VOICE AND BARGAINING POWER

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Abstract

We propose a formal concept of the power of voice in the context of a simple model where individuals form groups and trade in competitive markets. Individuals use outside options in two different ways. Actual outside options reflect the possibility to exit or to join other existing groups. Hypothetical outside options refer to hypothetical groups that are ultimately not formed. Articulation of hypothetical outside options in the bargaining process determines the relative bargaining power of the members of a group, which constitutes an instance of the power of voice. The adopted equilibrium concept endogenizes the outside options as well as the power of voice. In our illustrative example, there exists an equilibrium that uniquely determines the power of voice and the allocation of commodities.

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1 Introduction

Power has always been a prominent theme in the social sciences. The meaning of power can include everything from the ability to keep oneself alive to the ability of government to arrest people. A central conception of power is an individual's capacity to influence decisions taken by a group he or she belongs to.

The influential work of Hirschman (1970) has provided a convenient way of thinking about the power pertaining to collective decisions of social organizations. He distinguishes between power derived from the exit option and power based on voice. Although these concepts have been applied widely, the role of voice has proven extremely difficult to conceptualize. In particular, it is difficult to explain why voice — which may be merely cheap talk — would have any impact on material collective decisions.

In this paper we propose a concept of voice power.[1] The essential idea is as follows. In a society individuals use outside options in two different ways. First, the possibility to exit or to join other existing groups determines the actual outside options. Second, individuals also reason with reference to hypothetical groups, whose formation would require that more than one individual break away from their existing groups and form a new group. The possibility to form hypothetical, new groups is articulated in the bargaining process. The best possible hypothetical scenario for each person determines the relative bargaining power in existing groups. This impact of articulating one's conceivable opportunities in hypothetical groups is called the "power of voice". We show in a simple model with four individuals that there exists an equilibrium that uniquely determines the allocation of commodities and the power of voice.

Our paper is closely related to the theory of multilateral bargaining problems, when there are potential gains from forming coalitions but there is conflict over which coalition to form and how to distribute gains. The idea of antagonistic outside options

[1] Hirschman considers voice as a mechanism of recuperation and a means of influence. Here we focus on the second function.

appears already in Rochford (1984) who focuses on selections from the core. Bennett (1988, 1997) has pursued the idea further and has developed an intriguing approach to multilateral bargaining problems.[2] She considers an agreement within a coalition as a solution for the intra-coalitional bargaining process, if the agreement is consistent with the bargaining processes in all other coalitions. The outside option of an individual is the utility the individual would obtain from the agreement in his best alternative coalition.[3]

Our model shares the one important feature with Bennett (1997) that certain (hypothetical) outside options may not be disagreement outcomes because they are not jointly compatible. Our approach differs in other important aspects from the theory of Bennett (1988, 1997). In contrast to her, we consider outside options in a dual role for the bargaining process in a particular coalition. Coalitions belonging to the outcome and, thus, coalitions that will actually form determine outside options in the narrow sense. Hypothetical coalitions, that is those that ultimately are not formed, play a different role. They are used in the speeches of members in a particular coalition in order to articulate potential alternatives. Then the best hypothetical outside alternatives (or maximal complaints) determine the relative bargaining power inside the coalition.

Moreover, the research on group or coalition formation has highlighted that it is ultimately unclear how deviations from a proposed group structure should be modelled. Standard solutions such as Nash stability, individual or coalitional stability ignore any possible further deviations and thus may be myopic and may lack credibility.[4] Deviations can be followed by further deviations and thus it is plausible to allow a deviating coalition to reason about the ultimate result of its deviation. Such a reasoning

[2]Other analyses of multilateral bargaining problems have been proposed by Kalai and Samet (1985), Chatterjee et al. (1993) and Bennett and van Damme (1991). Bell (1991) provides a subtle discussion of the role of power and outside options in rural societies.

[3]The underlying non-cooperative model is an adaptation of the proposal-making model of Selten (1981).

[4]Moreover, in hedonic coalition formation games, the core may be empty. Bogomolnaia and Jackson (2002), Banerjeee et al. (2001), Alcalde and Revilla (2001) and Pápai (2000) provide conditions for the non-emptiness of the core.

of credibility and foresight has been initiated by von Neumann and Morgenstern's (1944) stable set and Harsanyi's (1974) indirect dominance. More recent formalizations of farsightedness and solution concepts include Greenberg (1990), Chwe (1994), Xue (1998), Diamantoudi and Xue (2003), and Barberà and Gerber (2003). They show that the answers depend on the behavioral characteristics of the individuals and that there are various plausible ways to formulate how deviations might induce further deviations. Given these difficulties we assume in the present paper that decision-makers use hypothetical outside options as arguments when they bargain over consumption bundles. The way in which such uncertain outside options impact the bargaining outcome is axiomatized through the "voice impact function" introduced in section 4.

At a more abstract level, our paper suggests a way to formalize how discussion among individuals can bring about a consensus. The role of communication in reaching a consensus in democratic societies has been stressed a great deal in political science (see e.g. Elster 1998) and philosophy (see e.g. Habermas 1987). In our context, discussion enables each side to convince the other of the feasibility of potential best alternatives. Each individual assesses the feasibility of hypothetical outside options of other group members. We assume that this deliberation and discussion transforms the best hypothetical outside alternative of one individual into concessions by the other individual and thus into relative bargaining power.

The paper is organized as follows. In the next section we introduce a simple model that serves as an illustrative example. In section 4 we derive the equilibria with free group formation. Voice is introduced and the equilibria with voice are derived in section 5. In section 6 we illustrate our findings with a numerical example. Section 7 offers final remarks.

2 Basic Feedback Mechanism

Here we outline the basic feedback mechanism that incorporates the impact of voice. We envision an economy where individuals can form two-person groups that trade in competitive markets. Individuals are denoted by i or j. It is assumed that at the prevailing market conditions, person j obtains utility U_j^0 from acting and living as a single. The utility levels U_j^0 are called actual outside options. If individuals 1 and 2 form group h, they enjoy respective bargaining power β_h and $1-\beta_h$, say, within the group. Maximization of the Nash product

$$N_h = (U_1 - U_1^0)^{\beta_h} \cdot (U_2 - U_2^0)^{1-\beta_h}$$

with respect to feasible utilities $(U_1, U_2) \geq (U_1^0, U_2^0)$ yields a solution (U_1^*, U_2^*). We shall continue to focus on the group $h = \{1, 2\}$, while an analogous treatment applies to any two-person group that actually forms.

The values U_j^0, $j \neq 1, 2$, also serve as reservation utility levels when individuals 1 or 2 form a hypothetical group with other individuals. These reservation utilities determine hypothetical outside option values $\widehat{U}_1$ and $\widehat{U}_2$, embodying the best conceivable outcomes that person 1 and 2, respectively, could expect when forming groups with third parties. Assuming $\widehat{U}_i > U_i^*$, the difference $\widehat{U}_i - U_i^*$ constitutes the maximal complaint $i = 1, 2$ can articulate vis-à-vis her partner. The power of voice manifests itself via the impact of $\widehat{U}_1 - U_1^*$ and $\widehat{U}_2 - U_2^*$ on the bargaining weight β_h. Namely, we postulate a **voice impact function** $f : \mathbb{R}_+ \to \mathbb{R}_+$ so that[5]

$$\beta_h = f\left(\frac{\widehat{U}_1 - U_1^*}{\widehat{U}_2 - U_2^*}\right), \tag{1}$$

which is tantamount to equation (3) below. Thus given actual outside option values $U_1^0, U_2^0, \ldots$ and actual two-person groups $g, h, \ldots$, one obtains a composite mapping B from the tuples of bargaining weights $(\beta_g, \beta_h, \ldots)$ to the tuples of bargaining weights. B has the following schematic form:

[5]The definition can be extended to the cases $\widehat{U}_i < U_i^*$ by setting $\beta_h = 1$ if $\widehat{U}_1 > U_1^*$ and $\widehat{U}_2 < U_2^*$ and $\beta_h = \frac{1}{2}$ if $\widehat{U}_i < U_i^*, i = 1, 2$, etc.

$$(\beta_g, \beta_h, \ldots) \nearrow \left.\begin{array}{c} (U_1^*, U_2^*, U_3^*, \quad \ldots) \\ \downarrow \\ (\widehat{U}_1, \widehat{U}_2, \widehat{U}_3, \quad \ldots) \end{array}\right\} \longrightarrow (\beta_g, \beta_h, \ldots)$$

A fixed point of B endogenizes the power of voice.

We envision such a scheme to apply to many socio-economic situations. In the illustrative example below, the maximizer of N_h will assume the parametric form $U_1^* = U_1(\alpha_h), U_2^* = U_2(\alpha_h)$ where α_h is the weight of individual 1 in the utalitarian welfare function of household h and the fixed point problem will be formulated in terms of the parameter-tuples $(\alpha_g, \alpha_h, \ldots)$; see equation (5). Moreover, the economic environment is going to depend on the price system p which in turn is determined by market clearing conditions.

3 The Model: An Illustrative Example

In this section, we describe the primitive data of our example. The model is an exchange economy where individuals can form groups in which they benefit from group externalities. Hence, we need to define consumers, group structures, commodities, endowments, allocations, and preferences.

3.1 Consumer Characteristics and Allocations

We consider a population of four consumers, represented by the set $I = \{1, \ldots, 4\}$. A generic consumer is again denoted by i or j. A population with four individuals proves to be just enough to define and illustrate the concept of power of voice.

The population I is partitioned into groups, i.e. there exists a partition P of I into non-empty subsets. We call any such partition P a group structure in I. A generic group is denoted h, g or k. We treat the group structure as an object of endogenous choice. Groups are endogenously formed so that some group structure P is ultimately realized.

We assume that there exist two commodities for private consumption. Each individual $i \in I$ has a consumption set $X_i = \mathbb{R}^2_+$ so that the **commodity allocation space** is $\mathcal{X} \equiv \prod_{j\in I} X_j$. The consumption of individual i is denoted by $x_i = (x_i^1, x_i^2)$ or $y_i = (y_i^1, y_i^2)$. The consumption allocation is denoted by $\mathbf{x}$. $\mathcal{X}_h = \prod_{i\in h} X_i$ is the consumption set for group h. $\mathcal{X}_h$ has generic elements $\mathbf{x_h} = (x_i)_{i\in h}$.

Preferences are represented by $U_i(\mathbf{x_h}; h) = U_i(x_i) + U_i^G(h) = U_i(x_i^1, x_i^2) + U_i^G(h)$ where x_i^k denotes the quantity of good k $(k = 1, 2)$ consumed by individual i. $U_i^G(h)$ captures the pure group externality contributing to the utility of individual i. Specifically, we assume $\gamma \in (0, 1)$, $b > 0$ and

$$U_i(\mathbf{x_h}; h) = \begin{cases} \gamma \ln x_i^1 + (1-\gamma) \ln x_i^2, & \text{in case } h = \{i\}; \\ \gamma \ln x_i^1 + (1-\gamma) \ln x_i^2 + v_{ij}, & \text{in case } h = \{i, j\} \text{ with } v_{ij} \geq 0,\ i \neq j; \\ \gamma \ln x_i^1 + (1-\gamma) \ln x_i^2 - b, & \text{in case } \#h = 3; \\ \gamma \ln x_i^1 + (1-\gamma) \ln x_i^2 - b, & \text{in case } \#h = 4. \end{cases}$$

Note that we adopt the assumption in Gersbach and Haller (2003) that an individual does not care about the features of an allocation beyond the boundaries of his own group. If a particular group structure is given, he is indifferent about the affiliation and consumption of individuals not belonging to his own group. Note that forming a three-person or four-person group exerts negative group externalities of $-G$ on everybody. Hence, such groups will never be formed in equilibrium.

We further assume individual endowments $w_i = (w_i^1, w_i^2)$. For a potential group h, its endowment is the commodity bundle $w_h \in \mathbb{R}^2$ given by the sum of the endowments of all participating individuals: $w_h = \sum_{i\in h} w_i$. The social or aggregate endowment is $w_S = \sum_{i\in I} w_i$. An allocation is a pair $(\mathbf{x}; P)$ specifying the consumption bundle and group membership of each consumer.

3.2 The Equilibrium Notion

The notion of voice power will act as a selection device for competitive exchange among groups. Hence, we first need to define an equilibrium notion in which the power of voice can be embedded. Among the several conceivable ways to formulate an equilibrium state of a model with variable group structure, we follow Gersbach and Haller (2003) and employ the concept of a competitive equilibrium with free group formation.

We briefly review the definition for a competitive equilibrium with free group formation. We consider a group $h \in P$ and a price system $p \in \mathbb{R}^2$. For $\mathbf{x_h} = (x_i)_{i \in h} \in \mathcal{X}_h$,

$$p * \mathbf{x_h} \equiv p \cdot \left(\sum_{i \in h} x_i \right)$$

denotes the expenditure of group h on group consumption plan $\mathbf{x_h}$ at the price system p. As p and $\mathbf{x_h}$ are of different dimension for multi-member groups, we use the $*$-product in lieu of the familiar inner product. Then group h's **budget set** is defined as

$$B_h(p) = \{\mathbf{x_h} \in \mathcal{X}_h \ : \ p * \mathbf{x_h} \leq p \cdot w_h\}.$$

We next define the **efficient budget set** $EB_h(p)$ as the set of $\mathbf{x_h} \in B_h(p)$ with the property that there is no $\mathbf{y_h} \in B_h(p)$ such that

(i) $U_i(\mathbf{y_h}; h) \geq U_i(\mathbf{x_h}; h)$ for all $i \in h$;

(ii) $U_i(\mathbf{y_h}; h) > U_i(\mathbf{x_h}; h)$ for some $i \in h$.

Further define a **state** of the economy as a triple $(p, \mathbf{x}; P)$ such that $p \in \mathbb{R}^2$ is a price system and $(\mathbf{x}; P) \in \mathcal{X} \times P$ is an allocation, i.e. $\mathbf{x} = (x_i)_{i \in I}$ is an allocation of commodities and P is an allocation of consumers (a group structure, a partition of the population into groups). A state $(p, \mathbf{x}; P)$ is a **competitive equilibrium with free group formation (CEFG)** if it satisfies the following conditions:

1. $\mathbf{x_h} \in EB_h(p)$ for all $h \in P$.

2. $\sum_i x_i = w_S$.

3. There is no $h \in P$, $i \in h$ and $y_i \in B_{\{i\}}(p)$ such that

   $U_i(y_i; \{i\}) > U_i(\mathbf{x_h}; h)$.

4. There are no h and $g \in P$, $i \in h$ and $\mathbf{y_{g\cup\{i\}}} \in B_{g\cup\{i\}}(p)$ such that

   $U_j(\mathbf{y_{g\cup\{i\}}}; g \cup \{i\}) > U_j(\mathbf{x_g}; g)$ for all $j \in g$;

   $U_i(\mathbf{y_{g\cup\{i\}}}; g \cup \{i\}) > U_i(\mathbf{x_h}; h)$.

Condition 1 reflects collective rationality. Efficient choice by the group refers to the individual consumption and welfare of its members, not merely to the aggregate consumption bundle of the group. Condition 2 requires market clearing. Condition 3 stipulates that no individual wants to leave a group and participate as a one-member group in the market at the going equilibrium prices. Condition 4 requires that no individual can leave a group and can propose a feasible consumption allocation to the members of a new group, created by the individual and another already existing group, which makes everybody in the new group better off at the going equilibrium prices.

4 Equilibria with Free Group Formation

To prepare a formal treatment of the power of voice, we first characterize equilibria with free group formation (CEFG). We observe that we can neglect group structures where the group size is larger than 2, since forming a four-person or a three-person group exerts negative group externalities on everybody. Accordingly, only group structures with two two-person groups prevail in CEFG.

Commodity prices are normalized so that $p_1 = 1$. We can represent the efficient decisions of a two-person group $h = \{i, j\}, i < j$, by assuming that the group maximizes a utilitarian social welfare function

$$W_h = \alpha_h U_i(x_i) + (1-\alpha_h) U_j(x_j)$$

subject to the budget constraint. The number α_h $(0 \leq \alpha_h \leq 1)$ is the utalitarian weight of individual i in household h. In this section we treat α_h as parametrically given. In section 5 the weight α_h will be endogenized. Given any p_2, identical homothetic preferences with respect to consumption imply that group demand as well as individual consumption bundles will be linear in income. Hence, we immediately obtain

Lemma 1 *CEFG exist and have the following properties:*

(i) Two two-person groups are formed.

(ii) The equilibrium price p_2^* is given by $p_2^* = (1-\gamma) \cdot w_S^1 / [\gamma \cdot w_S^2]$, with associated nominal social wealth $y_S^* = w_S^1 + p_2^* w_S^2$ and nominal income $y_k^* = w_k^1 + p_2^* w_k^2$ for any group k.

(iii) The equilibrium allocation for a group structure P, say $P = \{h, g\}$ with $h = \{1, 2\}$ and $g = \{3, 4\}$, is characterized by two numbers α_h and α_g $(0 < \alpha_h < 1$, $0 < \alpha_g < 1)$ and given by

$$\begin{array}{rclrcl} x_1^* &=& \alpha_h (y_h^*/y_S^*) w_S, & x_2^* &=& (1-\alpha_h)(y_h^*/y_S^*) w_S; \\ x_3^* &=& \alpha_g (y_g^*/y_S^*) w_S, & x_4^* &=& (1-\alpha_g)(y_g^*/y_S^*) w_S. \end{array}$$

To establish the boundaries for the numbers α_h and α_g we observe that we can neglect the joining option. Forming three-person groups does not create positive group externalities for the entrant and destroys existing benefits of group formation. Hence, exit dominates joining in all conceivable deviations from the CEFG candidate.

The exit option for individual i yields utility

$$U_i(x_i^0(p_2^*)) = \gamma \ \ln \Big(\gamma (w_i^1 + p_2^* w_i^2) \Big) + (1-\gamma) \ \ln \left((1-\gamma) \left(\frac{w_i^1 + p_2^* w_i^2}{p_2^*} \right) \right)$$

which establishes

Lemma 2

For a typical group structure that can qualify for a CEFG, say $P = \{h, g\}$ with $h = \{1, 2\}$ and $g = \{3, 4\}$, there exist $\underline{\alpha}_h < \overline{\alpha}_h$ and $\underline{\alpha}_g < \overline{\alpha}_g$ such that a CEFG with the properties described in lemma 1 exists if and only if

$$\underline{\alpha}_h \leq \alpha_h \leq \overline{\alpha}_h \text{ and } \underline{\alpha}_g \leq \alpha_g \leq \overline{\alpha}_g.$$

5 Voice Power

5.1 The Concept

The remaining question is how α_h and α_g are determined. For that purpose we introduce voice in the following sense: Every group member expresses the utility that he could achieve in a hypothetical group, i.e. in a group that does not currently exist and cannot be formed by exit or by joining another group. The potential gains relative to current utility that group members can identify in their speeches will then determine relative bargaining power through the power of voice.

To formulate the notion of voice power we start with the bargaining problem in a particular group. We assume that a group, say $h = \{1, 2\}$, maximizes the Nash product

$$N_h = \left\{U_1(\mathbf{x_h}; h) - U_1(x_1^0(p_2^*))\right\}^{\beta_h} \cdot \left\{U_2(\mathbf{x_h}; h) - U_2(x_2^0(p_2^*))\right\}^{1-\beta_h}.$$

To determine the values of β_h and α_h we proceed in two steps. In the first step, we determine the weight α_h that maximizes the Nash product for a given value of β_h. In the second step we determine the value of β_h for a given α_h through the power of voice. An equilibrium will be a pair (α_h^*, β_h^*) that solves the group bargaining problem and is consistent with voice power.

We start with the first step. To simplify the notation we use the following shortcuts. If α_h is the weight of the first member in the actual group h in the utilitarian welfare function, we can express the various utilities as follows:

$$\begin{array}{rcccl} U_1(\alpha_h) & := & U_1(\mathbf{x_h};h) & = & \gamma\ln\{\alpha_h\gamma y_h^*\}+(1-\gamma)\ln\{\alpha_h(1-\gamma)y_h^*\}+v_{12} \\ U_2(\alpha_h) & := & U_2(\mathbf{x_h};h) & = & \gamma\ln\{(1-\alpha_h)\gamma y_h^*\}+(1-\gamma)\ln\{(1-\alpha_h)(1-\gamma)y_h^*\}+v_{21} \\ U_1^0 & := & U_1(x_1^0(p_2^*)) & = & \gamma\ln\{\gamma y_1^*\}+(1-\gamma)\ln\{(1-\gamma)y_1^*\} \\ U_2^0 & := & U_2(x_2^0(p_2^*)) & = & \gamma\ln\{\gamma y_2^*\}+(1-\gamma)\ln\{(1-\gamma)y_2^*\} \end{array}$$

For any given β_h, the bargaining problem is well defined and can be solved for the utilitarian weights. Taking $\ln N_h$ and maximizing with respect to α_h yields the first-order condition:

$$\begin{aligned} &\beta_h\,\frac{1}{U_1(\alpha_h)-U_1^0}\,\left\{\gamma\frac{1}{\alpha_h}+(1-\gamma)\frac{1}{\alpha_h}\right\} \\ &\quad -(1-\beta_h)\,\frac{1}{U_2(\alpha_h)-U_2^0}\,\left\{\gamma\frac{1}{1-\alpha_h}+(1-\gamma)\frac{1}{1-\alpha_h}\right\}=0 \end{aligned}$$

or

$$\beta_h\,\frac{1}{(U_1(\alpha_h)-U_1^0)\alpha_h}-(1-\beta_h)\,\frac{1}{(U_2(\alpha_h)-U_2^0)(1-\alpha_h)}=0. \tag{2}$$

In the second step we determine β_h as a function of the utilitarian weight α_h through the power of voice. First, we need to be precise about the thinking of members in actual groups about allocations in hypothetical groups. We assume that individuals articulate situations in hypothetical groups to which they might belong and in which other members do not want to leave. We assume that $v_{i3}>v_{i4}, i=1,2$ and $v_{3i}>v_{4i}, i=1,2$. An individual $i=1,2$ can imagine being in a two-person group $\{i,3\}$ or $\{i,4\}$. We concentrate on the group $k=\{i,3\}$ since forming a group with individual 3 is the more attractive hypothetical group. If individual i imagines a group allocation such that individual 3 obtains his utility as a single, the maximal hypothetical utility for individual i, denoted by $\widehat{U}_i$, is determined by the system of equations

$$\begin{aligned} \widehat{U}_i &= \gamma\,\ln\left\{\gamma\overline{\alpha}_k\left(w_i^1+w_3^1+p_2^*(w_i^2+w_3^2)\right)\right\} \\ &+ (1-\gamma)\,\ln\left\{(1-\gamma)\overline{\alpha}_k\left(\frac{w_i^1+w_3^1+p_2^*(w_i^2+w_3^2)}{p_2^*}\right)\right\}+v_{i3}; \end{aligned}$$

$$
\begin{aligned}
U_3(x_3^0(p_2^*)) &= \gamma \ln\left\{\gamma(1-\overline{\alpha}_k)\left(w_i^1+w_3^1+p_2^*(w_i^2+w_3^2)\right)\right\} \\
&+ (1-\gamma)\ln\left\{(1-\gamma)(1-\overline{\alpha}_k)\left(\frac{w_i^1+w_3^1+p_2^*(w_i^2+w_3^2)}{p_2^*}\right)\right\}+v_{3i}.
\end{aligned}
$$

Note that $\overline{\alpha}_k$ is the highest possible weight individual i can have in household k without forcing the exit of individual 3. It is obvious that $\overline{\alpha}_k$ and $\widehat{U}_i$ are uniquely determined. Running through the same exercise for group $g = \{3, 4\}$, when individuals imagine forming groups with the first individual, yields imagined utilities $\widehat{U}_i$, $i = 3, 4$.

We assume that the utilities $\widehat{U}_i$ are used in the speeches of existing groups to express their members' aspirations and we further assume that these aspirations translate into relative bargaining power in existing groups. Hence, the relative bargaining power must be consistent with the potential utility gains that individuals can articulate for hypothetical groups. For that purpose we introduce the **voice impact function** $f : \mathbb{R}_+ \to \mathbb{R}_+$ which can be applied to groups h and g. We define the voice impact function by using group h. The difference $\widehat{U}_i - U_i(\alpha_h)$ compares individual i's maximal aspiration with the status quo. In a sense, the difference constitutes the (maximal) complaint i can articulate about his treatment in the status quo. We postulate that the first consumer's bargaining power in group h depends on the relative complaints via the voice impact function:

$$
\beta_h = f\left(\frac{\widehat{U}_1 - U_1(\alpha_h)}{\widehat{U}_2 - U_2(\alpha_h)}\right) \qquad (3)
$$

The voice impact function is assumed to satisfy the following requirements:

Properties of the Voice Impact Function

(1) $f(0) = 0$

(2) $f(x) + f(1/x) = 1$

(3) $\lim_{x\to\infty} f(x) = 1$

(4) $f' > 0$

The condition $f(x) + f(1/x) = 1$ supposes that both group members are equally able in transforming hypothetical but possible utility gains from forming other groups into bargaining power through articulation of their aspirations or complaints.

5.2 Equilibria

To satisfy all four axioms, we specify the voice impact function as $f(x) = x/(x+1)$. Applying the voice impact function (3) to group h amounts to

$$\left.\begin{aligned} \beta_h &= \frac{\widehat{U}_1 - U_1(\alpha_h)}{\widehat{U}_1 - U_1(\alpha_h) + \widehat{U}_2 - U_2(\alpha_h)}\,; \\ 1-\beta_h &= \frac{\widehat{U}_2 - U_2(\alpha_h)}{\widehat{U}_1 - U_1(\alpha_h) + \widehat{U}_2 - U_2(\alpha_h)}\,. \end{aligned}\right\} \tag{4}$$

Assuming that there exists a pair (α_h, β_h) with $0 \leq \alpha_h \leq 1$, $0 \leq \beta_h \leq 1$ that satisfies the above equations and inserting the voice power associated with β_h and $1-\beta_h$ into the group optimization rule (given by equation (2)) yields:

$$\frac{\widehat{U}_1 - U_1(\alpha_h)}{(U_1(\alpha_h) - U_1^0)\alpha_h} = \frac{\widehat{U}_2 - U_2(\alpha_h)}{(U_2(\alpha_h) - U_2^0)(1-\alpha_h)} \tag{5}$$

We obtain:

Proposition 1

Suppose there exists $\alpha_h \in [\underline{\alpha}_h, \overline{\alpha}_h]$ such that $\widehat{U}_1 > U_1(\alpha_h)$ and $\widehat{U}_2 > U_2(\alpha_h)$. Then there exist unique values $\alpha_h^* \in (0,1)$ and $\beta_h^* \in (0,1)$ that solve the group optimization problem and are consistent with voice power. α_h^* is determined by (5).

The proof of proposition 1 follows immediately from the observation that the left side of (5) is strictly decreasing in α_h whereas the right side of (5) is strictly increasing in α_h. Moreover, for $\alpha_h \to 0$ $(1-\alpha_h \to 0)$ the left side (right side) becomes infinite.

Proposition 1 shows how exit and voice power interact in determining the group allocation. *Ceteris paribus* considerations yield:

Corollary 1

$$\frac{\partial \alpha_h^*}{\partial \widehat{U}_1} > 0, \quad \frac{\partial \alpha_h^*}{\partial \widehat{U}_2} < 0; \frac{\partial \alpha_h^*}{\partial U_1^0} > 0, \quad \frac{\partial \alpha_h^*}{\partial U_2^0} < 0.$$

We note that exit and voice uniquely determine the group allocation. Given that equilibrium prices are independent of α_h^* and β_h^* we obtain:

Proposition 2
Suppose there exist utalitarian weights $\alpha_h \in [\underline{\alpha}_h, \overline{\alpha}_h]$ *and* $\alpha_g \in [\underline{\alpha}_g, \overline{\alpha}_g]$ *such that* $\widehat{U}_1 > U_1(\alpha_h)$, $\widehat{U}_2 > U_2(\alpha_h), \widehat{U}_3 > U_3(\alpha_g)$, *and* $\widehat{U}_4 > U_4(\alpha_g)$. *Then for* $P = \{\{1,2\},\{3,4\}\}$, *there exists a unique CEFG of the form* (p^*, x^*, P) *that satisfies the voice power consistency requirement.*

Observation 1.
We chose the above voice impact function $f(x) = x/(1+x)$ for the sake of convenience and transparency. Observe that any voice impact function f is determined by its restriction to $x \in [0,1]$, since $f(x) = 1 - f(1/x)$ for $x > 1$. Moreover, $f(1) = 1/2$. Conversely, any differentiable function $f : [0,1] \to \mathbb{R}_+$ with $f(0) = 0, f(1) = 1/2$ and $f' > 0$ can be extended to a voice impact function by setting $f(x) = 1 - f(1/x)$ for $x > 1$.

Observation 2.
Suppose all consumers have identical homothetic preferences for consumption, represented by a continuous, concave and weakly increasing utility function which is differentiable, strictly concave and strictly increasing on $\mathbb{R}^\ell_{++}$. Then an analogue of lemma

1 holds. Next consider household $h = \{1, 2\}$, say, with voice impact function f. Then given β_h and the equilibrium price system determined in lemma 1, maximization of the Nash product N_h yields α_h as a continuous function φ_1 of β_h. On the other hand, (3) determines β_h as a continuous function φ_2 of α_h. By Brouwer's fixed point theorem, the composition mapping $\varphi_1 \circ \varphi_2$ has a fixed point α_h^*. Hence there exist $\alpha_h^* \in [0, 1]$ and $\beta_h^* \in [0, 1]$ that solve the group optimization problem and are consistent with voice power. The value of β_h^* is obtained via (3). Application of the fixed point theorem does not yield uniqueness and interiority, which falls short of proposition 2.

6 A Numerical Example

To illustrate the working of proposition 2 we use the following parameter values:

$$\gamma = 1 - \gamma = \frac{1}{2}, \quad w_1 = w_2 = w_3 = w_4 = (1, 1)$$

Accordingly, $p_2^* = 1$. Moreover,

$$\begin{aligned}
U_1(\alpha_h) &= \frac{1}{2}\{\ln(2\alpha_h) + \ln(2\alpha_h)\} + v_{12} = \ln(2\alpha_h) + v_{12} \\
U_2(\alpha_h) &= \frac{1}{2}\{\ln[2(1-\alpha_h)] + \ln[2(1-\alpha_h)]\} + v_{21} = \ln[2(1-\alpha_h)] + v_{21} \\
U_1^0 &= \frac{1}{2}\{\ln 1 + \ln 1\} = 0 \\
U_2^0 &= 0 \\
\widehat{U}_1 &= \frac{1}{2}\{\ln[2\overline{\alpha}_k] + \ln[2\overline{\alpha}_k]\} + v_{13} = \ln[2\overline{\alpha}_k] + v_{13} \\
U_3(x_3^0(1)) &= 0 = \frac{1}{2}\{\ln(2(1-\overline{\alpha}_k)) + \ln(2(1-\overline{\alpha}_k))\} + v_{31} \\
\widehat{U}_2 &= \ln[2\overline{\alpha}_{k'}] + v_{23} \\
U_3(x_3^0(1)) &= 0 = \frac{1}{2}\{\ln(2(1-\overline{\alpha}_{k'}) + \ln(2(1-\overline{\alpha}_{k'})\} + v_{32}
\end{aligned}$$

where $k = \{1, 3\}$ and $k' = \{2, 3\}$. This implies:

$$2(1-\overline{\alpha}_k) = \mathrm{e}^{-v_{31}}, \quad \widehat{U}_1 = \ln(2 - \mathrm{e}^{-v_{31}}) + v_{13}$$

$$2(1-\overline{\alpha}_{k'}) = \mathrm{e}^{-v_{32}}, \quad \widehat{U}_2 = \ln(2 - \mathrm{e}^{-v_{32}}) + v_{23}$$

Using (5), we find that the group allocation satisfies:

$$\frac{\ln\left(2-\mathrm{e}^{-v_{31}}\right)+v_{13}-\ln(2\alpha_h)-v_{12}}{\alpha_h(\ln(2\alpha_h)+v_{12})}=\frac{\ln\left(2-\mathrm{e}^{-v_{32}}\right)+v_{23}-\ln[2(1-\alpha_h)]-v_{21}}{(1-\alpha_h)\left(\ln[2(1-\alpha_h)]+v_{21}\right)}$$

This equation determines α_h^*. We obtain

Corollary 2 *Suppose $v_{12}=v_{21}$. Suppose that there exists α_h such $\widehat{U}_1>U_1(\alpha_h)>0$ and $\widehat{U}_2>U_2(\alpha_h)>0$. Then there exists a unique value of α_h^*. Moreover, $\alpha_h^*>\frac{1}{2}$ if and only if*

$$\ln\left(2-e^{-v_{31}}\right)+v_{13}>\ln\left(2-e^{-v_{32}}\right)+v_{23}.$$

Intuitively, the higher v_{13} relative to v_{23} and the higher v_{31} relative to v_{32}, the higher the relative bargaining power of the first individual since her power of voice is comparatively larger.

7 Final Remarks

Via an example we have proposed and examined a concept of voice power. Numerous issues deserve further attention. Apart from incorporating voice power in more general models, a more detailed behavioral foundation of our concept should be taken up in future research. Moreover, here the power of voice relies on cardinal utility specifications since the voice impact function relies on cardinality. However, it would be desirable to have a clear view as to which properties of voice depend on ordinal properties of preferences and which properties depend on the cardinal representation.

References

Alcalde, J., and Revilla, P.: "Tops responsiveness, strategy-proofness and coalition formation problems", mimeo, 2001.

Banerjee et al.: "Core in a Simple Coalition Formation Game", *Social Choice and Welfare 18* (2001), 135-153.

Barberà, S., and Gerber, A.: "On Coalition Formation: Durable Coalition Structures", *Mathematical Social Sciences 45* (2003), 185-203.

Becker, G.S.: "Nobel Lecture: The Economic Way of Looking at Behavior", *Journal of Political Economy 101* (1993b), 385-409.

Bell, C: "Markets, Power and Productivity in Rural Asia: a Review Article", *Journal of Development Economics* (1991), 373-393.

Bennett, E.: "Multilateral Bargaining Problems", *Games and Economic Behavior 19* (1997), 151-179.

Bennett, E.: "Consistent Bargaining Conjectures in Marriages and Matching" *Journal of Economic Theory 45* (1988), 392-407.

Bennett, E., and van Damme, E.: "Demand Commitment Bargaining: The Case of Apex Games", in *Game Equilibrium Models*, Vol. III, *Strategic Bargaining*, (R. Selten, Ed.), (1991), 118-140, Berlin: Springer.

Bogomolnaia, A., Jackson, M.O.: 'The Stability of Hedonic Coalition Structures", *Games and Economic Behavior 38* (2002), 201-230.

Chatterjee, K., Dutta, B., Ray, D., and Sengupta, K.: "A Non-Cooperative Theory of Coalitional Bargaining", *Review of Economic Studies 60* (1993), 463-477.

Chwe, M.S.-Y.: "Farsighted Coalitional Stability ", *Journal of Economic Theory 63* (1994), 299-325.

Diamantoudi, E., and Xue, L.: "Farsighted Stability in Hedonic Games", *Social Choice and Welfare 21* (2003), 39-61.

Elster, J. (ed.): *Deliberative Democracy*. Cambridge University Press: Cambridge, UK, 1998.

Gersbach, H., and Haller, H.: "Collective Decisions and Competitive Markets," *Review of Economic Studies 68* (2001), 347-368.

Gersbach, H., and Haller, H.: "Competitive Markets, Collective Decisions and Group Formation," Working Paper, 2003.

Greenberg, J.: "The Theory of Social Situations: An Alternative Game-Theoretic Approach", Cambridge University Press: Cambridge, UK, 1990.

Habermas, J.: *Theory of Communicative Action*. Beacon Press: Boston, 1987.

Harsanyi, J.C.: "An Equilibrium-Point Interpretation of Stable Sets and a Proposed Alternative Definition." *Management Science 20* (1974), 1472-1495.

Hirschman, A.O.: *Exit, Voice, and Loyalty*. Harvard University Press, Cambridge, Massachusetts, 1970.

Kalai, E., and Samet, D.: "Monotonic Solutions to General Competitive Games", *Econometrica 53*, (1985), 307-327.

von Neumann, J., Morgenstern, O.: "Theory of Games and Economic Behavior", Princeton University Press, Princeton, 1944.

Pápai, S.: "Unique stability in simple coalition formation games", mimeo, 2000.

Rochford, S.C.: "Symmetrical Pairwise-Bargained Allocations in an Assignment Market", *Journal of Economic Theory 34*, (1984), 262-281.

Selten, R.: "A Noncooperative Model of Characteristic Function Bargaining", in *Essays in Game Theory and Mathematical Economics in Honor of Oskar Morgenstern* (V. Boehm and H. Nachtkamp, Eds.), (1981), 131-151. Vienna, Zurich; Wissenschaftsverlag Bibliographisches Institut Mannheim.

Xue, L.: "Coalitional Stability under Perfect Foresight.", in *Economic Theory 11* (1998), 603-627.

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