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OPTIMAL INVESTMENT STRATEGIES UNDER DEMAND AND TAX POLICY UNCERTAINTY

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Abstract

In this paper we offer an analysis of the effects of uncertainty about future tax policy on irreversible investment. The main message of the paper is that investment is not much affected by the degreee of tax policy uncertainty. This is true regardless of whether random tax changes are determined endogenously or exogenously. The paper therefore indicates that reducing tax policy uncertainty is probably no magic bullet to increase private investment spending.

Keywords: Investment, uncertainty, irreversibility, taxation

JEL Classification: D81, E22, H25

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1 Introduction

The impact of tax policy uncertainty on investment spending, a topic of obvious concern for policy-makers, has attracted considerable interest in the analytical public finance literature recently.[1] One reason for the policy relevance is that globalisation has created increasing tax competition in the OECD countries in which governments started a "race to lower tax rates" slashing taxes in tax-cut cum base-broadening tax reforms to lure global business.[2] Additionally, the severe limits on deficits and debt levels set by the "Growth and Stability Pact" means that several EMU members are still facing the need for major fiscal retrenchment. Altogether these ongoing fiscal policy debates have created substantial uncertainty about the future levels of corporate taxation and/or investment subsidies. In standard models of risk-neutral investors-firms the impact of uncertainty depends basically on the relationship between the expected marginal revenue product of capital and the stochastic variables - typically the relative output price. Consider, for example, the scenario of the constant-returns, perfectly competitive firm in which capital is the only fixed factor, while labour can be adjusted costlessly in the face of changing output prices. Price shocks then alter the optimal capital-labour mix, thus making the marginal revenue product of capital rise more (or fall less) then relative output price. In such conditions, marginal profitability is a convex function of output prices and Jensens inequality then implies that higher price uncertainty raises the expected profitability of capital, thereby decreasing the trigger value of investment.[3] The more recent literature pioneered by Dixit and Pindyck (1994) has emphasized the irreversible nature of most fixed investment projects combined with timing flexibility and risk neutrality. The analytical tactic is to value a corporate investment opportunity as an American call option. The potential investment to be made then corresponds to an options exercise price and the length of time the company can wait before it has to decide is like the call options time to expiration. The irreversibility assumption does not seem a widely unrealistic assumption, given the firm-specific nature of machinery, the costs of installation, bankruptcy, hiring and firing labour, and so on. It makes investment adjustment costs asymmetric - larger for downward than for upward adjustment. Under appropriate conditions, this creates a range of inaction: investment

[1] See, for example, Alvarez, Kanniainen & Södersten (1998), Alvarez & Kanniainen (1997), Hopenhavn & Muniagurria (1993), Jeong (1995), Niemann (1999) MacKie-Mason (1990), Hassett & Metcalf (1999), Pennings (2000) and Rodrik (1991). A simple textbook analysis of the effects of uncertainty about future tax policy is available in Romer (1996), pp. 365–369.

[2] For empirical evidence on tax competition and investment see Hines & Rice (1994), Hines (1999) and Althuser, Gruber & Newlon (1998). A comprehensive review of the relation between tax policy and investment spending is available in Hassett & Hubbard (1996).

[3] See, for example, Abel (1983) and Hartman (1972).

takes place only when expected profitability exceeds a certain threshold. From the preceding discussion it is clear, however, that irreversibility per se is not sufficient to turn around the positive uncertainty-investment link following from the convexity of the profit function. Indeed, even under irreversible investment it can be shown that optimal investment by a competitive firm operating under constant returns continues to be a non-decreasing function of uncertainty.[4] To reverse this counterintuitive result one has to bring in additional assumptions such as imperfect competition and/or decreasing returns to scale. When combined with irreversibility and timing flexibility this can create a wait-and-see-strategy-postponing investments until the future becomes clear.[5] The crucial assumption that the marginal profitability of capital declines with the stock of capital cannot, by definition, apply to a constant-returns perfectly competitive firm, for which the marginal product of capital is unrelated to the level of capital. On the contrary, such conditions hold for imperfectly competitive firms.[6] Hassett & Metcalf (1999) have recently used the real-options valuation model to analyse the impact of tax policy uncertainty on irreversible investment. Their theoretical paper gives the insight that higher tax uncertainty actually leads to more investment expenditures of a perfectly competitive firm. Since endogenously timed stationary investment allowances are likely to increase when they are low, the higher the uncertainty, the more firms are inclined to undertake an irreversible investment at a relatively low tax rate. This rosy picture is obviously in sharp contrast to the view of business economists that uncertainty about future corporate taxation cannot be beneficial.[7] Given the ambiguous analytical and numerical results of the investment literature summarized above, the paper offers a thorough re-examination of optimal investment strategies in uncertain

[4] See Caballero (1991) and Abel & Eberly (1994).

[5] The intuitive reason for this result is that irreversibility makes downside uncertainty more important than upside uncertainty because disinvestment is more costlier than investment. Favourable shocks have a smaller effect on profitability than adverse shocks, and therefore firms become ex ante reluctant to invest to reduce the risk of being stuck ex post with an unprofitable capital stock.

[6] So far the discussion has been limited to the risk-neutral case. One alternative approach take as starting point the case of risk-averse agents facing limited diversification possibilities in imperfect capital markets. Along these lines Zeira (1990) has again shown that the link between uncertainty and investment for perfectly competitive firms in ambiguous. On one hand the convexity of the profit function mentioned earlier increases investment, on the other hand, higher uncertainty decreases investment due to the investors risk aversion. An related approach is the "disappointment aversion" approach advocated by Aizenman (1995). They depart from the traditional expected utility framework by assuming that agents attach more weight to adverse outcomes than to favorable ones. They show that this can lead to a negative investment-uncertainty sign.

[7] In its recent benchmarking study of the UK economy, for example, the Department of Trade and Industry (DTI) has stated that "instability and uncertainty are bad for business. Volatile interest rates, high and unstable inflation and uncertainty over the future course of output and demand undermine business confidence and damages the incentives to invest and plan for the long term" [Department of Trade and Industry (1999) *Our Competitive Future - UK Competitiveness Indicators 1999*, London, p. 10].

tax environments in an extended model allowing for imperfect competition and/or decreasing returns to scale. In what follows, we lay out the pre-tax real-options model in Section 2. In Section 3, we turn to a model where demand uncertainty is described by geometric Brownian motion and tax policy is modelled as a stationary jump process (Poisson process). This work extends and complements recent work by Hassett & Metcalf (1999). We move to numerical simulations in Section 4. Finally, Section 5 concludes and discusses some possible extensions of our work.

2 The Basic Pre-Tax Model

We begin with a pre-tax model because it provides a useful benchmark against which we may judge the consequences of the additional complication of tax policy uncertainty. What follows is a partial-equilibrium framework for determining the level of optimal investment decisions surrounding uncertainty about future demand. The resulting business strategy tool kit is based upon a real-options valuation model that helps in correctly valuing irreversible investment opportunities.[8] Thoughout the paper, we will assume risk neutrality, i.e. agents will rely on the expectations of random variables. We consider a firm which faces an isoelastic demand function:

$$p(t) := Y(t)^{(1-\psi)/\psi} X(t) \qquad \psi \geq 1 \tag{1}$$

where p and Y respectively denote the price and the quantity of the good sold. ψ is an elasticity parameter that takes its minimum value of 1 under perfect competition. The stochastic term X evolves according to a geometric Brownian motion with drift μ_d and variance σ_d:

$$dX = \mu_d X dt + \sigma_d X dz \tag{2}$$

where dz is the increment of a standard Wiener process, with $E[dz] = 0$ and $E[(dz)^2] = dt$. The production technology is described by the Cobb-Douglas production function

$$Y = (AL^{\alpha} K^{1-\alpha})^{\gamma} \tag{3}$$

where L, K, and A are labour, capital, and the technology parameter at time t, respectively. The parameters α and γ are the constant labour share and an index of returns

[8]A similar continuous time optimization problem is drawn up in detail in Abel & Eberly (1994).

to scale, respectively. Since labour is assumed to be adjustable costlessly, we define Π as income minus labour costs.

$$\begin{aligned}\Pi &= max\{pY - wL\} \\ &= hX^{\eta_x}K^{\eta_k} \end{aligned} \tag{4}$$

where $\eta_x := \frac{1}{1-\alpha\xi} > 1$, $\eta_k := \frac{(1-\alpha)\xi}{1-\alpha\xi} \leq 1$, $h := (1-\alpha\xi)\left(\frac{\alpha\xi}{w}\right)^{\frac{\alpha\xi}{1-\alpha\xi}} A^{\frac{\xi}{1-\alpha\xi}} > 0$, and $\xi := \gamma/\psi$. The capital stock of the firm depreciates with the constant rate δ and is costly adjusted by positive or negative investment I. So, the law of motion for capital is given by

$$dK = (I - \delta K)dt \tag{5}$$

The costs of changing the stock of capital consist of the price per unit of installed capital p_k and the convex adjustment costs $c(I)$. The objective of the firm is to choose a sequence of investments to maximize its expected fundamental value V over an infinite horizon.

$$V = \max_I E\left[\int_0^\infty \left(hX^{\eta_x}K^{\eta_k} - Ip_k - c(I)\right)\exp(-rs)ds\right] \tag{6}$$

The present value satisfies the following Bellman equation:

$$rV = \max_I\left\{hX^{\eta_x}K^{\eta_k} - Ip_k - c(I) + \frac{E[dV]}{dt}\right\} \tag{7}$$

Using Itô's lemma, the stochastic term $E[dV]/dt$ is given by:

$$\frac{E[dV]}{dt} = (I-\delta K)\frac{\partial V}{\partial K} + \mu_d X\frac{\partial V}{\partial X} + \frac{1}{2}\sigma_d^2 X^2\frac{\partial^2 V}{\partial X^2}. \tag{8}$$

We now substitute equation (8) back into(7) which yields Resolving $E[dV]/dt$ yields

$$\begin{aligned} rV = \max_I \Big\{ & hX^{\eta_x}K^{\eta_k} - Ip_k - c(I) \\ & + (I-\delta K)\frac{\partial V}{\partial K} + \mu_d X\frac{\partial V}{\partial X} + \frac{1}{2}\sigma_d^2 X^2\frac{\partial^2 V}{\partial X^2}\Big\}. \end{aligned} \tag{9}$$

For simplicity we assume a quadratic adjustment cost function $c := (\gamma/2)I^2$ and complete irreversibility. The terms to be maximized in equation (9) are $-p_kI - (\gamma/2)I^2 + Iq$ with the first order condition $-p_k - \gamma I + q = 0$. Therefore, optimal investment is

$$I^* = \max\left\{\frac{q-p_k}{\gamma}, 0\right\} \tag{10}$$

where $q := \frac{\partial V}{\partial K}$ is Tobin's marginal q and the subscript $(^*)$ stands for optimal investment. Inserting I^* we write equation (9)

$$rV = hX^{\eta_x}K^{\eta_k} + \frac{(q-p_k)^2}{2\gamma} - \delta Kq + \mu_d X\frac{\partial V}{\partial X} + \frac{\sigma_d^2}{2}X^2\frac{\partial^2 V}{\partial X^2} \quad (11)$$

and

$$rV = hX^{\eta_x}K^{\eta_k} + \mu_d X\frac{\partial V}{\partial X} + \frac{\sigma_d^2}{2}X^2\frac{\partial^2 V}{\partial X^2} \quad (12)$$

for $q > p_k$ (positive investment) and $I^* = 0$, respectively.

In the general case there are no closed-form solutions of the investment model. For further insights it is therefore necessary to solve the discrete version of the model numerically. We simulate the model with two periods. The firm starts with an initial capital stock $K_0 = 100$ and the random variable X_0 is set such that optimal investment without uncertainty equals depreciation $I_{0,\sigma_d=0} = \delta K_{0,\sigma_d=0} = 10$.[9]

Figure 1: Investment as a Function of σ_d for Different Values of ξ

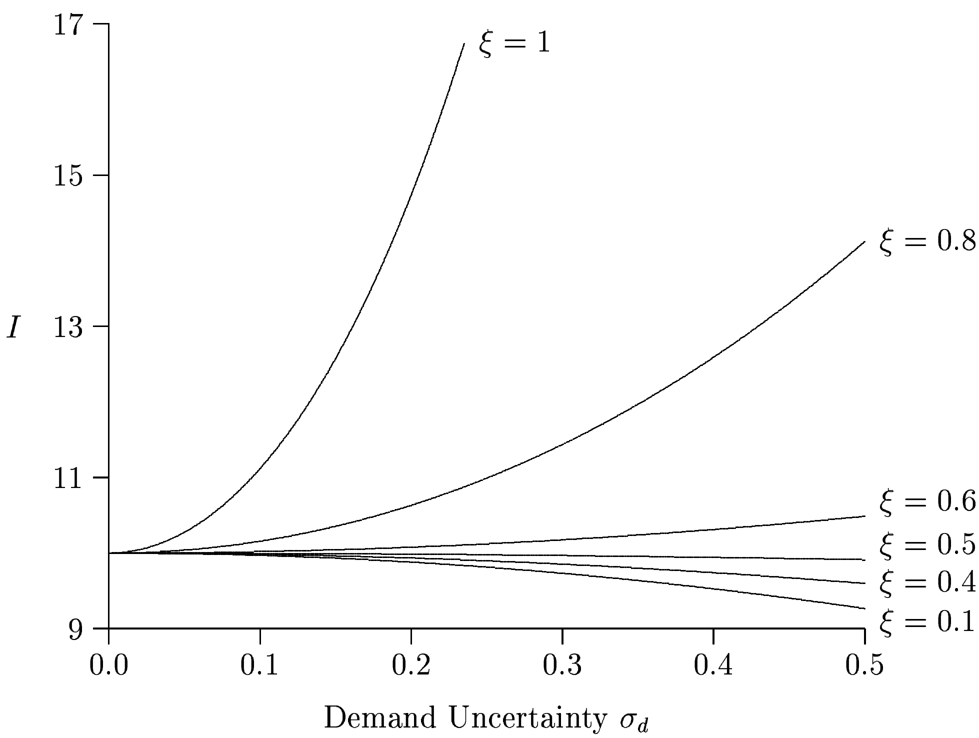


Figure 1 illustrates that with perfect (imperfect) competition the demand uncertainty investment link turns out to be strongly positive (slightly negative). This result confirms Caballero's (1991) demonstration that the existence of imperfect competition

[9]Our base choice of parameters is $p_k = 1$, $\gamma = 0.02$, $\alpha = 0.7$, $w = 0.7$, $A = 1$, $r = 0.02$, $\delta = 0.1$, $\mu = 0$. ξ is set to values explicitly specified. All computations in the paper have been produced using C.

and/or decreasing returns to scale is a necessary condition for the irreversibility-driven negative relationship between optimal investment and σ_d. Hence, simulations of models of irreversible investment should be based on degree of market power.[10] We elaborate on this point in the next Section.

3 Irreversible Investment with Demand and Tax Policy Uncertainty

What follows, next, is an extended framework for determining the features of tax uncertainty and for tailoring strategy to that uncertainty. The model is as in Section 2, with the only change that firm's are additionally facing uncertainty about future tax policy. Tax effects emerge when investment decisions change in light of the tax system, i.e. for a given realisation of the random tax variable it is optimal to exercise the option to invest without tax uncertainty and to continue waiting after the introduction of random taxes or vice versa. Following Hassett & Metcalf (1999), the uncertainty about future corporate taxation is described as one of two alternative outcomes. Investment tax credits, unlike output prices, tend to remain fixed for some time, and then change to new values. It is assumed that there is no tax other than the investment allowance analysed here.[11] The noise about timing is modelled by a continuous-time Poisson process switching between high investment allowances and low investment allowances. This seems to be a reasonable assumption about the stochastic nature of the period by period investment tax credits in many countries although tax reforms are typically preceded by time-consuming parliamentary and public debates. Nevertheless, the quantitative details and the timing of the reform often come as surprise despite the fact that the qualitative nature of the direction of the reform does not. Two recent examples come to mind. The massive temporary fiscal support programmes comprising investment allowances, investment grants and depreciation allowances which have given preferential treatment to investments in the New German Länder after unification are an example of frequent changes in tax parameters. The reason is that there was much uncertainty both about the timing and the size of the temporary measures because announced reductions of investment allowances have been broken several times. Given

[10] Empirical evidence for this industrial organisation insight that the sign of the uncertainty-investment relationship depends upon market power is available in Ghosal & Loungani (1996) and Guiso & Parigi (1999).

[11] This subsidy can be interpreted broadly as any policies that affect the cost of investment (e.g. subsidies, direct taxes and/or regulations affecting investments).

these experiences few firms must have felt able to predict with confidence investment allowances in East Germany after unification.The second example is the recent discussion about tax cuts promised by the German government. The major benefit to companies (both stock and limited liability) will be: a flat 25% tax on retained profits (now 40%), to encourage investment and job creation; and a top rate of 39% when the local trade tax and solidarity surcharge are included. The top level of personal income tax would be cut in steps from 51% to 45% by 2005. But unless the government can regain its majority in the Bundesrat (upper house), the government will have to reach a compromise with the Christian Social Union (CDU) which can block the bill and has already made a counterproposal of much more generous tax cuts. Therefore the current political process in Germany creates parameter uncertainty and noise about the precise timing of the reform.[12] In this environment, the capital installation costs functions are defined as $Ip_k(1-t^H)+c(I)$ and $Ip_k(1-t^L)+c(I)$ for the state of high (t^H) and low (t^L) investment tax credit, respectively. In other words, the investment tax credit in any particular period takes one of two values, t^L and t^H where $t^L < t^H$. The transition probabilities, which may depend on the stochastic process of the demand function X, are given by λ^H and λ^H, respectively. Demand uncertainty is again modelled by a Brownian motion according to Section 2. We therefore have a combination of a geometric Brownian motion and a Poisson (jump) process. The former goes on all time, the latter occurs infrequently. With this additional stochastic process the term term $E[dV]/dt$ in the mixed Brownian motion-jump process depends on whether we are in in the regime of high or low investment tax credit:[13]

$$\frac{E[dV]}{dt} = \begin{cases} (I-\delta K)q^H + \mu_d X \frac{\partial V^H}{\partial X} + \frac{1}{2}\sigma_d^2 X^2 \frac{\partial^2 V^H}{\partial X^2} + \lambda^H (V^L - V^H) \\ \\ (I-\delta K)q^L + \mu_d X \frac{\partial V^L}{\partial X} + \frac{1}{2}\sigma_d^2 X^2 \frac{\partial^2 V^L}{\partial X^2} + \lambda^L (V^H - V^L) \end{cases} \tag{13}$$

[12] Thorough cost of capital calculations illustrating the magnitude of the incentive effects of the fiscal support measures are available in Sinn (1995). His main conclusion is that most of the tax incentives are probably a waste of money. They have subsidised good investments(which needed no such help) or turned bad investments into profitable ones (although they should never see the light of day).

[13] The general Itô formula for jump processes is described in Malliaris & Brock (1982) , pp. 121-124 and Dixit & Pindyck (1994), pp. 85-87.

Rewriting equation (9) yields

$$\begin{aligned} rV^H &= \max_I \Big\{ hX^{\eta_x}K^{\eta_k} - I(1-t^H)p_k - c(I) + (I-\delta K)q^H \\ &\qquad + \mu_d X \frac{\partial V^H}{\partial X} + \frac{1}{2}\sigma_d^2 X^2 \frac{\partial^2 V^H}{\partial X^2} + \lambda^H (V^L - V^H) \Big\} \end{aligned} \tag{14}$$

and

$$\begin{aligned} rV^L &= \max_I \Big\{ hX^{\eta_x}K^{\eta_k} - I(1-t^L)p_k - c(I) + (I-\delta K)q^L \\ &\qquad + \mu_d X \frac{\partial V^L}{\partial X} + \frac{1}{2}\sigma_d^2 X^2 \frac{\partial^2 V^L}{\partial X^2} + \lambda^L (V^H - V^L) \Big\} \end{aligned} \tag{15}$$

for the high and low investment tax credit regime, respectively. Optimal investment is determined as:

$$I^* = \max \left\{ \frac{q-(1-t^i)p_k}{\gamma}, 0 \right\} \quad i = H, L \tag{16}$$

It is easy to show that q is independent of tax policy uncertainty in the case of perfect competition and constant returns to scale ($\xi = 1$) if λ^H, λ^L are constant and so is the optimal investment I.[14]

$$q^H = q^L = \frac{hX^{\eta_x}}{\eta_x \mu_d + \eta_x(1-\eta_x)\frac{1}{2}\sigma_d^2 - r - \delta} \tag{17}$$

Now we follow Hassett & Metcalf (1999) in the price dependency of the Poisson parameters λ^H and λ^L. This implies that the jump probabilities are endogenously determined by the profitability of investing.

$$\lambda^i := \begin{cases} 0 & \text{if } X \leq -\frac{\lambda_0^i}{\alpha^i} \\ \lambda_0^i + \alpha^i X & \text{if } -\frac{\lambda_0^i}{\alpha^i} \leq X \leq \frac{1-\lambda_0^i}{\alpha^i} \\ 1 & \text{if } X \geq \frac{1-\lambda_0^i}{\alpha^i} \end{cases} \qquad i = L, H, \qquad \alpha^L < 0 < \alpha^H \tag{18}$$

The logic behind (18) is that the probability to stay in or jump to the high investment tax credit regime therefore increases with declining profitability and vice versa.[15] Let us assume that increasing tax policy uncertainty is of the mean-preserving spread type, i.e $E[1-t^i], \quad i = H, L$ is constant. Since the sign of the impact of a mean preserving change in uncertainty cannot be determined unambiguously, it is not clear how a higher tax policy uncertainty will affect the probability of investing. We therefore have to use numerical results to illustrate whether irreversible investment is an increasing or decreasing function of tax policy uncertainty in the case of mixed Poisson and Wiener processes.

[14] The exact derivatives for this special case are reported in the Appendix.

[15] $\lambda^i := \lambda_0^i + \alpha^i \ln X$ would be more symmetrically.

4 Numerical Analysis of the Augmented Model

Thus far, we have presented two models of irreversible investment under uncertainty. In Section 2, investment was influenced by demand uncertainty. In Section 3, a mixed Brownian motion-jump process was assumed. As is well-known, models of our type seldom admit useful closed-form analytical solutions. We therefore now use numerical techniques to illustrate the features of the augmented model presented in the last Section. As a preliminary step, we rewrite the model in time-discrete form. The differential equation of the capital stock becomes the difference equation

$$K_{n+1} = (1-\delta)K_n + I_n \tag{19}$$

The discrete time equivalent to the stochastic process (2) is the random walk

$$X_n = X_{n-1} \exp \varepsilon \tag{20}$$

with ε distributed normally with mean $\mu_d - \sigma_d^2/2$ and variance σ_d^2. A second assumption is that the ε's are independent random variables. In conclusion the Poisson parameters λ^H and λ^L become transition probabilities between the two states of investment tax credit. With these formulas the value of the firm over a N period horizon is given by

$$V = \max_I E\left[\sum_{n=0}^{N} hX_n^{\eta_x} K_n^{\eta_k} - I_n p_k(1-t_n) - c(I_n)\right]. \tag{21}$$

With finite horizon this optimization problem can be solved recursively with

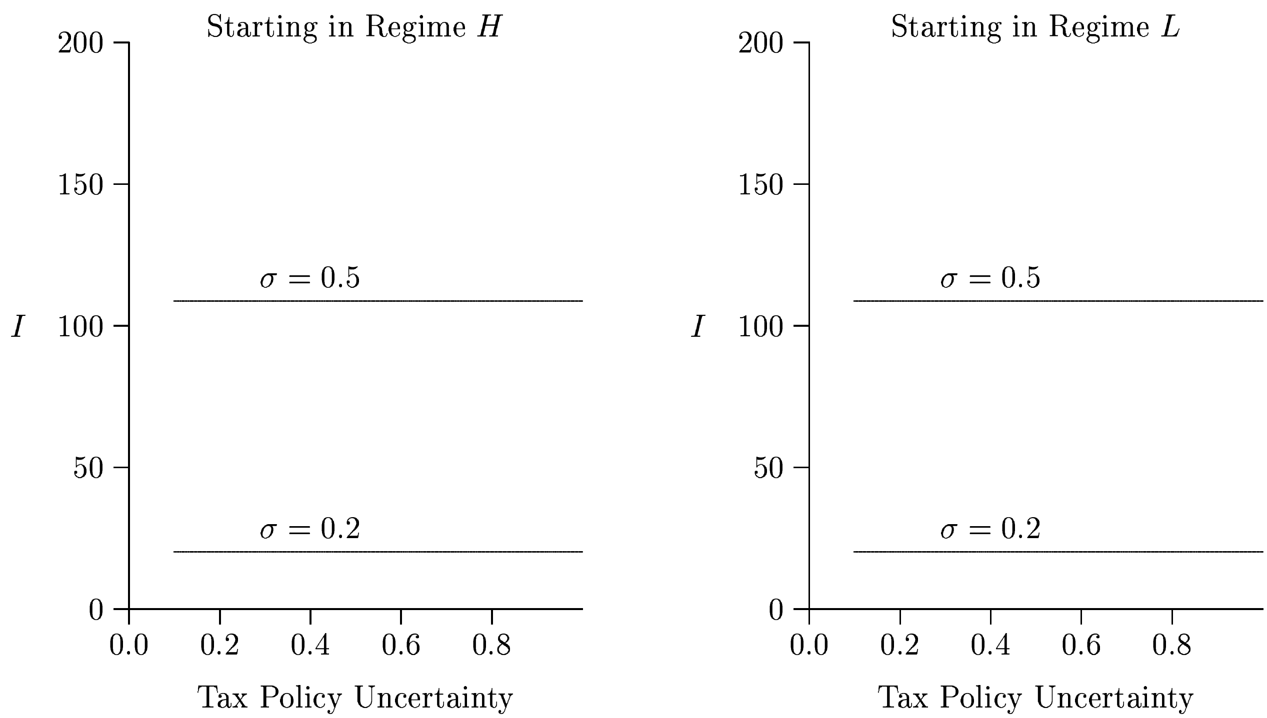
$$V_N(X_N, K_N) = hX_N^{\eta_x}((1-\delta)K_N)^{\eta_k} \tag{22}$$

and

$$\begin{aligned} V_n(X_n, K_n) &= \max_I \Big\{ hX_n^{\eta_x}((1-\delta)K_n)^{\eta_k} \\ &\qquad - I_n p_k(1-t_n) - c(I_n) + \frac{E[V_{n+1}(X_{n+1}, K_{n+1})]}{1+r} \Big\} \end{aligned}$$

The initial investment tax credit is $p_k(1-t^{start}) = 1$ and the firm is expecting a 50% change with probability 0.5. Other parameters are the same as in Figure 1. The parameter λ_0^i of the Poisson process is set to zero and α^i is shifted from $0.0/X_0$ to $1.0/X_0$ to vary the degree of tax policy uncertainty. For example with $\alpha^i = 0.4/X_0$ there is a 40% probability that the investment tax credit regime is changing if there is no change in demand. In the numerical computation we use the horizon $N = 2$. Given

Figure 2: Optimal Investment for $\xi = 1$ with a Price Dependent Poisson Process



the stochastic demand term X_0 the firm is facing tax policy uncertainty in period 1 and profit uncertainty in period 1 and 2.

How does a mean-preserving increase in tax policy uncertainty affect firm-level investment? Figure 2 – 6 summarize the effects of the tax policy uncertainty upon optimal investment spending. Figure 2 concerns optimal investment for $\xi = 1$ (perfect competition and/or constant returns to scale) and endogenously determined investment tax credit jumps according to (18). The two curves labeled $\sigma_d = 0.5$ and $\sigma_d = 0.2$ give the solutions for two alternative degrees of demand uncertainty, respectively. Two qualitative features are worth mentioning. First, the numerical results replicate the "counterintuitive" or "abnormal" feature of Figure 1 that a mean-preserving increase of σ_d increases optimal investment for $\xi = 1$. Second, the horizontal lines indicate that changing tax uncertainty has no impact upon optimal investment spending of the firm.[16]

Figure 3–6 give the optimal investment schedules of firms operating with different degrees of market power. The striking and illuminating result is that again the intensity of investment spending is not very much affected by different degrees of tax policy uncertainty. In other words, a reduction in tax policy uncertainty is no important investment stimulus. This is irrespective of whether the investment tax credit transition

[16]This feature is consistent with the rigorous analytical results in the Appendix.

Figure 3: Optimal Investment for $\xi = 0.5$ with a Price Dependent Poisson Process

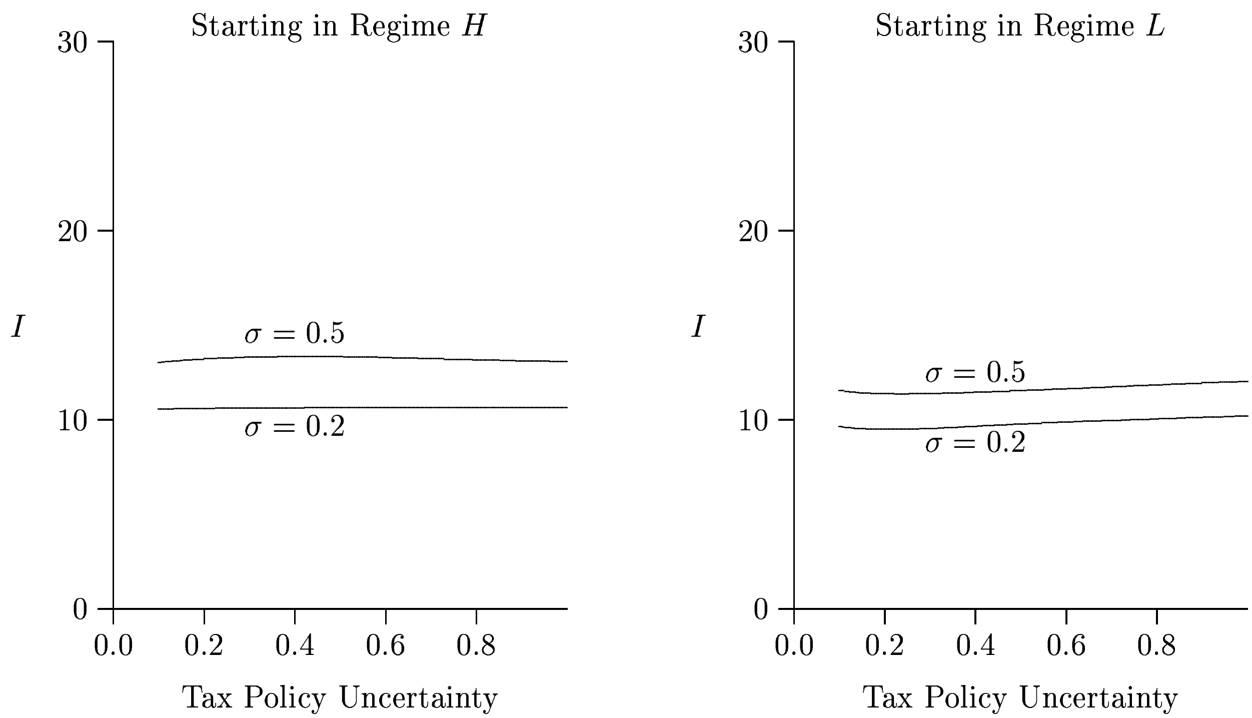


Figure 4: Optimal Investment for $\xi = 0.5$ with a Price Independent Poisson Process

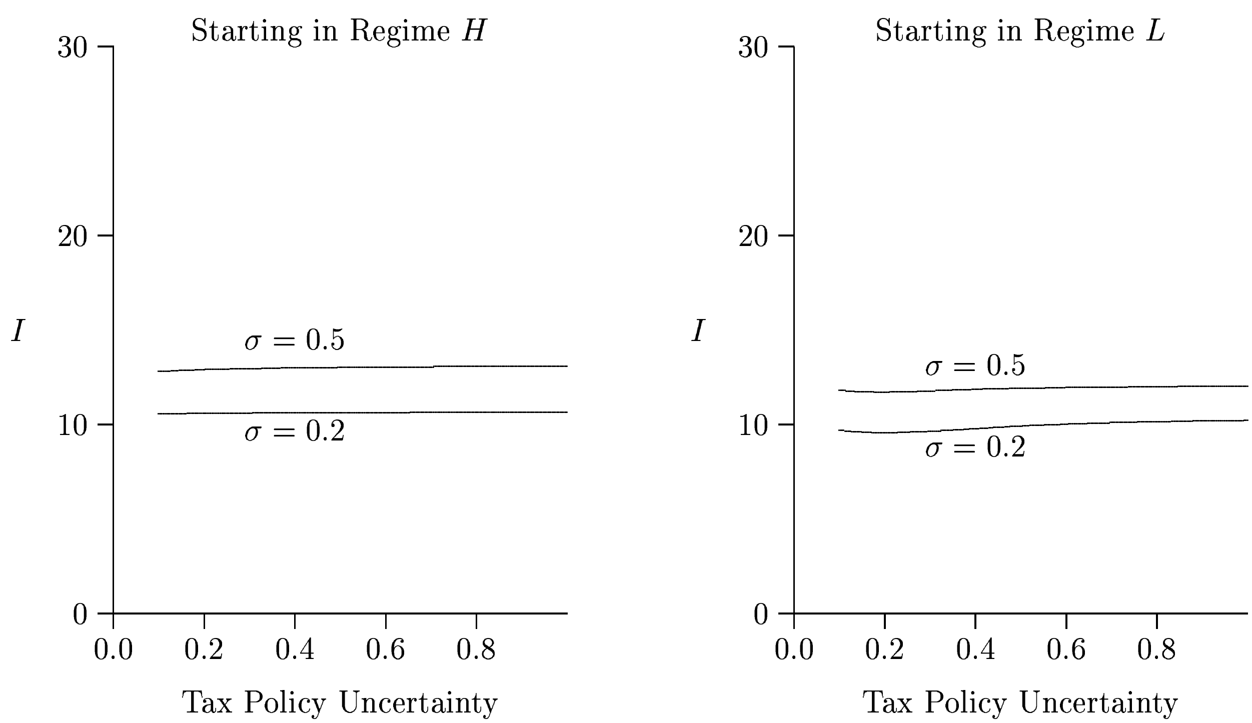


Figure 5: Optimal Investment for $\xi = 0.1$ with a Price Dependent Poisson Process

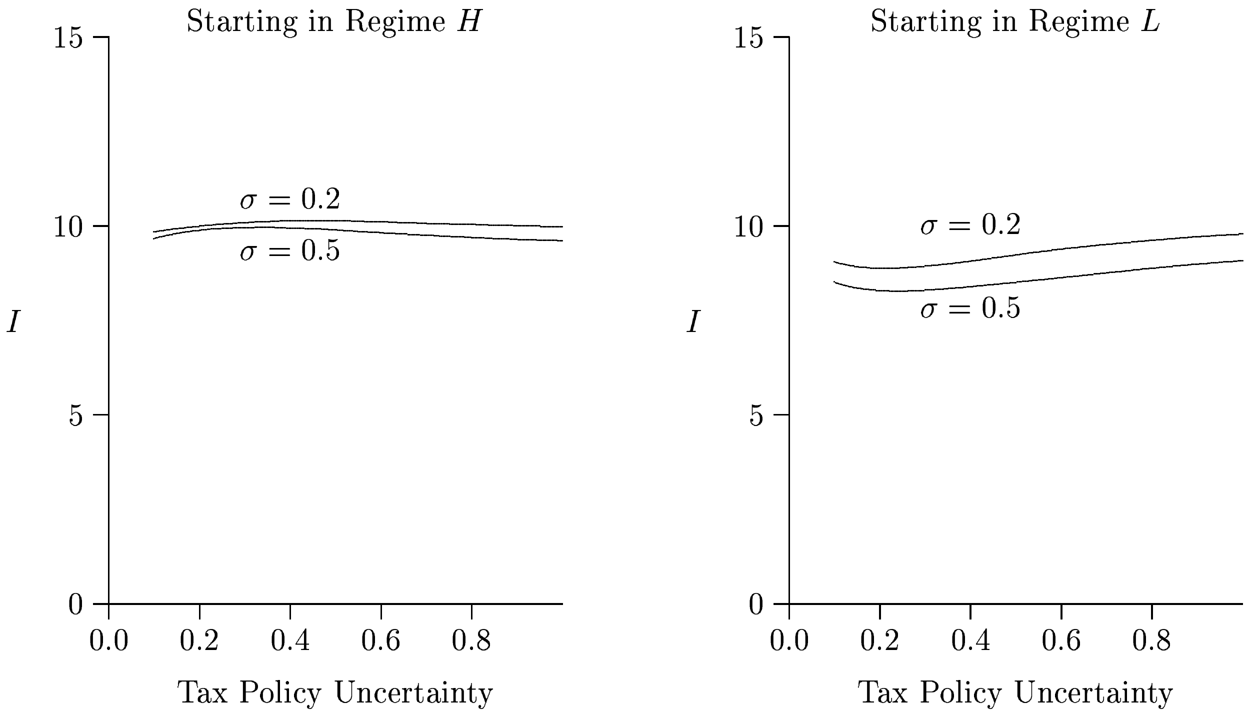
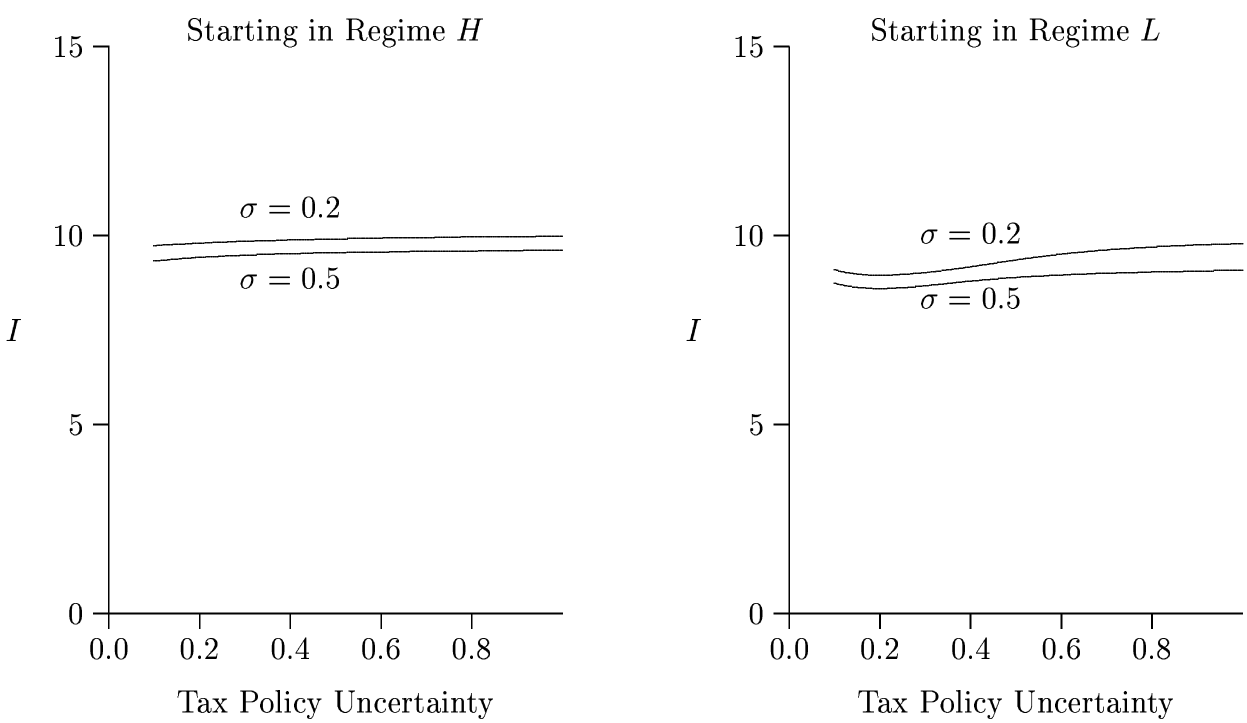


Figure 6: Optimal Investment for $\xi = 0.1$ with a Price Independent Poisson Process



probabilities λ^H and λ^L are constant (price independent Poisson process) or endogenously (price dependent Poisson process) determined according to (18). Therefore the Hassett & Metcalf (1999) "mean stationary tax policy uncertainty encourages investment" result becomes a "mean stationary tax policy uncertainty does not matter" result. Thus the interaction of tax policy uncertainty and investment spending seems to be rather sensitive to the structure of the underlying model and therefore there is no general answer.[17]

5 Conclusions

The idea that tax policy instability is detrimental to private investment spending conforms with the way businessmen see their world. According to their wisdom it is not enough to set corporate taxes at the "right" levels - uncertainty about the tax environment must also be minimized.[18] Contrary to this intuition, in everydays discussions Hassett and Metcalf (1999) have recently shown that a mean-preserving increase in tax policy uncertainty might actually speed up private investment spending. In other words, more uncertainty is not always bad for investment, because under certain conditions, it can actually increase the probability of investing. To summarize, we demonstrate that this feature of their model is not robust with respect to various modelling assumptions. The wisdom of the numerical results for the extended model presented above is that changing tax uncertainty has very limited effects on private investment spending. On the one hand from a practical, macroeconomic management point of view, this implies that tax policy uncertainty should not be blamed for current investment developments. On the other hand, reducing tax policy uncertainty is no magic bullet to increase the level of optimal private investment spending under risk-neutrality.[19] In future investigations, other types of fiscal uncertainty may merit

[17] This conclusion is consistent with the previous results in Alvarez et al. (1998). Empirical tests of the tax policy uncertainty-investment nexus are rare. Our qualitative feature is, however, consistent with the empirical evidence in Steigerwald & Stuart (1997). The authors develop a method for measuring the foresight firms have. They illustrate the method, examing investment around U.S. tax reforms. In this illustration, current investment appears to reflect currently available information but little foresight of already enacted policy changes.

[18] An interesting recent development has been the spread of tax stability laws in Latin America. They already exist in Columbia, Ecuador and Peru – and are discussed in Venezuela. These laws guarantee that the corporate tax environment is not going to get worse for a given period. Typically firms simply register to benefit from this – and often they may need to make a minimum investment to qualify. In Columbia, however, firms which want to benefit from tax stability must pay an extra 2 percentage point in the corporate tax.

[19] We would like to add a caveat here. What the article has investigated is the impact of tax uncertainty upon the investment trigger in a partial equilibrium model. However, from this result, it is not trivial to reach a conclusion regarding the effect on aggregate investment since this would imply

examination. For example, extending the model to incorporate government spending or government debt would clearly be desirable. When such additional complexities have been incorporated, the way would be clear for empirical analysis. The model presented here is therefore a first step, despite the fact that it is non-trivial analytically and numerically.

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that the model has to be extended to an industry general equilibrium setting.

Appendix: The Investment Schedule with Tax Policy Uncertainty for $\xi = 1$

In this appendix, we obtain an analytical solution for $\xi = 1$ an examin its properties. From equations (14) and (15) we get the differential equations

$$
\begin{aligned}
rV^i &= hX^{\eta_x}K + \frac{(q-(1-t^i)p_k)^2}{2\gamma}(1-t^i-q^i)^2 - \delta K q^i \\
&\quad + \mu_d X \frac{\partial V^i}{\partial X} + \frac{\sigma_d^2}{2} X^2 \frac{\partial^2 V^i}{\partial X^2} + \lambda^i (V^j - V^i) \qquad \text{(A1)}
\end{aligned}
$$

if $q > p_k$ and for zero investment

$$
rV^i = hX^{\eta_x}K + \mu_d X \frac{\partial V^i}{\partial X} + \frac{\sigma_d^2}{2} X^2 \frac{\partial^2 V^i}{\partial X^2} + \lambda^i (V^j - V^i) \qquad \text{(A2)}
$$

where q is independent of the capital stock K. The subscript i describes the regime H or L of the starting period while j is the complementary regime. Letting $V^i =: G^i + Kq^i$ decomposes the problem into two separate differential equations for G and q. Since G does not influence optimal investment (equation (16)) we consider the from equation (A1) extracted differential equation

$$
rq^i = hX^{\eta_x} - \delta q^i + \mu_d X \frac{\partial q^i}{\partial X} + \frac{\sigma_d^2}{2} X^2 \frac{\partial^2 q^i}{\partial X^2} + \lambda^i (q^j - q^i). \qquad \text{(A3)}
$$

Substituting new variables

$$
v_1 := q^H - q^L, \qquad v_2 := (q^H \lambda^L + q^L \lambda^H)/(\lambda^H + \lambda^L) \qquad \text{(A4)}
$$

yields

$$
\begin{aligned}
(r+\delta+\lambda^H+\lambda^L)v_1 &= X\mu_d \frac{\partial v_1}{\partial X} + \frac{1}{2} X^2 \sigma_d^2 \frac{\partial^2 v_1}{\partial X^2} \qquad \text{(A5)} \\
(r+\delta)v_2 &= hX_0^{\eta_X} + X\mu_d \frac{\partial v_2}{\partial X} + \frac{1}{2} X^2 \sigma_d^2 \frac{\partial^2 v_2}{\partial X^2} \qquad \text{(A6)}
\end{aligned}
$$

This differential equations can be handled analytically and we get the closed-form solutions

$$
\begin{aligned}
v_1 &= c_1 X^{\frac{2\mu_d - \sigma_d^2 + \sqrt{8(r+\delta+\lambda^H+\lambda^L)\sigma_d^2 + (2\mu_d - \sigma_d^2)^2}}{2\sigma_d^2}} \qquad \text{(A7)} \\
&\quad + c_2 X^{\frac{2\mu_d - \sigma_d^2 - \sqrt{8(r+\delta+\lambda^H+\lambda^L)\sigma_d^2 + (2\mu_d - \sigma_d^2)^2}}{2\sigma_d^2}}
\end{aligned}
$$

and

$$v_2 = \frac{hX^{\eta_x}}{\eta_x\mu + \eta_x(\eta_x - 1)\frac{1}{2}\sigma^2 - (r+\delta)} + c_3 X^{\frac{2\mu_d - \sigma_d^2 + \sqrt{8(r+\delta)\sigma_d^2 + (2\mu_d - \sigma_d^2)^2}}{2\sigma_d^2}} + c_4 X^{\frac{2\mu_d - \sigma_d^2 - \sqrt{8(r+\delta)\sigma_d^2 + (2\mu_d - \sigma_d^2)^2}}{2\sigma_d^2}} \quad \text{(A8)}$$

It is easy to verify that the solution that satisfies the boundary condition occurs when $c_1 = c_2 = c_3 = c_4 = 0$, which yields

$$q^H = \frac{v_1\lambda^H}{(\lambda^H + \lambda^L)} + v_2 = \frac{hX^{\eta_x}}{\eta_x\mu + \eta_x(\eta_x - 1)\frac{1}{2}\sigma^2 - (r+\delta)} \quad \text{(A9)}$$

and

$$q^L = -\frac{v_1\lambda^L}{(\lambda^H + \lambda^L)}] + v_2 = \frac{hX^{\eta_x}}{\eta_x\mu + \eta_x(\eta_x - 1)\frac{1}{2}\sigma^2 - (r+\delta)} \quad \text{(A10)}$$

In other words, the formula for optimal investment $I^* = \frac{q^i - p_k(1-t^i)}{\gamma}$ does not include the variables t^j and λ^i which determine tax policy uncertainty and therefore actual investment is independent of tax policy timing and parameter uncertainty.

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