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Abstract

This study develops a theoretical general equilibrium model to examine optimal externality tax policy in the presence of externalities linked to one another through markets rather than technical production relationships. Analytical results reveal that the second-best externality tax rate may be greater or less than the first-best rate, depending largely on the elasticity of substitution between the two externality-generating products. These results are explored empirically for the case of greenhouse gas from fossil fuel and nitrogen emissions associated with biofuels.

JEL-Code: D50, H23, Q58.

Keywords: second-best tax, multiple externalities, biofuel, GHG emissions, nitrogen leaching.

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OPTIMAL TAXATION OF EXTERNALITIES INTERACTING THROUGH MARKETS: A THEORETICAL GENERAL EQUILIBRIUM ANALYSIS

1. Introduction

This paper examines optimal externality tax policy in the presence of externalities linked to one another through markets rather than technical production relationships. In lieu of correlations between externalities arising through a single production process, we are interested in interaction that arises from market relationships between multiple processes embedded within the economy. We refer to these as "interacting externalities." Our analysis elaborates on the theory of the second best (Lipsey and Lancaster 1956). That theory states that if one of the Paretian conditions cannot be fulfilled, an optimal solution is likely to require departures from other Paretian conditions. As a corollary, if multiple market failures exist in the economy, eliminating one doesn't necessarily improve welfare. As described in Bennear and Stavins (2007), multiple market failures can be jointly ameliorating (correction of one market failure ameliorates welfare loss from the other), jointly reinforcing (correction of one market failure exacerbates welfare loss from the other), or neutral (correction of one market failure doesn't affect welfare loss from the other). With multiple market failures, the interrelationships can become complex, requiring explicit numerical examination to penetrate the web.

The theory of second best has received extensive study in the analytical environmental policy literature. Many studies examine interactions between an environmental externality and pre-existing distortions from labor or capital taxes (*e.g.*, Bovenberg and Goulder 1996, 1997; Fullerton and Metcalf 2001; Oates and Schwab 1988; and Parry 1995, 1997). With varying assumptions about policy instruments and revenue recycling measures, their results differ substantially. For example, a second-best tax on the externality can be either higher or lower than the first-best Pigouvian tax. The optimal environmental tax is a function of multiple terms: (1) a Ramsey term, which represents the revenue-raising function, and (2) the Pigouvian components that relate to each externality (*e.g*, Bovenberg and van der Ploeg 1994; and Sandmo 1975).

Very few studies consider corrective taxes in the presence of multiple simultaneous externalities. Caplan and Silva (2005) introduce the concept of "correlated externalities" to define multiple pollutants jointly produced by a single source that cause differentiated regional and global externalities. Within a multi-stage game theory framework, they find that non-cooperative, command-and-control environmental policies fail to achieve first-best optimality, but a joint permits mechanism can achieve a Pareto optimum. However, different externalities are usually regulated separately, or a single source of multiple externalities is regulated using a single instrument because a joint mechanism could face many political obstacles, especially for a global externality. For example, Peterson (1999) evaluates optimal agricultural land pricing policies considering pollution from agricultural land as well as non-market environmental benefits such as open space. Thus, one source, land, generates both a public good and a public bad. He finds the optimal land subsidy to correct the public goods is not equal the net extra-market regional values of the land amenities. Parry and Small (2005) evaluate the optimal gasoline tax considering externalities from traffic accidents, congestion, and air pollution. In a similar spirit, Khanna *et al.* (2008) develop a stylized economic model to evaluate the first-best and second-best ethanol policies in the presence of greenhouse gas (GHG) emissions and traffic congestion resulting from transportation uses of fuel. In each of these studies, a simple price-based policy instrument is applied to a single product to correct its multiple externalities.

This paper departs from the previous literature by developing a theoretical general equilibrium model incorporating two environmental externalities resulting from different industries that interact through market demands, in an economy with no government revenue requirement. The levels of the two externalities are determined not only by their individual production technologies, but also by the interaction between their sources in the market. In the model, two taxes are available to control the two environmental externalities, and the resulting revenues are transferred to consumers in lump-sum. Ideally, the tax rates for the two externalities are each set at its first-best level. However, if one of the externalities cannot be corrected fully, *i.e.*, one tax is constrained below the marginal environmental damage of the corresponding externality, the optimal tax rate for the other externality is unclear. Our results indicate that the optimal second-best policy depends on the nature of the market relationships between the two goods whose production causes the externalities. By explicitly modeling the production and market interaction of the two sources, this paper evaluates: (1) the effects of a small change in one tax, whether or not the tax rates are optimal, and (2) the optimal tax for one externality given a distortion from the other externality.

In an effort to illustrate this problem, our analysis is developed in the context of biofuel and fossil fuel production and the associated environmental externalities of greenhouse gases (GHGs) and nitrogen leaching. The biofuel industry has been developed rapidly in many countries, for its potential in greenhouse gas reductions and reduced dependence on foreign oil. Currently, the leading biofuel is ethanol. In 2009, the world's top ethanol producers were the United States (10.79 billion gallons) and Brazil (6.58 billion gallons), which account for about 89% of the world production (Renewable Fuel Association, 2010). As the most successful ethanol producer in the world, Brazil met 17.6% of its transportation energy requirements with ethanol in 2008, in term of energy balance (Empresa de Pesquisa Energética, 2009). As for the US, ethanol accounts for about 8% of the gasoline market by volume (ERS/USDA, 2010), but growing fast. Both fossil fuel and biofuel production processes emit GHGs, but in different amounts per unit of output. The biofuel production process emits less carbon, but U.S. corn-based ethanol production discharges nitrogen into the water environment.[1] Nitrogen in water can cause respiratory problems in infants and exacerbate algae growth and hypoxia in waterbodies. For the United States, the most severe problem associated with excessive nutrients is hypoxia in the Gulf of Mexico. A report released by National Oceanic and Atmospheric Administration (Rabalais et al. 1999) concluded that excess nitrogen from the Mississippi River combined with stratification of the Gulf's water is the cause of the hypoxia. Added production of nitrogen-intensive feedstocks, especially corn, to support increasing use of biofuel would add to the problem. Although the nitrogen discharge issue with sugarcane in Brazil is not as bad as with corn, the nitrogen application rate for sugarcane is still 80-100 kg/ha/year (Martinelli and Filoso, 2008), comparing to over 150 kg/ha/year for corn production in USA (ERS/USDA, http://www.ers.usda.gov/Data/FertilizerUse/). With a 60-80% leaching rate, the expansion of sugarcane ethanol production is responsible for eutrophication of dams and reservoirs in Brazil

[1] In addition to increasing soluble forms of nitrogen into surface water, fertilizer used in crop production generates N_2O which is a GHG. Policies designed to reduce nitrogen would affect N_2O emissions as well as nitrogen runoff. In this paper, the N_2O emissions are omitted to simplify the analysis. Further study with a more complex model would be required to analyze the effects of a nitrogen policy on total GHG emissions.

(Martinelli and Filoso, 2008). Given the global expansion of the ethanol market, the nitrogen leaching issue cannot be ignored.

So the two environmental externalities, carbon emissions and nitrogen leaching, are associated with two different products, and the two products are substitutes in the market. The interaction between the two pollutants acts through the relative demands for the two products. A first best tax for nitrogen leaching is unlikely because nitrogen leaching is a non-point source pollutant, and its accurate measurement is infeasible. Instead, this externality might be partially corrected by a fertilizer tax, or command-and-control policies applied to fertilizer management. Given a suboptimal policy for nitrogen discharges, the optimal tax for GHG may depend on how it affects nitrogen releases, which is mediated by the relationships between biofuel and fossil fuel. Following the theoretical analysis, we examine these relationships numerically to quantify the second-best GHG tax in the face of inability to apply the first-best tax on nitrogen.

The paper is organized as follows. Section 2 describes the basic model. Section 3 describes the method used to solve the system. Section 4 develops the analytical solutions for a small increase of GHG tax and characterizes the optimal GHG tax. Section 5 offers a numerical example to illustrate the nature of the interactions between policies, and it uses sensitivity analysis to determine effects of the most important parameters. Section 6 draws conclusions.

2. Model Structure

We continue the fossil fuel/biofuel metaphor in developing our analytical model. Consider an economy with n identical individuals who own one resource, a composite factor L (which can be a composite of labor, land and capital). The individuals receive utility from consumption of two goods: a composite commodity X and energy E. Energy E is consumers' energy demand, and it can be met by consuming fossil fuel F, biofuel B, or both. The final demand ratio of biofuel to fossil fuel can be viewed as the blend percentage of biofuel in liquid fuel. We assume this ratio can be any value between zero and one. In this model, $E = E(F, B)$ could be viewed as (1) a sub-utility function, (2) a home production function representing the consumer's production of energy, or (3) a firm's production function used to produce E for purchase by households. For concreteness, we take the last approach. All the uppercase letters refer to per-capita amounts.

Production of X is assumed to require the composite factor (in amount L_X) and energy (amount E_X). Fossil fuel F is produced using as an input.[2] Biofuel B is produced using an amount of the composite factor L_B. However, during the production and consumption processes, both fossil fuel and biofuel generate pollutants. Fossil fuel is a "dirty" product with pollutant emission C, representing CO_2 emissions. Biofuel is a substitute for fossil fuel. Combustion of biofuel also emits C, but the life cycle emissions from biofuel are less than those from fossil fuel. The emissions from fossil fuel are therefore measured by the *net* emission compared to biofuel. At the same time, production of biofuel induces nitrogen leaching, N. In this paper, the pollutant is treated as an input in the production process. The difference between inputs for F and B allows us to focus not only on the environmental effects but also on the different input requirements. With the assumptions of perfect competition and constant returns to scale, the production functions are assumed to be

$$X = X(L_X, E_X) \quad (1)$$

$$F = F(X_F, C) \quad (2)$$

$$B = B(L_B, N) \quad (3)$$

$$E = E(F, B) \quad (4)$$

All production functions are twice continuously differentiable and quasi-concave. The total emissions for each pollutant are summations across n identical individuals. Each consumer obtains utility from consumption of the composite commodity X (amount) and direct consumption of energy, E_U, and is affected by total emissions nC and nN:

$$U = U(X_U, E_U; nC, nN) \quad (5)$$

with $\partial U/\partial C < 0$ and $\partial U/\partial N < 0$. Utility is twice continuously differentiable and homothetic.

In this static model, the overall factor constraint is:

$$L_X + L_B = \bar{L} \quad (6)$$

where $\bar{L}$ is the total fixed endowment of the composite factor in the economy.

[2] It may seem odd to say that fossil fuel is "produced" using another produced composite commodity *X*, but the resulting model is equivalent to one where some of the domestic production of *X* is exported to another country in exchange for import of oil, at a fixed world price. Since *X* is chosen as numeraire, and the price of the imported oil is fixed relative to the price of *X*, the price of oil is equivalent to the price of *X*.

For the system, the market clearing conditions are

$$E_X + E_U = E \tag{7}$$

$$X_F + X_U = X \tag{8}$$

By the choice of X_U and E_U, each individual maximizes utility subject to a budget constraint

$$I = P_X X_U + P_E E_U = T + P_L \bar{L} \tag{9}$$

where T is the lump-sum transfer to the consumer from the government (where the amount will be $T = \tau_C C + \tau_N N$). The unit tax rates for GHG and nitrogen are represented by τ_C and τ_N, respectively. Market prices for the composite factor of production L and energy E are defined as P_L and P_E, respectively. The RHS of equation (9) is not chosen by the consumer, but it is endogenous to the economy. In this system, X is defined as numeraire. All the quantities and prices are endogenously determined except the tax rates for environmental externalities, τ_C and τ_N, which are exogenous.

3. Solution Strategy

The system is solved by totally differentiating relevant equations and solving the resulting system of linear differential equations.[3] First, totally differentiating the production functions and imposing perfect competition conditions, we have

$$\hat{X} = \theta_{XL}\hat{L_X} + \theta_{XE}\hat{E}_X \tag{10}$$

$$\hat{F} = \theta_{FX}\hat{X}_F + \theta_{FC}\hat{C} \tag{11}$$

$$\hat{B} = \theta_{BL}\hat{L_B} + \theta_{BN}\hat{N} \tag{12}$$

$$\hat{E} = \theta_{EF}\hat{F} + \theta_{EB}\hat{B} \tag{13}$$

where a hat (^) denotes a proportional change, *e.g.*,$\hat{X} = dX/X$. Parameter θ_{XL} refers to the expenditure share for input L in the total production costs of X, mathematically defined as $\theta_{XL} = \frac{P_L L_X}{P_X X}$. Other θ parameters are defined analogously. The detailed definition of each parameter is listed in Table 1.

Totally differentiating the factor constraint yields

$$\beta_{LX}\hat{L_X} + \beta_{LB}\hat{L_B} = 0 \tag{14}$$

[3] With this method, assumptions about specific functional forms for production and utility functions are not required. They just need to be continuous and differentiable. The procedure allows us to get analytical closed form solutions for the linearized model. However it uses derivatives and is therefore valid only for small changes.

where β_{LX} is the share of $\bar{L}$ that is used in the production of *X*, defined as $\beta_{LX} = L_X/\bar{L}$, with $\beta_{LB} = L_B/\bar{L}$.

The market clearing conditions in differentiated forms are:

$$\gamma_{EX}\hat{E_X} + \gamma_{EU}\hat{E_U} = \hat{E} \quad (15)$$

$$\gamma_{XF}\hat{X_F} + \gamma_{XU}\hat{X_U} = \hat{X} \quad (16)$$

where γ_{EX} is the quantity of *E* used in the production of *X* relative to the total quantity of *E* in the market, defined as $\gamma_{EX} = E_X/E$. All of the γ parameters refer to quantity shares and are defined analogously.

With perfect competition and constant return to scale, the zero profit conditions for the four production sectors can be written as

$$P_X X - P_L L_X - P_E E_X = 0$$

$$P_F F - P_X X_F - \tau_C C = 0$$

$$P_B B - P_L L_B - \tau_N N = 0$$

$$P_E E - P_F F - P_B B = 0$$

Rearranging and totally differentiating these conditions yields

$$\hat{P_X} + \hat{X} = \theta_{XL}(\hat{L_X} + \hat{P_L}) + \theta_{XE}(\hat{E_X} + \hat{P_E}) \quad (17)$$

$$\hat{P_F} + \hat{F} = \theta_{FX}(\hat{X_F} + \hat{P_X}) + \theta_{FC}(\hat{C} + \hat{\tau_C}) \quad (18)$$

$$\hat{P_B} + \hat{B} = \theta_{BL}(\hat{L_B} + \hat{P_L}) + \theta_{BN}(\hat{N} + \hat{\tau_N}) \quad (19)$$

$$\hat{P_E} + \hat{E} = \theta_{EF}(\hat{F} + \hat{P_F}) + \theta_{EB}(\hat{B} + \hat{P_B}) \quad (20)$$

Producers of X can substitute between the factor input and energy, depending on the prices they face, P_L and P_E, according to σ_X, the elasticity of substitution in the production technology.[4] The producer's response to changes in prices can be obtained from the definition of σ_X:

$$\sigma_X = \frac{d\ln(L_X/E_X)}{d\ln(P_E/P_L)}$$

[4] This parameter is a measure of curvature or substitutability at the initial equilibrium, for small changes in prices. The constant elasticity of substitution (CES) functional form is not required except for large changes in prices that might be considered using a computational general equilibrium (CGE) model.

With no taxes on factor *L* or energy generally, converting the above equation to the hat form yields:

$$\hat{L_X} - \hat{E_X} = \sigma_X(\hat{P_E} - \hat{P_L}) \tag{21}$$

Pollutant emissions are treated as necessary inputs in the production process. For fossil fuel production and use, both *X* and GHG are required. The elasticity of substitution between the two inputs in fossil fuel production is denoted σ_F. The definition of σ_F in hat form analogous to equation (21), is written as

$$\hat{X_F} - \hat{C} = \sigma_F(\hat{\tau_C} - \hat{P_X}) \tag{22}$$

where the tax is the only price for input *C*.

For biofuel feedstock production, nitrogen leaching might be reduced through improved fertilizer management strategies, genetic engineering that increases the nutrient conversion efficiency of crops, or substitution of cellulosic feedstocks for nutrient-intensive grains. We expect substitutability between nitrogen leaching and capital (part of the composite input *L*). Several studies have estimated a nonzero elasticity of substitution between fertilizer and other inputs in corn productions, a major biofuel feedstock in the United States (*e.g.*, Hertel *et al.* 1996; and Thompson *et al.* 2006). From the definition of the elasticity of substitution (analogous to equations (21) and (22)), we have

$$\hat{L_B} - \hat{N} = \sigma_B(\hat{\tau_N} - \hat{P_L}). \tag{23}$$

Energy, in this paper, is yielded by a production process with inputs of fossil fuel and biofuel. We assume fossil fuel and biofuel are imperfect substitutes with elasticity of substitution σ_F, due to the different energy contents, the need for vehicle modifications when the ratio of biofuel (ethanol) to fossil fuel (gasoline) exceeds a certain level, and environmental considerations. Analogous to equations (21), (22) and (23), we have

$$\hat{F} - \hat{B} = \sigma_E(\hat{P_B} - \hat{P_F}) \tag{24}$$

From the definition of the elasticity of substitution in utility, the relationship between consumption changes for X and F_U is:

$$\hat{X_U} - \hat{E_U} = \sigma_U(\hat{P_E} - \hat{P_X}) \tag{25}$$

By construction, $\sigma_U > 0$, so an increase in energy price index P_E will lead to more relative consumption of X (*i.e.*, more X_U relative to).

Totally differentiating the budget constraint yields:

$$\theta_{IX}\hat{X_U} + \theta_{IE}(\hat{P_E} + \hat{E_U}) = \frac{T}{I}\hat{T} + \frac{P_L\bar{L}}{I}\hat{P_L}$$

$$\hat{T} = \frac{\tau_C C}{T}(\hat{\tau_C} + \hat{C}) + \frac{\tau_N N}{T}(\hat{\tau_N} + \hat{N})$$

Combining the above two equations, we get

$$\begin{aligned} \theta_{IX}\hat{X_U} + \theta_{IE}(\hat{P_E} + \hat{E_U}) &= \frac{\tau_C C}{I}(\hat{\tau_C} + \hat{C}) + \frac{\tau_N N}{I}(\hat{\tau_N} + \hat{N}) + \frac{P_L\bar{L}}{I}\hat{P_L} \\ &= \theta_{IC}(\hat{\tau_C} + \hat{C}) + \theta_{IN}(\hat{\tau_N} + \hat{N}) + \theta_{IL}\hat{P_L} \qquad (26) \end{aligned}$$

Similar to previous definitions, θ_{IX} refers to the expenditure share for X in the consumer's total income, defined as $\theta_{IX} = P_X X/I$, and θ_{IE} is defined analogously. The mathematical definitions for θ_{IC}, θ_{IN}, and θ_{IL} are similar to θ_{IX} with different economic definitions. They refer to the income shares, rather than the expenditure shares. For example, θ_{IC} is the share of income from rebate of a GHG tax in total income.

The numeraire is *X*. Thus $P_X = 1$ and $\hat{P_X} = 0$. In this system, we have 16 variables ($\hat{X}$, $\hat{X_F}$, $\hat{X_U}$, $\hat{E}$, $\hat{E_X}$, $\hat{E_U}$, $\hat{B}$, $\hat{F}$, $\hat{L_X}$, $\hat{L_B}$, $\hat{C}$, $\hat{N}$, $\hat{P_L}$, $\hat{P_E}$, $\hat{P_F}$, and $\hat{P_B}$) 17 equations (equations (10) to (26)). Based on Walras' law, if all markets but one are in equilibrium, the last market must also be in equilibrium. Thus one of the market clearing conditions can be dropped. In this study, the market clearing condition for energy is dropped, *i.e.*, equation (15). This leaves us with 16 variables and 16 equations. Now we can solve the system for the changes of prices and quantities with corresponding changes of τ_C or τ_N. Since *C* and *N* are modeled symmetrically in the system, the results are similar for the two cases with ($\hat{\tau_C} > 0$, $\hat{\tau_N} = 0$) and ($\hat{\tau_C} = 0$, $\hat{\tau_N} > 0$). Thus only the GHG tax case ($\hat{\tau_C} > 0$, $\hat{\tau_N} = 0$) is explored here. Corresponding results for a change in the nitrogen tax ($\hat{\tau_C} = 0$, $\hat{\tau_N} > 0$) are provided in Appendix A.

4. Policy Implication

4.1 Effects of GHG Tax

We start the analysis by introducing a small exogenous increase of the GHG tax, $\hat{\tau_C} > 0$, while keeping the nitrogen tax constant ($\hat{\tau_N} = 0$).

Solving the system of equations in previous section, the changes in the prices and quantities of interest, induced by the change in the carbon tax rate, are:

$$\hat{P_F} = \theta_{FC}\hat{\tau_C} \tag{27}$$

$$\hat{P_B} = -\frac{\theta_{XE}\theta_{EF}\theta_{FC}\theta_{BL}}{D_1}\hat{\tau_C} \tag{28}$$

$$\hat{P_E} = \frac{\theta_{XL}\theta_{EF}\theta_{FC}}{D_1}\hat{\tau_C} \tag{29}$$

$$\hat{C} = [-\frac{A_4}{D_1 D_2}\sigma_X - \frac{\theta_{EF}\theta_{FC}A_5}{D_1 D_2}\sigma_U - \frac{\theta_{FC}A_6(D_2 - A_2)}{D_1 D_2}\sigma_E - \frac{\beta_{LX}A_3 + \theta_{FX}D_2}{D_2}\sigma_F - \frac{A_1 A_4}{\beta_{LX}D_1 D_2}\sigma_B]\hat{\tau_C} \tag{30}$$

$$\hat{F} = [-\frac{A_4}{D_1 D_2}\sigma_X - \frac{\theta_{EF}\theta_{FC}A_5}{D_1 D_2}\sigma_U - \frac{\theta_{FC}A_6(D_2 - A_2)}{D_1 D_2}\sigma_E - \frac{\beta_{LX}A_3}{D_2}\sigma_F - \frac{A_1 A_4}{\beta_{LX}D_1 D_2}\sigma_B]\hat{\tau_C} \tag{31}$$

$$\hat{N} = [-\frac{A_4}{D_1 D_2}\sigma_X - \frac{\theta_{EF}\theta_{FC}A_5}{D_1 D_2}\sigma_U + \frac{\theta_{FC}A_6 A_2}{D_1 D_2}\sigma_E - \frac{\beta_{LX}A_3}{D_2}\sigma_F - \frac{(A_1 + \theta_{BL}D_2)A_4}{\beta_{LX}D_1 D_2}\sigma_B]\hat{\tau_C} \tag{32}$$

$$\hat{B} = [-\frac{A_4}{D_1 D_2}\sigma_X - \frac{\theta_{EF}\theta_{FC}A_5}{D_1 D_2}\sigma_U + \frac{\theta_{FC}A_6 A_2}{D_1 D_2}\sigma_E - \frac{\beta_{LX}A_3}{D_2}\sigma_F - \frac{A_1 A_4}{\beta_{LX}D_1 D_2}\sigma_B]\hat{\tau_C} \tag{33}$$

$$\hat{E} = [-\frac{A_4}{D_1 D_2}\sigma_X - \frac{\theta_{EF}\theta_{FC}A_5}{D_1 D_2}\sigma_U + \frac{\theta_{FC}A_6(A_2 - \theta_{EF}D_2)}{D_1 D_2}\sigma_E - \frac{\beta_{LX}A_3}{D_2}\sigma_F - \frac{A_1 A_4}{\beta_{LX}D_1 D_2}\sigma_B]\hat{\tau_C} \tag{34}$$

where

$D_1 = \theta_{XL} + \theta_{XE}\theta_{EB}\theta_{BL}$

$D_2 = 1 - \beta_{LX}\gamma_{XU}\theta_{IL}$

$A_1 = \beta_{LB}\theta_{BN} - \beta_{LX}\gamma_{XU}\theta_{BL}\theta_{IN}$

$A_2 = \beta_{LX}\gamma_{XT} + \beta_{LX}\gamma_{XU}\theta_{IC}$

$A_3 = \gamma_{XT}\theta_{FC} - \gamma_{XU}\theta_{FX}\theta_{IC}$

$A_4 = \beta_{LX}\theta_{XE}\theta_{EF}\theta_{FC}$

$A_5 = \beta_{LX}\gamma_{XU}\theta_{XL}\theta_{IE}$

$A_6 = 1 - \theta_{XE}\theta_{BN}$

All of the parameters, βs and θs, are positive and less than one. Thus D_1, D_2, A_2, A_4, A_5, and A_6 are clearly positive. The signs for A_1 and A_3 are also positive, as shown in Appendix B. Thus, we can determine the signs of some effects on equilibrium quantities and prices, as summarized in Table 2.

First look at the last three rows of Table 2. Our model implies that the price change for each commodity is jointly determined by the price change of each of its inputs and the corresponding expenditure share of that input. For example, as shown in Equation (27), the percentage change of P_F is simply determined by the expenditure share of C in production of F, θ_{FC}, times the price change of C, $\hat{\tau_C}$ (since the other input, X, is numeraire). A positive $\hat{\tau_C}$ unambiguously increases P_F, since a positive $\hat{\tau_C}$ directly increases the production cost of F. Other prices are determined in the same manner, except the mathematical expressions are more complex because of the indirect effects of $\hat{\tau_C}$ on their inputs.

Equation (28) shows the solution of $\hat{P}_B$. Since its denominator and nominator are both positive, $\hat{P}_B$ is negative. Intuitively, we might expect an increase in the fossil fuel price to induce higher biofuel demand and thus a higher biofuel price. However, the demand for biofuel also depends on the change in total energy demand, for which the sign is ambiguous. We can explain the lower biofuel price due to a higher τ_C from the standpoint of input costs. Energy, whose price increases, is an input for X, the numeraire. Thus the price of L, the other input for X, has to fall for the producer of X to break even. Yet B is produced using L and N. An increase in τ_C reduces the relative price of L but has no effect on the price of nitrogen, since the nitrogen tax rate is exogenous. Thus, the final price of biofuel has to decline for the producer of B to break even.

The price of energy depends on the prices of both fossil fuel and biofuel, which change in opposite directions. The solution in equation (29) and the last row of Table 2 indicates that $\hat{P}_E$ is positive. Generally, in the current U.S. market, we might expect $\hat{P}_E$ to have the same sign as $\hat{P}_F$ because fossil fuel has a much larger market share than biofuel. Without any assumption about the relative values of θ_{EF} and θ_{EB}, however, our result still indicates a positive $\hat{P}_E$ for an increase of τ_C. The intuition is that the negative change in P_B is a "feedback effect" to the increase of P_F, where the increase of P_F is induced directly by the higher τ_C. Due to market adjustments, we expect that the “feedback effect” on $\hat{P}_B$ is smaller than the direct effect on $\hat{P}_F$, so $\hat{P}_E$ is positive even though $\hat{P}_B$ is negative.

The quantity changes in equations (30) to (34) are much more complicated and difficult to interpret. However, we can get some insights if we separate the expressions based on elasticities. For a change in , the sign for the coefficient on each elasticity is listed in a column in Table 2. As expected, a positive $\hat{\tau_C}$ always yields a negative $\hat{F}$ and $\hat{C}$. The coefficients on all of the elasticities for the solutions of these two variables are negative, as shown in the second and third rows in Table 2. Since we assume *F* and *B* are substitutes ($\sigma_E > 0$), we expect an increase in P_F to raise the demand for biofuel and the corresponding emissions, *N*. As shown in the fourth and fifth rows in Table 2, however, only the coefficient on σ_E is positive, and all the rest are negative. Thus, without additional assumptions, the effects of $\hat{\tau_C}$ on *B* and *N* are ambiguous.

An increase in τ_C increases the energy price, which causes the producer of X to substitute factor L for energy based on their relative prices and the value of σ_X. This substitution directly reduces total energy demand and thus the equilibrium quantities of F and its associated externality *C*, and B and its associated externality *N*. Thus the first term in each equation (30) to (34) is negative, as shown in the first column of Table 2. The same logic applies to σ_U, except that consumers substitute X for energy. Thus, with $\hat{\tau_C}$ positive, σ_U also has negative effects on F, *C*, B, *N* and E, as shown in the second column of Table 2.

The elasticity of substitution between *F* and *B*, σ_E, governs the final “blending ratio” of biofuel to fossil fuel in the market. With an increase in τ_C, P_F rises and P_B falls. This change in relative prices shifts the demand toward *B* and away from *F*. Thus, if τ_C increases, then a positive σ_E implies a negative $\hat{C}$ and a positive $\hat{N}$. Its effect on total energy demand *E* is ambiguous and depends on the market share parameter θ_{EF} (indicating *F* as a share of *E*).

The fossil fuel industry (*F*) can reduce GHG emissions via substitution into the other input X_F, as captured by σ_F. Increasing τ_C directly increases the price of *C*. As a result, the producer reduces emissions of *C* (treated as an input in the production process). On the other hand, because P_L falls, the elasticity of substitution between L and *N* in the biofuel production process, σ_B, implies a negative effect on *N*. The producer of *B* accordingly shifts from *N* to *L*.

With the elasticities that have definitive signs, an increase in τ_C reduces production of *F* and emissions of *C*. For the rest of the variables, including *B* and *N*, the changes are ambiguous. However, with assumptions about the parameter values, we find some definitive results under special cases. Before proceeding with special cases, the second-best optimal GHG tax rate (τ_C^*) is defined.

4.2 Optimal GHG Tax

To find the optimal GHG tax rate given a preexisting nitrogen tax, the following Langrangian equation can to be maximized by the choice of τ_C:

$$max\ \mathcal{L} = U(X_U, E_U; nC, nN) + \lambda(I - P_X X_U - P_E E_U) \tag{35}$$

where $I = \tau_C C + \tau_N N + P_L \bar{L}$. Given $d\bar{L} = 0$, fixed τ_N, and exogenous prices for the consumer, the total derivative of equation (35) with respect to τ_C is written as

$$\frac{1}{\lambda}\frac{d\mathcal{L}}{d\tau_C} = -\mu_C \frac{dC}{d\tau_C} - \mu_N \frac{dN}{d\tau_C} + \tau_C \frac{dC}{d\tau_C} + \tau_N \frac{dN}{d\tau_C}$$

where μ is "marginal environmental damage", and the subscript refers to the pollutant. For example, μ_C is the dollar value of disutility for all consumers from a marginal increase of GHG emissions, defined as $-n\frac{\partial U}{\partial C}/\lambda$, where λ is the marginal utility of income. As defined before, $\partial U/\partial C < 0$ and $\partial U/\partial N < 0$, so both μ_C and μ_N are positive.

The change in consumer utility includes the changes in damages from the environmental externalities (the first two terms on the RHS) and the offsetting environmental tax revenues (the last two terms). The optimal GHG tax rate is achieved when $dG/d\tau_C = 0$:

$$\tau_C^* = \mu_C + (\mu_N - \tau_N)\frac{dN}{d\tau_C} / \frac{dC}{d\tau_C} \tag{36}$$

If the nitrogen tax rate, τ_N, is set equal to μ_N, then τ_C^* equals the marginal damage of GHG, μ_C, which is the first-best combination of policies. More realistically, however, $\tau_N < \mu_N$, so the second-best τ_C^* is not equal to μ_C (the marginal damages). The restriction on τ_N precipitates the second-best policy problem.

To obtain the relationship between the second-best and the first-best tax rate, rewrite equation (36) to hat form as:

$$\tau_C^* = \mu_C + (\mu_N - \tau_N)\frac{N}{C}\frac{\hat{N}}{\hat{C}} \tag{37}$$

where *N* and *C* are the benchmark emission levels. Since μ_C, μ_N, C, and N all refer to the initial levels, and τ_N is exogenously defined, τ_C^* thus only depends on the ratio between percentage changes in nitrogen runoff and GHG emissions. If τ_N is too low ($\tau_N < \mu_N$) and the increased carbon tax reduces *C* while raising *N*, then equation (37) indicates that the second-best τ_C^* is below marginal environmental damage of carbon, μ_C. As discussed before, however, the sign of $\hat{N}$ is ambiguous. To get some definitive results, we explore some special cases.

4.3 Policy Implications with Special Cases

Case 1*:* As $\sigma_E \to \infty$, then $\tau_C^* < \mu_C$.

When the blend ratio is unconstrained, such as with greater market penetration of flex-fuel vehicles, we have practically perfect substitution between fossil fuel, *F*, and biofuel, *B*. With almost perfect substitution, σ_E is almost infinite. In each of the expressions, equations (30) to (34), compared to the term with σ_E, the terms with σ_X, σ_B, σ_F, and σ_U are numerically very small and accordingly inconsequential to the solution. Then an increase in τ_C, while keeping τ_N fixed, definitely reduces *F* and *C*, while raising *B* and *N*. With this case, the two externalities are jointly reinforcing, *i.e.*, correction of one market failure exacerbates welfare loss from the other. Then τ_C^* can be written as

$$\tau_C^* = \mu_C - (\mu_N - \tau_N)\frac{N}{C}\frac{D_2}{M - D_2}$$

Along with the assumption that $\mu_N - \tau_N > 0$, since $D_2 > 0$ and $M - D_2 > 0$, τ_C^* is less than μ_C, the marginal environmental damage from GHG. In this case, a larger distortion in the nitrogen market implies a smaller second-best τ_C^*.

Case 2*:* $\sigma_E \to 0$, then $\tau_C^* > \mu_C$

A very small value of σ_E represents the case with very low substitutability between fossil fuel and biofuel, such as when the mix ratio of ethanol reaches the "blend wall", and the consumer faces a very high cost to switch to alternative vehicles. Then the positive effects of σ_E in the corresponding production or emissions are negligible compared to the negative impacts from other elasticities of substitutions. The solutions become:

$$\hat{F} = [-\frac{A_4}{D_1 D_2}\sigma_X - \frac{\theta_{EF}\theta_{FC}A_5}{D_1 D_2}\sigma_U - \frac{\beta_{LX}A_3}{D_2}\sigma_F - \frac{A_1 A_4}{\beta_{LX}D_1 D_2}\sigma_B]\hat{\tau_C} < 0 \quad (38)$$

$$\hat{B} = [-\frac{A_4}{D_1 D_2}\sigma_X - \frac{\theta_{EF}\theta_{FC}A_5}{D_1 D_2}\sigma_U - \frac{\beta_{LX}A_3}{D_2}\sigma_F - \frac{A_1 A_4}{\beta_{LX}D_1 D_2}\sigma_B]\hat{\tau_C} < 0 \quad (39)$$

$$\hat{E} = [-\frac{A_4}{D_1 D_2}\sigma_X - \frac{\theta_{EF}\theta_{FC}A_5}{D_1 D_2}\sigma_U - \frac{\beta_{LX}A_3}{D_2}\sigma_F - \frac{A_1 A_4}{\beta_{LX}D_1 D_2}\sigma_B]\hat{\tau_C} < 0 \quad (40)$$

$$\hat{C} = [-\frac{A_4}{D_1 D_2}\sigma_X - \frac{\theta_{EF}\theta_{FC}A_5}{D_1 D_2}\sigma_U - \frac{\beta_{LX}A_3 + \theta_{FX}D_2}{D_2}\sigma_F - \frac{A_1 A_4}{\beta_{LX}D_1 D_2}\sigma_B]\hat{\tau_C} < 0 \quad (41)$$

$$\hat{N} = [-\frac{A_4}{D_1 D_2}\sigma_X - \frac{\theta_{EF}\theta_{FC}A_5}{D_1 D_2}\sigma_U - \frac{\beta_{LX}A_3}{D_2}\sigma_F - \frac{(A_1 + \theta_{BL}D_2)A_4}{\beta_{LX}D_1 D_2}\sigma_B]\hat{\tau_C} < 0 \quad (42)$$

Equations (38) to (40) indicate that $\hat{F} = \hat{B} = \hat{E}$. With no or very low substitution between *F* and *B*, a reduction in the production of *F* due to an increase in τ_C also reduce *B* at the same rate because of the fixed "blending ratio" (the technology to produce *E*). Thus an increase in τ_C reduces not only *C* but also *N*. Then the two externalities are jointly ameliorating, *i.e.*, correction of one market failure ameliorates welfare loss from the other. In this case, since both $\hat{C} < 0$ and $\hat{N} < 0$, the second term on the RHS in equation (37) is positive, so τ_C^* is higher than the marginal environmental damage of GHG emissions. A larger distortion is in the nitrogen market implies a larger τ_C^*.

Case 3: $0 < \sigma_E < +\infty$ and $\sigma_X = \sigma_B = \sigma_F = \sigma_U = \sigma > 0$

In general, substitution between gasoline and ethanol is neither perfect nor zero, since the consumers have at least some access to flex fuel vehicles that significantly relax the "blend wall". Whether the value of σ_E is large or small corresponds to consumers' willingness to switch to the flex fuel vehicles. With a generalized σ_E, one special case is when all other production and utility functions have the same elasticity value, *i.e.*, $\sigma_X = \sigma_B = \sigma_F = \sigma_U = \sigma$. Then the corresponding solutions of interest are:

$$\hat{F} \;\; = \;\; [-\frac{\theta_{FC}A_2 A_6 + \theta_{XE}\theta_{EF}\theta_{FC}\theta_{BN}D_2}{D_1 D_2}\sigma - \frac{\theta_{FC}A_6(D_2 - A_2)}{D_1 D_2}\sigma_E]\hat{\tau_C}$$

$$
\begin{aligned}
\hat{C} &= [-\frac{\theta_{FC}A_2A_6 + \theta_{XE}\theta_{EF}\theta_{FC}\theta_{BN}D_2 + \theta_{FX}D_1D_2}{D_1D_2}\sigma - \frac{\theta_{FC}A_6(D_2 - A_2)}{D_1D_2}\sigma_E]\hat{\tau_C} \\
\hat{B} &= [-\frac{\theta_{FC}A_2A_6 + \theta_{XE}\theta_{EF}\theta_{FC}\theta_{BN}D_2}{D_1D_2}\sigma + \frac{\theta_{FC}A_6A_2}{D_1D_2}\sigma_E]\hat{\tau_C} \\
\hat{N} &= [-\frac{\theta_{FC}A_2A_6 + \theta_{XE}\theta_{EF}\theta_{FC}D_2}{D_1D_2}\sigma + \frac{\theta_{FC}A_6A_2}{D_1D_2}\sigma_E]\hat{\tau_C} \\
\hat{E} &= [-\frac{\theta_{FC}A_2A_6 + \theta_{XE}\theta_{EF}\theta_{FC}\theta_{BN}D_2}{D_1D_2}\sigma + \frac{\theta_{FC}A_6(A_2 - \theta_{EF}D_2)}{D_1D_2}\sigma_E]\hat{\tau_C}
\end{aligned}
$$

Under this special case, while keeping τ_N fixed, a positive $\hat{\tau_C}$ reduces *F* and *C* (in the first two equations, both terms have the same sign). Its effects on *B* and *N* depend on the relative sizes of σ_E and σ. In the next two equations, the two terms have opposite signs. If $\sigma_E > (1 + \frac{\theta_{XE}\theta_{EF}D_2}{A_6A_2})\sigma$, the positive effects of σ_E on nitrogen leaching with an increased τ_C overcome the negative effects of σ_X, σ_B, σ_F, and σ_U, so an increase in τ_C increases nitrogen leaching. Then $\tau_C^* < \mu_C$, like Case 1. On the other hand, if $\sigma_E < (1 + \frac{\theta_{XE}\theta_{EF}D_2}{A_6A_2})\sigma$, an increase in τ_C reduces nitrogen leaching, so $\tau_C^* > \mu_C$, like Case 2. And, with a knife-edge situation where $\sigma_E = (1 + \frac{\theta_{XE}\theta_{EF}D_2}{A_6A_2})\sigma$, then $\tau_C^* = \mu_C$.

The above three cases cover only a fraction of the possibilities. In general, the knife-edge value of σ_E that defines whether τ_C^* is higher or lower than μ_C is in a much more complex expression and depends on the coefficients and the values of all of the elasticities. In the following section, plausible values are applied to the parameters to explore the likely size of the effects of a small change in τ_C on the economic equilibrium and the optimal value of τ_C^*.

5. Numerical Analysis

5.1 Parameter Impacts

The numerical analysis is based on US data for 2004. At that time, the major biofuel was corn ethanol, so we use data for gasoline and corn ethanol in this analysis. Our model is represented in the share forms, including the expenditure shares in production and consumption and the quantity shares in total demand. These values are calculated from a Social Accounting Matrix based on Global Trade and Analysis Program (GTAP) version 7.0 (Narayanan and Walmsley 2008).

Production data for the numeraire *X*, petrofuel *F*, and the factor costs for gasoline production are directly from GTAP 7.0 (Narayanan and Walmsley 2008). The numeraire X is the combination of all commodities produced apart from gasoline-related products. Factor inputs for ethanol production are from the GTAP_BIO developed by Taheripour *et al.* (2007). The environmental inputs (both GHG emissions and nitrogen leaching) for gasoline and corn ethanol are from a recent life cycle analysis (Khanna *et al.* 2009), which concludes that corn ethanol could reduce GHG emissions by 30% compared to gasoline. The benchmark GHG tax of $24.9/tonne of CO_2 equivalent is based on available carbon trading prices in European and East Asian markets (the World Bank 2005)[5] and then transformed into a 2004 value. No nitrogen externality market or tax exists in the United States. Based on a 2002 case study in the Long Island Sound Watershed done by EPA, the benchmark nitrogen permit cost is set at $1.73 per pound (USEPA 2002).[6] With these major data and related conversion factors, the required parameters can be calculated and are shown in Table 1.

The elasticity values are the most difficult to assign. Many studies have estimated elasticity of substitution values between different commodities or inputs in production processes. However, due to the extensive aggregation of sectors in our model, suitable elasticity values are not readily available in the literature. Instead of making assumptions about those values, the coefficients for these parameters in the model solutions are calculated and, based on those coefficients, the most important and sensitive elasticity values are determined. With the benchmark values documented in Table 1, the coefficients for the elasticities for each variable are listed in Table 3. For a shock in τ_C, each cell shows the coefficient for each elasticity (indicated by each column) in the solution for each variable (indicated by each row). A higher absolute value of the coefficient means a higher impact of this elasticity on that variable.

In Table 3, among all the coefficients for $\hat{C}$ shown in the second row, the coefficient of σ_F (column 5) is -0.9350, which departs from zero the most. This indicates that σ_F has the biggest impact on *C*, because τ_C directly increases the GHG price, and σ_F allows the producer of *F* to

[5] The United State has no federal level GHG tax. Although a gasoline tax could correct the GHG externality, the tax burden on GHG emissions from the US gasoline tax is less than the value used in this numerical example.

[6] This nitrogen tax applies directly to the nitrogen leaching. It could be a burden to farmers, but it might be an effective way to control the hypoxia problem.

shift away from *C*. In the next two rows, the changes in biofuel production, $\hat{B}$, and nitrogen leaching, $\hat{N}$, are affected the most by σ_F. As τ_C rises, P_F rises correspondingly. The price change for fossil fuel causes a demand shift between *F* and *B*, which is governed by σ_F. With the shock of τ_C, all the effects of other eleasticites on $\hat{B}$ and $\hat{N}$ are relatively indirect compared to the effect from σ_F. In terms of the change in total consumption of energy, $\hat{E}$, σ_U has the biggest impact among all elasticity values, although none of them are very big. Among all the elasticity values, σ_B, the elasticity of substitution between factor *L* and *N*, has the lowest impact on all the variables of concern, because τ_C affects the price of *L* only remotely and has no impact on the nitrogen price.

5.2 Policy Impact

In this section, we first discuss plausible elasticity values. Based on those values, we calculate the impacts of a change in τ_C on all of the variables and compute τ_C^* under the preexisting distortion in the nitrogen market.

In the production of biofuel, σ_B defines the elasticity of substitution between factor *L* and nitrogen leaching. No existing literature documents the substitutability between nitrogen runoff and other factors. However, nitrogen runoff is directly related to fertilizer usage in feedstock production. Thompson *et al.* (2006) estimate that the elasticity of substitution between fertilizer and other factors is nearly unity in U.S. corn production. Yasar and Uzunoz (2006) estimate the elasticity of substitution between fertilizer and other inputs is between 0.74 and 0.86 in sugar beet production in Turkey. In our model, ethanol producers can switch feedstocks, so we expect an even higher elasticity of substitution between the two inputs. As indicated in Table 3, the effects of σ_B on the system solutions are fairly low, so the result is not sensitive to this value. Thus, in this example, the Cobb-Douglas functional form is assumed for biofuel production, *i.e.*, $\sigma_B = 1$.

The elasticity of substitution between GHG and *X* in the production of fossil fuel, σ_F, can be fairly low. Most studies generally assume that GHG emissions are proportional to fossil fuel consumption. However, as more fuel-efficient technology/vehicles and carbon abatement technologies are developed, the substitution between X and GHG emissions becomes easier, and we expect a positive value of σ_F in this study. Since σ_F has the biggest impact on GHG emissions, the value of σ_F is very important. A small positive value of σ_F = 0.1 is assumed in the numerical example, and then sensitivity analysis is conducted on this value.

The value for the elasticity of substitution between energy and factor *L* in the production process for *X*, σ_X, is adopted from the value between capital and energy in the capital-energy composite in the GTAP_E model (Burniaux and Truong 2002). For elasticities of substitution between fossil fuel and biofuel, few studies have estimated σ_E due to inadequate data. In a modified GTAP_E model application by Birur *et al.* (2008), the elasticity of substitution between petroleum energy and biofuel for the US is defined as 3.75. With this value, they are able to reproduce biofuel production in accordance with the historical evidence between 2001 and 2006 with reasonable precision. We use their value in our model.

Concerning the elasticity of substitution between *X* and energy for the consumer, σ_U, generally, transportation energy demand is fairly inelastic. Two meta-analyses (Espey 1996; and Goodwin *et al.* 2004) find that the average price-elasticity of demand for gasoline is around -0.25 in the short run. Based on the consumption ratio of energy, the elasticity of substitution between energy and other commodities is less than 0.2. A more recent study estimates the price elasticities of gasoline demand for two periods of time, ranging from -0.034 to -0.077 during 2001 to 2006, versus -0.21 to -0.34 for 1975 to 1980 (Hughes, *et al.* 2007). It concludes that demand for gasoline has become less elastic over time. We use a value of 0.2 for σ_U. Although the coefficients of σ_U for all the variables shown in Table 3 are not so small that they can be ignored, given the fact that the expected value of σ_U is generally fairly low, the effects of σ_U on the variables of concern will be relatively small. Thus even though the value of σ_U is uncertain, sensitivity analysis is not essential. The elasticity values in the numerical example are summarized in Table 1.

With the assigned parameter values, the effects of a small change (1% increase) in τ_C are listed in Table 4. As expected, an increase in τ_C reduces *F*, *C*, and *E*. It increases *B* and *N*. The percentage increase in *N* is about two times the percentage reduction in *C*.

To evaluate the optimal GHG tax, τ_C^*, we need the marginal damage of both GHG emissions and nitrogen leaching. Both are very difficult to estimate. In this paper, only a specific value of μ_N is assumed. The optimal GHG tax is then presented as a function of the marginal damage from GHG emissions, μ_C, showing how much the second-best policy differs from the first-best.

In a survey of environmental damage estimates, Smith (1992) suggests that the economic damages of nitrogen leaching to the water system probably lie within a range of 0.27% to 18.24% of total crop value. Abrahams and Shortle (2004) use 10% of total crop value in their study, which is about the mid-point of the range reported by Smith. In our model, the crop sector is not explicitly modeled. With the GTAP data, the assumption that the environmental damage of nitrogen runoff is about 10% of total crop value implies that μ_N is approximately \$5.70/lb. Accordingly, the optimal GHG tax (\$/ton) is:

$$\begin{aligned}\tau_C^* &= \mu_C + (\mu_N - \tau_N)\frac{N}{C}\frac{\hat{N}}{\hat{C}} \\ &= \mu_C - 12.0\end{aligned}$$

This result indicates that the optimal GHG tax is \$12/ton equivalent of CO_2 less than the marginal damage of GHG emissions, given our benchmark values. If the nitrogen tax is less than the assumed value of \$1.73/lb, or if μ_N is higher than \$5.70/lb, then the optimal GHG tax, τ_C^*, is even further below the marginal damage of GHG emissions.

5.3 Sensitivity Analysis

As shown in Table 3, the values of σ_F and σ_E are fairly important to the changes of environmental emissions, which are our major concerns. In this section, ranges of values for these two parameters are tested to see the sensitivity of our results to these two parameters.

Ethanol and gasoline are highly substitutable commodities, and we expect an elasticity of substitution greater than unity. In the numerical example, the value for σ_E is set to 3.75, as in Birur *et al.* (2008). In the sensitivity analysis, the alternative values tested range from 0 to 5, to represent all possible cases discussed earlier. The upper value represents nearly perfect substitution. For σ_F, most studies generally assume the GHG emissions are proportional to fossil energy consumption. However, with new technology for carbon abatement, the possibility of a positive σ_F cannot be neglected. Since we expect a relatively low substitution level, we test a range of σ_F from 0 to 0.5. Table 5a shows the percentage change of production levels with a 1% increase of τ_C, for different values of σ_F, and Table 5b shows the optimal GHG tax. Table 6 documents the corresponding results with different σ_E.

Although other variables are not sensitive to the different values of σ_F as shown in Table 5a, GHG emissions respond significantly to it. Table 5b indicates that τ_C^* is also very sensitive to the value of σ_F, especially when σ_F is relatively low. Compare the values in the column for $\sigma_F = 0$ and the next column ($\sigma_F = 0.1$), in both Table 5a and Table 5b. If σ_F is 0, a 1% increase of τ_C reduces GHG emissions by only 0.03%, and τ_C^* is \$55/ton lower than the marginal damages of GHG emissions ($\tau_C^* = \mu_C - 54.9$). However, if σ_F is raised only from zero to 0.1, then a 1% increase of τ_C reduces GHG emission by 0.12%, and τ_C^* is \$12/ton lower than marginal damages ($\tau_C^* = \mu_C - 12.0$). Higher values of σ_F mean more reduction of GHG emissions from an increase in τ_C, and τ_C^* is closer to marginal damage. In another words, if the chosen value of σ_F is less than the true value, the effects of τ_C on GHG emissions are underestimated, and the calculated τ_C^* is less than optimal. The major concern regarding σ_F is that if σ_F is low, the optimal GHG tax is very sensitive to its value. A difference of 0.1 in the value of σ_F, from its baseline of 0.1, could result in more than a \$40/ton swing in the optimal GHG tax.

Concerning σ_E, none of the variables are as sensitive to σ_E as were GHG emissions to the size of σ_F. However, almost all outcomes are responsive to σ_E to some extent. Among all the outcome values, *B* and *N* are the two most sensitive to the values of σ_E; their signs change from negative to positive as σ_E increases (see Table 6a). With a small σ_E, *i.e.*, low substitution between *F* and *B*, a positive change in τ_C decreases *B* and *N*, as well as *F* and *C*. Correspondingly, the optimal τ_C is greater than the first best tax. With a greater σ_E, an increase in τ_C increases *B* and *N*. With a greater value of σ_E, a specific change in τ_C yields more nitrogen leaching. If *F* and *B* are close substitutes, optimal τ_C is smaller than the first best tax. A higher σ_E results in a lower τ_C^*, but the effect is limited. If the estimated value of σ_E is lower than the true value, nitrogen leaching is underestimated and the calculated τ_C^* is higher than the optimal value.

6. Caveats for This Analysis

This paper uses a general equilibrium model to address an important policy issue. All standard assumptions for general equilibrium analysis are applied: all markets are assumed to be perfectly competitive, production is assumed to be constant return to scale, all markets clear, factors are mobile, and all agents are well informed. We look at two externalities only, while abstracting from other market imperfections. These assumptions provide reasonable simplifications to enable study of the questions of interest. For this particular application, we employ a linearization procedure to solve our general equilibrium system. This method allows us to solve for analytical closed form solutions without making assumptions for specific functional forms, but it restricts our analysis to small changes around the assumed preexisting equilibrium.

As a stylized general equilibrium analysis, this paper highly aggregates the other commodity sectors while focusing on fossil fuel and biofuel markets. Many issues are left out, such as trade, imperfect markets, increasing returns to scale, inflexible prices, unemployment, other government regulations and other externalities. Any of these strong assumptions could be relaxed, with additional complications that might detract from the simple intuition provided here. For example, energy markets are often regulated rather than perfectly competitive. While any of those additional complications might add effects not studied here, they would not remove the effects we do study here. In particular, a carbon tax may reduce the GHG externality, but induce substitution into biofuels that cause a different externality.

7. Conclusions

This paper develops a general equilibrium model to address policy issues surrounding a special case of multiple externalities. Unlike previous studies, we incorporate two environmental externalities generated by different sources that also produce substitute goods. Two taxes are available to control the two externalities. Since the two externalites are connected through the fact that their sources are substitutes in the market, the two taxes interact. Emissions of both externalities are jointly determined by the two taxes. The direction of the effects of one tax on the other externality is analytically ambiguous.

Using this model, we examine the second-best taxes in the presence of connected externalities. The individually-first-best policy scheme sets the tax on each externality equal to its marginal environmental damage. However, the first-best policy may not be feasible, as seems likely for nitrogen leaching. Given a suboptimal tax for one externality, the optimal tax for the other externality depends on the remaining distortions. We find that the second-best tax on carbon could be lower or higher than the first-best tax, depending on the nature of the distortion in the other externality and the interactions between the final goods. Only in the knife-edge case is the second-best tax rate equal to the first-best rate (marginal environmental damage).

Because of ambiguity in the analytical results, we insert numerical parameter values to explore plausible empirical relationships between fossil fuels and biofuels, where greenhouse gases and nitrogen pollution are the externalities of concern. Our numerical results confirm that a GHG tax increases nitrogen leaching, under the assumption that gasoline and ethanol are close substitutes.

Our analytical solutions suggest that under certain circumstances, the optimal GHG tax could be higher than the marginal damage of GHG emissions. However, if the benchmark nitrogen tax is lower than its marginal environmental damage, and other parameters are set at plausible levels, then the optimal GHG tax is lower, and could be much lower, than the marginal environmental damage of GHG.

In our model, the levels of the two externalities are not affected solely by their individual production processes. The market interaction between the final goods also plays an important role in determining the emission levels. Our numerical example illustrates the relative importance of the technical production parameters relative to the market interaction. If τ_C increases, the technical substitution parameter associated with fossil fuel production, σ_F, has a significant impact on *C* but a small impact on *N*. On the other hand, the technical parameter associated with biofuel production, σ_B, has a very small impact on both externalities, because σ_B governs the substitution between *L* and *N* in production of *B* based on their relative price changes, while the change in τ_C has only a small impact on P_L with no impact on τ_N at all. Thus, with a change in τ_C, the effect of the technical parameter of production *B* to the system is minimal. The elasticity of substitution between F and B in production of energy, σ_E, is the most important parameter in determining the effect on N from an increase in τ_C.

Based on the sensitivity analyses, the parameter related to market interactions is the most important in determining the emission level of the other externality. Since the second-best policies are jointly determined by both emission levels, parameters affecting either or both emission levels matter to the policy design process. The second-best tax rate for one externality is most sensitive to the technical parameter in the production process associated with that externality and to the parameter that determines the substitution in utility between the two final goods.

Appendix A: Solutions with Nitrogen Tax Change ($\hat{\tau_N} > 0$ and $\hat{\tau_C} = 0$)

$$\hat{P_F} = 0$$

$$\hat{P_B} = \frac{\theta_{XL}\theta_{BN}}{D_1}\hat{\tau_N}$$

$$\hat{P_E} = \frac{\theta_{XL}\theta_{BN}\theta_{EB}}{D_1}\hat{\tau_N}$$

$$\hat{P_L} = -\frac{\theta_{XE}\theta_{BN}\theta_{EB}}{D_1}\hat{\tau_N}$$

$$\hat{F} = [-\frac{\theta_{BN}\theta_{EB}A_4}{\theta_{EF}\theta_{FC}D_1D_2}\sigma_X - \frac{\theta_{BN}\theta_{EB}A_5}{D_1D_2}\sigma_U + \frac{\theta_{XL}\theta_{BN}(D_2 - A_2)}{D_1D_2}\sigma_E - \frac{A_1A_7}{D_1D_2}\sigma_B]\hat{\tau_N}$$

$$\hat{C} = [-\frac{\theta_{BN}\theta_{EB}A_4}{\theta_{EF}\theta_{FC}D_1D_2}\sigma_X - \frac{\theta_{BN}\theta_{EB}A_5}{D_1D_2}\sigma_U + \frac{\theta_{XL}\theta_{BN}(D_2 - A_2)}{D_1D_2}\sigma_E - \frac{A_1A_7}{D_1D_2}\sigma_B]\hat{\tau_N}$$

$$\hat{B} = [-\frac{\theta_{BN}\theta_{EB}A_4}{\theta_{EF}\theta_{FC}D_1D_2}\sigma_X - \frac{\theta_{BN}\theta_{EB}A_5}{D_1D_2}\sigma_U - \frac{\theta_{XL}\theta_{BN}A_2}{D_1D_2}\sigma_E - \frac{A_1A_7}{D_1D_2}\sigma_B]\hat{\tau_N}$$

$$\hat{N} = [-\frac{\theta_{BN}\theta_{EB}A_4}{\theta_{EF}\theta_{FC}D_1D_2}\sigma_X - \frac{\theta_{BN}\theta_{EB}A_5}{D_1D_2}\sigma_U - \frac{\theta_{XL}\theta_{BN}A_2}{D_1D_2}\sigma_E - \frac{(A_1 + \theta_{BL}D_2)A_7}{D_1D_2}\sigma_B]\hat{\tau_N}$$

$$\hat{E} = [-\frac{\theta_{BN}\theta_{EB}A_4}{\theta_{EF}\theta_{FC}D_1D_2}\sigma_X - \frac{\theta_{BN}\theta_{EB}A_5}{D_1D_2}\sigma_U - \frac{\theta_{XL}\theta_{BN}(A_2 - \theta_{EF}D_2)}{D_1D_2}\sigma_E - \frac{A_1A_7}{D_1D_2}\sigma_B]\hat{\tau_N}$$

where $A_7 = 1 - \theta_{XE}\theta_{EF}$. The definitions of A_1 to A_5, D_1 and D_2 are the same as in the text.

Appendix B: Determining the Signs for Parameters

1): A_1

$$\begin{aligned} A_1 &= \beta_{LB}\theta_{BN} - \gamma_{XU}\beta_{LX}\theta_{BL}\theta_{IN} \\ &= \frac{L_B}{\bar{L}}\frac{\tau_N N}{P_B B} - \frac{X_U}{X}\frac{L_X}{\bar{L}}\frac{P_L L_B}{P_B B}\frac{\tau_N N}{I} = \frac{L_B \tau_N N}{\bar{L} P_B B}(1 - \frac{X_U L_X P_L}{XI}) > 0 \end{aligned}$$

Since $X_U < X$, $P_L L_X < P_L \bar{L} < I$, then $\frac{X_U L_X P_L}{XI} < 1$. Thus $A_1 > 0$

2): A_3

$$\begin{aligned} A_3 &= \theta_{FC}\gamma_{XT} - \theta_{FX}\theta_{IC}\gamma_{XU} \\ &= \frac{\tau_C C}{P_F F}\frac{X_T}{X} - \frac{P_X X_F}{P_F F}\frac{\tau_C C}{I}\frac{X_U}{X} = \frac{\tau_C C X_T}{P_F F X}(1 - \frac{X_U}{I}) > 0 \end{aligned}$$

Since $X_U = P_X X_U < I$, then $A_3 > 0$

3): $D_2 - A_2$

$$\begin{aligned} D_2 - A_2 &= 1 - \gamma_{XU}\beta_{LX}\theta_{IL} - \gamma_{XF}\beta_{LX} - \gamma_{XU}\beta_{LX}\theta_{IC} \\ &= 1 - \gamma_{XF}\beta_{LX} - \gamma_{XU}\beta_{LX} + \gamma_{XU}\beta_{LX} - \gamma_{XU}\beta_{LX}\theta_{IL} - \gamma_{XU}\beta_{LX}\theta_{IC} \\ &= 1 - \beta_{LX} + \gamma_{XU}\beta_{LX}\theta_{IN} \\ &= \beta_{LB} + \gamma_{XU}\beta_{LX}\theta_{IN} > 0 \end{aligned}$$

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Table 1. Major parameter definitions and baseline values

Parameter	Definition	Baseline value
θ_{XL}	Expenditure share of *L* in *X* production, = $P_L L_X / P_X X$	98%
θ_{XE}	Expenditure share of fuel in *X* production, = $P_E E_X / P_X X$	2%
θ_{EF}	Expenditure share of gasoline in total fuel consumption, =$P_F F / P_E E$	94%
θ_{EB}	Expenditure share of ethanol in total fuel consumption, =$P_B B / P_E E$	6%
θ_{FX}	Expenditure share of *X* in gasoline production, =$P_X X_F / P_F F$	93%
θ_{FE}	Expenditure share of emissions cost in gasoline production, =$\tau_C C / P_F F$	7%
θ_{BL}	Expenditure share of *L* in ethanol production, =$P_L L_B / P_B B$	89%
θ_{BE}	Expenditure share of emissions cost in ethanol production, =$\tau_N N / P_B B$	11%
λ_{XL}	Share of *L* usage in *X* production in total endowment, = $L_X / \bar{L}$	99%
λ_{BL}	Share of *L* usage in *B* production in total endowment, = $L_B / \bar{L}$	1%
θ_{IX}	Expenditure share of *X* in consumer's consumption = $P_X X_U / I$	98%
θ_{IE}	Expenditure share of *E* in consumer's consumption = $P_E E_U / I$	2%
θ_{IL}	Income share of *L* in total income = $\tau_C C / I$	99.7%
θ_{IC}	Income share of *C* in total income = $\tau_N N / I$	0.2%
τ_C	GHG tax rate ($/ton)	24.9
τ_N	Nitrogen tax rate ($/lb)	1.73
σ_X	Elasticity of substitution between inputs in *X* production	0.1
σ_B	Elasticity of substitution between inputs in *B* production	1
σ_F	Elasticity of substitution between inputs in *F* production	0.1
σ_E	Elasticity of substitution between fossil fuel and biofuel	3.75
σ_U	Elasticity of substitution between *X* and *E* for consumers	0.2

Table 2 Signs of Elasticity Coefficients for Different Variables Given Positive $\hat{\tau_C}$

<table>
<tr><th>Variables</th><th>σ_X</th><th>σ_U</th><th>σ_E</th><th>σ_F</th><th>σ_B</th></tr>
<tr><td>$\hat{F}$</td><td>−</td><td>−</td><td>−</td><td>−</td><td>−</td></tr>
<tr><td>$\hat{C}$</td><td>−</td><td>−</td><td>−</td><td>−</td><td>−</td></tr>
<tr><td>$\hat{B}$</td><td>−</td><td>−</td><td>+</td><td>−</td><td>−</td></tr>
<tr><td>$\hat{N}$</td><td>−</td><td>−</td><td>+</td><td>−</td><td>−</td></tr>
<tr><td>$\hat{E}$</td><td>−</td><td>−</td><td>Ambiguous</td><td>−</td><td>−</td></tr>
<tr><td>$\hat{P}_F$</td><td colspan="5">+</td></tr>
<tr><td>$\hat{P}_B$</td><td colspan="5">−</td></tr>
<tr><td>$\hat{P}_E$</td><td colspan="5">+</td></tr>
</table>

"+" indicates that σ has a positive effect on the variable from a positive change in the tax .
"-" indicates that σ has a negative effect on the variable from a positive change in the tax
"Ambiguous" indicates that we cannot identify the effect of σ on the variable from a positive change in the tax .

Table 3 Coefficients for Elasticities in Selected Variable Solutions (from)

Variables	σ_X	σ_U	σ_E	σ_F	σ_B
$\hat{F}$	-0.0249	-0.0366	-0.0044	-0.0012	0.0000
$\hat{C}$	-0.0249	-0.0366	-0.0044	-0.9350	0.0000
$\hat{B}$	-0.0249	-0.0366	0.0626	-0.0012	0.0000
$\hat{N}$	-0.0249	-0.0366	0.0626	-0.0012	-0.0007
$\hat{E}$	-0.0249	-0.0366	-0.0006	-0.0012	0.0000

Table 4 Effects of GHG Tax Change (for $\hat{\tau_C} = 1\%$)

Variables	Percentage Change
$\hat{F}$	-0.0264
$\hat{C}$	-0.1198
$\hat{B}$	0.2247
$\hat{N}$	0.2240
$\hat{E}$	-0.0101
$\hat{X}$	-0.0005
$\hat{P}_F$	0.0662
$\hat{P}_B$	-0.0007
$\hat{P}_L$	-0.0008

Table 5a. Percentage Change for Each Variable, when $\hat{\tau_C} = 1\%$, for Different σ_F

Variables	σ_F= 0	σ_F= 0.1	σ_F= 0.3	σ_F= 0.5
$\hat{F}$	-0.0263	-0.0264	-0.0266	-0.0269
$\hat{C}$	-0.0263	-0.1198	-0.3067	-0.4938
$\hat{B}$	0.2248	0.2247	0.2245	0.2243
$\hat{N}$	0.2241	0.2240	0.2237	0.2235
$\hat{E}$	-0.0100	-0.0101	-0.0103	-0.0106
$\hat{X}$	-0.0005	-0.0005	-0.0005	-0.0005

Table 5b. Optimal GHG Tax for Different σ_F

	σ_F= 0	σ_F= 0.1	σ_F= 0.3	σ_F= 0.5
τ_C^*	$\mu_C - 54.9$	$\mu_C - 12.0$	$\mu_C - 4.8$	$\mu_C - 3.0$

Table 6a. Percentage Change of for Each Variable, when $\hat{\tau_C} = 1\%$, for Different σ_E

Variables	σ_E= 0	σ_E= 0.1	σ_E= 0.2	σ_E= 1	σ_E= 2.5	σ_E= 3.75	σ_E= 5
$\hat{F}$	-0.0099	-0.0104	-0.0108	-0.0143	-0.0209	-0.0264	-0.0319
$\hat{C}$	-0.1033	-0.1037	-0.1042	-0.1077	-0.1143	-0.1198	-0.1253
$\hat{B}$	-0.0099	-0.0037	0.0026	0.0526	0.1465	0.2247	0.3029
$\hat{N}$	-0.0107	-0.0044	0.0018	0.0519	0.1458	0.2240	0.3022
$\hat{E}$	-0.0099	-0.0099	-0.0099	-0.0099	-0.0100	-0.0101	-0.0102
$\hat{X}$	-0.0001	-0.0001	-0.0001	-0.0002	-0.0004	-0.0005	-0.0007

Table 6b. Optimal GHG Tax for Different σ_E

	σ_E= 0	σ_E= 0.1	σ_E= 0.2	σ_E= 1	σ_E= 2.5	σ_E= 3.75	σ_E= 5
τ_C^*	$\mu_C + 0.7$	$\mu_C + 0.3$	$\mu_C - 0.1$	$\mu_C - 3.1$	$\mu_C - 8.3$	$\mu_C - 12.0$	$\mu_C - 15.6$