G_0}{\lambda+\left[1-\bar{\lambda}\right]}\left[\frac{\Gamma_0}{G_0}\right]^{\rho}\right]^{1-\alpha}, \tag{A55}$$

$$\frac{\bar{Y}_0}{\bar{K}_0}=\frac{\bar{A}}{k_0^{1-\alpha}}\left[\frac{\bar{G}_0}{\bar{\lambda}+[1-\lambda]}\left[\frac{\Gamma_0}{\bar{G}_0}\right]^{\rho}\right]^{1-\alpha}. \tag{A56}$$

In equilibrium $\frac{G_0}{\bar{G}_0}=\frac{\tau}{\bar{\tau}}$, so $\frac{Y_0}{K_0}\gtrless\frac{\bar{Y}_0}{\bar{K}_0}$ i.f.f. $\frac{\tau}{\lambda+\left[1-\bar{\lambda}\right]}\gtrless\frac{\bar{\tau}}{\bar{\lambda}+[1-\lambda]}$. Thus, there exists a completely symmetric equilibrium with $\tau=\bar{\tau}$ and $\lambda=\bar{\lambda}=1/2$. For the equilibrium with interior λ and $\bar{\lambda}=1$ it must be the case that $\frac{\tau}{\lambda}<\frac{\bar{\tau}}{2-\lambda}$, and for the equilibrium with interior λ and $\bar{\lambda}=0$ it must be the case that $\frac{\tau}{1+\lambda}>\frac{\bar{\tau}}{1-\lambda}$.

The growth rate in the domestic country is determined by

$$\frac{1}{\beta}[1+\gamma]=\lambda\frac{Y_0}{K_0}+[1-\bar{\lambda}]\frac{\bar{Y}_0}{\bar{K}_0}+[1-\delta_K]-\tau\lambda-\bar{\tau}[1-\bar{\lambda}], \tag{A57}$$

and in the foreign country by

$$\frac{1}{\beta}[1+\bar{\gamma}]=\bar{\lambda}\frac{\bar{Y}_0}{\bar{K}_0}+[1-\lambda]\frac{Y_0}{K_0}+[1-\delta_K]-\bar{\tau}\bar{\lambda}-\tau[1-\lambda]. \tag{A58}$$

Clearly, the completely symmetric equilibrium is consistent with $\gamma=\bar{\gamma}$.

A5.2 Cooperative equilibrium

Here γ is implicitly defined by

$$\gamma=-1+\beta\left[\lambda A\left[1+\frac{\bar{\tau}\left[\bar{\lambda}\bar{k}_0+[1-\lambda]k_0\right]}{\tau\left[\lambda k_0+[1-\bar{\lambda}]\bar{k}_0\right]}\right]^{\rho[1-\alpha]}\left[\tau\frac{1+\gamma}{\gamma+\delta_G}\right]^{1-\alpha}\right.$$
$$+[1-\delta_K]-\tau\lambda-\bar{\tau}[1-\lambda]$$
$$\left.+[1-\lambda]\bar{A}\left[1+\frac{\tau\left[\lambda k_0+[1-\bar{\lambda}]\bar{k}_0\right]}{\bar{\tau}\left[\bar{\lambda}\bar{k}_0+[1-\lambda]k_0\right]}\right]^{\rho[1-\alpha]}\left[\bar{\tau}\frac{1+\gamma}{\gamma+\delta_G}\right]^{1-\alpha}\right], \tag{A59}$$

and

$$\frac{Y_0}{K_0}=A\left[\tau\frac{1+\gamma}{\gamma+\delta_G}\left[1+\frac{\bar{\tau}\left[\bar{\lambda}\bar{k}_0+[1-\lambda]k_0\right]}{\tau\left[\lambda k_0+[1-\bar{\lambda}]\bar{k}_0\right]}\right]^{\rho}\right]^{1-\alpha}, \tag{A60}$$

$$\frac{\bar{Y}_0}{\bar{K}_0}=\bar{A}\left[\bar{\tau}\frac{1+\gamma}{\gamma+\delta_G}\left[1+\frac{\tau\left[\lambda k_0+[1-\bar{\lambda}]\bar{k}_0\right]}{\bar{\tau}\left[\bar{\lambda}\bar{k}_0+[1-\lambda]k_0\right]}\right]^{\rho}\right]^{1-\alpha}. \tag{A61}$$

A5.3 Non-cooperative equilibrium with redistribution

The domestic government chooses τ to maximize γ implicitly defined by

$$\gamma=-1+\beta\left[\lambda A\left[1+\frac{\bar{G}_0}{\tilde{\varphi}(\gamma,\tau)\left[\lambda k_0+[1-\bar{\lambda}]\bar{k}_0\right]}\right]^{\rho[1-\alpha]}\tilde{\varphi}(\gamma,\tau)^{1-\alpha}\right.$$
$$+[1-\delta_K]-\tau\lambda-\bar{\tau}[1-\lambda]$$

$$+\left[1-\lambda\right]\bar{A}\left[1+\frac{\tilde{\varphi}(\gamma,\tau)\left[\lambda k_0+\left[1-\bar{\lambda}\right]\bar{k}_0\right]}{\bar{G}_0}\right]^{\rho[1-\alpha]}\left[\frac{\bar{G}_0}{\bar{\lambda}\bar{k}_0+\left[1-\lambda\right]k_0}\right]^{1-\alpha}\Bigg]. \quad \text{(A62)}$$

where

$$\tilde{\varphi}(\gamma,\tau)=\frac{1+\gamma}{\gamma+\delta_G}\left[\left[1-\theta+\theta\mu\right]\tau+\bar{\theta}\mu\bar{\tau}\frac{\bar{K}_0}{K_0}\right], \quad \text{(A63)}$$

$$\frac{Y_0}{K_0}=A\left[\tilde{\varphi}(\gamma,\tau)\left[1+\frac{\bar{G}_0}{\tilde{\varphi}(\gamma,\tau)\left[\lambda k_0+\left[1-\bar{\lambda}\right]\bar{k}_0\right]}\right]^{\rho}\right]^{1-\alpha}, \quad \text{(A64)}$$

$$\frac{\bar{Y}_0}{\bar{K}_0}=\bar{A}\left[\frac{\bar{G}_0}{\bar{\lambda}\bar{k}_0+\left[1-\lambda\right]k_0}\left[1+\frac{\tilde{\varphi}(\gamma,\tau)\left[\lambda k_0+\left[1-\bar{\lambda}\right]\bar{k}_0\right]}{\bar{G}_0}\right]^{\rho}\right]^{1-\alpha}. \quad \text{(A65)}$$

The foreign government chooses $\bar{\tau}$ to maximize γ implicitly defined by

$$\gamma=-1+\beta\Bigg[\overline{\lambda A}\left[1+\frac{G_0}{\hat{\varphi}(\gamma,\bar{\tau})\left[\bar{\lambda}\bar{k}_0+\left[1-\lambda\right]k_0\right]}\right]^{\rho[1-\alpha]}\hat{\varphi}(\gamma,\bar{\tau})^{1-\alpha}$$

$$+\left[1-\delta_K\right]-\bar{\tau}\bar{\lambda}-\tau\left[1-\bar{\lambda}\right]$$

$$+\left[1-\bar{\lambda}\right]A\left[1+\frac{\hat{\varphi}(\gamma,\bar{\tau})\left[\bar{\lambda}\bar{k}_0+\left[1-\lambda\right]k_0\right]}{G_0}\right]^{\rho[1-\alpha]}\left[\frac{G_0}{\lambda k_0+\left[1-\bar{\lambda}\right]\bar{k}_0}\right]^{1-\alpha}\Bigg]. \quad \text{(A66)}$$

where

$$\hat{\varphi}(\gamma,\bar{\tau})=\frac{1+\gamma}{\gamma+\delta_G}\left[\left[1-\bar{\theta}+\bar{\theta}\left[1-\mu\right]\bar{\tau}+\theta\left[1-\mu\right]\tau\frac{\bar{K}_0}{K_0}\right]\right], \quad \text{(A67)}$$

where now

$$\frac{Y_0}{K_0}=A\left[\frac{G_0}{\lambda k_0+\left[1-\bar{\lambda}\right]\bar{k}_0}\left[1+\frac{\hat{\varphi}(\gamma,\bar{\tau})\left[\lambda\bar{k}_0+\left[1-\lambda\right]k_0\right]}{G_0}\right]^{\rho}\right]^{1-\alpha}, \quad \text{(A68)}$$

$$\frac{\bar{Y}_0}{\bar{K}_0}=\bar{A}\left[\hat{\varphi}(\gamma,\bar{\tau})\left[1+\frac{G_0}{\hat{\varphi}(\gamma,\bar{\tau})\left[\bar{\lambda}\bar{k}_0+\left[1-\lambda\right]k_0\right]}\right]^{\rho}\right]^{1-\alpha}. \quad \text{(A69)}$$

References

Barro, R.J. (1990) "Government spending in a simple model of endogenous growth", *Journal of Political Economy*, **98**, S103 – S125.

Bianconi, M. and Turnovsky, S.J. (1997) "International effects of government expenditure in interdependent economies", *Canadian Journal of Economics*, **30**, 57 – 84.

Chen J.-H., J.-Y. Shieh, C.-C. Lai, and J.-J. Chang (2005) "Productive public expenditure and imperfect competition with endogenous price markup", *Oxford Economic Papers*, **47**, 522 – 544.

Easterly, W. and Rebelo, S. (1993)"Fiscal policy and economic growth", *Journal of Monetary Economics*, **32**, 417 – 458.

Feinstein, C.H. (1972) *National Income, Expenditure and Output of the United Kingdom*, 1855 -- 1965 (Cambridge: Cambridge University Press).

Koester, R. and Kormendi, R. (1989) "Taxation, aggregate activity and economic growth: cross-country evidence on some supply-side hypotheses", *Economic Inquiry*, **27**, 367 – 386.

Myles, G.D. (2007) *Economic Growth and the Role of Taxation*, OECD.

Plosser, C. (1993) "The search for growth", in Federal Reserve of Kansas City symposium series, *Policies for Long Run Growth*, 57 – 86, (Kansas City).

Razin, A. and Yuen, C.-W. (1997) "Factor mobility and income growth: two convergence hypotheses", *Review of Development Economics*, **1**, 171 – 190.

Slemrod, J. (1995) "What do cross-country studies teach about government involvement, prosperity, and economic growth", *Brookings Papers on Economic Activity*, 373 – 431.

Turnovsky, S.J. (1999) "Productive government expenditure in a stochastically growing economy", *Macroeconomic Dynamics*, **3**, 544 – 570.