Public Inputs and Endogenous Growth

in the Agricultural Sector:

A Dynamic Dual Approach

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I. Introduction

Neoclassical models of growth (Solow, Ramsey) have been widely criticized because they cannot explain productivity changes. According to these models, growth is exogenously given by an unexplained rate of technical change. As a response, endogenous growth theories find that continuous growth is possible because of the existence of non-rival inputs of production (i.e., inputs that can be used by many firms at the same time or by the same firm repeatedly without additional cost). In these models, two necessary conditions for endogenous growth are: increasing returns to scale over all inputs and positive impacts of non-rival inputs on the returns to investment. The main contribution of this paper is to introduce a dynamic model of productivity measurement that incorporates public goods (non-rival by definition) as external factors to the firms. It also rationalizes the provision of public inputs by a benevolent social planner that internalizes the effects of them. Firms' minimize intertemporal costs taking public inputs as given. Additionally, estimable functions that allow testing the necessary conditions for endogenous growth are obtained.

Many other papers have focused on the effects of public goods on private production, and most of them have found a positive impact[1]. For example, Aschauer (1989) pioneer work estimates a single production function for the US economy including public infrastructure as factor of production. Lynde and Richmond (1992) and Berndt and Hansson (1992) have also used duality theory to estimate the role of infrastructure in private production in US and Sweden, respectively. Nadiri and Mamuneas (1994) estimate the impacts of public capital and R&D on the cost structure of twelve US manufacturing industries, and Morrison and Schwartz (1996) study the regional effects of public infrastructure on the US manufacturing sector. Both papers adopt a dual approach and find, in general, positive effects of public inputs on manufacturing productivity. The last paper also finds increasing returns to scale over all inputs (including infrastructure), but it does not include R&D.

For the agricultural sector, papers like Antle (1983) and Craig et al (1997) find positive effects of public infrastructure and research on agricultural productivity but their approach is based on estimating a single production function. Binswanger et al (1993) estimates the impacts of infrastructure and R&D in India. They consider, in a static framework, that public infrastructure investments are regionally allocated toward areas that are more productive. In contrast, the present paper develops a dynamic model of productivity measurement in which the provision of public inputs is done by a benevolent social planner that internalizes the effects of them. The dynamic demands for private inputs can be jointly estimated. This approach maintains producer rationality and allows examination of the impacts of public inputs on producer's behavior.

The model is tested with data for the U.S. agricultural sector. U.S. agricultural productivity has increased at an annual average rate of two percent over the 1948-1994 period. Some authors have found that productivity growth has been the main factor contributing to economic growth of the agricultural sector (Ball et al (1997)). In this country, the provision of public goods in the form of public research and extension, and

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[1] Exceptions are Garcia-Mila and McGuire (1992) and Holtz-Eakin (1994). They find insignificant effects

infrastructure has been sizable. In an atomistic environment, these public expenditures are traditionally justified because of their low degree of appropriability and high costs. Theoretically consistent firms' dynamic demands for inputs can be estimated. Stocks of public capital and R&D are used as quasi-fixed factors. The existence of economies of scale and the likely positive impact of public inputs on the steady state stocks of private capital can be tested.

There are several reasons to undertake this study. First, the possibility of endogenous growth in the agricultural sector may imply spillovers to other sectors and, in particular, may have important effects on the growth of regional economies based on agricultural activities. Second, by determining the substitution or complementarity between public and private inputs one may explain the recent evolution of private factors in the US agricultural sector. Ball et al (1997) show the increasing use of materials and the decreasing use of labor by the sector. Finally, the estimation of shadow prices for public capital and R&D stocks may provide an indicator to policy makers of the optimal provision of public investment.

The paper develops as follow. Section II presents a summary of the endogenous growth theory involving publicly provided goods and the related testable hypotheses using a dual approach. Section III introduces a dynamic model in which both producers' and government's behaviors are rationalized. The testable hypotheses are then revisited. Section IV introduces the empirical model and section V presents the preliminary results. Finally, conclusions and future lines of research are stated in section VI.

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of public infrastructure on private production.

II. Growth Theory and Testable Hypothesis

In the neoclassical models of growth (Solow, Ramsey), the rate of growth of per capita output is a decreasing function of the per capita stock of private capital. Without technical change and with a well-behaved neoclassical production function, the level of per capita output converges to a steady state where the growth of per capita private capital eventually stops. This result, implied by the assumption of decreasing returns to capital, has been one of the major criticisms to these models.

As a response to these empirically unsustainable results, endogenous growth theory arose proposing different hypotheses. One of the pioneer studies has been that by Romer (1986). In this paper, Romer specifies a production function $F(k_i, K, \mathbf{x}_i)$, being k_i and $\mathbf{x}_i$ firm-specific inputs ($\mathbf{x}$ can be seen as a vector of inputs) and K an input external to the firm ("the level of knowledge" defined as a function of the "firm-specific knowledge" k_i). By assuming that F is increasing in K and linear homogeneous in k_i and $\mathbf{x}_i$, a perfect competitive equilibrium is still possible, but the factor k_i no longer exhibits diminishing returns. Consequently, permanent endogenous growth of output per capita is allowed.

Barro (1990) has developed a similar model where K can be interpreted as the stock of public capital (hereafter G). The intuition is that publicly provided capital (like roads, sewer capital, etc.) has positive effects on private production affecting the productivity of the firm-specific inputs. Public capital is assumed a public input that can be used by additional producers without cost. Consequently, total stocks of public goods enter in the production function of each individual firm. In this context, two necessary conditions for the hypothesized constant endogenous growth are: existence of increasing returns to scale over all inputs and existence of constant returns to scale over factors that can be accumulated (private and public capital). A weaker requirement, alternative to the second condition, would be a positive impact of G on the demand for capital. Although not ensuring continuous growth, the presence of this nonrival input would imply a positive government's contribution to growth.

The above conditions can be tested using the theory of the firm. One can test the above mentioned conditions by estimating the total cost function of the firm including public inputs as quasi-fixed factors and all private inputs as variable factors. Thus, the cost function is $C(\mathbf{p}, \mathbf{G}, \mathbf{Y}) = \mathbf{p}'\mathbf{X}(\mathbf{p}, \mathbf{G}, \mathbf{Y})$, where **p** and **X** are the vectors of prices and quantities of inputs, **Y** is the vector of outputs, and **G** is the vector of quasi-fixed public inputs. In this context, increasing returns to scale can be directly tested. The effect of public inputs on the returns to capital, however, can be indirectly tested through the effect of these inputs on the demand for private capital.

Increasing returns to scale are evaluated by considering the elasticity of cost with respect to output (ε_{cy}). It is well known in the production economics literature that the elasticity of cost with respect to output is the dual expression of the elasticity of scale (η_y): $\varepsilon_{cy}=1/\eta_y$[2]. When the elasticity of cost with respect to output is less than one, firms exhibit economies of scale. However, in the presence of factors external to the firm, some adjustments should be made in order to obtain ε_{cy}. Following Morrison and Schwartz (1996), the Le Chatelier principle implies that the adjusted elasticity of cost with respect to output is[3]

[2] See Chambers (1988) for details.

[3] This result comes from the identity $C^A(\mathbf{P}, \mathbf{Pg}, Y) \equiv C(\mathbf{P},\mathbf{G}(\mathbf{P},\mathbf{Pg},Y),Y)$. Taking the derivative with respect to Y on both sides gives

$$\varepsilon_{CY}^{A} = \varepsilon_{CY} + \sum_{G} \varepsilon_{CG}\, \varepsilon_{GY} \tag{1}$$

where the superscript A indicates that the elasticity is "adjusted" to take into account public inputs; ε_{CG} is the elasticity of cost with respect to external factors; and ε_{GY} is the elasticity of "demand for external factors" with respect to output. This demand elasticity should be interpreted as a long-run one representing the change in external factors necessary to maintain the firm on the envelope long-run average cost curve after a change in output. Finally, a value of ε_{CY}^{A} less than one indicates the existence of increasing returns to scale over all inputs (private plus public).

The implied incentives to private investment are derived from the demand for private capital, K(**P**,**G**,**Y**). Thus, the derivative of K(**P**,**G**,**Y**) with respect to **G** gives the effect of public factors on the demand for private inputs. If this derivative is positive, then publicly provided inputs act as an incentive to the accumulation of private capital.

Another interesting result is the estimate of shadow prices of public inputs. Using Shephard's Lemma these shadow prices are easily calculated

$$\nabla_{G} C\,(p, G, Y) = -P_{G}^{*}(p, G, Y) \tag{2}$$

A positive P^{*}_{G} indicates that increases in public input G diminish cost of production and, consequently, have positive effects on private production.

$$\frac{\partial C^{A}}{\partial Y} = \frac{\partial C}{\partial Y} + \sum_{G} \frac{\partial C}{\partial G}\frac{\partial G}{\partial Y}$$

Finally, expressing as elasticities gives equation (1).

Note, however, that testing the hypothesized endogenous growth conditions with a cost function might imply specification errors due to the absence of dynamics. Models of growth consider capital as inputs that adjust gradually toward the steady state. In contrast, the static dual theory of the firm does not consider input dynamics and their transitional adjustments. The following section introduces then a dynamic model incorporating intertemporal optimization by firms and government.

III. The Model

A dynamic dual model of the firm is used to explain growth based on the existence of public inputs. As was hypothesized, public goods might have positive effects on firms' production. If the dual problem of the firms is considered, public inputs reduce cost of production given the level of firms' output. In this sense, increases of public inputs increase firms' productivity. This section introduces intertemporal optimization of economic agents: firms minimize intertemporal costs of production and the government (social planner) maximizes intertemporal welfare. Instantaneous adjustment of inputs is not possible because of the existence of cost of adjustment.

Firms take public inputs as given. Public inputs are considered quasi-fixed inputs of production that they cannot adjust to obtain the minimum possible cost. However, the government, behaving as a social planner, observes producers' surplus and provides public goods to maximize it, subject to the cost of providing the public input.

The model adopted in this paper assumes that the government knows the payoff function of the firms. This assumption implies that the government knows how the firms react when public inputs are changed. The government behaves as a 'leader' and optimizes first. Firms then take the level of public inputs as given and choose private inputs such that their cost of production is minimized.

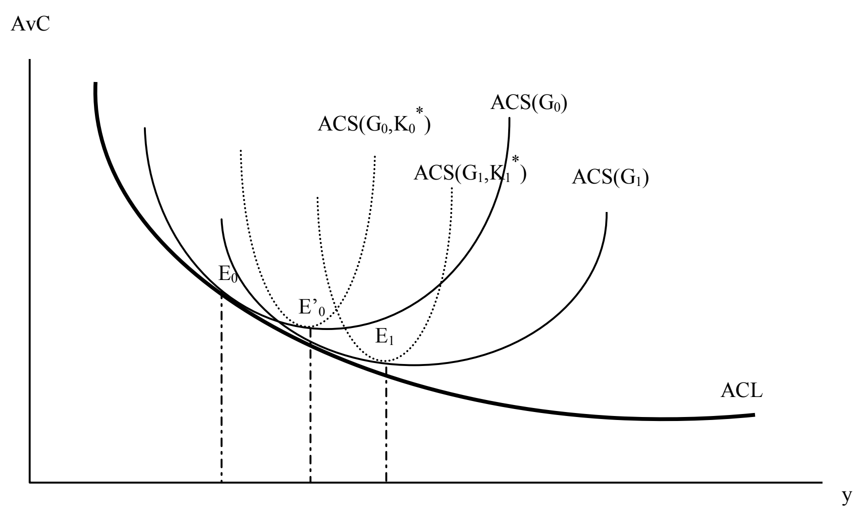
The dynamic of the model then works in the following way

1. The government decides the path of public investment I_g. This implies an optimal path of G (the stock of public inputs).

2. The firms observe the state variable G and determine the optimal path of private investment I (control variable of the firms). Consequently, the optimal path of private inputs Z (state variable) is also determined.[4]

The model can be solved by backward induction. First, the problem of the firms is solved. Then, once the payoff function of the firms is determined, the problem of the government is solved.

The following figure shows the dynamics for the firms.



[4] Note that for the firms, the controls (investment I) are strategies of the "closed-loop" type: the firms do not know the government behavior and the optimal path of I is adjusted every period with the changes in G.

Three average cost curves can be seen in the graph. $ACS(G_t, K_t)$ represents a very short-run average cost curve when private (capital (K) in this case) and public inputs are fixed. $ACS(G_t)$ is the short-run average cost curve when only public inputs are fixed. Finally, ACL is the long-run average cost curve when all inputs are adjusted. At each period t, the firms observe the public input stock G and choose the optimal path of I that allows them to reach the optimal steady state (SS) stock K*. When the stock of public inputs is G_0, firms choose an optimal path of I that allows the firm to reach K_0^* at the minimum cost. The firm moves from E_0 to E'_0. The path is adjusted the next period when the stock G_1 implies a new SS stock K_1^*. The firm then moves to E_1. The two conditions for the hypothesized endogenous growth of the firms can then be seen in the graph:

I. Increasing returns to scale over the long-run average cost curve (ACL): negative slope of ACL.

II. Positive effects of G on the SS stocks of the private capital (i.e. the private input "that can be accumulated"): the SS of K increases from K_0^* to K_1^*.

More formally, firms solve the following problem:

$$\begin{aligned} &\underset{I(t)>0}{\text{Min}} \int_0^\infty e^{-\rho t}[C(y,Z,I;G)+p'Z]dt \\ &\text{subject to } \dot{Z} = I - \delta Z \\ &Z(0) = Z_0 \\ &Z(t) > 0 \qquad \forall t \end{aligned} \tag{3}$$

where C(.) is the variable cost function, y is output, Z is the vector of stocks of quasi-fixed inputs, p is the rental price vector corresponding to Z, I is the vector of gross changes in quasi-fixed inputs, G is the vector of public inputs, and $\rho > 0$ is the firm's real rate of discount. It is assumed that there is one perfectly variable input whose price is normalized to one. Thus, the elements of p are relative rental prices.

Define now J(Z, y, p; G) as the value function that solves problem (3). Assuming that C(y, Z, I, p; G) satisfies the set of regularity conditions (A.1) – (A.6) and J(Z, y, p;G) satisfies properties (B1) – (B5) (see Appendix), duality between C(.) and J(.) can be established.

Duality between C(y, Z, I, p; G) and J(Z, y, p; G):[5] *any J(.) satisfying conditions (B) is the value function corresponding to C(.) that satisfies conditions (A) and is defined by*

$$C(y, Z, I; G) = \underset{p}{\mathrm{Max}}[\rho J(Z, y, p; G) - p'Z - J_z(Z, y, p; G)(I - \delta Z)] \tag{4}$$

or

$$\rho J(Z, y, p; G) = \underset{I}{\mathrm{Min}}[C(y, Z, I; G) + p'Z + J_z(Z, y, p; G)(I - \delta Z)] \tag{5}$$

These two equations provide the relationship between the cost function C(.) and the value function J(.). They allow expressing the parameters of C(.) in terms of the parameters of J(.) when firms minimize intertemporal costs. Thus, the derivative properties that characterize C(.) can be recovered from the parameters of J(.).

Derivative Properties

1. With respect to I:

$C_I(y, Z, I, p; G) = - J_z(Z, y, p; G)$, which must be positive from (A.2) or (B.2.ii).

Testing for $J_z(Z, y, p; G) = 0$ is equivalent to test for adjustment costs in inputs Z.

2. With respect to Z:

$C_z(y, Z, I; G) = (\rho + \delta) J_z(Z, y, p; G) - p - J_{zz}(Z, y, p; G)\dot{Z}^*(Z, y, p; G) < 0$ from (A.2).

This expression gives the shadow price of quasi-fixed inputs.

3. With respect to y:

$C_y(y, Z, I; G) = \rho J_y(Z, y, p; G) - J_{zy}(Z, y, p; G)\dot{Z}^*(Z, y, p; G) > 0$ from (A.2).

This expression represents the output supply of the firms.

4. With respect to p:

$$0 = \rho J_p(Z, y, p; G) - Z - J_{zp}(Z, y, p; G)\dot{Z}^*(Z, y, p; G)$$

Then,

$\dot{Z}^*(Z, y, p; G) = J_{pz}^{-1}(Z, y, p; G)[\rho J_p(Z, y, p; G) - Z]$, which is the dynamic demand for Z.

5. With respect to G:

$$C_G(y, Z, I; G) = \rho J_G(Z, y, p; G) - J_{ZG}(Z, y, p; G)\dot{Z}^*(Z, y, p; G)$$

This expression represents the shadow price of G when the firms are out of the SS. At the SS, the shadow price is

$$C_G(y, Z, I; G) = \rho J_G(Z, y, p; G)$$

If this expression is negative, the shadow price of G is positive, meaning that public inputs reduce cost of production.

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[5] Epstein (1983).

Given these derivative properties, the two previously mentioned endogenous growth conditions can be tested through estimation of parameters of J(Z, y, p; G).

Conditions for Endogenous Growth

1) The effect of G on

   a) The cost function: this is provided by the fifth derivative property in the previous section (positive shadow price of G means that public inputs reduce cost of production).

   b) On the dynamic demand for Z: it can be shown that the dynamic demand for Z can be expressed as

$$\dot{Z}^*(Z, y, p; G) = M(p, G)[Z - \bar{Z}(p, G)] \tag{6}$$

   where $\bar{Z}(.)$ is the SS stock of Z and M(.) is a stable adjustment matrix. This expression yields a flexible accelerator adjustment path for the stocks Z and is the reason for these dynamic models to be called “multivariate flexible accelerator models” (Epstein(1983)). The form of M(.) is determined by the functional form of C(.) but only under certain conditions can be successfully expressed as an explicit function of the parameters of C.

   The effect of G on the dynamic demand for Z can then be decomposed on the effect on the adjustment matrix and the effect on the SS stock of Z. The condition for endogenous growth would be for G to increase the SS stock of private capital K (one of the quasi-fixed factors of the firms). The effect on the adjustment matrix is only an effect on the speed of adjustment toward the SS. However, it is still required for this adjustment to be stable.

2) Scale Effects: there must be increasing returns to scale over all factors of production (public and private factors). The shadow price of the public input is an "inverse demand" for it: $P_G = P_G(Z, y, p; G)$. Then, solving for G, given P_G, gives the direct shadow demand for G that can be substituted into (4) to get

$$C(y, Z, I; G(\bar{P}_G, Z, y, p)) = \underset{p}{\mathrm{Max}}[\rho J(Z, y, p; G(\bar{P}_G, Z, y, p)) - pZ - \\ - J_z(Z, y, p; G(\bar{P}_G, Z, y, p))(I - \delta Z)]$$

Taking the derivative with respect to y, we obtain the adjusted effect of output on cost when the 'shadow demand' for G also changes with firms' output:

$$\frac{\partial C^A}{\partial y} = \rho J_y - J'_{zy}\dot{Z}^* + (\rho J'_G - J_{zG}\dot{Z}^*)G_y(\bar{P}_G, Z, y, p)$$

At the SS, this expression becomes

$$\frac{\partial C^A}{\partial y} = \rho J_y + \rho J'_G G_y(\bar{P}_G, Z, y, p)$$

$$= C_y + C'_G G_y(\bar{P}_G, Z, y, p)$$

Completing elasticities gives equation (1)

$$\varepsilon^A_{CY} = \varepsilon_{CY} + \sum_G \varepsilon_{CG}\varepsilon_{GY} \quad (7)$$

If this expression is less than one, then there are increasing returns to scale over all inputs.

Government Behavior

The government behaves as a benevolent social planner. It chooses the level of public inputs that maximizes welfare (producers' and consumers' surpluses minus the cost of providing public inputs). If a small open economy is considered, the output price is given. Assuming that public inputs do not affect utility of consumers, the problem of the government is reduced to the maximization of producers' surplus. Then, in a dynamic model, the government solves the following problem

$$\begin{aligned} &\underset{I_g>0}{\text{Min}} \int_0^{\infty} e^{-\theta t}[J(y,Z,p;G)+rG+AC(I_g)]dt \\ &\text{subject to } \dot{G} = I_g - \delta_g G \qquad (8) \\ &G(0) = G_0 \\ &G(t) > 0 \quad \forall t \end{aligned}$$

where J(.) is the value function of the firms that comes from their intertemporal cost minimization problem; I_g is the investment in public inputs which stocks are given by vector G; $AC(I_g)$ is the government's adjustment cost of G; δ_g is the depreciation rate of G; r the rental price of G; and θ is the government's rate of discount. The existence of adjustment costs is justified by the multiple activities the government does with given resources. Increasing I_g means that the government must reallocate funds and resources used in the provision of some other public goods, like goods that provide utility to consumers or are inputs for producers in other sectors. This reallocation of resources implies that some resources are wasted in the process. This loss can then be modeled as an adjustment cost.[6]

Define now $J^g(p, Z, y; r, G)$ as the government's value function that solves (8). Assuming that $J(Z, y, p; G) + AC(I_g)$ satisfies conditions (C.1) – (C.6) and $J^g(p, Z, y; r,G)$ satisfies conditions (D.1) – (D.5) (see Appendix), duality between $J(.) + AC(Ig)$ and $J^g(.)$ can be established.

Duality between $J(Z, y, p; G) + AC(I_g)$ and $J^g(p, Z, y; r, G)$: any $J^g(p, Z, y; r, G)$ satisfying conditions (D) is the value function corresponding to $J(Z, y, p; G) + AC(I_g)$ that satisfies conditions (C) and is defined by

$$J(Z,y,p;G) + AC(I_g) = \underset{r}{Max}[\theta J^g(Z,y,p,r,G) - rG - J^g_G(Z,y,p,r,G)(I_g - \delta_g G)] \qquad (9)$$

or

$$\theta J^g(Z,y,p,r,G) = \underset{I_g}{Min}[J(Z,y,p;G) + rG + AC(I_g) + J^g_G(Z,y,p,r,G)(I_g - \delta_g G)] \qquad (10)$$

These two expressions provide the relationship between J(.), the value function of the firms, and $J^g(.)$, the value function of the government. They allow expressing the parameters of J(.) in terms of the parameters of $J^g(.)$ and vice versa, when the government maximizes intertemporal welfare by minimizing producers' cost. Thus, the derivative properties that characterize J(.) can be recovered from the parameters of $J^g(.)$.

[6] Note that assuming the objective functions of consumers and producers are separable with respect to the objective function of problem (8), the government can decide the optimal provision of different public goods separately.

Derivative Properties

1. With respect to I_g:

$$0 = AC_{I_g} + J^g_G(p, Z, y; r, G)$$

or

$$-J^g_G(p, Z, y; r, G) = AC_{I_g} > 0,$$

This is positive given $AC_{Ig} > 0$.

2. With respect to G:

$$\theta J^g_G(p, Z, y; r, G) = J_G(Z, y, p; G) + r + J^g_{GG}(p, Z, y; r, G)\dot{G}^* - \delta_g J^g_G(p, Z, y; r, G)$$

or

$$J_G(Z, y, p; G) = (\theta + \delta_g)J^g_G(p, Z, y; r, G) - r - J^g_{GG}(p, Z, y; r, G)\dot{G}^*$$

This expression is the firms' willingness to pay for G (shadow price) when the firms are at the steady state. If the expression is negative (condition (D.2)(ii)), then public inputs reduce cost of production. When the government is also at the SS, we have

$$-J^g_G(p, Z, y; r, G) = \frac{-J_G(Z, y, p; G) - r}{\theta + \delta_g}$$

which could be interpreted as a 'social' shadow price: the net social benefit (the firms' shadow price of G minus the government's cost of providing G) adjusted by the 'social' discount rate plus the depreciation rate of public inputs.

3. With respect to r:

$$\theta J_r^g(p, Z, y; r, G) = G + J_{Gr}^g(p, Z, y; r, G)\dot{G}^*$$

or

$$\dot{G}^* = J_{Gr}^{g-1}(p, Z, y; r, G)[\theta J_r^g(p, Z, y; r, G) - G]$$

which gives the optimal path of G.

4. With respect to Z:

$$\theta J_z^g(p, Z, y; r, G) = J_z(Z, y, p; G) + J_{Gz}^g(p, Z, y; r, G)\dot{G}^*$$

or

$$J_z(Z, y, p; G) = \theta J_z^g(p, Z, y; r, G) - J_{Gz}^g(p, Z, y; r, G)\dot{G}^* < 0$$

where the sign is given by condition B.2(ii): the value function of the firm is decreasing in Z.

5. With respect to y:

$$\theta J_y^g(p, Z, y; r, G) = J_y(Z, y, p; G) + J_{Gy}^g(p, Z, y; r, G)\dot{G}^*$$

or

$$J_y(Z, y, p; G) = \theta J_y^g(p, Z, y; r, G) - J_{Gy}^g(p, Z, y; r, G)\dot{G}^* > 0$$

where the sign is given by condition B.2(iii): the value function of the firm is increasing in y. At the SS level of G (or with no adjustment cost of G),

$$J_y(Z, y, p; G) = \theta J_y^g(p, Z, y; r, G) > 0$$

The following section presents the empirical model.

IV. Empirical Implementation

This section presents the empirical implementation of the dynamic model presented in previous sections. Ideally, having data on r, the optimal path of I_g could be estimated. Nevertheless, the conditions for endogenous growth can still be tested through estimation of the dynamic demands for inputs. Adopting a flexible functional form for the value function, all parameters of interest can be recovered from the estimation of the dynamic demands for private quasi-fixed inputs and the demand for the variable input.

Consider, for example, the following normalized quadratic value function:

$$J(P,Z,Q) = a_0 + A'\begin{bmatrix} P \\ Z \\ Q \end{bmatrix} + \frac{1}{2}\begin{bmatrix} P & Z & Q \end{bmatrix} B \begin{bmatrix} P \\ Z \\ Q \end{bmatrix} \tag{11}$$

where Z is the vector of quasi-fixed factors, P is the corresponding vector of rental prices, and Q is the vector of output and public inputs; A and B are parameter matrices of appropriate order; a_0 is a scalar parameter.

The dynamic demands for quasi-fixed inputs can then be calculated from

$$\dot{Z}^*(Z,y,p;G) = J_{pz}^{-1}(Z,y,p;G)[\rho J_p(Z,y,p;G) - Z] \tag{12}$$

and the demand for the variable input (X*) is calculated from

$$X^*(Z,y,p;G) = \rho[J(Z,y,p;G) - J_p(Z,y,p;G)P] - [J_Z(Z,y,p;G) - p'J_{PZ}]\dot{Z}^* \tag{13}$$

Joint estimation of (12) and (13) gives all the parameters needed for testing the effects of public inputs on firms' costs, steady state stocks of capital, and scale. However, the large number of parameters to estimate does not allow us to estimate this model with available aggregate US agricultural data.[7] For this reason, previous estimations based on a static model are presented here.

The contribution of public capital and public R&D to US agricultural growth was then tested using a static duality approach. The study covers the period 1951 – 1992. A flexible cost function is specified, in particular, a translog cost function.[8]

In this study, the cost function includes: prices of private inputs (labor (N), intermediate inputs (M), and an aggregated measure of capital and land (K)); and stocks of quasi-fixed public inputs (public capital (G) and R&D (R)). Public capital stocks are constant-dollar values of federal, state, and local structures. Public R&D stocks are constructed from R&D spending using Huffman and Evenson's method (1989). Finally, the output (Y) is a Fisher index of all crops and livestock products. All data is in constant 1987 dollars.[9]

The adopted translog cost function is

$$\ln C = \alpha_0 + \alpha' X + \frac{1}{2} X' \beta X \tag{14}$$

where ln C represents the natural log of the cost and X is the following vector

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[7] Authors are currently working on estimation of this model with recently acquired data for US agriculture at the state level. This implies estimation of a panel of 48 states and 23 years. It is planned to have these estimations by July 2001.

[8] Normalized Quadratic and Generalized Leontief have also been used, but results showed that high multicolinearity affected the estimation of these functional forms.

$$X = \begin{bmatrix} \ln P_n \\ \ln P_k \\ \ln G \\ \ln R \\ \ln Y \end{bmatrix}$$

The cost C, the price of labor (P_n) and the price of capital (P_k) are normalized by the price of materials. The properties of symmetry and linear homogeneity in prices are imposed, and the parameters corresponding to the price of materials can be obtained from the appropriate combination of α and β.

By first-order differentiation of the translog cost function, one gets the share equations:

$$S = \alpha + \beta X \tag{15}$$

where S is a column vector consisting of private input shares, the output share, and the 'shadow' shares of the public inputs. Note again that the shares of output and private inputs must be positive, while the 'shadow' share of public inputs must be negative for the public inputs to reduce costs.[10]

Once the parameters of the cost function are estimated, the conditions hypothesized by the postulated endogenous growth theory can be tested. The condition of increasing returns to scale is tested by calculating the elasticities involved in (1). The possible positive effect of public inputs on the demand for private capital is checked by

[9] See Ball et al. (1997) for details on all agricultural data. Public capital stocks are from Survey of Current Business and include buildings, highways, streets, sewer structures etc. Military structures are excluded. R&D spending is from Alston and Pardey (1996).

obtaining Allen elasticities of substitution. In particular, if the Allen elasticities of substitution between private capital and public factors are positive, the condition of positive government contribution to private capital accumulation is satisfied.

The model also allows one to get another interesting result: the shadow prices of public inputs. Following equation (2), the implicit price of G and R is given by

$$P_{Qi}^{*} = -\frac{\partial C}{\partial Q_i} = -\frac{C}{Q_i}(\alpha_{Qi} + \beta_{nQi}\ln P_n + \beta_{kQi}\ln P_k + \beta_{QiQi}\ln Q_i + \beta_{QiQj}\ln Q_j + \beta_{Qiy}\ln Y) \qquad (16)$$

where Q represents G and R, and subscripts i and j are used to distinguish between them. A positive P_{Qi}^{*} implies that the external factor Q_i reduces the cost of private production.

V. Results

The translog cost function, the input shares of labor and capital, and the output share were jointly estimated imposing symmetry constraints among parameters. The cost and the prices of labor and capital were normalized by the price of materials. Since the paper uses highly aggregated data, iterative three stages least squares (I3SLS) was adopted. Iterations were shown by Barten (1969) to lead to estimates that are invariant to the equation chosen for deletion. Fitted values from the regressions of the logs of P_n/P_m, P_k/P_m and Y on a set of exogenous variables were used. The set of exogenous variables used as instruments includes total US population, number of non-farm workers, effective land tax ratios, interest rate charged by land banks, total agricultural exports, total non-agricultural exports, and time. The adjusted R^2s were 0.775, 0.926, and 0.975 for the logs of P_n/P_m, P_k/P_m and Y estimations, respectively.

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[10] This is the monotonicity property.

Table 1A presents the parameter estimates of the I3SLS estimation. In addition to the imposed properties of symmetry and homogeneity, monotonicity and concavity are properties of the cost function that cannot be imposed globally. However, they are checked at each data point. Monotonicity in output and private inputs is satisfied in all the observations. The concavity property, in contrast, is satisfied in 22 of the 42 years of the sample. This result implies that the parameter estimates are not consistent with the theory in almost half of the sample. Thus, estimates that are more reliable and consistent with the theory should be obtained.

Table 1B presents Bayesian parameter estimates that satisfy concavity and monotonicity in output and private inputs. The procedure developed by Geweke(1986) was used. The concavity and monotonicity properties were introduced as prior beliefs given by the neoclassical theory of the firm. In this way, these restrictions are introduced using Bayesian inference and inequality constrained estimation. The result of this procedure is a posterior distribution for parameters that is conditional on the observed data. Using the Monte Carlo method, 20,000 replications were drawn from a multivariate normal distribution with mean and covariance matrix given by the I3SLS parameter estimates and their covariance matrix. For each replication, the substitution matrix was obtained and the eigenvalues were computed.[11,12] The Bayesian estimate is the mean of the replications that satisfy the restrictions when a quadratic loss function is adopted. The number of replications that satisfy the curvature and monotonicity constraints is 12,626, which represents 63% of the replications. This proportion is interpreted as the probability

[11] The substitution matrix was evaluated at the mean.
[12] It should be noted that the monotonicity constraint was not binding.

that concavity holds. The parameter estimates in Table 1B are then the mean of the posterior distribution using the 12,626 replications.[13]

Table 2 presents the estimated shadow prices of public inputs using the Bayesian estimates. First, note that the shadow price of public capital is negative, although insignificant for some periods. This result contradicts the property of monotonicity in public inputs. This property is required under the assumption that the government behaves as a benevolent social planner that provides public inputs.[14] When this property was introduced as a prior belief in the estimation, no replication of the 12,626 satisfied it.

In contrast, the shadow price of public R&D has been positive and significantly different from zero for the entire sample period. This implies that public R&D reduces private costs of production. For example, an increase of 1 billion dollars in the stock of public R&D reduces the cost of the agricultural sector in 2 million dollars.

Endogenous growth conditions imply tests for long-run increasing returns to scale and impacts of G and R&D on the demand for capital. Table 3 presents Allen elasticities of substitution and output elasticities of demand. Note that Allen elasticities between private inputs and public inputs are negative, indicating substitutability between them. In particular, the substitutability between public inputs and capital is opposed to the hypothesis of complementarity given by the postulated endogenous growth theory: increases in public inputs reduce the demand for private capital.

From the same table, other results of interest can be inferred. For example, the substitutability between labor and public R&D helps to explain the evolution of labor

[13] For further details on Bayesian estimation, see Geweke (1986), Chalfant et al (1987), and Chalfant et al (1989).

[14] This is equivalent to condition (D.2)(ii) in the dynamic model.

usage over the sample period. In contrast, the substitutability between public R&D and materials does not support the increasing use of this private input. Hence, R&D seems to have been a substitute of labor, materials and capital.

Finally, Table 4 shows the adjusted elasticity of cost with respect to output and its components. Note that the traditional measure, ε_{cy}, has been less than one for the whole sample period indicating the possibility of increasing returns to scale. When the adjustment is done, increases in output, along with increases in public R&D, increase agricultural costs even in a smaller proportion (given the negative value of the elasticity of cost with respect to R&D, consistent with positive shadow prices). Thus, the returns to scale are 'even more increasing' when this measure is adjusted for the presence of public R&D, result that is consistent with the first hypothesis of the postulated endogenous growth theory. However, the adjusted elasticity of cost with respect to output presented in Table 4 has to be taken with care, given the insignificant effect of public capital on costs of production.

V. Conclusions

The paper has presented a dynamic model to measure the contribution of public inputs on productivity growth. It has also shown testable hypotheses related with the main postulates of a version of endogenous growth theory ('AK' models with public goods) using duality theory. In particular, two conditions have been postulated and preliminary tested with a static approach. One is the existence of increasing returns to scale over all inputs (private and public). The other one is the positive effect of public inputs on the demand for private factors that can be accumulated (capital). Although the second condition is not satisfied, there is some evidence that public R&D may have contributed to the existence of increasing returns to scale. With respect to the second condition, however, public R&D stocks appear to have been a substitute of private inputs. This helps to explain the decreasing use of labor, but it does not explain the decreasing use of materials. Additionally, public infrastructure is insignificant for some years. This result stands in contradiction to the monotonicity property necessary for public inputs under the hypothesized government's behavior. It also contradicts previous results obtained for the manufacturing sector.

Finally, more work is needed to overcome the limitations of this paper. The use of time series may originate problems due to the presence of nonstationary data. In this sense, future work should look at long-run relationships among variables. One way of overcoming this problem is to consider a cointegration approach. However, the large number of parameters to estimate relative to the sample size makes this task difficult. Another alternative approach is the use of panel data, estimating the model at the state level. This approach can improve this study in both theoretic and econometric aspects. In terms of theoretic aspects, panel estimation at the state level could also allow for the presence of spillover effects as well as different patterns of growth in each state. In terms of econometric, the larger number of degrees of freedom allows estimation of the dynamic model overcoming possible specification problems. Clearly, a model that introduces dynamics and panel data is the direction to follow.[15]

[15] Again, authors are currently working on this.

APPENDIX

This appendix presents conditions (A) to (D) that guarantee duality between cost and value functions of the firms, and between value function of the firms and the one of the government.

Conditions (A)

It is assumed that C(y, Z, I, p; G) satisfies the following set of regularity conditions:

(A.1) C(y, Z, I, p; G) ≥ 0.

(A.2) C(y, Z, I, p; G) is increasing in (y,I) and decreasing in Z.

(A.3) C(y, Z, I, p; G) is convex in I.

(A.4) For each (Z_0, y, p; G) a unique solution exists for (3). This means that there are well-defined factor demand functions associated with (3).

(A.5) For each (Z_0, y, p; G), problem (3) has a unique steady state (SS) stock $\bar{Z}(y,p;G)$ that is globally stable. (Uniqueness and stability of the SS).

(A.6) For any (Z_0, y, p; G), there exists $\hat{p}$ such that $\hat{I}$ is the optimal gross investment vector at t = 0 in (3) given (Z_0, y, p; G).

Conditions (B)

It is assumed that the value function J(Z, y, p; G) satisfies the following properties:

(B.1) J(Z, y, p; G) ≥ 0.

(B.2) (i) $(r+\delta)J_z(Z,y,p;G) - p - J_{zz}(Z,y,p;G)\dot{Z}^*(Z,y,p;G) < 0$. (this expression is dual to $C_z < 0$).

(ii) $J_z(.) < 0$. (dual to $C_I(.) > 0$)

(iii) $\rho J_y(.) - J_{yz}(.)\dot{Z}^*(.) > 0$ (dual to Cy(.) > 0), where

$$\dot{Z}^*(Z, y, p; G) = J_{pz}^{-1}(.)[\rho J_p(.) - Z]$$

(B.3) The following expression is concave in p:

$$\rho J(Z, y, p; G) - pZ - J_z(Z, y, p; G)\dot{Z}^*(Z, y, p; G)$$

Under some specific functional forms (like the normalized quadratic presented above), J_z is linear in p and the curvature requirement reduces to concavity of J(.) in p. This condition is dual to (A.3).

(B.4) $I^*(Z, y, p; G) \equiv \dot{Z}^*(Z, y, p; G) + \delta Z$ and $X^*(Z, y, p;G)$ are positive. $X^*(Z, y, p;G)$ is defined as the demand for the variable input.

(B.5) The stock profile that solves $\dot{Z}^*(.) = J_{pz}^{-1}(.)[\rho J_p(.) - Z]$, Z(0) > 0, has a unique globally stable SS $\bar{Z}(y, p; G)$.

Then, under conditions (A) and (B), duality between C(.) and J(.) can be established as in equations (4) and (5).

Conditions (C)

It is assumed that J(y, Z, p; G) + AC(I_g) satisfies the following conditions:

(C.1) J(y, Z, p; G) + AC(Ig) ≥ 0

(C.2) (i) J(y, Z, p; G) + AC(Ig) is increasing in Ig. Given that J(.) is independent of Ig, AC(.) must be increasing in Ig.

(ii) $J(y, Z, p; G) + AC(Ig)$ is decreasing in G. Given that AC(.) is independent of G, J(.) must be decreasing in G.

(C.3) $J(y, Z, p; G) + AC(Ig)$ is convex in Ig. Then, AC(.) must be convex in Ig.

(C.4) For each (Z, p, y, r, G_0), there exists a unique solution for (8). (There are well-defined supplies of public inputs).

(C.5) For each (Z, p, y, r, G_0), (8) has a unique steady state (SS) stock $\bar{G}(Z, p, y, r)$ that is globally stable.

For any (Z, p, y, r, G_0), there exists $\hat{r}$ such that $\hat{I}_g$ is the optimal public gross investment vector at t = 0 in (2), given (Z, p, y, r, G_0).

Conditions (D)

It is assumed that $J^g(y, Z, p; r, G)$ satisfies the following conditions:

(D.1) $J^g(y, Z, p; r, G) \geq 0$

(D.2) (i) $J^g_G(.) < 0$. This expression is dual to (C.2)(i): $AC_{Ig}(.) > 0$ (there are adjustment costs in the provision of public inputs).

(ii) $(\theta + \delta_g)J^g_G(.) - J^g_{GG}(.)\dot{G}^* < 0$. This expression is dual to (C.2)(ii): $J_G(.) < 0$ (positive shadow prices of public inputs). Given $J^g_G(.) < 0$, it is sufficient for (D.2)(ii) to hold that $-J^g_{GG}(.) < 0$ (increases of the public good decrease the shadow price of it).

(D.3) $\theta J^{g}(Z,y,p,r,G) - rG - J^{g}_{G}(Z,y,p,r,G)\dot{G}^{*}$ must be concave in r. This is dual to condition (C.3).

(D.4) $I^{*}_{g}(Z,p,y;r) \equiv \dot{G}^{*}(Z,p,y;r) + \delta_{g}G$ is positive.

(D.5) The stocks that solves $\dot{G}^{*}(Z,p,y;r) = J^{g-1}_{Gr}(.)[\theta J^{g}_{r}(.) - G]$, with G(0) > 0, has a unique globally stable SS $\bar{G}(Z,p,y;r)$.

Then, under conditions (C) and (D), duality between $J^{g}(.)$ and J(.) + AC(Ig) can be established as in equations (9) and (10).

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TABLE 1A
I3SLS PARAMETER ESTIMATES

Parameter	Estimate	Std. Error	Parameter	Estimate	Std. Error
α_0	-102.590*	57.881	β_{ky}	-0.044*	0.026
α_n	4.459**	0.532	α_g	36.865**	13.840
α_k	-0.639	0.728	α_r	-44.703**	12.686
β_{nn}	0.026	0.039	α_y	0.194	1.467
β_{nk}	-0.029**	0.011	β_{gg}	-4.942**	1.592
β_{ng}	-0.220**	0.044	β_{gr}	4.740**	1.272
β_{nr}	-0.130**	0.049	β_{gy}	-0.193*	0.106
β_{ny}	0.171*	0.098	β_{rr}	-3.281**	0.830
β_{kk}	0.089**	0.016	β_{ry}	0.468**	0.143
β_{kg}	0.011	0.074	β_{yy}	-0.367	0.311
β_{kr}	0.111**	0.050			

* Significant at 90%
** Significant at 95%

TABLE 1B
BAYESIAN PARAMETER ESTIMATES

Parameter	Estimate	Std. Error	Parameter	Estimate	Std. Error
α_0	-103.240*	58.098	β_{ky}	-0.044	0.027
α_n	4.462**	0.528	α_g	36.999**	13.887
α_k	-0.652	0.726	α_r	-44.789**	12.694
β_{nn}	0.026	0.038	α_y	0.207	1.459
β_{nk}	-0.029**	0.011	β_{gg}	-4.956**	1.598
β_{ng}	-0.221**	0.043	β_{gr}	4.758**	1.276
β_{nr}	-0.139**	0.048	β_{gy}	-0.194*	0.106
β_{ny}	0.171*	0.095	β_{rr}	-3.285**	0.835
β_{kk}	0.089**	0.016	β_{ry}	0.468**	0.143
β_{kg}	0.012	0.074	β_{yy}	-0.365	0.311
β_{kr}	0.111**	0.050			

TABLE 2
ESTIMATED SHADOW PRICES*

Period	Pg*	T-Ratio	Period	Pr*	T-Ratio
1951-1959	-.0000020	-1.98	1951-1959	0.00653	3.66
1960-1969	-.0000002	-0.39	1960-1969	0.00215	2.60
1970-1979	-.0000009	-1.57	1970-1979	0.00086	2.62
1980-1992	-.0000028	-3.45	1980-1992	0.00143	5.87
1951-1992	-.0000014	-2.69	1951-1992	0.00164	5.01

* Evaluated at sample means

TABLE 3
ALLEN ELASTICITIES OF SUBSTITUTION
AND OUTPUT ELASTICITIES*

Elasticity	1951-1959	1960-1969	1970-1979	1980-1992
σ_{nn}	-0.78	-1.30	-2.30	-3.30
σ_{mm}	-1.36	-1.26	-1.08	-1.07
σ_{kk}	-0.61	-1.72	-1.30	-1.02
σ_{nm}	1.02	1.02	1.03	1.04
σ_{nk}	0.49	0.66	0.65	0.62
σ_{km}	-0.54	0.27	0.54	0.62
σ_{ng}	-0.43	3.66	0.40	-0.46
σ_{mg}	-1.81	-5.75	-1.87	-1.25
σ_{kg}	-1.15	-1.47	-1.07	-1.02
σ_{nr}	-1.14	-1.44	-2.06	-1.58
σ_{mr}	-0.96	-0.90	-0.86	-0.94
σ_{kr}	-0.45	-0.34	-0.25	-0.75
σ_{ny}	1.45	1.61	1.83	1.99
σ_{my}	0.50	0.51	0.59	0.65
σ_{ky}	0.42	0.70	0.81	0.86

* Evaluated at sample means

TABLE 4
ADJUSTED ELASTICITY OF COST
WITH RESPECT TO OUTPUT*

Elasticity	1951-1959	1960-1969	1970-1979	1980-1992
ε_{cy}	0.70	0.69	0.75	0.85
ε_{cg}	0.72	0.12	0.57	2.02
ε_{cr}	-1.88	-0.78	-0.48	-1.18
ε_{gy}	-0.07	-0.023	-0.05	-0.218
ε_{ry}	0.10	0.015	-0.028	0.091
ε^{A}_{cy}	0.47	0.677	0.731	0.304

* Evaluated at sample means