In Search of the Privately Optimal Patent Breadth

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1. Introduction

Innovating firms choose to patent their innovations when patenting allows the appropriation of more rents than do other forms of intellectual property protection (e.g., trade secrecy). The level of innovation rents that can be captured by the patent is mainly determined by the breadth of patent protection. Thus, once the decision to patent has been made, the innovator needs to make another important decision, namely, how broad of a patent protection to claim. While the innovator's decision to patent the innovation or to keep it a secret has been examined in the literature (Horstmann et al. 1985, Waterson 1990), there is no formal framework of analysis of the innovator's patent breadth choice. Instead, the traditional assumption in the economic literature is that the innovator has an incentive to claim 'as much as possible'.

The effectiveness of a 'claim as much as possible' patent breadth strategy in maximizing the rents that can be appropriated by the patentee is questionable, however. A patent that is too broad increases the likelihood of both infringement and patent validity challenges by competitors and/or third parties (Merges and Nelson 1990). Consequently, broad patent protection may reduce the effective patent life and thus the innovation rents that can be captured with the patent, since patents are often revoked during infringement trials and patent validity challenges (Merges and Nelson 1990, Barton 2000). This concern is especially critical given the increase in patent litigation during the last decade, particularly in the field of biotechnology, and the increase in the number of patents that are invalidated after being challenged. While the broadest possible patent may not be optimal, neither is a very narrow patent, since narrow patents make it easier for rivals to enter the patentee's market and may not allow the patentee to capture enough returns to cover her R&D costs.

The purpose of this paper is to theoretically examine the patenting behavior of innovators that have generated patentable process innovations and have decided to seek patent protection. In specific, the paper determines the privately optimal patent breadth for drastic process innovations when the innovator faces the probability of a direct patent validity challenge by a third party and potential entry in his market by competitors that provide technologically equivalent processes. The paper also examines the affect of patent breadth on the competitors' incentive to generate a competing process (i.e., on the competitors' R&D spending), on the competitors' probability of success in the R&D process (i.e., the patentee's ability to deter entry) and on the timing that success is realized by competitors (i.e., the patentee's ability to delay entry).

The focus of the paper is on drastic process innovations because these innovations are usually granted broader protection by the Patent Office (EPO 2000a, USPTO 1999). Given that broad patents are challenged and invalidated more often than narrow ones (Waterson 1990, Lanjouw and Schankerman 2001, Merges and Nelson 1990), the innovator of a drastic process should be more careful when he determines the breadth of patent protection claimed as he cannot depend on the Patent Office for help in structuring his claims. In addition, drastic process innovations are associated with greater innovation rents, which increase the incentive of other parties to challenge the validity of the patent and to litigate (Harhoff and Reitzig 2000, Lanjouw and Schankerman 2001).

The innovating firm's patent breadth decision is determined in a sequential game between an incumbent innovator who decides on the breadth of patent protection claimed and a potential entrant who, having observed whether the patent was challenged or not, decides on whether to generate a competing process and how much to spend on R&D. The game is solved by backwards induction; the decisions of the potential entrant are examined first followed by the patentee's patent breadth decision.

Analytical results show that the optimal patent breadth depends on the affect of new entry on the incumbent's profits, the incumbent's legal costs incurred when the patent is challenged and on whether the incumbent operates under a short term or a long term horizon. A key result of the paper is that, even when a patent breadth that deters entry exists, it might not be profit maximizing for the incumbent to choose this patent breadth to deter entry. As well, claiming the maximum breadth of patent protection is never an optimal strategy for the incumbent patentee in this model. The analysis shows that the nature of the instantaneous probability of success is a critical factor in determining the optimal patent breadth as well as the affect of patent breadth on the rivals' R&D spending, the probability of success by rivals and the timing that success occurs.

The rest of the paper is organized as follows. Section 2 presents the theoretical framework developed to examine the innovator's strategic patenting behavior. The main assumptions of the model are stated in this section. Section 3 describes the analytical solution of the strategic patent breadth model. Finally, section 4 concludes the paper.

2. Theoretical Development of the Strategic Patent Breadth Model

The determination of the optimal patent breadth for a strategically behaving innovator who has invented a process innovation and seeks patent protection is modeled as a sequential game of complete information. The agents involved in the game are an incumbent/patentee who is a holder of a process patent and an entrant who considers entering into the incumbent's market. It is assumed that the process innovation that is generated by the incumbent meets the patentability requirements and that the regulator (i.e., Patent Office) always grants the patent as claimed. The regulator is thus not explicitly modeled. The model considers the determination of the optimal patent breadth when the innovator has no guidance from the Patent Office in structuring his claims; a realistic assumption for drastic innovations.

The game consists of two periods, period one, which takes place over the time interval T_G-−T_0, and period two, which takes place over the time interval T_0–∞. The expression T_G denotes the time that the patent is granted and the expression T_0 (T_0=0) the time that the incumbent markets the new product and the entrant enters the market. During the first period of the game, the incumbent, having developed a process innovation and having decided to seek patent protection, determines the breadth, b, of patent protection claimed. During this period the validity of the patent may be directly challenged. The outcome of the challenge determines whether the validity of the patent is upheld or not. If the validity challenge is successful and the patent is revoked, the entrant enters the market at time T_0 using the incumbent's process and the entrant and the incumbent choose their respective output levels and compete in the market. If the patent validity is not challenged or if it is challenged and the challenge is unsuccessful (i.e., the patent is found to be valid), then the entrant, starting at time T_0, determines the flow of R&D spending, x, that will enable her to generate a non-infringing process. The incumbent operates as a monopolist for as long as the entrant is not successful in generating the non-infringing process. Once the entrant succeeds in generating the non-infringing process, however, the incumbent and the entrant choose their respective output levels and compete in the market.

The single entrant assumption is made to simplify the analysis. The assumption implies that either there is a minimum efficient scale requirement in this industry or that large sunk costs not linked to the R&D process need to be incurred upon entry that prevent the market from becoming competitive even when the incumbent's patent is revoked. Thus, the sunk costs that need to be incurred by the players upon entry are exogenous (the level of the sunk costs is not affected by the players' decisions, e.g., regulatory costs) and their level is such that $\Pi_{(n=2)} \geq 0$ while $\Pi_{(n=3)} < 0$ (n denotes the number of players).

The incumbent's decision to invest in R&D and patent his product is not considered in this game. The above decisions are treated as exogenous. The only decision the incumbent makes is to determine the breadth of patent protection for his process. The length of patent protection is assumed to be fixed and for simplicity it is also assumed to be infinite. Thus, the patent will stay active unless it is invalidated during a patent validity challenge and is thus revoked. It is also assumed that the incumbent's patent does not infringe on any previous product or process patent and there is only one Patent Office where the incumbent can apply for patent protection. Time is modeled as being continuous and complete and perfect information are assumed. The incumbent acts strategically taking into consideration the entrant's response to different patent breadth choices when he determines the breadth of patent protection claimed.

A summary of the formal strategic patent breadth determination game is depicted diagrammatically in Figure 1. In period one the incumbent determines the breadth of patent protection claimed, b, and he is granted a process patent. The patent is then challenged by a third party with probability δ and during the challenge process the viability of the patent is determined. The patent is upheld with probability μ and it is revoked with probability 1-μ. The upholding or revoking of the patent marks the end of period one. In period two the product is marketed by the incumbent. If the validity of the patent is not challenged or if it is challenged and upheld, then at the beginning of period two the entrant chooses the optimal flow of R&D spending, x. The incumbent operates as a monopolist for as long as the entrant is not successful in generating her own non-infringing process. Once the entrant succeeds, however, the incumbent and the entrant choose their respective output levels and they each earn duopoly profits. The payoffs for the incumbent and the entrant when the patent is challenged and upheld are given by $E(\Pi_I)_U^C$ and $E(\Pi_E)_U^C$, respectively (see payoffs at A). If the patent is not challenged, the payoffs are given by $E(\Pi_I)^{NC}$ and $E(\Pi_E)^{NC}$, respectively (see payoffs at C). If the patent is revoked after it has been challenged, then starting at the beginning of period two the entrant produces the new non-patentable product using the incumbent's process and the incumbent and the entrant receive payoffs $E(\Pi_I)_R^C$ and $(\Pi_E)_R^C$, respectively (see payoffs at B).

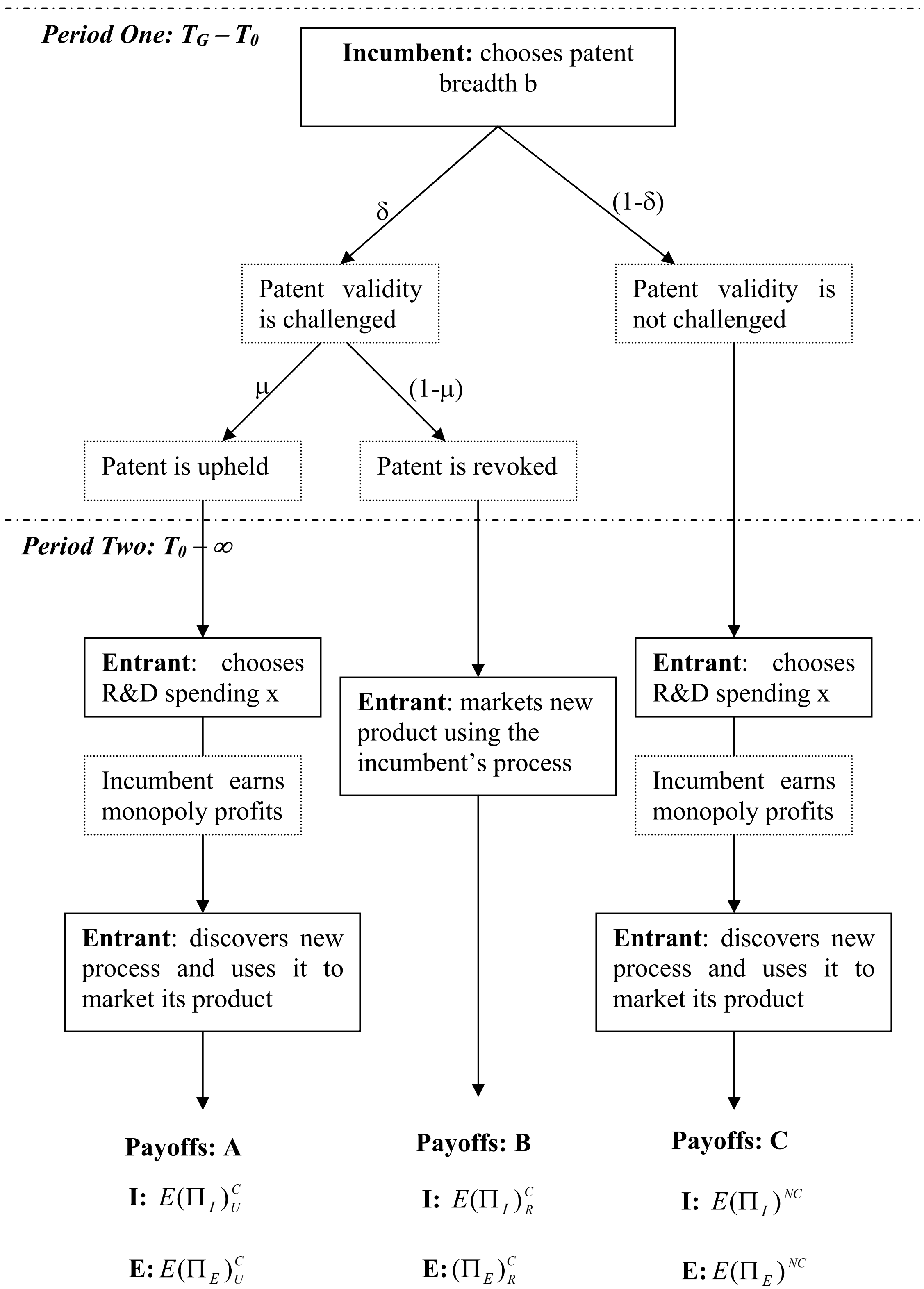

Figure 1 The Strategic Patent Breadth Game

In this model the entrant knows (with certainty) the outcome of the patent challenge when she decides on the level of R&D expenses to be incurred or when she decides on whether to enter the market using the incumbent's process. The incumbent on the other hand knows only the probability with which his patent will be challenged and the probability with which his patent will be upheld when challenged when he determines the breadth of patent protection claimed.

- **The Process Innovation Space, Patent Breadth and the R&D Process**

It is assumed that the patentable process is used for the production of a new non-patentable product (e.g., the t-PA drug in U.K.). The potential entrant is thus free to produce the new product by generating her own non-infringing process. This model assumes that if the entrant enters the market she will do so without infringing the incumbent's process. In addition, if the entrant is successful in generating her own process, she does not have to patent it since further entry is not anticipated.

The model assumes that the patented process results in zero per unit production costs and that if the entrant succeeds in generating her own non-infringing process, her process will be equally efficient (i.e., the entrant's process also results in per unit production costs of zero). Thus, the model is not a quality ladder model where one innovator supersedes the other in producing a better innovation. Both the incumbent/patentee and the potential entrant use their processes to produce the new non-patentable product, which is viewed by the consumers as a homogeneous product. In other words, consumers are indifferent as to whether the new product was made with the incumbent's or the entrant's process.

It is also assumed that there are no close substitutes for the new product. The above assumption implies that the incumbent will make monopoly profits for as long as the entrant fails to generate a non-infringing process. Once the entrant succeeds in generating her own non-infringing process, the incumbent and the entrant will share the market, each making duopoly profits.

The process innovation space and the breadth of patent protection are depicted in Figure 2. The line of unit length represents the process innovation space. Each point within this space represents a process that is capable of producing the product in question at the same production cost. Thus, different points on the unit length line refer to the different processes that can be used to produce the non-patentable product at a per unit cost of zero. The closer are two points in the process innovation space, the more similar are the processes in terms of the way they work in generating the given product.

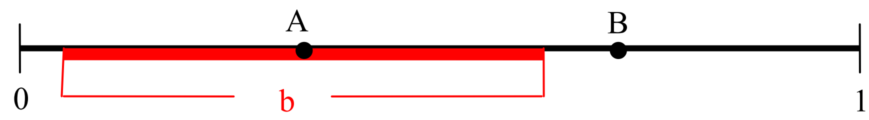

Figure 2 The Process Innovation Space and the Breadth of Patent Protection

Point A in Figure 2 refers to the patented process generated by the incumbent. Patent breadth refers to the area on the unit length line around point A which is protected by the patent. Patent breadth includes all the processes that, if they were developed by competitors, would infringe the patent. Patent breadth takes values in the interval $b \in (0,1]$. A patent breadth value close to zero ($b \to 0$) implies that the patent protects only against duplication of the patented process. On the other hand, a patent breadth value equal to one ($b=1$) implies that there is no other process that can be used to produce the non-patentable product without infringing the patent. It is assumed that patent breadth defines an exact border of protection (i.e., fencepost patent system).

To simplify the model it is assumed that it is a third party and not the potential entrant who directly challenges the validity of the patent.[1] Thus, in this model, the entrant benefits from the validity challenge without incurring the opposition costs. The costs incurred by the incumbent during a validity challenge are denoted by C_T and are assumed to be independent of the breadth of patent protection. In addition, it is assumed that the incumbent's opposition costs do not affect the probability that the patent will be challenged and the probability that the validity of the patent will be upheld.

The probability that the validity of the patent will be directly challenged is denoted by δ and it is a function of patent breadth. Recent empirical studies have found a positive relationship between the breadth of the patent, measured by the number of claims made, and the probability of validity challenges (Lanjouw and Schankerman 2001). In addition, Lentz (1988) and Merges and Nelson (1990) observe that the greater is the breadth of patent protection, the greater is the probability that the validity of the patent will be challenged. Following the above studies, this model assumes a positive relationship between patent breadth and the probability that the validity of the patent will be challenged. For simplicity, it is further assumed that when the maximum patent

[1] Third parties are allowed to challenge the validity of patents in the Patent Office without having to prove any special interest for doing so. Harhoff and Reitzig (2000) state that various interest groups are trying to influence the European patenting practice by filing opposition cases especially against biotechnology patents.

breadth is claimed (b_{max}=1), the validity of the patent is always challenged. These assumptions are captured by assuming that the probability that the validity of the patent will be directly challenged, δ, is equal to $\delta = b$.

The patent may not always be found valid during the patent validity challenge. There is a probability, denoted by μ, that the validity of the patent will be upheld during the validity challenge, where μ is given by $\mu = 1 - b$. Thus, the greater is the breadth of patent protection, the smaller is the probability that the validity of the patent will be upheld. The above assumption is justified by the fact that the greater is patent breadth, the harder it is to show novelty, nonobviousness and enablement (Cornish 1989). In addition, empirical evidence suggests that courts tend to uphold narrow patents and revoke broad ones (Merges and Nelson 1990, Waterson 1990). Thus, when patent breadth takes its maximum value (b_{max}=1), the patent is always found to be invalid (μ=0).

In statistical terms, the event that the patent will be challenged and the event that the validity of the patent will be upheld are treated as independent.[2] This assumption is valid given that the patent validity challenger is not the one who rules on whether the patent is valid. There is no evidence in the literature to suggest that there is a relationship between the probability that the patent will be challenged and the way the courts and/or the Patent Office rule on patent validity issues.

When the validity of the patent is not challenged, or when it is challenged and upheld, the entrant must invest in R&D to generate her own non-infringing process to produce the non-patentable product, if she wants to enter the market. To capture the uncertainty associated with the R&D process it is assumed that the innovation process is stochastic. Innovation in this model occurs according to the Poisson process. The research technology is ‘memoryless’; that is, the probability that the entrant will succeed in generating an innovation at any given point in time depends only on the current R&D expenditure, not on past R&D experience (Tirole 1988). This is a common assumption in the R&D literature and is made to simplify the analysis (Loury 1979, Lee and Wild 1980). The instantaneous probability of success is denoted by λ and is constant. The parameter λ shows that if the entrant has not succeeded by time τ in generating a non-infringing process then the probability of succeeding at the next instant, that is at $\tau + dt$, is λdt . The elapsed time, τ, before an innovation arrives has a probability density function described by the exponential

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[2] This assumption implies that the probability that the patent will be upheld given that is has been challenged is equal to the probability that the patent will be upheld, i.e., prob[μ | δ]=prob[μ].

distribution $f(\tau) = \lambda e^{-\lambda\tau}$ for $\lambda>0$ and $0\leq\tau\leq\infty$ and a cumulative probability function $F(\tau) = 1 - e^{-\lambda\tau}$. The cumulative distribution gives the probability that success will occur by time τ (i.e., $F(\tau) = prob[t \leq \tau]$).

In this model it is assumed that the instantaneous probability of success λ is a function of the entrant's R&D spending per unit of time, denoted by x, and the breadth of patent protection b, $\lambda = f(x,b)$. The flow rate of R&D spending, x, is assumed to be constant and it is incurred by the entrant for as long as it takes to realize a success. Following standard economic theory assumptions, it is assumed that the R&D spending per unit of time increases the probability of success at a decreasing rate; $\lambda_x>0$, $\lambda_{xx}<0$ and also $\lim_{x\to\infty}\lambda_x = 0$ and $\lambda_x(0)\to\infty$ (Loury 1979, Reinganum 1983).

The instantaneous probability of success λ is also a function of the breadth of patent protection since in this model success implies that the entrant will only be able to enter with a non-infringing process. That is, the entrant must generate a process outside the technological territory – i.e., the patent breadth – claimed by the incumbent. Given that the entrant has not already succeeded, it is assumed that the greater is the patent breadth, the smaller is the probability that the entrant will succeed at the next instant, in generating a non-infringing process for producing the new non-patentable product. It is thus assumed that the breadth of patent protection decreases the probability of success at an increasing rate; $\lambda_b<0$, $\lambda_{bb}>0$. The justification for this assumption is that since the entrant will enter with a non-infringing process, the greater is patent breadth the more dissimilar will be the two processes – the further away from the patentee's process the entrant's process will be in the process innovation space in Figure 2. This, in turn, implies that the greater is patent breadth, the less useful is the information disclosed by the patent for the entrant and thus the more difficult it becomes for the entrant to generate her non-infringing process.

To completely describe the instantaneous probability of success, λ, the instantaneous probability of success is assumed to be either additively or multiplicatively separable in the flow of R&D spending and in patent breadth, i.e., $\lambda = \varphi(x) + \psi(b)$ or $\lambda = \varphi(x)\cdot\psi(b)$. The functions $\varphi(x)$ and $\psi(b)$ satisfy all theoretical assumptions concerning the instantaneous probability of success, that is, $\varphi_x > 0$, $\varphi_{xx} < 0$, $\psi_b < 0$ and $\psi_{bb} > 0$. With the additively separable formulation, the marginal effect of R&D spending on the probability of success is independent of the breadth of the patent, $\lambda_{xb} = 0$. With the multiplicatively separable formulation the marginal effect of R&D

spending on the probability of success is inversely related to the breadth of the patent, $\lambda_{xb} < 0$ (see Proposition 2 for a formal proof).

Given the above, when $\lambda = \varphi(x) + \psi(b)$ the incumbent's patent breadth choice affects the entrant's probability of success, λ, only directly ($\lambda_b < 0$). When $\lambda = \varphi(x) \cdot \psi(b)$ the incumbent's patent breadth choice affects the entrant's probability of success, λ, both directly ($\lambda_b < 0$) and indirectly ($\lambda_{xb} < 0$). In this case, as patent breadth increases, the harder it becomes to generate a non-infringing process (i.e., direct effect) and the less effective R&D spending becomes in increasing the probability of success (i.e., indirect effect). An additively separable function and a multiplicatively separable function that satisfy all theoretical assumptions regarding the instantaneous probability of success are given by $f_1 : \lambda = x^{\theta} + \frac{1}{b}$ and $f_2 : \lambda = \frac{x^{\theta}}{b}$ respectively, where $\theta \in (0,1]$.

3. The Analytical Solution of the Strategic Patent Breadth Game

Given the assumption of complete and perfect information, the incumbent knows when he determines the breadth of patent protection claimed how patent breadth affects the probability that the patent will be challenged, δ, the probability that the validity of the patent will be upheld after challenge, μ, and the entrant's probability of succeeding at any given instant in generating a non-infringing process, λ. The incumbent chooses the breadth of patent protection that will induce the desired behavior from the entrant and will allow him to maximize the rents that he can appropriate from his innovation.

The optimal breadth of patent protection for the innovator is determined using backwards induction. The duopoly profits that are realized at the second period of the game when both the incumbent and the entrant operate in the market are determined first. The entrant's decision of the optimal R&D spending is determined next and the incumbent's optimal patent breadth choice is determined last.

- **Determination of the Duopoly Profits**

During the second period of the game (T_0–∞) both the incumbent and the entrant produce the product when either the entrant succeeds in generating the non-infringing process or when the patent is revoked after being challenged. Since production costs have been assumed to be zero, both players will produce the same output and will earn the same rate of instantaneous profits. These instant profits are given by $\Pi_I = \Pi_E = \Pi_d > 0$.[3] Although the entrant earns the same level of instantaneous profits the discounted profits will differ from those of the incumbent depending on the R&D expenditures and on the exogenous sunk costs she has to incur.

- **The Entrant's Optimal R&D Spending Decision**

Two cases emerge regarding the entrant's behavior depending on whether the patent is challenged and revoked or on whether the patent is not challenged or is challenged and upheld. The entrant's optimal decision when the patent is challenged and revoked is examined first.

- *The Patent is Challenged and Revoked*

The entrant does not have to make an investment decision if the patent is revoked after being challenged. Since generating a new process is costly for the entrant (i.e., positive R&D costs are required), the entrant simply uses the incumbent's process to produce the new product. When the entrant uses the incumbent's process to produce the new product her discounted profits are given by:

$$(\Pi_E)^R = \int_0^\infty e^{-rt}\Pi_d dt - F = \frac{\Pi_d}{r} - F \qquad (1)$$

where r is the discount rate and F are the exogenous sunk costs incurred by the entrant at time zero (T_0).

The entrant finds it optimal to enter the market when the patent is challenged and revoked if

$(\Pi_E)^R > 0 \Rightarrow \frac{\Pi_d}{r} > F$.

- *The Patent is Not Challenged or is Challenged and Upheld*

When the patent is not challenged, or is challenged and upheld, the entrant must decide on the flow of R&D spending that will enable her to generate the non-infringing process that will be used for the production of the new product. The entrant chooses the flow of R&D spending, x, that

[3] Given that the incumbent and the entrant compete in quantities and not in prices and that the unit production costs are zero the duopoly profits that they realize are positive.

maximizes the present value of her expected profits. Note that the entrant's expected profits are the same irrespective of whether the patent is not challenged or challenged and upheld ($E(\Pi_E)_U^C = E(\Pi_E)^{NC}$) since it is not the entrant but a third party that challenges the validity of the patent. The entrant's objective function is given by:

$$\max_x E(\Pi_E)_U^C = E(\Pi_E)^{NC} = \int_0^\infty e^{-rt} e^{-\lambda(x,b)t} (\lambda(x,b)\Pi_d - x)dt - F \qquad (2)$$

Equation (2) shows that if the entrant has not succeeded before time t in generating the non-infringing process, she then receives Π_d if she succeeds at time t. This event has probability density $\lambda(x,b)e^{-\lambda(x,b)t}$. The entrant pays R&D costs of x so long as no success has occurred. This event has probability $e^{-\lambda(x,b)t}$. Finally, the entrant pays costs F at time zero irrespective of whether she succeeds in generating the process.

Performing the indicated integrations the entrant's objective function can be expressed as:

$$\max_x E(\Pi_E)_U^C = E(\Pi_E)^{NC} = \frac{\lambda(x,b)\Pi_d - x}{r + \lambda(x,b)} - F \qquad (3)$$

The entrant chooses the flow of R&D spending that maximizes her objective function given in equation (3). Optimization of equation (3) yields the following first order conditions (F.O.C.) for a maximum:

$$\frac{\partial E(\Pi_E)_U^C}{\partial x} = \frac{\partial E(\Pi_E)^{NC}}{\partial x} = 0 \Rightarrow x - \frac{r + \lambda(x,b)}{\lambda_x} + r\Pi_d = 0 \Rightarrow x^* = x(b, r, \Pi_d) \qquad (4)$$

The F.O.C. yield the optimal flow of R&D spending expressed in terms of known parameters; the breadth of patent protection, the duopoly profits and the discount rate. The F.O.C. implicitly define the entrant's best response function, which shows how the entrant responds to different patent breadth choices.

To graphically characterize the optimal level of the flow of R&D spending let $g(x) = x$ and $h(x) = \frac{r + \lambda(x,b)}{\lambda_x} - r\Pi_d$. The F.O.C. can then be written as:

$$g(x) - h(x) = 0 \Rightarrow x = \frac{r + \lambda(x,b)}{\lambda_x} - r\Pi_d \qquad (5)$$

The second order conditions (S.O.C.) imply that for a maximum the condition given in equation (6) must be satisfied.

$$g_x - h_x < 0 \Rightarrow g_x < h_x \qquad (6)$$

Equation (6) shows that at the optimum the slope of $h(x)$ must be greater than the slope of $g(x)$; $h(x)$ must cut $g(x)$ from below at the optimum. Given that $g_x = 1 > 0$ equation (6) implies that $h(x)$ must be increasing in x and also that $h_x > 1$. It is easily verified that both the above conditions for the existence of an optimum are satisfied as $h_x = \frac{\lambda_x \lambda_x}{(\lambda_x)^2} - \frac{\lambda_{xx}(r+\lambda)}{(\lambda_x)^2} = 1 - \frac{\lambda_{xx}(r+\lambda)}{(\lambda_x)^2} > 1$, since $\lambda_{xx} < 0$. The S.O.C. are satisfied for both the additive and the multiplicative formulations of the instantaneous probability of success, λ. Also, $h(0) = -r\Pi_d$ since $\lambda_x(0) \to \infty$ which holds due to the theoretical properties of the instantaneous probability of success.

The slope of $h(x)$ is decreasing in the flow of R&D spending, x, for both the additive and the multiplicative formulations of the instantaneous probability of success, λ, that is, $h_{xx} = -\frac{\lambda_{xxx}(r+\lambda) + \lambda_x \lambda_{xx}}{(\lambda_x)^2} + \frac{2(\lambda_{xx})^2(r+\lambda)}{(\lambda_x)^3} \leq 0$. A formal proof is presented in the Appendix. Note that the determination of the curvature of $h(x)$ is not important for the results, it is necessary only for the graphical representation of the optimum. Figure 3 depicts the graphical representation of the determination of the optimal flow of R&D spending.

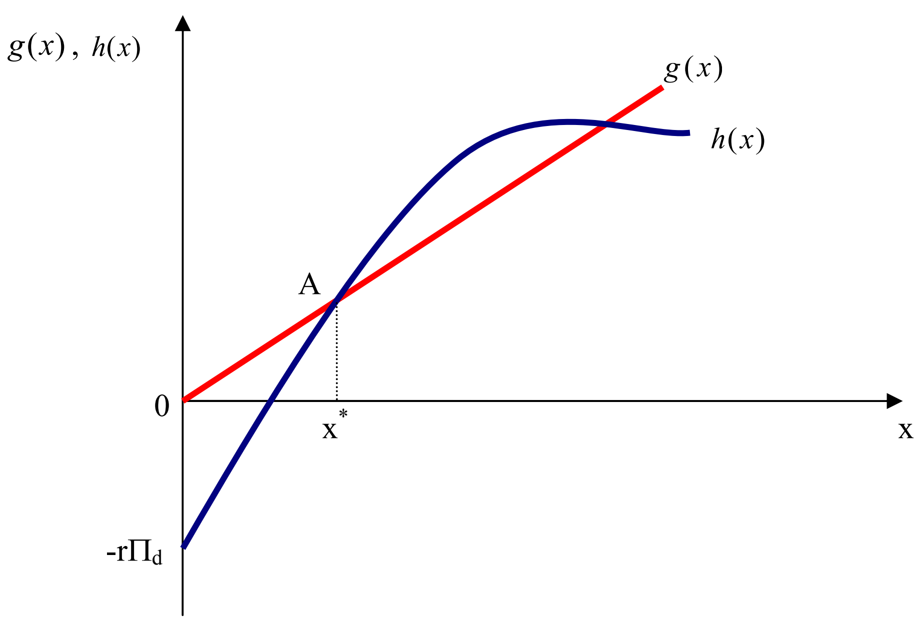

Figure 3 Graphical Representation of the Determination of the Optimal Flow of R&D Spending (x^*).

The entrant's expected profits when the patent is not challenged or when it is challenged and upheld are obtained by substituting the optimal flow of R&D spending into the entrant's expected profit function. The substitution yields the payoffs given by equation (7).

$$E(\Pi_E)_U^C = E(\Pi_E)^{NC} = \frac{\lambda(x^*,b)\Pi_d - x^*}{r+\lambda(x^*,b)} - F \tag{7}$$

The entrant will enter the market only if she realizes positive profits, that is, if $E(\Pi_E)_U^C = E(\Pi_E)^{NC} > 0$. This condition can be expressed as:

$$\frac{\lambda(x^*,b)\Pi_d - x^*}{r+\lambda(x^*,b)} - F > 0 \Rightarrow \Pi_d > \frac{x^* + (r+\lambda(x^*,b))F}{\lambda(x^*,b)} \tag{8}$$

Note that the entry condition in equation (8) is determined by the level of duopoly profits, the discount rate, the exogenous sunk costs and the incumbent's patent breadth decision. Equation (8) opens the possibility that a patent breadth value $\hat{b} \in (0,1]$ may exist such that the entry condition is not satisfied. If $\hat{b}$ exists and it is chosen by the incumbent, then the entrant will not enter the market.

When entry is not deterred the entrant's optimal flow of R&D spending is given by equation (4). This equation can be used to determine the effect of a change in the breadth of patent protection, the level of duopoly profits and the discount rate, on the optimal flow of R&D spending.

The effect of a change in patent breadth on the optimal flow of R&D spending is determined by examining $\frac{dx^*}{db}$. The expression for the term $\frac{dx^*}{db}$ is derived by totally differentiating the optimality condition $g(x) = h(x) \Rightarrow x = \frac{r+\lambda}{\lambda_x} - r\Pi_d$ (i.e., equation (5)), with respect to the optimal flow of R&D spending, x*, and patent breadth, b. The result of this differentiation is:

$$\begin{gathered}\frac{\partial g(x^*)}{\partial x^*}dx^* + \frac{\partial g(x^*)}{\partial b}db = \frac{\partial h(x^*)}{\partial x^*}dx^* + \frac{\partial h(x^*)}{\partial b}db \Rightarrow \\ (g_x - h_x)dx^* = (h_b - g_b)db \Rightarrow \frac{dx^*}{db} = \frac{h_b - g_b}{(g_x - h_x)}\end{gathered} \tag{9}$$

It is known from equation (6) that $g_x - h_x < 0$. Also, $g_b = 0$. Thus, the sign of the slope of the best response curve, $\frac{dx^*}{db}$, depends on the sign of the term h_b, where $h_b = \frac{\lambda_b\lambda_x - \lambda_{xb}(r+\lambda)}{(\lambda_x)^2}$.

The nature of the instantaneous probability of success, λ, must be specified before the relationship between the optimal level of R&D spending, x*, and patent breadth, b, can be determined. This is so because the term h_b depends on the term λ_{xb}, the sign of which depends on whether the instantaneous probability of success, λ, is additively or multiplicatively separable in the flow of R&D spending, x, and in patent breadth, b.

Proposition 1 *When the instantaneous probability of success, λ, is additively separable in patent breadth, b, and in the flow of R&D spending, x, (i.e., $\lambda = \varphi(x) + \psi(b)$), the effect of a change in patent breadth on the optimal flow of R&D spending is positive, i.e.,* $\frac{dx^*}{db} > 0$.

Proof:

When $\lambda = \varphi(x) + \psi(b)$, then $\frac{\partial \lambda_x}{\partial b} = \lambda_{xb} = 0$. Given that $\lambda_b < 0$ and $\lambda_x > 0$,

$h_b = \frac{\lambda_b \lambda_x - \lambda_{xb}(r + \lambda)}{(\lambda_x)^2}$<0. The slope of the best response function, given by equation (9), is thus

positive, i.e., $\frac{dx^*}{db} > 0$. □

The above result suggests that as patent breadth increases so does the flow of R&D spending. The intuition behind this result is as follows. The entrant responds to an increase in patent breadth with an increase in her flow of R&D spending trying to counterbalance the negative effect that the increase in patent breadth has on the probability of success.

Proposition 2 *When the instantaneous probability of success, λ, is multiplicatively separable in the patent breadth, b, and in the flow of R&D spending, x, (i.e., $\lambda = \varphi(x) \cdot \psi(b)$) the effect of a change in patent breadth on the optimal flow of R&D spending is negative, i.e.,* $\frac{dx^*}{db} < 0$.

Proof:

When $\lambda = \varphi(x) \cdot \psi(b)$, then $\lambda_{xb} < 0$, since $\frac{\partial \lambda}{\partial x} = \varphi_x \psi(b)$, $\frac{\partial \lambda}{\partial b} = \varphi(x)\psi_b$ and

$\frac{\partial \lambda_x}{\partial b} = \frac{\partial \lambda_b}{\partial x} = \lambda_{xb} = \varphi_x \psi_b < 0$. Thus, patent breadth affects the probability of success both directly

($\lambda_b < 0$) and indirectly ($\lambda_{xb} < 0$). The sign of the term h_b is positive, $h_b = \frac{\lambda_b \lambda_x - \lambda_{xb}(r+\lambda)}{(\lambda_x)^2} = -\frac{r}{\varphi_x \psi_b} > 0$. Given that $\frac{dx}{db} = \frac{h_b}{(g_x - h_x)}$ (from equation 9) and $g_x - h_x < 0$, the slope of the best response function is negative, i.e., $\frac{dx^*}{db} < 0$. □

The intuition behind the result presented in Proposition 2 is as follows. An increase in patent breadth has two effects on the entrant. First, she knows that a change in b means she will have to spend more to be able to succeed (to counterbalance the negative effect that an increase in the patent breadth has on the probability of success). Second, she also knows that the effect of the additional R&D spending on the probability of success will now be smaller (due to $\lambda_{xb} < 0$). Since an increase in patent breadth makes investment less efficient and more costly for the entrant, the entrant responds with a reduction in the optimal flow of R&D spending.

Having determined how patent breadth affects the flow of R&D spending under different assumptions about the relationship between the flow of R&D spending and patent breadth (i.e., $\lambda_{xb} = 0$ and $\lambda_{xb} < 0$), the effect of a change in patent breadth on the total expected R&D costs that the entrant must incur before a success is realized can be determined. The total expected R&D costs to be incurred by the entrant are given by:

$$TEC_E = \tau_e x = \frac{1}{\lambda(x,b)} x \qquad (10)$$

In equation (10), $\tau_e = \frac{1}{\lambda(x,b)}$ denotes the average elapsed time before success is realized; this is the mean of the exponential distribution $f(\tau) = \lambda e^{-\lambda\tau}$. The average elapsed time before success occurs is decreasing in the flow of the R&D spending, $\frac{\partial \tau_e}{\partial x} = \frac{\partial \tau_e}{\partial \lambda}\frac{\partial \lambda}{\partial x} = -\frac{1}{\lambda^2}\lambda_x < 0$ and increasing in the breadth of patent protection, $\frac{\partial \tau_e}{\partial b} = \frac{\partial \tau_e}{\partial \lambda}\frac{\partial \lambda}{\partial b} = -\frac{1}{\lambda^2}\lambda_b > 0$. Thus, on the one hand, the greater is the flow of R&D spending, the greater is the probability that success will be realized the next instant, and the shorter is the time that elapses before success occurs. On the other hand, the greater is patent breadth, the smaller is the probability that success will occur the next instant and thus the longer is the period that elapses before success occurs. The propositions that follow describe the relationship between patent breadth and the total expected R&D costs when the instantaneous probability of success is additively and multiplicatively separable in the flow of R&D spending and in patent breadth.

Proposition 3 *The total expected R&D costs are increasing in patent breadth,* $\frac{dTEC_E}{db} > 0$, *when the instantaneous probability of success, λ, is additively separable in patent breadth, b, and in the flow of R&D spending, x.*

Proof:

When $\lambda = \varphi(x) + \psi(b)$, $\frac{\partial TEC_E}{\partial b} = \frac{\partial \tau_e}{\partial b} x + \tau_e x_b = \frac{x_b \lambda - \lambda_b x}{\lambda^2} > 0$ since $x_b > 0$ as shown in Proposition 1 and $\lambda_b < 0$. □

The intuition behind the result presented in Proposition 3 is as follows. When the instantaneous probability of success is additively separable in the flow of R&D spending and in patent breadth, patent breadth affects the expected total R&D costs in two ways. First, a higher patent breadth increases the elapsed time before success is realized ($\frac{\partial \tau_e}{\partial b} > 0$), and second, greater patent breadth increases the flow of R&D spending ($x_b > 0$). Both outcomes imply that the expected total R&D costs to be incurred by the entrant are rising in b.

Proposition 4 *The effect of an increase in patent breadth on the total expected R&D costs when the instantaneous probability of success, λ, is multiplicatively separable in patent breadth, b, and in the flow of R&D spending, x, depends on whether an increase in patent breadth increases the elapsed time before success is realized more than it decreases the flow of R&D spending.*

Proof:

The effect of a change in patent breadth on the total expected R&D costs is given by $\frac{\partial TEC_E}{\partial b} = \frac{\partial \tau_e}{\partial b} x + \tau_e x_b = \frac{x_b \lambda - \lambda_b x}{\lambda^2}$. When $\lambda = \varphi(x) \cdot \psi(b)$ then $x_b < 0$ as shown in Proposition 2 and $\lambda_b < 0$. Given the above, the sign of $\frac{dTEC_E}{db}$ depends on the relative magnitudes of the terms $\frac{\partial \tau_e}{\partial b} x$ and $\tau_e x_b$, which are positive and negative respectively (or equivalently the terms $x_b \lambda$ and

$\lambda_b x$, which are both negative). When $\tau_e x_b > \frac{\partial \tau_e}{\partial b} x$ then $\frac{dTEC_E}{db} < 0$ while when $\tau_e x_b < \frac{\partial \tau_e}{\partial b} x$ then $\frac{dTEC_E}{db} > 0$. □

When the instantaneous probability of success is multiplicatively separable in the flow of R&D spending and in patent breadth, patent breadth affects the expected total R&D costs in two countervailing ways. On the one hand, an increase in patent breadth increases the elapsed time before success is realized; on the other hand, an increase in patent breadth decreases the flow of R&D spending. When $\frac{dTEC_E}{db} > 0$ then even though the amount spend on R&D per unit of time decreases as patent breadth increases this amount is now spread over a longer period making the total effect of the increase in patent breadth positive. When $\frac{dTEC_E}{db} < 0$ then even though the period over which the flow R&D costs are incurred increases as patent breadth increases, the decrease in the flow of R&D spending (caused by the patent breadth increase) is greater in absolute terms making the total effect of an increase in patent breadth on the total expected R&D costs negative.

The effect of a change in the anticipated level of duopoly profits on the flow of the R&D spending is determined by totally differentiating the optimality condition, $g(x) = h(x) \Rightarrow x = \frac{r+\lambda}{\lambda_x} - r\Pi_d$, with respect to the optimal flow of R&D spending, x^* and the anticipated duopoly profits, Π_d. The expression for $\frac{dx^*}{d\Pi_d}$ is given by:

$$\begin{aligned} &\frac{\partial g(x^*)}{\partial x^*} dx^* + \frac{\partial g(x^*)}{\partial \Pi_d} d\Pi_d = \frac{\partial h(x^*)}{\partial x^*} dx + \frac{\partial h(x^*)}{\partial \Pi_d} d\Pi_d \Rightarrow \\ &(g_x - h_x) dx^* = (h_{\Pi_d} - g_{\Pi_d}) d\Pi_d \Rightarrow \frac{dx^*}{d\Pi_d} = \frac{h_{\Pi_d} - g_{\Pi_d}}{(g_x - h_x)} \end{aligned} \tag{11}$$

Proposition 5 *The optimal level of the flow of R&D spending is increasing in the duopoly profits that the entrant anticipates to make if she succeeds in generating a non-infringing process,*

$\frac{dx^*}{d\Pi_d} > 0$, *for both the additive and the multiplicative formulations of the instantaneous probability of success, λ.*

Proof:

From the S.O.C., $g_x - h_x < 0$. Also, $g_{\Pi_d} = 0$ and $h_{\Pi_d} = -r < 0$, which implies that $\frac{dx^*}{d\Pi_d} > 0$.

Thus, as it would be expected, the entrant reacts to an increase in the anticipated duopoly profits by increasing the optimal flow of R&D spending. □

Proposition 6 *The total expected R&D costs that are incurred by the entrant before a success occurs are increasing in the duopoly profits that the entrant anticipates to make if she succeeds in generating a non-infringing process,* $\frac{dTEC_E}{d\Pi_d} > 0$.

Proof:

The change in the total expected R&D costs that follows a change in the anticipated duopoly profits is given by $\frac{\partial E(TC_E)}{\partial \Pi_d} = (\frac{\partial \tau_e}{\partial \lambda}\frac{\partial \lambda}{\partial x}\frac{\partial x}{\partial \Pi_d})x + \frac{\partial x}{\partial \Pi_d}\tau_e = (\frac{-r}{g_x - h_x})(\frac{\lambda - \lambda_x x}{\lambda^2})$. In this expression the term $(\frac{-r}{g_x - h_x}) = \frac{dx^*}{d\Pi_d}$ is positive as shown in Proposition 5. The term $\frac{\lambda - \lambda_x x}{\lambda^2}$ is also positive since $\frac{\lambda}{x} > \lambda_x$. To prove the last inequality the additively and multiplicatively separable functions $f_1 : \lambda = x^\theta + \frac{1}{b}$ and $f_2 : \lambda = \frac{x^\theta}{b}$, respectively, are used. When λ is described by f_1 then $\frac{\lambda}{x} > \lambda_x \Rightarrow x^{\theta-1} + \frac{1}{bx} > \theta x^{\theta-1} \Rightarrow 1 + \frac{1}{bx} > \theta$ which holds $\forall b \in (0,1]$, $x \geq 0$ since $\theta \in (0,1)$. When λ is described by f_2 then $\frac{\lambda}{x} > \lambda_x \Rightarrow \frac{x^{\theta-1}}{b} > \frac{\theta x^{\theta-1}}{b} \Rightarrow 1 > \theta$ which holds true $\forall b \in (0,1]$, $x \geq 0$ since $\theta \in (0,1)$.

□

The intuition behind the result presented in Proposition 6 is as follows. The increase in the anticipated duopoly profits causes the entrant to increase her flow of spending in R&D (see Proposition 5). The increase in the flow of R&D spending affects the expected total R&D costs in two countervailing ways. A greater flow of R&D spending directly obviously increases the expected total costs. It also indirectly decreases the expected total R&D costs by decreasing the average elapsed time before success is realized ($\frac{\partial \tau_e}{\partial x} < 0$). However, the positive direct effect is stronger than the negative indirect effect. The result is that an increase in the anticipated duopoly profits on the expected total R&D costs is positive.

The effect of a change in the discount rate on the optimal level of R&D spending is derived by totally differentiating the optimality condition $g(x) = h(x) \Rightarrow x = \frac{r+\lambda}{\lambda_x} - r\Pi_d$, with respect to the optimal flow of R&D spending, x*, and the discount rate, r. The expression for $\frac{dx^*}{dr}$ is given in equation (12).

$$\begin{aligned} &\frac{\partial g(x^*)}{\partial x^*}dx^* + \frac{\partial g(x^*)}{\partial r}dr = \frac{\partial h(x^*)}{\partial x^*}dx^* + \frac{\partial h(x^*)}{\partial r}dr \Rightarrow \\ &(g_x - h_x)dx^* = (h_r - g_r)dr \Rightarrow \frac{dx^*}{dr} = \frac{h_r - g_r}{(g_x - h_x)} \end{aligned} \tag{12}$$

From the S.O.C. it is known that $g_x - h_x < 0$. Also, $g_r = 0$. The sign of the term $h_r = \frac{1}{\lambda_x} - \Pi_d$ cannot be determined, however, without knowledge of the magnitude of the parameters in the expression. For instance, note that the impact of the discount rate on the optimal level of R&D spending depends on the magnitude of the anticipated duopoly profits, Π_d. When $\frac{1}{\lambda_x} > \Pi_d$ then an increase in the discount rate decreases the optimal level of the flow of R&D spending $\frac{dx}{dr} < 0$ while when $\frac{1}{\lambda_x} < \Pi_d$ an increase in the discount rate results in an increase in the optimal level of the flow of R&D spending $\frac{dx}{dr} > 0$. Finally, when $\frac{1}{\lambda_x} = \Pi_d$ a change in the discount rate causes no change in the optimal level of R&D spending. Given that the effect of a change in the discount rate on the optimal level of the flow of R&D spending is inconclusive, the effect of a change in the discount rate on the expected total R&D costs is also inconclusive.

- **The Incumbent's Optimal Patent Breadth Decision**

Given the assumption of complete information, the incumbent knows how patent breadth affects the entrant's optimal R&D spending decision. The incumbent can then choose the breadth of patent protection that induces the desired behavior from the entrant. This is the breadth of patent protection that maximizes the incumbent's discounted expected returns.

The incumbent's expected returns are a function of his expected returns when the patent is not challenged, $E(\Pi_I)^{NC}$, and the expected returns when the patent is challenged, $E(\Pi_I)^{C}$. Since the incumbent's patent is challenged with probability δ and it is not challenged with probability 1-δ, the incumbent's discounted expected profits are given by equation (13).

$$E(\Pi_I) = \delta E(\Pi_I)^C + (1-\delta)E(\Pi_I)^{NC} \qquad (13)$$

The incumbent's expected returns when the patent is challenged are a function of the incumbent's expected returns when the patent is challenged and upheld, $E(\Pi_I)_U^C$, and the expected returns when the patent is challenged and revoked, $E(\Pi_I)_R^C$. Given that the patent is challenged and upheld with probability μ and it is challenged and revoked with probability 1-μ, the incumbent's expected returns when the patent is challenged are given by equation (14). In equation (14) C_T denotes the legal costs incurred by the incumbent during the patent challenge process.

$$E(\Pi_I)^C = \mu E(\Pi_I)_U^C + (1-\mu)E(\Pi_I)_R^C - C_T \qquad (14)$$

The incumbent's expected returns when his patent is not challenged or when it is challenged and upheld are the same ($E(\Pi_I)^{NC} = E(\Pi_I)_U^C$) because in both cases the incumbent operates as a monopolist until the entrant succeeds in generating a non-infringing process. Once the entrant succeeds, the incumbent shares the market with the entrant, each making duopoly profits. The incumbent's discounted expected profits when his patent is not challenged or when it is challenged and upheld are given by equation (15).

$$E(\Pi_I)^{NC} = E(\Pi_I)_U^C = \int_0^\infty e^{-rt} e^{-\lambda(x^*,b)t} (\Pi_m + \lambda(x^*,b)\Pi_d)dt = \frac{\Pi_m + \lambda \Pi_d}{r+\lambda} = \Pi^u \qquad (15)$$

Equation (15) shows that the incumbent receives monopoly profits Π_m at t if by time t the entrant has not yet succeeded in generating a non-infringing process. This event has probability $e^{-\lambda(x^*,b)t}$. The incumbent receives duopoly profits Π_d at time t if, at t, the entrant succeeds in generating a non-infringing process. This event has a probability density function $\lambda(x^*,b)e^{-\lambda(x^*,b)t}$.

When the patent is challenged and revoked, the entrant enters using the incumbent's process and the incumbent shares the market with the entrant making duopoly profits. The incumbent's discounted profits when his patent is challenged and revoked are given by equation (16).

$$(\Pi_I)_R^C = \int_0^\infty e^{-rt}\Pi_d dt = \frac{\Pi_d}{r} = \Pi^R \tag{16}$$

It is assumed that the profits that the incumbent makes when his patent is not challenged or is challenged and upheld (Π^u) are greater than the profits that he makes when his patent is challenged and revoked (Π^R), that is, $\Pi^u > \Pi^R$. This assumption guarantees that the incumbent is not indifferent between receiving and not receiving patent protection for his process; the incumbent is better off when he receives patent protection.

Substitution of equations (15) and (16) into equation (14) yields the expression for the incumbent's discounted expected profits when the patent is challenged:

$$E(\Pi_I)^C = \mu\Pi^u + (1-\mu)\Pi^R - C_T \tag{17}$$

Substitution of equations (15) and (17) into equation (13) yields the expression for the incumbent's discounted profits when entry is not deterred. Recall that the probability of the patent being challenged is $\delta = b$ and the probability of the patent being found valid is $\mu = 1-b$.

$$\begin{aligned} E(\Pi_I)^{ND} &= \delta\{\mu\Pi^u + (1-\mu)\Pi^R - C_T\} + (1-\delta)\Pi^u = (\delta\mu + 1 - \delta)\Pi^u + \\ &(\delta - \delta\mu)\Pi^R - \delta C_T = (1-b^2)\Pi^u + b^2\Pi^R - bC_T \end{aligned} \tag{18}$$

The analysis so far has proceeded assuming that the entrant will always find it optimal to enter the market. It has been shown that the incumbent cannot deter entry when the patent is challenged and revoked since in this case the entrant's profits upon entry do not depend on the incumbent's patent breadth (see equation 1). Recall that the exogenous sunk costs (F) were assumed to be such as to allow a duopolistic market structure. It has also been shown that when the patent is not challenged, or is challenged and upheld, there may exist a patent breadth value $\hat{b} \in (0,1]$ such that the entry condition is not satisfied, that is, $\Pi_d \le \frac{x^*(\hat{b},r,\Pi_d) + (r + \lambda(x^*(\hat{b},r,\Pi_d),\hat{b}))F}{\lambda(x^*(\hat{b},r,\Pi_d),\hat{b})}$. If $\hat{b}$ exists and it is chosen by the incumbent, the entrant will not enter when the patent is not challenged or is challenged and upheld and the incumbent will make monopoly profits. The incumbent's profits when the patent is not challenged or is challenged and upheld and the incumbent chooses patent breadth $\hat{b}$ are given by equation (19).

$$(\Pi_I)^{NC} = (\Pi_I)_U^C = \int_0^{\infty} e^{-rt}\Pi_m dt = \frac{\Pi_m}{r} \tag{19}$$

The incumbent's discounted expected profits when patent breadth $\hat{b}$ that deters entry is chosen are given by substituting equations (14), (19) and (16) into equation (13). The incumbent's discounted expected profits when entry is deterred are given by equation (20).

$$\begin{aligned} E(\Pi_I)^D &= \delta\{\mu\frac{\Pi_m}{r} + (1-\mu)\Pi^R - C_T\} + (1-\delta)\frac{\Pi_m}{r} = \\ &(1-\hat{b}^2)\frac{\Pi_m}{r} + \hat{b}^2\Pi^R - \hat{b}C_T \end{aligned} \tag{20}$$

It should be noted that if a $\hat{b}$ that deters entry exists it will be chosen by the incumbent *if and only if* the incumbent's expected discounted profits when $\hat{b}$ is chosen are greater than or equal to his profits when entry is not deterred, $E(\Pi_I)^D \geq E(\Pi_I)^{ND}$. Thus, it may not always be optimal for the incumbent to deter entry in this model. To keep the model simple, the analysis proceeds assuming that either there is no patent breadth $\hat{b}$ that can deter entry or that if a patent breadth $\hat{b}$ exists, it is not optimal for the incumbent to deter entry because $\hat{b}$ does not satisfy the condition $E(\Pi_I)^D \geq E(\Pi_I)^{ND}$.

Given the assumption that entry will not be deterred the incumbent chooses the patent breadth that maximizes the expected discounted profits given by equation (18). His objective function is given by:

$$\max_b E(\Pi_I)^{ND} = (1-b^2)\Pi^u + b^2\Pi^R - bC_T \tag{21}$$

Optimization of equation (21) yields the F.O.C. for a maximum. The F.O.C. are given by equation (22).

$$\begin{aligned} &\frac{\partial E(\Pi_I)^{ND}}{\partial b} = 0 \Rightarrow (-2b)\Pi^u + (1-b^2)\frac{\partial \Pi^u}{\partial b} + 2b\Pi^R - C_T = 0 \Rightarrow \\ &\qquad b^* = b(\Pi_m, \Pi_d, C_T, r) \end{aligned} \tag{22}$$

The F.O.C. for the incumbent's optimization problem yield the optimal choice of patent breadth as a function of known parameters; the monopoly profits, the duopoly profits, the legal costs of the challenge process and the discount rate.

The interpretation of the F.O.C. given in equation (22) requires the determination of the sign of the term $\frac{\partial \Pi^u}{\partial b}$. The term $\frac{\partial \Pi^u}{\partial b}$ shows how the expected profits made by the incumbent when his patent is not challenged or when it is challenged and upheld are affected by the breadth of patent protection. The affect of patent breadth on Π^u does not depend on the nature of the instantaneous probability of success, as shown in the next proposition.

***Proposition** 7 The expected profits made by the incumbent when his patent is not challenged or when it is challenged and upheld are increasing in patent breadth (* $\frac{\partial \Pi^u}{\partial b} > 0$ *) for both the additive and the multiplicative formulations of the instantaneous probability of success, λ.*

Proof:

It is straight forward to prove that $\frac{\partial \Pi^u}{\partial b} > 0$ when the instantaneous probability of success is multiplicatively separable in the flow of R&D spending and in the patent breadth. In this case, an increase in patent breadth leads to a decrease in the flow of R&D spending, $x_b < 0$ (see Proposition 2). The term $\frac{\partial \Pi^u}{\partial b}$ is equal to $\frac{\partial \Pi^u}{\partial b} = \frac{(\lambda_b + \lambda_x x_b)(r\Pi_d - \Pi_m)}{(r+\lambda)^2}$, where the term ($r\Pi_d - \Pi_m$) is negative as duopoly profits are always smaller than monopoly profits and where $\lambda_b < 0$ and $\lambda_x > 0$ from the theoretical assumptions made about the instantaneous probability of success. The above conditions imply that $\frac{\partial \Pi^u}{\partial b} > 0$. When the instantaneous probability of success is additively separable in the flow of R&D spending and in the patent breadth, an increase in patent breadth leads to an increase in the flow of R&D spending, $x_b > 0$ (see Proposition 1). In this case, given that $r\Pi_d - \Pi_m < 0$, $\lambda_b < 0$ and $\lambda_x > 0$, the sign of the term $\frac{\partial \Pi^u}{\partial b}$ depends on the sign of the expression $(\lambda_b + \lambda_x x_b)$. To determine the sign of the term $\frac{\partial \Pi^u}{\partial b}$, the additively separable function $f_1 : \lambda = x^\theta + \frac{1}{b}$ is used. Using f_1 the expression for $\frac{\partial \Pi^u}{\partial b}$ is given by $\frac{\partial \Pi^u}{\partial b} = \frac{\Pi_m - r\Pi_d}{(1+b(r+x^\theta))^2}$ which is greater than zero ∀θ∈(0,1), b∈(0,1], x≥0 and r∈[0,1]. □

The intuition behind the result presented in Proposition 7 is as follows. When the instantaneous probability of success is multiplicatively separable in the flow of R&D spending and in the patent breadth, an increase in patent breadth affects the instantaneous probability of success both directly ($\lambda_b < 0$) and indirectly ($\lambda_{xb} < 0$). Since the entrant responds to an increase in patent breadth with a decrease in the flow of R&D spending ($x_b < 0$), it becomes more difficult for the entrant to succeed in generating the non-infringing process. The more difficult it is for the entrant to succeed, the longer the incumbent can operate as a monopolist and the greater are his expected profits (Π^u). When the instantaneous probability of success is additively separable in the flow of R&D spending and in the patent breadth, an increase in patent breadth affects the instantaneous probability of success only directly ($\lambda_b < 0$ and $\lambda_{xb} = 0$). In addition, the entrant responds to an increase in patent breadth with an increase in the flow of R&D spending ($x_b > 0$). The increase in the flow of the R&D spending, in turn, has a positive affect on the instantaneous probability of success ($\lambda_x > 0$). The total effect of an increase in patent breadth on the incumbent's expected profits (Π^u) is positive because the decrease in the probability of success caused by an increase in patent breadth is greater than the increase in the probability of success caused by the increase in the flow of R&D spending (i.e., $\lambda_b > \lambda_x$).

Having determined how patent breadth affects the incumbent's expected profits when his patent is not challenged or when it is challenged and upheld ($\frac{\partial \Pi^u}{\partial b}$), the F.O.C. can be interpreted. The F.O.C. demonstrate the trade off that the incumbent faces when he determines the optimal breadth of patent protection. An increase in patent breadth leads to an increase in the incumbent's expected returns by $(1-b^2)\frac{\partial \Pi^u}{\partial b} + 2b\Pi^R$; this increase represents the marginal benefit to the incumbent from an increase in patent breadth. At the same time, an increase in patent breadth leads to a decrease in the incumbent's expected returns by $2b\Pi^u + C_T$; this decrease represents the marginal cost to the incumbent from an increase in patent breadth. Given that as patent breadth increases so does the probability that the patent will be challenged and revoked, by increasing patent breadth the incumbent increases the likelihood that he will realize profits Π^R (i.e., profits earned when the patent is revoked) rather than Π^u (i.e., profits earned when the patent is not challenged or is challenged and upheld). In addition, by increasing patent breadth the incumbent increases the profits made when the patent is not challenged or is challenged and upheld (since $\frac{\partial \Pi^u}{\partial b} > 0$) but, at the same time, he increases the probability that the patent will be challenged and that he will have to incur the legal costs C_T. At the optimal patent breadth the marginal benefits will be equal to the marginal costs.

To graphically characterize the determination of the optimal patent breadth let $k(b) = C_T$ and $f(b) = (-2b)\Pi^u + (1-b^2)\frac{\partial \Pi^u}{\partial b} + 2b\Pi^R$.[4] The F.O.C. for a maximum can then be written as follows:

$$f(b) - k(b) = 0 \Rightarrow (-2b)\Pi^u + (1-b^2)\frac{\partial \Pi^u}{\partial b} + 2b\Pi^R = C_T \qquad (23)$$

The S.O.C. for a maximum imply that the following inequality must be satisfied

$$f_b - k_b < 0 \Rightarrow f_b < k_b \qquad (24)$$

Given that the $k_b = 0$, the S.O.C. imply that $f_b < 0$ which means that f_b must cut k_b from above at the optimum. It is easily verified that f_b is decreasing in b as $f(b \to 0) \approx \frac{\partial \Pi^u}{\partial b} > 0$ while $f(b=1) = -2\Pi^u + 2\Pi^R < 0$. To guarantee the existence of an optimum the increase in the incumbent's expected profits when the patent is not challenged or challenged and upheld, $\frac{\partial \Pi^u}{\partial b}$, should be greater than the legal costs incurred by the incumbent when the patent is challenged, C_T. The requirement that $\frac{\partial \Pi^u}{\partial b} > C_T$ guarantees that f_b cuts k_b from above.

Proposition 8 *Claiming the maximum breadth of patent protection (i.e., $b^*=1$) is never an optimal strategy for the incumbent in this model.*

Proof:

At b=1 $k(b=1) = C_T \geq 0$ and $f(b=1) = 2(-\Pi^u + \Pi^R) < 0$. The above imply that the curves $k(b)$ and $f(b)$ will never cross at b=1. The same result is of course derived when the marginal benefits and the marginal costs are compared for b=1. When b=1 the marginal costs are always greater than the marginal benefits, i.e., $2\Pi^u + C_T > 2\Pi^R$. Thus, b=1 is not a profit maximizing patent breadth choice for the incumbent in this model. □

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[4] Note that the functions $k(b)$ and $f(b)$ are not defined in terms of marginal benefits and marginal costs because the slope and the curvature of the marginal benefit curve cannot be determined without knowledge of the values of the parameters that affect it. The chosen formulation simplifies the analysis.

The graphical representation of the determination of the optimal patent breadth is depicted in Figure 4. In Figure 4 the slope of the curve $f(b)$ has been assumed to be decreasing in patent breadth, $f_{bb} < 0$.[5]

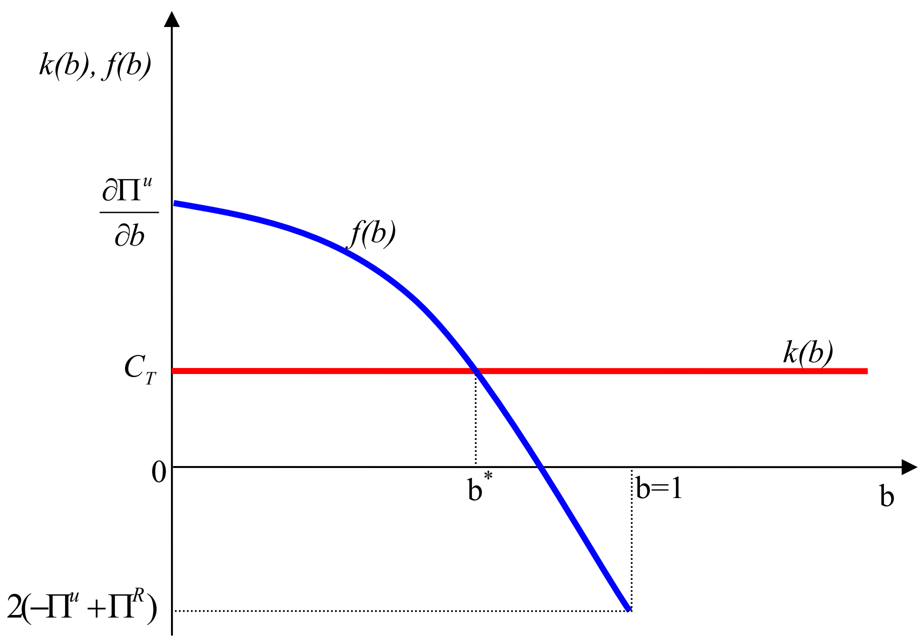

Figure 4 Graphical Representation of the Determination of the Optimal Patent Breadth

As shown in equation (22) the optimal patent breadth is a function of the following parameters, $b^* = (\Pi_m, \Pi_d, C_T, r)$. The effect of a change in the parameters of interest on the optimal patent breadth choice is determined by the signs of the following terms, $\frac{db^*}{d\Pi_m}, \frac{db^*}{d\Pi_d}$ $\frac{db^*}{dC_T}$ and $\frac{db^*}{dr}$.

The effect of a change in the monopoly profits on the optimal breadth is determined first. The expression for $\frac{db^*}{d\Pi_m}$ is derived by totally differentiating the optimality condition

[5] The curvature of *f*(b) cannot be determined without knowledge of the magnitude of the parameters that affect it. Note that the determination of the curvature of *f*(b) in not important for the results, it is necessary only for the graphical representation of the optimum.

$f(b)-k(b)=0 \Rightarrow (-2b)\Pi^{u}+(1-b^{2})\frac{\partial \Pi^{u}}{\partial b}+2b\Pi^{R}-C_{T}=0$ with respect to the optimal patent breadth, b^*, and the monopoly profits, Π_m:

$$\frac{\partial f(b^*)}{\partial b^*}db^* + \frac{\partial f(b^*)}{\partial \Pi_m}d\Pi_m = \frac{\partial k(b^*)}{\partial b^*}db^* + \frac{\partial k(b^*)}{\partial \Pi_m}d\Pi_m \Rightarrow$$
$$(f_b - k_b)db^* = (k_{\Pi_m} - f_{\Pi_m})d\Pi_m \Rightarrow \frac{db^*}{d\Pi_m} = \frac{k_{\Pi_m} - f_{\Pi_m}}{(f_b - k_b)} \quad (25)$$

From the S.O.C., $f_b - k_b < 0$. Since $k_{\Pi_m} = 0$ and $f_{\Pi_m} = -\frac{2b}{(r+\lambda)} - \frac{(1-b^2)(\lambda_b + \lambda_x x_b)}{(r+\lambda)^2}$, the sign of the term $\frac{db^*}{d\Pi_m}$ depends on the sign of the term f_{Π_m}.

Proposition 9 *An increase in the monopoly profits leads to an increase in the optimal patent breadth (*$\frac{db^*}{d\Pi_m} > 0$*), when* $b^* \in (0,\bar{b})$ *and to a decrease in the optimal patent breadth (*$\frac{db^*}{d\Pi_m} < 0$*), when* $b^* \in (\bar{b},1]$*. The patent breadth* $\bar{b} \in (0,1]$ *is the breadth of patent protection that makes the effect of a change in monopoly profits on the optimal patent breadth equal to zero,* $\frac{d(b^* = \bar{b})}{d\Pi_m} = 0$*. The patent breadth* $\bar{b}$ *exists for both the additive and multiplicative formulations of the instantaneous probability of success.*

Proof:

The additively and multiplicatively separable functions $f_1 : \lambda = x^{\theta} + \frac{1}{b}$ and $f_2 : \lambda = \frac{x^{\theta}}{b}$ are used, respectively, to prove Proposition 9. The detailed proof is presented in the Appendix. It is found that there exists a patent breadth $\bar{b} \in (0,1]$ such that $f_{\Pi_m}(\bar{b}) = 0$ for both f_1 and f_2. It is also found that f_{Π_m} is decreasing in patent breadth $\forall\theta\in(0,1)$, $x\geq 0$ and $r\in[0,1]$. The above imply that if patent breadth b^* is such that $b^* \in (0,\bar{b})$ then $f_{\Pi_m} > 0$ which implies that $\frac{db^*}{d\Pi_m} > 0$ while if patent breadth b^* is such that $b^* \in (\bar{b},1]$ then $f_{\Pi_m} < 0$ which implies that $\frac{db^*}{d\Pi_m} < 0$ (see equation (25)). □

The intuition behind the results of Proposition 9 is as follows. There are two countervailing effects that take place as patent breadth increases. On the one hand, a larger patent breadth makes it harder for the entrant to succeed in generating a non-infringing process, thus allowing the incumbent to make monopoly profits for a longer period. On the other hand, the probability that the patent will be challenged and the probability that it will be revoked increase, making it less likely for the incumbent to realize monopoly profits. There is a critical patent breadth value $\bar{b}$ which makes the two effects equal. When the breadth of patent protection is smaller than $\bar{b}$, the danger of having the patent challenged and revoked is relatively small and the incumbent tries to capture the (increased) monopoly profits by making it harder for the entrant to succeed. Thus, when $b^* < \bar{b}$, an increase in the anticipated monopoly profits results in an increase in the optimal breadth of patent protection. However, when initially the patent breadth is greater than $\bar{b}$, the risk of having the patent revoked (due to the large patent breadth) is now relatively large and the incumbent reduces the breadth of protection in order to reduce the probability that the patent will be revoked and that he will not have a chance to operate as a monopolist. Thus, when initially patent breadth is greater than $\bar{b}$, the incumbent responds to an increase in the anticipated monopoly profits with a decrease in the optimal patent breadth level.

The effect of a change in the duopoly profits on the optimal breadth of patent protection claimed, $\frac{db^*}{d\Pi_d}$, is determined by totally differentiating the optimality condition

$f(b) - k(b) = 0 \Rightarrow (-2b)\Pi^u + (1-b^2)\frac{\partial \Pi^u}{\partial b} + 2b\Pi^R - C_T = 0$ with respect to the optimal patent breadth, b^*, and the duopoly profits, Π_d. The expression for $\frac{db^*}{d\Pi_d}$ is given by equation (26).

$$\begin{aligned} &\frac{\partial f(b^*)}{\partial b^*}db^* + \frac{\partial f(b^*)}{\partial \Pi_d}d\Pi_d = \frac{\partial k(b^*)}{\partial b^*}db^* + \frac{\partial k(b^*)}{\partial \Pi_d}d\Pi_d \Rightarrow \\ &(f_b - k_b)db^* = (k_{\Pi_d} - f_{\Pi_d})d\Pi_d \Rightarrow \frac{db^*}{d\Pi_d} = \frac{k_{\Pi_d} - f_{\Pi_d}}{(f_b - k_b)} \end{aligned} \tag{26}$$

From the S.O.C. the term $f_b - k_b < 0$. In addition, $k_{\Pi_d} = 0$ and

$f_{\Pi_d} = -\frac{2b\lambda}{(r+\lambda)} + \frac{r(1-b^2)(\lambda_b + \lambda_x x_b)}{(r+\lambda)^2} + \frac{2b}{r}$ which implies that the sign of the term $\frac{db^*}{d\Pi_d}$ depends on the sign of the term f_{Π_d}. Given the above, if $f_{\Pi_d} > 0$ then $\frac{db^*}{d\Pi_d} > 0$ while if $f_{\Pi_d} \leq 0$ then $\frac{db^*}{d\Pi_d} < 0$.

Proposition 10 *An increase in the duopoly profits leads to a decrease in the optimal patent breadth ($\frac{db^*}{d\Pi_d} < 0$), when $b^* \in (0,\overline{\overline{b}})$ and to an increase in the optimal patent breadth ($\frac{db^*}{d\Pi_d} > 0$), when $b^* \in (\overline{\overline{b}},1]$. The patent breadth $\overline{\overline{b}} \in (0,1]$ is the breadth of patent protection that makes the effect of a change in duopoly profits on the optimal patent breadth equal to zero, $\frac{d(b^* = \overline{\overline{b}})}{d\Pi_d} = 0$. The patent breadth $\overline{\overline{b}}$ exists for both the additive and multiplicative formulations of the instantaneous probability of success.*

Proof:

The additively and multiplicatively separable functions $f_1 : \lambda = x^{\theta} + \frac{1}{b}$ and $f_2 : \lambda = \frac{x^{\theta}}{b}$ are used, respectively, to prove the above proposition. The detailed proof is presented in the Appendix. It is found that there exists a patent breadth $\overline{\overline{b}} \in (0,1]$ such that $f_{\Pi_d}(b^* = \overline{\overline{b}}) = 0$ for both f_1 and f_2. It is also found that the term f_{Π_d} is increasing in patent breadth, $\frac{\partial f_{\Pi_d}}{\partial b} > 0$, ∀θ∈(0,1), x≥0 and r∈[0,1]. The above imply that if patent breadth b^* is such that $b^* \in (0,\overline{\overline{b}})$ then $f_{\Pi_d} < 0$ which implies that $\frac{db^*}{d\Pi_d} < 0$ and if patent breadth b^* is such that $b^* \in (\overline{\overline{b}},1]$ then $f_{\Pi_d} > 0$ which implies that $\frac{db^*}{d\Pi_d} > 0$ (see equation (26)). □

The intuition behind the results of Proposition 10 is as follows. As discussed above two countervailing effects take place as patent breadth increases. On the one hand, it becomes harder for the entrant to succeed and on the other hand the probability that the patent will be challenged and the probability that it will be revoked increase. If the patent breadth is such that $b \in (0,\overline{\overline{b}})$, then the incumbent responds to an increase in duopoly profits by decreasing patent breadth to make it easier for the entrant to succeed and so that he can realize the duopoly profits. If the patent breadth is such that $b \in (\overline{\overline{b}},1]$, then the incumbent increases patent breadth to make it easier for his patent to be challenged and revoked, thus again increasing the probability of realizing the increased duopoly profits.

The effect of a change in the legal costs incurred by the incumbent on the optimal level of patent breadth is determined by totally differentiating the optimality condition $f(b) - k(b) = 0 \Rightarrow (-2b)\Pi^u + (1-b^2)\frac{\partial \Pi^u}{\partial b} + 2b\Pi^R - C_T = 0$ with respect to the optimal patent breadth, b*, and the legal costs C_T. The expression for $\frac{db^*}{dC_T}$ is given by:

$$\begin{aligned} &\frac{\partial f(b^*)}{\partial b^*}db^* + \frac{\partial f(b^*)}{\partial C_T}dC_T = \frac{\partial k(b^*)}{\partial b^*}db^* + \frac{\partial k(b^*)}{\partial C_T}db^* \Rightarrow \\ &(f_b - k_b)db^* = (k_{C_T} - f_{C_T})dC_T \Rightarrow \frac{db^*}{dC_T} = \frac{k_{C_T} - f_{C_T}}{(f_b - k_b)} \end{aligned} \tag{27}$$

Proposition 11 T*he effect of a change in the legal costs incurred by the incumbent on the optimal patent breadth is negative,* $\frac{db^*}{dC_T} < 0$, *for both the additive and the multiplicative formulations of the instantaneous probability of success, λ.*

Proof:

From the S.O.C. the term $f_b - k_b < 0$. In addition, $k_{C_T} = 1$ while $f_{C_T} = 0$ which imply that $\frac{db^*}{dC_T} < 0$. □

The results of Proposition 11 are as expected. The more expensive it becomes for the incumbent to defend the patent during a patent validity challenge, the less willing is the incumbent to risk having the patent challenged. The incumbent decreases the probability of having the patent challenged by decreasing the breadth of patent protection.

Finally, the effect of a change in the discount rate on the optimal patent breadth is determined by totally differentiating the optimality condition

$f(b) - k(b) = 0 \Rightarrow (-2b)\Pi^u + (1-b^2)\dfrac{\partial \Pi^u}{\partial b} + 2b\Pi^R - C_T = 0$ with respect to the optimal patent breadth, b*, and the discount rate, r. The expression for $\dfrac{db^*}{dr}$ is given by:

$$\begin{aligned} &\frac{\partial f(b^*)}{\partial b^*}db^* + \frac{\partial f(b^*)}{\partial r}dr = \frac{\partial k(b^*)}{\partial b^*}db^* + \frac{\partial k(b^*)}{\partial r}dr \Rightarrow \\ &(f_b - k_b)db^* = (k_r - f_r)dr \Rightarrow \frac{db^*}{dr} = \frac{k_r - f_r}{(f_b - k_b)} \end{aligned} \tag{28}$$

From the S.O.C. the term $f_b - k_b < 0$. In addition, $k_r = 0$ while

$$f_r = \frac{2(-1+b^2)(\lambda_b + \lambda_x x_b)(r\Pi_d - \Pi_m)}{(r+\lambda)^3} - \frac{2b\Pi_d}{r^2} + \frac{2b(\lambda\Pi_d + \Pi_m)}{(r+\lambda)^2} + \frac{(1-b^2)(\lambda_b + \lambda_x x_b)\Pi_d}{(r+\lambda)^2}.$$

The sign of the term f_r cannot be determined without knowledge of the magnitude of the parameters that affect it and thus the sign of the term $\dfrac{db^*}{dr}$ is inconclusive.

To summarize, the incumbent's optimal patent breadth choice depends on the level of monopoly profits that the incumbent realizes for as long as the entrant does not succeed in generating a non-infringing process, the level of duopoly profits realized by the incumbent once the entrant succeeds, the legal costs incurred during the patent challenge process and the discount rate. Claiming the maximum breadth of patent protection (b_{max}=1) is not a profit maximizing strategy for the entrant in this model. The effect of a change in the level of monopoly and duopoly profits on the optimal patent breadth depends on the initial optimal patent breadth value. The effect of a change in the legal costs incurred by the incumbent during the patent challenge process on the optimal patent breadth choice is always negative while the effect of a change in the discount rate on the optimal patent breadth choice is inconclusive.

4. Concluding Remarks

The paper uses a simple game theoretic model to model and to examine the determination of the optimal patent breadth for the innovator of a drastic process innovation. The optimal patent breadth for the innovator is the breadth of patent protection that maximizes the innovator's ability to appropriate innovation rents. The game consists of two players, an incumbent innovator who having generated a drastic process innovation and having decided to patent it determines the breadth of patent protection and an entrant who decides how much to spend on R&D to generate her own process.

The innovator in this model acts strategically and with foresight. That is, the innovator takes into consideration the entrant's response to his choice of patent breadth and the possibility that he may have to defend the validity of his patent when he determines the optimal breadth of patent protection claimed. The model allows for the probability that the patent will be challenged by a third party as soon as the patent is granted. The probability that the patent will be challenged and the probability that the validity of the patent will be upheld depend on the breadth of patent protection. The possibility of patent infringement is not considered in this model. It is thus assumed that if the entrant enters, she will do so without infringing the patent.

In this model, the R&D process is stochastic and the instantaneous probability of success is either additively or multiplicatively separable in the entrant's flow of R&D spending and in the incumbent's patent breadth choice. It is assumed that when success is realized by the entrant, her process is as efficient as the incumbent's process in producing the non-patentable product. Both players use their processes for the production of a new non-patentable product which is viewed as a homogenous product by consumers.

The results show that when the patent is revoked the entrant enters the market using the incumbent's process. When the patent is not challenged or is challenged and upheld, the entrant's optimal flow of R&D spending depends on the breadth of patent protection, the duopoly profits that the entrant will realize upon success and the discount rate. The effect of patent breadth on the entrant's optimal flow of R&D spending is positive or negative depending on whether the instantaneous probability of success is additively or multiplicatively separable, respectively, on the flow of R&D spending and on patent breadth. The duopoly profits have a positive effect on the optimal flow of R&D spending while the effect of the discount rate on the optimal flow of R&D spending is inconclusive.

The optimal breadth of patent protection depends on the level of monopoly profits realized by the incumbent during the period that the entrant undertakes R&D, the level of duopoly profits realized once the entrant succeeds, the legal costs incurred by the incumbent during the patent challenge process and the discount rate. The effect of the monopoly and the duopoly profits on the optimal patent breadth choice depends on the initial patent breadth value. The incumbent's legal costs have a negative effect on the optimal patent breadth while the effect of the discount rate on the optimal patent breadth is inconclusive.

The results show that there may exist a patent breadth that deters entry, but it may not be optimal for the incumbent to choose this patent breadth and deter entry. The results also show that claiming the maximum breadth of patent protection (b_{max}=1) is never an optimal strategy for the incumbent in this model. The results hold under the assumption of no patent infringement which implies that patent breadth affects the entrant's probability of success. If infringement was an option for the entrant then if the entrant found it optimal to infringe the patent, patent breadth would not have a binding effect on the entrant's probability of generating an infringing process. The results also depend on the assumption that the patent validity is challenged only by a third party. If the model allowed for a validity challenge by the entrant, as well as by a third party, then the optimal patent breadth might have been narrower. In addition, it has been assumed that there is only one entrant, that the patent life is infinite and that entry deterrence is either not possible or is not an optimal choice for the incumbent. Relaxing the above assumptions is the focus of future research.

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APPENDIX

- The curvature of the function $h(x) = \frac{r+\lambda(x,b)}{\lambda_x} - r\Pi_d$ is negative, $h_{xx} = -\frac{\lambda_{xxx}(r+\lambda)+\lambda_x\lambda_{xx}}{(\lambda_x)^2} + \frac{2(\lambda_{xx})^2(r+\lambda)}{(\lambda_x)^3} \leq 0$, for both the additive and the multiplicative formulations of the instantaneous probability of success, λ.

Proof:

To prove the above statement the additively separable function $f_1 : \lambda = x^\theta + \frac{1}{b}$ and the multiplicatively separable function $f_2 : \lambda = \frac{x^\theta}{b}$ are used. Both functions satisfy all the theoretical assumptions concerning the instantaneous probability of success λ.

- λ is additively separable in x and b, $f_1 : \lambda = x^\theta + \frac{1}{b}$.

In this case,

$$h_{xx} = (1-\theta)\{\frac{1}{x} - x^{-(1+\theta)}(\frac{1}{b} + r + x^\theta)\} = (1-\theta)\{\frac{1}{x} - (\frac{1}{bx^{(1+\theta)}} + \frac{r}{x^{(1+\theta)}} + \frac{1}{x})\} = (1-\theta)(-\frac{1+rb}{bx^{(1+\theta)}}) \leq 0$$

The above inequality holds ∀ θ∈(0,1), b∈(0,1] and r∈[0,1].

- λ is multiplicatively separable in x and b, $f_2 : \lambda = \frac{x^\theta}{b}$.

In this case, $h_{xx} = (1-\theta)\{\frac{1}{x} - x^{-(1+\theta)}b(r + \frac{x^\theta}{b})\} = (1-\theta)\{\frac{1}{x} - (\frac{rb}{x^{(1+\theta)}} + \frac{1}{x})\} = (1-\theta)(-\frac{rb}{x^{(1+\theta)}}) \leq 0$

The above inequality holds ∀ θ∈(0,1), b∈(0,1] and r∈[0,1]. □

Proposition 9

Proof:

To prove the above proposition the additively and multiplicatively separable functions $f_1 : \lambda = x^\theta + \frac{1}{b}$, $f_2 : \lambda = \frac{x^\theta}{b}$ are used, respectively. When the function f_1 is used $f_{\Pi_m} = -\frac{(b^2-1)}{(1+b(r+x^\theta))^2} - \frac{2b^2}{1+b(r+x^\theta)}$.

The patent breadth $\bar{b}$ that makes $f_{\Pi_m} = 0$ is given by $\bar{b} = -\frac{1}{2(r+x^\theta)} + \frac{3}{2^{2/3}(r+x^\theta)A} + \frac{A}{62^{1/3}(r+x^\theta)}$ where, $A = \left(-54 + 108(r+x^\theta)^2 + \sqrt{-2916 + (-54+108(r+x^\theta)^2)^2}\right)^{1/3}$. Performing simulations it is found that there are combinations of r∈[0,1], x≥0 and θ∈(0,1) values such that there exists a patent breadth $\bar{b} \in (0,1]$. The term f_{Π_m} is decreasing in patent breadth, $\frac{\partial f_{\Pi_m}}{\partial b} = -\frac{2\left(3b + r + x^\theta + 3b^2(r+x^\theta) + b^3(r+x^\theta)^2\right)}{\left(1+b(r+x^\theta)\right)^3} < 0$, ∀ b∈(0,1], θ∈(0,1), x≥0 and r∈[0,1]. The above imply that if patent breadth b* is such that $b^* \in (0,\bar{b})$ then $f_{\Pi_m} > 0$ which implies that $\frac{db^*}{d\Pi_m} > 0$ while if patent breadth b* is such that $b^* \in (\bar{b},1]$ then $f_{\Pi_m} < 0$ which implies that $\frac{db^*}{d\Pi_m} < 0$. When the multiplicatively separable function f_2 is used, $f_{\Pi_m} = -\frac{(b^2-1)x^\theta}{(br+x^\theta)^2} - \frac{2b^2}{br+x^\theta}$. The patent breadth $\bar{b}$ that makes $f_{\Pi_m} = 0$ is given by $\bar{b} = -\frac{x^\theta}{2r} + \frac{x^{2\theta}}{2rB} + \frac{B}{2r}$ where $B = \left(2r^2x^\theta - x^{3\theta} + 2\sqrt{r^4x^{2\theta} - r^2x^{4\theta}}\right)^{1/3}$. Performing simulations it is found that for certain r∈[0,1], x≥0 and θ∈(0,1) values $\bar{b} \in (0,1]$. The term f_{Π_m} is decreasing in patent breadth, $\frac{\partial f_{\Pi_m}}{\partial b} = -\frac{2\left(b^3r^2 + rx^\theta + 3b^2rx^\theta + 3bx^{2\theta}\right)}{\left(br+x^\theta\right)^3} < 0$, ∀ b∈(0,1], θ∈(0,1), x≥0 and r∈[0,1]. The above imply that if patent breadth b^* is such that $b^* \in (0,\bar{b})$ then $f_{\Pi_m} > 0$ which implies that $\frac{db^*}{d\Pi_m} > 0$ and if patent breadth b^* is such that $b^* \in (\bar{b},1]$ then $f_{\Pi_m} < 0$ which implies that $\frac{db^*}{d\Pi_m} < 0$. □

Proposition 10
Proof:

When $f_1 : \lambda = x^\theta + \frac{1}{b}$ is used $f_{\Pi_d} = \frac{2b}{r} + \frac{r(-1+b^2)}{(1+b(r+x^\theta))^2} - \frac{2b(1+bx^\theta)}{1+b(r+x^\theta)}$. The patent breadth $\bar{\bar{b}}$ that makes $f_{\Pi_d} = 0$ is given by $\bar{\bar{b}} = -\frac{C}{6D} - \frac{\left(-(C^2+12(-1+r)D)\right)}{E^{1/3}} + \frac{E^{1/3}}{62^{1/3}D}$ where, $C = \left(-4r + r^2 - 4x^\theta + 4rx^\theta\right)$, $D = -r^2 - 2rx^\theta + r^2x^\theta - x^{2\theta} + rx^{2\theta}$, $E = 32^{2/3}D\left((-2C^3) + 36(-1+r)CD - 108r^2D^2 + F\right)$ and $F = \sqrt{4\left((-C)^2 + 12(-1+r)D\right)^3 + \left(-2C^3 + 36(-1+r)CD - 108r^2D^2\right)^2}$. Performing simulations it is found that for certain θ∈(0,1), x≥0 and r∈[0,1] values $\bar{\bar{b}} \in (0,1]$. The term f_{Π_d} is increasing in patent breadth ∀ b∈(0,1], θ∈(0,1), x≥0 and r∈[0,1], $\frac{\partial f_{\Pi_d}}{\partial b} = \frac{2\begin{pmatrix}(1+bx^\theta)^3 - r(1+bx^\theta)^2(1+b(-3+x^\theta)) + r^3(1-b^3(-1+x^\theta)) + \\ r^2(x^\theta - 3b^2(-1+x^\theta) - b^3x^\vartheta(-3+2x^\theta))\end{pmatrix}}{r\left(1+b(r+x^\theta)\right)^3} > 0$. The above imply that if patent breadth b^* is such that $b^* \in (0,\bar{\bar{b}})$ then $f_{\Pi_d} < 0$ which implies that $\frac{db^*}{d\Pi_d} < 0$ and for if patent breadth b^* such that $b^* \in (\bar{\bar{b}},1]$ then $f_{\Pi_d} > 0$ which implies that $\frac{db^*}{d\Pi_d} > 0$. When the function $f_2 : \lambda = \frac{x^\theta}{b}$ is used, $f_{\Pi_d} = \frac{2b}{r} + \frac{(-1+b^2)rx^\theta}{(br+x^\theta)^2} - \frac{2bx^\theta}{br+x^\theta}$. The patent breadth $\bar{\bar{b}}$ that makes $f_{\Pi_d} = 0$ is given by $\bar{\bar{b}} = -\frac{(-4+r)x^\theta}{6r} - \frac{G}{32^{2/3}r^2(H+\sqrt{4G^3+H^2})^{1/3}} + \frac{(H+\sqrt{4G^3+H^2})^{1/3}}{62^{1/3}r^2}$ where $G = (-(-4+r)^2r^2x^{2\theta} - 12(-1+r)r^2x^{2\theta})$ and

$H=(108r^6x^{\theta}+2(-4+r)^3r^3x^{3\theta}+36(-4+r)(-1+r)r^3x^{3\theta})$. Performing simulations it is found that for certain θ∈(0,1), x≥0 and r∈[0,1] values $\overline{\overline{b}}\in(0,1]$. The term f_{Π_d} is increasing in patent breadth, $\frac{\partial f_{\Pi_d}}{\partial b}=\frac{2b^3r^3+3b^2r^2x^{\theta}+r^3x^{\theta}+3brx^{2\theta}+x^{3\theta}-rx^{3\theta}}{r(br+x^{\theta})^3}>0$, ∀ b∈(0,1], θ∈(0,1), x≥0 and r∈[0,1]. The above imply that if patent breadth b^* is such that $b^*\in(0,\overline{\overline{b}})$ then $f_{\Pi_d}<0$ which implies that $\frac{db^*}{d\Pi_d}<0$ and for if patent breadth b^* such that $b^*\in(\overline{\overline{b}},1]$ then $f_{\Pi_d}>0$ which implies that $\frac{db^*}{d\Pi_d}>0$. □