ATTENTION ECONOMIES

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Abstract

Attracting attention is a basic feature of economic life but no standard economic problem. A new theoretical model is developed which describes the general structure of competition for attention and characterizes equilibria. The exogenous fundamentals of an attention economy are the space of receiving subjects with their attention capacity, and the potential set of competing companies (senders) with their radiation technologies. The endogenous variables explained by the theory are equilibrium audiences (the clients belonging to a company), their signal exposure and attention, and the diversity of senders surviving the contest for attention. Application of the equilibrium analysis to changes in information technologies and globalisation suggests that international integration or range-increasing technical progress may decrease global diversity. Local diversity, perceived by the individual receivers, may increase nonetheless.

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I. Introduction

Suppose you had an idea but nobody except you is aware of it. Then, you and your idea don't exist in the scientific community. Descartes' *cogito ergo sum,* hardly applies to the real world. The case is only what is perceived. And for being perceived one has to attract the attention of recipients. The importance of being perceived and having impact is not only common experience in an economist's life but a basic feature of the (economic) world in general. To attract the attention of people is a prerequisite for doing profitable business as well as for winning elections and shaping society. It follows from the fact that only those ideas, goods, persons, firms matter, which are perceived. The purpose of this paper is to reflect this fact in an adequate theoretical model and to analyse its economic consequences.

What determines the set of items we perceive and why does the diversity of perceived objects change? – Suppose you are a reader of economic articles. Apart from your interests, you have individual characteristics like brain capacity or distractions by non-economic stuff. They put a limit to your processing of economic literature. Thus, you focus attention on important sites and outlets of publications. And important is who produces many powerful papers and gets the attention of the science community or, more generally, of the ensemble of producers and consumers of scientific work. Range and diversity of the important agents and places on which this ensemble focuses have changed over time. In particular, some local heroes have become global heroes and other local heroes have disappeared at all. Obviously, the circle that producers try to attract attention and consumers pay attention to producers who send powerful signals is not bound to scientific research but applies to many economic goods and services from simple things like clothes to entertainment and high-tech products. It determines the agenda of public debates as well as the actually relevant choice sets on our mind as economic agents.

Teachers (or parents) always have told the children to be attentive. And the teachers have been told that good teaching requires do attract the audience's attention. The new thing is that managing attention and attracting attention are becoming universal maxims in business and economics. An account of the many current trends of attention management and the practical implications for doing business was recently presented by Davenport and Beck [2001] under the title "The Attention Economy".[1] The title indicates that the issue of attracting attention really adds a new dimension to an economy. Herbert A. Simon pointed out that in an information-rich world a new scarcity problem arises, namely: "a scarcity of whatever it is that information consumes." According to him, what information consumes is obvious: "it consumes the attention of its recipients. Hence a wealth of information creates a poverty of attention" (Simon [1971], p. 40). Whereas Simon focussed on the question how economic agents can allocate their attention capacity efficiently among information sources, the focus of

1 See there for references on the psychological, technological and commercial backgrounds of the role of attention in business and management. The first version of this paper was written 2001 before I have got knowledge of their book. That I independently chose almost the same title underlines that the subject is topical. My interest in it was raised by the art journal *Kunstforum*, whose December 1999 issue focussed on "Ressource Aufmerksamkeit" (see also Goldhaber [1997] and Frank [1998]). Shapiro and Varian [1999] discuss business strategies for attracting attention by customizing information.

this paper is the contest of information sources for the attention of recipients in anonymous markets. Under this focus the central question is which and how many information sources succeed in getting enough attention to be viable. Put differently: Given that the agenda of people is crowded, who and what is on the agenda if an overabundant set of potential items compete for being there.[2] For answering this question, an economy is considered as a system of subjects (senders) who try to attract the attention of other subjects (receivers) by producing and distributing information packages (signals). The senders may be firms, news agencies, scientific networks, political parties etc., promoting products, persons or ideas under some logo. They are addressed also as companies or firms in this paper, since they typically consist of more than one individual. (However, the interior structure of companies is not considered.) The receiving subjects, on whom the signals are targeted, are called audiences. The central question to be answered is then: What is an equilibrium outcome if each company tries to attract the attention of subjects by exposing them to the signal "Look at me" or "Read my message"? As a result, we get characterizations of which companies survive the contest for attention and how big is their impact.

Although there is much economic jargon when people talk about attention, closer examination quickly shows that standard economic notions like supply, demand and exchange at the market price cannot be simply applied to attention as just another economic good. Attention is a prerequisite of any economic transaction. Before subjects can value and choose items they must get aware of them. This involves an interaction between producers and consumers beyond the exchange of specific products and contents.[3] The essential dimension in this "meta-interaction" is how many units of a receiver's attention capacity, e.g. time, are absorbed by signals targeted on her or him. To say that attention is a scarce resource means: The exposure of subjects to signals is so strong that having an impact by absorbing part of their attention capacity requires to send strong signals and to target them on audiences with relatively unexhausted perception capacity. In an attention economy many potential senders compete for attention in this way. This paper provides a formal description of the fundamentals of such an economy and a characterization of what is an equilibrium. Despite the abstract character of the model quite concrete predictions follow from the comparative-static analysis of equilibrium.

The key features of the model are: An attention economy is modelled as a family of *senders,* which employ costly *signals* to attract the attention of audiences and have an impact on them. An audience is a set of *receivers* characterized by their *attention capacity*. The attention capacity, formally represented by a relationship between stimulus exposure and attention level of an individual, constitutes one of the exogenous

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[2] Scarcity of attention capacity has not only implications for individual behaviour or business strategies but also consequences for the (equilibrium) outcome in markets or in the whole economy. To stress this aspect I use the label "attention economies" rather than "economics of attention" for addressing the subject of this paper.

[3] Also trash absorbs attention capacity before you can scrap it. As is well known, the purchase of information can never be based on perfect information. But even recognized noise, destroying the possibility to focus on other things, cannot always be turned off. This is an important difference between the problem of attention economies and Rosen's [1981] "Economics of Superstars", where consumers have a choice between services of different quality and their willingness to pay for a unit of service is higher if the service comes from a more talented producer.

fundamentals of the economy.[4] The other set of fundamentals describes the exogenous properties of potential senders. Each sender is characterized by its *radiation capacity* or *range*, which measures the maximal size of the audience that can be addressed. This allows us to deal with the fact that more or less local or global companies are competing for the attention of audiences. It is not required that the audience of a sender is a connected area or neighbourhood of receiving subjects. This complicates the formal structure of the presented model but is essential for an appropriate analysis. The prototype sender in an attention economy is not an emitter of electromagnetic waves covering a fixed geographic area. Modern techniques allow those who want to attract attention flexible targeting of their signals on attentive individuals. The location of the targeted receiver is of minor importance, or of none if we think of the Internet.

Given their capacity to address audiences of a certain size, the economic agents in search for attention can choose the targeted audience and the strength of the signals they focus on them. The purpose of attracting attention is to have an impact. And the impact of a sender is the bigger, the stronger the signal produced by the sender and the higher the level of attention of the targeted audience. Attention levels depend on the receivers' attention capacities and on the total volume of signals to which they are exposed. Single senders have zero measure and take attention levels as given when making their choices. The senders' simultaneous pursue of impact together with the individuals' attention capacity determine equilibrium signal exposure, equilibrium attention levels and equilibrium allocations of audiences to senders. Of course, audiences can overlap and an individual is typically a member of several audiences. This gives us a measure for the diversity of senders experienced by an individual. I call this the *local diversity*, since it varies across individuals with different attention characteristics. By contrast, the *global diversity* is given by the total measure of senders which survive in the equilibrium of the attention economy by achieving impact. Usual economic analysis concentrates on the specific choices out of the varieties that are available. Under that focus, diversity is closely related to interpersonal differences in tastes and talents as emphasized in Rosen [2002]. The following analysis points to another important aspect of diversity. Items that are not promoted powerfully enough, to be viable in the contest of attention, are not part of the choice set.[5] In the presented model equilibrium diversity is determined by attention capacities and radiation power.

The comparative-static analysis shows how changes in methods of impact generation or radiation capacities affect equilibrium signal strengths, attention levels and the measure of viable companies. As a main result, it is shown that an increase in the range of radiation, allowing to companies a wider diffusion of their signals, may diminish the equilibrium set of companies in the economy although each receiving subject has access to more varieties of senders than before. In other words, global diversity may decrease while a higher local diversity is experienced. The reason is that different local audiences turn into more homogeneous global audiences. Natural applications of this result on the

[4] Psychology or neurosciences tell us how people perceive signals and process information. Moreover, they may give advice how to concentrate and use our brain capacity optimally. Here the individual's perception psychology and attention capacity are taken as given.

[5] Even ideas or products that would be appreciated by all consumers – if they were aware of them – may not be viable since consumer attention is distracted to more powerful signals. In personal relationships you may be able to force others to listen, but in an anonymous market an audience of positive mass must be reached. This is impossible with radiation limited to personal relationships.

impact of extended radiation range are technical innovations like the Internet. According to evidence reported by Graham [2001], the number of sites that attract large shares of the time spent online substantially dropped in recent years. This doesn't mean a reduction of information received by the surfers, to the contrary. But more surfers focus on the same set of information sources. Apart from information and communication techniques, institutional restrictions may prevent senders from exhausting their radiation capacity by addressing for instance foreign audiences. Deregulation, in particular international integration, is thus another example to which the comparative-static result on the impact of an increase in the range of radiation can be applied.

The paper is organized as follows: In section II the general structure of attention economies is formalized. Section III determines equilibrium allocations of audiences to given families of senders. Section IV analyses the choice of signal strength in the contest for attention and characterizes equilibrium level of signal exposure and attention. In section V the equilibrium family of senders surviving in the contest for attention is derived and comparative-static results about equilibrium diversity are presented. Section VI summarizes.

II. The structure of an attention economy

An attention economy consists of two types of agents: The companies sending signals to earn attention, and the receivers exposed to the signals. The exogenous fundamentals of the economy are the *space of receivers* together with their *attention capacity* and the *space of potential senders* together with the *feasible audiences*. I present first the space of receivers and the space of senders.

II.1 Senders and audiences

Let all potential receivers be given by a set of subjects *S*. The potential audiences for senders are given by subsets $A \subset S$.[6] Examples of potential audiences are the set of Internet users with certain surfing characteristics, the subjects on the mailing list of a marketing firm, the set of subscribers to a broadcasting company or to the NBER working paper series. For a theatre company, the audience is located in a geographical space with a clear measure for the distances between actors and visitors. If less traditional media are used for attracting attention, the potential audiences have more complex structures. Visitors of web sites are scattered all over the world and companies may be interested in targeting their signals at isolated islands. Therefore, the length or the radius of some neighbourhood around a sender cannot characterize the size of an audience. A natural way to deal with this problem is to represent the *space of receivers* by a measure space $(S, \mathcal{A}, \mu)$, where $\mathcal{A}$ is a σ-field of measurable subsets of *S* and μ is a measure on $\mathcal{A}$. Every element of $\mathcal{A}$ is a subset $A \subset S$ representing a potential audience. The size of the audience is given by the measure $\mu(A)$. It is assumed that μ is finite. The following analysis focuses on contests for attention in competitive envi-

[6] $\subset$ denotes weak inclusion.

ronments. One requirement for competition is that there are many potential audiences for whose attention the companies can compete. In particular, it should be possible to pick also small sets of receivers. Formally, the following divisibility assumption is made: For any $A \in \mathcal{A}$ with $\mu(A) > 0$ and any constant $0 < c < \mu(A)$ there exist $B \in \mathcal{A}$ so that $\mu(B) = c$. (Take, for instance, the σ-field of Borel sets in a real interval S and the Lebesgue measure.)

Senders are the economic agents, for instance firms, who want to attract the attention of audiences. Let $(0, L]$ be the index set of potential senders in the economy. Thus, $t \in (0, L]$ is the name ("logo") under which a company, a scientific network or any other attention-seeking agent conveys information (signals) to receivers. In general, not all potential senders will be active. Actually, it is the purpose of the following analysis to determine the subset $T \subset (0, L]$ of companies surviving the competition for attention. The idea of a perfectly competitive contest for attention requires that single companies have zero measure. It is assumed that the potential sets of active senders are Lebesgue-measurable. That is, the *space of senders* is given by $((0, L], \mathcal{B}, \lambda)$ where $\mathcal{B}$, λ are the σ-field of Borel sets in $(0, L]$ and the Lebesgue measure, respectively. Every element of $\mathcal{B}$ is a subset $T \subset (0, L]$ representing a possible family of active senders. $\lambda(T)$ measures its size.

Each potential sender $t \in (0, L]$ is endowed with a *radiation capacity*. The radiation capacity is characterized by a real integrable function $\rho : (0, L] \to \mathbb{R}_{++}$.[7] $\rho(t)$ describes the maximal audience size (range) that can be addressed by company *t*. It is assumed that the senders are ordered according to their size, that is, ρ is non-increasing. Radiation capacity ρ determines the set of feasible audiences. Formally, this will be made precise in the next section. In reality, the maximal size of audiences that a company can handle depends on two factors: the available media and the resources attributed to the management of receiver relationship. Obviously, new media have dramatically changed the range of attention-seeking senders. Ceteris paribus, a larger audience can be addressed through the Internet than by phone calls. In general, such technological changes are exogenous to the single company. Also the means for entertaining the communication channels to the receivers, for instance the organizational infrastructure for maintaining mailing lists or evaluating the impact of distributed information on the addressed audience, are fixed in the short-run. The following analysis assumes that capacity $\rho(t)$ is an exogenously given characteristic of sender *t*. The comparative-static analysis of the impacts of changes in ρ on the diversity of companies and their impact on the receiving subjects will be an important part of the equilibrium analysis in section V.

7 $\mathbb{R}, \mathbb{R}_{+}, \mathbb{R}_{++}$ denote the sets of real numbers, non-negative real numbers, positive real numbers, respectively.

Whereas range ρ is exogenous, senders can choose their *signal strength*. This is the means by which a company attracts attention and which absorbs attention capacity of receivers. For any given set $T \in \mathcal{B}$ of senders, signal strength is represented by real integrable function $\sigma_T : T \to \mathbb{R}_+$. For every $t \in T$, $\sigma_T(t)$ describes volume and intensity of the information sent to the members of the audience of *t*, for instance, the number of mails to clients per period, or the loudness or conspicuousness of the transmitted signals. Production of signal strength is costly and the produced signals are sent to all members. Companies incur the cost for earning attention and achieving impact. This optimisation problem and the implied equilibrium signal exposure is analysed in section IV.

II.2 Audience allocations

The interaction of senders and receivers depends on which subjects are attracted by which companies. For characterizing the possible outcomes in an attention economy we must describe the assignment of receivers to companies. From the perspective of a signal sending company the question is which audience is reached by its signals. From the side of the receiving subjects the question is to which set of companies they pay attention. I call the assignment of audiences to senders an *audience allocation*. Formally, an audience allocation for a set of senders, $T \in \mathcal{B}$, is given by a relation $\mathfrak{a}_T \subset T \times S$, where $\mathfrak{a}_T$ is measurable $\mathcal{B} \times \mathcal{A}$. ($\mathcal{B} \times \mathcal{A}$ is the product space with measure $\lambda \times \mu$.) If a pair (t,s) is element of $\mathfrak{a}_T$, then sender t addresses subject s under audience allocation $\mathfrak{a}_T$. Thus, for any $t \in T$, the section $\mathfrak{a}_T(t) := \{s \in S | (t,s) \in \mathfrak{a}_T\}$ of $\mathfrak{a}_T$ defines the *audience* receiving the signals sent by *t*. From the perspective of a receiver $s \in S$, the section $M(s,\mathfrak{a}_T) := \{t \in T | (t,s) \in \mathfrak{a}_T\}$ of $\mathfrak{a}_T$ describes the set of perceived companies. I call $M(s,\mathfrak{a}_T)$ the *membership* of s under $\mathfrak{a}_T$. Note that $a_T \in \mathcal{B} \times \mathcal{A}$ implies that $M(s,\mathfrak{a}_T)$ and $\mathfrak{a}_T(t)$ are measurable $\mathcal{B}$ and $\mathcal{A}$, respectively. Not all audience allocations are feasible. The audience assigned to an active company must have positive measure and cannot be larger than the company's radiation capacity $\rho(t)$. Nor can it be larger than $\mu(S)$, the measure of all subjects in the economy. Since modern technologies have a very large range I don't want to exclude the case that senders have the capacity to cover the whole economy (i.e. $\rho(t) \geq \mu(S)$). In sum, an audience allocation $\mathfrak{a}_T$ for $T \in \mathcal{B}$ is *feasible* under radiation capacity ρ if for all $t \in T$: $0 < \mu(\mathfrak{a}_T(t)) \leq r_t$, where $r_t := \min\{\mu(S), \rho(t)\}$. The fact that only the size of audiences is limited by the radiation technology has far-reaching implications. It means that audiences of companies need not be connected areas or neighbourhoods in a geographical space. This would be the case if, for instance, a sender would have a fixed focus like a cone of light. In contrast to this, according to the given notion of feasibility, companies can target their signals fully flexibly, provided the targeted audiences are not larger than what they can handle by their given capacity. I think this is the adequate modelling of the media channels through which modern companies disseminate and promote information to anonymous audiences.

Membership $M(s,\mathfrak{a}_T)$ gives us a measure for *local diversity* under $\mathfrak{a}_T$. In an economy in which audience allocation $\mathfrak{a}_T$ is realised, $\lambda(M(s,\mathfrak{a}_T))$ measures the variety of companies perceived by subject *s*. By extending the notion to sets of subjects $A \in \mathcal{A}$, we can describe the variety of companies perceived in A by $M(A,\mathfrak{a}_T) := \{t \in T | (t,s) \in \mathfrak{a}_T \text{ for some } s \in A\}$. Then, $M(S,\mathfrak{a}_T)$ describes the total variety of companies perceived somewhere in the economy. By definition, $M(S,\mathfrak{a}_T) = T$ for any feasible $\mathfrak{a}_T$. I call $\lambda(T) = \lambda(M(S,\mathfrak{a}_T))$ the *global diversity* under $\mathfrak{a}_T$.[8] It puts an upper limit to local diversity. If for many subjects $\lambda(M(s,\mathfrak{a}_T))$ is small compared to $\lambda(T)$, then the variety of companies perceived by different subjects is relatively heterogeneous across subjects. If for all *s* local diversity coincides with global diversity, every subject experiences the same set of companies. For instance, if *T* is the set of active scientific networks in the world and each subject of the scientific community is also a member of all $t \in T$, then local diversity is equal to global diversity. There is a uniform international scientific community paying attention to the same set of paper series etc. If many networks $t \in T$ have only local membership, then local diversity perceived by a member of a local scientific community is small compared to the global diversity, which is experienced by a subject moving around through all local communities.

II.3 Attention, signal exposure and impact

The basic characteristic of subjects in their role as receivers in an attention economy is not taste or choice behaviour but the *attention capacity*. An agent may be in a more or less attentive state. If (s)he is more attentive (s)he processes the received information more carefully. On the one side, this capacity depends on individual psychological factors, which can vary from individual to individual since people are heterogeneous with respect to their abilities to concentrate. This ability may be the result of some self-management regarding a person's mental resources or time. In this analysis, it is taken as an exogenous individual characteristic. On the other side, the attentiveness of an individual with given psychological characteristics is also influenced by the strength and volume of signals to which she or he is exposed. People may almost fall into a doze if there is little stimulus. And they typically loose concentration when signal exposure is above a certain level. More generally, the attention level of a subject $s \in S$ depends on subjective characteristics but also on the environment of s, i.e. on the signal strength σ_T chosen by the active set of companies T and the prevailing audience allocation $_T$. This can be represented by a positive real function $\varphi(\cdot | \mathfrak{a}_T, \sigma_T)$ on the space of receivers. It is assumed that for all $\mathfrak{a}_T, \sigma_T$ function φ is measurable and bounded (see lemma 1 for a characterization of such functions). The function models the *attention*

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8 Note that for all A for which $M(A,\mathfrak{a}_T)$ is measurable $\lambda(M(A,\mathfrak{a}_T))$ can be seen as diversity function in the sense of Nehring and Puppe [2002], since $M(A,\mathfrak{a}_T) = \{t | \mathfrak{a}_T(t) \cap A \neq \varnothing\}$ and $\lambda(M(A,\mathfrak{a}_T)) = \int_M d\lambda$.

capacity of subjects. It assigns to every subject $s \in S$ the *attention level* $\upsilon = \varphi\left(s \middle| \mathfrak{a}_T, \sigma_T\right)$ of *s* when living in an economy with audience allocation $_T$ and signal strength σ_T. It is important to keep in mind that attention *capacities* are exogenous individual characteristics whereas attentive *levels* endogenously depend on environment $\mathfrak{a}_T, \sigma_T$. However, not all variations in the audience allocation or of signal strength matter. Any given $\mathfrak{a}_T, \sigma_T$ induces for $s \in S$ a certain total *signal exposure* given by the accumulated signal strength $\int_{M(s,\mathfrak{a}_T)} \sigma_T \, d\lambda$ over all senders to which *s* is exposed. This total signal exposure is the relevant factor for a subject's attention level under $\mathfrak{a}_T, \sigma_T$. Therefore, the following assumption is made with respect to the properties of the attention capacity.

Assumption AN (Anonymity). For all $s \in S$, if $T, \mathfrak{a}_T, \sigma_T$ and $T', \mathfrak{a}'_{T'}, \sigma'_{T'}$ induce the same signal exposure, i.e. if $\int_{M(s,\mathfrak{a}_T)} \sigma_T \, d\lambda = \int_{M(s,\mathfrak{a}'_{T'})} \sigma'_{T'} \, d\lambda$, then also the induced attention level is equal, i.e. $\varphi\left(s \middle| \mathfrak{a}_T, \sigma_T\right) = \varphi\left(s \middle| \mathfrak{a}'_{T'}, \sigma'_{T'}\right)$.

The assumption stipulates an anonymity property. The identity of the signal-sending source is irrelevant for the concentration of a subject exposed to signals. Only their strength matters for the absorption of attention capacity. Therefore, the attention level is invariant with respect to audience allocations leading to the same signal exposure. Assumption AN allows us the following characterization of attention capacities.

Lemma 1. φ satisfies AN if and only if there exists a measurable bounded function v: $S \times \mathbb{R}_+ \to \mathbb{R}_{++}$ so that $\varphi\left(s \middle| \mathfrak{a}_T, \sigma_T\right) = v(s, \tau_s)$, $\tau_s = \int_{M(s,\mathfrak{a}_T)} \sigma_T \, d\lambda$.

Proof. Appendix

The crucial mathematical point in the proof of lemma 1 is that signal exposure τ_s is a measurable function. The important economic assumption is AN. It allows the characterization of attention capacities by a function of signal exposure only. The attention level resulting when a subject with attention capacity v is exposed to an environment τ_s, is denoted by the variable υ. In the further analysis, AN is always assumed and, as a rule, representation $v(s, \tau_s)$ is used for describing attention capacities.

For various results of the paper, a further property of the attention capacity is required. It gives a precise meaning to the vague idea of attention as a scarce resource.

Assumption DA (Declining Attention). Attention capacities represented by $v(s, \tau_s)$ satisfy DA if for all $s \in S$ there is a threshold $\tau_s^+ \geq 0$ so that $v(s, \tau'_s) < v(s, \tau_s)$ if $\tau'_s > \tau_s \geq \tau^+$.

This assumption expresses the fact that too much signal exposure stresses people and destroys their concentration. The threshold τ_s^+ above which increasing signal exposure is harmful for a subject's level of attention can be different for different subjects.

There remains one element to be specified. By definition, in an attention economy companies send signals to earn the attention of receivers and to have an *impact* on them. Obviously a receiver can pay more attention if (s)he is concentrated, i.e. if her or his attention level is high. And a given attention level will more likely be focussed on a sender if the sender increases its signal strength, whereas a company of which no signal is perceived has no impact at all. Thus, the impact of $t \in T$ with strength $\sigma_T(t)$ on a receiver with attention level $\upsilon(>0)$ is given by a measurable non-negative bounded real function $z_t(\sigma_T(t), \upsilon)$, which is zero at $\sigma_T(t) = 0$, increasing in $\sigma_T(t)$ and, for $\sigma_T(t) > 0$, also increasing in υ. (Later, for some results, additional concavity and differentiability properties will be imposed.) z_t describes how signal strength and attention generate impact. Attention level υ is the only receiver characteristic which matters. There is no personalized interaction and apart from their attentiveness receivers are exchangeable. Note however that impact function z_t can vary across companies. The same signal strength may generate different impact for different types of senders. (σ_T comprises only that dimension of transmitted information which absorbs attention capacity. Any other aspects, which may be relevant for impact, for instance the used medium or the clarity of *t*'s presentation, are captured by z_t.) Whereas the input variable $\sigma_T(t)$ in impact function z_t is chosen by company *t*, the attention level $\upsilon = v(s, \tau_s)$ is exogenous to *t*. It is determined by total signal exposure, $\tau_s = \int_{M(s,\mathfrak{a}_T)} \sigma_T \, d\lambda$, on which single companies have no influence since they have zero measure.[9] Since the signals of company *t* reach all members of audience $\mathfrak{a}_T(t)$, the total impact of $t \in T$ under $\mathfrak{a}_T, \sigma_T$ is given by $V_t(\mathfrak{a}_T, \sigma_T) := \int_{\mathfrak{a}_T(t)} z_t(\sigma_T(t), v(s, \tau_s)) \, d\mu(s)$, where $\tau_s = \int_{M(s,\mathfrak{a}_T)} \sigma_T \, d\lambda$. Since v and z_t are non-negative, measurable and bounded, the integral exists (note $\mu(S) < \infty$).

II.4 Behaviour and equilibrium

Receivers react to signal exposure according to their exogenously given attention capacity v.[10] In the contest for attention, the active role is played by the senders. They

9 Formally, $\tau_s = \int_{M(s,\mathfrak{a}_T)} \sigma_T d\lambda = \int_{M(s,\mathfrak{a}_{T'})} \sigma_{T'} d\lambda$ for $T' = T - \{t\}$ and $\mathfrak{a}_T(t') = \mathfrak{a}_{T'}(t')$ for all $t' \in T'$.

10 Although the psychological processes behind v are taken as a black box, v is a rich box. Only very general properties are imposed by assumption AN and DA so that a wide variety of behavioural phenomena is covered. For instance, v may decline very sharply if changes in $\mathfrak{a}_T, \sigma_T$ lead to an

choose signal strength σ and select the audience of subjects at which their signals are targeted. The decision of an active sender about signal strength is a standard cost-benefit calculation. (It is assumed that the production of signal strength is costly whereas the diffusion of the signals is costless within the radiation capacity defined by range ρ. Setting up and running the radiation technology may imply fixed costs, which limits the set of active companies.) The selection of the audience is based on the following reasoning: As long as there is a choice between more or less attentive receivers, the impact of information can be increased by targeting it on subjects with higher attention. This requires that companies are able to observe the attention level of subjects. In a strict sense this is only possible with direct attention-monitoring methods evaluating brain waves or eye-ball movements. Although such methods exist and are further developed – Davenport and Beck [2001, p. 49] speak of the "Wire'em up principle" – observable proxies for the attention of audiences are more realistic. Media watch, download statistics, speed and frequency of response, self-reported data on attentiveness are examples.

For a given set of active companies, equilibrium is reached if a feasible audience allocation is realized, all companies choose their optimal signal strength and no company has an incentive to retarget its signals to more attentive audiences. The equilibrium set of active firms is determined by a viability condition. This condition will be derived from the premise that impact is the central determinant of the value of a sender. Therefore, only those companies survive which achieve a non-negative value of impact (net of the cost for generating the impact). By characterizing the properties of equilibrium sets of active companies, T, and of equilibrium allocations of audiences, $_{\mathrm{T}}$, we are able to assess the global and local diversity experienced in an attention economy by looking at $\lambda(T)$ and $\lambda\left(M\left(s,\mathfrak{a}_{\mathrm{T}}\right)\right)$, respectively. Moreover, we can provide explanations for changes in equilibrium signal exposure and equilibrium attention levels.

The further analysis proceeds as follows. In a first step, the properties of equilibrium audience allocations are determined for a given set of companies T with signal strength σ_T. It will be shown that under quite general conditions the attention level of subjects is uniquely determined by aggregate signal imission $X := \int_T r_t \sigma_T(t)\, dt$, $r_t = \min\{\mu(S), \rho(t)\}$. In a second step, I analyse, for a given set of senders T with aggregate signal level X, the optimal choice of signal strength σ_t^* of a single sender $t \in T$. Then I determine the equilibrium values of X, σ_T which are consistent with this choice. In the third step, I determine the size of equilibrium set T^* of companies which achieve sufficient impact to survive in an attention economy and describe the characteristics of companies in T^*. The results are used for characterizing the effects of changes in the technology of impact production or in the feasible range of radiation on sender diversity and attention level of receivers.

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increase in signal exposure. Subjects possibly concentrate on a certain amount of information and neglect the rest more or less.

III. Equilibrium allocation of audiences

Given a set of senders, $T \in \mathcal{B}$, with signal strength σ_T. Each company $t \in T$ can reach an audience of measure $\mu\left(\mathfrak{a}_T(t)\right) \le r_t = \min\left\{\mu(S), \rho(t)\right\}$. Depending on which feasible audience allocation is actually realised, total impact of *t* is $V_t\left(\mathfrak{a}_\mathrm{T}, \sigma_T\right)$. Suppose that there is another feasible audience allocation $\mathfrak{a}'_T$ under which company *t* would achieve a higher impact. Then *t* clearly would prefer $\mathfrak{a}'_T$ to $\mathfrak{a}_\mathrm{T}$. However, a single company hasn't the power to decide about which audience allocation is realised in the economy. It can only chose its own audience, i.e. the set of receivers $A \in \mathcal{A}$ on which the produced volume of signals, $\sigma_T(t)$, is targeted. Thus, for $t \in T$, a deviation from an audience allocation $\mathfrak{a}_\mathrm{T}$ to another audience allocation $\mathfrak{a}'_T$ is feasible if $\mathfrak{a}'_T$ results from $\mathfrak{a}_\mathrm{T}$ by exchanging $\mathfrak{a}_T(t)$ through $\mathfrak{a}'_T(t)$ and leaving all other audience assignments unchanged. This leads to the following definition of an equilibrium allocation of audiences.

Definition 1. For $T \in \mathcal{B}$ with σ_T, let $\mathfrak{a}_\mathrm{T}$ be a feasible audience allocation.

(i) An audience allocation $\mathfrak{a}'_T$ is a *feasible deviation* from $_T$ for $t \in T$, if $\mathfrak{a}'_T$ is a feasible audience allocation and $\mathfrak{a}'_T(t') = \mathfrak{a}_T(t')$ for all $t' \in T - \{t\}$.

(ii) $\mathfrak{a}_\mathrm{T}$ is an *equilibrium audience allocation* if for no $t \in T$ there is a feasible deviation $\mathfrak{a}'_T$ from $\mathfrak{a}_T$ with $V_t\left(\mathfrak{a}'_T, \sigma_T\right) > V_t\left(\mathfrak{a}_T, \sigma_T\right)$.

Whenever a company has an opportunity to increase its impact by retargeting its signals, the company will use the opportunity. It will skip less attentive receivers and use the radiation capacity to address subjects on which the produced signals have a higher impact. If audiences are allocated to senders in a way that offers no such opportunities, no company is interested in changing the audience allocation. If subjects have very heterogeneous attention capacities, it cannot be excluded that some of them are not covered at all, that is, no company targets its signals on them. Others may be fully covered by all companies. Let $U := \left\{s \middle| M\left(s, \mathfrak{a}_T\right) = \varnothing\right\}$, and $F := \left\{s \middle| M\left(s, \mathfrak{a}_T\right) = T\right\}$ be the (possibly empty) sets of uncovered and fully covered subjects, respectively. The following proposition characterizes the attention levels of covered and uncovered subjects in an equilibrium audience allocation.

Proposition 1. For $T \in \mathcal{B}$ with $\sigma_T > 0$ (i.e. $\sigma_T(t) > 0$ for all $t \in T$), let $\mathfrak{a}_T$ be a feasible audience allocation.

If $_T$ is an equilibrium audience allocation, the following conditions hold:

(a) For all $t \in T$: $v\left(s', \tau_{s'}\right) \le v\left(s, \tau_s\right)$ for almost all $s' \in U$, $s \in \mathfrak{a}_T(t)$.

(b) $v\left(s, \tau_s\right) \le v\left(s', \tau_{s'}\right)$, for almost all $s \in S - F$, $s' \in F$.

(c) For $\upsilon \in \mathbb{R}_{++}$, let $S_\upsilon^- := \{s | v(s,\tau_s) < \upsilon\}$, $S_\upsilon^+ := \{s | v(s,\tau_s) \geq \upsilon\}$. For all $t \in T$: If $\mu(\mathfrak{a}_T(t) \cap S_\upsilon^-) \neq 0$, then $\mu(S_\upsilon^+ - \mathfrak{a}_T(t)) = 0$.

(d) For all $t \in T$: $\mu(\mathfrak{a}_T(t)) = r_t$.

If $\mathfrak{a}_T$ satisfies conditions (c) and (d) then $\mathfrak{a}_T$ is an equilibrium audience allocation.
Proof. Appendix

The proposition provides necessary and sufficient conditions for an equilibrium. An immediate consequence of the sufficient conditions is that any audience allocation with equalized attention levels and fully utilized radiation range is an equilibrium.

Corollary 1. $\mathfrak{a}_T$ is an equilibrium audience allocation if $v(s,\tau_s) = v(s',\tau_{s'})$, for almost all s, $s' \in S$, and $\mu(\mathfrak{a}_T(t)) = \min\{\mu(S), \rho(t)\}$ for all $t \in T$.
Proof. Equal attention levels imply condition (c), and (d) holds by assumption.

The necessary conditions imply that, apart from extreme cases, attention levels *must* be equal, as the following discussion of conditions (a) – (d) shows.

Condition (a) says that only the least attentive subjects are possibly not addressed by any company, that is, are uncovered by T. Since they would pay less attention than other audiences, no company would wish to retarget its signals on them. In contrast, condition (b) deals with subjects which are in the focus of all companies. Such receivers, if there are any, must have a very high attention capacity. Despite full exposure (to all $t \in T$), their attention level is at least as high as the attention level of subjects which are not fully covered by T (i.e. for which $M(s,\mathfrak{a}_T)$ is a proper subset of T). Condition (c) is the most interesting one. Any company which has people with relatively low attention level among its receivers (i.e. for which $\mu(\mathfrak{a}_T(t) \cap S_\upsilon^-) \neq 0$) almost surely has also all receivers with higher attention in its audience $(\mu(S_\upsilon^+ - \mathfrak{a}_T(t)) = 0)$. Only a company that has exhausted all subjects with relatively high attention is willing to target its signals also on less attentive receivers. The alternative would be to leave part of the radiation capacity unused. But this would be no equilibrium, as stated by (d), since additional receivers, if they are feasible, always increase the total impact of a sender. If T contains enough small companies (i.e. $\rho(t)$ is small relative to the measure $\mu(S)$ of receivers), their chase after attentive audiences tends to equalize attention levels. To see this consider an audience allocation with two subject pools $A, A' \in \mathcal{A}$. Some companies target their signals on A, others on A' or on both sets. Small companies with $\rho(t) < \min\{\mu(A), \mu(A')\}$ can fully utilize their capacity by concentrating on one set. As long as attention levels are high in one set and low in the other, small companies can always switch to the more attractive subjects. If there are many such companies, signal exposure and attention levels change. The retargeting process only stops when attention levels in the two sets are equal or if all companies have switched to the more attentive audience. But the latter implies that this audience consists of an elite whose attention capacity is so high that they process any volume of signals better than others. The following corollary proves this intuitive argument. (A set $\tilde{U} \in \mathcal{A}$ is said to be almost completely uncovered if $\mu\left(\tilde{U} \cap \mathfrak{a}_T(t)\right) = 0$ for all $t \in T$. And $\tilde{F} \in \mathcal{A}$ is almost fully covered if $\mu\left(F - \mathfrak{a}_T(t)\right) = 0$ for any $t \in T$ [11].)

Corollary 2. If heterogeneity of radiation capacities is limited, in particular if $\rho(t) = \rho(t') \equiv \rho$ for all $t \in T$, then in any equilibrium allocation $\mathfrak{a}_T$ one of the following three conditions must be satisfied:

(a) $\nu(s,\tau_s) = \nu(s',\tau_{s'})$ almost everywhere or
(b) there exists an almost fully covered set of receivers with positive measure or
(c) there is a set of almost completely uncovered subjects with positive measure.

Proof. Appendix

The restriction on $\rho(t)$ used in the proof is that either all senders have a radiation range exceeding a certain size or all of them are smaller than this size. This is a sufficient condition for the result, not a necessary one. The corollary makes no restriction concerning heterogeneity of attention capacities. In sum, under quite general circumstances, in an attention economy in which all subjects are covered by some but not by all senders, attention levels are equal in an equilibrium allocation of audiences. Obviously, they may be equal in other circumstances as well, for example, if all subjects have identical attention capacities and are fully covered. The further analysis concentrates on audience allocations with equalized attention levels.

The following example describes a class of economies for which an equilibrium audience allocation with equal attention levels can be constructed explicitly.

Example 1. For $T, \sigma_T > 0$, let heterogeneity of senders be restricted by the assumption that there is a finite partition $T_k \in \mathcal{B},\ k = 1,\ldots,\ K,\ 0 < \lambda(T_k)\ < \lambda(T)$ with $\rho(t) = \rho_k$ and $\sigma_T(t) = \sigma_k$ for $t \in T_k$. Sender capacities satisfy the following divisibility condition: Senders in class k either cover all subjects (i.e. $\rho_k \geq \mu(S)$) or the subject space can be properly divided among them (i.e. $\mu(S)/\rho_k$ is an integer). Moreover, suppose that receivers' attention capacities $v(s,\tau_s)$ are given by a function $f(\tau_s) = v(s,\tau_s)$ almost everywhere.

Fact 1. For attention economies satisfying the properties of example 1, an equilibrium audience allocation exists with $\lambda\left(M(s,\mathfrak{a}_T)\right) = \sum_k \rho_k\ \lambda(T_k)/\mu(S)$, $\tau_s = \overline{\tau}$ and

[11] $\mu(\tilde{U})>0$ implies $\mu(U)>0$ but $\mu(\tilde{U})>0$ does not imply $\mu(U)=0$. The reason is that the union $\bigcup_{t\in T}\left(\tilde{U} \cap \mathfrak{a}_T(t)\right)$ over an uncountable T may have positive measure even if $\mu\left(\tilde{U} \cap \mathfrak{a}_T(t)\right) = 0$ for any t. Moreover, $\mu\left(\tilde{F}\right) = 0$ implies $\mu(F) = 0$ but not vice versa.

$v(s,\tau_s) = f(\overline{\tau})$, almost everywhere, where $\overline{\tau} = \sum_k \sigma_k r_k \, \lambda(T_k) / \mu(S)$, $r_k = \min\{\mu(S), \rho_k\}$.

Proof. Appendix

In the proof of fact 1 a concrete audience allocation is constructed satisfying the properties of equilibrium. Other audience allocations can fulfil these properties as well. However, the construction of the equilibrium audience allocation suggests that in any equilibrium audience allocation with equal attention levels, the equilibrium attention level is uniquely determined by average signal imission $\sum_k \sigma_k r_k \, \lambda(T_k) / \mu(S)$. (Note that $\sigma_k r_k$ is signal imission implied by sender k who spreads σ_k over range r_k. And $\sum_k \sigma_k r_k \, \lambda(T_k)$ is aggregate signal imission.) The following proposition shows that under DA this is generally the case for equilibrium audience allocations with equalized attention levels, i.e. for allocations $\mathfrak{a}_T$ which satisfy $\mu(\mathfrak{a}_T(t)) = r_t$, $t \in T$, and $v(s,\tau_s) = v(s',\tau_{s'})$ for almost all s, s'.

Proposition 2. For $T \in \mathcal{B}$, $\sigma_T > 0$ define $X = \int_T r_t \sigma_T(t)\, dt$. Under DA, there exists a decreasing real function $\overline{v}(\bullet)$ so that for any equilibrium audience allocation $\mathfrak{a}_T$ with $\tau_s \geq \tau_s^+$ for almost all s the following holds: If attention levels are equal under $\mathfrak{a}_T$, then they are given by $\overline{v}(X)$ (i.e. $v(s,\tau_s) = \overline{v}(X)$ almost everywhere)[12].

Proof. Appendix

Assumption DA and $\tau_s \geq \tau_s^+$ mean that signal exposure in the economy is sufficiently strong to make scarcity of attention relevant. If this is the case, in any equilibrium audience allocation with equalized attention levels the equilibrium attention level of almost all receivers is uniquely determined. There may be multiple equilibrium audience allocations. But the equilibrium attention level of receivers is given by $\overline{v}(X)$ which is independent of the signal strength $\sigma_T(t)$ of a single sender $(\lambda(\{t\}) = 0)$. Hence, each sender $t \in T$ can take the attention level of its receivers as given when deciding about signal strength. This decision and the implied equilibrium signal strength are analysed in the next section.

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[12] Like in all other statements $\mu(S)$ is fixed throughout the analysis. If $\mu(S)$ changes, a different function $\overline{v}(\bullet)$ is relevant. In general, the comparative-static effects of changes in $\mu(S)$ are ambiguous. But if attention capacities are identical as in example 1, equal attention levels imply $\tau_s = \tau = X / \mu(S)$. Then the equilibrium attention level is a function of $X / \mu(S)$. At the end of this paper this fact is used to illustrate possible consequences of international integration in an attention economy (see discussion of fact 3).

IV. Contest for attention and equilibrium signal exposure

The question solved so far was: Given a set of companies T with signal strength σ_T, what are the properties of equilibrium audience allocations under T, σ_T. This leaves open the questions: (i) Which sets of companies are equilibrium sets, and (ii) which signal strength do companies choose to have an optimal impact on their audience? In this section the second question is answered. Given an audience allocation $\mathfrak{a}_T$ for $T \in \mathcal{B}$, what are equilibrium choices $\sigma_T(t),\ t \in T$?

In an attention economy, the essential performance measure is impact. Formally, the impact of t sending with strength $\sigma \in \mathbb{R}_+$ on a receiver with attention level υ was modelled as $z_t(\sigma, \upsilon)$. The attention level υ of a subject depends on the environment to which it is exposed. It is given by $v(s, \tau_s)$ where signal exposure τ_s depends on the given audience allocation $\mathfrak{a}_T$ and on signal strength σ_T. The total impact achieved by t sending with strength σ is given by $V_t = \int_{\mathfrak{a}_T(t)} z_t(\sigma, v(s, \tau_s))\, d\mu(s)$. A single company has no influence on τ_s since it has measure zero. Thus, any $t \in T$ can take $v(s, \tau_s)$ as given when deciding about its signal strength.

Impact has economic value since only those who earn attention of audiences are in the trade, i.e. have a market, make money or earn reputation. Suppose that z_t is some physical measure of t's impact like frequency of citations or time spent on signals coming from t. Let the economic value of z_t be given by some measurable function $\pi_t(z_t)$. If π_t is bounded and monotonously increasing starting at $\pi_t(0) = 0$, $\tilde{z}_t = \pi_t(z_t)$ satisfies all properties that have been required from impact functions in section II.[13] Thus, provided one accepts the idea that economic performance is positively related to impact, it can be assumed without loss of generality that z_t and thus V_t measure the *economic* value of impact rather than impact in the physical sense.

It is assumed that production of strength σ requires resources. But the dissemination of σ to an audience is free, provided that the audience can be handled with t's radiation capacity $\rho(t)$. In view of modern technologies, this seems a reasonable assumption. Producing papers costs effort, sending them to a given mailing list is easy. Formally, for a company t the cost of exposing an audience A of measure $\mu(A) \le \rho(t)$ to signal strength $\sigma \ge 0$ are given by a function

$$C_t(\sigma, \mu(A)) = c_t(\sigma), \tag{4.1}$$

[13] The class of measurable functions $z_t : \mathbb{R}_+ \times \mathbb{R}_{++} \to \mathbb{R}_+$ with $z_t(\sigma, \upsilon) = 0$ for $\sigma = 0$, z_t bounded, increasing in σ and, for $\sigma > 0$, increasing in υ is invariant under measurable transformations $\pi_t : \mathbb{R}_+ \to \mathbb{R}_+$ with $\pi_t(0) = 0, \pi_t$ bounded and increasing.

where c_t is differentiable with $c_t' > 0,\ c_t'' \geq 0,$ starting at $c_t(0) \geq 0$. This allows for possible fixed costs $c_t(0) > 0$ and falling average costs. They may arise from setting up signal production or distribution capacity, i.e. the capacity to address audiences of size $\rho(t)$. It is assumed that c_t represents signal costs in units of the economic value of impact. Note that like impact function z_t also signal costs are allowed to be *t*-specific.

In equilibrium, each company chooses signal strength such that the net-value of its impact cannot be improved by choosing another strength. Let $V_t^n(\sigma) = \int_{\mathfrak{a}_T(t)} z_t(\sigma, v(s,\tau_s))\, d\mu(s) - c_t(\sigma)$ be the net-value of impact of σ. Signal strength σ is *optimal* for *t* under $\mathfrak{a}_T, \sigma_T$ if $V_t^n(\sigma') \leq V_t^n(\sigma)$ for $\sigma' \neq \sigma$. (Since *t* has measure zero no strategic aspects are involved in this choice.)

Definition 2 (Equilibrium signal strength). For $T \in \mathcal{B}$, let $\mathfrak{a}_T$ be a feasible audience allocation.

(i) σ_T is an *equilibrium* under $\mathfrak{a}_T$ if, for all $t \in T$, $\sigma_T(t)$ is optimal under $\mathfrak{a}_T, \sigma_T$.

(ii) $\mathfrak{a}_T, \sigma_T$ is an equilibrium for *T*, if $\mathfrak{a}_T$ is an equilibrium audience allocation for T, σ_T, and σ_T is an equilibrium under $\mathfrak{a}_T$.

As shown in proposition 2 under quite general assumptions, in an equilibrium audience allocation $\mathfrak{a}_T$ with signal strength σ_T attention levels of subjects are given by a decreasing function $v(s,\tau_s) = \overline{v}(X)$ of aggregate signal imission $X = \int_T r_t \sigma_T(t)\, dt$. And sender $t \in T$ has impact $z_t(\sigma_T(t), \overline{v}(X))$ on all subjects belonging to its audience $\mathfrak{a}_T(t)$. Since $\mu(\mathfrak{a}_T(t)) = r_t$ in an equilibrium audience allocation, the value of total impact $V_t(\mathfrak{a}_T, \sigma_T)$ which is achieved by $t \in T$ is given by

$$V_t = r_t z_t(\sigma_T(t), \overline{v}(X)). \tag{4.2}$$

According to (4.2), expanding signal strength $\sigma_T(t)$ has a positive effect on company *t*'s reception for any given *X*. But if a set of companies with positive measure increases signal strength, then *X* increases, which has an external effect on other companies. Under the assumptions of proposition 2, $\overline{v}(X)$ decreases in X so that the external effect is negative.[14] *t* ignores this effect when deciding about the optimal $\sigma_T(t)$. Determining equilibrium signal strength therefore requires two steps. First, one has to clarify which $\sigma_T(t)$ is optimal for individual companies $t \in T$ if aggregate signal

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[14] The idea to internalise such negative external effects is probably the reason why some people propose to impose a price on sending electronic mails. See Kraut, Sunder, Morris, Telang, Filer and Cronin [2002] for laboratory experiments on such proposals.

imission is *X.* Second, one has to look for an equilibrium which satisfies $X = \int_T r_t \sigma_T(t)\,dt$.

Given (4.1) and (4.2), for each $t \in T$, the choice of optimal strength σ is a standard optimisation problem $\max_{\sigma}[V_t - c_t]$. Since the innovation of this paper lies elsewhere, I want to keep this problem simple enough to apply differential calculus for deriving comparative-static results about the choice of signal strength. So far only boundedness of impact function $z_t(\sigma,\upsilon)$ was required. Moreover, $z_t(0,\upsilon)=0$, z_t increasing in σ and, for $\sigma > 0$, increasing in υ. From now on it is assumed that z_t is twice continuously differentiable and satisfies the following concavity property:[15]

$$\frac{\partial^2 z_t}{\partial \sigma^2} < 0,\ \frac{\partial^2 z_t}{\partial \upsilon\, \partial \sigma} \geq 0 \tag{4.3}$$

Moreover, $\lim_{\sigma \to 0} \frac{\partial z_t}{\partial \sigma} = \infty$ and $\lim_{\sigma \to \infty} \frac{\partial z_t}{\partial \sigma} = 0$.

With this specification the outcome of the optimisation of signal strength is a monotonous function of audience size and attention level.

<u>Proposition 3.</u> For T, σ_T let $\mathfrak{a}_T$ be a feasible audience allocation with $v(s,\tau_s) = \upsilon$ almost everywhere. If cost and impact functions satisfy (4.1) and (4.3), respectively, then:

(a) A company's optimal signal strength is given by a differentiable function $\sigma_t^*(\mu(\mathfrak{a}_T(t)),\upsilon)$ with $\partial\sigma_t^*/\partial\mu > 0$ and $\partial\sigma_t^*/\partial\upsilon \geq 0$.

(b) For all $t \in T$, the maximal net-value of impact is given by a differentiable increasing function of $\mu(\mathfrak{a}_T(t))$ and υ.

<u>Proof.</u> Appendix

The next task is to characterize equilibrium signal strength. For this purpose two further purely technical restrictions have to be imposed on the functions representing attention capacities, signal impacts and costs. First, it is assumed that the space of receivers $(S,\mathcal{A},\mu)$ is given by a real interval S with $\mathcal{A}$ the σ-field of *Borel* sets in S and μ the *Lebesgue* measure. Moreover, it is assumed that for all *s* the function representing attention capacities, $v(s,\tau)$, is differentiable with respect to τ. Secondly, it is assumed that impact and cost functions vary across *t* in a measurable way. Formally, for all σ,υ, the derivatives $\partial z_t(\sigma,\upsilon)/\partial\sigma$ and $c_t'(\sigma)$ are measurable functions of *t*. (Note that this is trivially fulfilled if z_t, c_t are identical for all *t*.) These properties guarantee the following facts which are useful for the derivation of equilibrium signal strength.

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15 These are sufficient conditions guaranteeing generally the existence of a positive optimal signal strength with comparative-static properties as required for an equilibrium. They need not hold in concrete examples, in which the solution can be calculated explicitly.

Lemma 2. Function $\overline{\upsilon}(\bullet)$ in proposition 2 is differentiable $(d\overline{\upsilon}/dX<0)$. Function $\sigma_T(t):=\sigma_t^*(\mu(\mathfrak{a}_T(t)),\upsilon)$ of optimal signal strength, derived in proposition 3, is measurable.
Proof. Appendix

With this preparation, equilibrium signal strength can be determined as follows: In an equilibrium audience allocation $\mu(\mathfrak{a}_T(t))=r_t$. Thus, according to proposition 3, optimal signal strength of $t\in T$ is given by $\sigma_t^*(r_t,\upsilon)$. Since r_t is an exogenous characteristic of *t*, argument r_t can be omitted without loss of information. Thus, in the further exposition I write $\sigma_t^*(\upsilon)$ instead of $\sigma_t^*(r_t,\upsilon)$. Equilibrium signal strength σ_T must satisfy two conditions: It must be consistent with the optimal choice of individual companies, i.e.:

$$\sigma_T(t)=\sigma_t^*(\upsilon) \tag{4.4}$$

for all $t\in T$. And σ_T must be consistent with attention level υ. If the assumptions of proposition 2 are fulfilled, this attention level is given by a decreasing function $\overline{\upsilon}(X)$, i.e.:

$$\upsilon=\overline{\upsilon}(X) \text{ with } X=\int_T r_t\sigma_T(t)\,dt. \tag{4.5}$$

(Lemma 2 guarantees that σ_t^* is measurable so that the integral defining *X* exists.) Combining the two conditions (4.4) and (4.5), we obtain the equation

$$X=Z(X,T,r_T) \tag{4.6}$$

with $Z(X,T,r_T):=\int_T r_t\sigma_t^*(\overline{\upsilon}(X))\,dt\geq 0$ and r_T denoting the function on *T* assigning to each $t\in T$ range r_t. According to lemma 2, $d\overline{\upsilon}/dX<0$. Since σ_t^* is differentiable and non-decreasing in υ, $\partial Z/\partial X\leq 0$ so that equation (4.6) defines for each T,r_T a unique equilibrium level $X^*[T,r_T]$ of aggregate signal imission. Together with (4.4) and (4.5), this defines also a unique attention level $\upsilon_T^*:=\overline{\upsilon}(X^*[T,r_T])$ and a unique equilibrium signal strength $\sigma_T^*(t):=\sigma_t^*(\upsilon_T^*)$. The following theorem summarizes this important result.

Theorem 1. Under (4.1), (4.3), the assumptions for lemma 2 and DA with τ_s^+ sufficiently small for all *s*, in any equilibrium $\mathfrak{a}_T,\sigma_T$ for $T\in\mathcal{B}$, in which attention levels are equalized, equilibrium signal strength σ_T^* and equilibrium attention level υ_T^* are uniquely determined by *T* and r_T.
Proof. Main text

Assumption DA with "τ_s^+ sufficiently small" means that receivers are strained by the prevailing signal exposure, so that their attention capacity is decreasing in the relevant range. For $\tau_s^+ = 0$ this is always guaranteed. Uniqueness of σ_T^* and υ_T^* allows us to derive comparative-static properties of equilibria in an attention economy. Regarding the attention level induced by a given set of active senders we have the following result.

Theorem 2. Under the assumptions of theorem 1: If $\rho(t) < \mu(S)$ and $\rho(t)$ increases to $\tilde{\rho}(t) > \rho(t)$ on a measurable subset $T_0 \subset T$, $0 < \lambda(T_0)$, then equilibrium attention υ_T^* declines.
Proof. Appendix

An increase in the range of radiation, allowing a wider diffusion of signals, induces companies to increase their signal strength in competing for attention. The reason is that diffusion of signals has zero marginal cost so that the production of impact by addressing signals on audiences is subject to economies of scale. At the same time, there is a second effect leading to higher signal exposure of subjects even without such economies of scale at the company level. Any given signal strength reaches more subjects when the radiation range of a company is extended. As long as the set of companies doesn't change, this necessarily means more overlap among audiences. For instance, if a national scientific network extends its range to the international level it has to address subjects belonging to other national or international communities. As a consequence, these subjects will be exposed to more publications. If signal exposure was high before, relative to the threshold τ_s^+ defined by DA, then the increased volume of signals will be perceived with less attentiveness. Of course, the described phenomenon isn't specific to scientific communities. Info-stress and attention deficit are not uncommon. According to theorem 2, a responsible economic fundamental is the possibility to address larger audiences.

The extension of the range of one company may wipe out other companies so that *T*, which up to now has been taken as given, changes. This brings us to the question of how equilibrium diversity is determined in an attention economy. This question is answered in section V. Before turning to this section, I want to illustrate theorem 1 and theorem 2 by the following example.

Example 2. Suppose that *T* can be partitioned in *K* measurable sets $T_1, \cdots, T_K$ with $\rho(t) = \rho_k$ for $t \in T_k$, where the divisibility condition of example 1 is satisfied. Like in example 1 let $v(s, \tau_s)$ be given by a non-increasing function $f(\tau_s)$. Moreover, for $t \in T_k$, $c_t(\sigma) = c_1^k \sigma + c_0^k$ for some constants $0 < c_1^k < r_k$ and, for $\sigma > 1$, $z_t(\sigma, \upsilon)$ $= g_k(\upsilon) \ln \sigma + h_k(\upsilon)$, where $g_k(\upsilon) \geq 1, h_k(\upsilon) \geq 0$ for $\upsilon > 0$, and $g_k' \geq 0$, $h_k' \geq 0$ with one inequality holding strictly. (For $\sigma \leq 1$, $z_t(\sigma, \upsilon) = 0$ is assumed.)

Fact 2. For any *T* satisfying the properties of example 2, (i) an equilibrium $\mathfrak{a}_T, \sigma_T$ with equalized attention levels exists, and (ii) in any such equilibrium $\sigma_k^* = r_k g_k(\upsilon^*) / c_1^k$, $\upsilon^* = f(\tau^*)$ where signal exposure of subjects is given by a function

$\tau^*\left(\lambda(T_1),\ldots,\ \lambda(T_k),\ \rho_1,\ldots,\rho_K\right)$ with $\partial\tau^*/\partial\lambda(T_k)>0$ and $\partial\tau^*/\partial\rho_k>0$ if $\rho_k<\mu(S),\ k=1,\ldots,K.$

Proof. Appendix

V. Viability and equilibrium diversity in an attention economy

The notion of a competitive environment requires: Agents must be viable, and free entry is allowed. In an attention economy, in which it is vital to attract attention, a natural notion of viability is that companies achieve a non-negative net-value $V_t^n\geq 0$ from sending costly signals for having impact on audiences. Free entry means that companies can participate in the contest for attention if they want.

For any single company the situation looks as follows: Given a set $T\in\mathcal{B}$ of active companies, a feasible audience allocation $\mathfrak{a}_T$ and some signal strength $\sigma_T>0$, the attention levels of subjects are $v(s,\tau_s)$ where τ_s depends on $\mathfrak{a}_\mathrm{T}$ and σ_T. For $t\in T$, the net-value of impact achieved under $T,\mathfrak{a}_\mathrm{T},\sigma_T$ is given by $\int\limits_{\mathfrak{a}_\mathrm{T}(t)} z_t\left(\sigma_T(t),\ v(s,\tau_s)\right)d\mu(s)-c_t\left(\sigma_T(t)\right)$. I denote this value by $V_t^n\left[\mathfrak{a}_\mathrm{T},\sigma_T\right]$.

Company $t\in T$ is viable if $V_t^n\left[\mathfrak{a}_\mathrm{T},\sigma_T\right]\geq 0$. Companies $t'\in L-T$ face the following entrance problem. If $t'\notin T$ participates by targeting $\sigma>0$ on an audience $A\in\mathcal{A}$, the set of active companies becomes $T'=T\cup\{t'\}$. Since $\lambda(\{t'\})=0$, this doesn't change attention levels of subjects. Thus, t' achieves maximal impact when picking an audience A^* with $\mu\left(A^*\right)=\min\{\mu(S),\ \rho(t')\}$ in such a way that $v(s,\tau_s)\geq v(s',\tau_{s'})$ for all $s\in A^*,\ s'\in S-A^*$, and maximal net-value of impact when choosing optimal signal strength $\sigma_{t'}^*=\arg\max\int\limits_{A^*} z_{t'}\left(\sigma,v(s,\tau_s)\right)d\mu(s)-c_{t'}(\sigma)$. Denote by $V_{t'}^*\left[\mathfrak{a}_T,\sigma_T\right]$ the maximal net-value of impact $\int\limits_{A^*} z_{t'}\left(\sigma_{t'}^*,v(s,\tau_s)\right)d\mu(s)-c_{t'}\left(\sigma_{t'}^*\right)$, which t' achieves when participating in the contest for attention in an optimal way. Company t' is viable if $V_{t'}^*\left[\mathfrak{a}_T,\sigma_T\right]\geq 0$. It definitely has an interest to enter if the achieved value is strictly positive. If $V_{t'}^*\left[\mathfrak{a}_T,\sigma_T\right]=0$, it is indifferent, with respect to entry.

Definition 3 (Free entry equilibrium). Let $(S,\mathcal{A},\mu)$, $(L,\mathcal{B},\lambda)$ be an attention economy with attention capacity v and radiation range ρ. $T,\mathfrak{a}_T,\sigma_T$ is a *free entry equilibrium* of the attention economy if $\mathfrak{a}_T,\sigma_T$ is an equilibrium for T (according to definition 2) and the following property is satisfied: For all $t\in T,\ V_t^n\left[\mathfrak{a}_T,\sigma_T\right]\geq 0,$ and $V_{t'}^*\left[\mathfrak{a}_T,\sigma_T\right]\leq 0$ for any $t'\in L-T$. ($V_t^n\left[\mathfrak{a}_T,\sigma_T\right]$ denotes the net-value of impact achieved by $t\in T$ under $\mathfrak{a}_T,\sigma_T$. $V_{t'}^*\left[\mathfrak{a}_T,\sigma_T\right]$ denotes the maximal net-value of impact which $t'\notin T$ could achieve when participating in the contest for attention.)

The further analysis is restricted to economies in which attention levels are equalized in equilibrium. In such an economy, equilibrium signal strength and equilibrium attention level are uniquely determined for a given set *T* of active companies (see theorem 1). They are given by

$$\sigma_T^*(t) = \sigma_t^*\left(\upsilon_T^*\right) \text{ with } \frac{\partial \sigma_t^*}{\partial \upsilon} \geq 0 \tag{5.1}$$

$$\upsilon_T^* = \overline{\upsilon}\left(X^*\right) \text{ with } d\overline{\upsilon} / dX^* < 0 \tag{5.2}$$

respectively, where X^* is implicitly defined by the condition (see (4.6)):

$$X^* = \int_T r_t \sigma_T^*(t)\, dt\,. \tag{5.3}$$

This implies that also the maximal net-value of impact achieved by a company $t \in T$ is unique in an equilibrium for *T*.

By definition, for any $t \in T$ facing an equalized attention level υ, the net-value which *t* achieves when addressing a maximally feasible audience with signal strength σ is given by[16] $V_t^n(\sigma,\upsilon) := r_t z_t(\sigma,\upsilon) - c_t(\sigma)$, where z_t, c_t satisfy (4.1), (4.3). The maximal net-value under υ is $V_t^n\left(\sigma_t^*(\upsilon),\upsilon\right)$. Thus, in an equilibrium with equalized attention levels υ_T^*, for every $t \in L$, the maximal net-value that can be achieved by *t* is

$$V_t^* \equiv V_t^n\left(\sigma_t^*\left(\upsilon_T^*\right),\upsilon_T^*\right) = r_t z_t\left(\sigma_t^*\left(\upsilon_T^*\right),\upsilon_T^*\right) - c_t\left(\sigma_t^*\left(\upsilon_T^*\right)\right). \tag{5.4}$$

(Note that no single company has an impact on υ and any company, $t \in T$ as well as $t' \in L - T$, is confronted with the same attention level υ).[17]

Theorem 1 has shown for a given set of active companies *T*, that in an equilibrium with equalized attention levels the equilibrium level of attention is unique. The following theorem shows that in any free-entry equilibrium with equalized attention levels the equilibrium level of attention is unique.

Theorem 3. For given fundamentals ($\rho(t), z_t, c_t,\ t \in L, \mu(S)$ and attention capacities v) there exists a unique $\upsilon^* \in \mathbb{R}_{++}$ so that in any free-entry equilibrium $T^*, \mathfrak{a}_{T^*}, \sigma_{T^*}^*$ full-filling the assumptions of theorem 1 (and thus (5.1) – (5.4)) the equilibrium level of attention, $\upsilon_{T^*}^*$, is equal to υ^*. Also aggregate signal imission X^* is uniquely determined.
Proof. Appendix

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16 So far function symbol V_t^n was used for $V_t^n(\sigma)$ or $V_t^n[\mathfrak{a}_T,\sigma_T]$ in a given context $\mathfrak{a}_T,\sigma_T$. Now, the relevant context is captured by a single variable υ. For saving notation I use the same symbol to denote net-value as a function of σ and υ.

17 The notational distinction between $V_t^n[\mathfrak{a}_T,\sigma_T]$, for $t \in T$, and $V_t^*[\mathfrak{a}_T,\sigma_T]$ for $t \in L - T$, is no longer necessary. Both values are given by $V_t^n\left(\sigma_t^*(\upsilon),\upsilon\right)$, where υ is the attention level implied by $\mathfrak{a}_T,\sigma_T$.

According to definition 3, for identifying a possible equilibrium set T^* of active companies we must check for which $t \in L$ the values given by (5.4) are positive, zero, or negative. Since companies may differ in capacity $\rho(t)$, impact function z_t or cost c_t, it is not possible to compare them without further restrictions. In the following an ordering on the set of potential companies L is assumed. I call *t more powerful* (or *stronger*) than t' under υ if $r_t\sigma_t^*(\upsilon) > r_{t'}\sigma_{t'}^*(\upsilon)$, i.e. if total signal imission coming from *t* is higher than from t' when both companies send with optimal strength. The following monotonicity property says that stronger companies are also more valuable.

Assumption M (Monotonicity). Heterogeneity of $\rho(t), z_t, c_t,\ t \in L$, is restricted in such a way that, for all $\upsilon \in \mathbb{R}_{++}$, $V_t^n(\sigma_t^*, \upsilon) >\ V_{t'}^n(\sigma_{t'}^*, \upsilon)$ if *t* is more powerful than t' under υ (i.e. if $r_t\sigma_t^*(\upsilon) > r_{t'}\sigma_{t'}^*(\upsilon)$).

Power is represented by a high $r_t\sigma_t^*$, that means, by sending widely and loudly. To assume that this is valuable in terms of impact seems to be natural in an attention economy. Being small and decent doesn't pay. Property M trivially holds if all potential companies have identical fundamentals so that no company is stronger than another. Also if they differ only in range $\rho(t)$ while having identical impact and cost functions, property M follows immediately from (5.4). But M is generally satisfied when the fundamentals of companies can be ranked in a clear way. I call *t non-inferior* to t' if $r_t \geq r_{t'}, \partial z_t / \partial\sigma \geq \partial z_{t'} / \partial\sigma,\ c_t' \leq c_{t'}'$.[18] If at least one inequality holds strictly, *t* is *superior* to t'. Thus, *t* is superior if it has a larger range or if it is better in the sense of having higher marginal impact or lower marginal cost of signal production. The following lemma shows how this ranking of companies in terms of fundamentals implies a ranking in terms of power and value.

Lemma 3. Suppose that *L* can be ordered so that *t* is non-inferior to t' if $t \leq t'$. Then,

(a) *t* is more powerful than t' if and only if *t* is superior to t'.

(b) If for all υ, t and t', $\partial z_t / \partial\sigma \geq \partial z_{t'} / \partial\sigma$ implies $z_t \geq z_{t'}$ and $c_t' \leq c_{t'}'$ implies $c_t \leq c_{t'}$, then property M is fulfilled.

Proof. Appendix

Part (a) shows that companies can be ranked according to their imission power if they can be ranked according to radiation capacity, marginal impact and marginal cost of signal provision. Part (b) shows that this ranking implies a ranking in terms of the net-values of impact which can be achieved by the respective companies. For instance, if companies have different radiation capacities $\rho(t)$ but have access to the some technology for signal and impact production so that c_t and z_t are identical, then companies with larger range send more powerfully and achieve a higher net-value of impact. In an analogous way, a ranking according to power and net-value is possible if

[18] Since z_t, c_t are functions of υ and σ, the respective inequalities can hold locally or globally. In the following analysis only the local properties at $\sigma_t^*(\upsilon)$ are relevant. However, we make comparative-static analysis with respect to υ.

companies have access to the same radiation technology so that $\rho(t)=\rho(t')$ for all $t,t'\in L$, but some of them have a cost advantage in signal production or an impact advantage in the sense that they achieve higher impact with the same signal strength. According to (b), a sufficient condition for M is that an advantage at the margin means also an advantage overall.

Theorem 4. Let $T^*, \mathfrak{a}_{T^*}, \sigma^*_{T^*}$ be a free-entry equilibrium fulfilling the assumptions of theorem 1 (and thus (5.1) – (5.4)). Under M,

(a) $\lambda(T^*)$ is uniquely determined by the fundamentals of the attention economy.

(b) If L can be ordered so that t is non-inferior to t' if $t \le t'$, then: t superior to t' and $t' \in T^*$ imply $t \in T^*$ and $V^*_t > V^*_{t'}$. (V^*_t evaluated at the (unique) equilibrium attention level υ^*.)

Proof. Appendix

Part (a) of the theorem is essential for comparative-static analysis about the determinants of equilibrium diversity. Even though in general there is no unique equilibrium set of active companies T^* and no unique equilibrium audience allocation, equilibrium attention υ^* and equilibrium measure $\lambda(T^*)$ are uniquely determined. Part (b) of the theorem shows that if there is ex ante heterogeneity of potential senders, in an equilibrium those companies dominate which are large or have a better technology for impact production. They send more powerfully (see lemma 3) and survive the contest of attention. An immediate consequence of (b) is that $t' \in L - T^*$ cannot be superior to $t \in T^*$.

In the following comparative-static analysis $\mu(S)$ and attention capacities are kept unchanged while the companies' fundamentals change from $b_t \equiv (r_t, z_t, c_t)$ to $\tilde{b}_t = (\tilde{r}_t, \tilde{z}_t, \tilde{c}_t)$, $t \in L$. I say that $\tilde{b}$ is a *progress* over b on $E \in \mathcal{B}$ if $\tilde{b}$ and b coincide on $L - E$ and, for every $t \in E$, $\tilde{b}_t$ is advantageous compared to b_t in the following sense: For $t \in E$, $\tilde{r}_t \ge r_t$, $\tilde{z}_t \ge z_t$, $\partial\tilde{z}_t/\partial\sigma \ge \partial z_t/\partial\sigma$, $\tilde{c}_t \le c_t$, $\tilde{c}'_t \le c'_t$ and at least one of the following properties hold: $\tilde{r}_t > r_t$, or $\tilde{z}_t > z_t$ and $\partial\tilde{z}_t/\partial\sigma > \partial z_t/\partial\sigma$, or $\tilde{c}_t < c_t$ and $\tilde{c}'_t < c'_t$. In words, companies in E experience progress in radiation capacity or in impact or signal production.

Theorem 5. Consider two (otherwise identical) attention economies with sender fundamentals b and $\tilde{b}$, respectively. Assume that under b as well as under $\tilde{b}$ assumption M is satisfied. Let $T^*, \mathfrak{a}_{T^*}, \sigma^*_{T^*}$ and $\tilde{T}, \mathfrak{a}_{\tilde{T}}, \tilde{\sigma}_{\tilde{T}}$ be free-entry equilibria for the respective economies satisfying the assumptions of theorem 1. Let $T_0 := \{t \in T^* \mid V^*_t = 0\}$ be the (possibly empty) set of marginal companies under b. If $\tilde{b}$ is a progress over b on some $E \subset T^*$, $\lambda(E) > 0$, then

(a) $\tilde{\upsilon} \le \upsilon^*$,

(b) $\lambda(\tilde{T}) < \lambda(T^*)$, if $\tilde{\upsilon} = \upsilon^*$ or if $\lambda(T_0 - E) > 0$.

($\tilde{\upsilon}, \upsilon^*$ denote the respective equilibrium attention levels. Their uniqueness is guaranteed by theorem 3.)
Proof. Appendix

Equilibrium attention certainly does not increase (part (a)) and could only remain unaffected if senders vanish (first if-clause in part (b)). Regardless of whether or not the attention level declines, the measure of active senders is reduced if not all marginal senders are subject to the considered progress (second if-clause in part (b)).[19] The economic mechanism behind this result is as follows: If companies get access to a radiation technology allowing a wider range of receivers, to a more powerful impact technology, or to less costly signal production, they produce and distribute stronger signals. Since the increased signal exposure leads to a decline in the receivers' attention level, other senders with weaker signals are no longer sufficiently perceived to achieve viable impact. From the point of view of receivers, this means that the diversity of senders changes. However, the impacts on global and local diversity must be carefully distinguished.

Global diversity addressed in theorem 4 and 5 is important from the perspective of an outside observer of the world. How many different senders – i.e. producers and emittors of signals pointing to ideas, products, issues – survive in a given attention economy? Why may small networks, local journals or national scientific communities vanish? By contrast to this global perspective, for every single receiver the experienced variety of senders is given by the measure $\lambda\left(M\left(s, \mathfrak{a}_{T^*}\right)\right)$ of the senders of which s is a member. I called it local diversity. By definition, for all $s \in S$, $M\left(s, \mathfrak{a}_{T^*}\right) \subset T^*$ so that local diversity is limited by global diversity. Nonetheless, $M\left(s, \mathfrak{a}_{T^*}\right)$ may increase for some or even all s even if $\lambda(T^*)$ is reduced. To see this, note that each company reaches an audience of subjects whose measure is equal to the radiation range of the company. If this range increases, more subjects are in the range of the surviving companies. More formally, let $\bar{m}(T^*) \equiv (1/\mu(S)) \int_S \lambda\left(M\left(s, \mathfrak{a}_{T^*}\right) d\mu(s)\right)$ denote average local diversity and $\bar{r}(T^*) \equiv \left(1/\lambda(T^*)\right) \int_{T^*} r_t dt$ denote average range. Since the measure of aggregate memberships is equal to the aggregate measure of audiences,[20] local diversity experienced on average is given by the equation $\bar{m}(T^*) = \bar{r}(T^*)\lambda(T^*)/\mu(S)$. Obviously, the effect of $\rho(t)$-changes on $\bar{m}(T^*)$ is ambiguous even if the effect on

19 Theorem 5 provides sufficient (not necessary) conditions for $\tilde{\upsilon} \le \upsilon^*$ and $\lambda(\tilde{T}) < \lambda(T^*)$. As example 3 will show, also a uniform increase in radiation possibilities for all potential senders or international integration may lead to declining attention and diversity. In general, however, such changes have ambiguous effects.

20 Set $\sigma_T = 1$ $\mu(\mathfrak{a}_T(t)) = r_t$ in lemma A1, to get $\int_S \lambda(M(s, \mathfrak{a}_T)) d\mu(s) = \int_T r_t dt$.

$\lambda(T^*)$ is negative. The following example illustrates that an increase of the radiation capacity of senders may lead to a decline in global diversity, while diversity experienced from the local perspective of every single receiver rises. Moreover, the example shows that ex ante-heterogeneity of potential senders is not a necessary requirement for the discussed comparative-static results.

Example 3. Suppose that potential senders are identical with $\rho(t) = \rho > 1$, $c_t(\sigma) = \sigma + c_0$ and $z_t(\sigma, \upsilon) = g(\upsilon)(\ln \sigma + \gamma)$ if $\sigma > 1$, where $c_0 > 0$, $0 \le \gamma \le 1$ and $g(\upsilon) = g_0 \upsilon^{\alpha}$, $g_0 \in \mathbb{R}_{++}$, $\alpha > 0$. (Note that these are instances of the cost- and impact functions discussed in example 2.) Moreover, for all $s \in S$, $v(s, \tau_s) = \tau_s^{-\beta}$, $\beta > 0$.

We know already that for such an economy an equilibrium with equalized attention levels exists for a given set $T \subset L$ of active companies. (It is assumed that L is sufficiently large so that not all potential senders are active in an equilibrium.) Moreover, since all potential senders are identical, M is satisfied and thus, according to theorem 4, the measure $\lambda(T^*)$ of active companies is unique in a free entry equilibrium. Both global and local diversity are characterized by the following fact.

Fact 3. In an attention economy as described in example 3,

(a) Global diversity is given by $\lambda^* = \lambda_0 \mu(S) r^{(1-\alpha\beta)/(\alpha\beta)}$, where $r = \min\{\mu(S), \rho\}$ and λ_0 is a positive constant which is positively related to γ, g_0 and negatively related to c_0.

(b) Local diversity is given by $m^* = \lambda_0\, r^{1/(\alpha\beta)}$.

Proof. Appendix

For the evaluation of comparative-static effects on equilibrium diversity two cases must be distinguished. If $\mu(S) \le \rho$, λ^* reduces to $\lambda_0\, \mu(S)^{1/(\alpha\beta)}$ and $m^* = \lambda^*$. If $\mu(S) > \rho$, then $r = \rho$ and λ^* decreases (increases) with ρ if $\alpha\beta > 1$ ($\alpha\beta \le 1$, respectively).

From an economic point of view the comparative-static analysis leads to important insights concerning effects of "globalisation" on diversity. Globalisation in an attention economy means that senders get access to a larger space of receivers. This can have two reasons: International integration or progress in the radiation technology.

International integration is relevant if the radiation technology is sufficiently advanced so that audiences larger than the population of an isolated economy can be addressed. Integration allows to senders to exploit these possibilities. To see the effects on diversity, consider two identical economies with receiver set S^i, $\mu(S^i) = \mu$, $i = H, F$. Senders have access to a radiation technology which allows them to cover more than S^i, but not the whole world $S^W = S^H \cup S^F$, $\mu(S^W) = 2\mu$. This means, they have identical radiation capacities with $\mu < \rho < 2\mu$. Thus, $r_a = \min\{\mu, \rho\} = \mu$ in the "closed economy" and $r_W = \min\{2\mu, \rho\} = \rho$ in the integrated world. Using this in fact 3, we obtain for equilibrium diversity in the closed economy: $\lambda_a^* = m_a^* = \lambda_0 \mu^{1/(\alpha\beta)}$. This gives for total diversity in the world: $\lambda_{tot}^* = 2\lambda_a^*$. In the integrated world with cross-border radiation, equilibrium diversity is given by $\lambda_W^* = \lambda_0\, 2\mu\, \rho^{(1-\alpha\beta)/(\alpha\beta)}$ and $m_W^* = \lambda_0\, \rho^{1/(\alpha\beta)}$. Comparing autarky diversity with diversity in the integrated world, we get: $m_W^* > m_a^*$ since $\rho > \mu$. Thus, local diversity experienced by an average receiver is larger under international integration than in a closed economy. However, with respect to global diversity the effect of integration is ambiguous. We have $\lambda_W^* > \lambda_{tot}^*$ if $1 > (\mu/\rho)^{1-1/(\alpha\beta)}$. Since $\mu < \rho$ was assumed, this implies $\lambda_W^* > \lambda_{tot}^*$ if $\alpha\beta > 1$. Although each subject is exposed to more different senders, fewer senders are active after integration. The reason is that senders have a radiation technology under which international radiation is feasible, and international integration allows full_use of the radiation range. As a consequence, more subjects are exposed to the same senders and some senders are driven out of the market.

The second source of globalisation in an attention economy is technical progress leading to an increase in the feasible range of radiation. To study the effect on diversity, suppose there is an integrated world with $\mu\left(S^W\right) = \mu_W$ and identical senders whose range increases from ρ_{I} to $\rho_{\mathrm{II}} > \rho_{\mathrm{I}}$ where $1 < \rho_{\mathrm{I}} < \mu_W$ is assumed. Then, according to fact 3, diversity under range ρ_{I} is given by $\lambda_I^* = \lambda_0\, \mu_W\, \rho_I^{(1-\alpha\beta)/(\alpha\beta)}$, $m_I^* = \lambda_0\, \rho_I^{1/(\alpha\beta)}$ whereas under range ρ_{II} diversity is $\lambda_{II}^* = \lambda_0\, \mu_W\, r_{II}^{(1-\alpha\beta)/(\alpha\beta)}$, $m_{II}^* = \lambda_0\, r_{II}^{1/(\alpha\beta)}$ where $r_{II} = \min\{\mu_W, \rho_{II}\} > \rho_I$. Thus, again $m_{II}^* > m_I^*$, and $\lambda_{II}^* < \lambda_I^* \left(\lambda_{II}^* \geq \lambda_I^*\right)$ if $\alpha\beta > 1$ ($\alpha\beta \leq 1$, respectively).

VI. Conclusion

The presented theory explains the basic mechanisms at work in an economy in which earning attention and achieving impact are prerequisites of economic viability.

The exogenous fundamentals are on the one side the space of receivers and their attention capacities, and on the other side the potential set of senders and their radiation and impact technologies.

The endogenous variables explained by the theory are equilibrium audiences (the clients belonging to a company), equilibrium signal exposure and attention, and the measure of active senders in a free-entry equilibrium. Although there are multiple equilibrium allocations of audiences to senders, the signal strength, the attention level and the equilibrium measure of active senders can be uniquely characterized.

Under the assumption that receivers are subject to disordered attention if exposed to high signal strength, the theory predicts that changes allowing to senders more powerful signal emission – for instance, an extension of feasible radiation ranges, cheaper signal production, more effective methods of impact generation, but also international integration – tend to decrease global diversity of senders and attention levels of subjects. Local diversity, measured by the variety of senders experienced by the individual receivers, may increase nonetheless.

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Appendix

We prove first two lemmas which are important in several proofs.

Lemma A1. For T, σ_T, $\mathfrak{a}_T \subset T \times S$, let $b(t,s) := \begin{cases} \sigma_T(t), & \text{if } (t,s) \in \mathfrak{a}_T \\ 0 & \text{, otherwise} \end{cases}$

be the signal imission on $s \in S$ from $t \in T$ under $\mathfrak{a}_T$. For any $\mathfrak{a}_T \in \mathrm{B} \times \mathrm{A}$,

(a) $b(t,s)$ is measurable $\mathcal{B} \times \mathcal{A}$.

(b) $\int\limits_{M(s,\mathfrak{a}_T)} \sigma_T(t)\,dt$ is a measurable function on *S*.

(c) $\int\limits_{S} \int\limits_{M(s,\mathfrak{a}_T)} \sigma_T(t)\,dt\,d\mu(s) = \int\limits_{T} \mu(\mathfrak{a}_T(t))\sigma_T(t)\,dt$.

Proof.

(a) Define $\tilde{\sigma}_T(t,s) = \sigma_T(t)$. For any $y \in \mathbb{R}_+$, the set $\{(t,s) | \tilde{\sigma}_T(t,s) > y\}$ = $\{t | \sigma_T(t) > y\} \times S$ is a measurable rectangle, since σ_T is measurable $\mathcal{B}$. Thus, $\tilde{\sigma}_T(t,s)$ is measurable $\mathcal{B} \times \mathcal{A}$. Since $b(t,s) = I_{\mathfrak{a}_T}\,\tilde{\sigma}_T$, where $I_{\mathfrak{a}_T}$ is the indicator function on $\mathfrak{a}_T$, also $b(t,s)$ is measurable $\mathcal{B} \times \mathcal{A}$.

Properties (b) and (c) follow from *Fubini's* theorem (see e.g. *Billingsley* [1995], p. 234). Note that $b(t,s)$ is non-negative. Moreover, $\int\limits_{T} b(t,s)\,dt = \int\limits_{M(s,\mathfrak{a}_T)} \sigma_T(t)\,dt$ and $\int\limits_{S} b(t,s)\,d\mu(s) = \int\limits_{\mathfrak{a}_T(t)} \sigma_T(t)\,d\mu(s) = \mu(\mathfrak{a}_T(t))\sigma_T(t)$, since $b(t,s) = 0$ for $t \notin M(s,\mathfrak{a}_T)$ or $s \notin \mathfrak{a}_T(t)$. QED.

Lemma A2. Suppose that $\mathfrak{a}_T$ is an equilibrium audience allocation for $T, \sigma_T > 0$. Under assumption AN, if there exist sets $A, A' \in \mathcal{A}$ of positive measure so that $v(s,\tau_s) < v(s',\tau_{s'})$ for all $s \in A$, $s' \in A'$, then: $\mu(A' - \mathfrak{a}_T(t)) = 0$ or $\mu(\mathfrak{a}_T(t) \cap A) = 0$, for all $t \in T$.

Proof.

Suppose that $\mu(A' - \mathfrak{a}_T(t)) > 0$ and $\mu(\mathfrak{a}_T(t) \cap A) > 0$ for some $t \in T$. The divisibility assumption imposed on $(S, \mathcal{A}, \mu)$ implies that $A' - \mathfrak{a}_T(t)$ and $\mathfrak{a}_T(t) \cap A$ contain measurable subsets B' and B, respectively, with $\mu(B) = \mu(B') > 0$. Let $\mathfrak{a}'_T$ be the audience allocation resulting from $\mathfrak{a}_T$ when $\mathfrak{a}_T(t)$ is replaced by $\mathfrak{a}'_T(t) := (\mathfrak{a}_T(t) - B) \cup B'$. Since $\mu(\mathfrak{a}'_T(t)) = \mu(\mathfrak{a}_T(t)) \le \rho(t)$, $\mathfrak{a}'_T$ is a feasible deviation for t. We show that it is also attractive for t to deviate from $\mathfrak{a}_T$ to $\mathfrak{a}'_T$. If t

retargets its signals from $\mathfrak{a}_T(t)$ to $\mathfrak{a}'_T(t)$, then for $s \in B$ membership changes to $M(s,\mathfrak{a}'_T) = M(s,\mathfrak{a}_T) - \{t\}$, whereas for $s \in B'$ membership changes to $M(s,\mathfrak{a}'_T) = M(s,\mathfrak{a}_T) \cup \{t\}$. Since singletons have zero measure, $\int_{M(s,\mathfrak{a}_T)} \sigma_T(t)\,dt = \int_{M(s,\mathfrak{a}'_T)} \sigma_T(t)\,dt$ for all $s \in S$. Thus, according to assumption AN, $\varphi(s|\mathfrak{a}_T,\sigma_T) = \varphi(s|\mathfrak{a}'_T,\sigma_T)$ for all $s \in S$. (For the sake of clarity, I use here the representation of attention capacities which explicitly refers to the audience allocation. See lemma 1 in text.) Since $B \subset A$ and $B' \subset A'$, we have $\varphi(s|\mathfrak{a}_T,\sigma_T) < \varphi(s'|\mathfrak{a}'_T,\sigma_T)$ for $s \in B$ and $s' \in B'$.

Moreover, since for $\sigma_T(t) > 0$ impact function z_t is increasing in attention level v we obtain $V_t(\mathfrak{a}'_T,\sigma_T) = \int_{\mathfrak{a}_T(t)} z_t(\sigma_T(t),\ \varphi(s|\mathfrak{a}'_T,\sigma_T))\,d\mu(s) - \int_B z_t(\sigma_T(t),\ \varphi(s|\mathfrak{a}_T,\sigma_T))\,d\mu(s) + \int_{B'} z_t(\sigma_T(t),\ \varphi(s|\mathfrak{a}'_T,\sigma_T))\,d\mu(s) > V_t(\mathfrak{a}_T,\sigma_T)$.

This is a contradiction to the assumption that $\mathfrak{a}_T$ is an equilibrium audience allocation. Q E D.

The rest of the appendix contains the proofs of the claims in the text.

<u>Proof of lemma 1.</u>

According to lemma A1 (b), for any $\mathfrak{a}_T,\sigma_T$, the integral $\int_{M(s,\mathfrak{a}_T)} \sigma_T\,d\lambda$ is a measurable function of s. Thus, for v measurable and bounded also the composition $v(s,\tau_s)$ is measurable and bounded. AN holds by construction. This proves that v represents attention capacities φ satisfying AN. For the only if part define for φ measurable and bounded $v(s,\tau) := \varphi(s|\mathfrak{a}_T,\sigma_T)$. Then v is measurable and bounded and $\varphi(s|\mathfrak{a}_T,\sigma_T) = v(s,\tau_s)$ by definition. QED.

<u>Proof of proposition 1.</u>

(a) Suppose there are $A,\ A' \in \mathcal{A}$ with positive measure so that for all $s \in A,\ s' \in A'$, $v(s,\tau_s) < v(s',\tau_{s'})$, $M(s',\mathfrak{a}_T) = \varnothing$ and $s \in \mathfrak{a}_T(t)$ for some *t*. According to lemma A2, $\mu(A' - \mathfrak{a}_T(t)) = 0$ or $\mu(A \cap \mathfrak{a}_T(t)) = 0$. The first clause implies that almost all $s' \in A'$ belong to $\mathfrak{a}_T(t)$ which contradicts $M(s',\mathfrak{a}_T) = \varnothing$. The second clause contracts the assumption that all $s \in A$ belong to $\mathfrak{a}_T(t)$.

(b) Suppose there are $A,\ A' \in \mathcal{A}$ with positive measure so that $v(s',\tau_{s'}) < v(s,\tau_s)$, $M(s',\mathfrak{a}_T) = T$ and $M(s,\mathfrak{a}_T) \neq T$ for all $s \in A$, $s' \in A'$. According to lemma A1, $\mu(A - \mathfrak{a}_T(t)) = 0$ or $\mu(A' \cap \mathfrak{a}_T(t)) = 0$ for all $t \in T$. However, the first clause cannot hold for $t \in T - M(s, \mathfrak{a}_T)$, and the second clause contradicts $M(s', \mathfrak{a}_T) = T$ for $s' \in A'$.

(c) Follows immediately from lemma A1.

(d) Suppose that $\mu(\mathfrak{a}_T(t)) < r_t$. The divisibility property imposed on $(S, \mathcal{A}, \mu)$ implies that a measurable set $A \subset S - \mathfrak{a}_T(t)$ exists with $\mu(A) = r_t - \mu(\mathfrak{a}_T(t))$. Let $\mathfrak{a}'_T$ be the audience allocation resulting from $\mathfrak{a}_T$ when $\mathfrak{a}_T(t)$ is replaced by $\mathfrak{a}'_T(t) := \mathfrak{a}_T(t) \cup A$. Since $\mu(\mathfrak{a}'_T(t)) = r_t$, $\mathfrak{a}'_T$ is a feasible deviation for *t*. Moreover, by targeting the unused radiation capacity $\rho(t) - \mu(\mathfrak{a}_T(t))$ on *A*, company *t* attracts additional attention implying $V_t(\mathfrak{a}'_T, \sigma_T) > V_t(\mathfrak{a}_T, \sigma_T)$. (Note that $\mu(A) > 0$ and $z_t(\sigma_T(t),\ \upsilon) > 0$ for $\sigma_T(t) > 0$.) This is a contradiction to the assumption that $\mathfrak{a}_T$ is an equilibrium allocation.

Sufficiency: Because of d), no company can increase the measure of its audience. Thus, according to AN, $V_t(\mathfrak{a}_T, \sigma_T)$ can only be increased if *t* retargets its signals $\sigma_T(t)$ from a measurable subset $A \subset \mathfrak{a}_T(t)$ to a measurable subset $A' \subset S - \mathfrak{a}_T(t)$, $\mu(A') \le \mu(A)$, so that $\int_A z_t(\sigma_T(t),\ \nu(s, \tau_s))\, d\mu(s) < \int_{A'} z_t(\sigma_T(t),\ \nu(s, \tau_s))\, d\mu(s)$. This is only possible if there exist measurable $B \subset A$, $B' \subset A'$ with $\mu(B) = \mu(B') > 0$ so that $\nu(s, \tau_s) < \nu(s', \tau_{s'})$ for almost all $s \in B$, $s' \in B'$. In sum, there must be B, B' with $\mu(B \cap \mathfrak{a}_T(t)) = \mu(B) > 0$, $\mu(B' \cap \mathfrak{a}_T(t)) = 0$ and higher attention levels in B' than *B*. This is in contradiction to condition (c). Thus no such deviation exists. QED.

<u>Proof of corollary 2.</u>

Take $S_\upsilon^-, S_\upsilon^+$ as defined in proposition 1.

Step 1: Since $\mu(\mathfrak{a}_T(t)) = \min\{\rho(t), \mu(S)\}$, $\rho(t) \ge \mu(S_\upsilon^+)$ implies $\mu(\mathfrak{a}_T(t) \cap S_\upsilon^-) \ne 0$ or $\mu(\mathfrak{a}_T(t)) = \mu(S_\upsilon^+)$. This implies $\mu(S_\upsilon^+ - \mathfrak{a}_T(t)) = 0$, according to proposition 1 (c). An analogous argument leads to $\mu(\mathfrak{a}_T(t) \cap S_\upsilon^-) = 0$ if $\rho(t) < \mu(S_\upsilon^+)$.

Step 2: By definition, $\mu(S_\upsilon^+)$ is a non-increasing function of υ starting at $\mu(S_0^+) = \mu(S)$ and eventually reaching zero since ν is bounded. $\mu(S_\upsilon^+) = \mu(S)$ implies that almost all subjects have at least attention level υ, whereas $\mu(S_\upsilon^+) = 0$ means that almost all subjects have lower attention level than υ. Hence, $\nu(s, \tau_s) = \upsilon^+$ for almost all s if and only if $\mu(S_\upsilon^+) = \mu(S)$ for all $\upsilon \le \upsilon^+$ and $\mu(S_\upsilon^+) = 0$ for all $\upsilon > \upsilon^+$. (The if part is obvious. For the only if part suppose that $0 < \mu(S_\upsilon^+) < \mu(S)$ for some $\upsilon > 0$. Then also $\mu(S_\upsilon^-) > 0$ and $\nu(s, \tau_s) \ge \upsilon > \nu(s', \tau_{s'})$ for $s \in S_\upsilon^+, s' \in S_\upsilon^-$.)

Step 3: Suppose that property (a) in corollary 2 does not hold. Then, according to step 2, $0 < \mu\left(S_{\upsilon}^{+}\right) < \mu(S)$ for some υ. Suppose that heterogeneity of radiation capacities is restricted so that either $\rho(t) \geq \mu\left(S_{\upsilon}^{+}\right)$ for all t or $\rho(t) < \mu\left(S_{\upsilon}^{+}\right)$ for all t. In the first case, step 1 implies $\mu\left(S_{\upsilon}^{+} - \mathfrak{a}_T(t)\right) = 0$ for all t and thus part (b) of the corollary holds for $\tilde{F} = S_{\upsilon}^{+}$. In the second case, step 1 implies (c) with $\tilde{U} = S_{\upsilon}^{-}$. QED.

Proof of fact 1.

For $k = 1, \ldots, K$, define $n_k := \max\{1, \mu(S)/\rho_k\}$ and $m_k := \lambda(T_k) r_k / \mu(S)$. By assumption, n_k is a natural number. Decompose S into n_k subsets $S_k^i, i = 1, \ldots, n_k$, of equal size $\mu\left(S_k^i\right) = r_k$ so that $\bigcup_i S_k^i = S$ and $S_k^i \cap S_k^j = \varnothing$ for $i \neq j$. Moreover, decompose T_k into n_k subsets T_k^i, $i = 1, \ldots, n_k$, of equal size $\lambda\left(T_k^i\right) = m_k$ so that $\bigcup_i T_k^i = T_k$ and $T_k^i \cap T_k^j = \varnothing$ if $i \neq j$. Then, the audience allocation $\mathfrak{a}_T$ defined by $\mathfrak{a}_T(t) = S_k^i$ if $t \in T_k^i$ satisfies the following properties:

(i) $\mu\left(\mathfrak{a}_T(t)\right) = r_k$ for all $t \in T$. (ii) For any $s \in S$ and $k \in \{1, \ldots, K\}$ there is $i(k,s) \in \{1, \ldots, n_k\}$, so that $s \in S_k^{i(k,s)}$. Moreover, $M(s, \mathfrak{a}_T) \cap T_k = T_k^{i(k,s)}$ and $\tau_s = \int_{M(s,\mathfrak{a}_T)} \sigma_T(t)\,dt = \sum_k \sigma_k\, \lambda\left(T_k^{i(k,s)}\right) = \sum_k \sigma_k\, m_k$. Thus, for all $s, v(s, \tau_s) = f\left(\sum_k \sigma_k\, m_k\right)$. Property (i) guarantees that no company can increase its impact by increasing the size of its audience and property (ii) implies equal attention levels. Thus, according to corollary 1, $\mathfrak{a}_T$ is an equilibrium. QED.

Proof of proposition 2.

Suppose that there are two audience allocations $\mathfrak{a}_T^1, \mathfrak{a}_T^2$ with the required properties, i.e. $v\left(s, \tau_s^i\right) = \tilde{\upsilon}_i$ for some constant $\tilde{\upsilon}_i$ and $\tau_s^i \geq \tau_s^+$, where $\tau_s^i = \int_{M\left(s,\mathfrak{a}_T^i\right)} \sigma_T(t)\,dt$, $i = 1,2$.

Suppose $\tilde{\upsilon}_1 \neq \tilde{\upsilon}_2$, say $\tilde{\upsilon}_2 < \tilde{\upsilon}_1$. Then, because of DA, $\tau_s^1 < \tau_s^2$ for almost all s. This implies $\int_S \tau_s^2 d\mu(s) > \int_S \tau_s^1 d\mu(s)$, in contradiction to lemma A1 (c), according to which $\int_S \tau_s d\mu(s) = X$ for any allocation $\mathfrak{a}_T$ with $\mu\left(\mathfrak{a}_T(t)\right) = \min\{\mu(S), \rho(t)\}$, $t \in T$. Hence, for any given $X, \tilde{\upsilon}_1 = \tilde{\upsilon}_2 \equiv \tilde{\upsilon}$. Define $\overline{\upsilon}(X) = \tilde{\upsilon}$. Suppose next that $\rho(t), t \in T$, or σ_T change, so that X increases to X'. Let $\mathfrak{a}_T, \mathfrak{a}_T'$ be audience allocations with the required properties (i.e. $\mu\left(\mathfrak{a}_T(t)\right) = r_t$, $\mu\left(\mathfrak{a}_T'(t)\right) = r_t'$, $\tau_s \geq \tau_s^+$, $\tau_s' \geq \tau_s^+$, $v(s, \tau_s) = \tilde{\upsilon}$, $v(s, \tau_s') = \tilde{\upsilon}'$, where notation is analogous to before). $X' > X$

implies $\int_S \tau_s' d\mu(s) > \int_S \tau_s d\mu(s)$ and thus $\tau_s' > \tau_s$ on a set $A \subset S$ with positive measure. Because of DA, $v(s,\tau_s') < v(s,\tau_s)$ for $s \in A$ and thus $\tilde{\upsilon}' < \tilde{\upsilon}$. QED.

<u>Proof of proposition 3.</u>

(a) By definition, $V_t^n(\sigma) = \mu(\mathfrak{a}_T(t)) z_t(\sigma,\upsilon) - c_t(\sigma)$, if $v(s,\tau_s) = \upsilon$ for almost all *s*. Thus, the first-order condition for $\max_\sigma V_t^n(\sigma)$ is given by

$$F \equiv \mu(\mathfrak{a}_T(t)) \frac{\partial z_t(\sigma,\upsilon)}{\partial \sigma} - c_t'(\sigma) = 0.$$

The assumptions in (4.1) and (4.3) guarantee that the equation has a solution and implicitly defines $\sigma_t^*(\mu(\mathfrak{a}_T(t)),\upsilon) > 0$ with $\partial \sigma_t^* / \partial \mu > 0$ and $\partial \sigma_t^* / \partial \upsilon \geq 0$. The second-order condition $\partial F / \partial \sigma < 0$ holds because of $\partial^2 z_t / \partial \sigma^2 < 0$ and $c_t'' \geq 0$.

(b) According to (a), we have $V_t^n(\sigma^*(\cdot)) = \mu(\mathfrak{a}_T(t))\ z_t(\sigma_t^*(\cdot),\upsilon) -\ c_t(\sigma_t^*(\cdot))$. Applying the envelope theorem, we obtain $\partial V_t^n / \partial \mu = z_t > 0$ and $\partial V_t^n / \partial \upsilon = \mu(\mathfrak{a}_T(t)) \partial z_t / \partial \upsilon > 0$. QED.

<u>Proof of lemma 2.</u>

For $\upsilon \in (0, v(s,\tau_s^+))$, the equation $v(s,\tau) = \upsilon$ defines for all *s* signal exposure $\tau(s,\upsilon) > \tau_s^+$ where $\tau(s,\upsilon)$ is differentiable in υ with $\partial \tau / \partial \upsilon < 0$. Thus, $\int_S \tau(s,\upsilon)\, ds$ is a differentiable and decreasing function of υ. Since $\int_S \tau(s,\upsilon)\, ds = X$, according to lemma 1, υ is a differentiable decreasing function of X.

According to the proof of proposition 3, for given $\mu(\mathfrak{a}_T(t)),\upsilon$, optimal signal strength $\sigma_t^*(\mu_T(\mathfrak{a}_T(t)),\upsilon)$ is determined by the first-order condition $F(t,\sigma) = \mu(\mathfrak{a}_T(t)) \frac{\partial z_t(\sigma,\upsilon)}{\partial \sigma} - c_t'(\sigma) = 0$. Since $\partial F / \partial \sigma < 0$, $\sigma_t^* \leq y$ if and only if $F(t,y) \leq 0$ for any $y \in \mathbb{R}_+$. Thus, $\{t | \sigma_T(t) \leq y\} = \{t | F(t,y) \leq 0\}$ which is a measurable set, if the functions $a(t) := \partial z_t(\sigma,\upsilon) / \partial \sigma$ and $b(t) := c_t'(\sigma)$ are measurable. QED.

Proof of theorem 2.

Only the impact on $X^*[T,r_T]$ has to be proved. The effect on υ_T^* follows from DA when τ_s^+ is not larger than τ_s for almost all s. According to proposition 3, $\sigma_t^*(\tilde{r}_t,\bar{\upsilon}(X)) > \sigma_t^*(\rho(t),\bar{\upsilon}(X))$ for all $t\in T_0$. (Note $\tilde{r}_t = \min\{\mu(S),\tilde{\rho}(t)\} > \rho(t)$.) This implies $Z(X,T,\tilde{r}_T) > Z(X,T,r_T)$, where Z is defined in (4.6). Since $\partial Z/\partial X \le 0$, the equilibrium level $\tilde{X}$ defined by the equation $Z(\tilde{X},T,\tilde{r}_T)=\tilde{X}$ is higher than the level defined by $Z(X,T,r_T)=X$. QED.

Proof of fact 2.

Let $\mathfrak{a}_T$ be the audience allocation constructed in the proof of fact 1. Chose υ and τ such that $\upsilon = f(\tau)$ and

$$\tau = \frac{1}{\mu(S)}\sum_k \lambda(T_k)\, r_k^2 g_k(\upsilon)/c_k^1 . \tag{A.1}$$

Then for $t\in T_k$:

$$V_t^n(\sigma) = r_k\left[g_k(\upsilon)\ln\sigma + h_k(\upsilon)\right] - c_1^k\,\sigma - c_0^k \tag{A.2}$$

and argmax $V_t^n = \sigma_k^*$ where

$$\sigma_k^* = r_k g_k(\upsilon)/c_k^1 . \tag{A.3}$$

For $\tau_s=\tau$, lemma A1, part (c), implies $\mu(S)\tau = \int_T r_t\sigma_t(t)\,dt = \sum_k r_k\sigma_k\,\lambda(T_k)$ and thus:

$$\tau = \frac{X}{\mu(S)} \text{ with } X = \sum_k r_k\sigma_k\,\lambda(T_k). \tag{A.4}$$

Substituting σ_k^* for σ_k we get (A.1) with $\upsilon = f(\tau)$. Thus, $\sigma_T(t)=\sigma_k^*,\ t\in T,$ is an equilibrium under $\mathfrak{a}_T$. Obviously, $\mathfrak{a}_T$ is an equilibrium for T,σ_T since $v(s,\mathfrak{a}_T)=\upsilon$ for all s and $\mu(\mathfrak{a}_T(t)) = r_t$ for all $t\in T$.

For proving (ii), note that in any equilibrium with equalized attention levels we must have $\tau_s=\tau,\ f(\tau)=\upsilon$ and $\mu(\mathfrak{a}_T(t)) = r_k,\ t\in T_k$. Thus, (A.3) and (A.4) hold. They imply equation (A.1). Since $\upsilon = f(\tau),\ f'\le 0,\ g_k'\ge 0$, this equation implicitly defines τ as a function $\tau^*(\lambda(T_1),\ldots,\lambda(T_k),\rho_1,\ldots,\rho_k)$ with $\partial\tau^*/\partial\lambda(T_k)>0$ and $\partial\tau^*/\partial\rho_k > 0\ (=0)$ if $\rho_k<\mu(S)\ (\ge\mu(S)$, respectively). (Substitute $r_k=\min\{\mu(S),\rho_k\}$ into (A.1).) (Moreover, $\tau^*(\cdot)$ is lower (higher) in an economy of size $\mu(S)<\mu(S')$ if $\rho_k<\mu(S)\left(\rho_k\ge\mu(S),\text{ respectively}\right)$ for all $k=1,\ldots,K$.) QED.

<u>Proof of theorem 3.</u>

Suppose that $T_i^*, \mathfrak{a}_{T_i^*}, \sigma^*_{T_i^*}$, $i = 1,2$, are free-entry equilibria with equilibrium attention levels $\upsilon^*_{T_i^*} := \overline{\upsilon}_i\left(X_i^*\right)$, respectively. For simplifying notation, set $\upsilon_i := \upsilon^*_{T_i^*}$ and $\sigma_t^i := \sigma_t^*\left(\upsilon_i\right)$, for any $t \in L$. Moreover, denote by τ_s^i signal exposure of s in equilibrium i and define $B_i := T_i^* - \left(T_1^* \cap T_2^*\right)$. Then,

$$X_1^* = \int_{T_1^* \cap T_2^*} r_t \sigma_t^1 dt + \int_{B_1} r_t \sigma_t^1 dt\,,$$

$$X_2^* = \int_{T_1^* \cap T_2^*} r_t \sigma_t^2 dt + \int_{B_2} r_t \sigma_t^2 dt\,. \tag{A.5}$$

Assume that $\upsilon_1 = \overline{\upsilon}_1\left(X_1^*\right) < \upsilon_2 = \overline{\upsilon}_2\left(X_2^*\right)$. (Since indices 1 and 2 can be exchanged, the following contradiction also applies to $\upsilon_2 < \upsilon_1$, which establishes $\upsilon_1 = \upsilon_2$.) Note first that $\upsilon_1 < \upsilon_2$ implies $V_t^2 \equiv V_t^n\left(\sigma_t^2, \upsilon_2\right) \geq V_t^n\left(\sigma_t^1, \upsilon_2\right) > V_t^n\left(\sigma_t^1, \upsilon_1\right) \equiv V_t^1$, for any $t \in L$. (The first inequality follows from the definition of σ_t^i as the optimal choice under υ_i. The second inequality follows from $\partial V_t^n / \partial \upsilon = r_t \partial z_t / \partial \upsilon > 0$, see proposition 3.) I show that $\upsilon_1 < \upsilon_2$ and $V_t^2 > V_t^1$, $t \in T_1^*$, lead to a contradiction:

(i) On the one hand, since DA with $\tau_s^i \geq \tau^+(s)$ is assumed, $\upsilon_1 < \upsilon_2$ implies $\tau_s^1 > \tau_s^2$ and $\int_S \tau_s^1 \, d\mu(s) > \int_S \tau_s^2 \, d\mu(s)$. Thus, according to lemma A1, $X_1^* > X_2^*$.

(ii) On the other hand, viability of $t \in T_1^*$ requires $V_t^1 \geq 0$ so that $V_t^2 > V_t^1$ implies $V_t^2 > 0$. Hence, $T_1^* \subset T_2^*$, $B_1 = \varnothing$, $B_2 = T_2^* - T_1^*$. Moreover, for all $t \in T_1^* = T_1^* \cap T_2^*$, $\sigma_t^1 \leq \sigma_t^2$ because of $\partial \sigma_t^i / \partial \upsilon \geq 0$ (see proposition 3). Combining this with (A.5), we conclude $X_1^* \leq X_2^*$. This contradicts (i).

Finally, $\upsilon_1 = \upsilon_2$ for all s implies $\tau_s^1 = \tau_s^2$ and thus $X_1^* = \int_S \tau_s^1 \, d\mu(s) = \int_S \tau_s^2 \, d\mu(s) = X_2^*$.

QED.

<u>Proof of lemma 3.</u>

(a) The first-order condition defining $\sigma_t^*(\upsilon)$ is $F(\sigma, \upsilon) \equiv r_t \partial z_t(\sigma, \upsilon) / \partial \sigma - c_t'(\sigma) = 0$. Since $\partial F / \partial \sigma < 0$, $\sigma_t^* \geq \sigma_{t'}^*$, if t is non-inferior to t'. $\sigma_t^* > \sigma_{t'}^*$, if t superior to t'. Thus, t superior to t' is sufficient for $r_t \sigma_t^* > r_{t'} \sigma_{t'}^*$. It is also necessary: Suppose t is not superior to t'. Then t is identical to t' or t' is superior to t, which would imply $r_t \sigma_t^* \leq r_{t'} \sigma_{t'}^*$.

(b) By definition, $V_t^n\left(\sigma_t^*, \upsilon\right) = r_t z_t\left(\sigma_t^*, \upsilon\right) - c_t\left(\sigma_t^*\right)$. According to (a), $r_t \sigma_t^* > r_{t'} \sigma_{t'}^*$ implies t is non-inferior to t' and (i) $r_t > r_{t'}$, or (ii) $\partial z_t / \partial \sigma > \partial z_{t'} / \partial \sigma$ or (iii)

$c_t' < c_{t'}'$. In case (i) $V_t^n > V_{t'}^n$ (evaluated at the respective arguments) follows from the assumption that non-inferiority implies $z_t \geq z_{t'}$ and $c_t \leq c_{t'}$. In the cases (ii), (iii), it follows from non-inferiority of t and the assumption that (ii) implies $z_t > z_{t'}$ and (iii) implies $c_t < c_{t'}$. QED.

<u>Proof of theorem 4.</u>

(a) Suppose that there are two equilibria T_1^*, T_2^* with $\lambda\left(T_1^*\right) < \lambda\left(T_2^*\right)$ and thus $0 \leq \lambda\left(B_1\right) < \lambda\left(B_2\right)$, where $B_i := T_i^* - \left(T_1^* \cap T_2^*\right)$. According to theorem 2, $\upsilon_1 = \upsilon_2 = \upsilon^*$, and thus $X_1^* = X_2^*,\ \sigma_t^1 = \sigma_t^2 \equiv \sigma_t$, $V_t^n\left(\sigma_t^i, \upsilon_i\right) = V_t^n\left(\sigma_t, \upsilon^*\right)$ for any $t \in L$. (Notation as in proof of theorem 3.) This implies

$$V_t^* \equiv V_t^n\left(\sigma_t, \upsilon^*\right) = 0 \text{ for } t \in B_1 \cup B_2. \tag{A.6}$$

(Note that $B_1 \subset T_1^*,\ B_2 \subset T_2^*$ imply $V_t^n\left(\sigma_t, \upsilon^*\right) \geq 0$ for $t \in B_1 \cup B_2$. But since $B_j \cap T_i^* = \varnothing,\ i \neq j,\ V_t^n\left(\sigma_t, \upsilon^*\right) = V_t^n\left(\sigma_t^i, \upsilon_i\right) \leq 0$ if $t \in B_j$.)

Moreover, according to (A.5), $X_1^* = X_2^*,\ \sigma_t^1 = \sigma_t^2$ imply

$$\int_{B_1} r_t \sigma_t dt = \int_{B_2} r_t \sigma_t dt. \tag{A.7}$$

Since $\lambda\left(B_1\right) < \lambda\left(B_2\right)$, equation (A.7) can only hold if $\bar{B}_i \subset B_i,\ \lambda\left(\bar{B}_i\right) > 0$, exist so that $r_t \sigma_t > r_{t'} \sigma_{t'}$ for $t \in \bar{B}_1,\ t' \in \bar{B}_2$. Thus, according to M, $V_t^n\left(\sigma_t, \upsilon^*\right) > V_{t'}^n\left(\sigma_{t'}, \upsilon^*\right) \geq 0$ for $t \in \bar{B}_1 \subset B_1,\ \ t' \in \bar{B}_2 \subset B_2$, in contradiction to (A.6).

(b) $t' \in T^*$ implies $V_{t'}^* \geq 0$. According to lemma 3 (a), t superior to t' implies t is more powerful than t' and thus, according to $M, V_t^* > V_{t'}^* \geq 0$. Hence, $t \in T^*$. QED.

<u>Proof of theorem 5.</u>

Theorem 3 and 4 guarantee that unique values for equilibrium attention $\upsilon^*, \tilde{\upsilon}$, aggregate signal imission $X^*, \tilde{X}$, and measures $\lambda\left(T^*\right),\ \lambda\left(\tilde{T}\right)$ of the set of active companies exist in the two equilibria.

(a) Assume that $\upsilon^* < \tilde{\upsilon}$: Denote $\tilde{V}_t^n(\sigma, \upsilon) := \tilde{r}_t \tilde{z}_t(\sigma, \upsilon) - \tilde{c}_t(\sigma)$ and $\tilde{\sigma}_t(\upsilon) := \operatorname{argmax} \tilde{V}_t^n(\sigma, \upsilon)$. Then, $\tilde{V}_t \equiv \tilde{V}_t^n\left(\tilde{\sigma}_t(\tilde{\upsilon}), \tilde{\upsilon}\right) > \ \tilde{V}_t^n\left(\tilde{\sigma}_t\left(\upsilon^*\right), \upsilon^*\right)$, since $\partial \tilde{V}_t^n / \partial \upsilon = \tilde{r}_t \partial \tilde{z}_t / \partial \upsilon > 0$, due to the envelope theorem. Moreover, for all $t \in L$, $\tilde{V}_t^n\left(\tilde{\sigma}_t\left(\upsilon^*\right), \upsilon^*\right) \ \geq \tilde{V}_t^n\left(\sigma_t^*\left(\upsilon^*\right), \upsilon^*\right) \ \geq V_t^n\left(\sigma_t^*\left(\upsilon^*\right), \upsilon *\right) \ \equiv V_t^*$. (The first inequality follows from the definition of $\tilde{\sigma}_t$ as the optimal choice under $\tilde{V}_t^n$. The second inequality follows from $\tilde{V}_t^n(\sigma, \upsilon) \geq V_t^n(\sigma, \upsilon)$ by construction ($\tilde{b}$ was assumed to be a progress over b). In sum, $\tilde{V}_t > V_t^*$ for all $t \in L$. According to the proof of theorem 3, $\tilde{V}_t > V_t^*$, $t \in T^*$ and $\upsilon^* < \tilde{\upsilon}$ lead to a contradiction. (Note that the argument did not rely on the underlying fundamentals.) Thus, $\upsilon^* \geq \tilde{\upsilon}$. This proves part (a).

(b) (i) By construction, for all $t \in E$, $\tilde{\sigma}_t(\upsilon) > \sigma_t^*(\upsilon)$ and $\tilde{r}_t\tilde{\sigma}_t(\upsilon) > r_t\sigma_t^*(\upsilon)$ for any υ. Moreover, for $t \in E$, $\tilde{V}_t^n(\sigma,\upsilon) = \tilde{r}_t\tilde{z}_t(\sigma,\upsilon) - \tilde{c}_t(\sigma) > V_t^n(\sigma,\upsilon) = r_t z_t(\sigma,\upsilon) - c_t(\sigma)$ for any σ,υ. For $t \in L-E$, $\tilde{\sigma}_t(\upsilon) = \sigma_t^*(\upsilon)$, $\tilde{r}_t\tilde{\sigma}_t(\upsilon) = r_t\sigma_t^*(\upsilon)$ and $\tilde{V}_t^n(\sigma,\upsilon) = V_t^n(\sigma,\upsilon)$.

(ii) Assume $\upsilon^* = \tilde{\upsilon}$: Then, $X^* = \tilde{X}$ (employ argument at the end of proof of theorem 3). According to (i), $\tilde{V}_t^n(\tilde{\sigma}_t(\upsilon),\upsilon) \geq \tilde{V}_t^n(\sigma_t^*(\upsilon),\upsilon) > V_t^n(\sigma_t^*(\upsilon),\upsilon)$ for all $t \in E$. Thus, $E \subset \tilde{T} \cap T^*$ since $E \subset T^*$ by assumption. Moreover, for all $t, \tilde{\sigma}_t \geq \sigma_t^*$ (I omit the argument $\tilde{\upsilon} = \upsilon^*$) with strict inequality for $t \in E$. This implies $\int_{\tilde{T}\cap T^*} \tilde{r}_t\tilde{\sigma}_t dt > \int_{\tilde{T}\cap T^*} r_t\sigma_t^* dt$. Using an analogous argument to (A.5), we obtain:

$$\int_{B^*} r_t\sigma_t^* dt > \int_{\tilde{B}} \tilde{r}_t\tilde{\sigma}_t dt, \tag{A.8}$$

where $B^* := T^* - (\tilde{T} \cap T^*)$ and $\tilde{B} := \tilde{T} - (\tilde{T} \cap T^*)$.

For $t \in B^*$, $t' \in \tilde{B}$, we have:

$$\tilde{r}_{t'}\tilde{\sigma}_{t'} \geq r_t\sigma_t^*. \tag{A.9}$$

(Suppose that $\tilde{r}_{t'}\tilde{\sigma}_{t'} < r_t\sigma_t^*$. Then, $\tilde{r}_{t'}\tilde{\sigma}_{t'} < \tilde{r}_t\tilde{\sigma}_t$, because of (i) and $\tilde{V}_t^n(\tilde{\sigma}_t,\upsilon) > \tilde{V}_{t'}^n(\tilde{\sigma}_{t'},\upsilon)$ due to M. Thus $t \in \tilde{T}$, since $t' \in \tilde{B} \subset \tilde{T}$. This contradicts $B^* \cap \tilde{T} = \varnothing$.) Using (A.9) in (A.8), we obtain $\lambda(B^*) > \lambda(\tilde{B})$ and thus $\lambda(T^*) > \lambda(\tilde{T})$.

(iii) Assume $\tilde{\upsilon} < \upsilon^*$: Then, $\tilde{V}_t^n(\tilde{\sigma}_t(\tilde{\upsilon}),\tilde{\upsilon}) = V_t^n(\sigma_t^*(\tilde{\upsilon}),\tilde{\upsilon}) < V_t^n(\sigma_t^*(\tilde{\upsilon}),\upsilon^*) \leq V_t^n(\sigma_t^*(\upsilon^*),\upsilon^*) \leq 0$ for every $t \in (T_0 - E) \cup (L - T^*)$. (The first equality follows from $t \notin E$. The next two inequalities follows for any $t \in L$ from $\partial V_t^n / \partial \upsilon > 0$ and the optimality of $\sigma_t^*(\upsilon^*)$. The last inequality holds for any $t \in T_0 \cup (L - T^*)$.) Thus, $(T_0 - E) \cup (L - T^*) \subset L - \tilde{T}$, $\lambda(L - \tilde{T}) \geq \lambda(T_0 - E) + \lambda(L - T^*)$, $> \lambda(L - T^*)$ and $\lambda(\tilde{T}) < \lambda(T^*)$, since $\lambda(T_0 - E) > 0$ by assumption. QED.

<u>Proof of fact 3.</u>

(a) Suppose that the set of active senders is T with $\lambda(T)$. Applying (A.2) and (A.3), we get $\sigma_t^* = r\,g(\upsilon)$ and $V_t^* = y \ln y + (\gamma - 1)y - c_0$ for $y \equiv r\,g(\upsilon)$. Thus,

$V^* >,=,< 0$ if $r\,g(\upsilon) >,=,< \overline{y}(c_0,\gamma)$ where $\overline{y}(c_o,\gamma)$ is defined by the condition $\ln \overline{y} = 1-\gamma + c_0/\overline{y}$. (Note that $\overline{y} > 1$ and $\overline{y}$ increases with c_0 and decreases with γ.) According to (A.1), $\tau = r^2 g(\upsilon)\lambda/\mu$ where λ denotes $\lambda(T)$ and μ denotes $\mu(S)$. Using this and $g(\upsilon) = g_0\upsilon^\alpha$ in $v(s,\tau_s) = \tau_s{}^{-\beta}$ we get $\tau^* = \left(r^2\lambda/\mu\right)^{1/(1+\alpha\beta)} g_0^{1/(1+\alpha\beta)}$, $\upsilon^* = \tau^{*-\beta}$ and $g\left(\upsilon^*\right) = \tilde{g}_0\left(\mu/\left(r^2\lambda\right)\right)^{\alpha\beta/(1+\alpha\beta)}$ with $\tilde{g}_0 \equiv g_0^{1/(1+\alpha\beta)}$. Substitution of $g\left(\upsilon^*\right)$ into the condition $V^* >,=,< 0$ gives us for $\lambda^* = \lambda\left(T^*\right) = \lambda_0\mu(S)\,r^{(1-\alpha\beta)/(\alpha\beta)}$ where $\lambda_0 \equiv \left(\tilde{g}_0/\overline{y}\right)^{1+1/(\alpha\beta)}$.

(b) Local diversity depends on the realised audience allocation. For the equilibrium audience allocation constructed in the proof of fact 1, the measure of membership of every $s \in S$ is given by $m^* = \lambda^* r/\mu(S)$. Since all active senders are identical, attention level must be equalized in any equilibrium audience allocation. Moreover, equal attention levels imply an equal measure of membership for all $s \in S$. Hence, $m^* = \lambda^* r/\mu(S)$ in any equilibrium. Substituting $\lambda^* = \lambda_0\mu(S)\,r^{(1-\alpha\beta)/(\alpha\beta)}$ into $m^* = \lambda^* r/\mu(S)$ we get $m^* = \lambda_0 r^{1/(\alpha\beta)}$. QED.

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