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SOCIAL SECURITY INCENTIVES AND HUMAN CAPITAL INVESTMENT

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Abstract

While the effect of social security systems on retirement decisions has received much attention, the impact of these systems on individuals' incentives to invest in their human capital has not been analyzed. We integrate human capital investment and retirement decisions in a simple analytical life-cycle model with full certainty and investigate how different social security schemes may a¤ect welfare, human capital investment and labor supply. We analyze and compare three different social security systems. Our results suggest that actuarial adjustment and the link between individual social security contributions and benefits increase human capital investment and postpone retirement.

Keywords: Social security, retirement, education, human capital, labor supply

JEL Classification: H55, I21, J26

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1 Introduction

The decrease in old age labor force participation rates is one of the most significant labor market trends in European countries and the United States over the last four decades; in some countries, the labor force participation rate for men aged 60 to 64 has fallen by up to 75%. Since life expectancy has increased at the same time, explaining this secular trend by worse health conditions is implausible. A more convincing reason suggested by Boskin (1977) is the development in social security systems. This explanation is supported by Gruber and Wise (1999), who find a strong correspondence in Western countries between early retirement and social security provisions. In particular, early retirement is widespread in countries with high implicit tax penalties on wage earnings after social security eligibility.

While the effect of social security systems on retirement decisions has received much attention, the impact of these systems on individual incentives to invest in human capital has not been analyzed. We integrate human capital investment and retirement decisions in a simple analytical life-cycle model and investigate how different social security schemes may affect welfare and the supply of labor services. Two important features of social security systems are highlighted here: (i) actuarial adjustment, and (ii) the link between individual social security contributions and benefits. We expect that actuarial adjustment encourages later retirement because the present value of social security benefits is unaffected by the retirement age, and we expect that the link between social security contributions and benefits has a positive effect on human capital investment because the return on human capital investment increases. Finally, the interaction between these two links is analyzed. How does actuarial adjustment affect human capital, and how does the link between social security contributions and benefits affect retirement behavior?

There is a trade-off between model complexity and robustness of results in dynamic life-cycle models. For example, Nielsen and Sørensen (1997) apply a two-period overlapping generations model with endogenous human capital formation to analyze efficiency effects of human capital taxation. Given a constant labor supply and a proportional tax rate on capital income, they find that a progressive tax rate on labor income may be defended on pure efficiency grounds – the reduced private return on human capital investment offsets the discrimination in favor of human capital investment if labor and capital income are taxed by the same proportional tax rate. Even in this simple model, Nielsen and Sørensen do not get unambiguous positive or negative results of the desirability of dual income taxation when the supply of labor is endogenous. We assume that lifetime utility is separable in lifetime consumption and leisure. The interest rate is normalized at zero, and we do not take stance on the allocation of consumption over the life-cycle. This specification allows us to classify different social security benefit systems according to human capital investment, retirement and welfare.

Pogue and Sgontz (1977) show that social security systems may affect public investment in human capital through intergenerational transfers. They argue that the unfunded "pay-as-you-go" social security system provides a stronger incentive for current working age generations to invest in the human capital of younger generations compared to a fully funded social security system. The pay-as-you-go social security system may therefore improve welfare for all generations if the rate of return on human capital investment exceeds the rate of return on financial investment in the fully funded system. We include the pay-as-you-go system with a balanced budget. However, the interest rate is equal to zero and there is no population growth, which implies that there is no distinction between this unfunded system and a funded system.

We analyze and compare the effects of three different social security components on private retirement and education decisions. Social security benefits are financed by a proportional tax rate on labor income in each system, and the three components include: (i) constant "old age benefits" paid to individuals who are older than a given entitlement age, (ii) "uniform retirement benefits" paid to retired individuals, and (iii) "income dependent benefits" paid to retired individuals as a proportion of wage income during a given period before retirement. The first component is actuarially fair, since the present value of social security benefits is independent of the retirement age. The last two components do not include any actuarial adjustment. The third component introduces a link between social security contributions and benefits, whereas social security benefits do not depend on contributions in the first two components.

Investment in human capital is viewed here as a private investment decision by individuals. The duration of education is kept constant, but the level of human capital depends on resources devoted by the individual to education. This representation of human capital investment allows us to capture the tax distortion attributable to non-deductible tuition fees, which is relevant in the United States. Education systems in Europe, on the other hand, are mainly financed by the public sector, and the opportunity cost of human capital investment thus mainly consists of lost after-tax wage income. Given the level of human capital, the agent then decides how long he or she will be active in the labor market and when to retire. A high level of human capital encourages individuals to retire later because the opportunity cost of retirement is high.

During retirement the agent receives social security benefits according to the social security system. The results suggest that actuarial social security schemes encourage later retirement and thus increase the incentive to invest in human capital compared to non-actuarial schemes. We also find that a stronger correspondence between earnings history and social security benefits increases the incentive to invest in human capital and postpones retirement.

2 Human Capital and Retirement without Social Security

To illustrate how different social security systems may affect the supply and quality of labor, we construct a simple life-cycle model with endogenous human capital formation and retirement. After completing education, the agent decides how long he/she will be active in the labor market and when to retire. In other words, the retirement age is endogenous in the model. All agents are identical, and we analyze the optimal behavior of a representative agent.

Human capital investment includes education obtained at universities and other institutions of higher learning, as well as any courses and other training obtained elsewhere. Some inputs have to be purchased, like tuition and books. While agents clearly decide on both time devoted to human capital investment and inputs purchased, we restrict our attention to inputs purchased and keep the time devoted to human capital investment constant. Including both time and expenditures as decision variables would require assumptions concerning how these two types of resource combine in producing human capital, a topic which is beyond the scope of our analysis.

The time horizon for each agent is normalized at unity, and there is no uncertainty about life expectancy or return on education. Since the duration of education is constant, we analyze only the allocation of time between work and retirement. Perfect competition prevails in each market, which implies that output and factor prices are given to all agents in the model. The interest rate is normalized at zero, and there is no market for physical capital. The homogeneous consumption good can be borrowed or lent internationally at a zero interest rate, so we need not restrict the distribution of consumption over time. We assume that lifetime utility $\widetilde{U}$ is separable in lifetime consumption and retirement:

$$\widetilde{U} = U(C) + V(R), \tag{1}$$

where U is a concave function of consumption C, and V is a concave function of the duration of retirement R. The wage rate, w, is a concave function of human capital investment H,

$w = w(H)$. We assume that $\lim_{C \to 0^+} U'(C) = \infty$, $\lim_{R \to 0^+} V'(R) = \infty$, and $\lim_{H \to 0^+} w'(H) = \infty$ in order to guarantee interior solutions.

The lifetime budget constraint states that the value of lifetime expenditures on consumption and human capital investment cannot exceed lifetime income from the supply of labor services:

$$(1 - R) \cdot w(H) = C + H, \tag{2}$$

where $(1 - R)$ is the duration of working life, as well as the point in time at which the individual retires from the labor market.

The representative agent maximizes lifetime utility (1) subject to the human capital production function and the lifetime budget constraint (2). The first-order condition with respect to human capital is:

$$(1 - R) \cdot w'(H) = 1,$$

where the left-hand side is the return on human capital investment, and the right-hand side is the opportunity cost in terms of foregone consumption. The first-order condition with respect to retirement is:

$$V'(R) = w(H) \cdot U'(C),$$

where the left-hand side is the marginal utility of retirement, and the marginal cost on the right-hand side is equal to foregone labor income times the marginal utility of consumption goods. These two equations determine optimal choices of human capital investment and the duration of retirement.

3 Human Capital Investment and Retirement with Social Security

We use the life-cycle model to analyze steady state effects of a social security system with three different components. Social security benefits are financed by a proportional tax rate on labor income, and the three components include (i) constant "old age benefits" paid to individuals who are older than a given entitlement age, (ii) "uniform retirement benefits" paid to retired individuals, and (iii) "income dependent benefits" paid to retired individuals as a proportion of wage income during a given period before retirement. The first component is actuarially fair, since the present value of social security benefits is independent of the retirement age. The last two components do not include any actuarial adjustment, and both systems effectively subsidize retirement since they drive the private cost of retirement below the net wage. The third component introduces a link between social security contributions and benefits, whereas social security benefits do not depend on past contributions in the first two components. Prices and quantities are constant in steady state because the interest rate and the growth rate are normalized at zero. Variables do therefore not carry time indices.

3.1 Introducing Different Benefit Schemes

In the first social security system, each person is entitled to "old age benefits" at age $\widehat{R}$. The old age benefits are constant and equal to z per unit of time, which implies that each individual receives a lump sum lifetime social security payment of $B = \widehat{R}z$ from the government. The social security payments are financed by a proportional tax rate on labor income, t_1, and the public budget constraint with respect to this component is:

$$t_1(1 - R) \cdot w(H) = \widehat{R}z = B. \tag{3}$$

The left-hand side is equal to tax payments from current generations who work, and the right-hand side reflects social security payments to current old generations.

The second social security system includes a uniform benefit flow, say a given monthly benefit to retired persons. The "uniform retirement benefits" are denoted by b, and the payments are financed by a proportional tax rate on labor income, t_2. In this case, the public budget constraint is:

$$t_2(1 - R) \cdot w(H) = Rb.$$

Again, the left-hand side is equal to tax payments from current generations who work, and the right-hand side is equal to social security payments to current old generations.

Finally, we introduce a social security system in which benefits depend on wage income during a given period before retirement. In particular, social security benefits are determined as a proportion, p, of wage income during a period, n, before retirement.[1] Defining $x \equiv np$, social security benefits for individual i are determined by:

$$b_i = x \cdot w(H_i),$$

where x is an exogenous fraction of wage rate. The "income dependent benefits" are financed by a proportional tax rate, t_3, on labor income, and the public budget constraint is given by:

$$t_3(1 - R) \cdot w(H) = Rx \cdot w(H),$$

where the left-hand side is equal to tax payments from individuals who work. All individuals are identical in the model, and the right-hand side is equal to aggregate social security payments to retired generations.

Combining the three social security systems, the budget constraint for the representative agent is:

$$(1 - t_1 - t_2 - t_3)(1 - R) \cdot w(H) + B + Rb + Rx \cdot w(H) = C + H. \tag{4}$$

All the three systems that we analyze introduce some form of distortion in the economic

[1] Since the wage rate is constant in the model, the length of the period is not important for the results, unless the period is sufficiently long to postpone retirement.

decision making. A system with perfect correspondence between an individual's own social security tax payments and benefits received would in our framework only replicate the solution without social security.

3.2 Incentive Effects of Social Security

The representative agent maximizes lifetime utility (1) subject to the lifetime budget constraint (4). The first-order condition with respect to human capital investment is:

$$[(1 - t_1 - t_2 - t_3)(1 - R) \cdot w' + Rx \cdot w' - 1]U' = 0,$$

and the first-order condition with respect to retirement is:

$$[-(1 - t_1 - t_2 - t_3) \cdot w + b + x \cdot w]U' + V' = 0.$$

The first-order condition with respect to human capital investment simplifies to:

$$(1 - t_1 - t_2 - t_3)(1 - R) \cdot w' + Rx \cdot w' = 1,$$

where the left-hand side is the return on human capital investment and the right-hand side is the opportunity cost in terms of foregone consumption. The second term on the left-hand side measures the return on human capital investment through its effects on social security benefits. Social security taxes decrease the return on human capital investment, whereas income dependent retirement benefits partially offset this decrease.[2] The first-order condition with respect to retirement can be written as:

$$V' = [(1 - t_1 - t_2 - t_3) \cdot w - b - x \cdot w]U',$$

where the left-hand side is the marginal utility of retirement, and the marginal cost on the right-hand side is equal to the net income loss due to retirement times the marginal utility of consumption goods. Note that social security taxes, "uniform retirement benefits" and "income dependent benefits" decrease the marginal cost of retirement.

Using Cramer's rule, we analyze and compare the three different social security components with respect to private retirement and education decisions. The results are derived in Appendix A and can be summarized as:

Proposition 1 *An increase in the tax rate to finance any component of the social security system discourages human capital investment and encourages early retirement.*

Proposition 2 *Increasing the share of "uniform retirement benefits" at the expense of either constant "old age benefits" or "income dependent benefits" discourages human capital investment and encourages early retirement.*

The social security system affects human capital investment in two ways. First, it may change the return on human capital investment at any given retirement age. The system in

[2] It is useful to contrast our results with Heckman (1976). Heckman assumes that the demand for leisure is constant and the opportunity cost of human capital investment is equal to foregone labor income. With those assumptions, labor income taxes are non-distortionary.

which social security benefits depend on wage income before retirement encourages human capital investment compared to systems without the link. Second, the social security system may indirectly affect human capital investment through the impact on retirement age, which affects the amortization period of human capital investment. We find that actuarial adjustment has a positive effect on human capital investment, because it postpones retirement. Hence, replacing "income dependent benefits" or constant "old age benefits" with "uniform retirement benefits" discourages human capital investment.

Retirement decisions are also affected in two ways by the social security system. First, social security benefits lower the private opportunity cost of retirement. The private opportunity cost of retirement is reduced by the retirement benefit in the two non-actuarial social security systems compared to the actuarial system. Replacing "old age benefits" with "uniform retirement benefits" thus encourages early retirement. Second, the social security system indirectly affects the retirement age through human capital investment, since the level of human capital affects individual productivity. Increasing the share of "uniform retirement benefits" compared to "income dependent benefits" therefore encourages early retirement.

It is not possible to say anything decisive about education and retirement decisions across the "old age benefit" and "income dependent benefit" components. We relegate this issue to the next section, where we restrict the lifetime utility function to be of the Cobb-Douglas variety.

4 Special Case: Cobb-Douglas Utility Specification

4.1 Incentive Effects of Social Security

We apply next a Cobb-Douglas specification of the utility function and assume that each agent maximizes:

$$\widetilde{U} = \ln(C) + \beta \ln(R), \tag{5}$$

where $\beta > 0$ is the relative weight of utility from retirement. The individual stock of human capital is determined by H^{α}, where $0 < \alpha < 1$. The marginal productivity of human capital investment is thus diminishing, which implies that human capital investment is strictly positive and bounded. The representative individual maximizes lifetime utility, (5) subject to the lifetime budget constraint:

$$(1 - t_1 - t_2 - t_3)(1 - R) \cdot H^{\alpha} + B + Rb + Rx \cdot H^{\alpha} = C + H. \tag{6}$$

The first term on the left-hand side is lifetime wage income after tax, the second term is the sum of old age benefits, the third term is the sum of uniform retirement benefits, and the fourth term is the sum of income dependent benefits. Solving the individual maximization problem, we find that the first-order conditions with respect to human capital investment and retirement simplify to

$$H = \left[\frac{\alpha(1 - t_1 - t_2 - t_3)(1 - t_1 - t_2)}{1 - t_1 + \beta(1 - \alpha(1 - t_1 - t_2))} \right]^{\frac{1}{1-\alpha}}, \tag{7}$$

$$R = \frac{t_2 + t_3 + \beta(1 - \alpha(1 - t_1 - t_2))}{1 - t_1 + \beta(1 - \alpha(1 - t_1 - t_2))}. \tag{8}$$

These two equations allow us to compare incentive effects across "old age benefit" and "income dependent benefit" components. The results are derived in Appendix B, and they show:

Proposition 3 *Increasing the share of "income dependent benefits" at the expense of "old age benefits" increases human capital investment and leads to earlier retirement.*

This proposition suggests that the link between social security contributions and benefits is more important than the actuarial link with respect to human capital investment, whereas actuarial adjustment is more important with respect to retirement decisions. The intuitive explanation is that first-order effects (the effect of linking benefits to wage level on human capital investment and the effect of actuarial adjustment on retirement age) are more important than second-order effects (the effect of retirement decision on human capital investment and the effect of human capital investment on retirement decision).

Using a Cobb-Douglas representation of the lifetime utility function, the results of propositions 2 and 3 can be summarized as:

Proposition 4 *Measured by human capital investment, the descending order of the three social security systems is: income dependent benefits, old age benefits and uniform retirement benefits. Measured by the retirement age, the descending order is: old age benefits, income dependent benefits and uniform retirement benefits.*

The ranking of the three systems with respect to human capital investment and retirement age are illustrated in Figures 1 and 2. In these examples, we set $\alpha = \beta = 0.5$.

FIGURES 1 AND 2

Figures 1 and 2 suggest that the differences between human capital investment and retirement age are magnified when the social security tax rate increases.

4.2 Welfare Effects of Social Security

Finally, we compare welfare across the three social security systems. Using the lifetime utility function (5), we can calculate the level of utility with three social security systems of only one component and without social security. Private consumption is equal to $(1-R)H^{\alpha} - H$, since the value of social security benefits is equal to social security contributions in the steady state equilibrium. The lifetime utility function thus simplifies to:

$$U = \ln((1-R)H^{\alpha} - H) + \beta \ln(R). \tag{9}$$

The utility of any given social security system is subsequently derived by substituting the expressions for H and R into the utility function (Appendix C provides a comparison of utility across the three different social security systems). The results can be summarized as:

Proposition 5 *Given the social security tax rate, uniform retirement benefits lead to lower utility than old age benefits and income dependent benefits. Depending on parameter values, old age benefits may lead to either lower or higher utility than income dependent benefits. In any of these systems, the utility level is decreasing with respect to the tax rate.*

This proposition suggests that welfare is improved by both actuarial adjustment and by the link between earnings history and social security benefits. The relative importance of these two links depends on parameter values. In Figure 3, $\alpha = \beta = 0.5$, and "old age benefits" lead to higher utility than "income dependent benefits" when tax rates are not extremely high. In Figure 4, $\alpha = 0.5$ and $\beta = 1.5$, and "income dependent benefits" lead to higher utility than "old age benefits". Furthermore, the crossing of utility curves associated with "old age benefits" and "income dependent benefits" in Figure 3 illustrates that the order of these systems may depend on the level of taxation.

FIGURES 3 AND 4

A caveat to be remembered in the interpretation of our welfare results is that they capture only distortions associated with incentive effects of social security, without including any potential benefits from these systems.[3]

5 Conclusions and Implications

We have analyzed the interaction between social security rules, human capital investment, and the timing of retirement. Our results highlight two important links in social security systems: (i) actuarial adjustment and (ii) the link between contributions made and benefits received. We find that actuarially adjusted systems lead to later retirement than systems with a weaker actuarial adjustment. This corresponds to the empirical finding by Börsch-Supan (2000), who suggests that retirement before age 60 would be reduced by more than a third if the German social security system were reformed and made actuarially fair. We also find that the link between benefits and contributions encourages human capital investment. The results stress the importance of incentives embedded in social security rules, since distortions arise even when agents are identical and there is no redistribution in equilibrium.

The results highlight the efficiency and welfare gains that may be available through a better planning of social security rules. However, our framework does not include uncertainty. The public finance literature identifies several ways in which redistribution may improve welfare and efficiency when uncertainty is present. For example, Eaton and Rosen (1980) show that proportional income taxation may produce efficiency gains if the return on human capital investment is uncertain, since redistributive taxation serves as a substitute for the

[3]Since our model can be solved only partially with a specification in which the wage rate contains a random term, we do not report the results here. Diamond and Mirrlees (1986) analyze the optimal structure of social security benefits with exogenous productivity and disability risk.

missing private market for income insurance. Diamond and Mirrlees (1986), on the other hand, analyze the optimal relationship between retirement age and retirement benefits, when workers with exogenous productivity face uncertainty about the length of their working lives. Since the government is not able to verify disability, workers are compensated for disutility of work through higher consumption compared to retirees. Diamond and Mirrlees argue that optimal benefits rise with the age of retirement because this increases the incentive to continue working for people who are able to work. However, actuarial adjustment is incomplete because redistribution for the disabled is a desired part of insurance. Finally, Sinn (1997) suggests that the private insurance market for career risk does not exist because of an adverse selection problem. When agents have private knowledge, any provider of voluntary income redistribution contracts would suffer from adverse selection. Redistributive taxation may therefore be used as a substitute for the missing private insurance market. An optimal social security system should balance these benefits of redistribution against the costs outlined in our study.

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Appendix A. Proofs of Propositions 1 and 2

To analyze incentive effects of different social security systems on human capital investment and retirement, we totally differentiate the system of the two first-order conditions with respect to the unknown individual decision variables H and R and social security tax rates t_1, t_2, and t_3. The social security parameters B, b and x are taken as given when we differentiate with respect to individual decision variables, whereas they are treated as endogenous when we differentiate with respect to tax rates (for details, see Poutvaara (2000)). The system of two equations with two unknown variables is represented in matrix form by:

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} dH \\ dR \end{bmatrix} = \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{21} & X_{22} & X_{23} \end{bmatrix} \begin{bmatrix} dt_1 \\ dt_2 \\ dt_3 \end{bmatrix}, \text{ where}$$

$$\begin{aligned}
A_{11} &= [(1-t_1-t_2-t_3)(1-R)\cdot w'' + Rx\cdot w'']U' \\
A_{12} &= A_{21} = [-(1-t_1-t_2-t_3)\cdot w' + x\cdot w']U' \\
A_{22} &= [-(1-t_1-t_2-t_3)\cdot w + b + x\cdot w]^2 U'' + V'' \\
X_{11} &= (1-R)\cdot w'U' \\
X_{12} &= (1-R)\cdot w'U' \\
X_{13} &= 0 \\
X_{21} &= -wU' \\
X_{22} &= -\frac{1}{R}\cdot wU' \\
X_{23} &= -\frac{1}{R}\cdot wU'.
\end{aligned}$$

An agent works only if the net wage exceeds potential social security benefits. Therefore, it must hold that:

$$(1 - t_1 - t_2 - t_3) \cdot w > b + x \cdot w,$$

which implies that $A_{12} = A_{21} < 0$. It is easy to check that an increase in the tax rate to finance any component of the social security system discourages human capital investment and encourages early retirement. Denote $D = A_{11}A_{22} - A_{12}A_{21}$. This determinant is positive, as H and R are chosen to maximize individual utility. For human capital investment, it always holds that:

$$\frac{dH}{dt_i} = \frac{\begin{vmatrix} X_{1i} & A_{12} \\ X_{2i} & A_{22} \end{vmatrix}}{D} < 0,$$

where $X_{1i}A_{22} \leq 0$ and $-X_{2i}A_{12} < 0$, $\forall i \in \{1, 2, 3\}$. For retirement, it always holds that:

$$\frac{dR}{dt_i} = \frac{\begin{vmatrix} A_{11} & X_{1i} \\ A_{12} & X_{2i} \end{vmatrix}}{D} > 0,$$

where $A_{11}X_{2i} > 0$ and $-A_{12}X_{1i} \geq 0$, $\forall i \in \{1, 2, 3\}$. These two equations prove proposition 1.

We next calculate the effects of changing two tax rates simultaneously such that the total tax burden does not change. The effect of increasing t_1 at the expense of t_2 is given by:

$$\frac{dH}{dt_1} + \frac{dH}{dt_2} \mid_{t_2=-t_1} = \frac{(1-\frac{1}{R}) \cdot wU'A_{12}}{D},$$

which is positive, since $(1-\frac{1}{R}) < 0$ and $A_{12} < 0$. It is also straightforward to show that:

$$\frac{dH}{dt_2} + \frac{dH}{dt_3} \mid_{t_3=-t_2} = \frac{(1-R) \cdot w'U'A_{22}}{D} < 0,$$

$$\frac{dR}{dt_1} + \frac{dR}{dt_2} \mid_{t_2=-t_1} = \frac{(\frac{1}{R}-1) \cdot wU'A_{11}}{D} < 0$$

and

$$\frac{dR}{dt_2} + \frac{dR}{dt_3} \mid_{t_3=-t_2} = \frac{-(1-R) \cdot w'U'A_{21}}{D} > 0.$$

However, it is not possible to order constant "old age benefits" and "uniform retirement benefits" according to the effect on human capital investment and retirement, because the signs of

$$\frac{dH}{dt_1} + \frac{dH}{dt_3} \mid_{t_3=-t_1} = \frac{(1-R) \cdot w'U'A_{22} + (1-\frac{1}{R}) \cdot wU'A_{12}}{D}$$

and

$$\frac{dR}{dt_1} + \frac{dR}{dt_3} \mid_{t_3=-t_1} = \frac{(\frac{1}{R} - 1) \cdot wU'A_{11} - (1 - R) \cdot w'U'A_{21}}{D}$$

are unclear.

Appendix B. Retirement and Human Capital Across Benefit Schemes

Maximizing the Cobb-Douglas specification of lifetime utility (5) subject to the lifetime budget constraint (6) yields the following the first-order conditions:

$$\frac{(1-t_1-t_2-t_3)(1-R)\alpha H^{\alpha-1}+R\alpha H^{\alpha-1}x-1}{C}=0, \tag{B1}$$

$$\frac{-(1-t_1-t_2-t_3)H^{\alpha}+b+H^{\alpha}x}{C}+\frac{\beta}{R}=0. \tag{B2}$$

The public budget constraint for the "old age benefits" component is $B=t_1(1-R)H^{\alpha}$, that for "uniform retirement benefits" is $b=t_2\frac{1-R}{R}H^{\alpha}$, and that for "income dependent benefits" is $x=t_3\frac{1-R}{R}$. When we substitute these expressions into (B1) and (B2), we obtain:

$$(1-t_1-t_2)(1-R)\alpha H^{\alpha-1}-1=0, \tag{B3}$$

$$-(1-t_1)RH^{\alpha}+t_2H^{\alpha}+t_3H^{\alpha}+\beta((1-R)H^{\alpha}-H)=0. \tag{B4}$$

(B3) and (B4) yield (7) and (8). We differentiate (7) and (8) with respect to t_1 and t_3 such that $dt_3=-dt_1$. These derivations reveal that $\frac{dH}{dt_1}\mid_{dt_3=-dt_1}<0$; and $\frac{dR}{dt_1}\mid_{dt_3=-dt_1}<0$.

Appendix C. Utility Comparisons

The level of utility from each different social security system can be found by substituting the associated levels of H and R into (9). To see how the social security systems compare to each other, we differentiate the utility difference between the systems with respect to t. The derivative of the utility difference between "old age benefits" and "uniform retirement benefits" is:

$$t\frac{\left[\begin{array}{c}(-1+\alpha-\alpha t)+(-(1-\alpha)(3-\alpha)+(\alpha^2 t-2\alpha t))\beta \\ +(-2(1-\alpha)^2-2\alpha t(1-\alpha))\beta^2\end{array}\right]}{\begin{array}{c}-\left(1-\alpha+\alpha t\right)\left(t+\beta-\beta\alpha+\beta\alpha t\right)\left(1-\alpha\right)\times \\ \left(1-t+\beta-\beta\alpha+\beta\alpha t\right)\left(1+\beta-\beta\alpha+\beta\alpha t\right)\end{array}}>0. \tag{C1}$$

Since (C1) is positive, "old age benefits" lead to higher utility than "uniform retirement benefits."

The derivative of the utility difference between "income dependent benefits" and "uniform retirement benefits" is:

$$\beta\alpha t\frac{\begin{array}{c}(-t-2\beta+2\beta\alpha-t\beta) \\ +(-2\beta^2+3\beta^2\alpha-\beta^2\alpha t-\beta^2\alpha^2+\beta^2\alpha^2 t)\end{array}}{\begin{array}{c}-\left(t+\beta-\beta\alpha\right)\left(t+\beta-\beta\alpha+\beta\alpha t\right)\times \\ \left(1-\alpha\right)\left(1+\beta-\beta\alpha+\beta\alpha t\right)\end{array}}>0,$$

which is positive.[4]

The derivative of the utility difference between "old age benefits" and "income dependent benefits" is:

$$t\frac{\begin{array}{c}(-1+\alpha-\alpha t+t)\\+(-2+2t+5\alpha-3\alpha^2-5\alpha t+4\alpha^2 t-\alpha^2 t^2+\alpha t^2)\beta\\+(3\alpha-5\alpha^2+5\alpha^2 t+2\alpha^3-3\alpha^3 t-2\alpha t-\alpha^2 t^2+\alpha^3 t^2)\beta^2\end{array}}{-(1-\alpha)(1-t)(1-\alpha+\alpha t)(1-t+\beta-\beta\alpha+\beta\alpha t)(t+\beta-\beta\alpha)} \gtreqless 0$$

Evaluating this at t close to zero, we see that the denominator is positive for $\alpha = 0.5, \beta = 1$ and negative for $\alpha = 0.5, \beta = 1.5$. Either system may therefore dominate, depending on values for α, β and t.

[4]The first term in the nominator is negative. The second term is negative because it increases with α, and it is equal to zero when α is equal to one. However, α is positive and less than one.

Figure 1. Social Security and Human Capital Investment

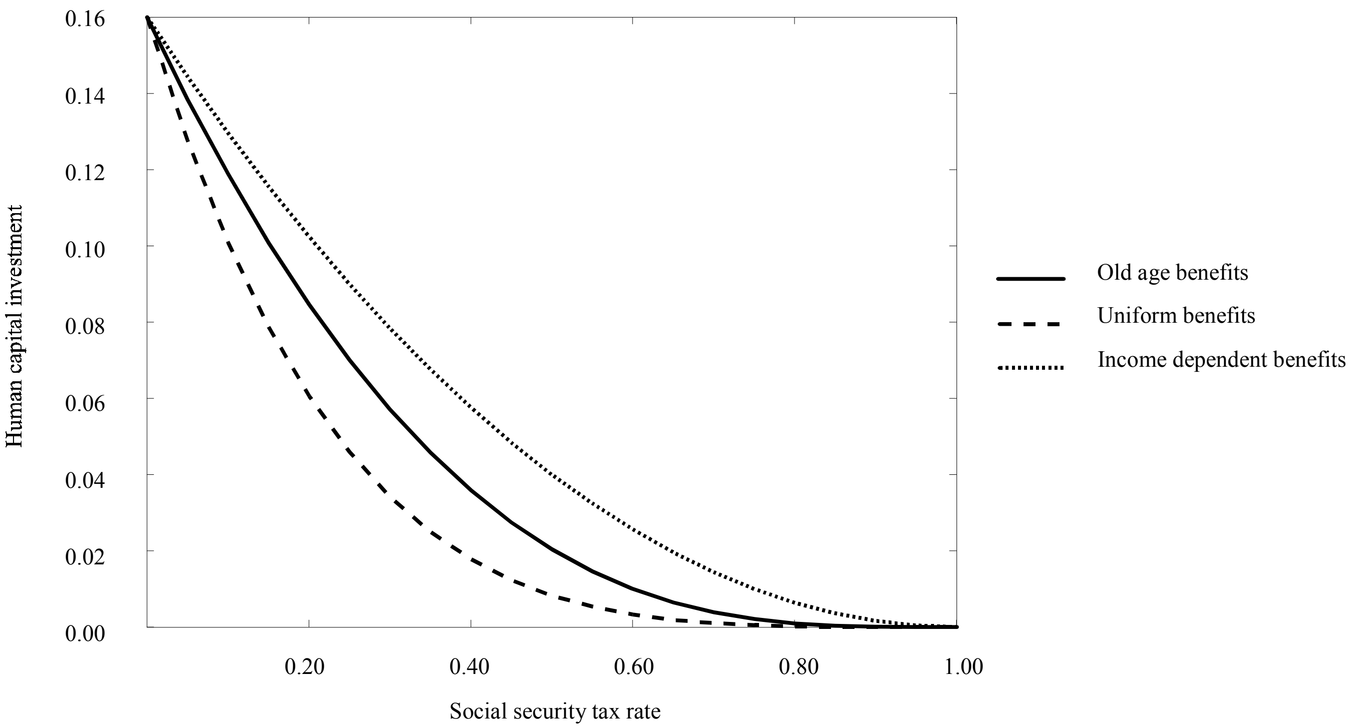

Figure 2. Social Security and Retirement

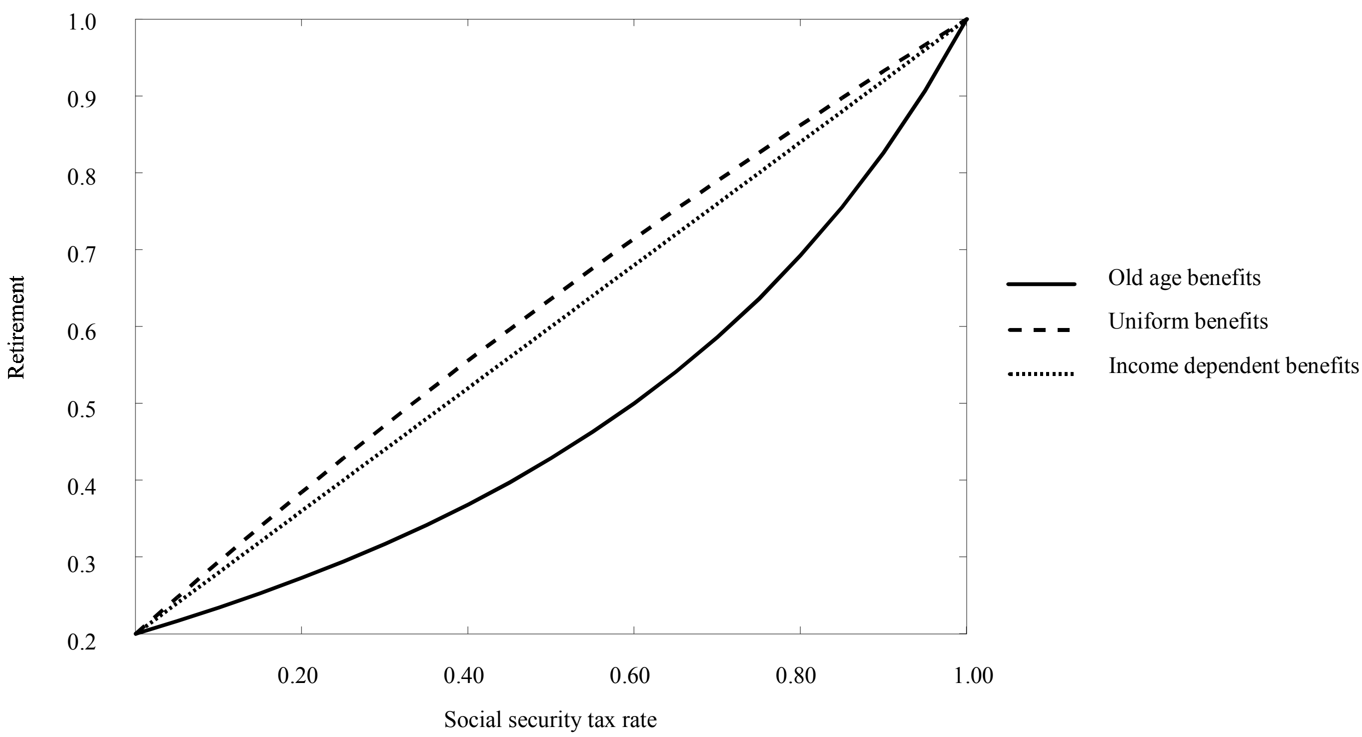

Figure 3. Social security and utility (alpha=0.5; beta=0.5)

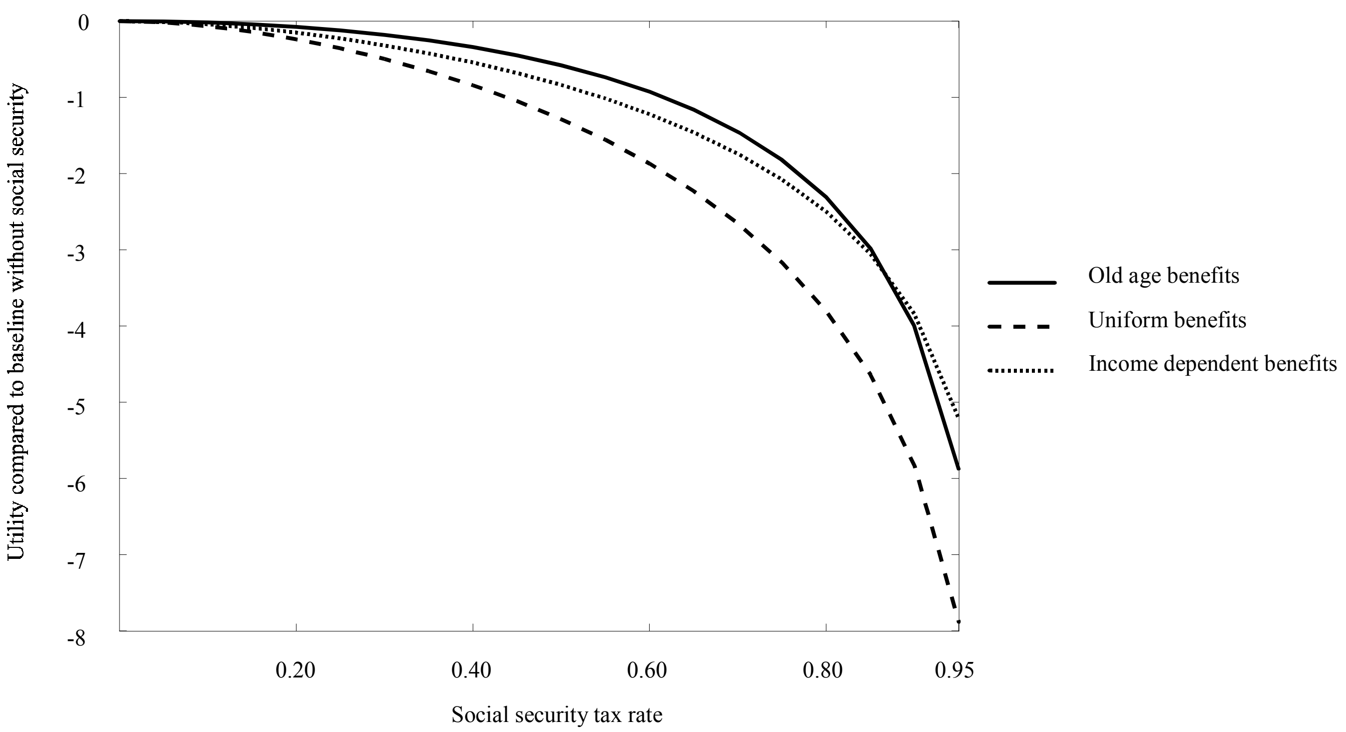

Figure 4. Social security and utility (alpha=0.5; beta=1.5)