

# The Cross-section of Firms over the Business Cycle: New Facts and a DSGE Exploration

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# The Cross-section of Firms over the Business Cycle: New Facts and a DSGE Exploration

## Abstract

Using a German firm-level data set, this paper is the first to jointly study the cyclical properties of the cross-sections of firm-level real value added and Solow residual innovations, as well as capital and employment adjustment. We find two new business cycle facts: 1) The cross-sectional standard deviation of firm-level innovations in the Solow residual, value added and employment is robustly and significantly countercyclical. 2) The cross-sectional standard deviation of firm-level investment is procyclical. We show that a heterogeneous-firm RBC model with quantitatively realistic countercyclically disperse innovations in the firm-level Solow residual and non-convex adjustment costs calibrated to the non-Gaussian features of the steady state investment rate distribution, produces investment dispersion that positively comoves with the cycle, with a correlation coefficient of 0.58, compared to 0.45 in the data. We argue more generally that the cross-sectional business cycle dynamics impose tight empirical restrictions on structural parameters and stochastic properties of driving forces in heterogeneous-firm models, and are therefore paramount in the calibration of these models.

JEL Code: E20, E22, E30, E32.

Keywords: Ss model, RBC model, cross-sectional firm dynamics, lumpy investment, countercyclical risk, aggregate shocks, idiosyncratic shocks, heterogeneous firms.

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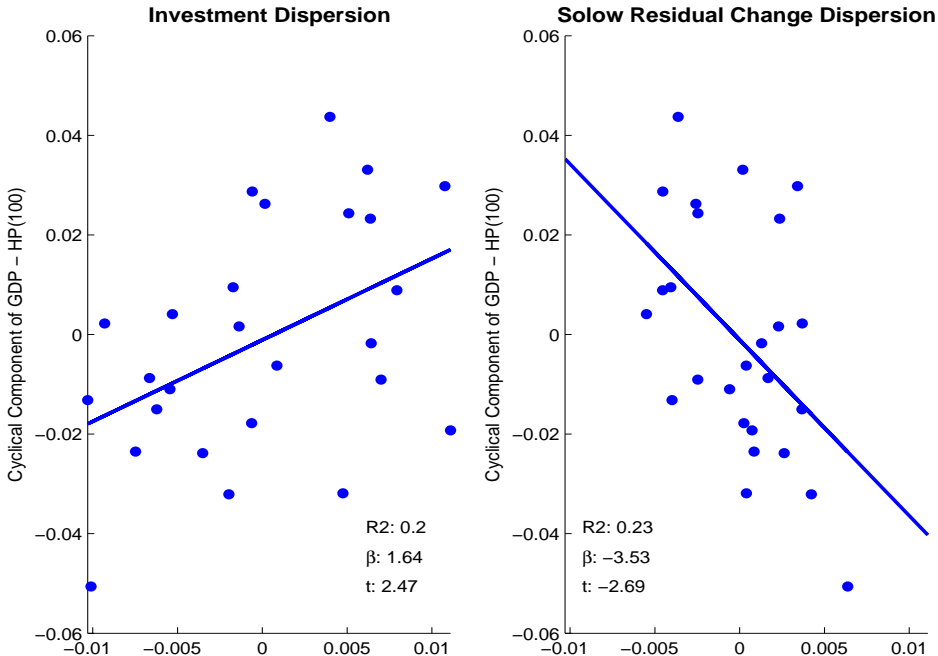
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# 1 Introduction

The cross-section of firms – more specifically the dispersions of change rates of firm-level output, capital, employment and Solow residuals – display stark cyclical patterns. This paper systematically documents the cyclical properties of these moments of the cross-section of firms. Using the balance sheet data set of Deutsche Bundesbank (USTAN) – a private sector, annual, firm-level data set that allows us to investigate 26 years of data (1973-1998), in which the cross-sections of the panel have over 30,000 firms per year on average –, we show that the cross-sectional standard deviations of the firm-level innovations in the Solow residual, value added and employment are robustly and significantly countercyclical, as measured by the contemporaneous correlation with the cyclical component of aggregate output. In contrast, the cross-sectional standard deviation of firm-level investment rates is robustly and significantly procyclical. These results hold when different filtering methods are used, as well as the cross-sectional interquartile range as a measure of dispersion. They are also robust to using cyclical indicators other than aggregate output and to various changes in the sample selection criteria. Figure 1 illustrates these two new business cycle facts (see Appendix A.5 for a time series graph of the investment rate dispersion):

Figure 1: Cross-sectional Dispersion of Firm-Level Investment Rates and Solow Residual Innovations



It is clear that this finding is incompatible with a simple frictionless model of the firm with ex ante homogeneous firms, as the latter would imply that the stochastic properties of the driving force – in this case dispersion in the innovations to firm-level Solow residuals – are at least qualitatively inherited by the outcome variables. Indeed, we show that such a model would lead to a counterfactual correlation of -0.54 between investment dispersion and aggregate output, compared to 0.45 in the data. We propose a heterogenous-firm RBC model with persistent idiosyncratic productivity shocks and lumpy capital adjustment to explain both qualitatively and quantitatively the procyclicality of investment dispersion, even in the presence of countercyclical second-moment shocks in the driving force. The basic intuition, why lumpy capital adjustment is at least qualitatively a suitable candidate to explain this fact, can be glanced from the simple Ss-model in Caplin and Spulber (1987):

**Proposition:**

*In a one-sided Ss-model a la Caplin and Spulber in steady state with a uniform gap- distribution, fixed optimal adjustment policy  $S - s$  and shock  $\Delta z > 0$ , the variance of adjustments is increasing in  $\Delta z$  if and only if the fraction of adjusters is smaller than 0.5.*

Proof:

As is well known, average adjustment in this environment is  $\Delta z$ . From this, it follows that the variance of adjustment is given by:  $(0 - \Delta z)^2 \left(1 - \frac{\Delta z}{S-s}\right) + ((S - s) - \Delta z)^2 \left(\frac{\Delta z}{S-s}\right) = \Delta z(S - s - \Delta z)$ , which is increasing in  $\Delta z$  if and only if  $\frac{\Delta z}{S-s} < 0.5$ , where  $\frac{\Delta z}{S-s}$  is the fraction of adjusters.

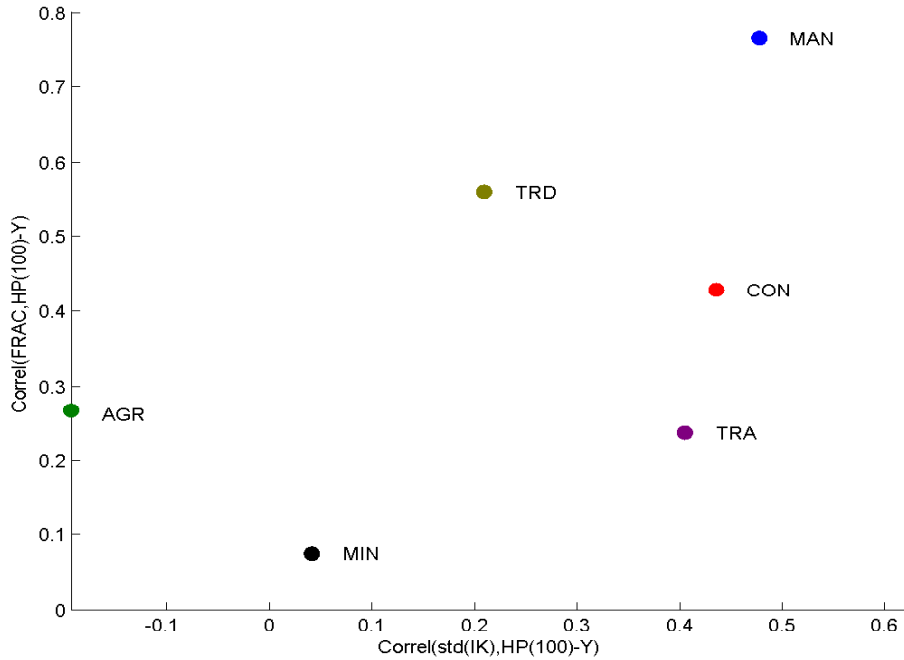
This example shows that with sufficient inertia the comovement of the extensive margin with the cycle leads to a procyclical dispersion of adjustment, as in this simple model all the dynamics are driven by the extensive margin. The intensive margin of adjustment,  $S - s$ , is fixed by assumption. We will show that in a more realistic model a positive extensive margin effect is still operative and can explain the observed procyclicality of investment dispersion.

Figure 2 displays sectoral variation that provides further suggestive evidence for this mechanism. It plots two correlation coefficients for the six one-digit sectors we observe in the USTAN database against each other. On the x-axis it displays the correlation coefficients of investment rate dispersion with the cyclical component of its own sectoral output. On the y-axis it displays the correlation coefficients of the fraction of adjusters with these same output measures.<sup>1</sup> The plot shows that there is a positive association between the procyclicality of investment dispersion and the procyclicality of the extensive margin. Incidentally, the contemporaneous correlation between the extensive margin and output for the aggregate USTAN data set is 0.73, a number even higher than for investment rate dispersion.

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<sup>1</sup>Using the convention of Cooper and Haltiwanger (2006), we define investing firms as those with annual investment rates of absolute value larger than 1%.

Figure 2: Sectoral Variation in the Procyclicality of the Extensive Margin and Investment Rate Dispersion



*Notes:* AGR: Agriculture; MIN: Mining & Energy; MAN: Manufacturing; CON: Construction; TRD: Trade (Retail & Wholesale); TRA: Transportation & Communication.  $HP(100) - Y$  refers to the cyclical component of the output of the corresponding sector. Both investment dispersion and the fraction of adjusters are linearly detrended.

We find more suggestive evidence that it is most likely lumpy capital adjustment that is generating this result: 1) we show that in a sector like manufacturing (see again Figure 2, left panel), where we would expect non-convex factor adjustment to be most prevalent, procyclicality of investment dispersion is particularly pronounced, even though the driving force is most starkly countercyclically disperse in this sector; 2) we also show that for smaller firms, i.e. firms that are likely incapable of outgrowing adjustment costs, investment dispersion is significantly more procyclical than for the largest firms. In contrast, conditional on firm size, finance variables do not seem to have a large impact on the cyclicity of investment dispersion. We conclude from this that the explanation does not lie in a financial friction. We also find no evidence of a composition effect in the sense that some large sectors or large firms have actually procyclical second-moment shocks that make the overall investment dispersion likewise procyclical. 3) Finally, we find that the dispersion of investment rates is countercyclical – -0.549 – just like the driving force, once we condition on large and lumpy investments as defined by a 20%-threshold (see Cooper and Haltiwanger, 2006), in order to measure the dispersion of the intensive margin. This is further evidence that the procyclicality of the unconditional investment rate dispersion is driven by movements in the extensive margin.

Why is this important? First, in our view explaining the business cycle dynamics of the higher cross-sectional moments of the underlying macroeconomic aggregates is just as important for our understanding of the business cycle as explaining these aggregates themselves. A fully fledged business cycle theory has to speak to these cross-sectional dynamics as well. This paper systematically documents the relevant facts and explains the most striking of them: procyclical investment dispersion in the presence of countercyclical second-moment shocks (see Figure 1 and Table 3 in Section 2.2). Secondly, heterogeneous-firm models have seen increased use both in the macroeconomic as well as international finance literature. We show in this paper that cross-sectional dynamics impose tight restrictions on structural parameters as well as on the nature and stochastic properties of the driving forces in these models.<sup>2</sup> For instance, we show that procyclical investment dispersion – generated by a procyclical extensive margin effect as in the above proposition – requires a large capital-curvature of the revenue function of the firm, for this procyclical extensive margin effect to be strong enough in the presence of countercyclical second-moment shocks. Only with a large capital-curvature rely firms mostly on the extensive margin for their capital adjustment (see Gourio and Kashyap (2007) for a related observation). We also document that the volatility of the countercyclical second-moment shocks must not be too strong to be compatible with procyclical investment dispersion. In particular, countercyclical second-moment shocks as large as suggested by Bloom (2009) and Bloom et al. (2009) and large enough to generate interesting business cycle dynamics are incompatible with this cross-sectional business cycle fact. That means cross-sectional dynamics have also strong implications for the nature of aggregate dynamics.

### ***Related Literature***

The empirical part of this paper, section 2, is most closely related to a series of papers by Higson and Holly et al. (2002, 2004), Doepke and Holly et al. (2005, 2008), Doepke and Weber (2006), as well as Holly and Santoro (2008). Higson and Holly et al. (2002), using Compustat data, study empirically the cyclicalities of the standard deviation, skewness and kurtosis of the sales growth rate distribution and find them to be countercyclical, countercyclical and procyclical, respectively. Higson and Holly et al. (2004) repeat this analysis for UK data on quoted firms, and Doepke and Holly et al. (2005) for Germany, using the USTAN database, with similar findings. Doepke and Weber (2006) study, again using USTAN data, the cyclicalities of transitions between sales growth regimes in firm-level data. In contrast to these papers, we focus on the cyclicalities of cross-sectional second moments only, but include value added, Solow residuals, investment rates and employment change rates into the analysis.<sup>3</sup> The quantitative-

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<sup>2</sup>Khan and Thomas (2005), in an earlier version of their 2008-paper, make a similar observation on the importance of general equilibrium in understanding cross-sectional firm dynamics. We confirm their conjecture here.

<sup>3</sup>Holly and Santoro (2008) as well as Doepke and Holly et al. (2008) start from the aforementioned empirical

theoretical part of this paper – sections 3, 4 and 5 – draws heavily on the recent literature on heterogenous-firm RBC models, developed in Khan and Thomas (2008), Bachmann et al. (2008), Bloom (2009), Bloom et al. (2009) as well as Bachmann and Bayer (2009). Finally, our work is related to the work by Eisfeldt and Rampini (2005), who show that capital reallocation is procyclical and explain this in a two-sector model with costly capital reallocation.

## 2 The Facts

In Section 2.1 we briefly describe the USTAN data set and the main sample selection criteria we use. Details are relegated to Appendix A.1. In Section 2.2 we present the baseline facts: the contemporaneous correlations of cyclical aggregate output and the cross-sectional standard deviations of firm-level Solow residual and real value added innovations as well as employment change rates are negative, while the contemporaneous correlation of cyclical aggregate output and the cross-sectional standard deviation of firm-level investment rates is positive. In Section 2.4 we perform extensive robustness checks and also show, how these facts depend on observable firm characteristics.

### 2.1 A Brief Data Description

#### 2.1.1 USTAN Data

USTAN is a large annual firm-level balance sheet data base (*Unternehmensbilanzstatistik*) collected by *Deutsche Bundesbank*. It is unique in its combination of size and coverage as well as detail of available variables. It provides annual firm level data from 1971 to 1998 from the balance sheets and the profit and loss accounts of over 60,000 firms per year (see Stoess (2001), von Kalckreuth (2003) and Doepke et al. (2005) for further details). In the days when the discounting of commercial bills were one of the principal instruments of German monetary policy, Bundesbank law required the Bundesbank to assess the creditworthiness of all parties backing a commercial bill put up for discounting. The Bundesbank implemented this regulation by requiring balance sheet data of all parties involved. These balance sheet data were then archived and collected into a database.

Although the sampling design – one’s commercial bill being put up for discounting – does not lead to a representative selection of firms in a statistical sense, the coverage of the sample is very broad. USTAN covers incorporated firms as well as privately-owned companies, which

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work and explore them in a monopolistically competitive model with financial frictions – the former – and in a monopolistically competitive model with simple Calvo-type price-stickiness – the latter.

distinguishes it positively from Compustat data.<sup>4</sup> Its sectoral coverage – while still somewhat biased to manufacturing firms – includes the construction, the service as well as the primary sectors. This makes it different from, for instance, the Annual Survey of Manufacturing (ASM) in the U.S.<sup>5</sup> The following table 1 displays the sectoral coverage of our final baseline sample.

Table 1: SECTORAL COVERAGE

1-digit Sector	Firm-year observations	Percentage
Agriculture - AGR	12,291	1.44
Mining & Energy - MIN	4,165	0.49
Manufacturing - MAN	405,787	47.50
Construction - CON	54,569	6.39
Trade (Retail & Wholesale) - TRD	355,208	41.59
Transportation & Communication - TRA	22,085	2.59

Moreover, while there remains a bias to somewhat larger and financially healthier firms, the size coverage is still fairly broad: 31% of all firms in our final baseline sample have less than 20 employees and 57% have less than 50 employees (see Table 21 in Appendix A.1 for details). Finally, the Bundesbank itself frequently used the USTAN data for its macroeconomic analyses and for cross-checking national accounting data. We take this as an indication that the bank considers the data as sufficiently representative and of sufficiently high quality. This makes the USTAN data an exceptionally suitable data source for the study of cross-sectional business cycle dynamics.

### 2.1.2 Selection of the Baseline Sample

From the original USTAN data, we select only firms that report complete information on payroll, gross value added and capital stocks. Moreover, we drop observations from East German firms to avoid a break of the series in 1990. In addition, we remove observations that stem from irregular accounting statements, e.g. when filing for bankruptcy or when closing operations. We deflate all but the capital and investment data by the implicit deflator for gross value added from the German national accounts.

Capital is deflated with one-digit sector- and capital-good specific investment good price deflators within a perpetual inventory method. Even though USTAN data can be considered as particularly high quality data, we cannot directly use capital stocks as reported. Tax motivated

<sup>4</sup>Davis et al. (2006) show that studying only publicly traded firms can lead to wrong conclusions, in particular when higher cross-sectional moments are concerned. See Appendix A.1 Table 23 for ownership coverage in our final sample.

<sup>5</sup>An additional advantage of these data is easy access: while access is on-site, it is practically free for researchers, so that results derived from this data base can be easily tested and checked.



depreciation and price developments of capital goods lead to a general understatement of the stock of capital a firm holds. Thus, capital stocks have to be recalculated using a perpetual inventory method (see Appendix A.2, for details). Similarly, we recover the amount of labor inputs from wage bills, as information on the number of employees (as opposed to payroll data) is only updated infrequently for some companies (see Appendix A.3, for details). Finally, the firm-level Solow residual is calculated from data on gross value added and factor inputs.

We remove outliers according to the following procedure: we calculate log changes in real gross value added, the Solow residual, real capital and employment, as well as the firm-level investment rate and drop all observations where a change falls outside a three standard deviations interval around the year-specific mean.<sup>6</sup> We also drop those firms for which we do not have at least five observations in first differences. This leaves us with a sample of 854,105 firm-year observations, which corresponds to observations on 72,853 firms, i.e. the average observation length of a firm in the sample is 11.7 years. The average number of firms in the cross-section per year is 32,850. We perform numerous robustness checks with respect to each of the selection criteria and measurement choices: we use sectoral deflators for value added, an aggregate investment good price deflator, change the cut-off rule to 2.5, 5 and 10 standard deviations and leave all firms in the sample with two and twenty observations in first differences, respectively. None of these choices change our baseline results.<sup>7</sup>

### 2.1.3 Calculating the Solow Residual and Factor Adjustments

We compute the firm-level Solow residual based on the following Cobb-Douglas production function in accordance with our model:

$$y_{i,t} = z_t \epsilon_{i,t} k_{i,t}^\theta n_{i,t}^\nu,$$

where  $\epsilon_{i,t}$  is firm-specific productivity, and  $z_t$  is aggregate productivity. We assume that labor input  $n_{i,t}$  is immediately productive, whereas capital  $k_{i,t}$  is pre-determined and inherited from last period. In our main specification, we estimate the output elasticities of the production factors,  $\nu$  and  $\theta$ , as median shares of factor expenditures over gross value added within each industry.<sup>8</sup>

For factor adjustment, we use the symmetric adjustment rate definition proposed in Davis et al. (1996). We thus define firm-level investment rates as  $\frac{i_{i,t}}{0.5*(k_{i,t}+k_{i,t+1})}$ <sup>9</sup> and firm-level em-

<sup>6</sup>This outlier removal is done after removing firm and sectoral fixed effects. Centering the outlier removal around the year mean is important to avoid artificial and countercyclical skewness of the respective distributions.

<sup>7</sup>See Appendix B for details. There we also discuss briefly the issue of sample selection.

<sup>8</sup>To check the robustness of our results, we try alternative specifications with predefined elasticities common across sectors. We also change the timing assumption to include a predetermined employment stock, as well as immediate adjustment in both factors. All results are very robust to the various ways of generating the firm-specific Solow residual (for a detailed discussion, see Bachmann and Bayer, 2009).

<sup>9</sup>Appendix A.1 compares the USTAN aggregate and manufacturing investment rate histogram with the U.S. one

ployment adjustment rates as  $\frac{\Delta n_{i,t}}{0.5*(n_{i,t-1}+n_{i,t})}$ .<sup>10</sup> We use log-differences in the Solow residual to capture Solow residual innovations, as the persistence of firm-level Solow residuals exhibits behavior close to a unit root. We remove firm fixed and sectoral-year<sup>11</sup> effects from these first-difference variables to focus on idiosyncratic fluctuations that do not capture differences in sectoral responses to aggregate shocks or permanent ex-ante heterogeneity between firms.

#### 2.1.4 Macro data

When combining this micro data with aggregate data, we have to take a stance on what sectoral aggregate we view as the empirical counterpart to our model. We chose to include firms from the following six sectors in our analysis: agriculture, mining and energy, manufacturing, construction, trade (both retail and wholesale) as well as the transportation and communication sector. This aggregate can be roughly characterized as the non-financial private business sector in Germany. Whenever we use the term aggregate in the following, we mean this sector.

German national accounting data per one-digit sector (see Appendix A.1 for a detailed description of the data sources used) allow us to compute real value added, investment, capital and employment data for this sectoral aggregate, and therefore also an aggregate Solow residual. Our USTAN sample captures on average 70% of sectoral value added, 44% of sectoral investment, 71% of its capital stock and 49% of sectoral employment.<sup>12</sup>

In addition to representing a large part of the non-financial private business sector in Germany, USTAN also represents its cyclical behavior very well, as the following Table 2 shows:<sup>13</sup>

Table 2: CYCLICALITY OF CROSS-SECTIONAL AVERAGES

Cross-sectional Moment	$\rho(\cdot, HP(100) - Y)$
$mean\left(\frac{i_{i,t}}{0.5*(k_{i,t}+k_{i,t+1})}\right)$	0.756
$mean(\Delta \log e_{i,t})$	0.592
$mean(\Delta \log y_{i,t})$	0.663
$mean\left(\frac{\Delta n_{i,t}}{0.5*(n_{i,t-1}+n_{i,t})}\right)$	0.602

Notes:

$\rho$ : correlation coefficient.

$HP(\lambda) - Y$ : Cyclical component of GDP after HP-filtering using smoothing parameter  $\lambda$ .

*mean*: cross-sectional average, linearly detrended.

from the Longitudinal Research Database, LRD. The similarities are remarkable, which suggests the generalizability of our results also to the U.S.

<sup>10</sup>The baseline within-transformed cross-sectional dispersion data for factor adjustments can be found in Table 27 in Appendix A.6.

<sup>11</sup>The sectoral fixed effects are essentially computed at the 2-digit level, see Table 20 in Appendix A.1 for details.

<sup>12</sup>The sectoral aggregate, in turn, captures 59% of real aggregate value added, 39% of aggregate investment, 26% of aggregate capital and 65% of aggregate employment.

<sup>13</sup>We further document the good representation properties of USTAN in Appendix A.1.

## 2.2 Main Facts

The following Table 3 presents the main new stylized facts about the cross-sectional dynamics of firms. Firm-level investment rates display *procyclical* dispersion, whereas the cross-sectional standard deviations of the (log)-changes in Solow residuals, output and employment are *countercyclical*.

Table 3: CYCLICALITY OF CROSS-SECTIONAL DISPERSION

Cross-sectional Moment	$\rho(\cdot, HP(100) - Y)$	5%	95%	Frac. w. opposite sign
$\sigma\left(\frac{i_{i,t}}{0.5*(k_{i,t}+k_{i,t+1})}\right)$	0.451	0.070	0.737	0.029
$\sigma(\Delta \log \epsilon_{i,t})$	-0.481	-0.678	-0.306	0.000
$\sigma(\Delta \log y_{i,t})$	-0.450	-0.675	-0.196	0.005
$\sigma\left(\frac{\Delta n_{i,t}}{0.5*(n_{i,t-1}+n_{i,t})}\right)$	-0.498	-0.717	-0.259	0.001

Notes:

$\sigma$ : cross-sectional standard deviation, linearly detrended.

The columns 5% and 95% refer to the top and bottom 5-percentiles in a parametric bootstrap of the correlation coefficient. The last column displays the fraction of simulations with the opposite sign of the point estimate. See further notes to Table 2.

The first column of Table 3 shows the contemporaneous correlation of the cyclical component of aggregate output<sup>14</sup> with the cross-sectional standard deviations of the firm-level investment rates, the percentage changes in the firm-level Solow residual and real value added as well as employment changes. The first is clearly procyclical, the latter three countercyclical. The next two columns show the 5% and 95% confidence bands from 10,000 parametric bootstrap simulations.<sup>15</sup> The last column displays the fraction of negative correlations for the standard deviation of the firm-level investment rates, and the fraction of positive correlations for the remaining three standard deviations in these bootstrap simulations. These three columns together show that the sign of all correlations is significant. In the following, we show that finding a procyclical investment rate dispersion is robust to the specific choices we have made in calculating the numbers in Table 3.

## 2.3 Robustness

Table 4 shows that procyclical investment dispersion is robust to the choice of the cyclical indicator.<sup>16</sup> The result stands irrespective of whether we choose as cyclical indicators output filtered

<sup>14</sup>For the baseline scenario we use log-output with an HP-parameter 100.

<sup>15</sup>We use a pairwise unrestricted VAR with one lag as the parametric model. The results from a nonparametric overlapping block bootstrap with a block size of four are similar to the parametric bootstrap.

<sup>16</sup>This is also true for the three other variables, and for  $\sigma(\Delta \log \epsilon_{i,t})$  and  $\sigma(\Delta \log y_{i,t})$ , we have documented this and other robustness tests elsewhere: Bachmann and Bayer (2009).

Table 4: PROCYCLICALITY OF CROSS-SECTIONAL INVESTMENT DISPERSION - ROBUSTNESS TO CYCLICAL INDICATOR

Cyclical Indicator	$\rho(\sigma(\frac{i_{i,t}}{0.5*(k_{i,t}+k_{i,t+1})}), \cdot)$
HP(6.25)-Y	0.370
Log-diff-Y	0.351
$mean(\frac{i_{i,t}}{0.5*(k_{i,t}+k_{i,t+1})})$	0.792
HP(100)-I	0.719
HP(100)-N	0.485
HP(100)-Solow Residual	0.387

Notes: See notes to Tables 2 and 3.  $I$  refers to aggregate investment,  $N$  to aggregate employment.

using a smaller smoothing parameter for the HP filter, following Ravn and Uhlig (2002), apply a log-difference filter to output, or use the linearly detrended average cross-sectional investment rate, or the HP(100)-filtered aggregate investment, employment or aggregate Solow residuals.

Table 5: PROCYCLICALITY OF CROSS-SECTIONAL INVESTMENT DISPERSION - MORE ROBUSTNESS

Cross-sectional Moment	$\rho(\cdot, HP(100) - Y)$
$IQR(\frac{i_{i,t}}{0.5*(k_{i,t}+k_{i,t+1})})$	0.567
$\sigma(\Delta \log k_{i,t})$	0.442
$\sigma(\frac{i_{i,t}}{0.5*(k_{i,t}+k_{i,t+1})})_{raw}$	0.451
$\sigma(\frac{i_{i,t}}{0.5*(k_{i,t}+k_{i,t+1})})_{quadratic \ detrending}$	0.555
$\sigma(\frac{i_{i,t}}{0.5*(k_{i,t}+k_{i,t+1})})_{cubic \ detrending}$	0.599
$\sigma(\frac{i_{i,t}}{0.5*(k_{i,t}+k_{i,t+1})})_{HP(100) \ detrending}$	0.618
$\sigma(\frac{i_{i,t}}{0.5*(k_{i,t}+k_{i,t+1})})_{1973-1990}$	0.297
$\sigma(\frac{i_{i,t}}{0.5*(k_{i,t}+k_{i,t+1})})_{1977-1998}$	0.359

Notes: See notes to Tables 2 and 3.  $IQR$  stands for interquartile range, which is linearly detrended.

Vice versa, our finding is also robust to the choices we have made for the other part of the correlation, see Table 5. One can use the interquartile range (IQR) as the dispersion measure, and one can study the firm level net percentage change in capital as opposed to the investment rate.<sup>17</sup> Moreover, it is not the removal of firm-level and sectoral fixed effects that induces this procyclicality, as row three of this table shows. The next three rows show that the choice of

<sup>17</sup>This variable has the advantage that it corresponds to the percentage innovations in the stock of capital. We can use a permanent-transitory decomposition to separate measurement error from true innovations to the capital stock. The resulting correlation coefficient of the standard deviation of these purified innovations with the cycle is 0.449, and thus the procyclicality of the dispersion of capital innovations is not driven by measurement error.

a linear trend for investment rate dispersion is conservative: using quadratic detrending and especially an HP-filter makes the procyclicality much stronger. This holds nearly uniformly for all the other variations and robustness checks as well as the significance numbers in Table 3. Finally, the last two rows demonstrate that the result is neither driven alone by the German reunification, nor by the strong recession in 1975.

Tables 6 and 7 show how the cyclicity of cross-sectional investment dispersion manifests itself across sectors and firm sizes. For the sectoral numbers we use the cross-sectional standard deviation of the firm-level investment rate and the HP(100)-filtered log-output of the corresponding sectors as inputs into the correlation measure. For the firm size numbers we use HP(100)-filtered log-output from the sectoral aggregate.

Table 6: CYCLICALITY OF CROSS-SECTIONAL INVESTMENT DISPERSION - SECTORS

1-digit Sector	$\rho(\sigma(\frac{i_{i,t}}{0.5*(k_{i,t}+k_{i,t+1})}), HP(100) - Y)$	$\rho(\sigma(\Delta \log \epsilon_{i,t}), HP(100) - Y)$
AGR	-0.192	-0.283
MIN	0.042	0.107
MAN	0.477	-0.397
CON	0.435	0.037
TRD	0.209	-0.387
TRA	0.404	0.034

Notes: See notes to Tables 2 and 3. See Figure 2 for the sectoral acronyms.

Table 6 shows that procyclicality of investment dispersion is strongly prevalent in the goods-producing sectors, in particular manufacturing. The trade sector exhibits a smaller effect, whereas in the primary sectors investment dispersion is nearly acyclical or weakly countercyclical.<sup>18</sup> To put these findings in perspective, we also display the cyclicity of the cross-sectional innovations-to-Solow-residual dispersion, which – despite the procyclicality of investment dispersion – is strongly countercyclical in the manufacturing and trade sectors. To sum up: manufacturing, a sector where nonconvex adjustment technologies can be expected to be most prevalent, has both the strongest countercyclically disperse driving force and the strongest procyclicality of investment dispersion.

As Table 7 shows, procyclicality of investment dispersion is driven mainly by the smaller firms, especially when size is measured by employment or value added. Large firms, in contrast, display only weakly procyclical to acyclical investment dispersion. This distinction is significant in the sense that at least if size is measured in terms of employment or value added, neither the

<sup>18</sup>Had we used the sectoral aggregate instead of own sector output as cyclical indicator, the results would be by and large the same, except that the procyclicality of investment dispersion in the trade sector would more than double with only half the countercyclicality in the dispersion of the driving force.

Table 7: CYCLICALITY OF CROSS-SECTIONAL INVESTMENT DISPERSION - FIRM SIZE

Size Class / Criterion	Employment	Value Added	Capital
Smallest 25%	0.583	0.601	0.391
25% to 50%	0.456	0.468	0.422
50% to 75%	0.366	0.330	0.387
Largest 25%	0.188	0.215	0.399
Largest 5%	0.050	0.048	0.184

point estimate for the smallest size class lies in the [5%, 95%]–bands of the largest size class nor vice versa. <sup>19</sup>

Finally, the last Table 8 shows that conditional on firm size – as measured by capital – the financial situation of a firm – as measured by the equity-asset-ratio – hardly matters for the cyclicity of investment dispersion:

Table 8: CYCLICALITY OF CROSS-SECTIONAL INVESTMENT DISPERSION - FINANCIAL SITUATION

Equity-Asset-Ratio Tercile	Smallest 95% - Capital	Largest 5% - Capital
First	0.369	-0.145
Second	0.273	0.034
Third	0.270	0.010

Tables 6 to 8 together with the finding that the Solow residual processes for small and large firms hardly differ both on average over time and in terms of cyclicity of their innovations,<sup>20</sup> at least suggests that the friction necessary to explain the differential cyclicity of the dispersions of firm-level innovations-to-Solow-residual and investment rates, respectively, can neither be found in financial constraints nor principally in different shock processes. It also does not appear to be driven by certain sectors and large firms. We relegate a discussion of potential cyclical sample selection to Appendix B and show there that it is not an issue.

Instead, we show in this paper that the presence of lumpy capital adjustment is a plausible cause for this aspect of the cross-sectional firm dynamics. Indeed, the fact that procyclical investment dispersion is mostly prevalent in the manufacturing sector as well as in smaller firms, i.e. firms where we would a priori expect non-convexities in the adjustment technology to be most relevant, is at least consistent with our explanation.

<sup>19</sup>See Appendix A.1 for detailed information on the size distribution of firms in our sample.

<sup>20</sup>See Bachmann and Bayer (2009) for an in-depth discussion of this fact.

### Other Data Sources

Naturally the question arises whether it is specific to the German data used that we find the dispersion of investment rates to be procyclical. To assess this we compare to data from the UK DTI-database and to Compustat data from the US. The UK data comprises 10,966 firm-year observations after removal of outliers and constraining the sample to firms with at least 5 observations, applying the same criteria as for the USTAN data. The UK data covers the period 1977-1990 and stems from a representative sample of firms (but over-sampling large firms) in the manufacturing and some selected non-financial service sectors in Britain. For the US we use Compustat annual accounts from 1968-2006. This yields for the US a final sample of 67,394 firm-year observations applying again the same sampling criteria as for the USTAN data.

Table 9 presents the results for these two data sets. The results are much in line with our findings for the USTAN data: For the UK data set, which comprises a larger fraction of smaller firms, we find a robust positive correlation of the dispersion of investment rates with the cycle, irrespective of how we measure dispersion and cycle.

For the Compustat sample, we find a lower (though positive) correlation of the investment rate dispersion with the cycle. This reflects that, in contrast to both the DTI data base and the USTAN data, Compustat covers only large, publicly traded companies. The firms in the Compustat sample are typically larger than even the top 5% largest firms in the USTAN data and we have seen for the USTAN data that for larger firms the correlation coefficient drops.<sup>21</sup>

Table 9: CYCLICALITY OF CROSS-SECTIONAL INVESTMENT DISPERSION - EVIDENCE FROM THE UK AND THE US

Cyclical Indicator	$\rho\left(\sigma\left(\frac{i_{it}}{0.5(k_{i,t-1}+k_{i,t})}\right), \cdot\right)$	$\rho\left(\text{IQR}\left(\frac{i_{it}}{0.5(k_{i,t-1}+k_{i,t})}\right), \cdot\right)$
UK: Cambridge DTI, 1977 - 1990		
HP(100)-Y	0.506	0.687
HP(6.25)-Y	0.488	0.749
Log-diff-Y	0.653	0.263
US: Compustat 1969 - 2006		
HP(100)-Y	0.326	0.649
HP(6.25)-Y	0.334	0.628
Log-diff-Y	0.259	0.421

*Notes:* Aggregate output data,  $Y$ , for the US refers to real gross value added in the non-financial private business sector. For the UK we use aggregate real gross-value-added instead, as the corresponding sectoral data is not publicly available for the corresponding time period. Dispersion measures are linearly de-trended.

<sup>21</sup>All results are robust to alternative detrending methods for the dispersion of investment rates such as fitting quadratic or cubic time-trends or HP-filtering. Results are available upon request.

## 2.4 The Extensive Margin

As we have argued in the Introduction, the mechanism by which the model generates a procyclical dispersion of investment rates is the procyclicity of the extensive margin of investment. Table 10 provides direct evidence on this. It displays the numbers for Figure 2 in the introduction as well as some robustness. The aggregate features strong procyclicity of the extensive margin,<sup>22</sup> as does the manufacturing sector. The service sectors have a middle place, whereas the primary sectors by and large display low and non-robust or zero procyclicity of the extensive margin. For the construction sector the results are somewhat mixed, but overall still show procyclicity.

Table 10: CYCLICALITY OF THE EXTENSIVE MARGIN

	$\rho(\text{Frac. of adj.}, HP(100) - Y)$	$\rho(\text{Frac. of lumpy adj.}, HP(100) - Y)$
Aggregate	0.727	0.614
AGR	0.267	0.100
MIN	0.075	-0.1233
MAN	0.765	0.646
CON	0.428	0.153
TRD	0.559	0.388
TRA	0.237	0.263

*Notes:* See notes to Tables 2 and 3. See Figure 2 for the sectoral acronyms. The fraction of adjusters (Frac. of adj.) is defined as firms with an investment rate of:  $|\frac{i_{i,t}}{0.5(k_{i,t}+k_{i,t+1})}| > 0.01$ . The fraction of lumpy adjusters (Frac. of lumpy adj.) is defined as firms with an investment rate of:  $|\frac{i_{i,t}}{0.5(k_{i,t}+k_{i,t+1})}| > 0.2$ . Both are linearly detrended.  $HP(100) - Y$  refers to the cyclical component of aggregate output in the first row, thereafter to the output of the corresponding sector.

## 3 The Model

In this section we describe our model economy. We start with the firm's problem, followed by a brief description of the households and the definition of equilibrium. We conclude with a sketch of the equilibrium computation. We follow closely Khan and Thomas (2008) and Bachmann et al. (2008). Since the model set up is discussed in detail there, we will be rather brief here.

The main departure from either papers is the introduction of a second exogenous aggregate state, the standard deviation of the distribution of idiosyncratic productivity shocks tomorrow,

<sup>22</sup>If we define the lumpiness threshold with 0.1, the aggregate number is 0.689.



$\sigma(\epsilon')$ . The motivation for this is both realism, as we find these second-moment shocks in the data, but also conservatism: we will show in Section 5.1 that without countercyclical second-moment shocks the investment rate dispersion is very procyclical, more so than in the data, even with very small fixed costs to adjustment. This comes as no surprise, as without countercyclical second-moment shocks there is no countervailing force that would undo the extensive margin effect that in turn causes the investment rate dispersion to be procyclical. Thus, since this is a quantitative exercise using the correct amount of second-moment volatility and countercyclicity in the driving force is important. Following Khan and Thomas (2008), we approximate this now bivariate aggregate state process with a discrete Markov chain.

### 3.1 Firms

The economy consists of a unit mass of small firms. We do not model entry and exit decisions. There is one commodity in the economy that can be consumed or invested. Each firm produces this commodity, employing its pre-determined capital stock ( $k$ ) and labor ( $n$ ), according to the following Cobb-Douglas decreasing-returns-to-scale production function ( $\theta > 0$ ,  $\nu > 0$ ,  $\theta + \nu < 1$ ):

$$y = z\epsilon k^\theta n^\nu, \quad (1)$$

where  $z$  and  $\epsilon$  denote aggregate and firm-specific (idiosyncratic) technology, respectively.

The idiosyncratic technology process has autocorrelation  $\rho_I$ . It follows a Markov chain, whose transition matrix depends on the aggregate state of its time-varying standard deviation,  $\sigma(\epsilon)$ . In contrast, its support is independent of the aggregate state. To also capture observed excess kurtosis in the idiosyncratic productivity shocks, we use a mixture of two Gaussian distributions in the Tauchen-approximation algorithm instead of the usual normal distribution.<sup>23</sup>

We denote the trend growth rate of aggregate productivity by  $(1-\theta)(\gamma-1)$ , so that aggregate  $y$  and  $k$  grow at rate  $\gamma-1$  along the balanced growth path. From now on we work with  $k$  and  $y$  (and later  $C$ ) in efficiency units. The linearly detrended logarithm of aggregate productivity levels as well as linearly detrended  $\sigma(\epsilon)$  evolve according to a VAR(1) process, with normal innovations  $v$  that have zero mean and covariance  $\Omega$ :

$$\begin{pmatrix} \log z' \\ \sigma(\epsilon'') - \bar{\sigma}(\epsilon) \end{pmatrix} = \varrho^A \begin{pmatrix} \log z \\ \sigma(\epsilon') - \bar{\sigma}(\epsilon) \end{pmatrix} + v, \quad (2)$$

where  $\bar{\sigma}(\epsilon)$  denotes the steady state standard deviation of idiosyncratic productivity innovations.<sup>24</sup>

<sup>23</sup>Tauchen (1986). For details, see Section 4.

<sup>24</sup>Specifying this process in terms of  $\log(\sigma(\epsilon))$ , in order to avoid negativity of the standard deviation of idiosyn-

Productivity innovations at different aggregation levels are independent. Also, idiosyncratic productivity shocks are independent across productive units. In contrast, we do not impose any restrictions on  $\Omega$  or  $\rho_A \in \mathbb{R}^{2 \times 2}$ .

Each period a firm draws from a time-invariant distribution,  $G$ , its current cost of capital adjustment,  $\xi \geq 0$ , which is denominated in units of labor.  $G$  is a uniform distribution on  $[0, \bar{\xi}]$ , common to all firms. Draws are independent across firms and over time, and employment is freely adjustable.

At the beginning of a period, a firm is characterized by its pre-determined capital stock, its idiosyncratic productivity, and its capital adjustment cost. Given this and the aggregate state, it decides its employment level,  $n$ , production and depreciation occurs, workers are paid, and investment decisions are made. Then the period ends.

Upon investment,  $i$ , the firm incurs a fixed cost of  $\omega\xi$ , where  $\omega$  is the current real wage rate. Capital depreciates at rate  $\delta$ . We can then summarize the evolution of the firm's capital stock (in efficiency units) between two consecutive periods, from  $k$  to  $k'$ , as follows:

	Fixed cost paid	$\gamma k'$
$i \neq 0$ :	$\omega\xi$	$(1 - \delta)k + i$
$i = 0$ :	0	$(1 - \delta)k$

Given the i.i.d. nature of the adjustment costs, it is sufficient to describe differences across firms and their evolution by the distribution of firms over  $(\epsilon, k)$ . We denote this distribution by  $\mu$ . Thus,  $(z, \sigma(\epsilon'), \mu)$  constitutes the current aggregate state and  $\mu$  evolves according to the law of motion  $\mu' = \Gamma(z, \sigma(\epsilon'), \mu)$ , which firms take as given.

Next we describe the dynamic programming problem of each firm. We will take two shortcuts (details can be found in Khan and Thomas, 2008). First, we state the problem in terms of utils of the representative household (rather than physical units), and denote by  $p = p(z, \sigma(\epsilon'), \mu)$  the marginal utility of consumption. Second, given the i.i.d. nature of the adjustment costs, continuation values can be expressed without explicitly taking into account future adjustment costs.

Let  $V^1(\epsilon, k, \xi; z, \sigma(\epsilon'), \mu)$  denote the expected discounted value—in utils—of a firm that is in idiosyncratic state  $(\epsilon, k, \xi)$ , given the aggregate state  $(z, \sigma(\epsilon'), \mu)$ . Then the expected value prior to the realization of the adjustment cost draw is given by:

$$V^0(\epsilon, k; z, \sigma(\epsilon'), \mu) = \int_0^{\bar{\xi}} V^1(\epsilon, k, \xi; z, \sigma(\epsilon'), \mu) G(d\xi). \quad (3)$$

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cratic productivity shocks is – given its high steady state value and relatively low variability (see Bachmann and Bayer, 2009) – an unnecessary precaution that does not change the results.

With this notation the dynamic programming problem is given by:

$$V^1(\epsilon, k, \xi; z, \sigma(\epsilon'), \mu) = \max_n \{CF + \max(V_{\text{no adj}}, \max_{k'}[-AC + V_{\text{adj}}])\}, \quad (4)$$

where  $CF$  denotes the firm's flow value,  $V_{\text{no adj}}$  the firm's continuation value if it chooses inaction and does not adjust, and  $V_{\text{adj}}$  the continuation value, net of adjustment costs  $AC$ , if the firm adjusts its capital stock. That is:

$$CF = [z\epsilon k^\theta n^\nu - \omega(z, \sigma(\epsilon'), \mu)n]p(z, \sigma(\epsilon'), \mu), \quad (5a)$$

$$V_{\text{no adj}} = \beta E[V^0(\epsilon', (1-\delta)k/\gamma; z', \sigma(\epsilon''), \mu')], \quad (5b)$$

$$AC = \xi \omega(z, \sigma(\epsilon'), \mu)p(z, \sigma(\epsilon'), \mu), \quad (5c)$$

$$V_{\text{adj}} = -ip(z, \sigma(\epsilon'), \mu) + \beta E[V^0(\epsilon', k'; z', \sigma(\epsilon''), \mu')], \quad (5d)$$

where both expectation operators average over next period's realizations of the aggregate and idiosyncratic productivity states, conditional on this period's values, and we recall that  $i = \gamma k' - (1-\delta)k$ . Also,  $\beta$  denotes the discount factor of the representative household.

Taking as given prices  $\omega(z, \sigma(\epsilon'), \mu)$  and  $p(z, \sigma(\epsilon'), \mu)$ , and the law of motion  $\mu' = \Gamma(z, \sigma(\epsilon'), \mu)$ , the firm chooses optimally labor demand, whether to adjust its capital stock at the end of the period, and the optimal capital stock, conditional on adjustment. This leads to policy functions:  $N = N(\epsilon, k; z, \sigma(\epsilon'), \mu)$  and  $K = K(\epsilon, k, \xi; z, \sigma(\epsilon'), \mu)$ . Since capital is pre-determined, the optimal employment decision is independent of the current adjustment cost draw.

### 3.2 Households

We assume a continuum of identical households that have access to a complete set of state-contingent claims. Hence, there is no heterogeneity across households. Moreover, they own shares in the firms and are paid dividends. We do not need to model the household side in detail (see Khan and Thomas (2008) for the details), and concentrate instead on the first-order conditions to determine the equilibrium wage and the marginal utility of consumption.

Households have a standard felicity function in consumption and (indivisible) labor:

$$U(C, N^h) = \log C - AN^h, \quad (6)$$

where  $C$  denotes consumption and  $N^h$  the household's labor supply. Households maximize the

expected present discounted value of the above felicity function. By definition we have:

$$p(z, \sigma(\epsilon'), \mu) \equiv U_C(C, N^h) = \frac{1}{C(z, \sigma(\epsilon'), \mu)}, \quad (7)$$

and from the intratemporal first-order condition:

$$\omega(z, \sigma(\epsilon'), \mu) = -\frac{U_N(C, N^h)}{p(z, \sigma(\epsilon'), \mu)} = \frac{A}{p(z, \sigma(\epsilon'), \mu)}. \quad (8)$$

### 3.3 Recursive Equilibrium

A *recursive competitive equilibrium* for this economy is a set of functions

$$(\omega, p, V^1, N, K, C, N^h, \Gamma),$$

that satisfy

1. *Firm optimality*: Taking  $\omega$ ,  $p$  and  $\Gamma$  as given,  $V^1(\epsilon, k; z, \sigma(\epsilon'), \mu)$  solves (4) and the corresponding policy functions are  $N(\epsilon, k; z, \sigma(\epsilon'), \mu)$  and  $K(\epsilon, k, \xi; z, \sigma(\epsilon'), \mu)$ .
2. *Household optimality*: Taking  $\omega$  and  $p$  as given, the household's consumption and labor supply satisfy (7) and (8).
3. *Commodity market clearing*:

$$C(z, \sigma(\epsilon'), \mu) = \int z\epsilon k^\theta N(\epsilon, k; z, \sigma(\epsilon'), \mu)^\nu d\mu - \int \int_0^{\bar{\xi}} [\gamma K(\epsilon, k, \xi; z, \sigma(\epsilon'), \mu) - (1 - \delta)k] dG d\mu.$$

4. *Labor market clearing*:

$$N^h(z, \sigma(\epsilon'), \mu) = \int N(\epsilon, k; z, \sigma(\epsilon'), \mu) d\mu + \int \int_0^{\bar{\xi}} \xi \mathcal{J}(\gamma K(\epsilon, k, \xi; z, \sigma(\epsilon'), \mu) - (1 - \delta)k) dG d\mu,$$

where  $\mathcal{J}(x) = 0$ , if  $x = 0$  and 1, otherwise.

5. *Model consistent dynamics*: The evolution of the cross-section that characterizes the economy,  $\mu' = \Gamma(z, \sigma(\epsilon'), \mu)$ , is induced by  $K(\epsilon, k, \xi; z, \sigma(\epsilon'), \mu)$  and the exogenous processes for  $z$ ,  $\sigma(\epsilon')$  as well as  $\epsilon$ .

Conditions 1, 2, 3 and 4 define an equilibrium given  $\Gamma$ , while step 5 specifies the equilibrium condition for  $\Gamma$ .

### 3.4 Solution

As is well-known, (4) is not computable, since  $\mu$  is infinite dimensional. Hence, we follow Krusell and Smith (1997, 1998) and approximate the distribution  $\mu$  by its first moment over capital, and its evolution,  $\Gamma$ , by a simple log-linear rule. In the same vein, we approximate the equilibrium pricing function by a log-linear rule, discrete aggregate state by discrete aggregate state:

$$\log \bar{k}' = a_k(z, \sigma(\epsilon')) + b_k(z, \sigma(\epsilon')) \log \bar{k}, \quad (9a)$$

$$\log p = a_p(z, \sigma(\epsilon')) + b_p(z, \sigma(\epsilon')) \log \bar{k}, \quad (9b)$$

where  $\bar{k}$  denotes aggregate capital holdings. Given (8), we do not have to specify an equilibrium rule for the real wage. As usual with this procedure, we posit this form and check that in equilibrium it yields a good fit to the actual law of motion. In contrast to models without second moment shocks, where it has been extensively shown that the first moment suffices, we show here that the pure  $R^2$  goodness-of-fit metric does not perform as well anymore:  $R^2$  below 0.9 are possible, as we shall see in Section 5.2. Nevertheless, Bachmann and Bayer (2009) show that the aggregate dynamics of such an economy are hardly affected, when higher moments of the capital distribution are included and the  $R^2$  are pushed closer to unity (see Bachmann et al. (2008) for a similar observation). We show here that also the cross-sectional dynamics are affected only to a small degree. And since we consistently find that not including higher moments leads to a slight underestimation of the procyclicality of investment dispersion, we prefer the increase in computational speed and report our results, unless otherwise noted, with the first moment only as a state variable.

Combining these assumptions and substituting  $\bar{k}$  for  $\mu$  into (4) and using (9a)–(9b), we have that (4) becomes a computable dynamic programming problem with policy functions  $N = N(\epsilon, k; z, \sigma(\epsilon'), \bar{k})$  and  $K = K(\epsilon, k, \xi; z, \sigma(\epsilon'), \bar{k})$ . We solve this problem via value function iteration on  $V^0$ .

With these policy functions, we can then simulate a model economy *without* imposing the equilibrium pricing rule (9b), but rather solve for it along the way. We simulate the model economy for 1,600 time periods and discard the first 100 observations, when computing any statistics. This procedure generates a time series of  $\{p_t\}$  and  $\{\bar{k}_t\}$  endogenously, with which assumed rules (9a)–(9b) can be updated via a simple OLS regression. The procedure stops when the updated coefficients  $a_k(z, \sigma(\epsilon'))$  and  $b_k(z, \sigma(\epsilon'))$ , as well as  $a_p(z, \sigma(\epsilon'))$  and  $b_p(z, \sigma(\epsilon'))$  are sufficiently close to the previous ones. We skip the details of this procedure, as this has been outlined elsewhere – see Khan and Thomas (2008) and Bachmann et al. (2008).

## 4 Calibration

The model period is a year – in congruence with the data frequency in USTAN. The following parameters have standard values:  $\beta = 0.98$  and  $\delta = 0.094$ , which we compute from German national accounting data for the sectoral aggregate that the USTAN sample corresponds to: the non-financial private business sector. Given this depreciation rate, we pick  $\gamma = 1.014$ , in order to match the time-average aggregate investment rate of 0.108. This number is also consistent with German long-run growth rates. The log-felicity function features an elasticity of intertemporal substitution (EIS) of one. The disutility of work parameter,  $A$ , is chosen to generate an average time spent at work of 0.33:  $A = 2$  for the baseline calibration.

We set the output elasticities of labor and capital to  $\nu = 0.5565$  and  $\theta = 0.2075$ , respectively, which correspond to the measured median labor and capital shares in manufacturing in the USTAN data base (see Appendix A.4). While our data also include a considerable amount of firms from other sectors, any weighted average or median of these shares would still be close to the manufacturing values, which is why we decided to use them in our baseline calibration. We discuss robustness to this parameter choice in Section 5.1 and Appendix A.4.<sup>25</sup>

Next, we have to choose the parameters of the two-state aggregate shock process. Here we simply estimate a bivariate, unrestricted VAR with the linearly detrended natural logarithm of the aggregate Solow residual<sup>26</sup> and the linearly detrended  $\sigma(\epsilon)$ -process from the USTAN data.<sup>27</sup> The parameters of this VAR are as follows:<sup>28</sup>

$$\varrho_A = \begin{pmatrix} 0.4474 & -3.7808 \\ 0.0574 & 0.7794 \end{pmatrix} \quad \Omega = \begin{pmatrix} 0.0146 & 0.1617 \\ 0.1617 & 0.0023 \end{pmatrix} \quad (10)$$

This process is discretized on a  $[5 \times 5]$ -grid, using a bivariate analog of Tauchen’s procedure.

We measure the steady state standard deviation of idiosyncratic technology innovations as  $\bar{\sigma}(\epsilon) = 0.1201$ . Since these innovations also exhibit mild excess kurtosis – 4.4480 on average over our time horizon –,<sup>29</sup> and since the adjustment cost parameter  $\bar{\xi}$  will be identified by the kurtosis of the firm-level investment rate (in addition to its skewness), we want to avoid attributing

<sup>25</sup>If one views the DRTS assumption as a mere stand-in for a CRTS production function with monopolistic competition, than these choices would correspond to an employment elasticity of the underlying production function of 0.7284 and a markup of  $\frac{1}{\theta+\nu} = 1.31$ . Given the regulated product markets in Germany, this is a reasonable value. The implied capital elasticity of the revenue function,  $\frac{\theta}{1-\nu}$  is 0.47. Finally, model simulations show that using the capital share as an estimate for the output elasticity of capital under the null hypothesis of the model leads to a small overestimation of the latter, which, as we will show in Section 5.1, leads to the the baseline calibration being conservative relative to the main result: procyclicality of investment dispersion.

<sup>26</sup>We use again  $\nu = 0.5565$  and  $\theta = 0.2075$  in these calculations.

<sup>27</sup>After firm-level and sectoral fixed effects have been removed.

<sup>28</sup>With a slight abuse of notation, but for the sake of readability,  $\Omega$  displays standard deviations on the main diagonal and correlations on the off diagonal.

<sup>29</sup>We find no skewness.

excess kurtosis in the firm-level investment rate to nonlinearities in the adjustment technology, when the driving force itself has kurtosis. Hence, we incorporate the measured excess kurtosis into the discretization process for the idiosyncratic technology state.<sup>30</sup> Finally, we set  $\rho_I = 0.95$ , in accordance with the high persistence of Solow residual innovations we find in the data. This process is discretized on a 19–state-grid, using Tauchen’s procedure with mixed Gaussian normals.<sup>31</sup>

Given the aforementioned set of parameters  $(\beta, \delta, \gamma, A, \nu, \theta, \rho_A, \Omega, \bar{\sigma}(\epsilon), \rho_I)$ , we then calibrate the adjustment costs parameter  $\bar{\xi}$  to minimize a quadratic form in the normalized differences between the time-average firm-level investment rate skewness produced by the model and the data, as well as the time-average firm-level investment rate kurtosis:<sup>32</sup>

$$\min_{\bar{\xi}} \Psi(\bar{\xi}) \equiv 0.5 \cdot \left[ \left( \left( \frac{1}{T} \sum_t \text{skewness} \left( \frac{i_{i,t}}{0.5 * (k_{i,t} + k_{i,t+1})} \right) (\bar{\xi}) - 2.1920 \right) / 0.6956 \right)^2 + \left( \left( \frac{1}{T} \sum_t \text{kurtosis} \left( \frac{i_{i,t}}{0.5 * (k_{i,t} + k_{i,t+1})} \right) (\bar{\xi}) - 20.0355 / 5.5064 \right) \right)^2 \right]. \quad (11)$$

As can be seen from (11), the distribution of firm-level investment rates exhibits both substantial positive skewness – 2.1920 – as well as kurtosis – 20.0355. Caballero et al. (1995) document a similar fact for U.S. manufacturing plants. They also argue that non-convex capital adjustment costs are an important ingredient to explain such a strongly non-Gaussian distribution, given a close-to-Gaussian shock process. We therefore use the deviation from Gaussianity in firm-level investment rates to identify  $\bar{\xi}$ .

The following Table 11 demonstrates identification of  $\bar{\xi}$ , as cross-sectional skewness and kurtosis of the firm-level investment rates are both monotonically increasing in  $\bar{\xi}$ . The minimum of the distance measure  $\Psi$  is achieved for  $\bar{\xi} = 0.3$ , our baseline case.<sup>33</sup> This implies costs conditional on adjustment equivalent to 15.4% of annual firm-level output on average, which is well in line with estimates from the U.S. (see Bloom, 2009). A description of the aggregate dynamics of the baseline calibration is relegated to Appendix C.

<sup>30</sup>We achieve this by using a mixture of two Gaussian distributions:  $N(0, 0.0777)$  and  $N(0, 0.1625)$  – the standard deviations are  $0.1201 \pm 0.0424$  – with a weight of 0.4118 on the first distribution.

<sup>31</sup>The cross-sectional results do not change significantly with either an increase in the fineness of the aggregate grid to  $[9 \times 9]$ , nor with one in the idiosyncratic grid to a 35–state-grid.

<sup>32</sup>The normalization constants in (11) are, respectively, the time series standard deviation of the investment rate skewness and the time series standard deviation of the investment rate kurtosis.

<sup>33</sup>We searched over a finer grid of  $\bar{\xi}$  than displayed in the table, in order to find the optimal  $\bar{\xi}$ .

Table 11: CALIBRATION OF ADJUSTMENT COSTS -  $\bar{\xi}$ 

$\bar{\xi}$	Skewness	Kurtosis	$\Psi(\bar{\xi})$	Adj. costs/ Unit of Output
0.01	0.7852	5.0389	11.5082	1.5%
0.05	1.5168	7.6444	6.0062	4.2%
0.1	1.9340	9.3327	3.9157	6.8%
0.2	2.4011	11.4056	2.547	11.3%
0.3 (BL)	2.6915	12.8042	2.2402	15.4%
0.5	3.0686	14.7669	2.5035	23.3%
1	3.5926	17.8112	4.2169	43.3%

## 5 Results

### 5.1 Baseline Results

Can a thus calibrated DSGE model with idiosyncratic productivity shocks, fixed adjustment costs to capital and countercyclical innovations to the dispersion of firm-level Solow residuals reproduce the cyclicity of the cross-sectional dynamics observed in the data?

Table 12 summarizes our main result numerically: in our baseline calibration the model matches the procyclicality of firm-level investment rate dispersion as well as the extensive margin reasonably well, even though it was calibrated to the steady state Non-Gaussianity of the investment rate distribution.<sup>34</sup> We use HP(100)-filtered aggregate model output as the cyclical indicator. The countercyclical dispersions of value added and employment changes are also captured very well.

Table 12: CYCLICALITY OF CROSS-SECTIONAL DISPERSION - BASELINE MODEL

Cross-sectional Moment	Data	Model
$\sigma\left(\frac{i_{i,t}}{0.5*(k_{i,t}+k_{i,t+1})}\right)$	0.451	0.580
$\sigma(\Delta \log y_{i,t})$	-0.450	-0.392
$\sigma\left(\frac{\Delta n_{i,t}}{0.5*(n_{i,t-1}+n_{i,t})}\right)$	-0.498	-0.436
Fraction of adjusters	0.727	0.485

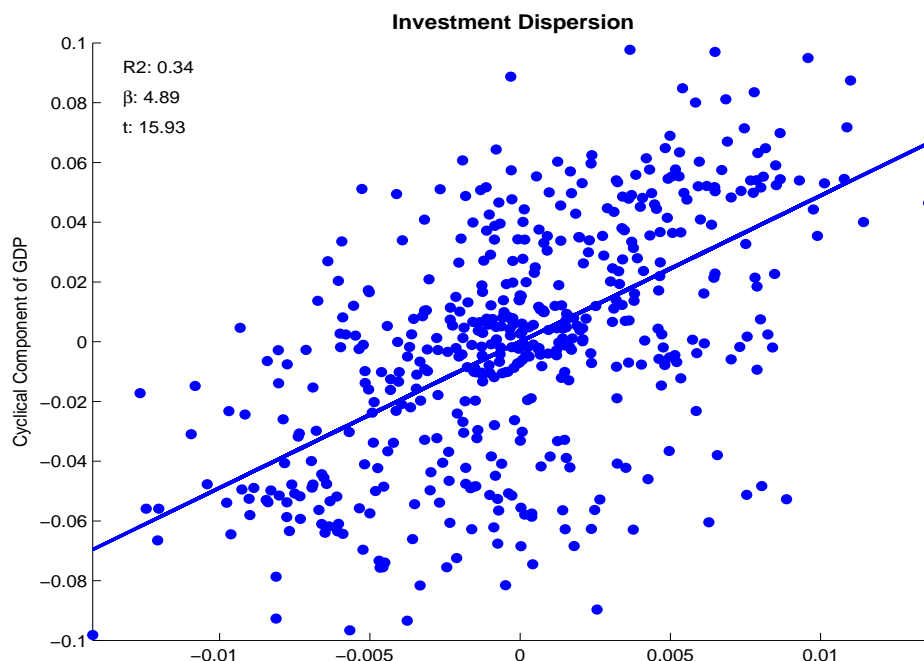
*Notes:* Correlation coefficients between HP(100)-filtered output and a cross-sectional standard deviation. Fraction of adjusters is defined as firms with an investment rate of:  $|\frac{i_{i,t}}{0.5(k_{i,t}+k_{i,t+1})}| > 0.01$ . The column ‘Model’ refers to the correlation coefficients from a simulation of the model over  $T = 1500$  periods.

<sup>34</sup>These numbers are obtained from a simulation of  $T = 1500$ . Using an even longer simulation of  $T = 3000$  and breaking it up into 60 pieces of  $T = 26$  (the length of the USTAN sample) independent time series produces an average value of 0.652 for the correlation between investment rate dispersion and cyclical output with a standard deviation of: 0.113. The range is [0.390, 0.845], which includes the point estimate of the data.



Figure 3 shows that indeed the model produces procyclical investment dispersion close to the one found in the data and shown in Figure 1 in the introduction. Likewise, Figure 8 in Appendix A.5 shows a simulated time path of investment dispersion that clearly exhibits positive comovement with aggregate output.

Figure 3: Cross-sectional Dispersion of Firm-Level Investment Rates and Solow Residual Innovations



*Notes:* Dispersion refers to the cross sectional standard-deviation.

The next Table 13 illustrates how lumpy capital adjustment and countercyclical second moment shocks interact to generate the procyclicality result.

Two findings are important: in the presence of countercyclical second moment shocks, the procyclicality of investment dispersion is a gradually and monotonically increasing function of the adjustment cost parameter. What is perhaps surprising is that the level of adjustment costs that best matches the cross-sectional average skewness and kurtosis of firm-level investment rates – two statistics that have been known to be related to the level of nonconvexities at the micro-level (see Caballero et al., 1995) – also leads to a model that almost matches an important time series moment of the cross-sectional business cycle dynamics. Had we matched the latter almost exactly, we would have chosen an adjustment cost parameter of 0.2, a value only somewhat below our baseline calibration. The table also shows that a more conservative calibration that calibrates to the cross-sectional skewness of firm-level investment rates only and

Table 13: ADJUSTMENT COSTS AND CYCLICALITY OF INVESTMENT DISPERSION AND THE EXTENSIVE MARGIN

$\bar{\xi}$	Full Model w. 2nd moment shocks	Model w/o. 2nd moment shocks
0	-0.543	-
0.0001	-0.536	-0.181
0.001	-0.456	0.674
0.005	-0.350	0.810
0.01	-0.283	0.832
0.05	-0.025	0.855
0.1	0.175	0.861
0.15 (skewness only)	0.334	0.870
0.2	0.444	0.873
0.3 (BL)	0.580	0.876
0.5	0.731	0.883
0.75	0.820	0.890
1	0.864	0.896

*Notes:* See notes to Table 12. Note that for the case with  $\bar{\xi} = 0$  and no second-moment shocks any time series variation of  $\sigma(\frac{i_{i,t}}{0.5(k_{i,t}+k_{i,t+1})})$  is a numerical artifact, which means that its correlation coefficient with output is not defined.  $\rho_A = 0.5259$  and  $\Omega = 0.0182$  for the univariate case.

puts zero weight on their kurtosis, still generates a sizeable level of procyclicality in investment dispersion. By contrast, the frictionless case merely replicates the countercyclicality of the dispersion of the driving force.

Moreover, the second column of this table shows that without second moment shocks, a very low level of non-convexity immediately generates procyclicality in investment dispersion – the gradient of procyclicality in the adjustment cost factor,  $\bar{\xi}$ , is extremely steep without countercyclical second moment shocks.<sup>35</sup> But it also makes the model overshoot this number considerably. Thus, countercyclical second moment shocks are an important part in understanding cross-sectional firm dynamics, both in generating countercyclical dispersions of value and employment changes, but also to generate realistic procyclicality in investment dispersion. Without them, it would simply be too easy to generate the latter. We view this as an important confirmation of our calibration and our mechanism: in the presence of quantitatively realistic countercyclicality of the dispersion of the driving force, it is a level of adjustment costs that matches best the nonlinear average moments of the investment rate distribution that also generates about the right correlation coefficient between the standard deviation of investment rates and aggregate output. Table 13 shows that this identification is rather tight.

<sup>35</sup>Notice that the proposition in the introduction suggests that for low enough adjustment cost parameters and for an important enough role of the extensive margin we should expect countercyclicality of investment rate dispersion even without countercyclical second moment shocks.

Table 14 illustrates how the procyclicality of the investment dispersion and the procyclicality of the extensive margin interact with the curvature of the revenue function in capital.

Table 14: FACTOR ELASTICITIES AND CYCLICALITY OF INVESTMENT DISPERSION

Cross-sectional Moment	Baseline (0.47)	Rev. Ela.=0.57	Rev. Ela.=0.63
$\sigma\left(\frac{i_{i,t}}{0.5*(k_{i,t}+k_{i,t+1})}\right)$	0.580	-0.102	-0.492
Fraction of Adjusters	0.485	-0.295	-0.603

*Notes:* See notes to Table 12. ‘Rev. Ela.’ stands for the revenue elasticity of capital in a reduced form revenue function, after labor has been maximized out. It is given by  $\frac{\theta}{1-\nu}$ . Using the convention of Cooper and Haltiwanger (2006), we define the fraction of adjusters as those firms that have annual investment rates of absolute value larger than 1%.

The results in columns two and three refer to setups with factor elasticities  $\nu = 0.5333$ ,  $\theta = 0.2667$  and  $\nu = 0.5556$ ,  $\theta = 0.2778$ , respectively, compared to  $\nu = 0.5565$ ,  $\theta = 0.2075$  in the baseline scenario.<sup>36</sup> It is clear that larger revenue elasticities in capital after labor has been maximized out, imply a lower procyclicality of the extensive margin and thus for the investment rate dispersion. Smaller revenue elasticities or higher curvature of the production function imply that the intensive margin of investment becomes less flexible: the range of the optimal capital return level in the baseline scenario is [0.0261, 41.9135], for the second column [0.0183, 98.6497] and [0.0073, 175.0381] for the third column; all with the same process for idiosyncratic technology. To achieve the optimal path for aggregate investment, the extensive margin becomes more important for the firms, the higher the curvature of the revenue function. This effect of curvature is well known and explained in detail in Gourio and Kashyap (2007).

Finally, Table 15 shows the effect of general equilibrium on both the procyclicality of the extensive margin as well as the procyclicality of investment dispersion. Real wage and interest rate movements lead to aggregate coordination and therefore to procyclicality of the fraction of adjusters. This in turn increases the cyclical comovement of both the investment rate dispersion, as the following Table 15 shows. As we have shown above, both quantities are strongly procyclical in the data. We thus confirm the conjecture in Khan and Thomas (2005) that general equilibrium price movements are important to quantitatively account for cross-sectional business cycle dynamics.

To sum up, the extent of both, the procyclicality of investment dispersion as well as the countercyclicality of the dispersion of firm-level Solow residual innovations, impose important

<sup>36</sup>In a monopolistic competition framework, column two implies a scenario with a CRTS-one-third-two-third production function and a markup of 1.25, column three a markup of 1.20. In each case, we recompute firm-level and aggregate Solow residuals, estimate a new driving process (2) and re-calibrate the adjustment cost parameter  $\bar{\xi}$  to minimize  $\Psi(\bar{\xi})$  in (11). For the second column this leads to  $\bar{\xi} = 0.45$ , and  $\bar{\xi} = 0.5$  for the third column.

Table 15: CYCLICALITY OF INVESTMENT DISPERSION AND GENERAL EQUILIBRIUM

Cross-sectional Moment	Baseline - GE	PE
$\sigma\left(\frac{i_{i,t}}{0.5*(k_{i,t}+k_{i,t+1})}\right)$	0.580	-0.1612
Fraction of Adjusters	0.485	-0.0222

*Notes:* See notes to Tables 12. ‘GE’ stands for general equilibrium and means a model simulation with market clearing wages and interest rates. ‘PE’ stands for partial equilibrium and means a model simulation, where wages and interest rates are held constant at the average level in the ‘GE’-simulation.

and very tight restrictions on important structural parameters, such as adjustment frictions and factor elasticities in the production function. More generally, this makes the study of cross-sectional business cycle dynamics important for the structure and calibration of heterogenous-firm models. We also confirm the conjecture in Khan and Thomas (2005) that general equilibrium price movements are important to quantitatively account for the cross-sectional business cycle dynamics observed in the data.

## 5.2 Robustness

In the following Table 16 we document robustness of our baseline result to some of the choices we have made in the baseline calibration.

Table 16: CYCLICALITY OF INVESTMENT DISPERSION - ROBUSTNESS

Scenario	$\rho\left(\sigma\left(\frac{i_{i,t}}{0.5*(k_{i,t}+k_{i,t+1})}\right), HP(100) - Y\right)$
<i>Baseline</i>	<i>0.580</i>
Double volatility of $\sigma(\Delta\epsilon_{i,t})$	-0.006
Quadruple volatility of $\sigma(\Delta\epsilon_{i,t})$	-0.283
<i>CRRA = 3</i>	<i>0.560</i>
Timing of $\sigma(\Delta\epsilon_{i,t})$	0.694
Log-weighting	0.731
<i>mean</i> ( $\Delta\epsilon_{i,t}$ )	<i>0.757</i>

*Notes:* See notes to Table 12.

In order to check robustness of our results to a potential underestimation of the volatility of the countercyclical second-moment shock, we double (and quadruple) it, while keeping its steady state value fixed at  $\bar{\sigma}(\epsilon) = 0.1201$ . To this end, we rescale the  $\sigma(\epsilon)$ -process by a factor of two (four) when simulating the model. As expected, now the ability of the procyclical extensive margin effect to overcome the countercyclical second-moment shocks is limited, because the latter fluctuates more. This drives down the correlation of the investment rate dispersion

and the cyclical component of aggregate output to zero and approximately  $-0.3$ , respectively. Notice, however, that it is still the case that non-convexities in capital adjustment cause an extensive margin effect that partially offsets the countercyclical second-moment shocks, as the frictionless counterparts of these two high volatility specifications feature correlation coefficients of investment dispersion with output of  $-0.551$  and  $-0.535$ , respectively. But it is also clear from this exercise that the strongly procyclical investment dispersion that we find in the data  $+0.451$  – is at odds with the even more, eightfold as volatile countercyclical second-moment shocks proposed in Bloom (2009) and Bloom et al. (2009) as important drivers of the business cycle. Next, we check whether our unity CRRA is driving our result by increasing the CRRA to 3. This leads to hardly any change.<sup>37</sup> Furthermore, we check whether the result is sensitive to the timing assumption about the revelation of the dispersion of the firm-level Solow residual innovation. The baseline model assumes that  $\sigma(\Delta\epsilon_{i,t+1})$  is revealed today in  $t$ , concomitantly with  $z_t$  and  $\epsilon_t$ , aggregate and idiosyncratic technology, which means investors know about the actual productivity risk tomorrow at the time of the investment decision. There is another plausible timing assumption: only  $\sigma(\Delta\epsilon_{i,t})$  is revealed today in  $t$ , and both  $z_t$  and  $\sigma(\Delta\epsilon_{i,t})$  predict the dispersion of the firm-level Solow residual innovation tomorrow through persistence in the VAR in equation (10). As the fourth row shows, this increases somewhat the procyclicality of investment dispersion, as the corresponding number from a frictionless model would be  $-0.3845$ , compared to the  $-0.5432$  in the frictionless counterpart of the baseline timing assumption. The effect through the countercyclical driving force in this case is simply more indirect. The next to last row shows that using a normalized calibration criterion,  $\Psi(\bar{\xi})$ , in (11) as opposed to, say, log-deviations between model simulated skewness and kurtosis of investment rates, was a conservative choice. Had we used the latter – and therefore calibrated  $\bar{\xi} = 0.5$  –, we would have found an even stronger procyclicality of the investment rate dispersion. Finally, we replace the aggregate Solow residual with the average firm-level Solow residual from USTAN in the bivariate aggregate driving force, which somewhat increases the procyclicality of investment dispersion.

### **Measurement Error**

Measuring Solow residuals is potentially fraught with error. Indeed, when we take measurement error in our firm-level Solow residual calculation into account, we find that the average standard deviation of the innovations to this Solow residual,  $\bar{\sigma}(\epsilon)$ , declines from 0.120 to 0.091.<sup>38</sup>

<sup>37</sup>Technically, with the separable felicity specification in (6) there is no balanced growth path with CRRA=3. The model remains consistent with balanced growth, if the disutility of leisure grows with the steady state growth rate,  $\gamma$ , and the fundamental discount rate is accordingly adjusted.

<sup>38</sup>Assuming additive classical measurement error in firm-level log-Solow residuals and a time-invariant measurement error variance, we can use the time-average of the difference between the one-period innovation to observed firm-level log-Solow residuals and half the variance of the two-period innovations to estimate the variance of measurement error. Subtracting this number (twice) from the observed variance of one-period innovations to

We therefore recompute and recalibrate our model with a driving force that is corrected for this measurement error. The adjustment cost parameter is lowered to  $\bar{\xi} = 0.25$ . As Table 17 shows, this increases dramatically the procyclicality of investment dispersion. The model overshoots. The next two lines explain why. We keep the fluctuations of the computed Solow residual innovation dispersion as in the baseline model, but lower mechanically its long-run average and recalibrate the adjustment cost parameter, respectively to  $\bar{\xi} = 0.25$  and  $\bar{\xi} = 0.15$ . There are two effects to consider: for a fixed adjustment cost parameter lowering the long-run average of the second-moment driving force mechanically increases the coefficient of variation of the latter, which in isolation drives down the procyclicality of investment dispersion. At the same time, lowering the long-run average dispersion of idiosyncratic productivity shocks also means that the adjustment costs parameter is relatively high compared to the remaining idiosyncratic uncertainty, which strengthens the extensive margin effect and on net the procyclicality of investment dispersion. The second effect means that this too high adjustment cost parameter overshoots now the optimal level to match skewness and kurtosis of the long-run investment rate distribution. That is why we recalibrate  $\bar{\xi}$  downward. However, this is not enough to offset the positive net effect on the procyclicality of investment dispersion.

Table 17: CYCLICALITY OF INVESTMENT DISPERSION - MEASUREMENT ERROR

Scenario	$\rho(\sigma(\frac{i_{i,t}}{0.5*(k_{i,t}+k_{i,t+1})}), HP(100) - Y)$
<i>Baseline</i>	<i>0.580</i>
Measurement Error	0.845
Three quarter $\bar{\sigma}(\epsilon)$	0.896
Half $\bar{\sigma}(\epsilon)$	0.954
Measurement Error + Rescaled volatility of $z_t$ (0.6)	0.585
Measurement Error + Double volatility of $\sigma(\Delta\epsilon_{i,t})$	0.414
Measurement Error + Quadruple volatility of $\sigma(\Delta\epsilon_{i,t})$	-0.078

Notes: See notes to Table 12.

One concern with this calibration is that we have treated idiosyncratic and aggregate Solow residuals differently, specifying the former with measurement error and the latter not. The fifth row addresses this concern which is reflected by the fact that aggregate volatility in our baseline calibration is too high (see Appendix C). If this high volatility was due to measurement error, then we would unduly increase the relative importance of first-moment shocks versus second-moment shocks. A calibration where we re-scale the volatility of aggregate Solow residuals – by a factor of 0.6 – to match the observed volatility of aggregate output in the German NIPA data in-

firm-level log-Solow residuals yields an estimate of the true variance of the innovations to firm-level log-Solow residuals.

deed lowers again the simulated procyclicality of investment dispersion. However, it is still very much in line with the data, in fact very close to our baseline calibration, where measurement error at any level was ignored.

The last two lines of Table 17 finally show that when we again take into account measurement error in the microeconomic driving force only, the procyclicality of investment dispersion still puts a sharp upper bound on the relative importance of countercyclical second-moment shocks proposed in Bloom (2009) and Bloom et al. (2009). They basically cannot exceed an unconditional time-series percentage standard deviation of twice 4.72%, where the latter is the percentage volatility of  $\sigma(\epsilon)$  after measurement error is taken into account. Notice that rescaling the aggregate Solow residual would only lower that number and make the bound sharper.

### ***Higher Moments in the Krusell and Smith Rules***

It remains to be shown that our result is not driven by the choice of only the average capital stock in the Krusell and Smith rules (9a) and (9b). While it is the case that in the presence of countercyclical second-moment shocks the conventional  $R^2$ -measure is fairly low – at least in some combinations of the discrete aggregate states, the minimum is 0.8701 –, and while it is also true that including the skewness of the capital distribution<sup>39</sup> leads to an average increase of the  $R^2$  for the capital regressions from 0.9378 to 0.9870 and for the marginal utility of consumption regressions from 0.9962 to 0.9986, neither the aggregate behavior (see Bachmann and Bayer (2009) for details) nor the cross-sectional dynamics of the model are significantly altered: the correlation between investment dispersion and cyclical aggregate output raises slightly from 0.580 to 0.639.<sup>40</sup> That means, if anything, our baseline numerical specification is somewhat conservative with respect to our main finding.<sup>41</sup> The bottom line, however, is that better forecasts do not necessarily induce the agents to behave differently (see Bachmann et al. (2008) for a similar finding).

The scatter plots in Figure 4 make this point graphically: the positive relationship between investment dispersion and cyclical aggregate output is nearly indistinguishable between a numerical specification where only average capital is used as a state variable and one, where also the skewness of firm-level capital is included in the forecasting rules.

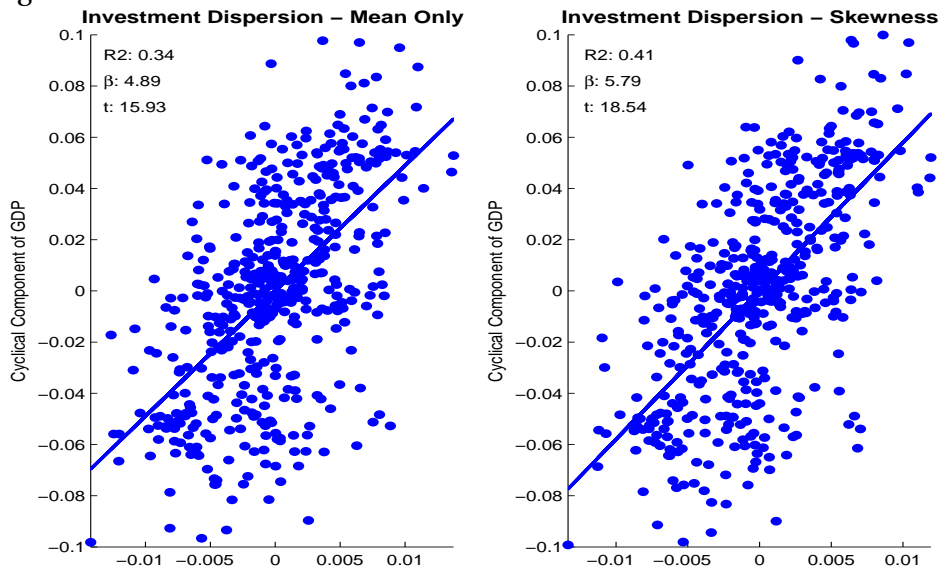
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<sup>39</sup>Including the standard deviation of capital does not yield any significant improvements in  $R^2$ . The average  $R^2$  over all discrete states for the skewness regression, that is analogous to (9a), is 0.9261.

<sup>40</sup>We find even somewhat better improvements in the  $R^2$  and a similarly small increase in the procyclicality of investment dispersion, when instead we include the standard deviation of log firm-level Solow residuals as an additional moment in the Krusell and Smith rules. Numbers are available on request.

<sup>41</sup>Our finding that the procyclicality of investment dispersion puts a sharp upper bound on the countercyclicality of the dispersion of firm-level Solow residual innovations is also robust to including higher moments. Numbers are available on request.

Figure 4: Cross-sectional Dispersion of Firm-Level Investment Rates and Solow Residual Innovations: Higher Moments



Notes: Dispersion refers to the cross sectional standard-deviation.

### 5.3 Results from Sectoral Calibrations

To test our mechanism further, we use the 1-digit sectoral variation in the procyclicality of investment dispersion and the extensive margin from Figure 2 in the introduction. We view this as an additional and suggestive exercise, given that for computational feasibility we have to make an important shortcut. Instead of calibrating and computing a realistic six-sector general equilibrium model of the German economy, we run six separate specifications of our baseline DSGE model, where we adjust crucial parameters to sectoral statistics, but otherwise treat the corresponding sector as the aggregate economy. Specifically, we calibrate the factor elasticities in the production function to sectoral income shares and use our sectoral USTAN results both for the long-run standard deviation (and the kurtosis)<sup>42</sup> of the innovations to the firm-level Solow residual,  $\bar{\sigma}(\epsilon)$ , as well as the time series process of the latter. We set the sectoral long-run growth rates to zero and calibrate the sectoral depreciation rates to match the sectoral long-run aggregate investment rate. In the baseline exercise we also use sectoral Solow residuals as the first moment shock in the VAR in equation (2).<sup>43</sup> Finally, given all these parameters we calibrate the adjustment costs parameter,  $\bar{\xi}$ , to match a variance-weighted quadratic form in sectoral skewness and kurtosis of the firm-level investment rates, analogous to (11). Table 30 in Appendix D

<sup>42</sup>As in the aggregate data we find mild excess kurtosis and no skewness across the one-digit sectors. The exact numbers are available from the authors on request.

<sup>43</sup>The exact specifications of these VARs for each sector are available from the authors on request. Table 31 in Appendix D displays the results, when we use aggregate Solow residuals in the estimation of the driving process and aggregate output as the cyclical indicator.



displays the main calibration parameters for each sector. For computational reasons we leave out the agricultural sector in this sectoral exercise. There we estimate the revenue elasticity of capital to be 0.935, which requires infeasibly fine and large grids for firm-level capital, a computational burden that is beyond the scope of this simple sectoral exercise. Given this high revenue elasticity of capital and our results in Table 14 in Section 5.1 about how curvature of the revenue function relates to the cyclicality of investment dispersion, the low and slightly negative correlation of investment dispersion in the agricultural sector – -0.192 (with own sectoral output, see Table 6 in Section 2.4) and 0.151 (with aggregate output) – should come as no surprise and, in fact, supports our mechanism without the aforementioned computational burden.<sup>44</sup>

Table 18: RESULTS FROM SECTORAL CALIBRATION

Sector	$\rho(\sigma(\frac{i_{i,t}}{0.5*(k_{i,t}+k_{i,t+1})}), HP(100) - Y)$		$\rho(\text{Fraction of Adjusters}, HP(100) - Y)$	
	Model	Data	Model	Data
<i>Aggregate</i>	0.580	0.451	0.485	0.727
MIN	0.050	0.042	0.058	0.075
MAN	0.771	0.477	0.754	0.765
CON	0.684	0.435	0.565	0.428
TRD	0.909	0.209	0.920	0.559
TRA	0.223	0.404	0.250	0.237

*Notes:* See notes to Tables 12. See Figure 2 for the sectoral acronyms.  $HP(100) - Y$  refers to the cyclical component of aggregate output in the first row, from the second row onwards to the cyclical component of the output of the corresponding sector.

Table 18 displays the results for the correlations of investment dispersion and the extensive margin, respectively, with own sector output.<sup>45</sup> The correlation coefficients between model simulations and data are 0.446 for investment rate dispersion and 0.901 for the extensive margin. The corresponding rank correlations are 0.4 and 0.9, respectively. The model captures the overall variation with mining and energy not displaying any cyclicality of either investment rate dispersion or the extensive margin, the transportation and communication sector featuring a middle position and manufacturing and construction having the strongest procyclicality. The models for the latter sectors slightly overestimate procyclicality, for the former one it is underestimated. The biggest exception is the trade sector, where the models produce an almost perfect procyclicality of both investment rate dispersion and the extensive margin, which is inconsistent with the data. The explanation is simple: as can be gathered from Table 30 in Appendix D,

<sup>44</sup>For similar reasons, in the computation for the mining and energy sector we scale down the measured factor elasticities by a factor of 0.9. This facilitates the numerics considerably without compromising our results.

<sup>45</sup>Tables 31 and 32 in Appendix D show the results with aggregate output and the own sector average cross-sectional investment rate, respectively. The latter show a higher congruence between data and model numbers.

we estimate the lowest revenue elasticity of capital in the trade sector – 0.403. This enormously facilitates the extensive margin mechanism to an extent that is obviously at odds with the data. It appears that we neglected a significant factor in trade that can be adjusted but does not appear in our specification of the production function. One advantage of using the cross-sectional dynamics of investment is that it helps to identify this flaw. Yet, it is beyond the scope of this paper to offer a remedy for this discrepancy. We nevertheless view the overall sectoral results in support of our basic mechanism, especially given the simplicity of the sectoral exercise, and leave the trade “puzzle” for future research.

## 6 Final Remarks

This paper studies the cyclical behavior of the second moments of the cross-sections of firm-level innovations to value added, Solow residuals, capital and employment. We show that even in the presence of countercyclically disperse Solow residual innovations the dispersion of investment rates is significantly and robustly procyclical. We also show that this can be quantitatively explained by realistically calibrated non-convex adjustment costs: a procyclical extensive margin effect dominates the countercyclical dispersion in the driving force. Other potential explanations, such as financial frictions, are ruled out. We finally argue that the understanding of the cross-sectional business cycle dynamics imposes important restrictions on structural parameters and driving forces. In particular, large countercyclical second moment shocks that could generate sizeable business cycle dynamics would be incompatible with procyclical investment dispersion.

We view this as just the beginning of a new research program that attempts to understand more comprehensively the time-series behavior of the entire cross-section of firms, not merely the cyclicity of second moments. This will ultimately lead to a better microfoundation of structural heterogeneous-firm models and contribute to making them suitable for policy analysis. We also plan to corroborate these new findings for more countries, in particular the U.S.

## References

- [1] Bachmann, R. and C. Bayer (2009). “Firm-specific Productivity Risk over the Business Cycle: Facts and Aggregate Implications”, mimeo.
- [2] Bachmann, R., Caballero, R. and E. Engel (2008). “Aggregate Implications of Lumpy Investment: New Evidence and a DSGE Model”, mimeo.
- [3] Bloom, N. (2009). “The Impact of Uncertainty Shocks”, *Econometrica*, forthcoming.
- [4] Bloom, N., M. Floetotto and N. Jaimovich (2009). “Really Uncertain Business Cycles”, mimeo.
- [5] Caballero, R., E. Engel and J. Haltiwanger (1995). “Plant-Level Adjustment and Aggregate Investment Dynamics”, *Brookings Paper on Economic Activity*, 1995, (2), 1–54.
- [6] Caplin, A. and D. Spulber (1987). “Menu Costs and the Neutrality of Money”, *Quarterly Journal of Economics*, **102**, 703–726.
- [7] Cooper, R. and J. Haltiwanger (2006). “On the Nature of Capital Adjustment Costs”, *Review of Economic Studies*, **73**, 611–633.
- [8] Davis, S., J. Haltiwanger and S. Schuh (1996). “Job Creation and Destruction”, Cambridge, MA: MIT Press.
- [9] Davis, S., J. Haltiwanger, R. Jarmin and J. Miranda (2006). “Volatility and Dispersion in Business Growth Rates: Publicly Traded and Privately Held Firms”, *NBER Macroeconomics Annual*.
- [10] Doepke, J. and S. Weber (2006). “The Within-Distribution Business Cycle Dynamics of German Firms”, *Discussion Paper Series 1: Economic Studies*, No 29/2006. Deutsche Bundesbank.
- [11] Doepke, J., M. Funke, S. Holly and S. Weber (2005). “The Cross-Sectional Dynamics of German Business Cycles: a Bird’s Eye View”, *Discussion Paper Series 1: Economic Studies*, No 23/2005. Deutsche Bundesbank.
- [12] Doepke, J., M. Funke, S. Holly and S. Weber (2008). “The Cross-Section of Output and Inflation in a Dynamic Stochastic General Equilibrium Model with Sticky Prices”, CWPE 0853.
- [13] Eisfeldt, A. and A. Rampini (2006). “Capital Reallocation and Liquidity”, *Journal of Monetary Economics*, **53**, 369–399.

- [14] Gourio, F. and A.K. Kashyap, (2007). “Investment Spikes: New Facts and a General Equilibrium Exploration”, *Journal of Monetary Economics*, **54**, 1–22.
- [15] Heckman, J. (1976). “The common structure of statistical models of truncation, sample selection, and limited dependent variables and a simple estimator for such models”, *Annals of Economic and Social Measurement*, **5**, 475–492.
- [16] Higson, C., S. Holly and P. Kattuman (2002). “The Cross-Sectional Dynamics of the US Business Cycle: 1950–1999”, *Journal of Economic Dynamics and Control*, **26**, 1539–1555.
- [17] Higson, C., S. Holly, P. Kattuman and S. Platis (2004): “The Business Cycle, Macroeconomic Shocks and the Cross Section: The Growth of UK Quoted Companies”, *Economica*, **71/281**, May 2004, 299–318.
- [18] Holly, S. and E. Santoro (2008). “Financial Fragility, Heterogeneous Firms and the Cross-Section of the Business Cycle”, CWPE 0846.
- [19] von Kalckreuth, U. (2003). “Exploring the role of uncertainty for corporate investment decisions in Germany”, *Swiss Journal of Economics*, Vol. 139(2), 173–206.
- [20] Khan, A. and J. Thomas, (2005). “Idiosyncratic Shocks and the Role of Nonconvexities in Plant and Aggregate Investment Dynamics”, *Federal Reserve Bank of Minneapolis - WP*.
- [21] Khan, A. and J. Thomas, (2008). “Idiosyncratic Shocks and the Role of Nonconvexities in Plant and Aggregate Investment Dynamics”, *Econometrica*, **76**(2), March 2008, 395–436.
- [22] Krusell, P. and A. Smith (1997). “Income and Wealth Heterogeneity, Portfolio Choice and Equilibrium Asset Returns”, *Macroeconomic Dynamics* **1**, 387–422.
- [23] Krusell, P. and A. Smith (1998). “Income and Wealth Heterogeneity in the Macroeconomy”, *Journal of Political Economy*, **106** (5), 867–896.
- [24] Ravn, M. and H. Uhlig (2002). “On Adjusting the Hodrick-Prescott Filter for the Frequency of Observations”, *The Review of Economics and Statistics*, **84** (2), 371–380.
- [25] Stoess, E. (2001). “Deutsche Bundesbank’s Corporate Balance Sheet Statistics and Areas of Application”, *Schmollers Jahrbuch: Zeitschrift fuer Wirtschafts- und Sozialwissenschaften (Journal of Applied Social Science Studies)*, **121**, 131–137
- [26] Tauchen, G. (1986). “Finite State Markov-Chain Approximations To Univariate and Vector Autoregressions”, *Economics Letters* **20**, 177–181.

# A Data Appendix

## A.1 Description of the Sample

The Bundesbank's corporate balance sheet database (*Unternehmensbilanzstatistik*, USTAN henceforth) has been originally created as a by-product of the bank's rediscounting activities, an important instrument of monetary policy before the introduction of the Euro. When a commercial bank wished to pledge a commercial bill of exchange to the Bundesbank, the commercial bank had to prove the creditworthiness of the bill. For that purpose the bank had to provide the Bundesbank with balance sheet information of all parties who backed the bill of exchange. By law, the Bundesbank could only accept bills backed by at least three parties known to be creditworthy. This procedure allowed the Bundesbank to collect a data set with information stemming from the balance sheets and the profit and loss accounts of firms (see Stoess (2001), von Kalckreuth (2003) and Doepke et al. (2005) for further details).

Quality standards of the data are particularly high. All mandatory data collected for USTAN have been double-checked by Bundesbank staff. Hence, the data should contain unusually few errors for a micro-data set. One drawback of USTAN is that with the introduction of the EURO, the Bundesbank stopped buying commercial bills and collected firm balance sheet data only irregularly and from publicly available sources. For this reason, the data set stops being useful in 1999. Therefore, we only use data from 1971 to 1998, which leaves us with essentially 26 year observations from 1973 to 1998 because of lagging and first-differencing.

The coverage of the sample is broad, although it is technically not a representative sample due to the non-random sample design. It was also more common to use bills of exchange in manufacturing and for incorporated companies, which biases our data somewhat towards these kinds of firms. And, of course, the Bundesbank would only rediscount bills with a good rating, so that the set of firms in USTAN is also somewhat biased to financially healthy and larger firms.

Nevertheless, USTAN covers a wide range of firms, since short-term financing through commercial bills of exchange was common practice for many German companies across all business sectors (see Table 20 below for the detailed sectoral composition of our final sample). USTAN also has a broad ownership coverage ranging from incorporated firms as well as privately owned companies, which distinguishes it from the Compustat data. Within the former group USTAN covers both untraded corporations (e.g. limited liability firms, *GmbH*) as well as publicly held companies (*AG*) – see Table 23 below. Finally, USTAN features also a relatively broad size coverage, as we will show in Table 21 below for our final sample, the creation of which we describe in some detail now.

We start out with the universe of observations in the USTAN data, merging the files for 1971-1986 and 1987-1998. In a first pass, we then drop all balance sheets that are irregular, e.g. bankruptcy or closing balance sheets, or stem from a group/holding (*Konzernbilanz*). This leaves us with only regular balance sheets (*Handelsbilanz* or *Steuerbilanz*). We also drop all firms with missing payroll data or missing or negative sales data, which are basically non-operating firms. A small amount of duplicate balance sheets is removed as well. And finally, we drop the following sectors: hospitality (hotels and restaurants), which has only a small amount of firms in the database, financial and insurance institutions, the mostly public health and education sectors, as well as other public companies like museums, etc. and some other small service industries, such as hair cutters, dry cleaners and funeral homes;<sup>46</sup> or when sectoral information was missing. The sectoral aggregate we are studying can be roughly characterized as the non-financial private business sector in Germany. This leaves us with an initial data set of 1,764,846 firm-year observations and 259,614 firms. The average number of firms per year is 63,030.

From this initial data set we remove step-by-step more observations, in order to get an economically reasonable data set. We first drop observations from likely East German firms to avoid a break of the series in 1990. We identify a West German firm as a firm that has a West German address or has no address information but enters the sample before 1990. Then we recompute capital stocks with a modified perpetual inventory method (PIM) and employment levels. In the modified PIM we drop a small amount of observations from the top and bottom of the distribution of correction factors for the initial capital stock, see Appendix A.2. Extreme correction factors indicate that constant depreciation is not a good approximation for this particular firm. Such a firm will have had an episode of extraordinary depreciation (e.g. fire, a natural disaster, etc.) and the capital stocks by PIM will be a bad measure of the actual capital after the disaster. We remove observations that do not have a log value added and a log capital stock after PIM. Another large part is removed due to not featuring changes in log firm-level employment, capital and real value added, which obviously requires us to observe firms for two consecutive years. Then we remove outliers in factor changes and real value added changes. Specifically, we identify as outliers in our sample a firm-year in which the firm level investment rate or log changes in firm-level real value added, employment and capital stock fall outside a three standard deviations band around the firm and sectoral-year mean. Then we compute firm-level Solow residuals (see Appendix A.4 for details) and similarly remove observations with missing log changes in Solow residuals as well as outliers therein. We finally remove – before and

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<sup>46</sup>The number of firms from the public sector and these small industries is tiny to begin with, as they did not use commercial bills as a financing instrument. We left out financial and insurance institutions, as they arguably have a very different production function and investment behavior.

after each step of the outlier removal – firms that have less than five observations in firm-level Solow residual changes. We conduct extensive robustness checks of our results to the choices for the outlier and observation thresholds (see Appendix B). Table 19 summarizes, how much observations are dropped in each step.

Table 19: SAMPLE CREATION

Criterion	Drops of Firm-Year Observations
East Germany	104,299
Outliers in PIM	7,539
Missing log value added	1,349
Missing log capital	31,819
Missing log-changes in N, K, VA	161,668
Outliers in factor and VA log-changes	41,453
Missing log-changes in Solow residual	126,086
Outliers in Solow residual log-changes	18,978
Not enough observations	417,550
Total	910,741

The final sample then consists of 854,105 firm-year observations, which amounts to observations on 72,853 firms and the average observation length of a firm in the sample is 11.7 years. The average number of firms per year is 32,850. The following Tables 20 and 21 as well as 22 and 23 show the average sectoral<sup>47</sup> and the size distributions in our sample, as well as the distributions over the number of observations and legal forms, respectively.

<sup>47</sup>WZ 2003 is the industry classification from 2003 that the German national accounting system (*Volkswirtschaftliche Gesamtrechnung, VGR*) uses.

Table 20: SECTORAL DISTRIBUTION

ID	Sector	Observations	Frequency	WZ 2003
10	Agriculture	12,291	1.44%	A, B
20	Energy & Mining	4,165	0.49%	C, E
31	Chemical Industry, Oil	14,721	1.72%	DE, DG
32	Plastics, Rubber	23,892	2.80%	DH
33	Glass, Ceramics	28,623	3.35%	DI
34	Metals	30,591	3.58%	DJ
35	Machinery	162,407	19.01%	DK, DL, DM, DN
36	Wood, Paper, Printing	61,672	7.22%	DD, DE
37	Textiles, Leather	46,173	5.41%	DB, DC
38	Food, Tobacco	37,708	4.41%	DA
40	Construction	54,569	6.39%	F
61	Wholesale Trade	213,071	24.95%	G51
62	Retail Trade & Cars	142,137	16.64%	G50, G51
70	Transportation & Communication	22,085	2.59%	I
	Total	854,105		

Table 21: SIZE DISTRIBUTIONS OF FIRMS

<b>Number of Employees</b>	1-4	5-9	10-14	15-19	20-49	50-99	100-249	250-499	500+
Fraction	6.14%	9.46%	8.24%	7.30%	26.28%	17.04%	14.37%	5.68%	5.49%
<b>Capital Stock</b> (in 1000 1991-Euro)	0-299	300-599	600-999	1,000-1,499	1,500-2,499	2,500-4,999	5,000-9,999	10,000-24,999	25,000+
Fraction	8.23%	9.01%	9.67%	9.36%	13.08%	17.71%	13.87%	11.08%	7.99%
<b>Real Value Added</b> (in 1000 1991-Euro)	0-299	300-499	500-999	1,000-1,499	1,500-2,499	2,500-4,999	5,000-9,999	10,000-24,999	25,000+
Fraction	8.17%	7.93%	16.38%	11.56%	14.45%	16.28%	11.20%	8.25%	5.79%



Table 22: OBSERVATION DISTRIBUTION

Obs. per Firm	Firms	Percent	Cum.	Obs. per Firm	Firms	Percent	Cum.
5	8,973	12.32	12.32	16	2,487	3.41	78.10
6	7,592	10.42	22.74	17	2,225	3.05	81.16
7	6,609	9.07	31.81	18	2,024	2.78	83.93
8	5,724	7.86	39.67	19	1,849	2.54	86.47
9	4,901	6.73	46.39	20	1,619	2.22	88.69
10	4,338	5.95	52.35	21	1,479	2.03	90.72
11	3,960	5.44	57.78	22	1,351	1.85	92.58
12	3,528	4.84	62.63	23	1,446	1.98	94.56
13	3,134	4.30	66.93	24	988	1.36	95.92
14	3,006	4.13	71.05	25	892	1.22	97.14
15	2,647	3.63	74.69	26	2081	2.86	100
				Total	72,853		

Table 23: LEGAL FORM DISTRIBUTION

Legal Form	Observations	Frequency
Publicly Traded (AG, KGaA, etc.)	18,582	2.18%
Limited Liability Companies (GmbH, GmbH&Co., etc.)	506,184	59.26%
Fully Liable Partnerships (OHG, KG, etc.)	327,526	38.35%
Other: unincorporated associations (e.V.) municipal agencies (Körperschaften öR) etc.	1,813	0.21%
Total	854,105	100%

How well does the USTAN aggregate represent the non-financial private business sector (NFPBS) in Germany? Table 24 shows that USTAN represents on average 70% of the value added of the NFPBS, which, in turn, comprises 59% of the aggregate real value added, 44% of its investment, etc. Moreover, USTAN replicates the capital-output ratio of NFPBS rather well, somewhat less so the other canonical ratios, such as the investment rate, average labor productivity and the labor share, which has obviously to do with our larger firm bias in the sample.<sup>48</sup>

Table 24: USTAN AND THE NFPBS

	USTAN/NFPBS	USTAN	NFPBS	NFPBS/Aggregate
Value Added	70%	-	-	59%
Investment	44%	-	-	39%
Capital	71%	-	-	26%
Employment	49%	-	-	65%
Payroll	54%	-	-	65%
Capital/Value Added	-	1.544	1.496	-
Investment/Value Added	-	0.099	0.158	-
Value Added/Employment	-	52828	36859	-
Payroll/Value Added	-	0.506	0.657	-

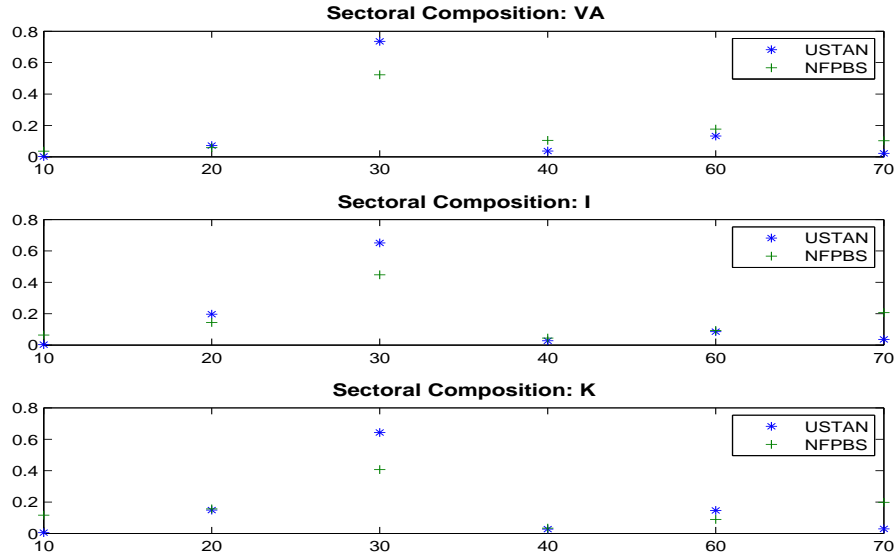
Figure 5 shows that except for a certain overrepresentation of manufacturing and a certain underrepresentation of the transportation and communication sector, USTAN represents the sectoral composition in NFPBS rather well.

Figure 6 demonstrates that also the cyclical behavior of USTAN and NFPBS is close. The correlation of the cyclical components of value added is 0.7671 and for the investment rate it is 0.7843.<sup>49</sup>

<sup>48</sup> To compute these time-average statistics we only average over the data from 1973 to 1990, because from then on German national accounting does no longer report West and East Germany separately. For the business cycle statistics we use the post-reunification data, but filter separately before and after this structural break. NFPBS value added is taken from *Bruttowertschoepfung in jeweiligen Preisen*, table 3.2.1 of *VGR*, deflated year-by-year by the implicit deflator for aggregate value added, table 3.1.1 of *VGR* (we apply the same deflator to USTAN data). The base year is always 1991. We experiment also with implicit sector-specific deflators for value added from table 3.2.1 and 3.2.2 of *VGR*, and results are robust to this. NFPBS investment is *Bruttoanlageinvestitionen in jeweiligen Preisen* from table 3.2.8.1, deflated with the implicit sector-specific investment price deflators given by *Bruttoanlageinvestitionen - preisbereinigt*, a chain index, from table 3.2.9.1, *VGR*. NFPBS capital is *Nettoanlagevermoeagen in Preisen von 2000* from table 3.2.19.1, *VGR*, re-chained to 1991 prices. In both the computation of investment and capital data for USTAN in the PIM we use the implicit sector and capital good specific (equipment and non-residential structures) deflators for investment: tables 3.2.8.2, 3.2.9.2., 3.2.8.3 and 3.2.9.3., *VGR*. We also experiment with deflating USTAN data with a uniform investment price deflator, the *Preisindex der Investitions-gueterproduzenten*, source: GP-X002, *Statistisches Bundesamt*. NFPBS employment is number of employed, *Arbeitnehmer*, from table 3.2.13, *VGR*. Finally, payroll is taken from *Arbeitnehmerentgelt*, table 3.2.10., *VGR*, deflated by the same general implicit deflator for aggregate value added that we use to deflate value added numbers.

<sup>49</sup> We take first differences of log value added and then take out both for it and the investment rate a deterministic

Figure 5: Sectoral Composition in USTAN and NFPBS



*Notes:*

Graphs display the fraction of the sum of real value added, investment and capital, respectively, over all firms by 1-digit sector within the USTAN sample over the NFPBS aggregate.

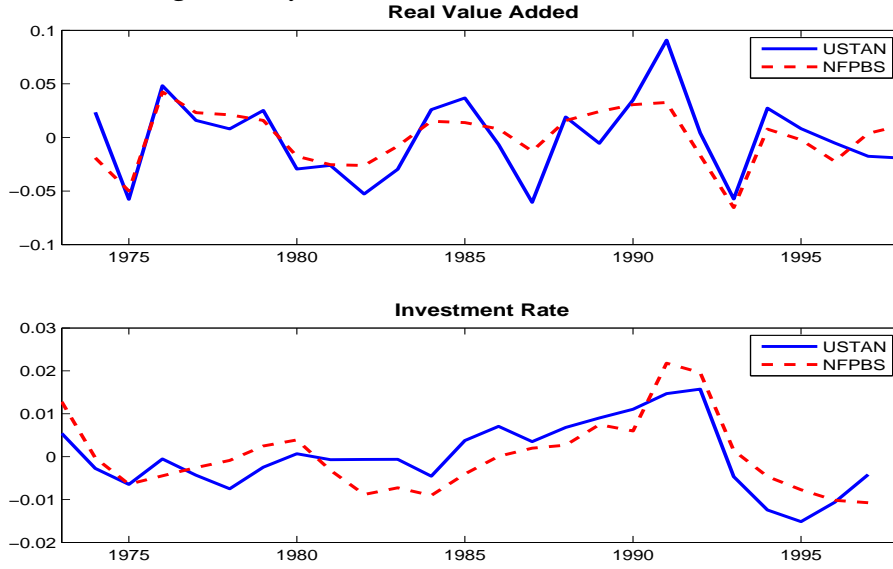
Finally, how does the USTAN investment rate cross-section compare to known data from the U.S.? The following Table 25 compares cross-sectional moments of the USTAN investment rates, as well as for the manufacturing sector in USTAN (for reasons of comparison with only  $k_{i,t}$  in the denominator) with the ones reported in Cooper and Haltiwanger (2006) for manufacturing plant-level data. Even though USTAN is a firm-level as opposed to a plant-level data set, these histograms are remarkably similar, which lends some optimism to the generalizability of our results to the U.S.

Table 25: USTAN AND LRD MOMENTS

Moment	USTAN	USTAN-Manufacturing	LRD
Negative Spike (<-20%)	0.3%	0.3%	1.8%
Negative Investment (-20%,-1%)	2.6%	2.0%	8.6%
Inaction (-1%,1%)	15.1%	11.4%	8.1%
Positive Investment (1%,20%)	67.7%	73.6%	62.9%
Positive Spike (> 20%)	14.2%	12.7%	18.6%

linear trend to remove the growth of the USTAN sample over time. The correlation between only the first differences in log value added is still 0.5348, and 0.4966, when an HP(100)-filter is applied. The correlation for the raw investment rate series is 0.7089.

Figure 6: Cyclical Behavior in USTAN and NFPBS



Notes:

Upper panel: time series for the sum of real value added over all firms in the USTAN sample and NFPBS after detrending with logarithmic first differences and a deterministic linear trend.

Lower panel: time series for the sum of investment over all firms in the USTAN sample and NFPBS, divided by the average of the beginning-of-period and end-of-period aggregate capital stocks in USTAN and NFPBS, respectively, after detrending with a deterministic linear trend.

## A.2 Capital Stocks

In order to obtain economically meaningful stocks of capital series for each firm, we have to re-calculate capital stocks in a Perpetual Inventory Method (PIM). The first step is to compute firm-level investment series,  $i_{i,t}$ , from the corporate balance sheets, which contain data only on accounting capital stocks,  $k_{i,t}^a$ , and accounting total depreciation,  $d_{i,t}^a$ . The following accumulation identity allows to back out nominal firm-level investment:<sup>50</sup>

$$k_{i,t+1}^a = k_{i,t}^a - d_{i,t}^a + p_t^I i_{i,t}. \quad (12)$$

The next step is to recognize that capital stocks from corporate balance sheets are not directly usable for economic analysis for two reasons: 1) accounting depreciation,  $d_{i,t}^a$ , in corporate balance sheets is often motivated by tax reasons and typically higher than economic depreciation,  $\delta_{i,t}^e$ , expressed as a rate; 2) accounting capital stocks are reported at historical

<sup>50</sup>Specifically,  $k_{i,t}^a$  is the sum of balance sheet items ap65, *Technische Anlagen und Maschinen*, and ap66, *Andere Anlagen, Betriebs- und Geschäftsausstattung*, for equipment; and balance sheet item ap64, *Grundstuecke, Bauten*, for structures. Since balance sheet data are typically end-of-year stock data, notice that  $k_{i,t}^a$  is the end-of-period capital stock in year  $t - 1$ .  $d_{i,t}^a$  is profit and loss account item ap156, *Abschreibungen auf Sachanlagen und immaterielle Vermoegensgegenstaende des Anlagevermoegens*. In contrast to  $k_{i,t}^a$ ,  $d_{i,t}^a$  is not given for each capital good separately. For the solution of this complication, see below.

prices. Both effects would lead to an underestimation of the real firm-level capital stock, if one were to simply deflate the current accounting capital stock,  $k_{i,t}^a$ , with a current investment price deflator,  $p_t^I$  (assuming that  $p_t^I$  increases over time). We therefore apply a Perpetual Inventory Method (PIM) to compute economic real capital stocks:

$$k_{i,1}^{(1)} = k_{i,1}^a. \quad (13)$$

$$k_{i,t+1}^{(1)} = (1 - \delta_t^e) k_{i,t}^{(1)} + \frac{p_t^I}{p_{1991}^I} i_{i,t}. \quad (14)$$

$k_{i,1}^a$  is the accounting capital stock in prices of 1991 at the beginning of an uninterrupted sequence of firm observations – if for a firm-year we have a missing investment observation, the PIM is started anew, when the firm appears again in the data set. We estimate  $\delta_t^e$  for each year from national accounting data, *VGR*, separately for equipment and non-residential structures (table 3.1.3, *VGR, Nettoanlagevermögen nach Vermoegensarten in jeweiligen Preisen, Ausrüstungen und Nichtwohnbauten*; table 3.1.4, *VGR, Abschreibungen nach Vermoegensarten in jeweiligen Preisen, Ausrüstungen und Nichtwohnbauten*). *VGR* contains sectoral and capital good specific depreciation data only after 1991, which is why we decided to use only capital good specific depreciation rates for the entire time horizon. For the data sources for investment price deflators see footnote 48. The drawback of this procedure is that we do not observe directly capital-good specific  $d_{i,t}^a$  in the balance sheets (differently from  $k_{i,t}^a$ ), so that (12) is not directly applicable for the two types of capital goods separately. We therefore split up  $d_{i,t}^a$  according to the fraction that each capital good accounts for in the book value of total capital, weighting each capital good by its *VGR* depreciation rate. Creating a capital series for both capital goods this way is mainly meant to provide a better estimate for total capital for each firm, because we finally aggregate up both types of capital into a single capital good at the firm-level.

There is a final complication, which comes through relying on  $k_{i,1}^a$  as the starting value of the PIM. It is typically not a good estimate of the productive real capital stock of the firm at that time. Therefore, we calculate the time-average factor  $\phi$  (for each sector), by which  $k_{i,t}^{(1)}$  is larger than  $k_{i,t}^a$ , and replace  $k_{i,1}^a$  by  $\phi k_{i,1}^a$  in the perpetual inventory method. We do this iteratively, until  $\phi$  converges, i.e. we calculate:

$$k_{i,t+1}^{(n)} = (1 - \delta_t^e) k_{i,t}^{(n)} + \frac{p_t^I}{p_{1991}^I} i_{i,t} \quad (15)$$

$$k_{i,1}^{(n)} = \phi^{(n-1)} k_{i,1}^{(n-1)} \quad (16)$$

$$\phi^{(n)} = (NT)^{-1} \sum_{i,t} \frac{k_{i,t}^{(n)}}{k_{i,t}^{(n-1)}} \quad (17)$$

where  $k_{i,t}^{(0)} = k_{i,t}^a$ ,  $\phi^{(0)} = 1$ . We stop when for each sector and each capital good category  $\phi < 1.1$ .

Since for our purposes we want to compute economic, i.e. productive, capital stocks, we then – as a final step – add to the capital stock series from this iterative PIM the net present value of the real expenditures for renting and leasing equipment and structures.<sup>51</sup>

### A.3 Labor Inputs

A more particular difficulty with USTAN data is that information on the number of employees is only updated infrequently for some companies, as it is not taken directly from balance sheets, but sampled from supplementary company information. Being no balance sheet item, the employment data is not constrained by legal accounting rules and did not undergo consistency checks by Bundesbank staff. However, in order to compute firm-level Solow residuals, we need some measure of employment.

We base this measure on the payroll data ( $wagebill_{i,t}$ ) from the profit and loss statements (item ap154, *Personalaufwand*). Payroll data is regulated by accounting standards and is checked for consistency by the Bundesbank using accounting identities. In contrast to the direct employment data, the payroll data is generally considered of high quality. Therefore, we exploit this data to construct a proxy measure for (log) employment  $n_{i,t}$  as follows (with a slight abuse of notation, we use  $n_{i,t}$  here for log employment).

The idea behind our proxy measure is that we can determine sectoral average wages even though firm level employment is measured with error. Since wage bargaining in Germany is highly centralized, the sectoral average wage is all we need then, since it is a good proxy for firm level wages. Therefore, dividing firm level payroll by the sectoral average wage recovers true firm level employment.

Specifically, we assume that the measurement error in reported log employment,  $n_{i,t}^*$ ,<sup>52</sup> is classical and additive:

$$n_{i,t}^* = n_{i,t} + \varepsilon_{i,t}. \quad (18)$$

Then we decompose the wage per employee,  $\omega_{i,t}$ , of firm  $i$  at time  $t$  into two effects. One is determined by a firm-time-specific wage component  $w_{i,t}$ , and the other one being region-,  $r(i, t)$ , sector-,  $j(i, t)$ , and size-class-specific,  $s(i, t)$ , where  $j(i, t)$ ,  $r(i, t)$  and  $s(i, t)$  denote that

<sup>51</sup> Specifically, we take item ap161, *Miet- und Pacht aufwendungen*, from the profit and loss accounts, deflate it by the implicit investment good price deflator, which we compute, in turn, from tables 3.2.8.1 and 3.2.9.1 from *VGR*, and then divide it by a measure of the user cost of capital. The latter is simply the sum of real interest rates for a given year, which - courtesy of the Bundesbank - we compute from nominal interest rates on corporate bonds and ex-post CPI inflation data (the series is available from the authors upon request), and the time-average, accounting capital-good weighted depreciation rate per firm.

<sup>52</sup> We use item ap34, *Beschäftigtenzahl im Durchschnitt des Geschäftsjahres*, to measure  $n_{i,t}^*$ , where available.

firm  $i$  belongs to sector  $j$ , region  $r$  and size-class  $s$  at time  $t$ , respectively.<sup>53</sup> Thus, we write

$$\omega_{i,t} = \bar{w}_{j(i,t),r(i,t),s(i,t),t} + w_{i,t}. \quad (19)$$

We denote all firms that belong to sector  $j$ , region  $r$  and size-class  $s$  at time  $t$  by  $I(j, r, s, t)$ . Then we can estimate a sector-region-size wage component,  $\bar{w}_{j,r,s,t}$ , as:<sup>54</sup>

$$\widehat{w}_{j,r,s,t} = \frac{1}{\#I(j, r, s, t)} \sum_{i \in I(j,r,s,t)} \left[ \log(\text{wagebill}_{i,t}) - n_{i,t}^* \right]. \quad (20)$$

We then use this estimate of the average wage rate to estimate employment on the basis of the firm's wage bill:

$$\hat{n}_{it} = \log \text{wagebill}_{it} - \widehat{w}_{j,r,s,t} \quad (21)$$

$$= n_{it} + \omega_{it} - \frac{1}{\#I(j, r, s, t)} \sum_{h \in I(j,r,s,t)} (n_{h,t} + \omega_{h,t} - (n_{h,t} + \varepsilon_{h,t})) \quad (22)$$

$$= n_{it} + \omega_{it} - \frac{1}{\#I(j, r, s, t)} \sum_{h \in I(j,r,s,t)} (w_{h,t} - \varepsilon_{h,t}) \quad (23)$$

$$= n_{it} + \omega_{it} + \frac{1}{\#I(j, r, s, t)} \sum_{h \in I(j,r,s,t)} \varepsilon_{h,t}. \quad (24)$$

The second equality stems from using (18). The next to last equality holds, because one can replace  $\omega_{it}$  by (19), realizing that the  $\bar{w}$ , which do not depend on a specific firm, cancel. The last equality holds, because, by construction, the average firm-level deviation from a sector-region-size bin is zero in every year. For  $\#I(j, r, s, t)$  large, the average measurement error term  $\left( \frac{1}{\#I(j,r,s,t)} \sum_{h \in I(j,r,s,t)} \varepsilon_{h,t} \right)$  is negligible. In addition, since wage bargaining is highly centralized in Germany, also the firm specific wage component,  $w_{it}$ , can be expected to be of lesser importance, i.e. the variance  $\sigma_w^2$  is small. In particular it can be expected to be smaller than the initial measurement error in employment stocks. Therefore our measure of employment,  $\hat{n}_{i,t}$ , should follow real employment,  $n_{i,t}$ , more closely than  $n_{i,t}^*$ .

To corroborate this claim, we checked our procedure using data from the German social

<sup>53</sup>Specifically, for sectors we use the 2-digit classification in Table 20 in Appendix A.1. For size classes we use terciles of the capital distribution in each year. For the region-specific wage component we proceed as follows: we divide West Germany into three regions, according to zip codes: South with zip codes starting with 7,8,9, except for 98 and 99; Middle with zip codes starting with 4,5,6, except for 48 and 59; North with zip codes starting with 2,3 as well as 48 and 59. However, not all balance sheets feature zip code information, which is why we compute  $\widehat{w}_{j,r,s,t}$  with and without a region component. For those firms that do not have zip code information or for those firms that are in sector-region-size bins with fewer than 50 observations in a given year, we take the estimate without the region component.

<sup>54</sup>To estimate  $\widehat{w}_{j,r,s,t}$  we of course use only those observations, where  $n_{i,t}^*$ , i.e. item ap34, *Beschaeftigtenzahl im Durchschnitt des Geschaeftsjahres*, is available.

security records at the *Institut fuer Arbeitsmarkt- und Berufsforschung (IAB)*, which provide information on the wage bill and employment at the establishment level. There we observe true employment and wage bills for all plants and the time 1975-2006. Constraining ourselves to the sample period 1975-1998 and to plants with more than 12 employees, i.e. to data comparable to the one of the USTAN data, we find the correlation between  $\hat{n}_{i,t}$  and  $n_{i,t}$  as well as between  $\Delta\hat{n}_{i,t}$  and  $\Delta n_{i,t}$  to be fairly high (98% and 94%, respectively). This means that the cross-sectional variance of the firm specific wage innovations  $\sigma_{\Delta w}^2$  is small (0.0026) compared to the cross-sectional variance of employment changes ( $\sigma_{\Delta n}^2 = 0.0163$ ,  $\sigma_{\Delta \hat{n}}^2 = 0.0162$ ). Finally, a correlation coefficient between  $mean(\Delta n_{i,t})$  in the USTAN data and the log-change in aggregate NFPBS employment of 0.653 shows also the quality of our employment measure.

#### A.4 Solow Residual Calculation

With the estimated firm-level capital stocks and employment levels we can now compute firm-level Solow residuals from the logged production function (1). In our baseline specification we estimate the factor elasticities,  $\nu$  and  $\theta$ , as 1-digit sector-specific median, pooled over all firm-year observations in a sector, expenditure shares.<sup>55</sup> Table 26 displays the estimated elasticities. Simulations show that under the null hypothesis of the model the labor elasticity is very accurately estimated by the labor share, whereas the capital elasticity is slightly overestimated by the capital share, which makes our simulations conservative, as we have shown that a lower capital elasticity, i.e. more curvature in the revenue function, will lead to a stronger extensive margin effect, that will make investment dispersion more procyclical (see Section 5.1 for details). Notice that for the aggregate Solow residual calculation in the baseline scenario, for which we use the data sources specified in Footnote 48 in Appendix A.1, we simply use the expenditure shares from manufacturing, as manufacturing is still the largest sector within NFPBS (had we used any weighted median of expenditure shares the result would have been the same). We experiment also with weighted average expenditure shares, both weighted with value added and with employment/capital and using USTAN and NFPBS weights. To come up with a single number for each factor elasticity, we simply take the median of these four weighted averages and use  $\nu = 0.5229$  and  $\theta = 0.2352$ . This requires a recalibration of the adjustment costs factor,  $\bar{\xi}$ , to 0.35, but the baseline result is not changed: the resulting procyclicality of investment dispersion is 0.584, a number very close to the 0.580 of the baseline scenario.

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<sup>55</sup>We use profit and loss account item ap153, *Rohergebnis*, for firm-level value added and deflate it in the baseline scenario with the aggregate value added deflator, but experiment also with sector-specific value added deflators, see Footnote 48 in Appendix A.1 for details. To compute firm-level expenditure shares, we proceed as follows: the labor share is simply total payroll divided by value added (ap154/ap153); capital expenditures, which are then again divided by value added, are the sum of the PIM capital stock and the net present value of renting and leasing expenditures multiplied by the user cost of capital as specified in Footnote 51 in Appendix A.2.

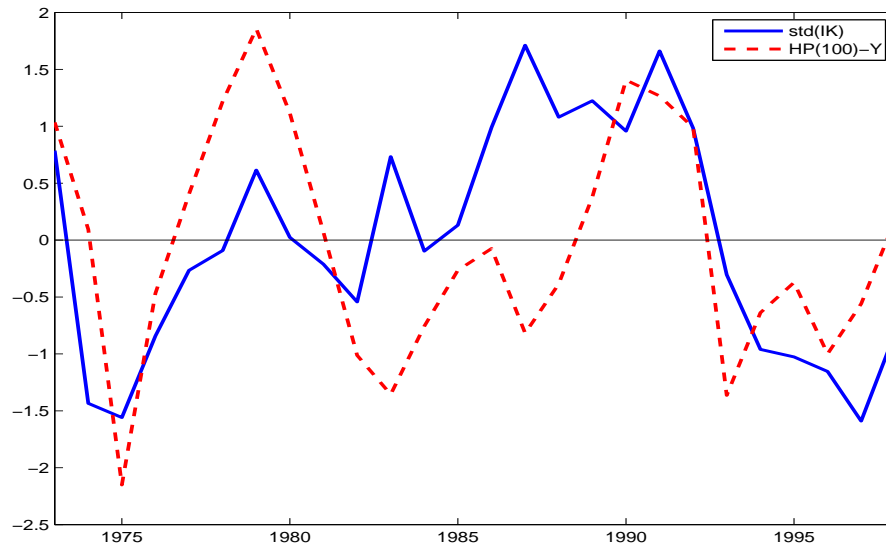


Table 26: SECTOR-SPECIFIC EXPENDITURE SHARES

ID	Sector	labor share $\nu$	capital share $\theta$
1	Agriculture	0.2182	0.7310
2	Energy & Mining	0.3557	0.5491
3	Manufacturing	0.5565	0.2075
4	Construction	0.6552	0.1771
6	Trade	0.4536	0.2204
7	Transport & Communication	0.4205	0.2896

## A.5 Two More Graphs

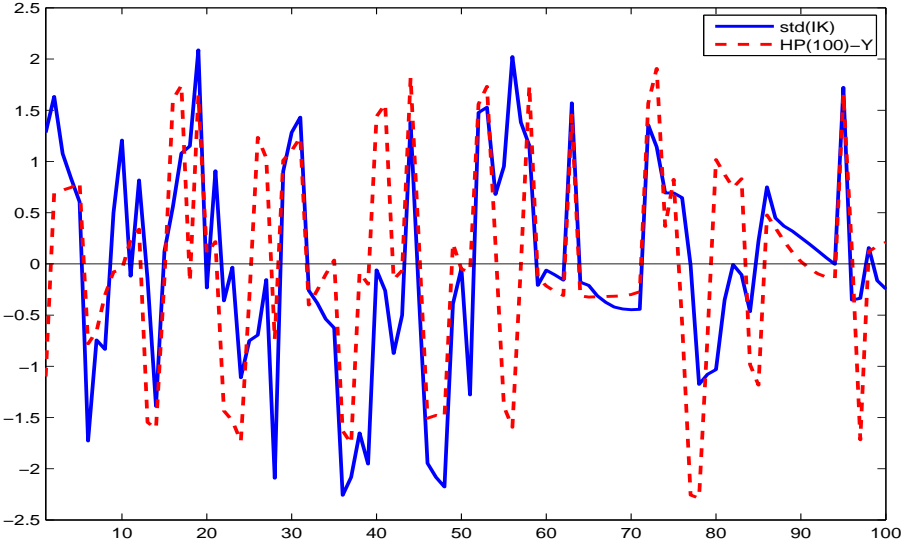
Figure 7: Data: Time Series of Investment Dispersion and Cyclical Component of GDP - Normalized by their STD



*Notes:*

Dispersion refers to the cross sectional standard-deviation. The cyclical component of GDP is the HP-filtered output series with a smoothing parameter of 100.

Figure 8: Baseline Model: Time Series of Investment Dispersion and Cyclical Component of GDP - Normalized by their STD



*Notes:*

Dispersion refers to the cross sectional standard-deviation. The cyclical component of GDP is the HP-filtered output series with a smoothing parameter of 100.

## A.6 Cross-sectional Dispersion Data

Table 27: CROSS-SECTIONAL DISPERSION DATA FOR THE INVESTMENT RATE AND THE EMPLOYMENT CHANGE RATE IN THE BASELINE EMPIRICAL SCENARIO

Year	$\sigma\left(\frac{i_{i,t}}{0.5*(k_{i,t}+k_{i,t+1})}\right)$	$\sigma\left(\frac{\Delta n_{i,t}}{0.5*(n_{i,t-1}+n_{i,t})}\right)$
1973	10.244	13.6663
1974	8.9526	14.4443
1975	9.0204	14.4376
1976	9.6333	13.93
1977	10.1548	13.2382
1978	10.4159	13.2087
1979	11.0221	13.1194
1980	10.7869	13.0973
1981	10.783	13.6914
1982	10.716	13.659
1983	11.6913	13.5832
1984	11.3018	13.2013
1985	11.5982	13.5816
1986	12.3032	13.2644
1987	12.9186	13.4395
1988	12.6576	13.0941
1989	12.8989	12.7371
1990	12.8748	13.3669
1991	13.4788	13.2751
1992	13.1874	12.9378
1993	12.5017	13.1612
1994	12.2241	12.9218
1995	12.3296	12.6971
1996	12.3953	12.8086
1997	12.2611	12.264
1998	12.9089	12.1935

*Notes:*  $\sigma$ : cross-sectional standard deviation of the within-transformed data. No detrending. The corresponding data for  $\sigma(\Delta \log \varepsilon_{i,t})$  and  $\sigma(\Delta \log y_{i,t})$  can be found in Bachmann and Bayer (2009).

## B Robustness of Cross-sectional Cyclicity

In this appendix we check the robustness of the main empirical finding of this paper – the procyclicality of investment dispersion – to sample selection and variable construction. First, we use an aggregate price deflator for investment goods (see Footnote 48 in Appendix A.1 for details) in the perpetual inventory method instead of sectoral deflators separately for equipment and structures. Second, we employ a stricter outlier removal criterion of 2.5 standard deviations around the firm- and year-specific mean in Solow residual and value added innovations, as well as investment rates and employment changes. Third, we use two more liberal outlier criteria using 5 and 10 standard deviations instead of 3.<sup>56</sup> Fourth, we employ a specification, where we assume that an outlier above 3 standard deviations means a merger and, subsequently, treat these firms as new firms in addition to removing them in the year, where the outlier occurs. Fifth, we restrict the sample to firms with at least 20 observations in first differences, in order to make sure that the cyclical effects we find are not due to cyclical variations in the sample composition.<sup>57</sup> Sixth, we use all the firms that we observe at least twice with first differences.<sup>58</sup> Finally, we carry out a more standard PIM that simply uses the reported capital stocks in the first year of observation for a firm, instead of solving a fixed point problem in correction factors (see Appendix A.2 for details). As one can see from Table 28, the results are robust to all these alternative sampling procedures; in particular, the robust procyclicality of investment dispersion is not driven by a change in the cyclicity of the dispersion of the driving force.

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<sup>56</sup>The latter variant lowers the number of dropped firm-year observations due to outliers in factor and value added changes from 41,453 to 4,240, and the ones due to outliers in Solow residual changes from 18,978 to 1,486. This leaves the total number of firm-year observations at 934,315 and the total number of firms in the sample at 78,092.

<sup>57</sup>Consistent with the slightly lower correlation of investment dispersion with aggregate output in this case, we find the same correlation coefficient to be 0.382, when we control for sample selection in the following way: we estimate a simple selection model, where lagged firm-level Solow residuals determine selection and the firm-level investment rate is modeled as a mean regression. We use the maximum likelihood estimator by Heckman (1976) to infer the selection-corrected variance of the residual in the firm-level investment rate equation. The latter is very close to the sample variance of firm-level investment rates, indicating that our results are not influenced by systematic sample drop-outs.

<sup>58</sup>This lowers the number of dropped firm-year observations due to not satisfying the minimum observation requirement from 417,550 to 158,950. This leaves the total number of firm-year observations at 971,308 and the total number of firms in the sample at 114,528.

Table 28: CYCLICALITY OF CROSS-SECTIONAL INVESTMENT DISPERSION - DATA TREATMENT

Treatment	$\rho(\sigma(\frac{\dot{i}_{i,t}}{0.5*(k_{i,t}+k_{i,t+1})}), HP(100) - Y)$	$\rho(\sigma(\Delta \log \epsilon_{i,t}), HP(100) - Y)$
<i>Baseline</i>	0.451	-0.481
Uniform price index for I-goods	0.427	-0.480
Stricter outlier removal	0.452	-0.499
Looser outlier removal	0.422	-0.476
Very loose outlier removal	0.427	-0.578
Stricter Merger Criterion	0.416	-0.486
Longer in sample	0.392	-0.341
Shorter in sample	0.439	-0.485
Standard Perpetual Inventory	0.563	-0.492

## C Aggregate Statistics

Table 29: AGGREGATE BUSINESS CYCLE STATISTICS FOR THE BASELINE CALIBRATION

Moment/Aggregate Quantity	<i>Y</i>	<i>C</i>	<i>I</i>	<i>N</i>
Standard Deviation	4.04% (2.30%)	1.27% (1.79%)	18.99% (4.37%)	3.01% (1.80%)
Relative Standard Deviation	1	0.32 (0.78)	4.71 (1.90)	0.74 (0.78)
Persistence	0.42 (0.48)	0.66 (0.67)	0.34 (0.42)	0.33 (0.61)
Correlation with <i>Y</i>	1	0.84 (0.66)	0.98 (0.83)	0.97 (0.68)

*Notes:*

Business cycle statistics of aggregate output, *Y*, consumption *C*, investment *I* and employment *N*. *N* in the model includes the amount of labor used to adjust the firms' capital stocks. All variables are logged and then HP-filtered with a smoothing parameter of 100. The first numbers in a column refer to a simulation of the model over  $T = 1500$  periods. Numbers in brackets refer to German aggregate NFPBS data. Persistence refers to the first order autocorrelation.

All variables are logged and then HP-filtered with a smoothing parameter of 100. The numbers in brackets are the statistics from the data, from the sectoral aggregate that corresponds to the USTAN data: the non-financial private business sector (NFPBS). They are gathered from German sectoral national accounting data (see Footnote 48 in Appendix A.1 for details). Real private consumption data are *private Konsumausgaben*, a chain index with base year in 1991, from table 3.2 in the *VGR*. The model employment variable includes the amount of labor used to adjust the firms' capital stocks.

In our baseline calibration, the economy is overall too volatile, which we attribute partly to

the fact that we compute the aggregate Solow residual process from the private non-financial business sector and not from the overall economy. Nevertheless, both the too high volatility numbers, as well as the too low persistence numbers as well as the discrepancy between model and the data in the relative standard deviations – relative to  $std(Y)$  – of aggregate consumption and aggregate investment show that there is not enough smoothing in the baseline calibration, which is a well-known problem of the standard RBC model. Our baseline model cannot improve that, as the level of non-convexities essentially puts it in a parameter range, where the Khan and Thomas neutrality result still holds (see Khan and Thomas, 2008). Since this paper is exclusively concerned with cross-sectional dynamics, for which – as we have shown – non-convexities matter already at a level, where they would be near-neutral for aggregate dynamics, we do not view this as a problem for our main result. More smoothing could be implemented through a standard quadratic adjustment cost element on top of the fixed cost, however at both a substantial computational burden and at the expense of cleanness of exposition. In fact, quadratic adjustment costs would work very similarly to an increase in curvature in the maximized-out revenue function, which, as we have shown, puts more emphasis on the procyclical extensive margin and would only strengthen our mechanism. Our robustness checks include a case, where we decrease the volatility of the aggregate Solow residual in order to match the volatility of aggregate output. This puts relatively more weight on the second-moment shocks, i.e. the countercyclicality of the dispersion in the Solow residual innovations, and would make it – all things equal – harder for the extensive margin effect in the lumpy model to generate procyclicality of investment dispersion. Row five in Table 17 in Section 5.2 shows that this does not invalidate our baseline result. To summarize: the aggregate shortcomings of the model are similar to the one in the standard RBC model, but based on our robustness checks we view them as mainly orthogonal to the cross-sectional dynamics that this paper focusses on.

## D Sectoral Calibration

Table 30: SECTORAL CALIBRATION

Sector	$\bar{\xi}$	$\bar{\sigma}(\epsilon)$	$skew(\frac{i_{i,t}}{0.5*(k_{i,t}+k_{i,t+1})})$	$kurt(\frac{i_{i,t}}{0.5*(k_{i,t}+k_{i,t+1})})$	$\theta$	$\nu$	$\delta$
<i>Aggregate</i>	0.3	12.01	2.1920	20.0355	0.208	0.557	0.108
MIN	0.5	11.56	1.3355	15.8334	0.549	0.356	0.093
MAN	0.25	11.47	2.2511	21.4518	0.208	0.557	0.119
CON	0.25	10.56	1.7684	20.9611	0.177	0.655	0.153
TRD	0.4	12.44	2.1091	17.6077	0.220	0.454	0.123
TRA	0.07	13.56	1.3315	10.6363	0.290	0.421	0.112

*Notes:* See Figure 2 for the sectoral acronyms.  $\bar{\xi}$  is the calibrated adjustment cost parameter.  $\bar{\sigma}(\epsilon)$  is the long-run standard deviation of the innovations to the firm-level Solow residual.  $\theta$  and  $\nu$  are the capital and employment, respectively, elasticity in the production function. In the computation for the mining and energy sector we scaled down the measured and reported factor elasticities by a factor of 0.9.  $\delta$  are sector-specific depreciation rates.

Table 31: RESULTS FROM SECTORAL CALIBRATION - AGGREGATE SOLOW RESIDUALS

Sector	$\rho(\sigma(\frac{i_{i,t}}{0.5*(k_{i,t}+k_{i,t+1})}), HP(100) - Y)$		$\rho(\text{Fraction of Adjusters}, HP(100) - Y)$	
	Model	Data	Model	Data
<i>Aggregate</i>	0.580	0.451	0.485	0.727
MIN	-0.097	0.011	-0.066	0.106
MAN	0.614	0.372	0.577	0.786
CON	0.335	0.357	0.154	0.675
TRD	0.900	0.452	0.917	0.575
TRA	0.239	0.473	0.255	0.592

*Notes:* See notes to Tables 12. See Figure 2 for the sectoral acronyms. Aggregate Solow residuals were used in the driving force.  $HP(100) - Y$  refers to the cyclical component of the output of the private non-financial business sector aggregate. The correlation coefficients between model simulations and data are 0.721 for investment rate dispersion and 0.560 for the extensive margin. The corresponding rank correlations are 0.4 and 0.3, respectively.

Table 32: RESULTS FROM THE SECTORAL CALIBRATION - OWN SECTOR SOLOW RESIDUALS - CORRELATION WITH AVERAGE INVESTMENT RATE

Sector	$\rho(\sigma(\frac{i_{i,t}}{0.5*(k_{i,t}+k_{i,t+1})}), \text{mean}(\frac{i_{i,t}}{0.5*(k_{i,t}+k_{i,t+1})}))$		$\rho(\text{Fraction of Adjusters}, \text{mean}(\frac{i_{i,t}}{0.5*(k_{i,t}+k_{i,t+1})}))$	
	Model	Data	Model	Data
<i>Aggregate</i>	0.621	0.792	0.736	0.847
MIN	-0.259	-0.674	-0.164	-0.312
MAN	0.799	0.845	0.800	0.874
CON	0.781	0.713	0.682	0.884
TRD	0.984	0.720	0.989	0.869
TRA	0.302	0.622	0.342	0.822

*Notes:* See notes to Tables 12. See Figure 2 for the sectoral acronyms. Own sector Solow residuals were used in the driving force. In the first row,  $\text{mean}(\frac{i_{i,t}}{0.5*(k_{i,t}+k_{i,t+1})})$  refers to the linearly detrended average investment rate in the USTAN sample. From the second row onwards it means the linearly detrended average investment rate in the corresponding sector. The correlation coefficients between model simulations and data are 0.899 for investment rate dispersion and 0.972 for the extensive margin. The corresponding rank correlations are 0.9 and 0.6, respectively.



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