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THE RIGHT MAN FOR THE JOB

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Abstract

This paper describes a search model with a continuum of worker and job types, transferable utility and an increasing returns to scale contact technology. We apply a second order Taylor expansion to characterize the equilibrium. One third of the increasing returns in contacts are absorbed by firms and workers being more choosy. Hence, strongly increasing returns in contact rates are consistent with weakly increasing returns in matching. The resulting equilibrium is not efficient. Unemployment benefits can reduce the loss by serving as a search subsidy. The loss caused by search frictions is higher when worker types are bad substitutes. Numerical simulations of the model show our Taylor expansions to be quite accurate.

Keywords: assignment, search, unemployment.

JEL Classification: J21, J23.

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1 Introduction

In recent years a flourishing literature on equilibrium search models has emerged. Important contributions include: Diamond (1982a), Pissarides (1990), Mortensen and Pissarides (1994). Those papers contributed a lot to our understanding of the mechanisms behind unemployment and job creation and their importance for the functioning of the labor market. Nevertheless, when going through most of this literature, one feels a bit uncomfortable because it is assumed that all matches can be summarized by an aggregate matching function, which maps the stocks of unemployed workers and vacancies into job-worker matches. This allows the researcher who is interested in outcomes for a representative worker only, to treat all workers and firms as if they are homogeneous.

The existence of such a single aggregate matching function is clearly not a realistic device for analyzing search equilibria. However, it might serve the goal of tractability. The relevant question is therefore whether important issues are left out by ignoring heterogeneity. We shall argue that this is the case. In the simplest version of the Pissarides (1990) framework, each contact between a job seeker and a vacancy results in a match.[1] Hence, the speed of the matching process is entirely a technical matter. Under heterogeneity, the mechanical contact process and the matching process are disentangled. Both job seekers and firms have to decide whether or not they want to give up the option value of continued search and engage in an employment relation with a particular contact.

Sattinger (1995) is to our knowledge the first who analyzed the effects of search frictions on the assignment of workers to jobs in a model with transferable utility. The structure of our model is however more closely related to Shimer and Smith (2000). They proof existence and derive, without explicitly characterizing the equilibrium, the weakest conditions for positive assortative matching to take place. There are however a number of important differences between our model and theirs. We allow for the free entry of vacancies and endogenous commodity prices. Moreover, our focus differs is different. Our main interest is to provide a characterization of the equilibrium so that we can talk about efficiency issues.

For modelling the production structure, we apply the continuous-type comparative advantage framework first developed in Rosen (1974) and Sattinger (1975) and later refined in Teulings (1995, 1999). In this structure, worker types are characterized by a single index, referred to as the skill level. Likewise, jobs are characterized by their complexity level. Both indices are continuous. High skilled workers have a comparative advantage in complex jobs, low-skilled workers in simple jobs. The comparative advantage structure provides a completely natural reason for search because each worker type has its own "best" job type, where her comparative advantages are best utilized. The employer and the worker have a joint interest in a good match, since the better the match, the larger the surplus that remains to be distributed. In contrast, models with universally "good" jobs would not survive a free entry condition for vacancies because only those "good" jobs would be created and the other jobs would disappear. The continuity of the skill and complexity index provides a much simpler framework for analyzing how matching sets change with the scale of the market than a discrete type framework, since it allows us to do comparative statics on marginal variations in the scale measure. The disadvantage of applying

[1] Pissarides (1990), chapter 5 distinguishes between contacts and matches. Workers and jobs are however ex ante homogeneous and they do not face the problem of defining matching sets. After they meet, their productivity is drawn from a distribution and the "bad draws", immediately dissolve.

this comparative advantage continuous type of search model is its analytical complexity. The Bellman equations for job seeker and vacancy types typically require the evaluation of integrals over matching sets where both the integrand and the boundaries of these integrals are endogenous functions. This complexity has made many researchers reluctant to apply this type of models.

An alternative interpretation of this final hurdle helps understanding the avenue that we intend to travel. Compared to the complexity of search models, a Walrasian model gives the annalist a great deal of convenience. First order conditions, reflecting for example the point of tangency between a wage-cost function and an iso-output function, provide a solid structure to market equilibrium. All higher order conditions can be ignored and the envelope theorem allows the researcher to ignore indirect effects. The first small step the annalist sets outside this Walrasian Utopia brings him into deep trouble. The wage-cost function falls below the iso-output function. Search decisions are driven by the integral over the matching set enveloped by both functions, see Figure 1 in section 3. Instead of a single first order effect, the annalist has to account for a multitude of higher order effects for the evaluation of this integral.

This interpretation alludes to a straightforward idea. Perhaps we can gain insight in a world with dirty search equilibria if we would add only the second order term. It is this idea that is investigated in the present paper. Our approach should not be viewed as just a mathematical trick, but has a number important economic implications. First, the closer we are to the Walrasian equilibrium, the smaller will be the area enveloped by the wage-cost function and the iso-output function and the better will therefore be our second order approximation. In the limit, higher order effects vanish, and the second order expansion gives a perfect description of the equilibrium. Interestingly, this limit is equivalent to raising the scale of the market to infinity.[2] Second, a number of elasticities (e.g. the elasticity of unemployment with respect to the scale) are fully determined by the order(s) of the first non-vanishing terms in the Bellman equations for workers and firms and flow equilibrium condition. Finally, the second order expansion links the theory of job search to the theory of substitution. Elasticities of substitution are indeed governed by the second derivative of cost functions, hence the same derivative which is applied here to evaluate search frictions. Teulings (1999) shows that the degree of substitutability between worker types in this type of comparative advantage model is driven by a parameter labeled the complexity dispersion parameter. In this paper, we show that the same parameter also determines the magnitude of search frictions. The larger this parameter, the harder it is to substitute between different worker types. There is a simple intuition for the relation between search frictions and substitutability of worker types. When firms cannot easily substitute between worker types, it becomes more important to have a precise assignment of workers to jobs. Therefore, for a given contact technology, it takes more time to find a suitable match partner. Hence, our analysis ends up with a simple equation that relates the size of search frictions to a small number of forces which can all be estimated and tested empirically.

We are also able to derive an explicit expression for total output loss due to search frictions. This loss is caused by three factors: mismatch, forgone production by unemployed workers and the costs of keeping vacancies open. The loss can itself be decomposed in: loss due to inadequate incentives (which can potentially be neutralized by adequate policies) and loss due to search

[2] Following these ideas, Ken Judd suggested to us an alternative way of deriving our results. Start describing the search equilibrium from the point where $1/\text{scale} = 0$ (hence: the Walrasian equilibrium) and then use the implicit function theorem to analyse the comparative statics of slight upward variations in $1/\text{scale}$. This method leads to similar results as the ones that are discussed in this paper.

frictions. Our solution also sheds light on the issue of optimal social insurance. In our model, unemployment benefits have a useful economic role in the sense that they provide adequate incentives for search and prevent workers from accepting jobs for which they are ill suited, see also Burdett (1979) and Marimon and Zilibotti (1999). While in Marimon and Zilibotti (1999), optimal unemployment insurance is zero when wages are set correctly (the Hosios-Pissarides condition is satisfied), in our model it is always positive. The reason is that we have an IRS contact technology so that no wage sharing rule will fully reward searchers for their effort.

Endogenizing the matching decision also affects the measurement of returns to scale in the matching function. The common wisdom is at the moment that there are no increasing returns. However, returns to scale are typically estimated by relating the stocks of job seekers and vacancies to the number of realized matches. This procedure ignores the fact that, under increasing returns, job seekers and firms become more choosy as the scale of the market increases. Our analysis shows that when both vacancies and unemployment increase by 1%, this leads to 2% more contacts but only to 1.66% more matches, the rest is absorbed by smaller matching sets.

Initially, our assumption of increasing returns in the contact process was motivated by the problem that arises in models with constant returns to scale in contacts and heterogeneity. In these models, all job seekers reduce the probability to find a vacancy for all other job seekers due to "congestion" externalities. Consequently, in an integrated labor market, low skilled immigrants would find it harder to find a hamburger job when a bunch of Harvard graduates enter the market. This implication of "congestion" externalities seems to be highly unrealistic. To avoid it, we assume a contact technology with increasing returns to scale. Finally, it is worth mentioning that economists are in good company when making the assumption that the contact probability for workers (vacancies) only depends on the amount of vacancies (workers) on the other side of the market. The theory of the velocity of chemical reactions in gasses is based on exactly the same premisses. Increasing pressure speeds up the reaction process. Likewise, the contact rate is higher in dense Manhattan than in Wyoming.

A potential risk in our approach is that the approximations are so bad that we really do not learn anything from them. Therefore, we confront our approximations with the numerical solution of the model. When search frictions are small, our approximations almost exactly mimic the numerical solutions. When search frictions become larger and when there is perfect substitution in the goods market, a problem which we label the "corner problem" arises. The problem arises from the fact that there are little incentives to open vacancies in the corners of the market because there are too few job seekers for these vacancies. This results in a spike of vacancies just above the lower corner and just below the upper corner which will attract all the low skilled workers. Given the existence of this spike, it becomes less attractive to open vacancies just above this spike. The spike crowds out neighboring jobs. As a consequence of this first spike, a second spike somewhere above the first one will arise and we get a process of endogenous segmentation. Burdett and Coles (1997) and Smith (1995) show that a similar process of endogenous segmentation arises in a non-transferable utility context. This process will not be picked up by our second order Taylor approximations. Our analytical results are therefore most precise for the middle groups, for labor markets with small search frictions, and for economies with less than perfect substitution on commodity markets.

The paper is organized as follows. Section 2 presents the assumptions underlying the model and discusses some of its implications regarding the efficiency of the bargaining process and compares the structure of the equilibrium with the Walrasian case. Section 3 discusses some general results regarding the existence of an equilibrium and the shape of the matching sets. Section 4 presents the main innovation of the paper, the use of second order Taylor expansions for the evaluation of the integrals over the matching sets. Next, Section 5 deals with the elasticities of the search surplus and unemployment with respect to the key parameters of the model. In this section, we also consider the size and nature of the efficiency losses. Section 6 discusses the corner problem. In Section 7, we compare the results from our Taylor expansions to full-fledged numerical solutions of the model. Our simulations are specified such that we can closely track empirical estimates of all key parameters. Finally, Section 8 discusses some relevant extensions of our approach like risk aversion.

2 Structure of the economy

2.1 Basic assumptions

In our economy, workers and jobs are characterized by a single index, referred to as skill level s and job-complexity level c respectively. Both indices vary continuously, so that there exists an infinitum of worker and job types:

$$\begin{aligned} s &\in [s^-, s^+] \\ c &\in [c^-, c^+],\ c^- > 0 \end{aligned}$$

Let $\underline{L}$ be the size of the labor force and $\underline{l}(s)$ be the density function of s; exogenous variables will be underlined throughout the paper. Furthermore, let $h(s)$ denote the number of unemployed workers of type s per unit of labor supply. Hence, $\frac{h(s)}{\underline{l}(s)}$ is the unemployment rate for workers of type s and the aggregate unemployment rate satisfies $u \equiv \int_{s^-}^{s^+} h(s)ds$. The supply of vacancies is determined by a free entry rule which drives the asset value of a vacancy to zero in equilibrium. We will denote the number of vacancies of type c per unit of labor supply by $g(c)$. The total number of vacancies per unit of labor supply follows then from $v \equiv \int_{c^-}^{c^+} g(c)dc$ while the total number of vacancies is $\underline{L}v$. Maintaining a vacancy is costly. We assume that those costs are independent of the job type and equal to K per period. We can think of those costs as advertisement costs. Let B denote unemployment benefits or the value of leisure, although the former interpretation is not fully consistent with the model, since we ignore the funding of these benefits. Nevertheless, we shall loosely refer to the ratio of B to reservation wages as the replacement rate.

Since each job type c produces its own commodity, we have to introduce a set of commodity prices $P(c)$. The productivity of a worker, s, at a job, c, is denoted by $F(s, c)$. The gross per period value of a match between s and c is therefore equal to $P(c)F(s, c)$. Search frictions enter the model by a simple linear contact rate $\lambda_{i \to j}$ for worker (job) type i to run into job (worker) type j:

$$\begin{aligned} \lambda_{s \to c} &\equiv \lambda^* \underline{L} g(c) \qquad (1) \\ \lambda_{c \to s} &\equiv \lambda^* \underline{L} h(s) \end{aligned}$$

where λ^* is a technology parameter which measures the efficiency of the matching process. For notational convenience we define $\lambda \equiv \lambda^* \underline{L}$. We can interpret λ then as the relevant scale of the labor market. Matches are destroyed at an exogenous rate δ. Finally, we assume for simplicity that both workers and firms are risk neutral. From these definitions, the value of search for a worker and an employer respectively, can be expressed as:

$$R(s) = B + \frac{\lambda}{\rho+\delta} \int_{m_c(s)} g(c) \left[W(s,c) - R(s)\right] dc \tag{2}$$

$$K = \frac{\lambda}{\rho+\delta} \int_{m_s(c)} h(s) \left[P(c)F(s,c) - W(s,c)\right] ds \tag{3}$$

where $R(s)$ is the reservation wage of worker type s, $W(s,c)$ is the wage of a worker of type s who is employed at a job type c, and ρ is the discount rate. Matching takes place when both firm and worker are better off. Since we allow for bargaining over the match surplus, any match with a value that exceeds the sum of the outside options of firm and worker is acceptable. The functions $m_c(s)$ and $m_s(c)$ define the subsets of c and s respectively with whom type s and c respectively are willing to form a match with. These subsets are determined by the condition that the match surplus is positive.

$$P(c)F(s,c) - R(s) > 0 \tag{4}$$

Wages are set by a simple Nash bargaining rule over the match surplus. Hence:

$$W(s,c) = \beta P(c)F(s,c) + (1-\beta)R(s) \tag{5}$$

where β denotes the workers' bargaining power. Substituting (5) in (2) and (3) yields:

$$R(s) = B + \frac{\beta\lambda}{\rho+\delta} \int_{m_c(s)} g(c) \left[P(c)F(s,c) - R(s)\right] dc \tag{6}$$

$$K = \frac{(1-\beta)\lambda}{\rho+\delta} \int_{m_s(c)} h(s) \left[P(c)F(s,c) - R(s)\right] ds \tag{7}$$

When the scale-parameter λ goes to infinity (the Walrasian case), the intergrals in both equations should vanish for all c in $m_c(s)$ for every s. This reflects the first order conditions to be discussed in Section 2.4. At this point, we have to be more specific about the shape of $F(s,c)$. We assume that $F(s,c)$ is log supermodular with the following functional form:[3]

$$F(s,c) \equiv e^{s\,c} \tag{8}$$

Log supermodularity is a crucial ingredient for comparative advantage, see Teulings (1995).[4] This can be seen most easily by comparing this specification with a simple supermodular function, for example $F = sc$, and analyzing a simple Walrasian model with free entry of vacancies.

[3] $F(s,c)$ is supermodular if $\forall\ c' > c$ and $\forall s' > s$ the following relation holds: $F(c,s) + F(c',s') > F(c,s') + F(c',s)$, and $F(s,c)$ is log supermodular if $f(s,c) = \log F(s,c)$ is supermodular, see e.g. Topkis (1998).

[4] Some commentators suggested that this specification also implies increasing returns to scale. However, s and c are just indices which can be transformed at will. E.g. nothing would be changed by specifiying $F(s,c) = \exp\left(\sqrt{sc}\right)$, except that s can then take no longer negative values.

The profit of a firm of type c is then equal to $P(c)\,sc - R(s)$. Under free entry, firms choose the profit maximizing c-type, that is the type that maximizes $P(c)\,sc$ or equivalently $P(c)\,c$ (because s enters multiplicatively). With perfect substitution on commodity markets and therefore exogenous commodity prices, this would imply that all firms would choose the same c-type. Hence, there can never be assortative matching, since there would only be a single firm type in the market. With imperfect substitution, commodity prices would adjust to make firms indifferent, that is: $P(c) = A/c$, where A is an arbitrary constant. Hence, in both cases, profits can be written as $As - R(s)$. Hence, there is no assortative matching since all assignments are equally profitable. In the case of log supermodularity, this argument breaks down. The c-type that maximizes profits now depends on the s type that is employed, which can be seen directly from the first order condition for the optimal c-type: $P'(c)\,e^{sc} + sP(c)\,e^{sc} = 0$, or: $P'(c)/P(c) = -s$. The crucial condition for positive assortative matching turns out to be that $F_c(s,c)/F(s,c)$ increases in s, which is equivalent to log supermodularity. Hence, in our model, there is no such thing as "good" or "bad" jobs which are independent of the skills of the worker. For each worker type s there exists one perfect match $c(s)$ that maximizes the joint surplus by making optimal use of the comparative advantages of that worker type. Moreover, the notion of one particular type of job being best, independent of s, is inconsistent with commodity market competition and the free entry condition for vacancies. If a particular type of job would be more profitable than others, a corner solution would result where only "good" vacancies would be opened. However, under product market competition the price of the output of that job would be driven down till it is no longer best.

Given our assumption on the wage bargaining, we can write the matching sets for worker type, s, and job type, c, respectively as:

$$\begin{aligned} m_c(s) &= \{c\}_{\,|\,x(s,c)>0} \\ m_s(c) &= \{s\}_{\,|\,x(s,c)>0} \end{aligned} \tag{9}$$

where the log match surplus $x(s,c) \equiv p(c) + sc - r(s)$. Hence $x(s,c)$ has the neat interpretation of being the relative surplus of value added above the reservation wage. In the sequel, lower cases denote the logarithm of the corresponding upper cases. The definition implies: $c \in m_c(s) \Leftrightarrow s \in m_s(c)$, that is: if c is in the matching set of s, then s is in the matching set of c. The condition $x(s,c) > 0$ is equivalent to the condition $P(c)F(s,c) - R(s) > 0$, that has been applied before. Furthermore, we assume that $p(c^-) + s^-c^- > \ln B$, implying that there is a positive surplus from producing, even for the least productive worker.[5]

Since we consider a stationary economy, the number of workers finding a job must equal the number loosing their job. Hence:

$$\delta\,[\underline{l}(s) - h(s)] = \lambda\, h(s) \int_{m_c(s)} g(c)\, dc \tag{10}$$

Physical output per job type can be derived from the inflow of new workers of type s times their productivity in job type c times the expected duration of the employment relation $1/\delta$:

[5] Hence, we rule out structural unemployment. Obviously, the model can deal with structural unemployment by simply adding agents whose productivity will never exceed the value of leisure, but this does not generate further insights. Note that this assumption is only fully appropriate in the case that $\eta = \infty$ (perfect substitution in the good market, this will be discussed below) since otherwise $p(c^-)$ is endogenous.

$$Y(c) = \frac{\lambda}{\delta} g(c) \int_{m_s(c)} h(s) e^{sc} ds \tag{11}$$

The outputs of each job type are traded on commodity markets. Assume that the demand for commodities is given by a continuous type CES utility function such that:

$$(1-\eta)p = \ln \left[\int_{c^-}^{c^+} \exp \left[(\eta+1) \underline{q}(c) + (1-\eta)p(c)) \right] dc \right] \tag{12}$$

where p is the log price index of consumption (or alternatively, the price of the composite consumption good), $\underline{q}(c)$ is the weight of type c in consumption and η is the elasticity of substitution. From (12) we can derive that the demand for good c satisfies:

$$y(c) - y - \underline{q}(c) = \eta[p - p(c) + \underline{q}(c)]$$

where y denotes log aggregate consumption. Let p be the numeraire ($p = 0$). We will confine our attention to two special cases, perfect substitution ($\eta = \infty$) and zero substitution ($\eta = 0$). In the first case, prices are effectively exogenous since $p(c) = \underline{q}(c)$. In the second case, the distribution of output per job type is exogenous $y(c) = \underline{q}(c)$. Summarizing:

$$\begin{aligned} \eta &= \infty : p(c) = \underline{q}(c), \forall c : \underline{q}''(c) < 0, \ \underline{q}'(c^-) = s^-, \ \underline{q}'(c^+) = s^+ \\ \eta &= 0 : y(c) = \underline{q}(c), \forall c : \underline{q}(c) > 0, \end{aligned} \tag{13}$$

The assumption $\underline{q}(.)$ being positive in the case that $\eta = 0$ implies that there is positive demand for all job types c. The assumption $\underline{q}''(.)$ being negative in the case $\eta = \infty$ has the same effect, as will be discussed in section 2.4. In that section, we will also motivate the lower and upper bound of $\underline{q}'(c)$ in the case of $\eta = \infty$. This completes the description of the structure of the economy.

2.2 Increasing returns?

It is useful to compare our matching technology with the CRS matching technology which is frequently used in the literature (e.g. Pissarides 1990). Let m be the flow of contacts between workers and firms. Then, we have by (1):

$$m = \lambda \int_{s^-}^{s^+} h(s) \int_{c^-}^{c^+} g(c) \, dc \, ds = \lambda \, u \, v \tag{14}$$

The matching elasticities with respect to u and v are both equal to one, hence their sum is two, which is twice as large as in the constant returns specification of Pissarides (1990). Hence, in our model, the contact rate of a job seeker, $m/u = \lambda v$, is independent of the number of other job seekers on the market, and mutatis mutandis the same for vacancies. This rules out congestion effects, as discussed in the introduction. However, most of the empirical evidence suggests that returns to scale are close to constant, see e.g. Petrongolo and Pissarides (1999).[6] How do we square our assumption with these empirical results? We offer three arguments.

[6] Not all the evidence rejects IRS. Yashiv (1996) finds IRS in the Israelian matching function and Shimer (1999) gives demographical evidence for the US that supports the thick market externality arguments.. Burdett et al. (1994) argue that increasing returns to scale are obscured because mismeasurement (due to aggregation and frequency problems in the data) of the true matching function.

First, the empirical research refers to the number of realized matches, while our technology refers to the number of contacts between workers and firms, or even better, potential contacts. Not all contacts will yield a match however. The agents in our model respond to the greater efficiency of the contact process not only by reducing their search spells, but also by becoming choosier (resulting in a better match quality). Hence, increasing returns in the contact technology does not translate one-to-one into returns to scale in realized matches. It could be argued that this issue can be resolved by using data on contacts, or referrals in the wording of Berman (1997). This trick will not work either since when a Wall Street stock broker sees a "help wanted" sign in a Hamburger restaurant, he is unlikely to report this event as a referral. This type of self selection avoids a lot of potential contacts to turn into referrals. It also explains why so few job offers get rejected. Disappointing as it is, there is no direct way to establish the returns to scale elasticity in the contact technology from any existing data set. An important goal of our analysis is to establish the size of the effect of an increased contact rate on the match quality and on the number of accepted matches, so that our assumptions can be tested indirectly.

Second, one has to consider: what is scale? Obviously, saying that scale matters is not the same as saying that the US labor market with 200 million inhabitants is more efficient than that of the Netherlands with only 15 million inhabitants. A more useful way to analyze the effect of scale on the efficiency of the search process is to interpret it as the density of the labor market. In that case, we can compare the functioning of labor markets in for example Manhattan and Wyoming. Per job opening, the number of potential candidates for filling a vacancy is much higher in Manhattan than in the mountains of Wyoming.

Finally, all existing estimates of the returns to scale parameter are based on cyclical variation in the number of vacancies and unemployed. This variation is affected by all kinds of out-of-equilibrium processes, which usually are not fully modelled. We suspect that this variation does therefore not adequately reflect differences in the scale (or: density) of a labor market, leading to a downwardly biased estimate of the scale elasticity.

2.3 The Walrasian benchmark and complexity dispersion

As a benchmark for future reference, we reiterate some of the previous results on the frictionless continuous type comparative advantage model, see e.g. Teulings (1995, 1999). In this simple Walrasian world, the profit maximizing strategy for an employer with a vacancy of type c is simply to minimize (log) cost per efficiency unit, that is: $r(s) - sc$. When there are no mass points in the distribution of either labor supply or product demand, this cost curve has a unique minimum for each c. Hence, the assignment of workers to jobs can be described by a one-to-one correspondence between s and c. The first order condition of the cost minimization problem reads:

$$r'\left[s(c)\right] = c \tag{15}$$

where $s(c)$ is the optimal assignment for job c. Differentiating this first order condition with respect to c yields: $r''[s(c)]s'(c) = 1$. Since our supermodular production function implies that $s'(c) > 0$, it must also be that $r''(s) > 0$. This is the second order condition of the cost minimization problem: the (log) costs of hiring a worker with an additional unit of skill are increasing while the log returns are constant, namely equal to the value of c for that firm.

The zero profit condition of firms reads: $p(c) + s(c)c - r[s(c)] = 0$. Since this condition applies identically for all c, its first derivative must also apply. Hence (the effect via $s(c)$ drops out by the envelope theorem):

$$-p'(c) = s(c) \tag{16}$$

Differentiating a second time yields: $-p''(c) = s'(c) = 1/r''(s)$. Hence: $p''(c) < 0$. For the case $\eta = \infty$, $p(c) = \underline{q}(c)$ and hence $p''(c) = \underline{q}''(c)$. This result motivates the assumption on $\underline{q}$" (c) discussed in relation to equation (13). Moreover, the lower and upper support of $\underline{q}(c)$ for $\eta = \infty$ guarantee that the lowest skill type will be employed at the simplest job types and mutatis mutandis the same for the highest skill type.

A crucial variable in this model is the second derivative of the reservation wage function. Since $r''[s(c)] = 1/s'(c)$, it is a measure of job heterogeneity. The higher $r''(s)$, the more variation there exists in job complexity per unit of s. This explains why $r''(s)$ is the main determinant of the elasticities of substitution and complementarity between skill types. The higher $r''(s)$ is, the more heterogeneous jobs are and the less easy workers are substitutable, since their relative productivities vary between job types.

The empirical implications of the model are invariant to a linear transformation of s, since we have not yet defined the units of measurement of commodities of various c types. Any multiplicative transformation of s can be absorbed by an opposite transformation of c and any additive transformation of c can be absorbed by a compensating redefinition of commodity prices $p(c)$. However, $r''(s)$ will be affected by a linear transformation of s, which makes it unsuitable as a summary statistic that can be meaningfully compared across time and space.

Teulings and Vieira (1998) therefore introduced the complexity dispersion parameter: $\gamma(s) \equiv r''(s)/r'(s)^2$. As can be checked easily, this parameter is invariant to a linear transformation of s.[7] The parameter basically determines the degree of substitutability of worker types in this type of economy. For the case that $\eta = 0$, it also has the nice interpretation of being the compression elasticity, measuring the percentage decrease in the return to human capital per percent increase in the value of human capital in the economy. It governs therefore the compression of wage differentials in course of the accumulation of human capital, see Teulings (1999). We shall show later on that this complexity dispersion parameter is also crucial in determining the magnitude of search frictions in the economy. There is a simple intuition for this result. The more easy worker types can be substituted between job types (substitution is perfect when the complexity dispersion parameter equals zero)[8], the lower the productivity loss due to suboptimal matching will be, and hence, the wider the matching sets will be. Because workers become less choosy, the expected unemployment duration will fall. We will apply this concept when discussing the simulation results.

3 Some characteristics of the equilibrium

[7] Consider the linear transform $s^+ = \sigma_0 + \sigma_1 s$. For the calculation of $\gamma(s^+) = r\text{"}[s^+(s)]/r'[s^+(s)]$, the parameter σ_0 drops out by differentiation and the parameter σ_1 cancels in the numerator and denominator.

[8] This concept should be distinguished from η, which measures substitution between the products of various c-types on commodity markets, see Teulings (1999).

Equations (6), (7) and (10) immediately reveal that it is difficult to achieve analytical results. They all involve the calculation of integrals of which both the integrand and the integration boundaries are endogenous. It is precisely this complexity which has prohibited progress in this type of models. Therefore, we attack this problem by focusing on second order Taylor expansions which will be discussed in detail in the next section. The following Definition and Theorem establish a number of characteristics of the equilibrium that are be crucial for our Taylor expansions to make sense.

Definition *A steady state search equilibrium is defined as a sextet* $\{r(s), g(c), h(s), p(c), m_c(s), m_s(c)\}$, *satisfying the functional equations (6), (7), (12), the two equations in (9), and (10).*

Since our production function is nonnegative, log supermodular, symmetric, continuous and at least twice differentiable, we refer to Shimer and Smith (1999) for an existence proof.[9]

Theorem 1 *Under the assumptions of 2.1, the following conditions hold in search equilibrium:*

1. $r(s)$, $p(c)$, *and* $x(s,c)$ *are twice differentiable;*
2. $r'(s) > 0$, $r''(s) > 0$, $p''(c) < 0$;
3. $x_{ss}(s,c) < 0$ *and* $x_{cc}(s,c) < 0$;
4. *the sets* $m_c(s)$ *and* $m_s(c)$ *are connect;*
5. *let* $c^-(s)$ *be the lower bound of* $m_c(s)$ *and let* $c^+(s)$ *be the upper bound; then* $c^{-'}(s) > 0$ *and* $c^{+'}(s) > 0$.

The proof of part 5 requires some notation which will play a crucial role in the subsequent analysis. We introduce functions which denote the values of c and s that maximize the match surplus $x(s,c)$ for a particular value of s and c respectively:

$$\begin{aligned} c(s) &\equiv \widetilde{c}\,|\,\{x(s,\widetilde{c}) \geq x(s,c), \forall c\} \\ s(c) &\equiv \widetilde{s}\,|\,\{x(\widetilde{s},c) \geq x(s,c), \forall s\} \end{aligned} \tag{17}$$

By this definition and part 1 of Theorem 1, $x_c[s, c(s)] = 0$ and $x_s[s(c), c] = 0$. Hence: $-p'[c(s)] = s$, $-p''[c(s)]\,c'(s) = 1$, $r'[s(c)] = c$, and $r''[s(c)]\,c'(s) = 1$. For future reference, we define the inverse function of $s(c)$: $s[d(s)] \equiv s$. By this definition we have $r'(s) = d(s)$ and $r''(s) = d'(s)$. The proof can be found in Appendix 1.

Theorem 1 provides a number of important characteristics of the equilibrium. Part 2 states that the characteristics of the Walrasian equilibrium (15) and the second order condition $r''(s) >$ 0 and the condition $p''(c) < 0$ carry over to the search equilibrium. The conditions on the second

[9] The proof is based on a fixed point argument of an appropriate map of the agent's continuous value functions into itself. Translated to our model: Given the CES utility function (12), condition (4) maps the reservation wages into matching sets, condition (10) maps matching sets $m_c(s)$ and $m_s(c)$ into unmatched densities and conditions (6), and (7) are the composite maps of values mapping unmatched workers and vacancies into new values. A search equilibrium is a fixed point of these mappings.

derivatives imply that the surplus $x(s, c)$ has at most a single interior maximum in c (and s) keeping s (and c) fixed (part 3 of theorem 1). Those maxima are defined above as $c\,(s)$ and $s\,(c)$ respectively. An inspection of (6) shows that this maximum must necessarily be positive, if vacancies of that c-type exist. Hence, there are at most two solutions for the equation $x(s, c) = 0$ (keeping either s or c fixed). By this feature and by the differentiability of $x(s, c)$, matching sets are connect. Their upper and lower bounds are upward sloping, so that "on average" better skilled workers are matched to more complex jobs. As mentioned before, Shimer and Smith (1999) give a more general proof for positive assortative matching to arise. They show that log-supermodularity of the production function is a sufficient general condition

Under the assumptions made, the best worker for a job of type c is an interior point of the matching set of c. The intuition for this is the following. Some workers in the matching set of this job will have more skills than the worker who generates the highest output. The threat point of those workers in the wage bargain is so high that the firm receives only a small share of the output. On the other hand, some workers in the job's matching set are less productive than the firm's favorite worker. Although the firm has a strong bargaining position when bargaining with such a low productivity worker, the total surplus is too small. The firm's favorite worker type lies in between the above described cases.

The situation is depicted graphically in Figure 1-3.

FIGURE 1 ABOUT HERE

Figure 1 shows the surplus (the area between the (log) value-added-locus and the reservation-wage-locus for a given job type c. The surplus reaches a maximum when the job is occupied by a type $s(c)$ worker. In the Walrasian case, all type c jobs will be matched with type $s(c)$ workers because only there, both loci are tangent ($r'(s) = c$ by the first order condition, and $p\,(c) + s\,(c)\,c = r\,[s\,(c)]$ by the zero profit condition). Figure 2 shows the surplus for a given worker type, s.

FIGURE 2 ABOUT HERE

When such a worker, s, meets a job type, $c^+(s)$ or $c^-(s)$, he is indifferent between this match or remaining unemployed. When he meets a job in $< c^+, c^- >$ the match value exceeds the option value of waiting for a better match plus his value of leisure B. Figure 3 shows the functions $s(c)$, $c(s)$, $c^-(s)$ and $c^+(s)$ in the s, c-space, for the economy as a whole.

FIGURE 3 ABOUT HERE

Obviously, the maxima $s(c)$ and $c(s)$ are in between the upper and the lower bound. The shaded area represents the surplus for worker type s_i. When $c = c\,(s)$ for a particular s, then it is not necessarily true that $s = s(c)$ for that particular c. In other words, the inverse of $s(c)$, denoted $d(s)$, is not necessarily equal to $c(s)$. Here, the following result will prove to be helpful:

$$d(s) = c(s) \Leftrightarrow \frac{d\,x\,[s, c(s)]}{ds} = 0 \tag{18}$$

The proof of this result is simple. By the definition of $c(s)$, $x_c\,[s, c(s)] = 0$. Hence, the result applies if $x_s\,[s, c(s)] = 0$. By the definition of $s(c)$, this is the case when $s = s\,[c(s)]$, or equivalently, when $d(s) = c(s)$. Q.E.D.

4 Using Taylor expansions for the integrals

The main obstacle to achieve progress in the analysis of equilibrium search models, of which the model in this paper is an example, is the evaluation of the integrals of the type that show up in the right hand side of equation (6). Our innovation is that we use a second order Taylor expansion to approximate the value of this integral. For this purpose, we benefit from the characteristics of the surplus $x(s,c)$, which has a negative second derivative in both its arguments. Since the calculation of the integral requires the evaluation of $x(s,c)$ around its maximum, it is a straightforward idea to approximate this function by a parabola. This approximation requires the maximum surplus $x[s,c(s)]$ to be relatively small. In the Walrasian case, this surplus equals zero. Hence, our approximation applies as long as we do not move too far from the competitive equilibrium. Using this idea for the integral in equation (6), and (3) yields:

$$\int_{m_c(s)} g(c)\left[e^{x(s,c)}-1\right] dc \cong \frac{4\sqrt{2}}{3} g\left[c(s)\right] \sqrt{c'(s)\, x\left[s,c(s)\right]^3} \tag{19}$$

$$\int_{m_s(c)} h(s)\left[e^{x(s,c)}-1\right] dc \cong \frac{4\sqrt{2}}{3} h\left[s(c)\right] \sqrt{s'(c)\, x\left[s(c),c\right]^3} \tag{20}$$

The derivation of these relations can be found in Appendix 2. Figure 4 provides a simple intuition for this result.

FIGURE 4 ABOUT HERE

First, we apply an approximation for the integrand: $e^{x(s,c)}-1 \simeq x(s,c)$. This approximation applies, like all our approximations, for small $x(s,c)$, that is, close the Walrasian optimum. Next, the maximum of the integrand of the domain of integration can be approximated by a second order Taylor expansion: $x[s,c(s)] \simeq -\frac{1}{2}x_{cc}[s,c(s)]\,\Delta^2$, where $\Delta = c^+(s)-c(s) = c(s)-c^-(s)$. We apply $x_{cc}[s,c(s)] = p''[c(s)] = -c'(s)^{-1}$. Hence, the surface of the rectangle 2(A+B) in Figure 4 equals $2\Delta x[s,c(s)] = 2\sqrt{2c'(s)x[s,c(s)]^3}$. Integration over a parabola yields the result that two thirds of this surface is below the parabola. The above argument implies that the average surplus of a worker of type s above her log reservation wage $r(s)$,area A, can be approximated by $\frac{2}{3}x[s,c(s)]$, or by a complementary argument, the average loss relative to the Walrasian optimum, area B is: $\frac{1}{3}x[s,c(s)]$.

By substituting these expansions in (6), (7), and (10), the model can be written as a system of three functional equations:

$$\begin{aligned} 1-\frac{B}{R(s)} &= \frac{4\sqrt{2}}{3}\frac{\lambda\beta}{\rho+\delta} g\left[c(s)\right]\sqrt{c'(s)\, x\left[s,c(s)\right]^3} \\ \frac{K}{R(s)} &= \frac{4\sqrt{2}}{3}\frac{\lambda(1-\beta)}{\rho+\delta} h(s)\sqrt{d'(s)^{-1}\, x\left[s,d(s)\right]^3} \\ \delta\left[\underline{l}(s)-h(s)\right] &= 2\sqrt{2}\lambda h(s) g\left[c(s)\right]\sqrt{c'(s)\, x\left[s,c(s)\right]} \end{aligned} \tag{21}$$

where we have substituted $d(s)$ for c for and $s[d(s)] = s$ for $s(c)$ in equation (7).

Although equation (21) is a great simplification compared to the system of equations (6), (7) and (10), the calculation of the equilibrium is still a huge task. We need to apply therefore two further approximations, which greatly simplify our system of equations. The first of these approximations has a clear justification, the next applies only under a particular assumption. Our approach will be to discuss the approximations and their implications extensively here and to test their applicability by comparing them with the numerical equilibrium for the full model. There is no harm in revealing at this point that our approximations do surprisingly well.

First, we apply the standard assumption that the unemployment rate is close to zero: $\frac{u}{1-u} \cong u$, and that this holds for each worker type, s: $\frac{h(s)}{\underline{l}(s)-h(s)} \cong h(s)$. Second, we assume that $d(s) = c(s)$. A legitimation for this assumption is that both $c(s)$ and $d(s)$ are in the set $m_c(s)$. In a frictionless labor market, this set converges to a single point, so that the assumption is no longer an approximation. This is the one-to-one correspondence result for the Walrasian version of the model which we discussed in Section 2.4. However, in the system of equations (21) both $x[s, c(s)]$ and $x[s, d(s)]$ show up. Since in the Walrasian case both $x[s, c(s)]$ and $x[s, d(s)]$ converge to zero, we have no clue about their relative magnitude. Equation (18) shows that the assumption that $c(s) = d(s)$ applies if and only if $x[s, c(s)]$ does not vary with s. These simplifications allow for the full characterization of the equilibrium that will be discussed in the next section.

5 The analysis of the equilibrium

5.1 The scale of the labor market

Using the relation $c(s) = d(s)$ and $\frac{h(s)}{\underline{l}(s)-h(s)} \cong h(s)$, the system (21) simplifies to:

$$x^*(s)^5 = \frac{81}{128} Q^2 B^*(s)^2 K^*(s)^2 \underline{l}(s)^{-2} c'(s) \tag{22}$$

$$\frac{h(s)}{\underline{l}(s)} = \frac{2}{3} \frac{\delta\beta}{\rho+\delta} B^*(s)^{-1} x^*(s) \tag{23}$$

$$g[c(s)] = \frac{2}{3} \frac{\delta(1-\beta)}{(\rho+\delta)} K^*(s)^{-1} x^*(s) c'(s)^{-1} \tag{24}$$

where $x^*(s) \equiv x[s, c(s)]$, $Q \equiv \frac{(\rho+\delta)^2}{\delta\beta(1-\beta)\lambda}$, $B^*(s) \equiv 1 - \frac{B}{R(s)}$, and $K^*(s) \equiv \frac{K}{R(s)}$. Equations (22) and (23) give a full characterization of the equilibrium for $\eta = \infty$.[10] The system gives a simple account of the forces governing the size of the search-surplus $x^*(s)$ in the economy, and its relation to the rate of unemployment, $\frac{h(s)}{\underline{l}(s)}$, and the number of vacancies per unit of labor supply, $g[c(s)]$. Since $x^*(s)$ is equal to the relative difference between maximum output attainable and the minimum output acceptable (i.e.: the reservation wage), it is a measure of the inverse of the match quality. The surplus $x^*(s)$ depends on a composite parameter Q. This parameter reflects a number of factors. First, the costs of waiting for a better match relative to the expected duration of the match, $\frac{(\rho+\delta)^2}{\delta}$, enter positively, because agents accept a lower match quality when the costs of waiting rise. Second, the distribution of bargaining power matters, $\beta(1-\beta)$. Match quality is maximal for $\beta = \frac{1}{2}$. Finally, the size of the labor market λ enters negatively. This captures the effect that agents trade a higher match quality for a lower

[10] For $\eta = 0$, $c\prime(s)$ remains to be determined.

acceptance probability when the offer arrival rate goes up due to an increase in the size of the market.

Besides Q, the surplus depends positively on one minus the replacement rate $B^*(s)$ and the ratio of capital costs to reservation wages $K^*(s)$. These effects capture the idea that match quality falls when the costs of unemployment and the costs of maintaining a vacancy go up. The distribution of labor supply $\underline{l}(s)$ enters negatively. This is a mirror image of the effect of λ. Whether one increases the size of a particular segment of the market, either by a general increase in supply or by a concentration of supply in that particular segment, in both cases match quality goes up by the same amount. For many distribution functions, there will be a lower density in both tails. Hence, search frictions are larger in both extremes of the skill distribution. Finally, the dispersion in job complexity per unit of the skill distribution, $c'(s)$, increases the surplus. Here, the intuition is that when complexity dispersion increases, the less substitutable will be worker types across jobs and the larger will be the cost of sub-optimal assignment. We shall return to this issue in greater depth in Section 7.1.

A remarkable conclusion of equation (22) is that the elasticities of the relations discussed above do not depend on any of the model's parameters. For example, one minus the replacement rate enters with an elasticity of $\frac{2}{5} = 0.4$. This result underscores the economic usefulness of applying Taylor expansions: the elasticities in this equilibrium search model are determined by the powers of the first non-vanishing terms of the three structural equations of the model, the equations for the values of search for job-seekers and firms and the flow equilibrium condition, see equation (21).

Both the unemployment and vacancy equations (23) and (24) respectively, are related to the surplus by simple linear relations, where we have to account for the Jacobian of the transformation from c to s, $c'(s)$, in the case of vacancies. The intuition for these linear relations is again that agents respond to any increase in search frictions along two channels: they spend more time in the search state, which results in an increase in unemployment and vacancies, and they reduce their reservation match quality, which yields an increase in $x^*(s)$. Only a change in β yields an asymmetric response of unemployment and vacancies. Again, there is a clear intuition for this result. The higher β, the larger the workers expected return to search is and the greater therefore their willingness to invest in search. This explains why the model breaks down if we attribute the whole surplus either to the workers ($\beta = 1$) or to firms ($\beta = 0$). In that case, the other side has no incentives for search whatsoever, so nobody will enter the market. This also explains why match quality is largest for $\beta = \frac{1}{2}$. Finally, an increase in the interest rate or a decrease in the expected duration of a job reduces unemployment relative to match quality because the returns on the investment in search are discounted more heavily.

An important issue is the magnitude of the returns to scale in the matching process that is implied by the model. Our contact technology implies a returns to scale elasticity of 2: doubling both the number of job seekers and vacancies quadruples the number of contacts. However, part of the effect of this increase in contacts on the number of realized matches will be undone by job seekers becoming more choosy about the jobs they are willing to accept (and mutatis mutandis the same for firms). The size of this effect can be obtained by considering the implications of a change in the parameter measuring total labor supply, $\lambda = \lambda^* \underline{L}$. By equations (22) and (23), where λ enters through Q, the elasticities of the surplus $x^*(s)$ and hence the unemployment rate $h(s)$ with respect to λ are equal to 0.4. The equation for the number of vacancies per unit of labor supply, $g[c(s)]$, has the same elasticity with respect to λ of 0.4. Hence, a 1 % increase in total labor supply increases the total number of vacancies and unemployed by (1-0.4)% = 0.6%. By the flow equilibrium and the constancy of the separation rate δ, the number of matches varies proportionally to λ (almost, up to a factor $1-u$). A 0.6 % increase in both inputs in the matching process yields therefore a 1 % increase in the number of matches. The returns to scale elasticity can then be calculated to be equal to $\frac{1}{0.6} = 1.66$. Hence, one third of the increasing returns in the contact technology are absorbed by a greater choosiness of job seekers and firms. Accounting for some downward bias in most estimates due to the mismeasurement of the scale concept and due to aggregation bias over time (see Burdett et al. (1994)), this number might well be within the confidence interval of most empirical studies of the matching function[11]

5.2 Efficiency of the bargaining process

In a single worker-single job world, efficiency can be achieved when the Hosios (1990) condition is satisfied: the workers' share of the match surplus is equal to his marginal contribution to the matching process. In a world with increasing returns, efficiency can never be achieved because there is insufficient output to reward each factor by its marginal contribution. The issue of efficiency gets more complicated when the heterogeneity of the workforce is taken into account because in that case, the distribution of the surplus over the different worker types also becomes an issue. However, matters are simplified by the fact that the utility function (12) is homothetic, which implies that we can separate between efficiency and distribution issues.

When we assume that costless redistribution policies are available, any efficient outcome must always maximize net aggregate output. Since there are no choice variables involved in ongoing matches and since employers always get their outside option due to the free entry condition, maximizing aggregate output is equivalent to maximizing the sum of the asset values of job seekers minus the costs for maintaining vacancies. This function $S[g(.), h(.)]$ therefore satisfies:

$$\begin{aligned} S[g(.), h(.)] &= \int_{s^-}^{s^+} h(s) R^o(s)\, ds - \int_{c^-}^{c^+} g(c) K\, dc = \\ & \int_{s^-}^{s^+} h(s) \left\{ B + \frac{\lambda}{\rho+\delta} \left[\int_{m_c^o(s)} g(c) \left[P(c) F(s,c) - R^o(s) \right] dc \right] \right\} ds \\ & - \int_{c^-}^{c^+} g(c) K\, dc \end{aligned} \tag{25}$$

where $R^o(s)$ denotes the asset value of a job seeker and where $m_c^o(s)$ denotes the matching set that is based on this asset value. The task of a social planner would be to choose $R^o(s)$ and $g(c)$ such that $S(\cdot)$ is maximized. The first order condition for $g(c)$ reads $S_{g(c)}[g(.), h(.)] = 0$. By duality, the first order condition for $R^o(s)$ is identical to the condition $S_{h(s)}[g(.), h(.)] = R^o(s)$, where the suffices refer to the partial derivatives of the integrands for each c and s respectively.[12]

[11] Increases in L, independent of λ^* are rare in reality. The following example illustrates this.When the EC countries opened their borders for workers from other countries, L increased enormously. However, the probability for a worker in Lisbon to meet a vacancy in Berlin is many times smaller than to meet a vacancy in Lisbon. In other words, $\lambda = \lambda^* L$, did not change very much.

[12] The analogue of the equation for $R(s)$ follows from the first order condition for $h(s)$. For the condition for K, the following equality is applied:
$\int_s h(s) \int_{m_c^o(s)} g(c)\, dc\, ds = \int_c g(c) \int_{m_s^o(c)} h(s)\, ds\, dc$ where $m_s^o(c)$ is defined similarly to $m_c^o(s)$.

The first order conditions for this problem can be rewritten in the form of equations (6) and (7), except that we have to replace β and $(1-\beta)$ by unity in the equations for $R(s)$ and K respectively because only then, workers and firms receive the full rewards of their marginal contribution of the matching process. There is a simple intuition for this result. In Pissarides' constant returns to scale world, a job seeker entering the labor market imposes a positive externality on employers (since their contact rate goes up) and a negative externality on other job seekers (since their contact rate goes down). In our increasing returns world, there is no negative externality, since the contact rate for workers is independent of the number of other job seekers that are wandering around. Hence, we should award workers the full surplus of the match to reward them for the positive externality they impose on employers. Mutatis mutandis the same argument applies for employers. In section 5.3, we present a simple technique which yields an approximation of the efficiency loss.

5.3 The efficiency loss due to search frictions

A second issue regarding the equilibrium is the implied efficiency loss due to search frictions. This loss as a fraction of total output consists of three parts: the lost production due to unemployment, the cost of maintaining vacancies and the cost of misallocation that arises because search frictions induce workers to lower their reservation wage so that they end up accepting jobs that are not fully optimal. The first factor can simply be calculated from the unemployment rate $\frac{h(s)}{l(s)}$ times the costs of unemployment relative to the reservation wage.[13] These costs are equal to one minus the replacement rate, $B^*(s)$. The second factor can be calculated from the number of vacancies that is maintained per unit s, $g\left[c\left(s\right)\right]c'(s)$, multiplied by the relative costs of maintaining a vacancy, $K^*(s)$. For the calculation of the final term, recall from Section 4 that the average loss relative to the optimal Walrasian allocation is ${}^1/{}_3x^*(s)$. Hence:

$$Loss_{Wal} \simeq \frac{1}{3}x^*(s)\left[\frac{2\delta\beta}{\rho+\delta}+\frac{2\delta(1-\beta)}{\rho+\delta}+1\right] \tag{26}$$

The three terms in square brackets reflect the respective losses. All losses vary therefore proportional to $x^*(s)$. Remarkably, the relative importance of unemployment and vacancies in the average loss compared to the Walras equilibrium are independent of $B^*(s)$ and $K^*(s)$. The ratio between both cost types is fully determined by the bargaining power parameter β. This is due to the fact that firms keep investing in vacancies till the costs of keeping the vacancy open are equal to their expected share in the future surpluses from an employment relation. Similarly, workers adjust their reservation wages such that the share in the expected surplus from search is equal to their reservation wage. The ratio of the costs of misallocation relative to the cost of unemployment and vacancies is $\frac{\rho+\delta}{2\delta}$: the higher the separation rate δ, the more often employment relations will dissolve and the more often workers will experience an unemployment spell; the higher the discount rate ρ, the more costly search is and hence, the greater will be misallocation.

[13] Here (and below, when discussing the cost of a vacancy), the proper normalization would be to use the actual wage $W(s,c)$ instead of $R(s)$. >From section 4: $\mathrm{E}_{m_c(s)}\left[\frac{W(s,c)}{R(s)}-1\right] \simeq \frac{1}{3}x\left[s,c(s)\right]$. Hence, the proper cost are: $B^*(s)+\frac{1}{3}x\left[s,c(s)\right]$. Substitution in equation (26) reveals that this is a second order effect in $x\left[s,c(s)\right]$.

From the expression for Q, it can be seen that the surplus $x^*(s)$ (and therefore also the efficiency loss) are proportional to the inverse of $\beta(1-\beta)$; hence, both are minimized for $\beta = 0.5$.[14] This result implies that according to most estimates of the distribution of bargaining power (e.g. Abowd and Lemieux, 1991) workers bargaining power is too low! Interestingly, recent estimates suggest workers' bargaining power to be lower in continental Europe with more centralized wage bargaining regimes (Teulings, 1998; Nickell, 1998) which suggests that the European problem is not that workers have too much bargaining power. From equation (23), unemployment is proportional to $\beta^{0.6}(1-\beta)^{-0.4}$; hence, it is minimized by letting $\beta \to 0$. However, this would not be optimal from a welfare point of view because the workers would accept jobs at which they are not very productive and employers would spend too many resources on maintaining vacancies.

5.4 Efficiency loss due to inadequate incentives

There is another way to decompose the efficiency loss: the costs of search frictions versus the costs of inadequate incentives due to the IRS contact technology, see the discussion in Section (2.3). Theoretically, a social planner could undo this second component by letting workers and firms participate as if they would get the full match surplus. The magnitude of this second component can then be calculated from the first order conditions of the maximization of (25). Deriving the equivalent of (22) for these equations is equivalent to replacing the factor $\beta(1-\beta)$ in the denominator of Q by unity. Hence, the surplus $x^*(s)$ would go down by a factor $\beta^{0.4}(1-\beta)^{0.4}$ if a social planner could implement adequate incentives. In the same vein, unemployment and vacancies change by: $\beta^{-0.6}(1-\beta)^{0.4}$, and $\beta^{0.4}(1-\beta)^{-0.6}$, respectively. The welfare loss of the decentralized search equilibrium compared to the social planner's search optimum reads[15]:

$$Loss_{SP} \simeq \frac{1}{3}\left[\frac{\rho+3\delta}{\rho+\delta} - \beta^{0.4}(1-\beta)^{0.4}\frac{\rho+5\delta}{\rho+\delta}\right] x^*(s) \tag{27}$$

This loss is minimized by setting β equal to one half, which mimics the conclusion of the previous paragraph. Comparing equation (26) to equation (27) reveals that for $\beta = 0.5$ and $\delta/\rho \to 0$, the inefficiency introduced by the lack of incentives would be compensated for by quadrupling the size of the labor market, i.e. the parameter λ. For other values of β, the inefficiencies are larger. The ratio of the social planners level of unemployment and the decentralized equilibrium level of unemployment is $\beta^{-0.6}(1-\beta)^{0.4}$, which is 1.15 for $\beta = 0.5$ or 1.41 for $\beta = 0.4$. Social planner's unemployment is therefore higher than unemployment in the decentralized market equilibrium. There is too low a reward for search activities which are produced under increasing returns. Hence, job seekers accept jobs too easily.

5.5 The trade off between bargaining power and the replacement rate

Burdett (1979), Diamond (1981), and Marimon and Zilibotti (1999) have shown that unemployment compensation can increase welfare, even when agents are risk neutral, by decreasing

[14] That large surpluses yield high inefficiencies might be strange at first sight. However, surplusses are defined relative to the reservation wage of the worker. Large surplus are due to low reservation wages.

[15] The first term within brackets is simply equal to Loss_{Wal} while the last term within the brackets is Loss_{Wal} with β and $(1-\beta)$ set equal to 1 (which would reflect unemployment and the stock of vacancies when incentives were adequate).

mismatch. In this section, we show that also in our model, unemployment compensation can fulfill a useful role by improving the expected match quality.

First note that the elasticity of unemployment with respect to one minus the replacement rate is -0.6 (-1 directly in the unemployment equation and 0.4 indirectly via $x^*(s)$). Meyer (1990) finds an elasticity of unemployment with respect to the benefit level of up to about minus unity for the United States. His source of variation is mainly structural variation in legislation between states. Hence, we feel comfortable to interpret his estimate as reflecting the elasticity of equilibrium unemployment. This estimate is consistent with our model when the replacement rate is 0.60. The model implies that the detrimental effect on unemployment goes up with every percent further increase in B. This might explain the European problems that have arisen for a large part after the construction of an extensive social security system.

A social planner has few instruments to implement an allocation that takes away the losses embodied in (27). Unemployment benefits might be the only instrument available that can alleviate the consequences of reservation wages being below their optimal value. For an analysis of this issue, we have to drop the interpretation of B as being the value of leisure since it needs to be funded. Suppose we pay unemployment benefits from an insurance premium that is proportional to earned wages. When we define $B^*(s)$ relative to the net reservation wage, this will have no further impact on the model, since the difference between the gross and the net value of $r(s)$ is a constant. However, our loss function, equation (26), has to take into account that the cost of unemployment relative to the reservation wage is no longer equal to $B^*(s)$ but to unity. In order to account for this difference, the first term between square brackets has to be divided by $B^*(s)$. Substituting equation (23) for $x^*(s)$ and minimizing the resulting expression with respect to $B^*(s)$ yields:

$$B^*(s) = \frac{3\delta\beta}{\rho + \delta + 2\delta(1-\beta)} \tag{28}$$

The optimal net replacement rate, $B/R(s) = 1 - B^*(s)$ is therefore a negative function of the bargaining power of workers. This fits the intuition: when workers have a high bargaining power, it does not make sense to further strengthen their position by providing them a luxurious outside option. For realistic parameter values, the optimal replacement rate is about 50 %.[16]

If a social planner could set both β and $B^*(s)$ jointly, he would make workers as choosy as possible by setting $B(s) = R(s) - \varepsilon$, to minimize mismatch. At the same time he would stimulate vacancy supply by rewarding the employers with almost the full share of the match surplus, hence $\beta = 0 + \varepsilon$.

Note however that in our model, B is the same for all worker types, hence a proper interpretation of (28) is that it is the optimal replacement rate for the average worker. Consequently, setting B at its optimal level (given β), may lead to a substantial replacement rate for lower educated workers. This is one explanation for the relatively high unemployment rates for low skilled workers in Europe.

5.6 Standard deviation of wages conditional on the reservation wage

In the Walrasian case, actual wages and reservation wages coincide. With frictions, actual log wages, denoted by $w(s)$, exceed $r(s)$ in almost all employment relations. This difference can be

[16] We took: $\beta = 0.4$, $\delta = 0.15$ and $\rho = 0.10$.

interpreted as the rents from search. The average rents can be calculated, again by a second order Taylor expansion analogous to the way we calculated the efficiency loss. This surplus, which is the complement of the efficiency loss, equals ${}^{2}/_{3}\beta\, x^{*}(s)$. Using the same technique of Taylor expansions, we can calculate the standard deviation of rents to be:

$$\text{Std.dev.}\,[w(s) - r(s)] = \frac{2}{3\sqrt{5}}\beta\, x^{*}(s)$$

For $\beta = 0.40$, this expression equals $0.12\, x^{*}(s)$. For reasonable values of $x^{*}(s)$ this is a small number, in particular when compared with the standard deviation of industry differentials. Krueger and Summers (1988) report a value of about 0.15. When these differentials are indeed rents, these results suggest that they cannot be attributed fully to search frictions. However, a complete analysis of the implications of search frictions for industry differentials requires a more complicated model than the one discussed here.

6 The corner problem

Burdett and Coles (1997) and Smith (1995) show in the context of a non-transferable utility (but otherwise similar) model how endogenous segmentation arises. The intuition for this result is that both sides of the market are only willing to match with all types which are higher than their reservation types. Consequently, the highest types are in the position of being most selective and will therefore only match with each other. When the highest types are no longer available for the middle types, the middle types can do no better than match with each other and finally, the lowest types are left over and will also match with each other.

At first sight, this segmentation result is a peculiarity of the non-transferable utility assumption, which does not transfer to models with Nash bargaining. This view is mistaken. Though less extreme than the model of Burdett and Coles, the model in this paper contains similar problems. The problem can best be explained by a graph. Figure 5 depicts the type space $\underline{L}(s), J(c)$ ($\underline{L}\,(s)$ is the distribution function associated with $\underline{l}$, $J\,(c)$ is the distribution of occupied jobs), that is: we transformed s and c such that every point in the graph has equal density.

FIGURE 5 ABOUT HERE

For simplicity, we assume that the Walrasian outcome is represented by the diagonal. The matching sets $m_c(s)$ can be plotted in the figure by their upper and lower bound, $c^{+}(s)$ and $c^{-}(s)$ respectively, which results in a band which goes from the south-west corner to the north-east corner, see panel A. As a thought experiment, consider how this band would look like when every worker has an equal probability of being matched. Since the upper and the lower bound are drawn to be parallel, the matching probabilities are constant for all s, except for the corners. Now look at the problem from the point of view of vacancies. In panel B, the most and the least complex vacancy will have a zero matching probability. Hence such a vacancy will never be opened in equilibrium. Only the vacancies in the interval (c^{--}, c^{++}) have a constant matching probability. Outside this interval, in the south-west and north-east corners of the type space, the matching probabilities decline quickly. Panel C depicts the case where each vacancy has an equal matching probability, but it is easy to see now that worker types at both ends of the support face the same problem as vacancies did before. Panel D sketches the only outcome that would give every job type and every worker type an equal matching probability, that is, Burdett and Coles complete segmentation result. By Theorem 1, this outcome can never be an equilibrium of the model in this paper, since for this, $c^{-}(s)$ and $c^{+}(s)$ have to be differentiable. Nevertheless, panel D is illustrative for the type of force that is at work in this economy. We refer to this phenomenon as the corner problem.

We offer a intuitive argument on how this economy deals with the corner problem. The way in which the economy deals with the problem depends on whether $\eta = \infty$ or $\eta = 0$. An example of the former case is sketched in Panel E. In that case, there is no necessity to open vacancies of all types. Let $c^{-,\lambda}$ be the least complex vacancy that will be opened with search frictions λ. Since the expected value of s in the matching set of this vacancy will be larger than of the least skilled worker, $\mathrm{E}\left[s|s \in m_s(c^{-,\lambda})\right] > s^{-}$, and since this vacancy is best adapted to its matching set if it, loosely speaking, maximizes the surplus of the expected value of its matching set. Hence $c^{-,\lambda} > c^{-}$

A similar argument applies to the north-east corner of the type-space. Search frictions will therefore truncate both the upper and the lower support of the domain of the vacancies that are opened. The density of jobs is depicted in the right quadrant of Panel E. There is a bulk at $c^{-,\lambda}$ and $c^{+,\lambda}$, to compensate for the deleted vacancies at the extremes of the support. This clustering in vacancies will lead to a clustering in of matching sets, similar to the market segmentation result of Burdett and Coles (1997). When search frictions are large, we have simulation results which show a further clustering, with a concentration of vacancies for some job types and no vacancies for neighboring job types. When $\eta = 0$, vacancies of all types are opened, since each job type is by the nature of the CES utility function a necessity in the composite consumption good. Hence, the corner problem for vacancies at both ends of the domain needs to be resolved by price adjustment. Commodity prices at both ends will therefore be higher with search frictions than they are in a friction-free Walrasian world.

The corner problem and the implied non-linearities in $c^{-}(s)$ and $c^{+}(s)$, depicted in Panel E, are obviously not picked up by the Taylor expansions presented in the previous section. These expansions ignore completely the boundary conditions imposed by the upper and lower support of s and c and which are the cause of the corner problem. However, although the corner problem leaves its marks in the corners of the simulations, the Taylor expansions turn out to do a good job in predicting $x^{*}(s)$ and $h(s)$.

7 Simulations

The value of numerical simulations is highly sensitive to their practical relevance. It is not very interesting to present all kinds of simulations for parameter values and functional forms which are completely unrealistic in practice. The simulation results in this section will therefore be shaped such that we have a reasonable idea about all the parameters that are involved. For this purpose, we proceed in three steps. First, we specify equation (22) using the log wage distribution instead of the skill distribution. While we have no idea about the shape of the skill distribution, we can directly observe the log wage distribution. This specification implies introducing the complexity dispersion parameter that has been discussed in section 2.4. This complexity dispersion parameter will be assumed constant along the domain of s: $\gamma(s) = \gamma$. Empirical estimates for γ are available from other studies. Next, we specify the model in terms of transformed skill and complexity variables s^{*} and c^{*} such that -for the assumption of a constant complexity dispersion parameter- they exhibit a linear relation with log wages in the Walrasian equilibrium, see the appendix for the details. Finally, log wages will be assumed to be distributed normally in the Walrasian equilibrium. Due to their relation with the transformed skill variable s^*, the latter will also be distributed normally.

7.1 The complexity dispersion parameter

Whereas equation (22) is convenient from an analytical point of view, it is less suitable for an empirical evaluation, since we have no idea about the units of measurement of s. As discussed in section 2.4, the scale of measurement of s can be changed by a linear transformation without changing the empirical implications of the model. It is therefore more useful to write the model in terms of an effect in log wages where we have no dispute about its unit of measurement. Applying this transformation to equation (22) yields:

$$\widehat{x}(r)^5 = \frac{81}{128} Q^2 \widehat{B}^* (r)^2 \widehat{K}^* (r)^2 \underline{f}(r)^{-2} \widehat{\gamma}(r) \tag{29}$$

where $\underline{f}(r)$ is the density function of log reservation wages r (weighted by persons) in the Walrasian equilibrium and where $\widehat{x}^* [r(s)] \equiv x^*(s)$, and mutatis mutandis the same for B^*, K^* and γ; $\gamma(s) \equiv r"(s)/r'(s)^2 = c'(s)/c(s)^2$, which is the complexity dispersion parameter discussed in Section 2.4. The denominator of $\widehat{\gamma}(r)$ comes in as the Jacobian from a transformation of variable: $\underline{l}(s) = \underline{f}[r(s)]\, r'(s)$. In economies that are characterized by comparative advantage of worker types across job types, the complexity dispersion parameter turns out to be the crucial determinant both of elasticities of substitution and complementarity and of the search frictions in this type of economy.

Teulings and Vieirra (1998) review the value for the complexity dispersion parameter implied by estimation results for the Netherlands, Portugal, and the United States. They conclude that its value is in the range 2.5-5, with the higher values being the more likely. The term parameter might be somewhat misleading here, for there is no reason why it should be constant either along the domain of r or across economies that differ in their skill distribution. Actually, Teulings (1999) shows that the complexity dispersion parameter increases due to the accumulation of human capital and decreases due to skill biased technological progress. Teulings and Vieirra (1998) report it to be upward sloping in Portugal. However, we are reluctant to take this as an established fact. Since we are not sure about the slope, we feel free to assume this parameter to be invariant with respect to r. We apply a value of 5 throughout the simulations in this section. In the appendix we transform s, and c such that their relation is lineair. In addition we make the assumption of a constant complexity dispersion parameter. The distribution of log wages is reasonably described by a log normal distribution. From the linearity of the equation of $r^*_{Wal}(s^*)$ in (40), we know that in that case s^* is also distributed normally. This distribution is conveniently specified as: $s^* \tilde{} N\left[0, (\gamma\sigma)^2\right]$. Hence, σ measures the standard deviation of log wages in the Walrasian equilibrium. This parameter is in the range of $0.30 - 0.60$ for most OECD countries. We shall apply a value of 0.60 in our simulations.

7.2 Simulation results

Our numerical simulations are based on a grid for s^* and c^* ranging from minus three till plus three times their standard deviation. We divide the domain of both variables in 100 intervals per standard deviation, yielding a matrix of 601×601. For $\eta = \infty$, the calculation requires two iteration loops. First, we take the distribution of vacancies, $g^*(c^*)$, as given and solve for $r^*(s^*)$. In the second loop, we adjust $g^*(c^*)$ in order to set expected returns on a vacancy equal to K/ρ. We take as a starting value for $g^{*,\text{start}}(c^*) = 0.10\, \underline{f}\,(r_{Wal})\,[c^* + \ln\gamma]$. This iteration procedure converges to an equilibrium quickly. As will be discussed later on, changes in the precision of the stopping criteria for the simulations have important consequences for the results. This is due to corner problem.

When $\eta = 0$, we have a third loop. We start by taking $p^*(c^*)$ and $g^*(c^*)$ as given and proceed in solving $r^*(s^*)$ in exactly the same way as for $\eta = \infty$. After having solved for $r^*(s^*)$, we calculate the implied $Y^*(c^*)$ and change log prices $p^*(c^*)$ to adjust the assignment of workers to jobs to solve for excess supply or demand of various commodities. Finally, after having solved for $r^*(s^*)$ and $p^*(c^*)$, we adjust $g^*(c^*)$ to satisfy the free entry condition that the value of a vacancy must be equal to K. This iteration procedure converges much slower. An important hurdle is the corner problem discussed in the previous section. The iteration procedure finds it difficult to adjust prices such that there is adequate supply in the tails of the distribution.

Since the model exhibits increasing returns, multiple equilibria cannot be ruled out. In that sense our simulations are merely illustrative. Whether we did encounter them in practice is not always easy to say, since it is hard to distinguish between multiple equilibria and lack of numerical precision. For high values of λ, there did not seem to be a problem but for low $\lambda's$, some peculiarities (crowding out effects of vacancies due to the corner problem) strongly suggest that there must be multiple equilibria. However, since the computed equilibria are all close to the analytical equilibrium of equation (29), we feel save to interpret our analytical equilibrium as point of gravitation for the numerical solutions.

The model has nine parameters: four in Q $(\delta, \rho, \lambda, \beta)$, the replacement rate B, the cost of a vacancy K, the elasticity of substitution η, the complexity dispersion parameter γ, and the standard deviation of log wages σ. We think of the unit of time for our simulations to be equal to one year. The baseline parameter values are: $\delta = 0.15$, $\beta = 0.40$, $\rho = 0.10$, $B = 0.09$, $K = 0.50$, $\gamma = 5.0$, $\sigma = 0.60$; we shall simulate the model for a range of values for λ using both $\eta = \infty$ and $\eta = 0$. The value of δ corresponds to an average job duration of seven years, which seems reasonable from an empirical point of view. The worker share in the surplus squares with the evidence regarding the United States and Canada, see Abowd and Lemieux (1993). By a previous assumption, the mean wage equals unity (since r_{Wal} is distributed normally with zero mean) in the Walrasian equilibrium. Hence, a worker earning a wage twice the standard deviation below the mean (0.30), faces a replacement rate of 0.09/0.30 = 30 %. In a more general model that includes capital as a factor of production, K would measure the reward for the production factor capital. For the mean wage of unity, $K = 0.5$ implies therefore that capital accounts for one third of value added $(\frac{0.5}{1+0.5})$. The value of the standard deviation of log wages and the complexity dispersion parameter have been discussed before.

Table 1 shows aggregate outcomes for wage dispersion, output loss, and aggregate unemployment and vacancies for different values of λ and for $\eta = 0, \infty$. We reduce λ by a factor 4 in every simulation, starting from a high value of 2500. We present the standard deviation of log reservations wages, the relative output loss compared to the Walrasian optimum (= log output under Walras - log {output in the numerical equilibrium - $uB + vK$}, see (26)), the unemployment rate and the number of vacancies per unit of labor supply (both in %). According to the analytical solution, the loss, unemployment and vacancies should increase linearly with $x^*(.)$, see equation (26), (23), and (24).

TABLE 1 ABOUT HERE

Furthermore, each reduction of λ by a factor 4 should increase these variables by a factor $4^{0.4} = 1.74$, see equation (22). Another prediction of our analytical approximations regards the u/v-ratio. According to (23) and (24), this ratio equals $(1-\beta)B^*/\beta K^*$, which is about 1.21. Finally, consider (26), which captures the total loss due to search frictions. Under the assumed parameter values, this loss equals 0.73 $x(s)$. Table 1 provides strong support for these implications. Based on the value of unemployment, $\lambda = 156$ is a reasonable value ($u \cong 5\%$). This implies that the average workers has $\lambda v \cong 15$ contacts a year. In traditional search models one needs much larger search frictions to get this level of unemployment.

Table 2 gives the simulation results for values of s^* with each column covering one standard deviation of the skill distribution.

TABLE 2 ABOUT HERE

We present the unemployment rate $u(s)$, the reservation wage, the maximum surplus, and the prediction errors of unemployment when applying the analytical approximations of equations (22) and (23).

First, consider the effect of search frictions on reservation wages. This effect can reasonably well be approximated by the effect on $x^*(s)$. Since for $\eta = \infty$, $p(c)$ is determined exogenously, the only factor that can account for a change in $x^*(s)$ in response to an increase in search frictions is the reservation wage. Hence:

$$\frac{d\,r(s)}{d\,\lambda} = -\frac{d\,x^*(s)}{d\,\lambda} \tag{30}$$

$x^*(s)$ is not the only channel along which $r(s)$ enters in equation (22); $r(s)$ also enters via $B^*(s)$ and $K^*(s)$. However, in the limiting case close to the Walrasian equilibrium, $x^*(s) \ll B^*(s), K^*(s)$. Hence, the relative effect of a 1% decline in real wages on $x^*(s)$ is much larger than the effect on $B^*(s)$ or $K^*(s)$. We shall therefore ignore the latter two effects. The results in Table 2 give strong support for this notion: for any value of λ, adding up $r(s)$ and $x(s)$ yields the value of $r_{Wal}(s)$. For $\eta = 0$, this simple relation between $r(s)$ and $x(s)$ breaks down, because in that case $p(c)$ is endogenous, see Table 3 and $r(s)$ reacts strongly to search frictions in response to the corner problem, as will be discussed below.

TABLE 3 ABOUT HERE

Second, the pattern of $x(s^*)$ along the domain of s^* is U-shaped. By the linear relation between $x(s^*)$ and $u(s^*)$, this pattern is repeated in the unemployment rate by type. The explanation for this phenomenon can be found in equation (29), where three factors depend on r_{Wal}: B^*, K^*, and $\underline{f}$. Numerically, the latter factor tends to dominate. The skill density is the highest around the mean and the lowest in the tails of the distribution. This implies that search frictions will be largest in the tails of the skill distribution. Due to increasing returns in contacts, search frictions are largest in the tails of the distribution where the market is relatively thin. These frictions come to surface both in a larger decline of reservation wages relative to the Walrasian benchmark and in a higher unemployment rate for the workers in the tails of the distribution. The effect is even stronger in the lower tail, due to the higher replacement rate and higher capital costs which are implied by the constancy of both B and K. This mechanism yields important implications in the field of regional economics, which deserve empirical testing. When search frictions are more important in the tails of the distribution, one would expect both high and low-skilled workers to cluster in high density regions (that is: in cities), so that the high value of λ can compensate for the low value of $\underline{f}$. The fact that wages are depressed in both tails of the distribution implies that the wage distribution is skewed toward the left compared to the Walrasian benchmark.[17] The overall effect of search frictions on the wage distribution is moderate for $\eta = \infty$, see Table 1.

Third, Figures 6 and 7 plot the matching sets and $c^* (s^*)$ and $d^* (s^*)$ in the s^*, c^*-space for $\eta = \infty$ and $\lambda = 2500$ and $\lambda = 39$, respectively, while Figure 8 and 9 plot matching sets for the same values of λ, but now for $\eta = 0$.

FIGURE 6 ABOUT HERE

FIGURE 7 ABOUT HERE

FIGURE 8 ABOUT HERE

FIGURE 9 ABOUT HERE

By equation (40), the Walrasian benchmark is represented by the diagonal. The larger search frictions, the wider is the matching set. Originally, we also plotted $c^* (s^*)$ and $d^* (s^*)$ but they lied so close to each other and to the Walrasian benchmark that it is not instructive to include them. Hence, the assumption $c^* (s^*) = d^* (s^*)$ is not a bad approximation. The waves in the matching sets in Figure 7 are due to corner problem that will be discussed extensively below. In the simulations with $\eta = 0$ we see first of all that the matching sets are much smoother. Figure 8 shows the case where $\lambda = 2500$ while in Figure 9 with $\lambda = 39$ we see that larger search frictions lead to wider matching sets. Due to imperfect substitution, the matching sets are much smoother now and the corner problem does not manifest itself in vacancy clustering.

Next, consider the accuracy of our analytical expansions. First, when $\eta = \infty$, our approximations are surprisingly accurate for $\lambda = 2500$. The maximum deviation for unemployment is 3.9% while for $x(s)$ it is 7.5% (both in the lower corners). As the search frictions get higher, our approximations become less accurate. This is mainly due to the corner problem, as can be seen from Figure 10 and 11.

FIGURE 10 ABOUT HERE

FIGURE 11 ABOUT HERE

Figure 10 portrays the output for various job types c relative to the output in the Walrasian equilibrium for $\lambda = 2500$. For a broad range of jobs in the middle of the distribution, this output ratio is close to unity. Only in the corners, clustering due to the corner problem is clearly observable. For $\lambda = 39$, search frictions are so important that the process of clustering is clearly manifest along the whole range of c^*.[18]. For $\eta = 0$, our approximations for unemployment are fairly accurate. For small search frictions the model almost exactly mimics the simulation outcomes. Only when search frictions are very large our prediction for the lowest skill groups become less accurate. The predictions of $x(s)$ are also good. The reason for the lower accuracy of our approximations $\eta = \infty$ is that our Taylor approximations do not account for this clustering of vacancies. However, in the more realistic case with imperfect substitution, there will be no clustering because jobs of all types have to be opened. Commodity prices will then take care of the corner problem. This is documented in Figure 12, which plots commodity prices relative to their level in the Walrasian equilibrium.

[17] Interesting, this mechanism provides a countervailing force against the skewness to the right implied by the comparative advantage mechanism, see Sattinger (1975).

[18] Only for a really sharp stop criterium for the simulations of 0.00001, this pattern emerges in our simulations. We therefore expect that for values of η slightly above 0, the clustering will disappear.

FIGURE 12 ABOUT HERE

Prices are higher in both corners than in the intermediate range, in particular for the upper tail. Obviously, the substantial price increases in the upper tail pushes up reservation wages. Hence, equation (30) does not apply for the case that $\eta = 0$, which can be checked from Table 3 ($r(s) + x(s) \neq r_{Wal}(s)$). Hence, the combination of search frictions and imperfect substitution leads to a substantial increase in wage dispersion, see Table 1.

8 Final Remarks

The main contribution of this paper is to supply a new tool for analyzing equilibrium search models. Our method of applying second order Taylor expansions to evaluate the integrals over the matching sets proved to be useful in the sense that it gave us an approximate characterization of a search equilibrium which was hitherto completely untractable. This approximation turns out to be accurate, in particular for the middle groups, for low search frictions and for the case where the produced commodities are less than perfect substitutes. This approach also gives us various predictions on economic relations between unemployment, vacancies, efficiency losses due to various causes, the effect of benefits on the unemployment rate, the optimal benefit level and the like.

An important extension to the model that comes to mind is to allow for on the job search. Since workers match with their favorite job only by accident, there are incentives to continue search. In the extreme case when on the job search is costless, workers will accept any job that pays a wage which is higher than B and continue searching for better jobs. A related extension is to endogenize the separation rate δ. Consider for example a simple model with automatic on-the-job accumulation of skill, that is, the standard Mincer model: the skill level s goes up as the worker accumulates experience. Due to the comparative advantage structure, the complexity of the optimal job type will also move up. Sooner or later, the present job will no longer satisfy the reservation match constraint and the worker will separate to look for a better match.

In addition, it is interesting to further explore the implications of this model for the spacial structure of the economy. Other things equal, search frictions have the strongest downward effect on wages in both tails of the skill distribution. Hence, the wage distribution is skewed to the left relative to its Walrasian counterpart. Since search frictions are more severe in rural areas than in densely populated metropolitan labor markets, wages will be most depressed in the tails of the distribution in rural areas, again, other things equal. This will stimulate mobility. Both low and high skilled workers will leave the countryside and move towards the big cities, leading to a wider skill distribution in metropolitan areas. Hence, one explanation for the existence of cities is that they have a comparative advantage in matching extreme types. In Teulings and Gautier (2000), we give evidence that this is indeed the case.[19]

9 Literature

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[19] This does not only hold for the labor market but also for example for the marriage market and the goods market.

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1 Appendices

1.1 The proof of Theorem 1

Part 1: The continuity of $\mathbf{R}(\mathbf{s})$:

By equation (6) we have:

$$\lim_{h\to 0}\Delta R(s) = \lim_{h\to 0}\frac{\lambda\beta}{\rho+\delta}\left\{\begin{array}{c}\int_{m_c(s)} -g(c)\Delta R(s)\,dc+\\ \int_{m_c(s+h)-m_c(s)} g(c)\left[P(c)F(s+h,c)-R(s+h)\right]dc\end{array}\right\} \tag{31}$$

where $\Delta R(s) \equiv R(s+h) - R(s)$. Suppose that $\Delta R(s) \geq 0$. Then the first term on the right hand side is negative. The second term is zero, because a necessary condition for c to be in the set $m_c(s+h) - m_c(s)$ is $x(s+h,c) - x(s,c) > 0$. Since $\lim_{h\to 0} x(s+h,c) - x(s,c) = -\Delta R(s)$, this condition is never satisfied. Hence, the only way for equation (31) to apply is that $\lim_{h\to 0}\Delta R(s) = 0$. Similar arguments prove the other statements in part 1 of theorem. Q.E.D.

Part 2: The sign of the derivatives of $\mathbf{R(s)}$ and $\mathbf{P(c)}$:

Differentiating equation (6) yields:

$$r'(s)R(s) = \frac{\lambda\beta}{\rho+\delta}\int_{m_c(s)} g(c)\left[cP(c)F(s,c) - r'(s)R(s)\right]dc + \text{ effect via } m_c(s) \tag{32}$$

The second term on the right hand side is zero by the envelope theorem (by the definition of $m_c(s)$, $P(c)R(s,c) - R(s) = 0$ at the boundaries of the integration interval). Bringing the term with $r'(s)$ in the integrand to the right hand side of the equation shows $r'(s) > 0$. Q.E.D.

Dividing equation (32) by $R(s)$ and differentiating the result yields:

$$r''(s) = \frac{\lambda\beta}{\rho+\delta}\int_{m_c(s)} g(c)\left[c\left\{c - r'(s)\right\}\frac{P(c)F(s,c)}{R(s)} - r''(s)\right]dc + \text{ effect via } m_c(s) \tag{33}$$

The effect via the boundaries of integration is necessarily positive in this case: ignoring the effect via the boundaries yield the second derivative of the value of job search when workers would not adjust their search strategy to the increase in s. Adjusting the search strategy to the optimal strategy can only increase the value of search. Rewriting (32) as:

$$\begin{aligned} r'(s) &= \frac{\lambda\beta}{\rho+\delta}\int_{m_c(s)} g(c)\left[\{c - r'(s)\}\frac{P(c)F(s,c)}{R(s)} + r'(s)\left\{\frac{P(c)F(s,c)}{R(s)} - 1\right\}\right]dc = \\ &= \frac{\lambda\beta}{\rho+\delta}\int_{m_c(s)} g(c)\{c - r'(s)\}\frac{P(c)F(s,c)}{R(s)}dc + r'(s)\left[1 - \frac{B}{R(s)}\right] \end{aligned} \tag{34}$$

where the second equality follows from substituting the final term by the original equation (6). Write this integral as: $Q\int_{m_c(s)} \frac{q(c)}{Q}\left\{c - r'(s)\right\}dc$,or $Q\ \mathrm{E}_q\left[c - r'(s)\right]$where: $Q = \int_{m_c(s)} q(c)\,dc$ and where E_q is the expectation operator with respect to the density function $\frac{q(c)}{Q}$. Likewise, the first term of the integral in (33) can be written as: $Q\int_{m_c(s)}\frac{q(c)}{Q}c\left\{c - r'(s)\right\}dc = V_q(c) + E_q(c)^2 - E_q(c)r'(s) = V_q(c) + E_q(c)\left[E_q(c) - r'(s)\right]$, where $V_q(c)$ is the variance with respect to the density function $\frac{q(c)}{Q}$. Finally note that: $Q\left\{\mathrm{V}_q\left[c\right] + \mathrm{E}_q\left[c - r'(s)\right]\right\} > Q\ \left[E_q c - r'(s)\right] > 0$. Q.E.D.

The proof of $p''(c) < 0$ follows immediately by differentiating twice equation (6) for K with respect to c. Q.E.D.

Part 3: $x_{ss}(s,c) = -r''(s) < 0$, $x_{cc}(s,c) = p''(c) < 0$. Q.E.D.

Part 4: the connectedness of $m_c(s)$ and $m_s(c)$:

$m_c(s)$ is defined by $x(s,c)>0$. By part 1 and part 3, $x(s,c)$ has a unique maximum (either interior or exterior). If $m_c(s)$ is non-empty, this maximum has to be positive. Also by part 1 and 3, the equation $x(s,c)=0$ for fixed s has at most two roots, one above the location of the maximum (defined as $c^+(s)$) and one below it ($c^-(s)$). All c's in between are element of $m_c(s)$, all others c's are not. A similar argument applies to $m_s(c)$. Q.E.D.

Part 5: $c^{-'}(s)>0$ and $c^{+'}(s)>0$:

Taking the total differential of functional equation $x\left[s,c^-(s)\right]=0$ yields $c^{-'}(s)=-\frac{x_s\left[s,c^-(s)\right]}{x_c\left[s,c^-(s)\right]}$. Since $c^-(s)<c(s)$ and by part 3, $x_c\left[s,c^-(s)\right]>0$. By part 1 and 2 and the definition $s(c)$, $s'(c)>0$. Hence: $s<s\left[c^-(s)\right]$, or: $s^+[c^-(s)]=s$ (the point $c^-(s)$ is the lower bound of $m_c(s)$ but the upper bound of $m_s(c)$). Hence, by part 3, $x_s[s,c^-(s)]<0$. Q.E.D.

1.2 The Taylor expansion of the integrals

Rewrite (6) as:

$$1-\frac{B}{R(s)}=\frac{\beta\lambda}{\rho+\delta}\int_{m_c(s)}g(c)\left[e^{x(s,c)}-1\right]dc$$

and note that $x_c\left[s,c(s)\right]=0$ and that $x_{cc}\left[s,c(s)\right]=p''\left[c(s)\right]=-1/c'(s)$. Define $\Delta=c-c(s)$, $\Delta^-=c^-(s)-c(s)$, and $\Delta^+=c^+(s)-c(s)$. Then:

$$\begin{aligned}&\int_{m_c(s)}g(c)\left[e^{x(s,c)}-1\right]dc \qquad (35)\\ =\ & g\int_{\Delta^-}^{\Delta^+}\left[1+\frac{g'}{g}\Delta+o\left(\Delta\right)\right]\left[E\left(1-\frac{1}{2c'}\Delta^2+o\left(\Delta^2\right)\right)-1\right]d\Delta\end{aligned}$$

where $E\equiv e^x$ with $x\equiv x\left[s,c(s)\right]$ and where g and $g'$are evaluated at $c(s)$ and where c' denotes $c'(s)$. The arguments of these functions are suppressed for convenience. The integrand can be written as: $(E-1)\left[1+\frac{g'}{g}\Delta+o\left(\Delta\right)\right]-\frac{1}{2}c'E\left[\Delta^2+\frac{g'}{g}\Delta^3+o\left(\Delta^3\right)\right]$. Furthermore $E-1=x+o\left(x\right)$ and $x[s,c(s)]=\frac{1}{2c'}\Delta^{\#^2}+o\left(\Delta^{\#^2}\right)$, where $\Delta^{\#}\equiv c^{\#}(s)-c\left(s\right), (\#)=(-),(+)$. Hence:

$$E-1=\frac{1}{2c'}\Delta^{\#^2}+o\left(\Delta^{\#^2}\right)$$

Furthermore $-\frac{1}{2c'}\Delta^{+^2}+o(\Delta^{\#^2})=\frac{1}{2c'}\Delta^{-^2}+o(\Delta^{\#^2})$ and therefore:

$$-\frac{1}{2c'}(\Delta^++\Delta^-)(\Delta^+-\Delta^-)=o(\Delta^{\#^2})\Rightarrow\Delta^+-\Delta^-=o(\Delta^{\#})$$

Substituting these results in equation (35) yields:

$$\begin{aligned}&g\int_{\Delta^-}^{\Delta^+}\left[1+\frac{g'}{g}\Delta+o\left(\Delta\right)\right]\left[E\left(1-\frac{1}{2c'}\Delta^2+o\left(\Delta^2\right)\right)-1\right]d\Delta \qquad (36)\\ =\ & g\left[(E-1)\left(\Delta+o(\Delta)\right)-\frac{1}{2c'}E\left(\frac{1}{3}\Delta^3+o\left(\Delta^3\right)\right)\right]_{\Delta=\Delta^-}^{\Delta^+}\\ =\ & \frac{1}{2c'}g\left[\frac{2}{3}\Delta^3\right]_{\Delta=-\Delta^-}^{\Delta^+}+o\left(\Delta^{\#^3}\right)=\frac{2}{3c'}g\Delta^{\#^3}+o\left(\Delta^{\#^3}\right) \qquad (37)\end{aligned}$$

Applying $\Delta^{\#} = \sqrt{2xc' + o\left(\Delta^{\#}\right)}$ yields equation (19) in the text. Q.E.D.

1.3 The respecification in terms of s^* and c^*

We look for a specification of $\underline{q}\left(c\right)$ which satisfies the assumption of a constant complexity dispersion parameter. It is convenient to start with the case $\eta = \infty$, since then output prices are effectively exogenous: $p\left(c\right) = \underline{q}\left(c\right)$. The following specification meets our demands:

$$\underline{q}(c) = \frac{1}{\gamma}\ln\left(\gamma c\right) - c \tag{38}$$

Hence, for the Walrasian benchmark, $p'\left[c(s)\right] = \frac{1}{\gamma}c(s)^{-1} - 1 = -s$, or $c(s) = \frac{1}{\gamma}(1-s)^{-1}$. The complexity dispersion parameter, $\gamma(s) = \frac{c'(s)}{c(s)^2}$, is therefore indeed constant:

$$\gamma(s) = \gamma \tag{39}$$

Integrating $r'(s) = c(s)$ and applying the zero profit constraint $p(c) + sc - r(s) = 0$ yields the locus of log reservation wages in the frictionless Walrasian world, denoted by $r_{Wal}(s)$:

$$r_{Wal}(s) = -\frac{1}{\gamma}\ln\left(1-s\right)$$

It is convenient for the presentation of the simulation results to transform both s and c: $s^* \equiv -\ln\left(1-s\right)$ and $c^* \equiv \ln c$. Applying this transformation yields a model which is almost linear:

$$\begin{aligned} c^*\left(s^*\right) &= s^* - \ln\gamma \\ r^*_{Wal}\left(s^*\right) &= \frac{1}{\gamma}s^* \end{aligned} \tag{40}$$

where $c^*(s^*) \equiv c^*$ and $r^*(s^*) \equiv r$. Hence, the relations between s^*, c^*, and r are linear under the assumption of a constant complexity dispersion parameter. The slope of $c^*\left(s^*\right)$ is even equal to unity. We present the simulation results for the matching sets in the s^*, c^*-space instead of s, c-space, since the linearity of the relations in the Walrasian equilibrium makes the results more easy to judge.

For $\eta = 0$, the above derivation cannot be applied directly, since commodity prices are endogenous. However, it is a straightforward task to derive the implied function for $\underline{q}\left(c\right)$ from the assumption above, using equation (13). For this model we present simulation results in the next section.

2 Figures

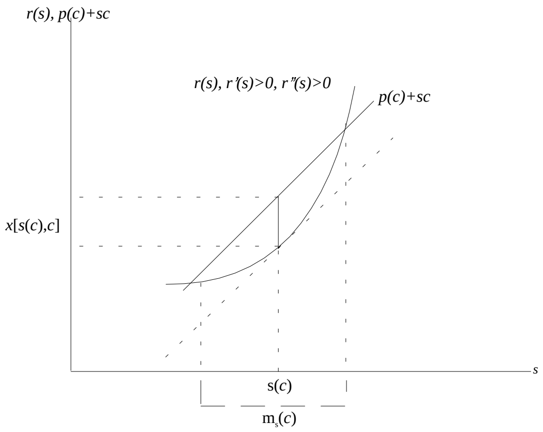

Figure 1: Walras versus search frictions for a given job type c.

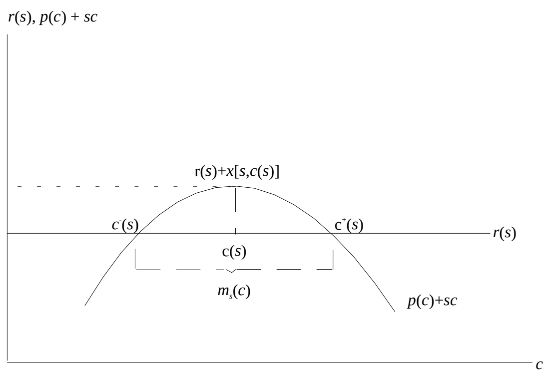

Figure 2: Approximation of the search surplus for a given worker type s

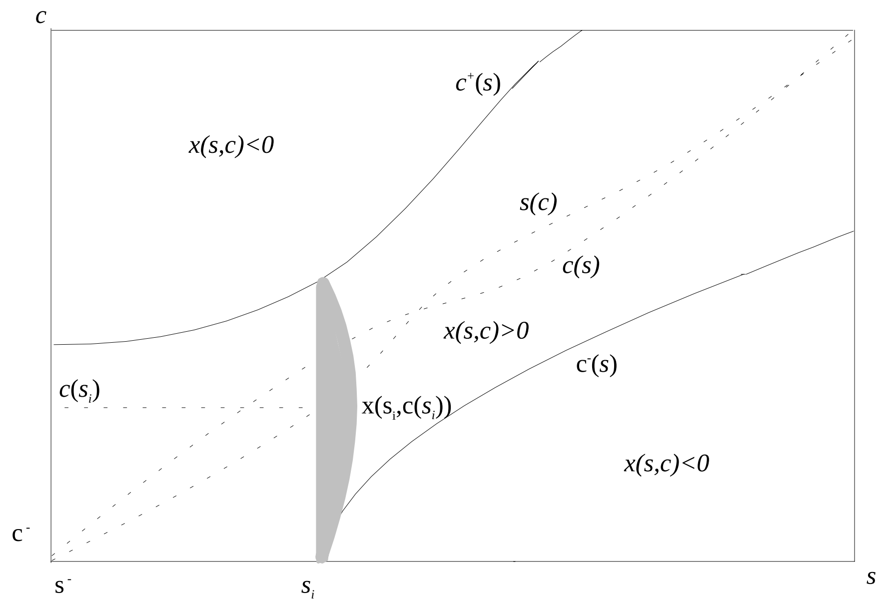

Figure 3: Matching sets, $c(s)$, and $s(c)$

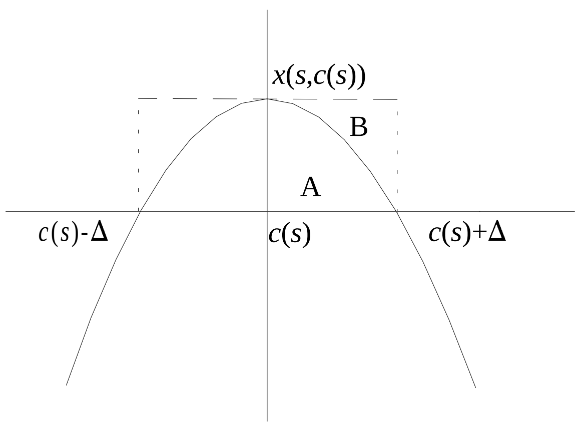

Figure 4: Taylor approximation of the surplus for a given worker type s

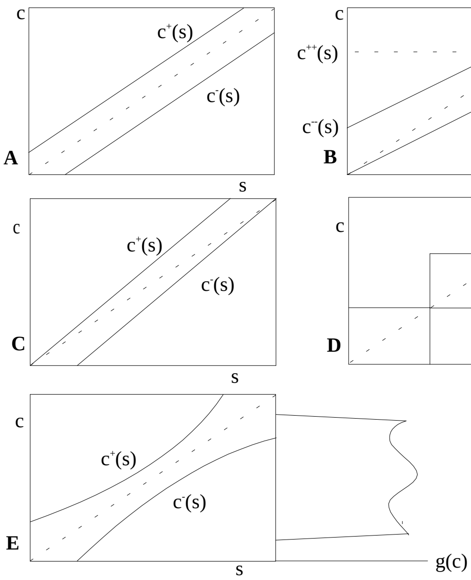

Figure 5: Illustration of the corner problem

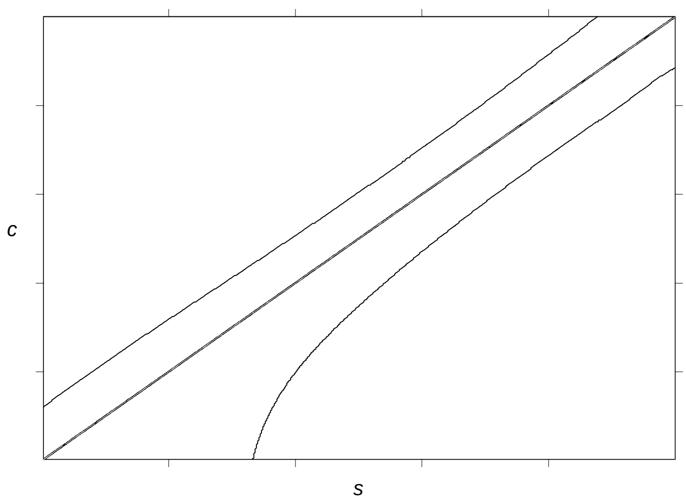

Figure 6: Matching sets for $\lambda = 39$ and $\eta = 0$

3 Tables

η	λ	2,500	625	156	39
∞	stdev r	0.61	0.62	0.63	0.63
	loss	0.05	0.09	0.15	0.26
	aggr. u(%)	1.5	2.8	5.3	9.9
	aggr. g(%)	3.7	6.1	10.2	14.6
0	stdev r	0.65	0.69	0.73	0.78
	loss	0.04	0.06	0.11	0.17
	aggr. u(%)	1.6	3.0	5.7	11.2
	aggr. g(%)	3.8	6.4	10.6	16.5

Note: loss is output loss (in logs) due to search frictions

Table 1: Aggregate outcomes for different values of λ and η

	s^*	-6.0	-3.0	0.0	3.0	6.0
$\lambda = 2500$	u(%)	4.8	1.8	1.1	1.0	1.4
	$r(s)$	-1.39	-0.69	-0.06	0.54	1.12
	$x(s)$	0.18	0.09	0.06	0.06	0.09
	error $x(s)$ %	-7.5	-3.5	-2.4	-2.3	-5.9
	error $u(s)$ %	-3.9	-1.2	-0.8	-1.1	-0.6
$\lambda = 625$	u(%)	10.2	3.9	1.8	1.8	2.9
	$r(s)$	-1.47	-0.75	-0.10	0.48	1.07
	$x(s)$	0.27	0.14	0.09	0.10	0.12
	error $x(s)$ %	-22.10	-13.9	-19.8	-8.4	-23.5
	error $u(s)$ %	10.0	23.4	-4.0	1.0	15.0
$\lambda = 156$	u(%)	25.3	7.6	2.7	3.0	3.1
	$r(s)$	-1.75	-0.90	-0.16	0.41	0.96
	$x(s)$	0.55	0.30	0.16	0.19	0.23
	error $x(s)$ %	-10.7	1.6	-13.6	0.37	-18.2
	error $u(s)$ %	21.8	25.2	-19.3	-7.0	-33.0
$\lambda = 39$	u(%)	70.3	10.4	5.6	3.4	6.8
	$r(s)$	-2.2	-0.97	-0.25	0.25	0.72
	$x(s)$	0.94	0.36	0.23	0.34	0.25
	error $x(s)$ %	10.2	-30.2	-31.5	-3.5	-52.6
	error $u(s)$ %	-13.3	-5.4	-9.1	-42.6	-23.7

Table 2: Simulation results for different worker skill groups when $\eta = \infty$

Table 3: Simulation results for different worker skill groups when $\eta = 0$

	s^*	-6.0	-3.0	0.0	3.0	6.0
$\lambda = 2500$	$u(\%)$	5.3	1.9	1.1	1.0	1.4
	$r(s)$	-1.48	-0.77	-0.10	0.55	1.22
	$x(s)$	0.18	0.09	0.06	0.06	0.09
	error $x(s)$ %	-8.21	-3.86	0.30	-1.68	2.83
	error $u(s)$ %	-1.17	0.16	0.65	0.41	0.58
$\lambda = 625$	$u(\%)$	10.8	3.5	2.0	1.8	2.5
	$r(s)$	-1.64	-0.88	-0.18	0.51	1.21
	$x(s)$	0.31	0.16	0.11	0.11	0.15
	error $x(s)$ %	-12.21	-4.58	-0.82	1.15	2.07
	error $u(s)$ %	-3.9	-1.2	-0.8	-1.1	-0.6
$\lambda = 156$	$u(\%)$	23.0	6.9	3.7	3.3	4.3
	$r(s)$	-1.89	-1.08	-0.31	0.43	1.20
	$x(s)$	0.52	0.29	0.20	0.19	0.26
	error $x(s)$ %	-14.19	-7.27	-1.94	0.27	3.35
	error $u(s)$ %	-6.31	2.15	2.34	3.88	4.44
$\lambda = 39$	$u(\%)$	45.6	14.6	7.4	6.1	7.7
	$r(s)$	-2.14	-1.39	-0.55	0.28	1.15
	$x(s)$	0.80	0.50	0.36	0.34	0.46
	error $x(s)$ %	-14.65	-13.30	-5.65	-0.44	1.53
	error $u(s)$ %	-30.44	0.73	3.18	3.83	4.14