**Preprints of the
Max Planck Institute for
Research on Collective Goods
Bonn 2010/40**

Too Much Information Sharing? Welfare Effects of Sharing Acquired Cost Information in Oligopoly

Juan José Ganuza
Jos Jansen

MAX PLANCK SOCIETY

Preprints of the
Max Planck Institute
for Research on Collective Goods Bonn 2010/40

Too Much Information Sharing? Welfare Effects of Sharing Acquired Cost Information in Oligopoly

Juan José Ganuza / Jos Jansen

September 2010

Max Planck Institute for Research on Collective Goods, Kurt-Schumacher-Str. 10, D-53113 Bonn
http://www.coll.mpg.de

Too Much Information Sharing? Welfare Effects of Sharing Acquired Cost Information in Oligopoly*

Juan José Ganuza†, and Jos Jansen‡

September 2010

Abstract

By using general information structures and precision criteria based on the dispersion of conditional expectations, we study how oligopolists' information acquisition decisions may change the effects of information sharing on the consumer surplus. Sharing information about individual cost parameters gives the following trade-off in Cournot oligopoly. On the one hand, it decreases the expected consumer surplus for a given information precision, as the literature shows. On the other hand, information sharing increases the firms' incentives to acquire information, and the consumer surplus increases in the precision of the firms' information. Interestingly, the latter effect may dominate the former effect.

Keywords: Information acquisition, Information sharing, Information structures, Oligopoly, Consumer surplus.
JEL classification numbers: D82, D83, L13, L40.

*We are grateful for comments by Larry Ausubel, Christoph Engel, Maria Angeles de Frutos, Matthias Lang, Johannes Münster, and participants of the JEI (Vigo), the Madrid Summer Workshop on Economic Theory (2009), the ACLE meeting on "Information, Communication and Competition" (Amsterdam), the 11th SFB/TR 15 Conference (Caputh), the UPF internal microeconomics seminar (Barcelona), and the EEA Congress (Glasgow). Naturally, all errors are ours.
†Corresponding author. Universitat Pompeu Fabra, Department of Economics and Business, Ramon Trias Fargas, 25-27, 08005 Barcelona, Spain; e-mail <juanjo.ganuza@upf.edu>.
‡Max Planck Institute for Research on Collective Goods, Kurt-Schumacher-Str. 10, D-53113 Bonn, Germany; e-mail <jansen@coll.mpg.de>

1 Introduction

The role of trade associations in facilitating firms sharing information, has always been an important and controversial topic for economic theorists, practitioners and antitrust authorities. On the practical side, the controversy starts with some contradictory decisions taken by US Courts (e.g., see Vives (1990) for details). Currently, although antitrust authorities do not forbid explicitly the exchange of information (as long as it is not used to facilitate collusion or deter entry), suspicions remain.[1] On the theoretical side, there is a classical literature (see, e.g., Kühn and Vives (1995), Raith (1996), and Vives (1999) for surveys) that provides a taxonomy of regulatory recommendations depending on the type of strategic interaction, and the type of information.

This paper tries to look at this old issue with new methodological techniques and taking the new perspective of information economics, which allows agents to some extent to determine their information structures. In particular, we focus on a simple question that has a clear answer in the previous literature: Should Cournot oligopolists be allowed to share information about their private costs of production? The answer of the classical literature on information sharing in oligopoly seems to be unambiguously negative. In the first place, information sharing among competing firms decreases the consumer surplus when the firms compete in quantities (Shapiro, 1986, Sakai and Yamato, 1989). Moreover, information sharing may facilitate collusion between firms, which also hurts consumers.[2] Hence, a policy maker, who maximizes expected consumer surplus, should prohibit agreements among Cournot oligopolists to share information about their costs. However, we do not observe many regulatory restrictions on information sharing in reality.[3]

However, this conclusion was drawn in settings where firms receive information exogenously. In this paper we show that the policy conclusion may become ambiguous when information is endogenous, i.e., firms invest in acquiring information.[4] This may provide a rationale for the

[1]See Kühn and Vives (1995), Vives (1990), and Kühn (2001) for a discussion on the decisions of antitrust authorities regarding information-sharing policies.

[2]Information sharing may help firms to detect deviations from collusive agreements (Green and Porter (1984)).

[3]We claim that this result should lead to regulatory constraints not only because there are economic sectors as automobile industry that are typically associated with Cournot competition. But also, since with Bertrand competition it can be shown that competing firms do not have an interest in sharing information.

[4]There can be several situations where a firm may not have complete information about its cost of production. For example, the production process of the car industry is complex, and the decisions about how much to produce is taken before some of the production contracts are signed and some input costs are known. In other words, the production decisions are based on expected cost. Further, a firm may have developed a process innovation, and it may not be clear how far this innovation reduces the firm's cost. In addition, there may be incomplete information

observed lenient antitrust policy regarding information sharing. This counter-intuitive result is based on the fact that allowing firms to share information has two effects on consumer surplus. On the one hand, as previous literature pointed out, there is a negative direct effect. For an exogenously given level of information precision, allowing information sharing between firms has a negative effect (positive effect) on the consumer surplus (profits of the firms). However, on the other hand, there is a positive indirect effect of sharing information. This is due to the fact that the incentives of acquiring information are larger when firms are allowed to share information. The higher investments by firms that share information have a positive effect on the consumer surplus. On top of that, we provide an example in which allowing firms to share information increases the consumer surplus, i.e., the positive indirect effect dominates the negative direct effect.

One of the main difficulties in translating the theoretical results of information sharing to regulatory policies is that it is difficult to obtain unambiguous results. We have to acknowledge that, although making information structures endogenous is a necessary step in understanding the welfare consequences of information-sharing policies, it may make this task more complex. However, the second main contribution of this paper is to clarify the driving forces of our and existing results. Basically, we show that most of the results are due to the fact that the objective functions of the firms (i.e., profits) and antitrust authority (i.e., consumer surplus) are convex functions of the firms' outputs. This implies that the dispersion of the firms' outputs has a clear impact on the outcomes. In turn, information sharing and information acquisition strategies determine the quality of information held by firms (i.e., the distribution of posterior beliefs), and indirectly, the dispersion of outputs.

Recently, Ganuza and Penalva (2010) has provided a family of precision criteria for ranking information structures according to the effect that information has on the dispersion of conditional expectations. The basic principle of these precision criteria is that a more accurate information structure leads to a more disperse distribution of the conditional expectation. Applying these informativeness measures allows us to obtain very general results in terms of the information structures under consideration.

Besides this conceptual contribution, we attempt to contribute to the literature on information acquisition in oligopoly. Li *et al.* (1987), Hwang (1995), Hauk and Hurkens (2001) study the

about geological or meteorological conditions for firms that extract natural resources.

information acquisition incentives of Cournot oligopolists. These papers assume that firms do not share their acquired information, and make complementary comparisons.[5] By contrast, we focus on the interaction between the incentives to acquire information and to share information. Fried (1984), Kirby (2004), and Jansen (2008) study the effects of this interaction on the expected profits of firms. In contrast to these papers, we focus on welfare effects, and we consider more general information structures.

Persico (2000) studies the interaction between information acquisition and information aggregation in an auction model with affiliated values. For a given information structure the second price auction yields a higher expected revenue to an auctioneer than the first price auction. But the first price auction gives a greater incentive to acquire information, which may reverse the expected revenue ranking. As in our paper, Persico (2000) also considers general information structures but ordered according to an alternative informativeness criterion (i.e., Lehmann (1986)).

In the next section we describe the model. Section 3 defines the concept of Integral Precision for signals. Section 4 briefly describes the equilibrium strategies. Section 5 compares expected consumer surplus levels in equilibrium. Section 6 extends the analysis in some relevant directions. Finally, section 7 concludes the paper. All proofs are relegated to the Appendix. The Supplementary Appendix presents some results related to the model's extensions.

2 The Model

2.1 Preferences and Technology

Consider an industry where two risk-neutral firms (i.e., firms 1 and 2) compete in quantities of differentiated goods.[6] The representative consumer's gross surplus from consuming (x_1, x_2) is:

$$u(x_1, x_2) \equiv \alpha(x_1 + x_2) - \frac{1}{2}(x_1 + x_2)^2 + (1 - \beta)x_1 x_2, \tag{1}$$

with $0 < \beta \leq 1$. Hence, the inverse demand function for good i is linear in the outputs, i.e., $P_i(x_i, x_j) = \alpha - x_i - \beta x_j$. The demand intercept α is sufficiently high. The parameter β represents

[5] For example, Hwang (1995) observes that information acquisition incentives are important for the welfare comparison between perfect competition, oligopoly, and monopoly. Although perfect competition yields the highest expected welfare for any exogenously given precision of information, it may fail to do so when the precision is determined endogenously, since firms in perfectly competitive markets may have a lower incentive to acquire information. Whereas Hwang changes the mode of competition while keeping information sharing constant, we do the opposite.

[6] Section 6 considers an oligopoly with N firms ($N \geq 2$). All duopoly results also hold with more than two firms.

the degree of substitutability between goods 1 and 2. For $\beta = 1$ the goods are perfect substitutes, while for $\beta = 0$ the markets for the goods are independent. The consumption of the bundle (x_1, x_2) gives the representative consumer a net surplus of:

$$v(x_1, x_2) \equiv u(x_1, x_2) - \sum_{i=1}^{2} P_i(x_i, x_j)x_i = \frac{1}{2}(x_1 + x_2)^2 - (1-\beta)x_1x_2. \tag{2}$$

Firms have constant marginal costs of production. Firm i's profit of producing quantity x_i at marginal cost θ_i is simply: $\pi_i(x_i, x_j; \theta_i) = [P_i(x_i, x_j) - \theta_i]\, x_i$ for $i, j = 1, 2$ and $j \neq i$.

2.2 Firms' Information Structures

Firms' marginal costs are initially unknown. The cost of firm i is distributed according to c.d.f. $F_i : [0, \theta^h] \to [0, 1]$ with mean $\overline{\theta}_i$. Firm i can acquire a costly signal S_i^δ about θ_i, where $S_i^\delta \in \mathcal{S}$ for some set $\mathcal{S}$. Signal S_i^δ is characterized by the family of distributions $\{H_\delta(s|\theta_i)\}_{\theta_i}$. That is, given the marginal cost θ_i, which is a realization of the random variable Θ_i, S_i^δ is represented by the conditional distribution $H_\delta(s|\theta_i) = \Pr(S_i^\delta \le s|\Theta_i = \theta_i)$. The prior distribution $F_i(\theta)$ and the signal distribution $\{H_\delta(s|\theta_i)\}_{\theta_i}$ define the information structure, i.e., the joint distribution of (Θ_i, S_i^δ). Parameter δ orders the signals in the sense of Integral Precision (see section 3). We denote the cost of acquiring a signal S_i^δ of precision δ by $c(\delta)$, where c is increasing in δ.

We assume that $H_\delta(s|\theta_i)$ admits a density $h_\delta(s|\theta_i)$. The marginal distribution of S_i^δ is denoted by $H_i^\delta(s)$ and satisfies:

$$H_i^\delta(s) = \int_{\{y \in \mathcal{S}|y \le s\}} \int_0^{\theta^h} h_\delta(y|\theta) dF(\theta) dy.$$

Let $F_i^\delta(\theta_i|s_i^\delta)$ and $E_i[\theta|s_i^\delta]$ denote the posterior distributions and the conditional expectation of Θ_i conditional on $S_i^\delta = s_i^\delta$

2.3 Firms' Information Sharing Policies

If the antitrust authority allows information sharing between firms, the firms simultaneously choose their information-sharing policy *vis-à-vis* their competitor before they acquire the signal.[7]

[7] In other words, firms unilaterally choose whether to precommit to information sharing. Alternative assumptions could be to allow the firms to precommit cooperatively to share information (through a *quid pro quo* agreement), or to assume that firms make strategic information sharing choices (i.e., each firm chooses whether to share information after it learns its signal). As it turns out, in equilibrium the information sharing choices are not affected by these changes of assumptions (e.g., see Okuno-Fujiwara *et al.* (1990) on strategic information disclosure).

As is common in the literature (e.g., Raith (1996)), a firm either shares its information truthfully or it keeps the information secret. We focus on a parametric family of information-sharing policies. Firm i chooses $\rho_i \in [0,1]$, which implies that firm j receives the informative message, $m_i = s_i^\delta$ (the private realization of the signal S_i^δ), with probability ρ_i, and the non-informative message, $m_i = \varnothing$, with the complementary probability, $1 - \rho_i$.

2.4 Timing

1. Initially, an antitrust authority chooses whether to allow or prohibit information sharing between the firms in the industry. The authority maximizes the expected consumer surplus.

2. In the second stage, firms simultaneously choose their information-sharing policy *vis-à-vis* their competitor, $\rho_i \in [0,1]$, taking into account the decision of the antitrust authority.

3. The marginal costs of firms 1 and 2 are determined by two independent draws from their corresponding distributions F_1 and F_2, respectively.

4. Firms simultaneously choose information acquisition investments: δ_i at a cost of $c(\delta_i)$, with c increasing in δ_i for $i = 1, 2$. Firm i's investment δ_i determines the precision of the firm's cost signal S_i^δ. Signal S_i^δ is characterized by the family of distributions $\{H_\delta(s|\theta_i)\}_{\theta_i}$.

5. Firms send messages about their signal in accordance with their information-sharing policies in stage 2. If firm i precommitted to share its information in accordance with ρ_i, then firm j will receive an informative message $m_i = s_i^\delta$ (the private realization of the signal S_i^δ) with probability ρ_i and an uninformative message $m_i = \varnothing$, with probability $1 - \rho_i$.

6. In the final stage firms simultaneously choose their output levels, $x_i \geq 0$ for firm i, to maximize the expected value of $\pi_i(x_i, x_j; \theta_i)$, i.e., firms are Cournot competitors.

We solve the game backwards, and restrict the analysis to perfect Bayesian equilibria. Before solving the model, we want to discuss how the choice of information acquisition investment δ_i determines the information structure.

3 Information Criteria: Integral Precision

In this paper we assume that the parameter δ_i rank signals according to Integral Precision. Precision criteria (introduced by Ganuza and Penalva, 2010) are based on the principle that an information structure, i.e., the joint distribution of the state of the world and the signal, is more informative (*more precise*) than another if it generates more dispersed conditional expectations. This dispersion effect arises because the sensitivity of conditional expectations to the realized value of the signal depends on the informational content of the signal. If the informational content of the signal is low, conditional expectations are concentrated around the expected value of the prior. When the informational content is high, conditional expectations depend to a large extent on the realization of the signal which increases their variability.

In our context, given the prior distribution $F_i(\theta)$, we assume that if $\delta_i > \delta_i'$ then $E_i[\theta|S_i^\delta]$ is "more spread out" than $E_i[\theta|S_i^{\delta'}]$. In the present paper, we use the *Integral Precision* criterion, which combines this approach with the convex order (Ganuza and Penalva, 2010):

Definition 1 (Convex Order) *Let Y and Z be two real-valued random variables with distribution F and G respectively. Then Y is greater than Z in the* convex order *($Y \geq_{cx} Z$) if for all convex real-valued functions ϕ, $E[\phi(Y)] \geq E[\phi(Z)]$ provided the expectation exists.*

Using the convex order, Ganuza and Penalva define Integral Precision to order signals in terms of their informativeness:

Definition 2 (Integral Precision) *Given a prior $F_i(\theta)$ and two signals S_1 and S_2, signal S_1 is more* integral precise *than S_2 if $E_i[\theta|S_1]$ is greater than $E_i[\theta|S_2]$ in the convex order.*

Ganuza and Penalva (2010) show that *Integral Precision* is weaker than (is implied by) all common informativeness orders based on the value of information for a decision maker (Blackwell, 1951, Lehmann, 1988, and Athey and Levin, 2001). In other words, if S_1 is more valuable for a decision maker than S_2, then S_1 is more integral precise than S_2. The following information models are consistent with *Integral Precision.*

Normal Experiments: Let $F_i(\theta) \sim \mathcal{N}(\mu, \sigma_v^2)$ and $S_i^\delta = \theta_i + \epsilon_\delta$, where $\epsilon_\delta \sim \mathcal{N}(0, \sigma_\delta^2)$ and is independent of θ_i. The variance of the noise, σ_δ^2, orders signals in the usual way: we assume that

$\delta > \delta' \iff \sigma_\delta^2 < \sigma_{\delta'}^2$ and the signal with a noise term that has lower variance is more informative in terms of Integral Precision.

Linear Experiments: Let the signal be perfectly informative, $S_i^\delta = \theta_i$, with probability δ, and the signal is pure noise, $S_i^\delta = \epsilon$ where $\epsilon \sim F_i(\theta)$ and is independent of θ_i, with probability $1-\delta$. Let S_i^δ and $S_i^{\delta'}$ be two such signals. If $\delta > \delta'$, i.e. S_i^δ reveals the truth with a higher probability than $S_i^{\delta'}$, then S_i^δ is more informative than $S_i^{\delta'}$ in terms of Integral Precision.

Binary Experiments: Let θ_i be equal to θ^h with probability q and θ^l with probability $1-q$. The signal, S_i^δ, can take two values h or l, where $\Pr[S_i^\delta = k|\theta_i = \theta^k] = \frac{1}{2}(1+\delta_i)$ for $i \in \{1,2\}$ and $k \in \{l,h\}$, where $0 \leq \delta_i \leq 1$. The parameter δ_i orders signals in the usual way: higher δ implies greater Integral Precision.

Uniform Experiments: Let $F_i(\theta)$ be the uniform distribution on $[0,1]$ and let $H_\delta(s|\theta_i)$ be uniform on $\left[\theta_i - \frac{1}{2\delta}, \theta_i + \frac{1}{2\delta}\right]$. For any δ, δ' with $\delta > \delta'$, S_i^δ is more informative than $S_i^{\delta'}$ in terms of Integral Precision.

Partitions: Let $F_i(\theta)$ have support equal to $[0,1]$. Consider two signals generated by two partitions of $[0,1]$, $\mathcal{A}$ and $\mathcal{B}$, where $\mathcal{B}$ is finer than $\mathcal{A}$.[8] Using these partitions, one can define signals S_i^δ and $S_i^{\delta'}$ in the usual way: signal S_i^δ $[S_i^{\delta'}]$ tells you which set in the partition $\mathcal{A}$ $[\mathcal{B}]$ contains θ_i.[9] If a larger δ means a finer partition, δ orders signals according to Integral Precision.

4 Solving the Model: Equilibrium Strategies

First, we characterize the equilibrium output levels. Second, we analyze the information acquisition choices of firms. Finally, we analyze the information-sharing choices of the firms.

4.1 Output Levels

Each firm chooses its output level on the basis of its own information, s_i, and the information received from its competitor, $m_j \in \{s_j, \varnothing\}$. In order to save notation we do not make explicit the dependence of s_i on δ_i. The expected cost given the uninformative message $m_j = \varnothing$ is:

[8] A partition, $\mathcal{A}$, divides $[0,1]$ into disjoint subsets, $\mathcal{A} = \{A_1, .., A_k\}$, i.e., $\cup_{j=1}^k A_j = [0,1]$ and $A_i \cap A_j = \emptyset$ for all $i, j = 1, .., k$ with $i \neq j$. Partition $\mathcal{B}$ is finer than $\mathcal{A}$, when for all $B \in \mathcal{B}$, there exists $A \in \mathcal{A}$ such that $B \subseteq A$.

[9] However, observing A_j $[B_j]$ does not allow you to distinguish between different states of the world within that set.

$E\{\theta_j|\varnothing\} = \overline{\theta}_j$.

For any combination of messages m_i and m_j, firm i with signal s_i maximizes its expected profit, which yields the following first-order condition:

$$x_i(s_i) = \frac{1}{2}\Big(\alpha - E\{\theta_i|s_i\} - \beta E\{x_j(s_j)|m_j\}\Big) \tag{3}$$

for $i, j = 1, 2$ with $i \neq j$. Solving the system of equations (3) for $i = 1, 2$ gives the following equilibrium output level of firm i (for $i, j = 1, 2$ with $i \neq j$):

$$x_i^*(s_i; m_i, m_j) = \frac{1}{4-\beta^2}\left((2-\beta)\alpha - 2E\{\theta_i|s_i\} + \beta E\{\theta_j|m_j\} + \frac{\beta^2}{2}\left[E\{\theta_i|s_i\} - E\{\theta_i|m_i\}\right]\right) \tag{4}$$

where $E\{\theta_i|m_i\} = E_{s_i}\{E(\theta_i|s_i)|m_i\}$. First, notice that the last term is the distortion due to the asymmetric information between firms. If $m_i = s_i$, there is no distortion. If $m_i = \varnothing$ and s_i gives bad news (high θ_i), the term is positive since the firm j is reacting to the average cost, producing less than it would have produced with perfect information. Conversely, concealed good news gives a negative distortion. Second, notice that the expected equilibrium output level is independent of the information acquisition and information sharing variables:

$$\overline{x_i}^* \equiv E_{s_i,m_i}\left\{E_{s_j,m_j}[x_i^*(s_i; m_i, m_j)]\right\} = \frac{1}{4-\beta^2}\Big((2-\beta)\alpha - 2\overline{\theta_i} + \beta\overline{\theta_j}\Big) \tag{5}$$

where

$$E_{s_i,m_i}\left\{E_{s_j,m_j}[x_i^*(s_i; m_i, m_j)]\right\} = \rho_i E_{s_i}\left\{E_{s_j,m_j}[x_i^*(s_i; s_i, m_j)]\right\} + (1-\rho_i)E_{s_i}\left\{E_{s_j,m_j}[x_i^*(s_i; \varnothing, m_j)]\right\}$$

and $E_{s_j,m_j}[.]$ is defined likewise. Hence, information acquisition and sharing have no effect on the average output level. They only have an effect on the output dispersion.

The expected equilibrium product market profits of firm i with signal s_i, and messages m_i and m_j equals: $\pi_i^*(s_i; m_i, m_j) = x_i^*(s_i; m_i, m_j)^2$. Hence, the firm's expected profit equals:

$$\Pi_i(\delta_i, \delta_j; \rho_i, \rho_j) \equiv E_{s_i,m_i}\left\{E_{s_j,m_j}[x_i^*(s_i; m_i, m_j)^2]\right\} - c(\delta_i) \tag{6}$$

Notice that the profit function of firm i is convex in its own output. This feature of the objective function is important for our future results. It implies that firms prefer more dispersed individual outputs. As we show below, the information sharing policies as well as the information acquisition strategies affect the dispersion of the outputs.

4.2 Information Acquisition

In this subsection, we study the effects of information acquisition investments on firms. First, we analyze the effects of information acquisition investments on the expected profit.

PROPOSITION 1 *Firm i's expected product market profit* $E_{s_i,m_i}\left\{E_{s_j,m_j}[x_i^*(s_i;m_i,m_j)^2]\right\}$ *is: (i) increasing in the own information acquisition investment* δ_i*, and (ii) weakly increasing in the competitor's investment* δ_j*.*

Proposition 1(i) confirms that a firm generates a positive revenue by acquiring information. The firm trades off this marginal revenue from investment (i.e., $\partial E_{s_i,m_i}\left\{E_{s_j,m_j}[x_i^*(s_i;m_i,m_j)^2]\right\}/\partial\delta_i$) against the marginal cost of investment $c'(\delta_i)$. In Proposition 1(ii) we show that also the information acquisition investment of the competitor increases a firm's expected profit.

Second, we analyze the relationship between the information acquisition incentives and the information sharing policy.

LEMMA 1 *The expected profit* $\Pi_i(\delta_i,\delta_j;\rho_i,\rho_j)$ *is supermodular in* (δ_i,ρ_i)*.*

In other words, for $\rho_i > \rho_i'$, the profit difference $\Pi_i(\delta_i,\delta_j;\rho_i,\rho_j) - \Pi_i(\delta_i,\delta_j;\rho_i',\rho_j)$ is weakly increasing in δ_i for all δ_j and ρ_j. This implies that information-sharing firms have a greater incentive to acquire information than concealing firms.

PROPOSITION 2 *Firm i's equilibrium information acquisition investment,* δ_i^**, is: (i) independent of the competitor's information sharing choice,* ρ_j*, and (ii) increasing in the own choice,* ρ_i*.*

Proposition 2(i) is due to the independence of the firms' costs and signals. The next section analyzes incentives of firms to share information, and gives intuition for Propositions 1 and 2(ii).

4.3 Information Sharing

For a given precision, information sharing is a dominant strategy for a firm (Gal-Or (1986), Shapiro (1986)). We confirm that sharing information is also a dominant strategy in our model.

PROPOSITION 3 *The expected profit* $\Pi_i(\delta_i,\delta_j;\rho_i,\rho_j)$ *is increasing in* ρ_i *and* ρ_j *for all* (δ_i,δ_j)*.*

We cannot directly apply this result since in our model the precision is not exogenously given, but it depends on the information-sharing choices of the firms. The effect of information sharing on a firm's own profit can be decomposed as follows:

$$\frac{d\Pi_i(\delta_i^*, \delta_j^*; \rho_i, \rho_j)}{d\rho_i} = \frac{\partial\Pi_i(\delta_i^*, \delta_j^*; \rho_i, \rho_j)}{\partial\delta_i} \cdot \frac{\partial\delta_i^*}{\partial\rho_i} + \frac{\partial\Pi_i(\delta_i^*, \delta_j^*; \rho_i, \rho_j)}{\partial\delta_j} \cdot \frac{\partial\delta_j^*}{\partial\rho_i} + \frac{\partial\Pi_i(\delta_i^*, \delta_j^*; \rho_i, \rho_j)}{\partial\rho_i} > 0. \quad (7)$$

The first term of (7) is zero, since the firm chooses its information acquisition investment optimally (i.e., $\partial\Pi_i/\partial\delta_i = 0$). Also the second term of (7) is zero. This follows from Proposition 2(i), which shows that the competitor's equilibrium information acquisition investment is independent of the firm's information sharing choice (i.e., $\partial\delta_j^*/\partial\rho_i = 0$). Finally, Proposition 3 shows that the last term of (7) is positive. Hence, information sharing is also a dominant strategy in our model.

The propositions have another implication. We can decompose the effects of information sharing on the profit of a firm's competitor as follows:

$$\frac{d\Pi_i(\delta_i^*, \delta_j^*; \rho_i, \rho_j)}{d\rho_j} = \frac{\partial\Pi_i(\delta_i^*, \delta_j^*; \rho_i, \rho_j)}{\partial\delta_i} \cdot \frac{\partial\delta_i^*}{\partial\rho_j} + \frac{\partial\Pi_i(\delta_i^*, \delta_j^*; \rho_i, \rho_j)}{\partial\delta_j} \cdot \frac{\partial\delta_j^*}{\partial\rho_j} + \frac{\partial\Pi_i(\delta_i^*, \delta_j^*; \rho_i, \rho_j)}{\partial\rho_j} > 0. \quad (8)$$

As before, Proposition 2(i) implies that the first term of (8) is zero (i.e., $\partial\delta_i^*/\partial\rho_j = 0$). The second term of (8) captures an indirect effect of information sharing. This effect is non-negative for the following reasons. First, a firm's profit is weakly increasing in the competitor's information acquisition investment (i.e., $\partial\Pi_i/\partial\delta_j \geq 0$), as Proposition 1(ii) shows. Second, Proposition 2(ii) shows that the competitor's investment is increasing in a firm's information-sharing probability (i.e., $\partial\delta_j^*/\partial\rho_j > 0$). Finally, the third term of (8) is positive due to Proposition 3. Hence, the overall effect of information sharing on a competitor's expected profit is positive. This observation implies that firms would also have an incentive to share information cooperatively (e.g., by entering a *quid pro quo* agreement), since information sharing increases the industry profits.

In short, firms will share information if they are allowed to do so.[10] In the remainder of this section we illustrate the intuition of Propositions 1-3 by means of a simple example.

[10]This result may also hold in a model in which the firms would make strategic information-sharing decisions (i.e., they choose whether to share or conceal after receiving their private signal), and the firms' signals are informative, see Okuno-Fujiwara *et al.* (1990).

Consider a simple version of our model in which two risk-neutral firms compete in quantities of a homogenous goods ($\beta = 1$). There is uncertainty only regarding firm 1's cost. Nature draws θ_1 from the set $\{\theta^l, \theta^h\}$ with equal probability and sends a private signal to firm 1:

$$S^\delta = \begin{cases} \theta_1 & \text{with probability } \delta_1 \\ \varnothing & \text{with probability } 1 - \delta_1. \end{cases}$$

Firm 2's cost θ_2 is common knowledge. Information sharing policies and information acquisition strategies are binary, i.e. $\delta_1, \rho_1 \in \{0, 1\}$.

Illustration of the Profit Results. In our binary example, we have to consider three regimes. First, there is the information sharing regime (s), in which firm 1 learns perfectly its cost and shares this information with its competitor (i.e., $\delta_1 = 1$, $\rho_1 = 1$). This allows both firms to adjust their outputs to the true productivity of firm 1. Fig. 1(a) illustrates this. In particular, if the firms learn that $\theta_1 = \theta^h$, then firm 1's best response is $r_1(x_2; \theta^h)$. Firm 2's best response is the bold curve $r_2(x_1; \theta_2)$. The equilibrium corresponds to point A. Similarly, if the firms learn that $\theta_1 = \theta^l$, then firm 1 expands its output by adopting best response $r_1(x_2; \theta^l)$ while firm 2 reduces its output, and they reach equilibrium point B. The output adjustments of the firms create output dispersion $\Delta x_1^s \equiv x_1^s(\theta^l) - x_1^s(\theta^h)$ for firm 1, and dispersion $\Delta x_2^s \equiv x_2^s(\theta^h) - x_2^s(\theta^l)$ for firm 2.

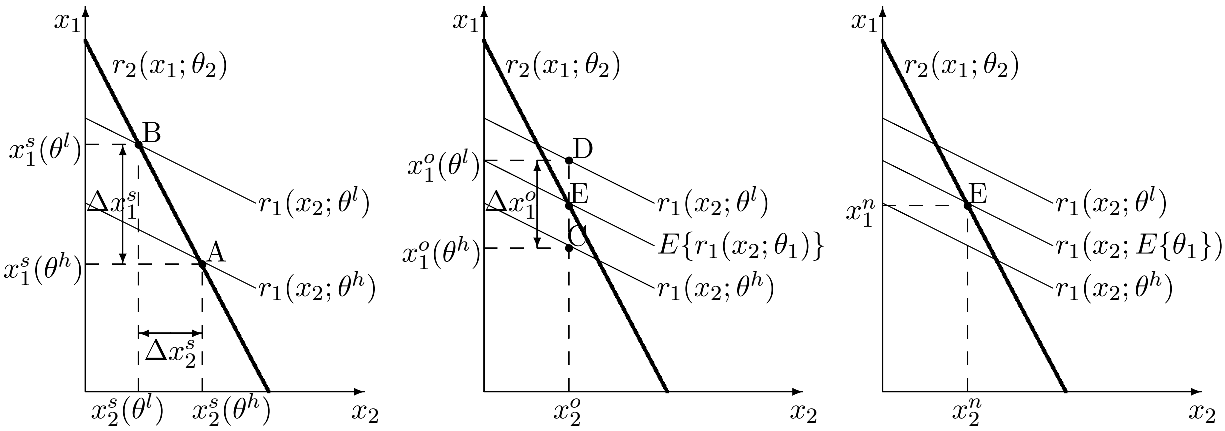

Fig. 1(a): regime (s) Fig. 1(b): regime (o) Fig. 1(c): regime (n)

Figure 1: Equilibrium output levels

Second, in the information concealment regime (o) firm 1 learns perfectly its cost and keeps this information secret from its competitor (i.e., $\delta_1 = 1$, $\rho_1 = 0$). Then, firm 1 adjusts its output level to its productivity while firm 2 can only base its output decision on the average productivity of 1. That is, firm 2 plays a best response against the expected best response of firm 1, $E\{r_1(x_2;\theta_1)\}$. This gives equilibrium output x_2^o for firm 2, which corresponds to point E in Fig. 1(b). In turn, firm 1 plays a best response against x_2^o, which is $x_1^o(\theta^h)$ if $\theta_1 = \theta^h$ (corresponding to point C), and $x_1^o(\theta^l)$ if $\theta_1 = \theta^l$ (i.e., point D). Fig. 1(b) shows that the dispersion of firm 1's output in regime (o) is smaller than in regime (s), i.e., $\Delta x_1^o \equiv x_1^o(\theta^l) - x_1^o(\theta^h) < \Delta x_1^s$. The greater dispersion in regime (s) is due to the fact that the output adjustments of firm 2 augment the adjustments of firm 1 towards its information. The distortion of equilibrium output (4) also captures this.

Finally, in the no information regime (n) firm 1 does not learn its cost and there is not information to transmit ($\delta_1 = 0$). Uninformed firms base their output choices on the average technology of firm 1. This gives firm 1 the best response $r_1(x_2; E\{\theta_1\})$, and yields the equilibrium in point E in Fig. 1(c). In this case, there is a single output level for firm 1 (i.e., $\Delta x_1^n = 0$).

The profit function of a firm (6) is convex in the firm's output level. Hence, firms 1 and 2 prefer regime (s) to regime (o) since the dispersions of their outputs are larger in the former regime (Proposition 3), i.e., $\Delta x_1^s > \Delta x_1^o$ and $\Delta x_2^s > 0 = \Delta x_2^o$. For the same token, Propositions 1 and 2(ii) are captured in this example, by comparing the increase in firm 1's profits from the no information regime to either the regime (s) or regime (o). These increases in profits are also related to the increase in output dispersion. Clearly, information acquisition gives more dispersed outputs for the firms (Proposition 1), i.e., $\Delta x_1^r > 0 = \Delta x_1^n$ and $\Delta x_2^r \geq 0 = \Delta x_2^n$ for $r \in \{s, o\}$. Moreover, firm 1's profit increases more (and consequently the firm has a bigger incentive to invest in acquiring information) when it moves from regime (n) to regime (s) than when it moves from regime (n) to regime (o), i.e., $\Delta x_1^s - \Delta x_1^n > \Delta x_1^o - \Delta x_1^n$, as Proposition 2(ii) shows in general.

5 Expected Consumer Surplus

Using the definition of the surplus v in (2) for a bundle of outputs (x_1, x_2), we denote the expected consumer surplus for exogenously given information acquisition levels as follows:

$$\begin{aligned} V(\delta_i, \delta_j; \rho_i, \rho_j) &\equiv E_{s_i,m_i}\left\{E_{s_j,m_j}\left[v\left(x_1^*(s_1; m_1, m_2), x_2^*(s_2; m_2, m_1)\right)\right]\right\} \\ &= \frac{1}{2}E_{s_i,m_i}\left\{E_{s_j,m_j}\left[\left(x_i^*(s_i; m_i, m_j) + x_j^*(s_j; m_j, m_i)\right)^2\right]\right\} \\ &\quad -(1-\beta)E_{s_i,m_i}\left\{E_{s_j,m_j}\left[x_i^*(s_i; m_i, m_j) x_j^*(s_j; m_j, m_i)\right]\right\} \end{aligned} \tag{9}$$

Below we analyze the effects of information sharing and acquisition on this expected surplus.

5.1 Consumer Surplus Properties

The next proposition establishes a basic property of the consumer surplus in our framework.

PROPOSITION 4 *Surplus* $V(\delta_i, \delta_j; \rho_i, \rho_j)$ *is decreasing in* ρ_k *and increasing in* δ_k *for any* $k \in \{i, j\}$.

These surplus results are consequences of the quantity adjustment effect, and the preference for variety effect (e.g., Kühn and Vives (1995)). Below we explain and illustrate the results in greater detail by means of the binary example.

Illustration of the Surplus Results. We return to the binary example of section 4.4. Figure 2 illustrates the first part of Proposition 4 by comparing the surpluses in regimes (s) and (o).

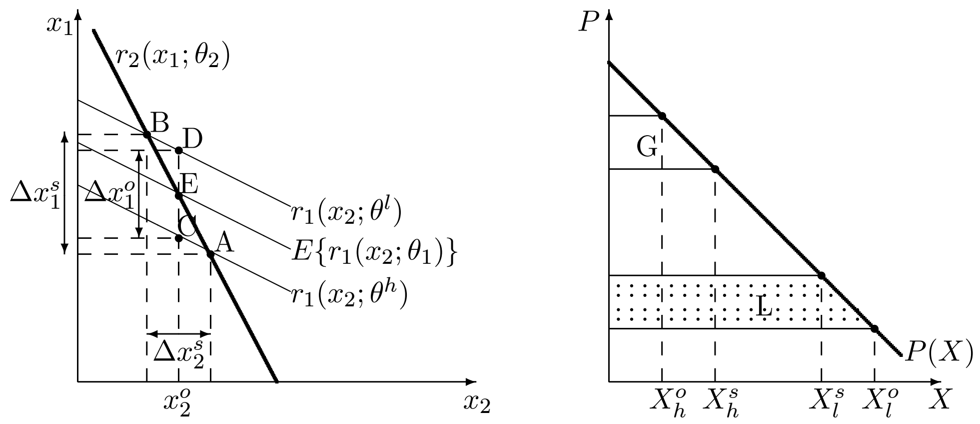

Fig. 2(a): Equilibrium Output Variability Fig. 2(b): Effects on consumer surplus

Figure 2: Consumer surplus and information sharing

In our example, where goods are homogenous, only the first term of (9) matters. As a consequence, the consumer surplus is simply increasing in the dispersion of the total industry output since it is a convex function of $x_1^* + x_2^*$. Figure 2(a) illustrates that the dispersion in industry output is lower in regime (s) than in regime (o), since $\Delta X^s = \Delta x_1^s - \Delta x_2^s < \Delta x_1^o = \Delta X^o$. In words, output adjustment by firm 2 in the information-sharing regime, countervails firm 1's adjustment, which creates lower variability of industry outputs than in the concealing regime. Fig. 2(b) illustrates how a lower dispersion of output in the regime (s) leads to a lower consumer surplus than in regime (o). The areas L and G represent respectively the loss (when $\theta_1 = \theta^l$) and the gain (when $\theta_1 = \theta^h$) between regime (s) and (o), and it is clear that area L is larger than area G. Hence, on average consumers are worse off in regime (s).

The illustration of the second part of the proposition is direct in our example, given the convexity of the consumer surplus function. There is no dispersion of output in the no information regime (n). Hence, the consumer surplus is lower than in either regime (s) or (o). These effects, related to the first term of (9), capture the quantity adjustment effect (Kühn and Vives (1995)).

Notice that the second term of (9) is related to covariance of the firms' output levels and conflicts with the effects illustrated in the example and the statement of Proposition 4. Sharing information and acquiring information both reduce the covariance of firms output which increases consumer surplus. This is called the preference for variety effect (Kühn and Vives (1995)). The proof of Proposition 4 shows that the quantity adjustment effect due to the first term of (9) dominates this second effect.

5.2 Consumer Surplus Trade-off

The effect of information sharing on the consumer surplus can be decomposed as follows:

$$\frac{dV(\delta_i^*, \delta_j^*; \rho_i, \rho_j)}{d\rho_i} = \frac{\partial V(\delta_i^*, \delta_j^*; \rho_i, \rho_j)}{\partial \delta_i} \cdot \frac{\partial \delta_i^*}{\partial \rho_i} + \frac{\partial V(\delta_i^*, \delta_j^*; \rho_i, \rho_j)}{\partial \delta_j} \cdot \frac{\partial \delta_j^*}{\partial \rho_i} + \frac{\partial V(\delta_i^*, \delta_j^*; \rho_i, \rho_j)}{\partial \rho_i}. \quad (10)$$

This decomposition yields an interesting trade-off. On the one hand, information sharing has a negative direct effect on the consumer surplus, as we show in Proposition 4. The last term of (10) captures this effect (i.e., $\partial V/\partial \rho_i < 0$). Therefore, if the precision were exogenously given, then information sharing should be prohibited. On the other hand, information sharing has a positive indirect effect on the consumer surplus. It increases the incentives to invest in information acquisition (i.e., $\partial \delta_i^*/\partial \rho_i > 0$ as Proposition 2(ii) shows). Higher investments increase the expected consumer surplus (i.e., $\partial V/\partial \delta_i > 0$ by Proposition 4). The first term of (10) captures this positive, indirect effect. The second term of (10) is zero, since $\partial \delta_j^*/\partial \rho_i = 0$ by Proposition 2(i).

Hence, when the signal's precision is not exogenous, but determined endogenously by information acquisition investments, the antitrust authority's choice (between allowing and disallowing information sharing) should depend on the trade-off between these two conflicting effects. In fact, it is possible that the second effect outweighs the first effect, as we illustrate below.

Illustration of the Trade-off: Information Sharing May Increase Consumer Surplus. In the example of section 4.4, the expected consumer surpluses under information sharing ($\rho_1 = 1$) and concealment ($\rho_1 = 0$) are, respectively:

$$
\begin{aligned}
V(\delta_1; 1) &= \frac{1}{2}\left(\delta_1 E_{\theta_1}\left\{[x_1^*(\theta_1; \theta_1, \theta_2) + x_2^*(\theta_2; \theta_2, \theta_1)]^2\right\} + (1-\delta_1)\,[x_1^*(\varnothing; \varnothing, \theta_2) + x_1^*(\theta_2; \theta_2, \varnothing)]^2\right) \\
V(\delta_1; 0) &= \frac{1}{2}\left(\delta_1 E_{\theta_1}\left\{[x_1^*(\theta_1; \varnothing, \theta_2) + x_2^*(\theta_2; \theta_2, \varnothing)]^2\right\} + (1-\delta_1)\,[x_1^*(\varnothing; \varnothing, \theta_2) + x_1^*(\theta_2; \theta_2, \varnothing)]^2\right)
\end{aligned}
$$

We illustrate these surpluses by means of Figure 3. The figure illustrates that information sharing decreases the surplus (i.e., $V(\delta_1; 1) \leq V(\delta_1; 0)$ for any δ_1), and information acquisition increases the surplus (i.e., $V(1; \rho_1) > V(0; \rho_1)$ for any ρ_1), as Proposition 4 shows in general.

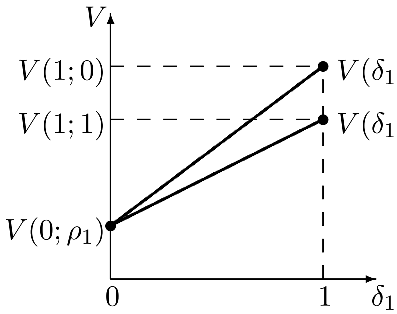

Figure 3: Trade-off for consumers

Figure 3 suggests that the indirect effect of information sharing may be the dominant effect. The next proposition gives a condition such that our example delivers this result.

Proposition 5 *Consider the binary example of section 4.4 with the cost of acquiring information* $c(0) = 0$ *and* $c(1) = \lambda$. *Then* $V(\delta_1^*(1); 1) > V(\delta_1^*(0); 0)$, *if* $\frac{(\theta^h - \theta^l)^2}{16} < \lambda < \frac{(\theta^h - \theta^l)^2}{9}$.

Whereas the direct effect of information sharing generates a lower surplus for given investments in information acquisition than concealment (i.e., $V(\delta_1; 1) \leq V(\delta_1; 0)$ as illustrated for $\delta_1 = 1$), the indirect effect favors information sharing. When λ satisfies the condition of Proposition 5, firm 1 acquires information only if it is allowed to share information, i.e., $\delta_1^*(1) = 1 > 0 = \delta_1^*(0)$. This favors information sharing, since the more information firm 1 acquires, the larger the surplus (i.e., $V(1; 1) > V(0; 1)$). The indirect effect dominates, since $V(1; 1) > V(0; 1) = V(0; 0)$.

By contrast, if λ is lower than $\frac{(\theta^h - \theta^l)^2}{16}$, firm 1 always acquires information, and then the direct effect implies that $V(1; 1) < V(1; 0)$. If λ is larger than $\frac{(\theta^h - \theta^l)^2}{9}$, the firm acquires information neither with information sharing nor without it, and then $V(0; 1) = V(0; 0)$.

6 Extensions

In this section, we extend our analysis in three directions. First, we allow for more than two firms. Second, we analyze the effects of competition in prices (Bertrand competition) instead of outputs. Finally, we discuss the effects of introducing correlation between the firms' costs.

6.1 Oligopoly

Our model assumes that there is competition between only two firms. This is without loss of generality, since an oligopoly model yields qualitatively identical results.

In a model with N firms and goods, with $N \geq 2$, the representative consumer's gross surplus from consuming bundle $(x_1, .., x_N)$ is:

$$u(x_1, .., x_N) \equiv \alpha \sum_{\ell=1}^{N} x_\ell - \frac{1}{2}\left(\sum_{\ell=1}^{N} x_\ell\right)^2 + \frac{1}{2}(1-\beta)\sum_{\ell=1}^{N} x_\ell \sum_{k\neq\ell} x_k.$$

As before, the inverse demand function for good i is linear: $P_i(x_i, x_{-i}) = \alpha - x_i - \beta \sum_{j\neq i} x_j$, where $x_{-i} \equiv (x_1, .., x_{i-1}, x_{i+1}, .., x_N)$. Firm i's profit of producing quantity x_i is simply $\pi_i(x_i, x_{-i}; \theta_i) = [P_i(x_i, x_{-i}) - \theta_i]\, x_i$ for $i = 1, .., N$. For any combination of messages $m_1, .., m_N$, firm i with signal s_i maximizes its expected profit, which yields the following first-order condition:

$$x_i(s_i) = \frac{1}{2}\left(\alpha - E\{\theta_i|s_i\} - \beta \sum\nolimits_{j\neq i} E\{x_j(s_j)|m_j\}\right) \tag{11}$$

for $i, j = 1, .., N$ with $j \neq i$. Solving the system of equations (11) for $i = 1, .., N$ gives the following equilibrium output level of firm i (for $i, j = 1, .., N$ with $j \neq i$):

$$\begin{aligned} x_i^*(s_i; m_i, m_{-i}) \quad = \quad & \frac{1}{[2+(N-1)\beta](2-\beta)}\Bigg((2-\beta)\alpha - [2+(N-2)\beta]E\{\theta_i|s_i\} + \beta \sum_{j\neq i} E\{\theta_j|m_j\} \\ & +(N-1)\frac{\beta^2}{2}\left[E\{\theta_i|s_i\} - E\{\theta_i|m_i\}\right]\Bigg), \end{aligned} \tag{12}$$

where $E\{\theta_i|m_i\} = E_{s_i}\{E(\theta_i|s_i)|m_i\}$ and $m_{-i} \equiv (m_1, .., m_{i-1}, m_{i+1}, .., m_N)$. As before, information concealment creates a distortion, as captured by the last term of (12). This distortion dampens the sensitivity of the equilibrium outputs to the precision of information, which gives supermodular expected profits, and equilibrium strategies as in section 4 (see Appendix B).

The consumption of the bundle $(x_1, .., x_N)$ gives the representative consumer a net surplus of:

$$v(x_1, .., x_N) \equiv u(x_1, .., x_N) - \sum_{i=1}^{N} P_i(x_i, x_{-i})x_i = \frac{1}{2}\left[\left(\sum_{\ell=1}^{N} x_\ell\right)^2 - (1-\beta)\sum_{\ell=1}^{N} x_\ell \sum_{k\neq\ell} x_k\right]. \quad (13)$$

As before, the first term captures the quantity adjustment effect, and the second term is related to the preference for variety effect. This gives essentially the same trade-off for consumers as we described in the duopoly model. In Appendix B we show formally that the same qualitative results emerge with more than two firms.

6.2 Bertrand Competition

We briefly consider the model where firms choose prices, $p_i \geq 0$ for $i = 1, 2$ (Bertrand competition), instead of output levels. The system of inverse demand functions gives the following direct demand function (for $i, j = 1, 2$ and $i \neq j$):

$$D_i(p_i, p_j) = \frac{1}{1-\beta^2}\bigg((1-\beta)\alpha + \beta p_j - p_i\bigg) \quad (14)$$

Firm i maximizes its profit $\pi_i(p_i, p_j; \theta_i) \equiv (p_i - \theta_i)D_i(p_i, p_j)$.

6.2.1 Equilibrium Choices Each firm chooses its price on the basis of its own information, s_i, and the information received from its competitor, $m_j \in \{s_j, \varnothing\}$. We adopt the same notation for conditional and unconditional expectations as before without making explicit the dependence of s_i on δ_i.

For any combination of messages m_i and m_j, firm i with signal s_i maximizes its expected profit, which yields the following first-order condition:

$$p_i(s_i) = \frac{1}{2}\bigg((1-\beta)\alpha + E\{\theta_i|s_i\} + \beta E\{p_j(s_j)|m_j\}\bigg) \quad (15)$$

for $i, j = 1, 2$ with $i \neq j$. Solving the system of equations (15) for $i = 1, 2$ gives the following equilibrium price of firm i (for $i, j = 1, 2$ with $i \neq j$):

$$p_i^*(s_i; m_i, m_j) = \frac{1}{4-\beta^2}\left((2+\beta)(1-\beta)\alpha + 2E\{\theta_i|s_i\} + \beta E\{\theta_j|m_j\} - \frac{\beta^2}{2}\left[E\{\theta_i|s_i\} - E\{\theta_i|m_i\}\right]\right) \quad (16)$$

where $E\{\theta_i|m_i\} = E_{s_i}\{E(\theta_i|s_i)|m_i\}$. In equilibrium, firm i's output level and profit relate as follows to the equilibrium price-cost margin:

$$x_i^b(s_i;m_i,m_j) = \frac{p_i^*(s_i;m_i,m_j) - E\{\theta_i|s_i\}}{1-\beta^2} \text{ and} \tag{17}$$

$$\pi_i^b(s_i;m_i,m_j) = \frac{[p_i^*(s_i;m_i,m_j) - E\{\theta_i|s_i\}]^2}{1-\beta^2}, \tag{18}$$

respectively. Hence, the expected profit of firm i is:

$$\Pi_i^b(\delta_i,\delta_j;\rho_i,\rho_j) \equiv \frac{1}{1-\beta^2} E_{s_i,m_i}\left\{E_{s_j,m_j}\left[(p_i^*(s_i;m_i,m_j) - E\{\theta_i|s_i\})^2\right]\right\} - c(\delta_i) \tag{19}$$

These equilibrium profits determine the firm's incentive to acquire and share information. In particular, we can show the following property (see Appendix B).

LEMMA 2 *The expected profit* $\Pi_i^b(\delta_i,\delta_j;\rho_i,\rho_j)$ *is submodular in* (δ_i,ρ_i).

In other words, for $\rho_i > \rho_i'$, the difference $\Pi_i(\delta_i,\delta_j;\rho_i,\rho_j) - \Pi_i(\delta_i,\delta_j;\rho_i',\rho_j)$ is weakly decreasing in δ_i for all δ_j and ρ_j. This implies that information-sharing firms have a smaller incentive to acquire information than concealing firms. Notice that this is the reverse result of Lemma 1. As it turns out, this property reverses the indirect effect of information sharing on the expected consumer surplus.

6.2.2 Consumer Surplus The qualitative properties of the expected consumer surplus in equilibrium are identical to those in Proposition 4 (see Appendix B). This gives the following overall effect of information sharing on the expected consumer surplus. For a given precision of the firms' signals, information sharing decreases the expected consumer surplus. This is a direct effect of information sharing. Moreover, information sharing reduces the firms' investments in acquiring information (Lemma 2), which reduces the expected surplus even further. In other words, the indirect effect reinforces the direct effect of information sharing on the surplus when firms compete in prices.

6.3 Correlated Costs

We have analyzed an independent private value framework. In this framework, information acquisition creates an indirect effect of information sharing on the consumer surplus. This gives a trade-off between a negative direct effect and positive indirect effect. Now we briefly discuss the effects of introducing cost correlation.

Analyzing a model of imperfect positive correlation is complex, for the reasons mentioned below. However, it is tractable and illuminating to analyze a setting in which firms have perfectly correlated costs. In such a situation, information sharing also yields a trade-off between a direct and indirect effect on the consumer surplus. However, as Figure 4 illustrates, the directions of both effects are reversed.

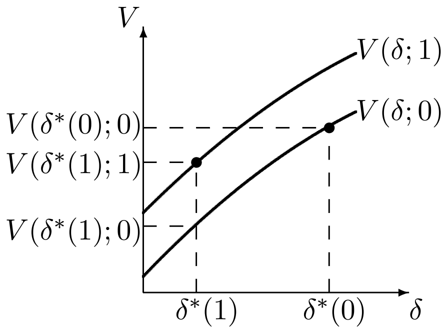

Figure 4: Perfect positive correlation

Vives (1984) shows that the direct effect of information sharing on consumer surplus is positive (i.e., $V(\delta;1) > V(\delta;0)$ as Figure 4 illustrates for $\delta = \delta^*(1)$). The paper shows that, with perfect correlation, the dispersion of the industry output is larger when both firms adjusts their output levels to a common cost shock (similarly to a common demand shock). Information sharing also creates an indirect effect, as in the model with independent costs. Vives (1984) also shows that, similarly to present model, the more accurate is the firms' information, the larger is the consumer surplus (i.e., $V(\delta;1)$ and $V(\delta;0)$ are increasing in δ). Previously, information sharing gives the firms a greater incentive to acquire information, since the output adjustments of the competitor increases a firm's own output dispersion. However, the more correlated are the firms' costs, the less important is this effect. In addition, cost correlation creates a free-riding problem, since firms may use the information of their competitors to learn about their own cost. Jansen (2008) shows that with perfectly correlated costs the free-rider effect dominates, and sharing information about a common cost parameter leads to lower information acquisition investments ($\delta^*(1) < \delta^*(0)$ as in Figure 4). Therefore, information sharing has a negative indirect effect on the consumer surplus.[11]

[11] In fact, it can be shown that in the model corresponding to the binary example, the indirect effect can dominate the direct effect of information sharing, as Figure 4 illustrates.

In other words, the introduction of perfect cost correlation reverses the direction of both the direct and indirect effects compared with independent private value setting. This make the task of analyzing an imperfect correlation framework very difficult, since it is likely that the signs of the direct and indirect effect are going to depend not only on the degree of cost correlation but also on the information structure that we use to set up the model.

7 Conclusion

We have shown that the incentives of acquiring information are larger when firms in a Cournot oligopoly are allowed to share information. A higher information acquisition investment increases the consumer surplus. These observations have important implications for an antitrust authority's choice between allowing and disallowing information sharing. Whereas conventional wisdom predicts that information sharing reduces consumer surplus, our observations predict a surplus increase from information sharing. Overall, the trade-off between the positive and negative effects of information sharing can make the consumer surplus larger when firms are allowed to share information.

In the paper we used the expected consumer surplus as welfare measure. This enables us to distinguish the effects on firms from the effects on consumers. A more general welfare measure would be a weighed sum of consumer surplus and producer surplus (i.e., industry profits). The adoption of such a general welfare function would not change the main conclusion of the paper, since information sharing gives a positive indirect effect on the producer surplus too. Information sharing gives higher information acquisition investments than information concealment (Proposition 2(ii)). The higher investment under information sharing increases the industry profit gain from information sharing (Proposition 1(ii)). In other words, the indirect effects of information sharing on consumers and producers are aligned, and favor information sharing.

Finally, we want to stress that we have undertaken the analysis using general information structures and new information orderings based on dispersion measures. This methodological approach allows us to show that our results and the results of previous literature crucially depend on the convexity of consumers' and firms' objective functions over output, as well as on the effect of information on the dispersion of equilibrium output.

A Appendix

We make repeated use of the following result.

Lemma 3 *If δ ranks signals according to Integral Precision, then the variance of $E_i[\theta|S_i^\delta]$ is increasing in δ.*

Proof of Lemma 3:
The variance of $E_i[\theta|S_i^\delta]$ is equal to: $\text{Var}(E_i[\theta|S_i^\delta]) = E\{(E_i[\theta|S_i^\delta] - \overline{\theta_i})^2\}$. Given that $(E_i[\theta|S_i^\delta] - \overline{\theta_i})^2$ is a convex function of $E_i[\theta|S_i^\delta]$, the result is a direct implication of the definitions of the convex order and integral precision. ■

Notice that: $\text{Var}(E_i[\theta|S_i^\delta]) = E\{(E_i[\theta|S_i^\delta] - \overline{\theta_i})^2\} = E\{E_i[\theta|S_i^\delta]^2\} - \overline{\theta_i}^2$. Then, by Lemma 3, $E\{E_i[\theta|S_i^\delta]^2\}$ is increasing in δ.

For the proofs of Lemma 1, and Propositions 1-3 it is convenient to rewrite the expected profit (6). First, by using the definition of $x_i^*(s_i; m_i, m_j)$ in (4), we can rewrite $E_{s_i}\left\{E_{s_j,m_j}[x_i^*(s_i; \varnothing, m_j)^2]\right\}$ as follows:

$$E_{s_i}\left\{E_{s_j,m_j}[x_i^*(s_i;\varnothing,m_j)^2]\right\} = E_{s_i}\left\{E_{s_j,m_j}\left[\left(x_i^*(s_i;s_i,m_j) + \frac{\beta^2}{2(4-\beta^2)}\left[E\{\theta_i|s_i\} - \overline{\theta_i}\right]\right)^2\right]\right\}$$

$$\begin{aligned}
&= E_{s_i}\left\{E_{s_j,m_j}\left[x_i^*(s_i;s_i,m_j)^2\right]\right\} \\
&\quad + \frac{\beta^2}{2(4-\beta^2)} E_{s_i}\left\{\left[E\{\theta_i|s_i\} - \overline{\theta_i}\right] E_{s_j,m_j}\left[2x_i^*(s_i;s_i,m_j) + \frac{\beta^2}{2(4-\beta^2)}\left(E\{\theta_i|s_i\} - \overline{\theta_i}\right)\right]\right\} \\
&= E_{s_i}\left\{E_{s_j,m_j}\left[x_i^*(s_i;s_i,m_j)^2\right]\right\} \\
&\quad + \frac{\beta^2}{4(4-\beta^2)^2} E_{s_i}\left\{\left[E\{\theta_i|s_i\} - \overline{\theta_i}\right]\left[4(2-\beta)\alpha + 4\beta\overline{\theta_j} - \beta^2\overline{\theta_i} - (8-\beta^2)E\{\theta_i|s_i\}\right]\right\} \\
&= E_{s_i}\left\{E_{s_j,m_j}\left[x_i^*(s_i;s_i,m_j)^2\right]\right\} - \frac{\beta^2(8-\beta^2)}{4(4-\beta^2)^2}\left(E_{s_i}\left\{E\left[\theta_i|s_i\right]^2\right\} - \overline{\theta_i}^2\right). \qquad (20)
\end{aligned}$$

In this last simplification, we use the property that $E_{s_i}\left\{E\left[\theta_i|s_i\right] - \overline{\theta_i}\right\} = 0$. Then, any constant multiplied by $E_{s_i}\left\{E\left[\theta_i|s_i\right] - \overline{\theta_i}\right\}$ is also equal to 0. Second, we can rewrite $E_{s_i,m_i}\{x_i^*(s_i;m_i,\varnothing)^2\}$ as follows:

$$E_{s_i,m_i}\{x_i^*(s_i;m_i,\varnothing)^2\} = E_{s_i,m_i}\left\{E_{s_j}\left[\left(x_i^*(s_i;m_i,s_j) - \frac{\beta}{4-\beta^2}\left[E\{\theta_j|s_j\} - \overline{\theta_j}\right]\right)^2\right]\right\}$$

$$\begin{aligned}
&= E_{s_i,m_i}\left\{E_{s_j}[x_i^*(s_i;m_i,s_j)^2]\right\} \\
&\quad - \frac{\beta}{4-\beta^2} E_{s_j}\left\{\left[E\{\theta_j|s_j\} - \overline{\theta_j}\right] E_{s_i,m_i}\left[2x_i^*(s_i;m_i,s_j) - \frac{\beta}{4-\beta^2}\left[E\{\theta_j|s_j\} - \overline{\theta_j}\right]\right]\right\} \\
&= E_{s_i,m_i}\left\{E_{s_j}[x_i^*(s_i;m_i,s_j)^2]\right\} \\
&\quad - \frac{\beta}{(4-\beta^2)^2} E_{s_j}\left\{\left[E\{\theta_j|s_j\} - \overline{\theta_j}\right]\left(2(2-\beta)\alpha - 4\overline{\theta_i} + \beta\left[E\{\theta_j|s_j\} + \overline{\theta_j}\right]\right)\right\} \\
&= E_{s_i,m_i}\left\{E_{s_j}[x_i^*(s_i;m_i,s_j)^2]\right\} - \left(\frac{\beta}{4-\beta^2}\right)^2\left(E_{s_j}\left\{E\left[\theta_j|s_j\right]^2\right\} - \overline{\theta_j}^2\right). \qquad (21)
\end{aligned}$$

As before, in the last two simplifications, we use the property that $E_{s_j}\left\{E\left[\theta_j|s_j\right]-\overline{\theta_j}\right\}=0$ for any j. Using (20) and (21), the expected profit (6) simplifies as follows:

$$\begin{aligned}
\Pi_i(\delta_i,\delta_j;\rho_i,\rho_j) &= \rho_i E_{s_i}\left\{E_{s_j,m_j}[x_i^*(s_i;s_i,m_j)^2]\right\}+(1-\rho_i)E_{s_i}\left\{E_{s_j,m_j}[x_i^*(s_i;\varnothing,m_j)^2]\right\}-c(\delta_i)\\
&= E_{s_i}\left\{E_{s_j,m_j}[x_i^*(s_i;\varnothing,m_j)^2]\right\}+\rho_i\frac{\beta^2(8-\beta^2)}{4(4-\beta^2)^2}\mathrm{Var}(E_i[\theta|S_i^\delta])-c(\delta_i)\\
&= \rho_j E_{s_i}\left\{E_{s_j}[x_i^*(s_i;\varnothing,s_j)^2]\right\}+(1-\rho_j)E_{s_i}\left\{x_i^*(s_i;\varnothing,\varnothing)^2\right\}\\
&\quad+\rho_i\frac{\beta^2(8-\beta^2)}{4(4-\beta^2)^2}\mathrm{Var}(E_i[\theta|S_i^\delta])-c(\delta_i)\\
&= E_{s_i}\left\{x_i^*(s_i;\varnothing,\varnothing)^2\right\}+\rho_j\left(\frac{\beta}{4-\beta^2}\right)^2\mathrm{Var}(E_j[\theta|S_j^\delta])\\
&\quad+\rho_i\frac{\beta^2\left(8-\beta^2\right)}{4\left(4-\beta^2\right)^2}\mathrm{Var}(E_i[\theta|S_i^\delta])-c(\delta_i).
\end{aligned}\tag{22}$$

Proof of Proposition 1:

Using (4), we can rewrite the first term of (22) as follows:

$$\begin{aligned}
E_{s_i}\left\{x_i^*(s_i;\varnothing,\varnothing)^2\right\} &= \frac{1}{\left(4-\beta^2\right)^2}E_{s_i}\left\{\left((2-\beta)\alpha-2\overline{\theta_i}+\beta\overline{\theta_j}-\frac{1}{2}\left(4-\beta^2\right)\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]\right)^2\right\}\\
&= \left(\frac{(2-\beta)\alpha-2\overline{\theta_i}+\beta\overline{\theta_j}}{4-\beta^2}\right)^2+\frac{1}{4}E_{s_i}\left\{\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]^2\right\}.
\end{aligned}\tag{23}$$

In the last simplification, we use the property that $E_{s_i}\left\{\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]\right\}=0$.

(i) Lemma 3 implies that (23) is increasing in δ_i. Further, the second term of (22) is independent of δ_i, while the third term is increasing in δ_i. Hence, $\Pi_i(\delta_i,\delta_j;\rho_i,\rho_j)+c(\delta_i)$ is increasing in δ_i.

(ii) The second term of (22) is (weakly) increasing in δ_j by Lemma 3. The remaining terms are independent of δ_j (see (23) for the first term). Hence, Π_i is weakly increasing in δ_j. ■

Proof of Lemma 1:

For any $\rho_i,\rho_i'\in[0,1]$, expression (22) gives:

$$\Pi_i(\delta_i,\delta_j;\rho_i,\rho_j)-\Pi_i(\delta_i,\delta_j;\rho_i',\rho_j)=(\rho_i-\rho_i')\frac{\beta^2(8-\beta^2)}{4(4-\beta^2)^2}\mathrm{Var}(E_i[\theta|S_i^\delta]).$$

For $\rho_i>\rho_i'$, this expression is increasing in δ_i by Lemma 3. ■

Proof of Proposition 2:

(i) We want to show that $\partial^2\Pi_i(\delta_i,\delta_j;\rho_i,\rho_j)/(\partial\delta_i\partial\rho_j)=0$. It follows from (22) that:

$$\frac{\partial\Pi_i(\delta_i,\delta_j;\rho_i,\rho_j)}{\partial\rho_j}=\left(\frac{\beta}{4-\beta^2}\right)^2\left(E_{s_j}\left\{E\left[\theta_j|s_j\right]^2\right\}-\overline{\theta_j}^2\right).$$

As $\partial\Pi_i/\partial\rho_j$ is independent of δ_i, we have $\frac{\partial^2\Pi_i(\delta_i,\delta_j;\rho_i,\rho_j)}{\partial\delta_i\partial\rho_j}=0$ which concludes the proof.

(ii) The result follows from Theorem 4 of Milgrom and Shanon (1994) and Lemma 1. ■

Proof of Proposition 3:
It follows directly from (22) that:

$$\begin{aligned}\frac{\partial \Pi_i(\delta_i,\delta_j;\rho_i,\rho_j)}{\partial \rho_i} &= \frac{\beta^2(8-\beta^2)}{4(4-\beta^2)^2}\mathrm{Var}(E_i[\theta|S_i^\delta]) \geq 0, \text{ and}\\ \frac{\partial \Pi_i(\delta_i,\delta_j;\rho_i,\rho_j)}{\partial \rho_j} &= \left(\frac{\beta}{4-\beta^2}\right)^2\mathrm{Var}(E_j[\theta|S_j^\delta]) \geq 0.\end{aligned}$$

■

Proof of Proposition 4:
The expected consumer surplus can be rewritten as follows (for $i,j=1,2$ and $i \neq j$):

$$\begin{aligned}V(\cdot) = \rho_i E_{s_i}\left\{E_{s_j,m_j}\left[\frac{1}{2}\left(x_i^*(s_i;s_i,m_j)+x_j^*(s_j;m_j,s_i)\right)^2-(1-\beta)x_i^*(s_i;s_i,m_j)x_j^*(s_j;m_j,s_i)\right]\right\}\\ +(1-\rho_i)E_{s_i}\left\{E_{s_j,m_j}\left[\frac{1}{2}\left(x_i^*(s_i;\varnothing,m_j)+x_j^*(s_j;m_j,\varnothing)\right)^2-(1-\beta)x_i^*(s_i;\varnothing,m_j)x_j^*(s_j;m_j,\varnothing)\right]\right\}\end{aligned}$$

Differentiating with respect to ρ_i gives (for $i,j=1,2$ and $i \neq j$):

$$\begin{aligned}\frac{\partial V}{\partial \rho_i} &= \frac{1}{2}E_{s_i}\left\{E_{s_j,m_j}\left[\left(x_i^*(s_i;s_i,m_j)+x_j^*(s_j;m_j,s_i)\right)^2-\left(x_i^*(s_i;\varnothing,m_j)+x_j^*(s_j;m_j,\varnothing)\right)^2\right]\right\}\\ &\quad -(1-\beta)E_{s_i}\left\{E_{s_j,m_j}\left[x_i^*(s_i;s_i,m_j)x_j^*(s_j;m_j,s_i)-x_i^*(s_i;\varnothing,m_j)x_j^*(s_j;m_j,\varnothing)\right]\right\} \qquad (24)\end{aligned}$$

The first line of this expression can be simplified by using the following for $m_j \in \{s_j,\varnothing\}$:

$$\begin{aligned}&E_{s_i}\left\{E_{s_j,m_j}\left[\left(x_i^*(s_i;\varnothing,m_j)+x_j^*(s_j;m_j,\varnothing)\right)^2\right]\right\}\\ &= E_{s_i}\left\{E_{s_j,m_j}\left[\left(x_i^*(s_i;s_i,m_j)+x_j^*(s_j;m_j,s_i)-\frac{\beta(2-\beta)}{2(4-\beta^2)}\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]\right)^2\right]\right\}\\ &= E_{s_i}\left\{E_{s_j,m_j}\left[\left(x_i^*(s_i;s_i,m_j)+x_j^*(s_j;m_j,s_i)\right)^2\right]\right\}\\ &\quad -\frac{\beta}{2(2+\beta)}E_{s_i}\left\{\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]E_{s_j,m_j}\left[2\left(x_i^*(s_i;s_i,m_j)+x_j^*(s_j;m_j,s_i)\right)-\frac{\beta\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]}{2(2+\beta)}\right]\right\}\\ &= E_{s_i}\left\{E_{s_j,m_j}\left[\left(x_i^*(s_i;s_i,m_j)+x_j^*(s_j;m_j,s_i)\right)^2\right]\right\}+\frac{\beta(4+\beta)}{4(2+\beta)^2}E_{s_i}\left\{\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]^2\right\} \qquad (25)\end{aligned}$$

The second line of (24) can be simplified by using the following for $m_j \in \{s_j,\varnothing\}$:

$$\begin{aligned}&E_{s_i}\left\{E_{s_j,m_j}\left[x_i^*(s_i;\varnothing,m_j)x_j^*(s_j;m_j,\varnothing)\right]\right\}\\ &= E_{s_i}\left\{E_{s_j,m_j}\left[\left(x_i^*(s_i;s_i,m_j)+\frac{\beta^2\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]}{2(4-\beta^2)}\right)\left(x_j^*(s_j;m_j,s_i)-\frac{\beta\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]}{4-\beta^2}\right)\right]\right\}\\ &= E_{s_i}\left\{E_{s_j,m_j}\left[x_i^*(s_i;s_i,m_j)x_j^*(s_j;m_j,s_i)\right]\right\}+E_{s_i}\left\{\frac{\beta^2\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]}{2(4-\beta^2)}E_{s_j,m_j}\left[x_j^*(s_j;m_j,s_i)\right]\right\}\\ &\quad -E_{s_i}\left\{\frac{\beta\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]}{4-\beta^2}E_{s_j,m_j}\left[x_i^*(s_i;s_i,m_j)\right]\right\}-\frac{\beta^3}{2(4-\beta^2)^2}E_{s_i}\left\{\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]^2\right\}\\ &= E_{s_i}\left\{E_{s_j,m_j}\left[x_i^*(s_i;s_i,m_j)x_j^*(s_j;m_j,s_i)\right]\right\}+\frac{2\beta}{(4-\beta^2)^2}E_{s_i}\left\{\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]^2\right\} \qquad (26)\end{aligned}$$

Substitution of (25) and (26) for $m_j \in \{s_j, \varnothing\}$ in (24) gives:

$$\begin{aligned}
\frac{\partial V}{\partial \rho_i} &= -\frac{1}{2}\cdot\frac{\beta(4+\beta)}{4(2+\beta)^2}E_{s_i}\left\{\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]^2\right\}+(1-\beta)\frac{2\beta}{(4-\beta^2)^2}E_{s_i}\left\{\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]^2\right\} \\
&= \frac{-\beta}{2(4-\beta^2)^2}\left[(4+\beta)\left(1-\frac{\beta}{2}\right)^2-4(1-\beta)\right]E_{s_i}\left\{\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]^2\right\} \\
&= \frac{-\beta^2}{2(4-\beta^2)^2}\left(1+\frac{\beta^2}{4}\right)E_{s_i}\left\{\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]^2\right\}<0
\end{aligned}$$

To prove that the expected surplus is increasing in δ_i, it is sufficient to show that all terms of V are increasing in δ_i. First, we show the first term of V is increasing in δ_i by rewriting its first component as follows:

$$\begin{aligned}
&\tfrac{1}{2}E_{s_i}\left\{E_{s_j,m_j}\left[\left(x_i^*(s_i;s_i,m_j)+x_j^*(s_j;m_j,s_i)\right)^2\right]\right\} \\
&\quad=\frac{1}{2}\left(\frac{2-\beta}{4-\beta^2}\right)^2E_{s_i}\left\{E_{s_j,m_j}\left[\left(2\alpha-E\{\theta_i|s_i\}-E\{\theta_j|s_j\}-\frac{\beta}{2}\left[E\{\theta_j|s_j\}-E\{\theta_j|m_j\}\right]\right)^2\right]\right\} \\
&\quad=\frac{1}{2(2+\beta)^2}E_{s_i}\left\{E_{s_j,m_j}\left[\left(2\alpha-E\{\theta_j|s_j\}-\frac{\beta}{2}\left[E\{\theta_j|s_j\}-E\{\theta_j|m_j\}\right]\right)^2\right.\right. \\
&\qquad\left.\left.-2\left(2\alpha-E\{\theta_j|s_j\}-\frac{\beta}{2}\left[E\{\theta_j|s_j\}-E\{\theta_j|m_j\}\right]\right)E\{\theta_i|s_i\}+E\{\theta_i|s_i\}^2\right]\right\}
\end{aligned}$$

Notice that only the last term depends on δ_i (i.e., $E_{s_i}\{E\left[\theta_i|s_i\right]^2\}$), and is increasing in δ_i. This immediately implies that $\frac{1}{2}E_{s_i}\left\{E_{s_j,m_j}\left[\left(x_i^*(s_i;s_i,m_j)+x_j^*(s_j;m_j,s_i)\right)^2\right]\right\}$ is increasing in δ_i. The remaining component of the first term equals:

$$\begin{aligned}
&(1-\beta)E_{s_i}\left\{E_{s_j,m_j}\left[x_i^*(s_i;s_i,m_j)x_j^*(s_j;m_j,s_i)\right]\right\} \\
&= \frac{1-\beta}{(4-\beta^2)^2}E_{s_i}\left\{E_{s_j,m_j}\left[\left((2-\beta)\alpha-2E\{\theta_i|s_i\}+\beta E\{\theta_j|m_j\}\right)\right.\right. \\
&\qquad\left.\left.*\left((2-\beta)\alpha-2E\{\theta_j|s_j\}+\beta E\{\theta_i|s_i\}+\frac{\beta^2}{2}\left[E\{\theta_j|s_j\}-E\{\theta_j|m_j\}\right]\right)\right]\right\} \\
&= \frac{1-\beta}{(4-\beta^2)^2}E_{s_i}\left\{E_{s_j,m_j}\left[\left((2-\beta)\alpha-2E\{\theta_i|s_i\}+\beta E\{\theta_j|m_j\}\right)\right.\right. \\
&\qquad\left.\left.*\left((2-\beta)\alpha-2E\{\theta_j|s_j\}+\frac{\beta^2}{2}\left[E\{\theta_j|s_j\}-E\{\theta_j|m_j\}\right]\right)\right]\right\} \\
&\quad+\frac{\beta(1-\beta)}{(4-\beta^2)^2}E_{s_i}\left\{E\{\theta_i|s_i\}E_{s_j,m_j}\left[(2-\beta)\alpha+\beta E\{\theta_j|m_j\}\right]-2E\{\theta_i|s_i\}^2\right\}
\end{aligned}$$

Again, only the last term depends on δ_i (i.e., $E_{s_i}\{E\left[\theta_i|s_i\right]^2\}$), and is increasing in δ_i. This immediately implies that $(1-\beta)E_{s_i}\left\{E_{s_j,m_j}\left[x_i^*(s_i;s_i,m_j)x_j^*(s_j;m_j,s_i)\right]\right\}$ is decreasing in δ_i. Subtracting the latter component from the former component immediately implies that the first term of V is increasing in δ_i. It is straightforward to show that the second term is also increasing in δ_i, by using the decompositions (25) and (26) in combination with the observation that the first term of V is increasing in δ_i. This proves that $\partial V/\partial\delta_i > 0$. ■

PROOF OF PROPOSITION 5:
We first compare the information acquisition incentives under information sharing ($\rho_1 = 1$) and information concealment ($\rho_1 = 0$). In our binary example, the expected equilibrium profit (6) reduces to:

$$\begin{aligned}\Pi_1(\delta_1;\rho_1) &= \delta_1\left[\rho_1 E_{\theta_1}\left\{x_1^*(\theta_1;\theta_1,\theta_2)^2\right\} + (1-\rho_1)E_{\theta_1}\left\{x_1^*(\theta_1;\varnothing,\theta_2)^2\right\}\right] \\ &\quad +(1-\delta_1)x_1^*(\varnothing;\varnothing,\theta_2)^2 - \lambda\delta_1.\end{aligned}$$

When firm 1 shares information, then its marginal profit from information acquisition is:

$$\begin{aligned}\Pi_1(1;1) - \Pi_1(0;1) &= E_{\theta_1}\left\{x_1^*(\theta_1;\theta_1,\theta_2)^2 - x_1^*(\varnothing;\varnothing,\theta_2)^2\right\} - \lambda \\ &= -\frac{2}{3}E_{\theta_1}\left\{\left[\theta_1 - \overline{\theta}_1\right]\left[x_1^*(\theta_1;\theta_1,\theta_2) + x_i^*(\varnothing;\varnothing,\theta_2)\right]\right\} - \lambda \\ &= \frac{4}{9}E_{\theta_1}\left\{\left[\theta_1 - \overline{\theta}_1\right]\theta_1\right\} - \lambda = \frac{(\theta^h - \theta^l)^2}{9} - \lambda \qquad (27)\end{aligned}$$

Hence, in equilibrium the information acquisition choice for firm 1 with $\rho_1 = 1$ is:

$$\delta_1^*(1) = \begin{cases} 1, & \text{if } \lambda \le \frac{(\theta^h-\theta^l)^2}{9} \\ 0, & \text{otherwise} \end{cases} \qquad (28)$$

When firm 1 conceals information, then its marginal profit from information acquisition is:

$$\begin{aligned}\Pi_1(1;0) - \Pi_1(0;0) &= E_{\theta_1}\left\{x_1^*(\theta_1;\varnothing,\theta_2)^2 - x_1^*(\varnothing;\varnothing,\theta_2)^2\right\} - \lambda \\ &= -\frac{1}{2}E_{\theta_1}\left\{\left[\theta_1 - \overline{\theta}_1\right]\left[x_1^*(\theta_1;\varnothing,\theta_2) + x_1^*(\varnothing;\varnothing,\theta_2)\right]\right\} - \lambda \\ &= \frac{1}{4}E_{\theta_1}\left\{\left[\theta_1 - \overline{\theta}_1\right]\theta_1\right\} - \lambda = \frac{(\theta^h - \theta^l)^2}{16} - \lambda \qquad (29)\end{aligned}$$

The evaluation of (29) gives the following information acquisition choice in equilibrium for $\rho_1 = 0$:

$$\delta_1^*(0) = \begin{cases} 1, & \text{if } \lambda \le \frac{(\theta^h-\theta^l)^2}{16} \\ 0, & \text{otherwise} \end{cases} \qquad (30)$$

If $\frac{(\theta^h-\theta^l)^2}{16} < \lambda < \frac{(\theta^h-\theta^l)^2}{9}$, then an information-sharing firm acquires information whereas an information-concealing firm acquires no information, i.e., $\delta_1^*(1) = 1 > 0 = \delta_1^*(0)$. Therefore, in this case, information sharing gives a higher expected surplus:

$$\begin{aligned}V(\delta_1^*(1);1) &= \frac{1}{2}E_{\theta_1}\left\{\left[x_1^*(\theta_1;\theta_1,\theta_2) + x_2^*(\theta_2,\theta_2,\theta_1)\right]^2\right\} \\ &> \frac{1}{2}\left[x_1^*(\varnothing;\varnothing,\theta_2) + x_2^*(\theta_2,\theta_2,\varnothing)\right]^2 = V(\delta_1^*(0);0).\end{aligned}$$

■

B Supplementary Appendix

Here we derive the results for the extensions of the model. First, we extend the results to an oligopoly with N risk-neutral firms that compete in quantities of differentiated goods (with $N \geq 2$). Second, we analyze competition in prices.

B.1 Cournot Oligopoly

First, for the profit results, it is convenient to rewrite $E_{s_i}\left\{E_{s_{-i},m_{-i}}[x_i^*(s_i;\varnothing,m_{-i})^2]\right\}$ as follows by using (12):

$$
\begin{aligned}
&E_{s_i}\left\{E_{s_{-i},m_{-i}}[x_i^*(s_i;\varnothing,m_{-i})^2]\right\}\\
&= E_{s_i}\left\{E_{s_{-i},m_{-i}}\left[\left(x_i^*(s_i;s_i,m_{-i}) + \frac{(N-1)\beta^2\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]}{2[2+(N-1)\beta](2-\beta)}\right)^2\right]\right\}\\
&= E_{s_i}\left\{E_{s_{-i},m_{-i}}\left[x_i^*(s_i;s_i,m_{-i})^2\right]\right\} + \frac{(N-1)\beta^2}{2[2+(N-1)\beta](2-\beta)}E_{s_i}\Bigg\{\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]\\
&\qquad *E_{s_{-i},m_{-i}}\left[2x_i^*(s_i;s_i,m_{-i}) + \frac{(N-1)\beta^2\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]}{2[2+(N-1)\beta](2-\beta)}\right]\Bigg\}\\
&= E_{s_i}\left\{E_{s_{-i},m_{-i}}\left[x_i^*(s_i;s_i,m_{-i})^2\right]\right\} + \frac{(N-1)\beta^2}{4[2+(N-1)\beta]^2(2-\beta)^2}E_{s_i}\Bigg\{\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]\\
&\qquad *\left[4(2-\beta)\alpha + 4\beta\sum_{j\neq i}\overline{\theta_j} - (N-1)\beta^2\overline{\theta_i} - \left(4[2+(N-2)\beta]-(N-1)\beta^2\right)E\{\theta_i|s_i\}\right]\Bigg\}\\
&= E_{s_i}\left\{E_{s_{-i},m_{-i}}\left[x_i^*(s_i;s_i,m_{-i})^2\right]\right\}\\
&\qquad -\frac{(N-1)\beta^2\left[4(2-\beta)+(N-1)\beta(4-\beta)\right]}{4[2+(N-1)\beta]^2(2-\beta)^2}\left(E_{s_i}\left\{E\left[\theta_i|s_i\right]^2\right\}-\overline{\theta_i}^2\right).
\end{aligned}
\tag{31}
$$

In the last simplification, we use that $E_{s_i}\left\{E\left[\theta_i|s_i\right]-\overline{\theta_i}\right\}=0$. Then, any constant multiplied by $E_{s_i}\left\{E\left[\theta_i|s_i\right]-\overline{\theta_i}\right\}$ also equals 0. Second, we rewrite $E_{s_i,m_i}\{E_{s_{-i},m_{-i}}[x_i^*(s_i;m_i,\varnothing,m_{-ij})^2]\}$, by using (12), as:

$$
\begin{aligned}
&E_{s_i,m_i}\{E_{s_{-i},m_{-i}}[x_i^*(s_i;m_i,\varnothing,m_{-ij})^2]\}\\
&= E_{s_i,m_i}\left\{E_{s_{-i},m_{-i}}\left[\left(x_i^*(s_i;m_i,s_j,m_{-ij}) - \frac{\beta\left[E\{\theta_j|s_j\}-\overline{\theta_j}\right]}{[2+(N-1)\beta](2-\beta)}\right)^2\right]\right\}\\
&= E_{s_i,m_i}\left\{E_{s_{-i},m_{-i}}[x_i^*(s_i;m_i,s_j,m_{-ij})^2]\right\}\\
&\quad - \frac{\beta}{[2+(N-1)\beta]^2(2-\beta)^2}E_{s_j}\Bigg\{\left[E\{\theta_j|s_j\}-\overline{\theta_j}\right]E_{s_{-j},m_{-j}}\Bigg[\Bigg(2(2-\beta)\alpha - 2[2+(N-2)\beta]E\{\theta_i|s_i\}\\
&\quad +2\beta\sum\nolimits_{h\neq i,j}E\{\theta_h|m_h\} + 2(N-1)\frac{\beta^2}{2}\left[E\{\theta_i|s_i\}-E\{\theta_i|m_i\}\right] + \beta\left[E\{\theta_j|s_j\}+\overline{\theta_j}\right]\Bigg)\Bigg]\Bigg\}\\
&= E_{s_i,m_i}\left\{E_{s_{-i},m_{-i}}[x_i^*(s_i;m_i,s_j,m_{-ij})^2]\right\}
\end{aligned}
$$

$$
\begin{aligned}
& - \frac{2\beta}{[2+(N-1)\beta]^2(2-\beta)^2} E_{s_j}\Big\{ \left[E\{\theta_j|s_j\} - \overline{\theta_j}\right]\Big((2-\beta)\alpha - [2+(N-2)\beta]\overline{\theta_i} \\
& \qquad\qquad + \beta \textstyle\sum_{h\neq i,j} \overline{\theta_h} + \frac{\beta}{2}\left[E\{\theta_j|s_j\} + \overline{\theta_j}\right]\Big)\Big\} \\
= \;& E_{s_i,m_i}\left\{E_{s_{-i},m_{-i}}[x_i^*(s_i;m_i,s_j,m_{-ij})^2]\right\} - \frac{\beta^2}{[2+(N-1)\beta]^2(2-\beta)^2}\left(E_{s_j}\left\{E[\theta_j|s_j]^2\right\} - \overline{\theta_j}^2\right). \quad (32)
\end{aligned}
$$

In the last two simplifications, we use the property $E_{s_h}\left\{\left[E\{\theta_h|s_h\} - \overline{\theta_h}\right]\right\} = 0$ for any $h = 1,..,N$. Using (31) and (32), we can rewrite the expected profit Π_i as follows:

$$
\begin{aligned}
& \Pi_i(\delta_i, \delta_{-i}; \rho_i, \rho_{-i}) \\
& = \rho_i E_{s_i}\left\{E_{s_{-i},m_{-i}}[x_i^*(s_i;s_i,m_{-i})^2]\right\} + (1-\rho_i)E_{s_i}\left\{E_{s_{-i},m_{-i}}[x_i^*(s_i;\varnothing,m_{-i})^2]\right\} - c(\delta_i) \\
& = E_{s_i}\left\{E_{s_{-i},m_{-i}}[x_i^*(s_i;\varnothing,m_{-i})^2]\right\} - c(\delta_i) \\
& \quad +\rho_i \frac{(N-1)\beta^2\left[4(2-\beta)+(N-1)\beta(4-\beta)\right]}{4[2+(N-1)\beta]^2(2-\beta)^2}\mathrm{Var}(E_i[\theta|S_i^\delta]) \\
& = \rho_j E_{s_i}\left\{E_{s_j}E_{s_{-ij},m_{-ij}}[x_i^*(s_i;\varnothing,s_j,m_{-ij})^2]\right\} \\
& \quad +(1-\rho_j)E_{s_i}\left\{E_{s_{-ij},m_{-ij}}\left[x_i^*(s_i;\varnothing,\varnothing,m_{-ij})^2\right]\right\} - c(\delta_i) \\
& \quad +\rho_i \frac{(N-1)\beta^2\left[4(2-\beta)+(N-1)\beta(4-\beta)\right]}{4[2+(N-1)\beta]^2(2-\beta)^2}\mathrm{Var}(E_i[\theta|S_i^\delta]) \\
& = E_{s_i}\left\{E_{s_{-ij},m_{-ij}}\left[x_i^*(s_i;\varnothing,\varnothing,m_{-ij})^2\right]\right\} + \rho_j\frac{\beta^2}{[2+(N-1)\beta]^2(2-\beta)^2}\mathrm{Var}(E_j[\theta|S_j^\delta]) \\
& \quad +\rho_i \frac{(N-1)\beta^2\left[4(2-\beta)+(N-1)\beta(4-\beta)\right]}{4[2+(N-1)\beta]^2(2-\beta)^2}\mathrm{Var}(E_i[\theta|S_i^\delta]) - c(\delta_i) \\
& = \ldots = E_{s_i}\left\{x_i^*(s_i;\varnothing,..,\varnothing)^2\right\} + \frac{\beta^2}{[2+(N-1)\beta]^2(2-\beta)^2}\sum_{j\neq i}\rho_j \mathrm{Var}(E_j[\theta|S_j^\delta]) \\
& \qquad + \rho_i \frac{(N-1)\beta^2\left[4(2-\beta)+(N-1)\beta(4-\beta)\right]}{4[2+(N-1)\beta]^2(2-\beta)^2}\mathrm{Var}(E_i[\theta|S_i^\delta]) - c(\delta_i) \quad (33)
\end{aligned}
$$

Proof of Proposition 1 for oligopoly:
Using (12), we can rewrite the first term of (33) as follows:

$$
\begin{aligned}
E_{s_i}\left\{x_i^*(s_i;\varnothing,..,\varnothing)^2\right\} &= \left(\frac{1}{[2+(N-1)\beta]\,(2-\beta)}\right)^2 E_{s_i}\Bigg\{\Bigg((2-\beta)\alpha - [2+(N-2)\beta]\,\overline{\theta_i} \\
& \quad +\beta\sum_{j\neq i}\overline{\theta_j} - \frac{1}{2}[2+(N-1)\beta]\,(2-\beta)\left[E\{\theta_i|s_i\} - \overline{\theta_i}\right]\Bigg)^2\Bigg\} \\
&= \left(\frac{(2-\beta)\alpha - [2+(N-2)\beta]\,\overline{\theta_i} + \beta\sum_{j\neq i}\overline{\theta_j}}{[2+(N-1)\beta]\,(2-\beta)}\right)^2 + \frac{1}{4}E_{s_i}\left\{\left[E\{\theta_i|s_i\} - \overline{\theta_i}\right]^2\right\}.
\end{aligned}
$$

In the last simplification, we use the property that $E_{s_i}\left\{\left[E\{\theta_i|s_i\} - \overline{\theta_i}\right]\right\} = 0$. Lemma 3 implies that $E_{s_i}\left\{x_i^*(s_i;\varnothing,..,\varnothing)^2\right\}$ is increasing in δ_i and independent of δ_j.

(i) The first and third terms of (33) are increasing in δ_i, while the second term is independent of δ_i. Hence, $\Pi_i(\delta_i, \delta_j; \rho_i, \rho_j) + c(\delta_i)$ is increasing in δ_i.
(ii) The second term of (33) is (weakly) increasing in δ_j by Lemma 3. The remaining terms are independent of δ_j. Hence, Π_i is weakly increasing in δ_j. ■

PROOF OF LEMMA 1 FOR OLIGOPOLY:
For any $\rho_i, \rho_i' \in [0,1]$, expression (33) gives:

$$\Pi_i(\delta_i, \delta_{-i}; \rho_i, \rho_{-i}) - \Pi_i(\delta_i, \delta_{-i}; \rho_i', \rho_{-i}) = (\rho_i - \rho_i') \frac{(N-1)\beta^2 \left[4(2-\beta) + (N-1)\beta(4-\beta)\right]}{4[2+(N-1)\beta]^2(2-\beta)^2} \mathrm{Var}(E_i[\theta|S_i^\delta]).$$

For $\rho_i > \rho_i'$, this expression is increasing in δ_i by Lemma 3. ■

PROOF OF PROPOSITION 2(i) FOR OLIGOPOLY:
(i) We want to show that $\partial^2\Pi_i(\delta_i, \delta_{-i}; \rho_i, \rho_{-i})/(\partial\delta_i\partial\rho_j) = 0$ for any $j \neq i$. It follows from (33) that (for any $j \neq i$):

$$\frac{\partial\Pi_i(\delta_i, \delta_{-i}; \rho_i, \rho_{-i})}{\partial\rho_j} = \left(\frac{\beta}{[2+(N-1)\beta](2-\beta)}\right)^2 \left(E_{s_j}\left\{E\left[\theta_j|s_j\right]^2\right\} - \overline{\theta}_j^2\right).$$

As $\partial\Pi_i/\partial\rho_j$ is independent of δ_i, this gives $\partial^2\Pi_i(\delta_i, \delta_{-i}; \rho_i, \rho_{-i})/(\partial\delta_i\partial\rho_j) = 0$ for any $j \neq i$. ■

PROOF OF PROPOSITION 3 FOR OLIGOPOLY:
It follows directly from (22) that:

$$\begin{aligned}
\frac{\partial\Pi_i(\delta_i, \delta_{-i}; \rho_i, \rho_{-i})}{\partial\rho_i} &= \frac{(N-1)\beta^2 \left[4(2-\beta) + (N-1)\beta(4-\beta)\right]}{4[2+(N-1)\beta]^2(2-\beta)^2} \mathrm{Var}(E_i[\theta|S_i^\delta]) \geq 0, \text{ and} \\
\frac{\partial\Pi_i(\delta_i, \delta_{-i}; \rho_i, \rho_{-i})}{\partial\rho_k} &= \left(\frac{\beta}{[2+(N-1)\beta](2-\beta)}\right)^2 \mathrm{Var}(E_k[\theta|S_k^\delta]) \geq 0
\end{aligned}$$

for any $k \neq i$. ■

After substitution of the equilibrium output levels in the expected consumer surplus (13), we obtain the following (by slightly abusing notation):

$$\begin{aligned}
V(\boldsymbol{\delta}, \boldsymbol{\rho}) &\equiv E_{s_i,m_i}\left\{E_{s_{-i},m_{-i}}\left[v\left(x_1^*(s_1; m_1, m_{-1}), .., x_N^*(s_N; m_N, m_{-N})\right)\right]\right\} \\
&= \frac{1}{2} E_{s_i,m_i}\Bigg\{E_{s_{-i},m_{-i}}\Bigg[\Bigg(x_i^*(s_i; m_i, m_{-i}) + \sum_{j\neq i} x_j^*(s_j; m_{-i}, m_i)\Bigg)^2 \\
&\qquad - (1-\beta) x_i^*(s_i; m_i, m_{-i}) \sum_{j\neq i} x_j^*(s_j; m_{-i}, m_i) \\
&\quad - (1-\beta)\sum_{j\neq i} x_j^*(s_j; m_{-i}, m_i)\Bigg(x_i^*(s_i; m_i, m_{-i}) + \sum_{h\neq i,j} x_h^*(s_h; m_{-i}, m_i)\Bigg)\Bigg]\Bigg\} \qquad (34)
\end{aligned}$$

Proof of Proposition 4 for oligopoly:

The expected consumer surplus (34) can be rewritten as follows (for $i, j, h = 1, .., N$):

$$\begin{aligned} V(\cdot) &= \rho_i \frac{1}{2} E_{s_i} \left\{ E_{s_{-i},m_{-i}} \left[\left(x_i^*(s_i; s_i, m_{-i}) + \sum_{j \neq i} x_j^*(s_j; m_{-i}, s_i) \right)^2 \right.\right. \\ & \qquad - (1-\beta) x_i^*(s_i; s_i, m_{-i}) \sum_{j \neq i} x_j^*(s_j; m_{-i}, s_i) \\ & \qquad \left.\left. - \ (1-\beta) \sum_{j \neq i} x_j^*(s_j; m_{-i}, s_i) \left(x_i^*(s_i; s_i, m_{-i}) + \sum_{h \neq i,j} x_h^*(s_h; m_{-i}, s_i) \right) \right] \right\} \\ & \quad + (1-\rho_i) \frac{1}{2} E_{s_i} \left\{ E_{s_{-i},m_{-i}} \left[\left(x_i^*(s_i; \varnothing, m_{-i}) + \sum_{j \neq i} x_j^*(s_j; m_{-i}, \varnothing) \right)^2 \right.\right. \\ & \qquad - (1-\beta) x_i^*(s_i; \varnothing, m_{-i}) \sum_{j \neq i} x_j^*(s_j; m_{-i}, \varnothing) \\ & \qquad \left.\left. - \ (1-\beta) \sum_{j \neq i} x_j^*(s_j; m_{-i}, \varnothing) \left(x_i^*(s_i; \varnothing, m_{-i}) + \sum_{h \neq i,j} x_h^*(s_h; m_{-i}, \varnothing) \right) \right] \right\} \end{aligned}$$

Differentiating with respect to ρ_i gives (for $i, j = 1, .., N$ and $i \neq j$):

$$\begin{aligned} \frac{\partial V}{\partial \rho_i} &= \frac{1}{2} E_{s_i} \left\{ E_{s_{-i},m_{-i}} \left[\left(x_i^*(s_i; s_i, m_{-i}) + \sum_{j \neq i} x_j^*(s_j; m_{-i}, s_i) \right)^2 \right.\right. \\ & \qquad\qquad - \left(x_i^*(s_i; \varnothing, m_{-i}) + \sum_{j \neq i} x_j^*(s_j; m_{-i}, \varnothing) \right)^2 \\ & \quad -(1-\beta) \left(x_i^*(s_i; s_i, m_{-i}) \sum_{j \neq i} x_j^*(s_j; m_{-i}, s_i) - x_i^*(s_i; \varnothing, m_{-i}) \sum_{j \neq i} x_j^*(s_j; m_j, \varnothing) \right) \\ & \quad -(1-\beta) \left(\sum_{j \neq i} x_j^*(s_j; m_{-i}, s_i) \left(x_i^*(s_i; s_i, m_{-i}) + \sum_{h \neq i,j} x_h^*(s_h; m_{-i}, s_i) \right) \right. \\ & \qquad \left.\left.\left. - \sum_{j \neq i} x_j^*(s_j; m_{-i}, \varnothing) \left(x_i^*(s_i; \varnothing, m_{-i}) + \sum_{h \neq i,j} x_h^*(s_h; m_{-i}, \varnothing) \right) \right) \right] \right\} \end{aligned} \tag{35}$$

The first two lines of this expression can be simplified by using the following:

$$\begin{aligned} & E_{s_i} \left\{ E_{s_{-i},m_{-i}} \left[\left(x_i^*(s_i; \varnothing, m_{-i}) + \textstyle\sum_{j \neq i} x_j^*(s_j; m_{-i}, \varnothing) \right)^2 \right] \right\} \\ & \quad = E_{s_i} \left\{ E_{s_{-i},m_{-i}} \left[\left(x_i^*(s_i; s_i, m_{-i}) + \sum_{j \neq i} x_j^*(s_j; m_{-i}, s_i) - \frac{(N-1)\beta \left[E\{\theta_i | s_i\} - \overline{\theta_i} \right]}{2[2 + (N-1)\beta]} \right)^2 \right] \right\} \end{aligned}$$

$$
\begin{aligned}
&= E_{s_i}\left\{E_{s_{-i},m_{-i}}\left[\left(x_i^*(s_i;s_i,m_{-i})+\sum\nolimits_{j\neq i}x_j^*(s_j;m_{-i},s_i)\right)^2\right]\right\}\\
&\quad -\frac{(N-1)\beta}{2[2+(N-1)\beta]}E_{s_i}\Bigg\{\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]\\
&\qquad *E_{s_{-i},m_{-i}}\left[2\left(x_i^*(s_i;s_i,m_{-i})+\sum\nolimits_{j\neq i}x_j^*(s_j;m_{-i},s_i)\right)-\frac{(N-1)\beta\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]}{2[2+(N-1)\beta]}\right]\Bigg\}\\
&= E_{s_i}\left\{E_{s_{-i},m_{-i}}\left[\left(x_i^*(s_i;s_i,m_{-i})+\sum\nolimits_{j\neq i}x_j^*(s_j;m_{-i},s_i)\right)^2\right]\right\}\\
&\quad +\frac{(N-1)\beta[4+(N-1)\beta]}{4[2+(N-1)\beta]^2}E_{s_i}\left\{\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]^2\right\}
\end{aligned}
\tag{36}
$$

The third line of (35) can be simplified by using the following:

$$
\begin{aligned}
&E_{s_i}\left\{E_{s_{-i},m_{-i}}\left[x_i^*(s_i;\varnothing,m_{-i})\sum\nolimits_{j\neq i}x_j^*(s_j;m_{-i},\varnothing)\right]\right\}\\
&\quad = E_{s_i}\Bigg\{E_{s_{-i},m_{-i}}\Bigg[\left(x_i^*(s_i;s_i,m_{-i})+\frac{(N-1)\beta^2\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]}{2(2-\beta)[2+(N-1)\beta]}\right)\\
&\qquad\qquad *\left(\sum\nolimits_{j\neq i}x_j^*(s_j;m_{-i},s_i)-\frac{(N-1)\beta\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]}{(2-\beta)[2+(N-1)\beta]}\right)\Bigg]\Bigg\}\\
&\quad = E_{s_i}\left\{E_{s_{-i},m_{-i}}\left[x_i^*(s_i;s_i,m_{-i})\sum\nolimits_{j\neq i}x_j^*(s_j;m_{-i},s_i)\right]\right\}\\
&\qquad +E_{s_i}\left\{\frac{(N-1)\beta\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]}{(2-\beta)[2+(N-1)\beta]}E_{s_{-i},m_{-i}}\left[\frac{\beta}{2}\sum\nolimits_{j\neq i}x_j^*(s_j;m_{-i},\varnothing)-x_i^*(s_i;s_i,m_{-i})\right]\right\}\\
&\quad = E_{s_i}\left\{E_{s_{-i},m_{-i}}\left[x_i^*(s_i;s_i,m_{-i})\sum\nolimits_{j\neq i}x_j^*(s_j;m_{-i},s_i)\right]\right\}\\
&\qquad +\frac{(N-1)\beta\,[2+(N-2)\beta]}{(2-\beta)^2[2+(N-1)\beta]^2}E_{s_i}\left\{\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]^2\right\}
\end{aligned}
\tag{37}
$$

Finally, the last two lines of (35) can be simplified by using the following:

$$
\begin{aligned}
&E_{s_i}\left\{E_{s_{-i},m_{-i}}\left[x_j^*(s_j;m_{-i},\varnothing)\left(x_i^*(s_i;\varnothing,m_{-i})+\sum\nolimits_{h\neq i,j}x_h^*(s_h;m_{-i},\varnothing)\right)\right]\right\}\\
&= E_{s_i}\Bigg\{E_{s_{-i},m_{-i}}\Bigg[\left(x_j^*(s_j;m_{-i},s_i)-\frac{\beta\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]}{(2-\beta)[2+(N-1)\beta]}\right)\\
&\quad *\left(x_i^*(s_i;s_i,m_{-i})+\sum_{h\neq i,j}x_h^*(s_h;m_{-i},s_i)+\frac{\left[(N-1)\frac{\beta}{2}-(N-2)\right]\beta\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]}{(2-\beta)[2+(N-1)\beta]}\right)\Bigg]\Bigg\}\\
&= E_{s_i}\left\{E_{s_{-i},m_{-i}}\left[x_j^*(s_j;m_{-i},s_i)\left(x_i^*(s_i;s_i,m_{-i})+\sum\nolimits_{h\neq i,j}x_h^*(s_h;m_{-i},s_i)\right)\right]\right\}\\
&\quad +E_{s_i}\Bigg\{\frac{\beta\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]}{(2-\beta)[2+(N-1)\beta]}E_{s_{-i},m_{-i}}\Bigg[\left((N-1)\frac{\beta}{2}-(N-2)\right)x_j^*(s_j;m_{-i},s_i)\\
&\qquad -x_i^*(s_i;s_i,m_{-i})-\frac{(N-1)\frac{\beta^2}{2}\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]}{(2-\beta)[2+(N-1)\beta]}-\sum\nolimits_{h\neq i,j}x_h^*(s_h;m_{-i},\varnothing)\Bigg]\Bigg\}
\end{aligned}
$$

$$
\begin{aligned}
= \quad & E_{s_i}\left\{E_{s_{-i},m_{-i}}\left[x_j^*(s_j;m_{-i},s_i)\left(x_i^*(s_i;s_i,m_{-i})+\sum\nolimits_{h\neq i,j}x_h^*(s_h;m_{-i},s_i)\right)\right]\right\}\\
& +\frac{2\beta}{(2-\beta)^2[2+(N-1)\beta]^2}E_{s_i}\left\{\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]^2\right\} \qquad (38)
\end{aligned}
$$

Substitution of (36), (37) and (38) in (35) gives:

$$
\begin{aligned}
\frac{\partial V}{\partial \rho_i} \quad = \quad & -\frac{1}{2}\left(\frac{(N-1)\beta[4+(N-1)\beta]}{4[2+(N-1)\beta]^2}E_{s_i}\left\{\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]^2\right\}\right.\\
& \quad -(1-\beta)\frac{(N-1)\beta\,[2+(N-2)\beta]}{(2-\beta)^2[2+(N-1)\beta]^2}E_{s_i}\left\{\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]^2\right\}\\
& \quad \left.-(1-\beta)\frac{2(N-1)\beta}{(2-\beta)^2[2+(N-1)\beta]^2}E_{s_i}\left\{\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]^2\right\}\right)\\
= \quad & \frac{-(N-1)\beta E_{s_i}\left\{\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]^2\right\}}{2(2-\beta)^2[2+(N-1)\beta]^2}\left[\left(1-\frac{\beta}{2}\right)^2[4+(N-1)\beta]-(1-\beta)\,[4+(N-2)\beta]\right]\\
= \quad & \frac{-(N-1)\beta^2\left[1+\frac{1}{4}(N-1)\beta^2\right]}{2(2-\beta)^2[2+(N-1)\beta]^2}E_{s_i}\left\{\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]^2\right\}<0.
\end{aligned}
$$

To prove that the expected surplus is increasing in δ_i, it is sufficient to show that all terms of V are increasing in δ_i. First, we show the first term of V is increasing in δ_i by rewriting it as follows:

$$
\begin{aligned}
& E_{s_i}\left\{E_{s_{-i},m_{-i}}\left[\left(x_i^*(s_i;s_i,m_{-i})+\sum\nolimits_{j\neq i}x_j^*(s_j;m_{-i},s_i)\right)^2\right]\right\}\\
= \quad & \frac{1}{[2+(N-1)\beta]^2}E_{s_i}\left\{E_{s_{-i},m_{-i}}\left[\left(N\alpha-E\{\theta_i|s_i\}-\sum\nolimits_{j\neq i}E\{\theta_j|s_j\}\right.\right.\right.\\
& \qquad\qquad \left.\left.\left.-(N-1)\frac{\beta^2}{2}\sum\nolimits_{j\neq i}[E\{\theta_j|s_j\}-E\{\theta_j|m_j\}]\right)^2\right]\right\}\\
= \quad & \frac{1}{[2+(N-1)\beta]^2}E_{s_i}\left\{E_{s_{-i},m_{-i}}\left[\left(N\alpha-\sum_{j\neq i}\left(E\{\theta_j|s_j\}-(N-1)\frac{\beta^2}{2}\left[E\{\theta_j|s_j\}-E\{\theta_j|m_j\}\right]\right)\right)^2\right.\right.\\
& \quad \left.\left.-2\left(N\alpha-\sum_{j\neq i}\left(E\{\theta_j|s_j\}-(N-1)\frac{\beta^2}{2}\left[E\{\theta_j|s_j\}-E\{\theta_j|m_j\}\right]\right)\right)E\{\theta_i|s_i\}+E\{\theta_i|s_i\}^2\right]\right\}
\end{aligned}
$$

Notice that only the last term depends on δ_i (i.e., $E_{s_i}\{E\left[\theta_i|s_i\right]^2\}$), and is increasing in δ_i. This immediately implies that $\frac{1}{2}E_{s_i}\left\{E_{s_{-i},m_{-i}}\left[\left(x_i^*(s_i;s_i,m_{-i})+\sum_{j\neq i}x_j^*(s_j;m_{-i},s_i)\right)^2\right]\right\}$ is increasing in δ_i. Similarly, it is easy to show that the second and third terms of V are increasing in δ_i. It is straightforward to show that the remaining terms are also increasing in δ_i, by using the decompositions (36), (37) and (38) in combination with the observation that the first three terms of V are increasing in δ_i. This proves that $\partial V/\partial\delta_i > 0$. ■

Proof of Lemma 2:
The proof is similar to the proof of Lemma 1. For any $\rho_i, \rho_i' \in \{0,1\}$, we have:

$$\Pi_i^b(\delta_i,\delta_j;\rho_i,\rho_j) - \Pi_i^b(\delta_i,\delta_j;\rho_i',\rho_j) = (\rho_i-\rho_i')\frac{1}{1-\beta^2}E_{s_i}\left\{E_{s_j,m_j}[P_i^*(s_i;s_i,m_j)^2 - P_i^*(s_i;\varnothing,m_j)^2]\right\}$$

where $P_i^*(s_i,m_i,m_j) \equiv p_i^*(s_i;m_i,m_j) - E\{\theta_i|s_i\}$ is the equilibrium price-cost margin. By using (16), We can rewrite $E_{s_i}\left\{E_{s_j,m_j}[P_i^*(s_i;\varnothing,m_j)^2]\right\}$ as follows:

$$\begin{aligned}
E_{s_i}\left\{E_{s_j,m_j}[P_i^*(s_i;\varnothing,m_j)^2]\right\} &= E_{s_i}\left\{E_{s_j,m_j}\left[\left(P_i^*(s_i;s_i,m_j) - \frac{\beta^2}{2(4-\beta^2)}\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]\right)^2\right]\right\} \\
&= E_{s_i}\left\{E_{s_j,m_j}\left[P_i^*(s_i;s_i,m_j)^2\right]\right\} \\
&\quad -\frac{\beta^2}{2(4-\beta^2)}E_{s_i}\left\{\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]E_{s_j,m_j}\left[2P_i^*(s_i;s_i,m_j) - \frac{\beta^2}{2(4-\beta^2)}\left(E\{\theta_i|s_i\}-\overline{\theta_i}\right)\right]\right\} \\
&= E_{s_i}\left\{E_{s_j,m_j}\left[P_i^*(s_i;s_i,m_j)^2\right]\right\} \\
&\quad -\frac{\beta^2}{4(4-\beta^2)^2}E_{s_i}\left\{\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]\left[4(2+\beta)(1-\beta)\alpha + 4\beta\overline{\theta_j} + \beta^2\overline{\theta_i} - (8-3\beta^2)E\{\theta_i|s_i\}\right]\right\} \\
&= E_{s_i}\left\{E_{s_j,m_j}\left[P_i^*(s_i;s_i,m_j)^2\right]\right\} + \frac{\beta^2(8-3\beta^2)}{4(4-\beta^2)^2}\left(E_{s_i}\left\{E\left[\theta_i|s_i\right]^2\right\} - \overline{\theta_i}^2\right)
\end{aligned}$$

In this last simplification, we use the property that $E_{s_i}\left\{E_{s_j,m_j}[E\{\theta_i|s_i\}-\overline{\theta_i}]\right\} = 0$. Then, any constant multiplied by $E_{s_i}\left\{E_{s_j,m_j}[E\{\theta_i|s_i\}-\overline{\theta_i}]\right\}$ will be also equal to 0. Therefore,

$$\begin{aligned}
\Pi_i(\delta_i,\delta_j;\rho_i,\rho_j) - \Pi_i(\delta_i,\delta_j;\rho_i',\rho_j) &= -(\rho_i-\rho_i')\frac{\beta^2(8-3\beta^2)}{4(4-\beta^2)^2(1-\beta^2)}\left(E_{s_i}\left\{E\left[\theta_i|s_i\right]^2\right\} - \overline{\theta_i}^2\right) \\
&= -(\rho_i-\rho_i')\frac{\beta^2(8-\beta^2)}{4(4-\beta^2)^2(1-\beta^2)}\mathrm{Var}(E_i[\theta|S_i^\delta])
\end{aligned}$$

For $\rho_i > \rho_i'$, this expression is decreasing in δ_i by Lemma 3 (see Appendix A). ■

Proof of Proposition 4 with Bertrand competition:
The proof is analogous to the original proof (with Cournot competition). Differentiating the expected surplus V with respect to ρ_i gives (24) for $i,j = 1,2$ and $i \neq j$. The first line of this expression can be simplified by using (16) and (17) for $m_j \in \{s_j, \varnothing\}$:

$$\begin{aligned}
&E_{s_i}\left\{E_{s_j,m_j}\left[\left(x_i^*(s_i;\varnothing,m_j) + x_j^*(s_j;m_j,\varnothing)\right)^2\right]\right\} \\
&= E_{s_i}\left\{E_{s_j,m_j}\left[\left(x_i^*(s_i;s_i,m_j) + x_j^*(s_j;m_j,s_i) - \frac{\beta(2+\beta)}{2(4-\beta^2)(1-\beta^2)}\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]\right)^2\right]\right\} \\
&= E_{s_i}\left\{E_{s_j,m_j}\left[\left(x_i^*(s_i;s_i,m_j) + x_j^*(s_j;m_j,s_i)\right)^2\right]\right\} \\
&\quad -\frac{\beta}{2(2-\beta)(1-\beta^2)}E_{s_i}\left\{\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]E_{s_j,m_j}\left[2\left(x_i^*(s_i;s_i,m_j) + x_j^*(s_j;m_j,s_i)\right) - \frac{\beta\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]}{2(2-\beta)(1-\beta^2)}\right]\right\} \\
&= E_{s_i}\left\{E_{s_j,m_j}\left[\left(x_i^*(s_i;s_i,m_j) + x_j^*(s_j;m_j,s_i)\right)^2\right]\right\} + \frac{\beta(8-2\beta-3\beta^2)}{4(2-\beta)(4-\beta^2)(1-\beta^2)^2}E_{s_i}\left\{\left[E\{\theta_i|s_i\}-\overline{\theta_i}\right]^2\right\} \quad (39)
\end{aligned}$$

The second line of (24) can be simplified by using (16) and (17) for $m_j \in \{s_j, \varnothing\}$:

$$\begin{aligned}
&E_{s_i}\left\{E_{s_j,m_j}\left[x_i^*(s_i;\varnothing,m_j)x_j^*(s_j;m_j,\varnothing)\right]\right\} \\
&= E_{s_i}\left\{E_{s_j,m_j}\left[\left(x_i^*(s_i;s_i,m_j) - \frac{\beta^2\left[E\{\theta_i|s_i\} - \overline{\theta_i}\right]}{2(4-\beta^2)(1-\beta^2)}\right)\left(x_j^*(s_j;m_j,s_i) - \frac{\beta\left[E\{\theta_i|s_i\} - \overline{\theta_i}\right]}{(4-\beta^2)(1-\beta^2)}\right)\right]\right\} \\
&= E_{s_i}\left\{E_{s_j,m_j}\left[x_i^*(s_i;s_i,m_j)x_j^*(s_j;m_j,s_i)\right]\right\} - E_{s_i}\left\{\frac{\beta^2\left[E\{\theta_i|s_i\} - \overline{\theta_i}\right]}{2(4-\beta^2)(1-\beta^2)}E_{s_j,m_j}\left[x_j^*(s_j;m_j,s_i)\right]\right\} \\
&\quad -E_{s_i}\left\{\frac{\beta\left[E\{\theta_i|s_i\} - \overline{\theta_i}\right]}{(4-\beta^2)(1-\beta^2)}E_{s_j,m_j}\left[x_i^*(s_i;s_i,m_j)\right]\right\} + \frac{\beta^3 E_{s_i}\left\{\left[E\{\theta_i|s_i\} - \overline{\theta_i}\right]^2\right\}}{2(4-\beta^2)^2(1-\beta^2)} \\
&= E_{s_i}\left\{E_{s_j,m_j}\left[x_i^*(s_i;s_i,m_j)x_j^*(s_j;m_j,s_i)\right]\right\} + \frac{\beta(2-\beta^2)}{(4-\beta^2)^2(1-\beta^2)^2}E_{s_i}\left\{\left[E\{\theta_i|s_i\} - \overline{\theta_i}\right]^2\right\} \qquad (40)
\end{aligned}$$

Substitution of (39) and (40) for $m_j \in \{s_j, \varnothing\}$ in (24) gives:

$$\begin{aligned}
\frac{\partial V}{\partial \rho_i} &= -\left(\frac{\beta(8-2\beta-3\beta^2)}{8(2-\beta)(4-\beta^2)(1-\beta^2)^2} - (1-\beta)\frac{\beta(2-\beta^2)}{(4-\beta^2)^2(1-\beta^2)^2}\right)E_{s_i}\left\{\left[E\{\theta_i|s_i\} - \overline{\theta_i}\right]^2\right\} \\
&= \frac{-\beta}{8(4-\beta^2)^2(1-\beta^2)^2}\Big((2+\beta)(8-2\beta-3\beta^2) - 8(1-\beta)(2-\beta^2)\Big)E_{s_i}\left\{\left[E\{\theta_i|s_i\} - \overline{\theta_i}\right]^2\right\} \\
&= \frac{-\beta^2(20-11\beta^2)}{8(4-\beta^2)^2(1-\beta^2)^2}E_{s_i}\left\{\left[E\{\theta_i|s_i\} - \overline{\theta_i}\right]^2\right\} < 0.
\end{aligned}$$

To prove that the expected surplus is increasing in δ_i, it is sufficient to show that all terms of V are increasing in δ_i. First, we show the first term of V is increasing in δ_i by rewriting its first component as follows (by using (16) and (17)):

$$\begin{aligned}
&\tfrac{1}{2}E_{s_i}\left\{E_{s_j,m_j}\left[\left(x_i^*(s_i;s_i,m_j) + x_j^*(s_j;m_j,s_i)\right)^2\right]\right\} \\
&= \frac{1}{2}\frac{(2+\beta)^2}{(4-\beta^2)^2(1-\beta^2)^2}E_{s_i}\Bigg\{E_{s_j,m_j}\Bigg[\Bigg((1-\beta)\left[2\alpha - E\{\theta_i|s_i\} - E\{\theta_j|s_j\}\right] \\
&\qquad -\frac{\beta}{2}\left[E\{\theta_j|s_j\} - E\{\theta_j|m_j\}\right]\Bigg)^2\Bigg]\Bigg\} \\
&= \frac{1}{2(2-\beta)^2(1-\beta^2)^2}E_{s_i}\Bigg\{E_{s_j,m_j}\Bigg[\left((1-\beta)\left[2\alpha - E\{\theta_j|s_j\}\right] - \frac{\beta}{2}\left[E\{\theta_j|s_j\} - E\{\theta_j|m_j\}\right]\right)^2 \\
&\quad -2\left((1-\beta)\left[2\alpha - E\{\theta_j|s_j\}\right] - \frac{\beta}{2}\left[E\{\theta_j|s_j\} - E\{\theta_j|m_j\}\right]\right)(1-\beta)E\{\theta_i|s_i\} + (1-\beta)^2 E\{\theta_i|s_i\}^2\Bigg]\Bigg\}
\end{aligned}$$

Notice that only the last term depends on δ_i (i.e., $E_{s_i}\{E\left[\theta_i|s_i\right]^2\}$), and is increasing in δ_i. This immediately implies that $\frac{1}{2}E_{s_i}\left\{E_{s_j,m_j}\left[\left(x_i^*(s_i;s_i,m_j) + x_j^*(s_j;m_j,s_i)\right)^2\right]\right\}$ is increasing in δ_i. By using (16) and (17), the remaining component of the first term equals can be written as:

$$
\begin{aligned}
&(1-\beta)E_{s_i}\left\{E_{s_j,m_j}\left[x_i^*(s_i;s_i,m_j)x_j^*(s_j;m_j,s_i)\right]\right\}\\
&= \frac{1-\beta}{(4-\beta^2)^2(1-\beta^2)^2}E_{s_i}\Bigg\{E_{s_j,m_j}\Bigg[\Bigg((2+\beta)(1-\beta)\alpha-(2-\beta^2)E\{\theta_i|s_i\}+\beta E\{\theta_j|m_j\}\Bigg)\\
&\qquad *\left((2+\beta)(1-\beta)\alpha-(2-\beta^2)E\{\theta_j|s_j\}+\beta E\{\theta_i|s_i\}-\frac{\beta}{2}\left[E\{\theta_j|s_j\}-E\{\theta_j|m_j\}\right]\right)\Bigg]\Bigg\}\\
&= \frac{1-\beta}{(4-\beta^2)^2(1-\beta^2)^2}E_{s_i}\Bigg\{E_{s_j,m_j}\Bigg[\Bigg((2+\beta)(1-\beta)\alpha-(2-\beta^2)E\{\theta_i|s_i\}+\beta E\{\theta_j|m_j\}\Bigg)\\
&\qquad\qquad *\left((2+\beta)(1-\beta)\alpha-(2-\beta^2)E\{\theta_j|s_j\}-\frac{\beta}{2}\left[E\{\theta_j|s_j\}-E\{\theta_j|m_j\}\right]\right)\Bigg]\Bigg\}\\
&\quad + \frac{\beta(1-\beta)}{(4-\beta^2)^2(1-\beta^2)^2}E_{s_i}\Bigg\{E_{s_j,m_j}\Bigg[E\{\theta_i|s_i\}\Bigg((2+\beta)(1-\beta)\alpha+\beta E\{\theta_j|m_j\}\Bigg)-(2-\beta^2)E\{\theta_i|s_i\}^2\Bigg]\Bigg\}
\end{aligned}
$$

Again, only the last term depends on δ_i (i.e., $E_{s_i}\{E\left[\theta_i|s_i\right]^2\}$), and this makes the second component of the first term increasing in δ_i. We can obtain similar results for the second term of V. ■

References

Athey, S. and Levin, J. (2001) "The Value of Information in Monotone Decision Problems", *mimeo*, Stanford University

Blackwell, D. (1951) "Comparisons of Experiments", *Proceedings of the Second Berkeley Symposium on Mathematical Statistics*, 93-102

Fried, D. (1984) "Incentives for Information Production and Disclosure in a Duopolistic Environment", *Quarterly Journal of Economics* 99, 367-381

Gal-Or, E. (1986) "Information Transmission – Cournot and Bertrand Equilibria", *Review of Economic Studies* 53, 85-92

Ganuza, J.-J. and Penalva, J.S. (2010) "Signal Orderings Based on Dispersion and the Supply of Private Information in Auctions", *Econometrica* 78, 1007-1030

Green, E.J. and Porter, R.H. (1984) "Noncooperative Collusion under Imperfect Price Information", *Econometrica* 52, 87-100

Hauk, E. and Hurkens, S. (2001) "Secret Information Acquisition in Cournot Markets", *Economic Theory* 18, 661-681

Hwang, H-S. (1995) "Information Acquisition and Relative Efficiency of Competitive, Oligopoly and Monopoly Markets", *International Economic Review* 36, 325-340

Jansen, J. (2008) "Information Acquisition and Strategic Disclosure in Oligopoly", *Journal of Economics & Management Strategy* 17, 113–148

KIRBY, A.J. (2004) “The Product Market Opportunity Loss of Mandated Disclosure”, *Information Economics and Policy* 16, 553–577

KÜHN, K-U. AND VIVES, X. (1995) “Information Exchanges among Firms and their Impact on Competition”, Luxembourg: *Office for Official Publications of the European Communities*

KÜHN, K-U. (2001) “Fighting Collusion by Regulating Communication Between Firms”, *Economic Policy* 16, 167-204

LEHMANN, E.L. (1988): “Comparing Location Experiments”, *Annals of Statistics* 16, 521-533

LI, L., MCKELVEY, R.D. AND PAGE, T. (1987) “Optimal Research for Cournot Oligopolists”, *Journal of Economic Theory* 42, 140-166

MILGROM, P. AND SHANON, C. (1994) “Monotone Comparative Statics”, *Econometrica* 62, 157-180

OKUNO-FUJIWARA, M., POSTLEWAITE, A., AND SUZUMURA, K. (1990) “Strategic Information Revelation”, *Review of Economic Studies* 57, 25-47

PERSICO, N. (2000) “Information Acquisition in Auctions”, *Econometrica* 68 (1), 135-48

RAITH, M. (1996) “A General Model of Information Sharing in Oligopoly”, *Journal of Economic Theory* 71, 260–88

SAKAI, Y. AND YAMATO, T. (1989) “Oligopoly, Information and Welfare”, *Journal of Economics* 49, 3-24

SHAPIRO, C. (1986) “Exchange of Cost Information in Oligopoly”, *Review of Economic Studies* 53, 433-446

VIVES, X. (1984) “Duopoly Information Equilibrium: Cournot and Bertrand”, *Journal of Economic Theory* 34, 71-94

VIVES, X. (1990) “Trade Association Disclosure Rules, Incentives to Share Information and Welfare”, *RAND Journal of Economics* 21, 409-430

VIVES, X. (1999) *Oligopoly Pricing: Old Ideas and New Tools*, Cambridge, MA: MIT Press