INDIRECT UTILITY FUNCTIONS AND TESTABLE CONDITIONS

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We develop testable hypotheses for utility maximization given risk averse producers based on a general specification of the utility function. This is a direct expansion of the model posed by Pope (1978). Empirical tests using production data with a translog specification indicate that utility maximization does not always hold.

Key words: production, utility maximization

Introduction

In the late 1970's and early 1980's there was considerable research in the field of agricultural economics for determining the efficiency of economic agents. One of the principal researchers in this area was Rulon D. Pope. Published in 1978 in the AJAE he presented his work on the expected utility hypothesis and demand-supply restrictions. In his research, Pope treats the case of a risk averse agent in the most general of utility functions (that is he does not assume any specific functional from) in his theoretical derivations or any particular distribution of returns. In the derivation of the matrix of second partials, of the expected price, he concludes that there is little that can be determine about the own price slopes of demand and supply functions for the general form case. He finds that the implied demand and supply functions are not homogenous of degree zero in expected price, whereas under the case of risk neutrality and when risk preferences are linear these conditions do hold. Thus, he abandons the general case in

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an effort to focus on specific classes of utility functions, which generate tractability in their results (Foster 2003).

In Pope's paper there is a result similar to "Roy's Identity" in the consumer problem that can be obtained from the first order conditions that Pope fails to address concerning the general utility function, but can be found in other models with uncertain prices. We name this the "Paris Identity" in honor of Professor Q. Paris who discovered this identity in a similar context (date), which allows for the development of a testable Hessian Matrix. From the second order conditions of the Primal-dual problem we know that the matrix is Positive Semi-Definite (PSD), which is a condition that can be evaluated at specific data values but not imposed on a globally, for any know functional forms. This occurs because the elements of the matrix are data dependent for any reasonable specification of the indirect utility function.

Literature

Much research has been conducted showing that the inclusion of uncertainty about output prices affects many of the testable relationships that are tractable in the certainty case. One problem that exists is the fact that we are unable sign the slopes of either output supply or input demands, therefore the uncertainty destroys the Hicksian reciprocity and zero homogeneity (Paris 1988) that is found in the certainty case. It is often the case that in the short-run under the guise of uncertainty we observe input demand functions, which are upward sloping and supply functions, which are downward sloping. One possible explanation for this phenomenon is that when risk is present in output prices, a producer who is risk averse, may factor their aversion into the production of output, which in turn may cause a violation of slopes of input demand and output supply conditions that are testable under a static model with certainty. In order to get results for the comparative statics problem, which is, econometrically tractable certain assumptions must be imposed upon the utility function including but not limited to homotheticity and decreasing risk avers ion (Paris 1998). However, the results from imposing such restrictions are less than satisfactory because the imposition of more and more assumptions takes us away from what we are truly after and this a general testable framework of comparative statics. Therefore, utility maximization in the short run as modeled Pope and others provides few specific testable comparative statics result under uncertainty. However, in this paper we examine the testable implications in a broader approach.

Data and Methodology

The indirect utility function V can be represented in the following form:

$$(1)\ \ln V = \ln V\left(p_1, p_2, ..., p_m\right)$$

Using this we can derive the logarithmic form of Roy's identity, which is the budget share for the j[th] commodity from the identity:

$$(2)\ \frac{r_j X_j}{r_j q_j} = \frac{\partial \ln V}{\partial \ln r_j} \Big/ \frac{\partial \ln V}{\partial \ln p_j} \qquad \forall \mathrm{j} = 1,2,...,\mathrm{m}$$

We will now approximate the logarithm of the indirect utility function with the translog utility function, which is quadratic in the logarithms of the ratios of prices to the value of the total expenditure (Christensen et al. 1975):

$$(3)\ \begin{aligned} \ln V = {} & \alpha_o + \sum_i^1 B_i \ln p_i + \sum_j^3 \eta_j \ln r_j + \frac{1}{2}\sum_i^1 \sum_j^1 \beta_{ij} \ln p_i \ln p_j \\ & + \sum_j^3 \sum_i^1 \delta_{ji} \ln r_j \ln p_i + \frac{1}{2}\sum_j^3 \sum_m^3 \eta_{ji} \ln r_j \ln r_m + \varepsilon \end{aligned}$$

where Q_j: Quantity index of crop products

X_1: Quantity index of inputs

X_1: Quantity index of family and hired labor

X_2: Quantity index of land, structures, durable equipment, animal capital, and inventories

X_3: Quantity index of materials (energy, feed and seed, chemicals, and miscellaneous inputs)

p_1: Price index of the crop output

rj: Price index of inputs

r_1: Price index of family and hired labor

r_2: Price index of land, structures, durable equipment, animal capital, and inventories

r_3: Price index of materials (energy, feed and seed, chemicals, and miscellaneous inputs)

The data used to test the slopes of the supply and demand functions along with the definiteness of the matrix, is the data set developed by Capalbo, Vo, and Wade (1985) for measuring agricultural productivity and characterizing the structure of US agriculture.

To determine the proper first and second derivatives for use in the econometric estimation, the above equation was programmed into Maple®. The following equations represent a form of Roy's Identity (as presented before in the general model) and are the system of equations that are estimated in the econometric model:

(4)

$$ROYS_{1,1} := -\frac{(\eta_1 + \delta_{1,1}\ln(p_1) + \eta_{1,1}\ln(r_1) + \eta_{2,1}\ln(r_2) + \eta_{3,1}\ln(r_3))\,p_1}{r_1\,(\beta_1 + \beta_{1,1}\ln(p_1) + \delta_{1,1}\ln(r_1) + \delta_{2,1}\ln(r_2) + \delta_{3,1}\ln(r_3))}$$

(5)

$$ROYS_{2,1} := -\frac{(\eta_2 + \delta_{2,1}\ln(p_1) + \eta_{2,1}\ln(r_1) + \eta_{2,2}\ln(r_2) + \eta_{3,2}\ln(r_3))\,p_1}{r_2\,(\beta_1 + \beta_{1,1}\ln(p_1) + \delta_{1,1}\ln(r_1) + \delta_{2,1}\ln(r_2) + \delta_{3,1}\ln(r_3))}$$

(6)

$$ROYS_{3,1} := -\frac{(\eta_3 + \delta_{3,1}\ln(p_1) + \eta_{3,1}\ln(r_1) + \eta_{3,2}\ln(r_2) + \eta_{3,3}\ln(r_3))\,p_1}{r_3\,(\beta_1 + \beta_{1,1}\ln(p_1) + \delta_{1,1}\ln(r_1) + \delta_{2,1}\ln(r_2) + \delta_{3,1}\ln(r_3))}$$

η_1, δ_{11}, η_{11}, η_{21}, η_{31}, β_1 (if β1 is not assumed to have a value), β_{11}, δ_{21}, δ_{31}, η_2, η_{22}, η_{32}, η_3, and η_{33} are the parameters, which are estimated econometrically. Multiple specifications of β1 were necessary to combat the problem of expenditure share equations being homogeneous of degree zero in parameters, thus estimates of these parameters using least squares are not unique (any multiple of the least squares multiple will yield the same result as the data) (Foster 2003). Therefore, the following values of β1 were used -1, 0, and 1, prior to estimation of the model. In addition, the coefficient for β1 was also estimate in the model.

The above equations were estimated in Shazam using a nonlinear seemingly unrelated regression (SUR) model. A seemingly unrelated regression model is chosen, due to the likelihood that there are common factors among the error terms that have been omitted as explanatory variables for all three of the Roy's Identity equations, i.e., there is contemporaneous correlation between errors in the different equations.

When working with a nonlinear SUR model, autocorrelation is often present, therefore a Durbin-Watson statistic was used to test for the presence of autocorrelation. Autocorrelation was found to be present in the in the model. Thus, in an effort to correct for the presence of autocorrelation in the model, the aforementioned system of equations was first differenced (elevating the problem of non-stationary). The following equations were subtracted form the previous Roy's Identity equations to obtain the first difference equations

(7)

$$ROYS_{1,1,t-1} := -\frac{(\eta_1 + \delta_{1,1}\ln(p_{1,t-1}) + \eta_{1,1}\ln(r_{1,t-1}) + \eta_{2,1}\ln(r_{2,t-1}) + \eta_{3,1}\ln(r_{3,t-1}))\,p_{1,t-1}}{r_{1,t-1}\,(\beta_1 + \beta_{1,1}\ln(p_{1,t-1}) + \delta_{1,1}\ln(r_{1,t-1}) + \delta_{2,1}\ln(r_{2,t-1}) + \delta_{3,1}\ln(r_{3,t-1}))}$$

(8)

$$ROYS_{2,1,t-1} := -\frac{(\eta_2 + \delta_{2,1}\ln(p_{1,t-1}) + \eta_{2,1}\ln(r_{1,t-1}) + \eta_{2,2}\ln(r_{2,t-1}) + \eta_{3,2}\ln(r_{3,t-1}))\,p_{1,t-1}}{r_{2,t-1}\,(\beta_1 + \beta_{1,1}\ln(p_{1,t-1}) + \delta_{1,1}\ln(r_{1,t-1}) + \delta_{2,1}\ln(r_{2,t-1}) + \delta_{3,1}\ln(r_{3,t-1}))}$$

(9)

$$ROYS_{3,1,t-1} := -\frac{(\eta_3 + \delta_{3,1}\ln(p_{1,t-1}) + \eta_{3,1}\ln(r_{1,t-1}) + \eta_{3,2}\ln(r_{2,t-1}) + \eta_{3,3}\ln(r_{3,t-1}))\,p_{1,t-1}}{r_{3,t-1}\,(\beta_1 + \beta_{1,1}\ln(p_{1,t-1}) + \delta_{1,1}\ln(r_{1,t-1}) + \delta_{2,1}\ln(r_{2,t-1}) + \delta_{3,1}\ln(r_{3,t-1}))}$$

The resulting equations represent the new system of equations that are estimated to obtain the estimated coefficients for the parameters in the model.

The first derivatives of the indirect translog utility function were then differentiated for the second time with respect to r_j and p_i to obtain the second derivatives of the indirect translog utility function. These second derivatives allow us to form the Hessian Matrix of the indirect translog utility function that will be tested for positive definiteness and positive semi-definiteness for every observation. The Hessian Matrix is:

(10)

$$H := \begin{bmatrix} -\frac{\eta_1}{r_1^2} - \frac{\delta_{1,1}\ln(p_1)}{r_1^2} + \frac{\eta_{1,1}}{r_1^2} - \frac{\eta_{1,1}\ln(r_1)}{r_1^2} - \frac{\eta_{2,1}\ln(r_2)}{r_1^2} - \frac{\eta_{3,1}\ln(r_3)}{r_1^2}, & \frac{\eta_{2,1}}{r_2 r_1}, & \frac{\eta_{3,1}}{r_3 r_1}, & \frac{\delta_{1,1}}{r_1 p_1} \\ \frac{\eta_{2,1}}{r_2 r_1}, & -\frac{\eta_2}{r_2^2} - \frac{\delta_{2,1}\ln(p_1)}{r_2^2} - \frac{\eta_{2,1}\ln(r_1)}{r_2^2} + \frac{\eta_{2,2}}{r_2^2} - \frac{\eta_{2,2}\ln(r_2)}{r_2^2} - \frac{\eta_{3,2}\ln(r_3)}{r_2^2}, & \frac{\eta_{3,2}}{r_3 r_2}, & \frac{\delta_{2,1}}{r_2 p_1} \\ \frac{\eta_{3,1}}{r_3 r_1}, & \frac{\eta_{3,2}}{r_3 r_2}, & -\frac{\eta_3}{r_3^2} - \frac{\delta_{3,1}\ln(p_1)}{r_3^2} - \frac{\eta_{3,1}\ln(r_1)}{r_3^2} - \frac{\eta_{3,2}\ln(r_2)}{r_3^2} + \frac{\eta_{3,3}}{r_3^2} - \frac{\eta_{3,3}\ln(r_3)}{r_3^2}, & \frac{\delta_{3,1}}{r_3 p_1} \\ \frac{\delta_{1,1}}{r_1 p_1}, & \frac{\delta_{2,1}}{r_2 p_1}, & \frac{\delta_{3,1}}{r_3 p_1}, & -\frac{\beta_1}{p_1^2} + \frac{\beta_{1,1}}{p_1^2} - \frac{\beta_{1,1}\ln(p_1)}{p_1^2} - \frac{\delta_{1,1}\ln(r_1)}{p_1^2} - \frac{\delta_{2,1}\ln(r_2)}{p_1^2} - \frac{\delta_{3,1}\ln(r_3)}{p_1^2} \end{bmatrix}$$

The second derivatives necessary to form the Hessian Matrix of the indirect translog utility function were also calculated in an alternative method by manipulating the original Roy's Identity to get the following result:

$$(11)\quad V_{\bar{r}_j} = -V_{\bar{p}_i}\frac{r_j x_j}{p_i q_i}(\bar{p},\bar{r})$$

$$(12)\quad V_{\bar{p}_i} = -V_{\bar{r}_j}\frac{r_j x_j}{p_i q_i}(\bar{p},\bar{r})$$

These equations can be used to determine the second derivatives, which take the following form:

$$(13)\quad V_{\bar{r}\bar{p}} = -V_{\bar{p}\bar{p}}\frac{x}{y} - V_{\bar{p}}\frac{\partial\left(x/y\right)}{\partial\bar{p}},$$

where $\frac{x}{y}$ *and* $\partial\left(x/y\right)$ are the Roy's Identity for the input price which the first derivative is taken with respect to,

$$(14)\quad V_{\bar{r}\bar{r}} = -V_{\bar{p}\bar{r}}\frac{x}{y} - V_{\bar{p}}\frac{\partial\left(x/y\right)}{\partial\bar{r}},$$

where $\frac{x}{y}$ *and* $\partial\left(x/y\right)$ are the Roy’s Identity for the input price which the first derivative is taken with respect to, $V_{\bar{r}\bar{p}} = V_{\bar{p}\bar{r}}$ by the symmetry condition, in addition, $V_{\bar{p}\bar{p}}$ is the same second derivative used earlier. We find that the Hessian Matrix is identical to the prior Hessian, therefore proving that the methods for deriving the Hessian matrix for testing are identical.

(15)

$$H := \begin{bmatrix} -\frac{\eta_1}{r_1^2} - \frac{\delta_{1,1}\ln(p_1)}{r_1^2} + \frac{\eta_{1,1}}{r_1^2} - \frac{\eta_{1,1}\ln(r_1)}{r_1^2} - \frac{\eta_{2,1}\ln(r_2)}{r_1^2} - \frac{\eta_{3,1}\ln(r_3)}{r_1^2}, & \frac{\eta_{2,1}}{r_2 r_1}, & \frac{\eta_{3,1}}{r_3 r_1}, & \frac{\delta_{1,1}}{r_1 p_1} \\ \frac{\eta_{2,1}}{r_2 r_1}, & -\frac{\eta_2}{r_2^2} - \frac{\delta_{2,1}\ln(p_1)}{r_2^2} - \frac{\eta_{2,1}\ln(r_1)}{r_2^2} + \frac{\eta_{2,2}}{r_2^2} - \frac{\eta_{2,2}\ln(r_2)}{r_2^2} - \frac{\eta_{3,2}\ln(r_3)}{r_2^2}, & \frac{\eta_{3,2}}{r_3 r_2}, & \frac{\delta_{2,1}}{r_2 p_1} \\ \frac{\eta_{3,1}}{r_3 r_1}, & \frac{\eta_{3,2}}{r_3 r_2}, & -\frac{\eta_3}{r_3^2} - \frac{\delta_{3,1}\ln(p_1)}{r_3^2} - \frac{\eta_{3,1}\ln(r_1)}{r_3^2} - \frac{\eta_{3,2}\ln(r_2)}{r_3^2} + \frac{\eta_{3,3}}{r_3^2} - \frac{\eta_{3,3}\ln(r_3)}{r_3^2}, & \frac{\delta_{3,1}}{r_3 p_1} \\ \frac{\delta_{1,1}}{r_1 p_1}, & \frac{\delta_{2,1}}{r_2 p_1}, & \frac{\delta_{3,1}}{r_3 p_1}, & -\frac{\beta_1}{p_1^2} + \frac{\beta_{1,1}}{p_1^2} - \frac{\beta_{1,1}\ln(p_1)}{p_1^2} - \frac{\delta_{1,1}\ln(r_1)}{p_1^2} - \frac{\delta_{2,1}\ln(r_2)}{p_1^2} - \frac{\delta_{3,1}\ln(r_3)}{p_1^2} \end{bmatrix}$$

The data used to test the slopes of the supply and demand functions along with the PSD of the matrix, is the data set developed by Capalbo, Vo, and Wade (1985) for measuring agricultural productivity and characterizing the structure of US agriculture.

Results

The results for the parameter values estimated from the nonlinear SUR model given the four different specifications are found in Table 1.1. The following eigenvalue test was used to determine the definiteness of the Hessian matrix, note the matrix must be symmetric (Sydsaeter and Hammond 1995):

1. The matrix is positive definite ↔ all eigenvalues of the matrix are positive.
2. The matrix is positive semidefinite ↔ all eigenvalues of the matrix are ≥ 0.
3. The matrix is negative definite ↔ all eigenvalues of the matrix are negative.

4. The matrix is negative semidefinite ↔ all eigenvalues of the matrix are ≤ 0.
5. The matrix is indefinite ↔ the matrix has at least two eigenvalues with opposite signs.

The definiteness of the matrix was also tested by checking the signs of the determinants for the leading principal minors according to the following (Sydsaeter and Hammond 1995):

Let the matrix=$(a_{ij})_{nxn}$ be a symmetric matrix with leading principal minors D_k (k=1,2,...,n) Then

1. The matrix is positive definite ↔ $D_k>0$ for k=1,2,…,n.
2. The matrix is positive semidefinite ↔ if and only if all of the principal minors of the in the matrix are ≥ 0.
3. The matrix is negative definite ↔ $(-1)^k D_k>0$ for k=1,2,…,n.
4. The matrix is negative semidefinite ↔ if and only if all of the principal minors of order k in the matrix, have the same sign as $(-1)^k$.
5. The matrix is indefinite ↔ if the determinate for any two of the leading principal minors have opposite signs.

For the data set used in our model we find that the both the tests indicate that the matrix in indefinite for each observation (Tables 2 and 3 for the eigenvalues and Tables 4 and 5 for the derivatives). The results provide in the table are an indication that our underlying assumptions that the producer who is risk averse must be an expected utility should be reexamined and/or revaluated.

Conclusion

The results of the eigenvalue and determinant tests provide indication that there is a violation of the expected utility maximization principal due to the indefiniteness of the Hessian Matrix. As stated previously that the results of the model are dependent upon data and the data may have been the sole and/or major contributor to the above result. However, the results are intriguing nonetheless. When uncertainty is introduced, especially for the short-run due, our assumptions concerning the slopes of supply and demand functions actually hold (Paris et. al. 1993), or does the uncertainty create non-convexities in these functions? Another possible reason for the indefiniteness of the matrix is that the share equations concern both input prices and output prices. We might expect that the denominator (output prices) would determine the overall sign of the share equation. In addition, input and output prices should possibly carry different signs i.e. output prices would increase utility and input prices would decrease utility. The aforementioned results indicate that more research and understanding of the inner workings of the model need to be conducted.

We realize that there exists a possibility that a there may be a misspecification of the correct coefficient value for the parameter β_1, which could affect the results of the eigenvalue and determinant tests. Although, we have taken precautions in making sure that the results were robust by estimating the model were the coefficients of β_1 was set to the following values of -1, 0, and1; and with the coefficient being determined in the model. As stated previously the results for all four specifications of the coefficients of β_1 yielding eigenvalues and determinants that resulted in indefinite matrices for every series of observations, i.e., the results are consistent over alternative choices for β_1. Further research, which conducts sensitivity analysis about the correct coefficient value for β1, may provide insight into the definiteness of the matrix.

There exist several possible avenues of future research that could be conducted by using the methodology presented in this research; in addition, many of these future research opportunities are limitations of the research presented above. One of the more interesting avenues of research is to test the above methodology using multiple data sets and see what conclusions can be reached i.e., are the matrices negative semi-definite as we would expect or do other data sets behave in a similar manner as the data used in our estimation. A second prospect for future research would be test the model with other generic flexible indirect utility functions, including other general functions, which are second order numerical approximations such as the Generalized Leontief, Generalized Cobb Douglass, and the Generalized Box Cox. If the results indicate that the matrix is either negative or negative semi-definite, then perhaps the indirect translog utility function is an incorrect general representation of the specific indirect utility function.

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Table 1: Coefficient Estimates For Parameters

	Neg Beta				Zero Beta		
	COEFFICIENT	ST. ERROR	T-RATIO		COEFFICIENT	ST. ERROR	T-RATIO
ETA1	2.54E-06	1.93E-04	1.32E-02	**ETA1**	0.008986	0.0063716	1.4103
DELTA11	-2.02E-07	1.58E-05	-1.28E-02	**DELTA11**	-7.62E-04	5.50E-04	-1.39E+00
ETA11	-1.24E-08	1.05E-05	-1.18E-03	**ETA11**	1.57E-04	5.56E-04	2.82E-01
ETA21	2.38E-07	2.90E-06	8.21E-02	**ETA21**	2.66E-04	2.46E-04	1.08E+00
ETA31	1.3303E-07	0.000001403	0.09482	**ETA31**	2.61E-04	5.92E-04	4.40E-01
BETA11	1.07E-01	0.34529	3.09E-01	**BETA11**	2.00E+01	4.70E+00	4.25E+00
DELTA21	-3.09E-02	0.96077	-0.032141	**DELTA21**	-5.39E+01	1.76E+01	-3.06E+00
DELTA31	-0.060424	0.15617	-0.38693	**DELTA31**	2.56E-01	6.62E+01	3.86E-03
ETA2	1	1	1	**ETA2**	1.00E+00	1.00E+00	1.00E+00
ETA22	1	1	1	**ETA22**	1	1	1
ETA32	1	1	1	**ETA32**	1	1	1
ETA3	1	1	1	**ETA3**	1	1	1
ETA33	1	1	1	**ETA33**	1	1	1

	Pos Beta				No Beta		
	COEFFICIENT	ST. ERROR	T-RATIO		COEFFICIENT	ST. ERROR	T-RATIO
ETA1	1.57E+07	9.84E+12	1.59E-06	**ETA1**	1.57E+10	9.61E+15	1.63E-06
DELTA11	-1.33E+06	8.35E+11	-1.59E-06	**DELTA11**	-1.25E+09	7.64E+14	-1.63E-06
ETA11	2.75E+05	1.73E+11	1.59E-06	**ETA11**	6.64E+08	4.06E+14	1.63E-06
ETA21	4.63E+05	2.91E+11	1.59E-06	**ETA21**	5.63E+08	3.45E+14	1.63E-06
ETA31	4.52E+05	2.84E+11	1.59E-06	**ETA31**	1.49E+08	9.15E+13	1.63E-06
BETA11	3.47E+10	2.18E+16	1.59E-06	**BETA1**	-1.65E+15	1.01E+21	-1.63E-06
DELTA21	-9.36E+10	5.88E+16	-1.59E-06	**BETA11**	2.00E+14	1.22E+20	1.63E-06
DELTA31	3.05E+08	1.92E+14	1.59E-06	**DELTA21**	-1.15E+14	7.05E+19	-1.63E-06
ETA2	1	1	1	**DELTA31**	-1.04E+14	6.39E+19	-1.63E-06
ETA22	1	1	1	**ETA2**	1	1	1
ETA32	1	1	1	**ETA22**	1	1	1
ETA3	1	1	1	**ETA32**	1	1	1
ETA33	1	1	1	**ETA3**	1	1	1
				ETA33	1	1	1

Table 2: Eigenvalue Test Results

OBS		Negative Beta				Zero Beta		
3	5.941496	0.4519707	-9.88E-07	-0.2150054	2526.535	-1.75E-03	-2.081706	-979.5055
4	3.544734	-9.75E-07	-4.86E-02	-0.160207	1744.625	-1.60E-03	-1.980347	-782.0948
5	2.491516	-1.00E-06	-0.1328606	-0.2847548	1164.246	-1.57E-03	-2.258539	-664.7151
6	3.54018	-8.22E-07	-5.74E-03	-0.139051	1567.767	-1.39E-03	-2.299849	-642.4967
7	4.085026	0.1331936	-7.44E-07	-0.1307693	1757.451	-1.32E-03	-2.313185	-612.2487
8	4.184979	0.1790594	-1.06E-06	-0.163055	1721.473	-1.86E-03	-2.424771	-832.5206
9	3.915079	0.1216215	-9.79E-07	-0.1533514	1585.395	-1.69E-03	-2.517665	-751.8711
10	3.608199	6.33E-02	-9.21E-07	-0.1182916	1295.012	-1.65E-03	-2.905314	-632.079
11	3.198635	-8.46E-07	-6.82E-02	-8.84E-02	1114.074	-1.55E-03	-2.988686	-530.2228
12	2.57905	-8.81E-07	-6.60E-02	-0.26473	857.0411	-1.59E-03	-3.092415	-479.577
13	2.690321	-9.98E-07	-7.82E-02	-0.2319574	881.5223	-1.78E-03	-3.137641	-551.0727
14	2.576665	-8.32E-07	-6.74E-02	-0.2622854	913.0691	-1.50E-03	-2.924672	-468.2672
15	2.497697	-6.87E-07	-5.37E-02	-0.2844668	935.2401	-1.28E-03	-2.772937	-388.6993
16	2.36769	-6.79E-07	-4.97E-02	-0.3218763	882.4732	-1.26E-03	-2.782129	-373.3477
17	2.191296	-5.87E-07	-3.92E-02	-0.370184	841.9311	-1.10E-03	-2.699164	-314.8477
18	1.989344	-5.30E-07	-2.76E-02	-0.4239672	744.4079	-1.01E-03	-2.748027	-265.7846
19	1.494064	-4.20E-07	-1.88E-02	-0.5301111	568.762	-7.83E-04	-2.70022	-190.7701
20	1.430568	-4.76E-07	-1.54E-02	-0.5365972	479.997	-8.83E-04	-2.979032	-199.3705
21	1.24E+00	-4.13E-07	-8.59E-03	-5.57E-01	391.8274	-7.72E-04	-3.114497	-158.4548
22	0.9235019	-3.67E-07	-3.05E-03	-0.5833307	305.0066	-6.79E-04	-3.009571	-126.9224
23	6.54E-01	-3.81E-07	-2.65E-03	-5.87E-01	233.5127	-6.71E-04	-2.914383	-120.6021
24	0.6613386	-3.62E-07	-9.72E-04	-0.5866191	232.755	-6.48E-04	-2.939858	-113.4778
25	3.72E-01	-3.73E-07	-6.03E-03	-5.84E-01	2.18E+02	-6.07E-04	-2.09E+00	-1.18E+02
26	0.1425784	-2.87E-07	-1.59E-02	-0.5262674	174.3852	-3.91E-04	-1.496369	-93.65145
27	6.46E-04	-2.64E-07	-1.52E-02	-4.34E-01	111.6535	-3.18E-04	-1.134256	-76.22405
28	-2.38E-07	-6.34E-03	-2.82E-02	-4.05E-01	89.8483	-3.13E-04	-1.069143	-60.17177
29	-1.92E-07	-2.68E-03	-4.01E-02	-3.94E-01	74.92389	-2.65E-04	-1.095964	-44.81254
30	-1.79E-07	-1.03E-03	-4.59E-02	-3.96E-01	63.00462	-2.53E-04	-1.165623	-38.81379
31	-1.83E-07	-8.84E-04	-5.90E-02	-3.73E-01	46.48446	-2.48E-04	-1.094599	-36.10958
32	4.40E-04	-1.55E-07	-6.21E-02	-3.39E-01	29.23515	-2.04E-04	-0.9403591	-25.94047
33	1.90E-03	-1.49E-07	-5.21E-02	-2.72E-01	16.21299	-1.88E-04	-0.6779725	-20.2605
34	4.92E-03	-1.79E-07	-3.99E-02	-2.96E-01	10.86431	-2.30E-04	-0.7190559	-20.17355
35	4.01E-03	-1.40E-07	-3.99E-02	-2.86E-01	10.70096	-1.84E-04	-0.6890773	-15.72055
Count Positive	29	5	0	0	33	0	0	0
Count Negative	4	28	33	33	0	33	33	33
Count Zero	0	0	0	0	0	0	0	0

Table 3: Eigenvalue Test Results

OBS	Positive Beta				Beta Coefficient Estimated			
3	4.38293E+12	-3.03E+06	-3475420000	-1.70189E+12	5.39348E+15	1.19E+15	-3703933000	-1.55071E+15
4	3.02802E+12	-2.78E+06	-2879790000	-1.35886E+12	3.72618E+15	9.85E+14	-3449238000	-1.21853E+15
5	2.02132E+12	-2.73E+06	-3045520000	-1.15494E+12	2.48736E+15	1.04E+15	-3491099000	-1.03158E+15
6	2.72092E+12	-2.41E+06	-3389720000	-1.11635E+12	3.34827E+15	1.16E+15	-3150530000	-1.01475E+15
7	3.04968E+12	-2.30E+06	-3544262000	-1.0638E+12	3.75283E+15	1.21E+15	-3029301000	-9.64203E+14
8	2.9871E+12	-3.23E+06	-3766159000	-1.44654E+12	3.67582E+15	1.29E+15	-4017725000	-1.28749E+15
9	2.75114E+12	-2.94E+06	-3848832000	-1.30642E+12	3.38545E+15	1.32E+15	-3757444000	-1.17321E+15
10	2.24734E+12	-2.87E+06	-4383780000	-1.09831E+12	2.7655E+15	1.50E+15	-3737404000	-9.70323E+14
11	1.9336E+12	-2.69E+06	-4356921000	-9.21324E+11	2.37942E+15	1.49E+15	-3520392000	-7.96444E+14
12	1.48786E+12	-2.77E+06	-4233690000	-8.33325E+11	1.83091E+15	1.45E+15	-3595217000	-6.97832E+14
13	1.53029E+12	-3.10E+06	-4346866000	-9.57562E+11	1.88312E+15	1.49E+15	-3975111000	-8.05917E+14
14	1.58513E+12	-2.62E+06	-4006075000	-8.13662E+11	1.9506E+15	1.37E+15	-3406497000	-6.85798E+14
15	1.62367E+12	-2.22E+06	-3767313000	-6.75396E+11	1.99803E+15	1.29E+15	-2924179000	-5.65238E+14
16	1.53216E+12	-2.19E+06	-3725270000	-6.48721E+11	1.88542E+15	1.27E+15	-2882785000	-5.39772E+14
17	1.46189E+12	-1.92E+06	-3541562000	-5.47068E+11	1.79896E+15	1.21E+15	-2551442000	-4.50534E+14
18	1.2927E+12	-1.76E+06	-3515782000	-4.61819E+11	1.59075E+15	1.20E+15	-2347033000	-3.71538E+14
19	9.88002E+11	-1.36E+06	-3219016000	-3.31473E+11	1.2158E+15	1.10E+15	-1860873000	-2.65039E+14
20	8.33864E+11	-1.54E+06	-3501040000	-3.46426E+11	1.19802E+15	1.03E+15	-2076544000	-2.70274E+14
21	6.80807E+11	-1.34E+06	-3537223000	-2.75334E+11	1.21041E+15	8.38E+14	-1829100000	-2.09034E+14
22	5.30116E+11	-1.18E+06	-3220697000	-2.2054E+11	1.10212E+15	6.52E+14	-1612530000	-1.6112E+14
23	4.05986E+11	-1.17E+06	-2940132000	-2.09556E+11	1.00611E+15	5.00E+14	-1596178000	-1.52693E+14
24	4.04665E+11	-1.13E+06	-2970131000	-1.97177E+11	1.01638E+15	4.98E+14	-1539898000	-1.41197E+14
25	3.79E+11	-1.06E+06	-1.96E+09	-2.06E+11	6.72E+14	4.67E+14	-1.44E+09	-1.55E+14
26	3.03484E+11	-6.80E+05	-1286064000	-1.62695E+11	4.40096E+14	3.73E+14	-997682300	-1.40214E+14
27	1.94442E+11	-5.53E+05	-884988800	-1.32408E+11	3.02849E+14	2.39E+14	-830449300	-1.17453E+14
28	1.5651E+11	-5.44E+05	-804738100	-1.04521E+11	2.75391E+14	1.93E+14	-783701900	-8.34057E+13
29	1.30532E+11	-4.62E+05	-808650600	-77841140000	2.76734E+14	1.61E+14	-664315900	-5.87945E+13
30	1.09778E+11	-4.40E+05	-848000900	-67421840000	2.90203E+14	1.35E+14	-631415300	-4.88522E+13
31	81020570000	-4.31E+05	-764008600	-62722480000	2.61459E+14	9.97E+13	-621018600	-4.52989E+13
32	50983600000	-3.55E+05	-614859800	-45055310000	2.10421E+14	6.27E+13	-512949100	-3.07072E+13
33	28296450000	-3.26E+05	-406243800	-35184100000	1.39028E+14	3.48E+13	-464667700	-2.12834E+13
34	18967590000	-4.01E+05	-416364200	-35032100000	1.42491E+14	2.33E+13	-555103500	-1.62321E+13
35	18683330000	-3.20E+05	-396995800	-27298740000	1.35864E+14	2.30E+13	-446946100	-1.23464E+13
Count Positive	33	0	0	0	33	33	0	0
Count Negative	0	33	33	33	0	0	33	33
Count Zero	0	0	0	0	0	0	0	0

Table 4: Determinant Test Results

	Negative Beta				Zero Beta			
OBS	DET H1	DET H2	DET H3	DET H4	DET H1	DET H2	DET H3	DET H4
3	-9.88E-07	-4.81E-06	-2.65E-06	5.70E-07	-1.75E-03	-4.41E+00	9.18E+00	-9.00E+03
4	-9.75E-07	-2.43E-06	1.68E-07	-2.69E-08	-1.60E-03	-2.80E+00	5.54E+00	-4.33E+03
5	-1.00E-06	-1.31E-06	7.09E-07	-9.43E-08	-1.57E-03	-1.83E+00	4.14E+00	-2.75E+03
6	-8.22E-07	-1.91E-06	1.67E-08	-2.32E-09	-1.39E-03	-2.17E+00	5.00E+00	-3.21E+03
7	-7.44E-07	-2.12E-06	-4.05E-07	5.29E-08	-1.32E-03	-2.33E+00	5.38E+00	-3.30E+03
8	-1.06E-06	-3.05E-06	-7.93E-07	1.29E-07	-1.86E-03	-3.20E+00	7.75E+00	-6.45E+03
9	-9.79E-07	-2.51E-06	-4.66E-07	7.15E-08	-1.69E-03	-2.69E+00	6.76E+00	-5.09E+03
10	-9.21E-07	-1.87E-06	-2.10E-07	2.49E-08	-1.65E-03	-2.14E+00	6.22E+00	-3.93E+03
11	-8.46E-07	-1.35E-06	1.84E-07	-1.63E-08	-1.55E-03	-1.72E+00	5.15E+00	-2.73E+03
12	-8.81E-07	-8.95E-07	6.01E-07	-3.97E-08	-1.59E-03	-1.36E+00	4.22E+00	-2.02E+03
13	-9.98E-07	-1.09E-06	6.23E-07	-4.87E-08	-1.78E-03	-1.57E+00	4.92E+00	-2.71E+03
14	-8.32E-07	-9.03E-07	5.62E-07	-3.79E-08	-1.50E-03	-1.37E+00	4.02E+00	-1.88E+03
15	-6.87E-07	-7.42E-07	4.88E-07	-2.62E-08	-1.28E-03	-1.19E+00	3.31E+00	-1.29E+03
16	-6.79E-07	-6.55E-07	5.17E-07	-2.57E-08	-1.26E-03	-1.11E+00	3.09E+00	-1.15E+03
17	-5.87E-07	-4.98E-07	4.76E-07	-1.87E-08	-1.10E-03	-9.29E-01	2.51E+00	-7.89E+02
18	-5.30E-07	-3.54E-07	4.47E-07	-1.24E-08	-1.01E-03	-7.53E-01	2.07E+00	-5.50E+02
19	-4.20E-07	-1.37E-07	3.33E-07	-6.27E-09	-7.83E-04	-4.45E-01	1.20E+00	-2.29E+02
20	-4.76E-07	-1.15E-07	3.66E-07	-5.62E-09	-8.83E-04	-4.24E-01	1.26E+00	-2.52E+02
21	-4.13E-07	-5.46E-08	2.86E-07	-2.46E-09	-7.72E-04	-3.02E-01	9.42E-01	-1.49E+02
22	-3.67E-07	-3.60E-09	1.98E-07	-6.03E-10	-6.79E-04	-2.07E-01	6.24E-01	-7.92E+01
23	-3.81E-07	2.58E-08	1.46E-07	-3.87E-10	-6.71E-04	-1.57E-01	4.57E-01	-5.51E+01
24	-3.62E-07	2.37E-08	1.40E-07	-1.37E-10	-6.48E-04	-1.51E-01	4.43E-01	-5.03E+01
25	-3.73E-07	5.78E-08	8.10E-08	-4.88E-10	-6.07E-04	-1.32E-01	2.77E-01	-3.28E+01
26	-2.87E-07	6.35E-08	2.16E-08	-3.43E-10	-3.91E-04	-6.82E-02	1.02E-01	-9.56E+00
27	-2.64E-07	5.72E-08	7.40E-11	-1.12E-12	-3.18E-04	-3.56E-02	4.03E-02	-3.07E+00
28	-2.38E-07	4.71E-08	-2.71E-09	1.72E-11	-3.13E-04	-2.81E-02	3.00E-02	-1.81E+00
29	-1.92E-07	3.40E-08	-3.04E-09	8.16E-12	-2.65E-04	-1.99E-02	2.18E-02	-9.76E-01
30	-1.79E-07	2.79E-08	-3.26E-09	3.36E-12	-2.53E-04	-1.59E-02	1.86E-02	-7.21E-01
31	-1.83E-07	2.38E-08	-4.04E-09	3.57E-12	-2.48E-04	-1.15E-02	1.26E-02	-4.56E-01
32	-1.55E-07	1.51E-08	-3.25E-09	-1.43E-12	-2.04E-04	-5.97E-03	5.62E-03	-1.46E-01
33	-1.49E-07	1.00E-08	-2.11E-09	-4.00E-12	-1.88E-04	-3.04E-03	2.06E-03	-4.18E-02
34	-1.79E-07	8.72E-09	-2.12E-09	-1.04E-11	-2.30E-04	-2.50E-03	1.80E-03	-3.63E-02
35	-1.40E-07	6.80E-09	-1.60E-09	-6.42E-12	-1.84E-04	-1.97E-03	1.35E-03	-2.13E-02
Count Positive	0	13	20	9	0	0	33	0
Count Negative	33	20	13	24	33	33	0	33
Count Zero	0	0	0	0	0	0	0	0

Table 5: Determinant Test Results

	Negative Beta				Zero Beta			
OBS	**DET H1**	**DET H2**	**DET H3**	**DET H4**	**DET H1**	**DET H2**	**DET H3**	**DET H4**
3	-3.03E+06	-1.33E+19	-7.85E+25	4.61E+13	-3.70E+09	-2.00E+25	-2.38E+25	3.68E+25
4	-2.78E+06	-8.43E+18	-3.30E+25	2.43E+13	-3.45E+09	-1.29E+25	-1.27E+25	1.54E+25
5	-2.73E+06	-5.53E+18	-1.94E+25	1.68E+13	-3.49E+09	-8.68E+24	-9.05E+24	9.33E+24
6	-2.41E+06	-6.55E+18	-2.48E+25	2.22E+13	-3.15E+09	-1.05E+25	-1.22E+25	1.24E+25
7	-2.30E+06	-7.01E+18	-2.64E+25	2.49E+13	-3.03E+09	-1.14E+25	-1.38E+25	1.33E+25
8	-3.23E+06	-9.63E+18	-5.25E+25	3.63E+13	-4.02E+09	-1.48E+25	-1.90E+25	2.45E+25
9	-2.94E+06	-8.10E+18	-4.07E+25	3.12E+13	-3.76E+09	-1.27E+25	-1.68E+25	1.97E+25
10	-2.87E+06	-6.46E+18	-3.11E+25	2.83E+13	-3.74E+09	-1.03E+25	-1.55E+25	1.50E+25
11	-2.69E+06	-5.20E+18	-2.09E+25	2.27E+13	-3.52E+09	-8.38E+24	-1.25E+25	9.95E+24
12	-2.77E+06	-4.12E+18	-1.45E+25	1.74E+13	-3.60E+09	-6.58E+24	-9.54E+24	6.65E+24
13	-3.10E+06	-4.74E+18	-1.97E+25	2.06E+13	-3.98E+09	-7.49E+24	-1.11E+25	8.97E+24
14	-2.62E+06	-4.15E+18	-1.35E+25	1.66E+13	-3.41E+09	-6.64E+24	-9.11E+24	6.25E+24
15	-2.22E+06	-3.60E+18	-9.17E+24	1.36E+13	-2.92E+09	-5.84E+24	-7.53E+24	4.26E+24
16	-2.19E+06	-3.35E+18	-8.10E+24	1.25E+13	-2.88E+09	-5.44E+24	-6.93E+24	3.74E+24
17	-1.92E+06	-2.81E+18	-5.43E+24	9.93E+12	-2.55E+09	-4.59E+24	-5.56E+24	2.51E+24
18	-1.76E+06	-2.27E+18	-3.69E+24	7.99E+12	-2.35E+09	-3.73E+24	-4.49E+24	1.67E+24
19	-1.36E+06	-1.35E+18	-1.44E+24	4.33E+12	-1.86E+09	-2.26E+24	-2.49E+24	6.61E+23
20	-1.54E+06	-1.28E+18	-1.55E+24	4.49E+12	-2.08E+09	-2.13E+24	-2.55E+24	6.90E+23
21	-1.34E+06	-9.14E+17	-8.90E+23	3.23E+12	-1.83E+09	-1.53E+24	-1.85E+24	3.88E+23
22	-1.18E+06	-6.27E+17	-4.45E+23	2.02E+12	-1.61E+09	-1.05E+24	-1.16E+24	1.87E+23
23	-1.17E+06	-4.74E+17	-2.92E+23	1.39E+12	-1.60E+09	-7.97E+23	-8.02E+23	1.23E+23
24	-1.13E+06	-4.56E+17	-2.67E+23	1.36E+12	-1.54E+09	-7.67E+23	-7.79E+23	1.10E+23
25	-1.06E+06	-4.00E+17	-1.62E+23	7.87E+11	-1.44E+09	-6.74E+23	-4.53E+23	7.02E+22
26	-6.80E+05	-2.06E+17	-4.32E+22	2.65E+11	-9.98E+08	-3.73E+23	-1.64E+23	2.30E+22
27	-5.53E+05	-1.08E+17	-1.26E+22	9.52E+10	-8.30E+08	-1.99E+23	-6.02E+22	7.07E+21
28	-5.44E+05	-8.51E+16	-7.16E+21	6.85E+10	-7.84E+08	-1.51E+23	-4.16E+22	3.47E+21
29	-4.62E+05	-6.03E+16	-3.79E+21	4.87E+10	-6.64E+08	-1.07E+23	-2.95E+22	1.74E+21
30	-4.40E+05	-4.83E+16	-2.76E+21	4.10E+10	-6.31E+08	-8.53E+22	-2.48E+22	1.21E+21
31	-4.31E+05	-3.49E+16	-1.67E+21	2.67E+10	-6.21E+08	-6.19E+22	-1.62E+22	7.33E+20
32	-3.55E+05	-1.81E+16	-5.02E+20	1.11E+10	-5.13E+08	-3.22E+22	-6.77E+21	2.08E+20
33	-3.26E+05	-9.23E+15	-1.32E+20	3.75E+09	-4.65E+08	-1.62E+22	-2.25E+21	4.79E+19
34	-4.01E+05	-7.60E+15	-1.11E+20	3.17E+09	-5.55E+08	-1.30E+22	-1.85E+21	3.00E+19
35	-3.20E+05	-5.97E+15	-6.47E+19	2.37E+09	-4.47E+08	-1.03E+22	-1.40E+21	1.72E+19
Count Positive	0	0	0	33	0	0	0	33
Count Negative	33	33	33	0	33	33	33	0
Count Zero	0	0	0	0	0	0	0	0