A MICROFOUNDED SECTORAL MODEL FOR OPEN ECONOMIES

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Abstract

In this paper we derive a microfounded macro New Keynesian model for open economies, be them large or small. We consider habit formation in consumption, sectoral linkages for tradable and non-tradable goods, capital stock investments with variable capital utilization, domestic and foreign governments, imperfect (exchange rate) pass-through in import prices and incomplete international financial markets. Sticky nominal prices and wages are modeled in Calvo and Taylor staggered ways. The model economy is composed of a continuum of infinitely-lived consumers and producers of final and intermediate goods. We provide a very general log-linearization method, from which we can easily obtain various special cases, as trend inflation or steady-state log-linearizations.
Numerical simulations of the two-country sectoral model are provided for a relatively large number of structural shocks as domestic and foreign productivity shocks in final tradables and non-tradables, money demand shocks and a shock in the exchange rate. Such a model is well suited for monetary policy analysis at the international level and risk analysis.

JEL Code: E31, D21, F41, P24.

Keywords: New Keynesian open economy model, tradable and non-tradable sectors, final and intermediate goods, log-linearization.

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1 Introduction

During the recent years the theoretical and empirical research in New Keynesian (NK) macroeconomics has been extended steadily and produced a whole new series of results and insights about the workings of the macroeconomy. Essential starting point of the NK approach is the explicit derivation of macroeconomic relations by their microeconomic foundations, in particular the inclusion of optimizing consumers and producers. It shares this principle with New Classical macroeconomics, although it differs from the latter by considering various imperfections in the goods and labor markets that contain an amount of flexibility, especially in the short run.

Recently, much interest is also directed at the modeling and testing of the effects and interactions produced by the foreign sector, e.g. rigidities of import and export pricing may be of significant importance to an open economy. Moreover, fluctuations in import prices of important intermediate goods such as oil and steel will have strong effects on domestic firms of an open economy. In that perspective, the exchange rate will play an important role in the transmission of such price fluctuations to the domestic economy. NK models with a worked out foreign sector are often referred to as *New Open Economy Macroeconomics* (NOEM) models.

This paper derives *a microfounded macroeconomic NK model for open economies*, which tries to extend the NK/NOEM literature in several respects. We provide a detailed modeling of the consumers' and producers' decisions. In particular, firms in this model produce two types of goods: *final* and *intermediate* goods. Final goods can be consumption and capital (investment) goods. Moreover, both types of goods can be either *tradable* or *non-tradable* depending on whether they can be traded internationally.[1] Firms in every sector are assumed to be characterized by nested CES production functions whose arguments are technology, labor force, domestic tradable and non-tradable capital goods and intermediate goods and imported foreign capital goods and intermediate goods. It is assumed that intermediate goods producers do not use other intermediate goods as their inputs, but notice that capital goods producers may use all kinds of inputs. Each firm is assumed to possess some price-setting power on relevant markets, reflecting *monopolistic competition.* Consumers are assumed to purchase a bundle of domestically produced tradable and non-tradable final goods and imported final goods and are assumed to act as monopolistic suppliers of labor services (with sticky wages). Moreover, consumers own capital that is rented out to firms and the consumer budget constraint is subject to variable capital utilization costs. Consumption allocation is assumed to be shaped by *habit formation.* Financial markets are assumed to be complete domestically and incomplete internationally (see Benigno (2001)).[2] Domestic consumers' and firms' decisions, and financial markets build the model of the home economy. Altogether, we distinguish 21 (final and intermediate) goods markets with which domestic firms are confronted. The foreign economy is modeled in a parallel manner so that import/export prices and quantities and other relevant variables are endogenized in our approach. In particular, it makes the model more flexible since both small and large economy settings can be studied in this framework.

Such an extensive modeling of intermediate goods sectors is important especially in the context of exchange rate policies. Dellas (2005) points out that the presence of intermediate goods has vital consequences for the ability of monetary authorities to manipulate nominal exchange rates. When there is no production interdependence between countries (i.e. only consumption goods are traded) changes in a nominal exchange rate do not affect production costs. However, when there is trade in intermediate and capital goods as in the real world, an exchange rate depreciation/appreciation has adverse direct effects on the cost of domestic production. Consequently, it makes the exchange rate instrument less useful.[3]

An important feature of the model is the method of log-linearization. Almost the complete existing NOEM literature approximates the model by log-linearization around steady state values of variables. These studies usually assume that the long-run anchor for inflation expectation is zero, which is a very convenient but counterfactual assumption. Long-run inflation expectations should converge to the natural inflation of monetary policy, where there are no restrictions. This natural inflation is a (relatively small) positive rate in most modern industrialized

[1] The distinction between tradable and non-tradable goods in macroenomic models is still controversial. On the one hand, Chari *et al.* (2002) find that only 2% of the variance of real exchange rates is due to fluctuations in the relative prices of non-traded to traded goods using data for the United States and an aggregate of Europe. Consequently, they ignore the non-tradable sector in their model. On the other hand, our approach hinges upon the explicit introduction of non-tradable goods and incomplete markets with a stationary net foreign assets position. Betts and Kehoe (2005) and Burstein *et al.* (2005) stress that the variance of the real exchange rates explained by (that of) the relative prices of non-traded to traded goods is not 2% but about one third of the total variance (see also Selaive and Tuesta (2006), pp. 1 and 2). Hence, a modeling with a separate treatment of non-tradable goods seems to be important.

[2] The assumption that international financial markets are incomplete originates from the fact that a risk premium on external borrowing exists and is related to net foreign assets. Hence, the (exchange rate) pass-through is assumed to be incomplete due to underlying nominal rigidity in the buyer's currency position (see e.g. Adjémian *et al.* (2004)). The proposed specification in this paper is flexible enough to discriminate between the polar cases of Producer Currency Pricing (PCP), where the law of one price holds and there is perfect pass-through, and Local Currency Pricing (LCP), where the pass-through is zero in the short run.

[3] The relevance of intermediate goods in the production processes is nicely illustrated by the annual input-output tables of (for instance) the Dutch economy, where they account for up to more than 50% of all inputs in several industries.

economies, but typically time-varying. Consequently, some recent studies departure from the zero steady state inflation rate, e.g. Ascari (2004), who considers trend inflation, and Kozicki and Tinsley (2002) and Bakhshi *et al.* (2003), who also provide derivations of an NK Philips curve with a non-zero steady state inflation rate. We propose a more general approach of log-linearization around *time-dependent paths* of economic variables. Starting from this general formulation, we introduce several restrictions in increasing order of limitation. First, we assume that time-dependent variables around which we log-linearize satisfy the first order conditions, where there are no restrictions or rigidities on the variables. Such time-dependent variables are called *natural* (or *flexible*) *values*. Second, we restrict that all variables in natural values fluctuate at the same time-dependent rate. Such paths of variables are called *(time-)varying rate paths*. Third, we further restrict our attention to constant rate paths, i.e. to (constant) *trend paths* of (natural) variables (see, for example, Ascari's (2004) constant *trend inflation*). Finally, the conventional *steady state* is the simplest special case of all the above log-linearizations, where the constancy of natural values is assumed. Moreover, if also the equilibrium conditions are satisfied, time-dependent paths of variables are called *time-dependent equilibrium paths*. In this paper we log-linearize around time-dependent paths of variables and derive the above mentioned special cases from them. The log-linearization is performed in detail in the appendix.

In this paper we focus on *nominal* price and wage rigidities in all relevant markets.[4] Given adequate interpretation, *sticky wages* on labor markets can be modeled in a similar way as sticky prices on product markets. This way of modeling prices and wages is now widely used in empirical Dynamic Stochastic General Equilibrium (DSGE) models; see, for example, Altig *et al.* (2002), Amato and Laubach (2003), Smets and Wouters (2003, 2004), Adjémian *et al.* (2004) and Christiano *et al.* (2005).

Modeling nominal price and wage rigidities can generally be subdivided according to three main hypotheses (see also Klein (2002)):

(i) Grossman (1974), introduced the model of price adjustment, which was named the "P-bar" model by McCallum (1994). The idea is that the degree of disequilibrium in prices indicates inflationary pressure in a "sticky" price world and prices are set before trading; hence, price setters can only adjust at time t to the movements in equilibrium prices expected at $t-1$. This specification was used by Barro and Grossman (1976), McCallum (1979,1994), Mussa (1981a,1981b,1982), Obstfeld and Rogoff (1984), Flood and Hodrick (1986), and Chadha and Prasad (1993).

(ii) Taylor's (1980) staggered price (wage) setting model is a second well known tractable model of slow price (wage) adjustment. It is based on the theory of price and wage contracting, where periodically such contracts are renegotiated.

(iii)The Calvo-type staggered price setting (see Rotemberg (1982) and Calvo (1983)) allows a subset of firms to choose their own product price randomly with a constant probability. From this partial adjustment behavior price dispersion emerges. Calvo's price adjustment model is extremely tractable and also very popular in DSGE models such as Rotemberg (1987), Hairault and Portier (1993), Kimball (1995), King and Watson (1996), King and Wolman (1996), Rotemberg (1996), Yun (1996), Ireland (1997), Woodford (1998) and Kim (2000). A Calvo-type staggered wage setting by consumers can also be studied similarly; see e.g. Erceg *et al.* (2000) who present a Calvo model for both price and wage settings.

Interrelationships of these three price adjustment models can be found in Kiley (2002). In this paper we concentrate on the last two nominal price and wage adjustment models. In this respect, some recent studies have pointed out deficiencies of the Calvo (1983) approach and have challenged it. For example, when using the Calvo price setting model, it is typically assumed that steady state inflation is zero, which is a very restrictive assumption *per sé* as mentioned above. Moreover, Bakhshi *et al.* (2003) argue that firms in a low inflation environment behave in different ways than firms in a high inflation environment; consequently, inflation dynamics driven in the Calvo-type model by a constant probability of resetting prices should not be considered as constant. They show that for standard calibrations, the Calvo price setting model with constant probability of resetting prices can be used only when annual trend inflation is lower than 5.5%, a condition that is not met for inflation time-series data of several OECD countries over the last 25 years.

This paper is organized as follows. In Section 2 the domestic households' problem is defined, where habit

[4] *Real* rigidities occur when relative prices react only slowly to changes in relative demand, implying that reallocation of (factors of) production is slow. There are numerous possible sources of real rigidities. A selection includes efficiency wages, wage indexation, hysteresis, insider-outsider workers and other labor market institutions, distortionary taxation and other forms of government intervention and regulation. In a reduced form, real rigidities imply amongst others a low parameter in the price adjustment function of the real variables, principally output. In other words, the output supply function is flatter, i.e. shifts in demand lead to adjustment in output rather than in prices (see Blanchard and Fisher (1989)). Similarly, wages would not react much to unemployment. In addition, real rigidities may imply that long-run equilibrium is achieved at below-equilibrium full employment output. In the presence of (nominal and) real rigidities, the economy may therefore reach a steady state that is marked by both positive long-run inflation ("core" inflation) and divergences between long-run equilibrium real output and employment and potential output and employment.

formation in consumption is assumed and domestic households' consumption demands and investment supplies are derived in Subsections 2.1 and 2.2, respectively. The foreign households' consumption demands and investment supplies are analyzed in Appendices A.1 and A.2, respectively. Section 3 introduces a domestic government, while the foreign government's demands are summarized in Appendix A.3. The firms' supply linkages in production, distinguishing intermediate goods from final goods and tradable from non–tradable goods, are shortly discussed in Section 4. The domestic households' and firms' optimization problems are explicitly solved in Section 5. Equilibrium conditions for the home economy are derived in Section 6. In Section 7, the net foreign asset position of the aggregate domestic economy is derived. Regarding price and wage formation, we consider stickiness with Calvo and Taylor staggered price and wage settings in Section 8. Monetary policy rules are discussed in Section 9. Some numerical simulations demonstrating the functioning of the model will be discussed in Section 10. Finally, concluding remarks are in Section 11.

2 Domestic households

Household i is assumed to enjoy an expected level of intertemporal utility represented by a concave, differentiable and strongly separable function, which positively depends on present and past consumption (the latter due to habit formation), leisure and real money balances. Hence, household i's utility can be written as:[5]

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t \left[U_1 \left(C_t(i), C_{t-1}(i) \right) - U_2(L_t(i)) + U_3\left(\frac{M_{t+1}(i)}{P_t^C(i)} \right) \right] \right], \tag{1}$$

where E_0 denotes the conditional expectations' operator depending on the information available at period 0; β is the household's discount factor satisfying $0 < \beta < 1$.

The first (utility) felicity $U_1(.)$ represents utility from consumption $C(i)$ of both domestic and foreign goods. We assume that household i is interested not only in current consumption but also in past values of it because of habit formation (see Abel (1990, 1999), Caputo (2003), Choi and Jung (2003), and Lindé *et al.* (2004)). The idea is that, if a household's consumption increases today owing to a shock, this household will experience a higher utility from an additional unit of consumption tomorrow. Intuitively, under habit formation, a consumer gets 'used' to a higher level of consumption and the marginal utility of consumption gets 'renormalized' at this higher reference level.

The second (utility) felicity $U_2(.)$ in (1) reflects a domestic consumer i's disutility derived from supplying labor services $L_t(i)$ to the (domestic) firms remunerated at the rate $W_t(i)$; hence, period t utility is a negative function of labor effort $L_t(i)$.[6] This labor effort can be subdivided across four different sectors that produce tradable final goods ($L_t^{FT}(i)$), non-tradable final goods ($L_t^{FN}(i)$), tradable intermediate goods ($L_t^{VT}(i)$) and non-tradable intermediate goods ($L_t^{VN}(i)$),[7] or

$$L_t(i) \equiv L_t^{FT}(i) + L_t^{FN}(i) + L_t^{VT}(i) + L_t^{VN}(i), \tag{2}$$

and corresponding nominal wages in each sector are $W_t^{FT}(i)$, $W_t^{FN}(i)$, $W_t^{VT}(i)$ and $W_t^{VN}(i)$, respectively. Hence, household i's gross nominal labor income equals,

$$W_t(i)L_t(i) = \sum_{m=FT,FN,VT,VN} W_t^m(i)L_t^m(i), \tag{3}$$

so that $W_t(i)$ is consumer i's weighted aggregate wage rate.

The third (utility) felicity $U_3(.)$ in (1) reflects utility from holding real money balances by consumer i at the beginning of period $t+1$, $\frac{M_{t+1}(i)}{P_t^C(i)}$, with $M_{t+1}(i)$ and $P_t^C(i)$ consumer i's nominal money balances at the beginning of period $t+1$ and consumer i's prices at (the end of) period t, respectively.

For sector $m = FT, FN, VT, VN$, let us assume that consumer i owns physical capital, $K_t^m(i)$, which is rented out to firms, and decides on its utilization rates, $\mathcal{Z}_t^m(i)$. Consequently, capital services (or rental services from capital) supplied by consumer i to sector m at period t are:[8]

$$\mathcal{K}_t^m(i) \equiv \mathcal{Z}_t^m(i) K_t^m(i). \tag{4}$$

[5] The infinite time horizon implied by utility function (1) is a simplication. It can be rationalized as if there are dynasties of individual consumers who are concerned about the future of their children.

[6] Thus, leisure $1 - L_t(i)$ is the residual of the individual's time endowment (with a normalized value of 1 for total time endowment).

[7] This sectoral split is motivated from Natalucci and Ravenna (2005) and Ortega and Rebei (2006). As pointed out in the Introduction, a modeling with a separate treatment of non-tradable goods seems to be very relevant.

[8] Even though the installed capital in sector m is utilized at a variable rate, it is costly to modify the utilization rate in the short run. Moreover, to expand the capital available in sector m, investors must face sector-specific installation costs. The implied asymmetric

The total supply of real investment goods by domestic consumer i is defined as:

$$I_t(i) \equiv \sum_{m=FT,FN,VT,VN} I_t^m(i), \tag{5}$$

where sector m's physical capital at the beginning of period $t+1$ is subject to the following laws of capital accumulation (see e.g. Smets and Wouters (2003, p. 1128) and Coenen and Straub (2005, p. 13)):

$$K_{t+1}^m(i) = (1 - d_t^m(i))K_t^m(i) + \left(1 - \Upsilon\left(\frac{I_t^m(i)}{I_{t-1}^m(i)}\right)\right) I_t^m(i) \quad \text{for } m = FT, FN, VT, VN, \tag{6}$$

where $d_t^m(i)$ is consumer i's depreciation rate (in perunages) for sector m, $I_t^m(i)$ is the investment expenditure in sector m by consumer i at period t and the function $\Upsilon(.)$ captures the (adjustment) costs due to changes in investment rates and acts as a proxy to installation costs of investment.[9]

Each household i is assumed to face a (nominal) consumer budget constraint (CBC) in home currency units:[10]

$$\begin{aligned} &P_t^C(i)C_t(i) + P_t^I(i)I_t(i) + Q_{h,t,t+1}B_{h,t+1}(i) + S_t\hat{Q}_{f,t,t+1}B_{f,t+1}(i) + M_{t+1}(i) \\ &+P_t^I(i)\left[\sum_{m=FT,FN,VT,VN} \Psi(\mathcal{Z}_t^m(i))K_t^m(i)\right] \leq (1-\tau_t^w)\, W_t(i)L_t(i) + T_t(i) \\ &+ \left(1-\tau_t^k\right)\left[\sum_{m=FT,FN,VT,VN} R_t^m(i)\mathcal{K}_t^m(i)\right] + B_{h,t}(i) + S_tB_{f,t}(i) + M_t(i) + \mathcal{B}_t(i), \end{aligned} \tag{7}$$

where consumer i's outlays of resources are on the left hand side, while her disposable resources are on the right hand side; moreover, $P_t^C(i)$ and $P_t^I(i)$ indicate aggregate consumption and investment goods prices, respectively.[11]

Among the outlays we include nominal consumption, nominal investment, the portfolio of assets (bonds and stocks) and money holdings and the nominal cost of the underutilization of sectoral physical capital, which is given by the function $\Psi(\mathcal{Z}_t^m(i))$ and is zero when capacity is fully utilized.[12] Regarding assets, according to Woodford (2003) and Ambler *et al.* (2003, 2004), $Q_{h,t,t+1} \equiv (1 + r_{t,t+1})^{-1}$ and $\hat{Q}_{f,t,t+1} \equiv (1 + \hat{r}^*_{t,t+1})^{-1}$ are the domestic consumer's one-period ahead discount factors for (nominal) domestic and foreign asset payoffs, respectively.[13] Following Benigno (2001) financial markets are assumed to be complete domestically and incomplete internationally. Hence, a risk premium exists only on foreign assets so that the discount factor for foreign asset payoffs is stochastic and the expected (nominal) market price at date t of an asset portfolio, held at the beginning of period $t+1$, is given by $Q_{h,t,t+1}B_{h,t+1}(i) + E_t\left[S_t\hat{Q}_{f,t,t+1}B_{f,t+1}(i)\right]$, where $B_{h,t+1}(i)$ ($B_{f,t+1}(i)$) are consumer i's domestic (foreign) asset holdings denominated in local currency.[14]

Concerning the net resources, τ_t^w and τ_t^k are proportional tax rates applied on nominal labor and capital incomes, respectively, $R_t^m(i)$ is consumer i's rental rate on capital services supplied to sector m, $T_t(i)$ is consumer i's nominal lump-sum transfer from the government and $\mathcal{B}_t(i)$ are consumer i's net benefits from firms' ownerships.

Summarizing, households' behavior consists in an intertemporal smoothing of consumption, a supply of rental services from capital and a monopolistic supply of labor services under a staggered wage setting. This behavior will be analyzed in the following (sub)sections.

2.1 Domestic households' consumption demands

Reminding that subscript $h(f)$ denotes the home (foreign) economy and that, if necessary, the country of origin is indicated as a lower index and the country of destination (usage) as an upper index, the total domestic households'

treatment of inputs, namely labor and capital in terms of their underemployment, hinges on the fact that we assume that labor can move freely among sectors, whereas capital stock is fixed once it is allocated in a sector (see Natalucci and Ravenna (2005)). For a model that introduces labor unemployment in the equilibrium integrating search and matching with NK models (by considering real frictions in the labor market), see Blanchard and Galí (2006).

[9] Smets and Wouters (2003) and Christiano *et al.* (2005) assume that $\Upsilon(.) = \Upsilon'(.) = 0$ in the steady state (no fixed adjustment costs). In addition, they assume that $\Upsilon''(.)$ is non-negative then.

[10] Notice that the nominal exchange rate (the price of one unit of foreign currency in home currency) S_t is used to translate asset returns both at the beginning and at the end of period t so that, *de facto*, it represents the average exchange rate during period t; see also equation (20) in Selaive and Tuesta (2006). Therefore, S_t is used to change both $B_{f,t+1}$ and $B_{f,t}$ into the home currency.

[11] See Smets and Wouters (2003, p. 1128) for the consideration of the (real) terms containing capital stocks and depreciation rates.

[12] Or, $\Psi(1) = 0$; see Christiano *et al.* (2005).

[13] Alternatively, these discount factors can also be considered as prices of one (home and foreign) asset paid by the domestic consumer (owner of assets) at time $t+1$.

[14] See Section 7 in this paper for an exact definition of the stochastic effective interest rate $\hat{r}^*_{t,t+1}$.

expenditure on consumption can be written as:

$$P_t^C C_t \equiv \int_0^1 \int_0^1 P_{h,t}^{CT}(z,i)\, C_{h,t}^T(z,i)\, dzdi + \int_0^1 \int_0^1 P_{h,t}^{CN}(z,i)\, C_{h,t}^N(z,i)\, dzdi + \int_0^1 \int_0^1 P_{f,t}^{CT}(z,i)\, C_{f,t}^T(z,i)\, dzdi, \quad (8)$$

where $P_{h,t}^{CT}(z,i)$, $P_{h,t}^{CN}(z,i)$ and $P_{f,t}^{CT}(z,i)$ are the prices of the home produced tradable and non-tradable consumption goods and foreign produced tradable consumption goods purchased by domestic household i, which consumes domestically produced and imported goods z at quantities $C_{h,t}^T(z,i)$, $C_{h,t}^N(z,i)$ and $C_{f,t}^T(z,i)$, respectively. Consumer prices are assumed to be expressed in the home currency ('pricing to market' or LCP with zero (exchange rate) pass-through as opposed to PCP with perfect (exchange rate) pass-through). As suggested by Dixit and Stiglitz (1977), these consumption quantities can be aggregated through CES (constant elasticity of substitution) aggregators over a continuum $[0,1]$ of differentiated goods:

$$C_{k,t}^l(i) \equiv \left(\int_0^1 C_{k,t}^l(z,i)^{\frac{\theta_{Cl}^{(i)}-1}{\theta_{Cl}^{(i)}}} dz \right)^{\frac{\theta_{Cl}^{(i)}}{\theta_{Cl}^{(i)}-1}} \quad \text{for } (k,l) = (h,T),\ (h,N) \text{ and } (f,T), \quad (9)$$

with household i's intratemporal elasticity of substitution between two different goods given by $\theta_{Cl}^{(i)} > 1$.[15]

Consumer i takes the differentiated consumption goods prices as given and minimizes purchases of differentiated goods $C_{k,t}^l(z,i)$ over $[0,1]$ for each consumption category (k,l); the solutions to this minimization of total expenditure costs (8) subject to (9) are provided by the minimands:

$$\arg \min_{\{C_{k,t}^l(z,i)\}} \left\{ \int_0^1 P_{k,t}^{Cl}(z) C_{k,t}^l(z,i) dz + \lambda_{k,t}^{Cl}(i) \left(C_{k,t}^l(i) - \left(\int_0^1 C_{k,t}^l(z,i)^{\frac{\theta_{Cl}^{(i)}-1}{\theta_{Cl}^{(i)}}} dz \right)^{\frac{\theta_{Cl}^{(i)}}{\theta_{Cl}^{(i)}-1}} \right) \right\}, \quad (10)$$

where the Lagrange multiplier $\lambda_{k,t}^{Cl}(i)$ is household i's cost-minimizing (shadow) price of one unit of consumption goods in each particular category (k,l). Solving for the Lagrange parameters, we obtain consumer i's aggregate consumption price index for each consumption category (k,l) expressed in domestic currency:

$$\lambda_{k,t}^{Cl}(i) = P_{k,t}^{Cl}(i) = \left(\int_0^1 (P_{k,t}^{Cl}(z,i))^{1-\theta_{Cl}^{(i)}} dz \right)^{\frac{1}{1-\theta_{Cl}^{(i)}}} \quad \text{for } (k,l) = \{(h,T),(h,N),(f,T)\}. \quad (11)$$

Hence, household i's expenditure minimizing demands for each differentiated good z and each consumption category (k,l) are:

$$C_{k,t}^l(z,i) = \left(\frac{P_{k,t}^{Cl}(z)}{P_{k,t}^{Cl}(i)} \right)^{-\theta_{Cl}^{(i)}} \left(\left(\int_0^1 C_{k,t}^l(z,i)^{\frac{\theta_{Cl}^{(i)}-1}{\theta_{Cl}^{(i)}}} dz \right)^{\frac{\theta_{Cl}^{(i)}}{\theta_{Cl}^{(i)}-1}} \right) = \left(\frac{P_{k,t}^{Cl}(z)}{P_{k,t}^{Cl}(i)} \right)^{-\theta_{Cl}^{(i)}} C_{k,t}^l(i) \quad (12)$$

for $i, z \in [0,1]$ and $(k,l) = (h,T),(h,N)$ and (f,T).

Defining household i's aggregate consumption basket (index) of tradable and non-tradable goods as a nested CES specification (Dixit-Stiglitz aggregator of indices of traded and non-traded goods):

$$C_t(i) \equiv \left[\left(\alpha_{CT}^{(i)}\right)^{\frac{1}{\eta_{CT}^{(i)}}} \left(C_t^T(i)\right)^{\frac{\eta_{CT}^{(i)}-1}{\eta_{CT}^{(i)}}} + \left(1-\alpha_{CT}^{(i)}\right)^{\frac{1}{\eta_{CT}^{(i)}}} \left(C_{h,t}^N(i)\right)^{\frac{\eta_{CT}^{(i)}-1}{\eta_{CT}^{(i)}}} \right]^{\frac{\eta_{CT}^{(i)}}{\eta_{CT}^{(i)}-1}}, \quad (13)$$

where $C_t^T(i)$ is household i's aggregate consumption basket of tradable domestic and imported goods bundles, defined as $C_t^T(i) \equiv \left[\left(\alpha_{Ch}^{(i)}\right)^{\frac{1}{\eta_{Ch}^{(i)}}} \left(C_{h,t}^T(i)\right)^{\frac{\eta_{Ch}^{(i)}-1}{\eta_{Ch}^{(i)}}} + \left(1-\alpha_{Ch}^{(i)}\right)^{\frac{1}{\eta_{Ch}^{(i)}}} \left(C_{f,t}^T(i)\right)^{\frac{\eta_{Ch}^{(i)}-1}{\eta_{Ch}^{(i)}}} \right]^{\frac{\eta_{Ch}^{(i)}}{\eta_{Ch}^{(i)}-1}}$, with $\eta_{CT}^{(i)}$ and $\eta_{Ch}^{(i)}$ being

[15] The elasticity of substitution is larger than one to ensure that firms' markup factors are always positive.

domestic consumer i's intratemporal elasticities of substitution between (the bundle of) tradable consumption goods and (the bundle of) domestic non-tradable consumption goods, and between domestic and imported tradable consumption goods, respectively ($\eta_{Ch}^{(i)} > \eta_{CT}^{(i)} > 0$); $\alpha_{CT}^{(i)}$ and $\alpha_{Ch}^{(i)}$ are domestic consumer i's shares of tradable goods in her total domestic consumption and of domestically produced tradable goods in her total domestic tradable goods consumption, respectively.[16]

By using the definition of household i's total domestic consumption (13), we get the aggregate demand relationships of the optimal intratemporal consumption allocations (12):

$$C_{h,t}^{T}(i) = \alpha_{Ch}^{(i)} \left(\frac{P_{h,t}^{CT}(i)}{P_{t}^{CT}(i)} \right)^{-\eta_{Ch}^{(i)}} C_{t}^{T}(i) \quad , \quad C_{f,t}^{T}(i) = \left(1-\alpha_{Ch}^{(i)}\right) \left(\frac{P_{f,t}^{CT}(i)}{P_{t}^{CT}(i)} \right)^{-\eta_{Ch}^{(i)}} C_{t}^{T}(i),$$

$$C_{h,t}^{N}(i) = \left(1-\alpha_{CT}^{(i)}\right) \left(\frac{P_{h,t}^{CN}(i)}{P_{t}^{C}(i)} \right)^{-\eta_{CT}^{(i)}} C_{t}(i) \text{ and } C_{t}^{T}(i) = \alpha_{CT}^{(i)} \left(\frac{P_{t}^{CT}(i)}{P_{t}^{C}(i)} \right)^{-\eta_{CT}^{(i)}} C_{t}(i), \tag{14}$$

where $P_t^C(i)$ is household i's aggregate consumption price index and $P_t^{CT}(i)$ ($P_t^{CN}(i)$) is household i's aggregate consumption price index of tradable (non-tradable) goods. These price indices can be directly derived by minimizing the cost of purchasing one unit of the aggregate nested consumption bundle by household i and are dual equations to CES specifications (13):

$$P_{t}^{C}(i) = \left[\alpha_{CT}^{(i)} \left(P_{t}^{CT}(i)\right)^{1-\eta_{CT}^{(i)}} + \left(1-\alpha_{CT}^{(i)}\right) \left(P_{h,t}^{CN}(i)\right)^{1-\eta_{CT}^{(i)}} \right]^{\frac{1}{1-\eta_{CT}^{(i)}}} \text{ and} \tag{15}$$

$$P_{t}^{CT}(i) = \left[\alpha_{Ch}^{(i)} \left(P_{h,t}^{CT}(i)\right)^{1-\eta_{Ch}^{(i)}} + \left(1-\alpha_{Ch}^{(i)}\right) \left(P_{f,t}^{CT}(i)\right)^{1-\eta_{Ch}^{(i)}} \right]^{\frac{1}{1-\eta_{Ch}^{(i)}}}. \tag{16}$$

2.2 Domestic households' investment supplies

Similarly to (8) for the total domestic households' consumption, the total domestic households' expenditure on investment can be written as:

$$P_t^I I_t \equiv \int_0^1 \int_0^1 P_{h,t}^{IT}(z,i)\, I_{h,t}^{T}(z,i)\, dzdi + \int_0^1 \int_0^1 P_{h,t}^{IN}(z,i)\, I_{h,t}^{N}(z,i)\, dzdi + \int_0^1 \int_0^1 P_{f,t}^{IT}(z,i)\, I_{f,t}^{T}(z,i)\, dzdi, \tag{17}$$

where $I_{h,t}^T(z,i)$, $I_{h,t}^N(z,i)$ and $I_{f,t}^T(z,i)$, and corresponding prices $P_{h,t}^{IT}(z,i)$, $P_{h,t}^{IN}(z,i)$ and $P_{f,t}^{IT}(z,i)$ are the quantities and prices of the home produced tradable and non-tradable investment goods, and tradable foreign investment goods purchased by domestic household i, respectively.

Considering CES aggregators for individual investment quantities as for consumption quantities in (9) as constraints, the solutions of minimizing (17) over the interval $[0,1]$, with household i's intratemporal elasticity of substitution between two different investment goods given by $\theta_{Il}^{(i)} > 1$, involve Lagrange multipliers $\lambda_{k,t}^{Il}(i)$ (similar to (11)).

Defining consumer i's aggregate investment basket of tradable and non-tradable investment goods similarly to (13) as:

$$I_t(i) \equiv \left[\left(\alpha_{IT}^{(i)}\right)^{\frac{1}{\eta_{IT}^{(i)}}} \left(I_t^T(i)\right)^{\frac{\eta_{IT}^{(i)}-1}{\eta_{IT}^{(i)}}} + \left(1-\alpha_{IT}^{(i)}\right)^{\frac{1}{\eta_{IT}^{(i)}}} \left(I_{h,t}^N(i)\right)^{\frac{\eta_{IT}^{(i)}-1}{\eta_{IT}^{(i)}}} \right]^{\frac{\eta_{IT}^{(i)}}{\eta_{IT}^{(i)}-1}}, \tag{18}$$

with household i's aggregate investment basket of tradable domestic and imported investment goods bundles, $I_t^T(i) \equiv \left[\left(\alpha_{Ih}^{(i)}\right)^{\frac{1}{\eta_{Ih}^{(i)}}} \left(I_{h,t}^T(i)\right)^{\frac{\eta_{Ih}^{(i)}-1}{\eta_{Ih}^{(i)}}} + \left(1-\alpha_{Ih}^{(i)}\right)^{\frac{1}{\eta_{Ih}^{(i)}}} \left(I_{f,t}^T(i)\right)^{\frac{\eta_{Ih}^{(i)}-1}{\eta_{Ih}^{(i)}}} \right]^{\frac{\eta_{Ih}^{(i)}}{\eta_{Ih}^{(i)}-1}}$, with $\eta_{IT}^{(i)}$ and $\eta_{Ih}^{(i)}$ being consumer i's intratemporal elasticities of substitution between tradable and non-tradable investment goods, and between domestic and imported tradable investment goods, respectively ($\eta_{Ih}^{(i)} > \eta_{IT}^{(i)} > 0$); $\alpha_{IT}^{(i)}$ and $\alpha_{Ih}^{(i)}$ are domestic consumer i's

[16] Hence, $\alpha_{CT}^{(i)}$ is household i's *tradable goods bias* in its total consumption, whereas $\alpha_{Ch}^{(i)}$ is its *home bias* in tradable consumption.

shares of tradable goods in her total domestic investment and of domestically produced tradable goods in her total domestic tradable goods investment, respectively.[17]

Proceeding in a way similar to the previous subsection, the aggregate optimal intratemporal investment demands are:

$$I_{h,t}^{T}(i) = \alpha_{Ih}^{(i)} \left(\frac{P_{h,t}^{IT}(i)}{P_t^{IT}(i)} \right)^{-\eta_{Ih}^{(i)}} I_t^T(i) \quad , \quad I_{f,t}^{T}(i) = \left(1-\alpha_{Ih}^{(i)}\right) \left(\frac{P_{f,t}^{IT}(i)}{P_t^{IT}(i)} \right)^{-\eta_{Ih}^{(i)}} I_t^T(i),$$

$$I_{h,t}^{N}(i) = \left(1-\alpha_{IT}^{(i)}\right) \left(\frac{P_{h,t}^{IN}(i)}{P_t^{I}(i)} \right)^{-\eta_{IT}^{(i)}} I_t(i) \text{ and } I_t^T(i) = \alpha_{IT}^{(i)} \left(\frac{P_t^{IT}(i)}{P_t^{I}(i)} \right)^{-\eta_{IT}^{(i)}} I_t(i), \tag{19}$$

where $P_t^I(i)$ is household i's aggregate investment price index and $P_t^{IT}(i)$ ($P_t^{IN}(i)$) is household i's aggregate investment price index of tradable (non-tradable) goods. These price indices can be directly derived by minimizing the cost of purchasing one unit of the investment bundle $I_t(i)$ by household i as (these are the dual equations to the CES definitions (18)):

$$P_t^I(i) = \left[\alpha_{IT}^{(i)} \left(P_t^{IT}(i)\right)^{1-\eta_{IT}^{(i)}} + \left(1-\alpha_{IT}^{(i)}\right) \left(P_{h,t}^{IN}(i)\right)^{1-\eta_{IT}^{(i)}} \right]^{\frac{1}{1-\eta_{IT}^{(i)}}} \text{ and} \tag{20}$$

$$P_t^{IT}(i) = \left[\alpha_{Ih}^{(i)} \left(P_{h,t}^{IT}(i)\right)^{1-\eta_{Ih}^{(i)}} + \left(1-\alpha_{Ih}^{(i)}\right) \left(P_{f,t}^{IT}(i)\right)^{1-\eta_{Ih}^{(i)}} \right]^{\frac{1}{1-\eta_{Ih}^{(i)}}} . \tag{21}$$

The foreign consumers solve similar consumption and investment problems as the domestic consumers do. These problems are developed in detail in Appendices A.1 and A.2, respectively.

3 Domestic government

The expenditure of the domestic government usually comprises public consumption and public investment goods. According to the literature about DSGE models, the government only purchases consumption goods (see e.g. Smets and Wouters (2003)). Following Leith and Malley (2003), the domestic government considers the same differentiated consumption goods prices as the domestic private consumers do. Hence, it minimizes for each consumption category (k,l) purchases $G_{k,t}^l(z)$ of a differentiated good at a price $P_{k,t}^{Cl}(z)$ over the interval $[0,1]$. The solution to this total budget minimization subject to a CES aggregator similar to (9) results in minimands and Lagrange multipliers similar to (10) and (11), respectively. Thus, we get for $z \in [0,1]$ (in the sense of (12)):

$$G_{k,t}^l(z) = \left(\frac{P_{k,t}^{Cl}(z)}{P_{k,t}^{Cl}(G)} \right)^{-\theta_{Cl}^{(G)}} G_{k,t}^l \text{ for } (k,l) = (h,T), (h,N) \text{ and } (f,T) \tag{22}$$

with $\theta_{Cl}^{(g)} > 1$ the relevant elasticity of substitution and

$$P_{k,t}^{Cl}(G) = \left(\int_0^1 \left(P_{k,t}^{Cl}(z,G)\right)^{1-\theta_{Cl}^{(G)}} dz \right)^{\frac{1}{1-\theta_{Cl}^{(G)}}} \tag{23}$$

the domestic government consumption price index.

In order to get the domestic government's aggregate demand, we propose CES aggregators, similar to equations (13) with $\eta_{CT}^{(G)}$ and $\eta_{Ch}^{(G)}$ and $\alpha_{CT}^{(G)}$ and $\alpha_{Ch}^{(G)}$ having similar interpretations as in Subsection 2.1; hence, the domestic government demand is:

$$G_t \equiv \left[\left(\alpha_{CT}^{(G)}\right)^{\frac{1}{\eta_{CT}^{(G)}}} \left(G_t^T\right)^{\frac{\eta_{CT}^{(G)}-1}{\eta_{CT}^{(G)}}} + \left(1-\alpha_{CT}^{(G)}\right)^{\frac{1}{\eta_{CT}^{(G)}}} \left(G_{h,t}^N\right)^{\frac{\eta_{CT}^{(G)}-1}{\eta_{CT}^{(G)}}} \right]^{\frac{\eta_{CT}^{(G)}}{\eta_{CT}^{(G)}-1}}, \tag{24}$$

[17] Parallel to the previous footnote, $\alpha_{IT}^{(i)}$ is household i's tradable goods bias in its total investment, whereas $\alpha_{Ih}^{(i)}$ is its home bias in tradable investment.

where G_t^T is the aggregate domestic government demand of tradable home and imported goods bundles, defined as $G_t^T \equiv \left[\left(\alpha_{Ch}^{(G)}\right)^{\frac{1}{\eta_{Ch}^{(G)}}} \left(G_{h,t}^T\right)^{\frac{\eta_{Ch}^{(G)}-1}{\eta_{Ch}^{(G)}}} + \left(1-\alpha_{Ch}^{(G)}\right)^{\frac{1}{\eta_{Ch}^{(G)}}} \left(G_{f,t}^T\right)^{\frac{\eta_{Ch}^{(G)}-1}{\eta_{Ch}^{(G)}}} \right]^{\frac{\eta_{Ch}^{(G)}}{\eta_{Ch}^{(G)}-1}}$.

Similarly to (14), the optimal demand equations for domestic government expenditure are:

$$G_{h,t}^T = \alpha_{Ch}^{(G)} \left(\frac{P_{h,t}^{CT}(G)}{P_t^{CT}(G)} \right)^{-\eta_{Ch}^{(G)}} G_t^T, \quad G_{f,t}^T = \left(1-\alpha_{Ch}^{(G)}\right) \left(\frac{P_{f,t}^{CT}(G)}{P_t^{CT}(G)} \right)^{-\eta_{Ch}^{(G)}} G_t^T,$$

$$G_{h,t}^N = \left(1-\alpha_{CT}^{(G)}\right) \left(\frac{P_{h,t}^{CN}(G)}{P_t^{C}(G)} \right)^{-\eta_{CT}^{(G)}} G_t \text{ and } G_t^T = \alpha_{CT}^{(G)} \left(\frac{P_t^{CT}(G)}{P_t^{C}(G)} \right)^{-\eta_{CT}^{(G)}} G_t, \tag{25}$$

where $P_t^C(G)$ is the domestic government's aggregate consumption price index and $P_t^{CT}(G)$ ($P_t^{CN}(G)$) is the domestic government's aggregate consumption price index of tradable (non-tradable) goods, derived as:

$$P_t^C(G) = \left[\alpha_{CT}^{(g)} \left(P_t^{CT}(G)\right)^{1-\eta_{CT}^{(g)}} + \left(1-\alpha_{CT}^{(g)}\right) \left(P_{h,t}^{CN}(G)\right)^{1-\eta_{CT}^{(g)}} \right]^{\frac{1}{1-\eta_{CT}^{(g)}}} \text{ and} \tag{26}$$

$$P_t^{CT}(G) = \left[\alpha_{Ch}^{(g)} \left(P_{h,t}^{CT}(G)\right)^{1-\eta_{Ch}^{(g)}} + \left(1-\alpha_{Ch}^{(g)}\right) \left(P_{f,t}^{CT}(G)\right)^{1-\eta_{Ch}^{(g)}} \right]^{\frac{1}{1-\eta_{Ch}^{(g)}}}. \tag{27}$$

The domestic government is confronted with the government budget constraint (GBC), i.e. the total amount of taxes plus the net variation of the outstanding debt must equalize the total purchases in any period t, which is implied by (24) and (26). Formally, the GBC is given by:

$$\begin{aligned} &\tau_t^w \left[\sum_{m=FT,FN,VT,VN} \int_0^1 W_t^m(i) L_t^m(i) di \right] + \tau_t^k \left[\sum_{m=FT,FN,VT,VN} \int_0^1 R_t^m(i) \mathcal{K}_t^m(i) di \right] \\ &+ \int_0^1 \left[M_{t+1}(i) - M_t(i)\right] di + Q_{h,t,t+1} \int_0^1 B_{h,t+1}(i) di - \int_0^1 B_{h,t}(i) di \\ &+ \hat{Q}_{h,t,t+1}^* \int_0^1 B_{h,t+1}^*(i) di - \int_0^1 B_{h,t}^*(i) di \geq \int_0^1 T_t(i) di + P_t^C(G) G_t. \end{aligned} \tag{28}$$

Equation (28) includes on the left hand side labor and capital tax revenues, money creation and net domestic and foreign borrowing, while on the right hand side outlays of government revenues (transfers and goods purchases) are considered.

Similarly, the foreign government has a set of demands, which are considered in Appendix A.3, leading to a foreign GBC similar to (28).

4 Firms

Following Galí and Monacelli (2002), Smets and Wouters (2003), Choi and Jung (2003), Jung (2004) and Lindé *et al.* (2004), it is assumed that suppliers of inputs are price setters under profit maximization and demanders for inputs are price takers under cost minimization. All goods markets are characterized by monopolistic competition and profit maximization in the output markets. An alternative interpretation is that the firms' problem in open economies can be disentangled in two stages: one for final goods (being consumption and investment goods) and one for intermediate goods. The inspiration for an intermediate goods versus final goods situation can be found in Clarida *et al.* (2002), Smets and Wouters (2003) and Plasmans *et al.* (2004).[18] We assume that intermediate and final goods and tradable and non-tradable goods are produced in a sectoral framework.

Home goods are assumed to be produced in each sector by a continuum of monopolistically competitive firms, indexed by $j \in [0,1]$, while imported final and intermediate goods are bought (at marginal cost) in the foreign market by importing firms in import sectors MF and MV, respectively, and are repacked or rebranded and sold in

[18] Non-tradable intermediate goods include intermediate goods which are relatively too expensive to be transported, e.g. sand, water, various kinds of services.

the domestic market, also under monopolistic competition. Hence, firms in the monopolistically competitive import goods sectors turn the foreign goods, bought at their given world price marginal cost, into differentiated final and intermediate import goods.

Each domestic firm j is assumed to produce one differentiated final or intermediate good which can be either tradable or non-tradable. Six main sectors are distinguished: 1. the tradable final goods (FT) sector; 2. the non-tradable final goods (FN) sector; 3. the tradable intermediate goods (VT) sector; 4. the non-tradable intermediate goods (VN) sector; 5. the sector of imported (tradable) final goods (MF) and 6. the sector of imported (tradable) intermediate goods (MV). The latter two sectors are characterized by firms that import, repack or rebrand the imported good, adapting it for the home market.

In the **final goods** production sectors $m = FT, FN$, we assume variable returns to scale and a nested CES production technology with labor, $L_t^m(j)$, capital services, $\mathcal{Z}_t^m(i)K_t^m(j)$, and intermediate goods, $V_t^m(j)$, as inputs and symmetric production function parameters for all firms within the same sector:[19]

$$Y_t^m(j) \equiv (\Omega_{m,t} Z_t^m(j))^{\varpi_m} \equiv \left\{ \Omega_{m,t} \left[\begin{array}{c} \upsilon_{Lm}^{\frac{1}{\gamma_m}} \left(L_t^m(j)\right)^{\frac{\gamma_m - 1}{\gamma_m}} + \upsilon_{Km}^{\frac{1}{\gamma_m}} \left(\mathcal{Z}_t^m K_t^m(j)\right)^{\frac{\gamma_m - 1}{\gamma_m}} \\ + (1 - \upsilon_{Km} - \upsilon_{Lm})^{\frac{1}{\gamma_m}} \left(V_t^m(j)\right)^{\frac{\gamma_m - 1}{\gamma_m}} \end{array} \right]^{\frac{\gamma_m}{\gamma_m - 1}} \right\}^{\varpi_m}, \tag{29}$$

where

$$K_t^m(j) \equiv \left[\kappa_{m1}^{\frac{1}{\epsilon_m}} \left(K_{h,t}^{T,m}(j)\right)^{\frac{\epsilon_m - 1}{\epsilon_m}} + \kappa_{m2}^{\frac{1}{\epsilon_m}} \left(K_{h,t}^{N,m}(j)\right)^{\frac{\epsilon_m - 1}{\epsilon_m}} + (1 - \kappa_{m1} - \kappa_{m2})^{\frac{1}{\epsilon_m}} \left(K_{f,t}^{T,m}(j)\right)^{\frac{\epsilon_m - 1}{\epsilon_m}} \right]^{\frac{\epsilon_m}{\epsilon_m - 1}} \tag{30}$$

and

$$V_t^m(j) \equiv \left[\nu_{m1}^{\frac{1}{\chi_m}} \left(V_{h,t}^{T,m}(j)\right)^{\frac{\chi_m - 1}{\chi_m}} + \nu_{m2}^{\frac{1}{\chi_m}} \left(V_{h,t}^{N,m}(j)\right)^{\frac{\chi_m - 1}{\chi_m}} + (1 - \nu_{m1} - \nu_{m2})^{\frac{1}{\chi_m}} \left(V_{f,t}^{T,m}(j)\right)^{\frac{\chi_m - 1}{\chi_m}} \right]^{\frac{\chi_m}{\chi_m - 1}}, \tag{31}$$

for $j \in [0, 1]$, where γ_m, ϵ_m and χ_m are the (home country) intratemporal elasticities of substitution between the final goods inputs, different capital goods inputs and different intermediate goods inputs, respectively.

In (29-31), υ_{Lm} and υ_{Km} are the shares of labor and capital stock inputs in total input, κ_{m1} and κ_{m2} are the shares of domestically produced tradable and non-tradable capital stocks in total capital stock input of a domestic firm, while ν_{m1} and ν_{m2} are the shares of domestically produced tradable and non-tradable intermediate goods in total intermediate goods input of that firm in sector m; $\Omega_{m,t}$ is a domestic technology shock in period t, which, according to learning characteristics, is assumed to satisfy an $AR(1)$ process: $\ln \Omega_{m,t} \equiv \omega_{m,t} = \rho_{m,\omega}\omega_{m,t-1} + \xi_{m,\omega,t}$ with $-1 < \rho_{m,\omega} < 1$ and $\xi_{m,\omega,t}$ an independently and identically distributed (iid) error term; in addition, ϖ_m is the returns to scale parameter of the production function in sector m in the home country.

Moreover, the aggregate quantities $L_t^m(j)$, $K_{h,t}^{T,m}(j)$, $K_{h,t}^{N,m}(j)$, $K_{f,t}^{T,m}(j)$, $V_{h,t}^{T,m}(j)$, $V_{h,t}^{N,m}(j)$ and $V_{f,t}^{T,m}(j)$ are defined, using appropriate Dixit-Stiglitz CES aggregators similar to (9), as bundles over all labor, capital stock and intermediate goods types (varieties), respectively:

$$L_t^m(j) \equiv \left(\int_0^1 L_t^m(j,i)^{\frac{\varrho_{Lm}^{(j)} - 1}{\varrho_{Lm}^{(j)}}} di \right)^{\frac{\varrho_{Lm}^{(j)}}{\varrho_{Lm}^{(j)} - 1}}, \; K_{k,t}^{l,m}(j) \equiv \left(\int_0^1 K_{k,t}^{l,m}(j,z)^{\frac{\varrho_{Kl,m}^{(j)} - 1}{\varrho_{Km}^{(j)}}} dz \right)^{\frac{\varrho_{Kl,m}^{(j)}}{\varrho_{Kl,m}^{(j)} - 1}} \tag{32}$$

and

$$V_{k,t}^{l,m}(j) \equiv \left(\int_0^1 V_{k,t}^{l,m}(j,z)^{\frac{\varrho_{Vl,m}^{(j)} - 1}{\varrho_{Vl,m}^{(j)}}} dz \right)^{\frac{\varrho_{Vl,m}^{(j)}}{\varrho_{Vl,m}^{(j)} - 1}} \qquad \text{for } (k,l) = (h,T), (h,N) \text{ and } (f,T), \tag{33}$$

where for any category of final goods, $m = FT$, FN, $\varrho_{Lm}^{(j)}$, $\varrho_{Kl,m}^{(j)}$ and $\varrho_{Vl,m}^{(j)}$ are the intratemporal elasticities of substitution, which should be larger than one to assure positive markups, $L_t^m(j,i)$ is company j's demand for the labor supplied by household i at period t, $K_{k,t}^{l,m}(j,z)$ is company j's demand for capital stock produced by company

[19]Notice that this assumed symmetry of production function parameters (for firms within the same sector) is similar to the usually assumed symmetry of preferential parameters in the household's optimization problem (*cf. ultra*).

Moreover, it is noted that the rate of utilization $Z_t^m$is not a firm j's choice variable so that it is independent of j.

z in regime (k,l) at time t or before and $V_{k,t}^{l,m}(j,z)$ is company j's demand for intermediate goods produced by company z in regime (k,l) at period t.

Intermediate good firms in production sectors $m = VT, VN$ combine capital stock and labor according to the production function with symmetric parameters for all firms within the same sector:

$$Y_t^m(j) \equiv (\Omega_{m,t} Z_t^m(j))^{\varpi_m} = \left\{ \Omega_{m,t} \left[\nu_{Lm}^{\frac{1}{\gamma_m}} \left(L_t^m(j)\right)^{\frac{\gamma_m - 1}{\gamma_m}} + (1-\nu_{Lm})^{\frac{1}{\gamma_m}} \left(K_t^m(j)\right)^{\frac{\gamma_m-1}{\gamma_m}} \right]^{\frac{\gamma_m}{\gamma_m-1}} \right\}^{\varpi_m}, \quad (34)$$

where γ_m is the intratemporal elasticity of substitution between capital and labor and where $L_t^m(j)$ and $K_t^m(j)$ are defined as in (32) and (30), respectively, with ϖ_m being the returns to scale parameter in the production function for intermediate goods in sector m.

For simplicity, we assume no explicit production function for importers of final and intermediate goods because they are assumed to only rebrand and repack the same goods.

5 Optimization

5.1 The domestic households' optimization

Plugging household i's aggregate consumption demand functions (14) into its CBC (7), the constrained maximization of domestic household i's expected utility (1) leads to an Euler equation for consumption that is unsolvable.[20] Therefore, following, e.g. Ambler *et al.* (2004), we specify household i's period utility function in (1) as:[21]

$$U_1\left(C_t(i), C_{t-1}(i)\right) - U_2(L_t(i)) + U_3\left(\frac{M_{t+1}(i)}{P_t^C(i)}\right) = \frac{1}{1-\sigma}\left(\frac{C_t(i)}{\left(C_{t-1}(i)\right)^{\kappa}}\right)^{1-\sigma} - \frac{\left(L_t(i)\right)^{1+\phi}}{1+\phi} + \chi_M \frac{\left(\frac{M_{t+1}(i)}{P_t^C(i)}\right)^{1-\frac{1}{\chi}}}{\left(1-\frac{1}{\chi}\right)}, \quad (35)$$

where $\sigma > 0$ is a parameter of constant relative risk aversion of domestic households in the home country being equal to the inverse of the intertemporal elasticity of substitution in consumption, κ is the home consumers' habit persistence parameter,[22] and ϕ is the elasticity of the willingness to work,[23] $\frac{M_{t+1}(i)}{P_t^C(i)}$ is household i's real money balances at the beginning of period $t+1$, χ_M is a preferential constant for (real) money balances and χ is the elasticity of substitution of real money balances in the home country.

We maximize the expected discounted sum of household i's utility flows (35) subject to its CBC (7) and the law of motion (6) of its sectoral capital. In the domestic CBC we used definition (5) for consumer i's aggregate investment supply and expression (3) for consumer i's gross nominal labor income. The resulting first order conditions (FOCs) for $C_t(i)$, $L_t^m(i)$, $I_t^m(i)$, $B_{h,t+1}(i)$, $B_{f,t+1}(i)$, $M_{t+1}(i)$, $K_{t+1}^m(i)$ and $\mathcal{Z}_{t+1}^m(i)$ for $m = FT, FN, VT, VN$ are derived

[20] Beyer and Farmer (2004) discuss the general lack of identification in linear rational expectations models and apply this to NK models.

[21] This is a Constant Rate of Risk Aversion (CRRA) utility function that allows for habit formation as in Kozicki and Tinsley (2002). Moreover, notice that following the international literature about DSGE models, we assume that the parameters in this utility function are equal over consumers.

[22] Instead of using an *ad hoc* rule of thumb as e.g. in Amato and Laubach (2003 and 2004), we propose a completely microfounded aggregate demand equation, based on the hypothesis of habit formation. Recent micro-level studies report mixed evidence of the impact of habit formation in consumption (see Ravina (2005)), while studies conducted with aggregate data find substantial evidence, e.g. Christiano *et al.* (2005) emphasize the role of habit persistence in the explanation of the hump-shaped behavior of aggregate consumption in response to a monetary policy shock. Notice that habit formation in consumption vanishes and consumption is as the usual CRRA ulitily if $\kappa = 0$.

[23] Which under perfect competition might be measured by the elasticity of labor supply w.r.t. the real wage.

in Appendix B as follows:[24]

$$\Gamma_t(i) = \frac{U_{C_t(i)}}{P_t^C(i)} = \frac{(C_t(i))^{-\sigma}}{P_t^C(i)\,(C_{t-1}(i))^{\kappa(1-\sigma)}} - E_t\left[\beta\kappa\frac{(C_{t+1}(i))^{1-\sigma}}{P_t^C(i)\,(C_t(i))^{[\kappa(1-\sigma)+1]}}\right], \tag{36}$$

$$\frac{\varrho_{Lm}}{\varrho_{Lm}-1}\,(L_t(i))^{\phi} = \Gamma_t(i)\,(1-\tau_t^w)\,W_t^m(i), \tag{37}$$

$$P_t^I(i)\Gamma_t(i)\text{-}\mathcal{Q}_t^m(i)\Gamma_t(i)\left[1\text{-}\Upsilon(.)\text{-}\Upsilon'(.)\frac{I_t^m(i)}{I_{t-1}^m(i)}\right] = \beta E_t\left[\mathcal{Q}_{t+1}^m(i)\Gamma_{t+1}(i)\left[\Upsilon'(.)\frac{\left(I_{t+1}^m(i)\right)^2}{\left(I_t^m(i)\right)^2}\right]\right], \tag{38}$$

$$E_t\left[\Gamma_t(i)Q_{h,t,t+1} - \beta\Gamma_{t+1}(i)\right] = 0, \tag{39}$$

$$E_t\left[\Gamma_t(i)\hat{Q}_{f,t,t+1}S_t - \beta\Gamma_{t+1}(i)S_{t+1}\right] = 0, \tag{40}$$

$$\chi_M\frac{1}{P_t^C(i)}\left[\frac{M_{t+1}(i)}{P_t^C(i)}\right]^{-\frac{1}{\chi}} = -\Gamma_t(i) + \beta E_t\left[\Gamma_{t+1}(i)\right], \tag{41}$$

$$\mathcal{Q}_t^m(i)\Gamma_t(i) = \beta E_t\left[\Gamma_{t+1}(i)\left((1\text{-}d_{t+1}^m(i))\mathcal{Q}_{t+1}^m(i) + (1\text{-}\tau_{t+1}^k)R_{t+1}^m(i)\mathcal{Z}_{t+1}^m(i)\text{-}P_{t+1}^I(i)\Psi\left(\mathcal{Z}_{t+1}^m(i)\right)\right)\right], \tag{42}$$

$$(1-\tau_{t+1}^k)R_{t+1}^m(i) = P_{t+1}^I(i)\Psi'\left(\mathcal{Z}_{t+1}^m(i)\right), \tag{43}$$

where the Lagrange multipliers $\Gamma_t(i)$ and $\mathcal{Q}_t^m(i)\Gamma_t(i)$ for $m = FT, FN, VT, VN$, are the marginal utility of household i's wealth and sector m's shadow prices of physical capital installed, respectively. Moreover, $\varrho_{Lm} > 1$ is the intratemporal elasticity of substitution for different types of labor demanded by firms in sector $m = FT, FN, VT, VN$. Equation (36) implies that the marginal utility of a particular household's consumption good equals the marginal utility of its wealth. Equation (37) relates the household i's marginal utility of leisure to its marginal utility of the nominal wage in every sector. Equation (38) refers to household i's investment decisions in sector m, which depend on the adjustment costs of investment growth in present and future periods; equations (39) and (40) refer to the household i's intertemporal decision involving the allocation of home and foreign financial assets and equation (41) is the optimal real money balances demand. Equation (42) states that the value of installed capital depends on its expected future value net of the depreciation rate and the expected rental rate multiplied by the utilization rate of capital. Finally, equation (43) equalizes the marginal adjustment cost of the utilization of capital to the (after tax) rental rate of capital.[25]

Combining FOCs (36) and (39), we may obtain a stochastic consumption Euler equation, which we may also derive by appropriate log-linearization. The complete log-linearization of FOCs (36) - (43) is worked out in Appendix D.

Each foreign consumer solves a similar problem as presented above, with the only difference providing an asterisk for the variables to denote foreign variables; foreign consumers must satisfy a set of FOCs analogous to (36 - 43).

5.2 The domestic firms' optimization: optimal demands for inputs

5.2.1 Final goods producers' input demands

Since demanders for inputs are price takers under minimization of costs, defined as $TC_t^m(j) \equiv W_t^m(j)L_t^m(j) + R_t^m(j)\mathcal{Z}_t^m K_t^m(j) + P_t^{V,m}(j)V_t^m(j)$, company j's derived demands for labor, capital stock and intermediate goods, given production function (29) subject to (30) and (31), satisfy (see Appendix E):

$$L_t^m(j) = \upsilon_{Lm}\left(\frac{W_t^m(j)}{P_t^{Z,m}(j)}\right)^{-\gamma_m}\frac{\left[Y_t^m(j)\right]^{\frac{1}{\varpi_m}}}{\Omega_{m,t}}, \tag{44}$$

$$K_t^m(j) = \upsilon_{Km}\left(\frac{\mathcal{Z}_t^m R_t^m(j)}{P_t^{Z,m}(j)}\right)^{-\gamma_m}\frac{\left[Y_t^m(j)\right]^{\frac{1}{\varpi_m}}}{\Omega_{m,t}} \quad \text{and} \tag{45}$$

$$V_t^m(j) = (1-\upsilon_{Lm}-\upsilon_{Km})\left(\frac{P_t^{V,m}(j)}{P_t^{Z,m}(j)}\right)^{-\gamma_m}\frac{\left[Y_t^m(j)\right]^{\frac{1}{\varpi_m}}}{\Omega_{m,t}}, \tag{46}$$

[24] Any consumer can own homogeneous capital and can rent it out to firms in domestic industries. Capital services can be increased either by new capital or by augmenting the utilization rate of existing capital.

[25] In the interpretation of Smets and Wouters (2003) for instance, a higher rental rate of capital induces a higher utilization rate of it up to the point where the extra gains equalize the extra input costs.