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CESifo Working Papers

NATIONAL AND SUPRANATIONAL BUSINESS CYCLES (1960-2000): A MULTIVARIATE DESCRIPTION OF CENTRAL G7 AND EURO15 NIPA AGGREGATES

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CESifo Working Paper No. 658 (5)

January 2002

Category 5: Fiscal Policy, Macroeconomics and Growth

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ISSN 1617-9595

An electronic version of the paper may be downloaded
• from the SSRN website: www.SSRN.com
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* Comments are appreciated. So far, I thank Claude Hillinger for his comments and suggestions.

CESifo *Working Paper No. 658*
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Abstract

This paper applies volatility measures and VAR spectral analytic techniques to give a thorough description of the salient business cycle characteristics of central NIPA aggregates for the G7. Furthermore, their role in contributing to the supranational G7 and EURO15 cyclic dynamics is investigated. Several refinements of conventional methods are suggested. Three different detrending filters are used, including ones that tend to distort the spectrum in opposite directions. As advocated in the literature, features of the spectra which survive these different detrending procedures are considered to be robust. The study reveals evidence of significant classical business cycles with frequencies corresponding to about two to four and about seven to ten years period lengths that are both reflected also in the spectra of the two supranational economies.The central findings are summarized with regard to (i) national product share, contribution-to-variance and volatility charateristics, (ii) explained variance and prominence of the cyclicalities contained in the different macro-aggregates and (iii) lead-lag and coherence relationships with national and supranational product cycles.

JEL Classification: E32.

Keywords: business cycle dynamics, spectral analysis, volatility.

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1. Introduction

Recently, Christiano and Fitzgerald (1998) note: "Good fiscal and monetary policy requires a clear understanding of the workings of the economy, especially what drives the business cycle ... Since at least the late 1800s, a full swing from the start of an economic expansion to a recession and back to the start of another expansion has generally taken between two to eight years. Every citizen is keenly aware of the state of the economy, whether it is in prosperity or in recession," p. 56. This is especially important with regard to the recent early stages of implementation of the EMU and the impending second 'Delors Stage' of the Maastricht Treaty. Only a clear understanding of the national business cycle dynamics in relation to a 'supranational' economy like the EMU, based on a thorough and sophisticated descriptive analysis, will show the potential effectiveness, range of coverage and limits of a concerted fiscal and monetary policy.[1] The present study aims to undertake a step in this direction. Related studies are *inter alia* the ones based on time domain analysis by Artis and Zhang (1999), Duecker and Wesche (1999) and Gallegatti and Gallegatti (2000) and, based on time and frequency domain analysis, the ones by Croux, Forni and Reichlin (2001), Forni and Reichlin (1999), Forni et al. (2001) and Den Haan and Sumner (2001).

When dealing with the determination of business cycle stylized facts, it seems natural to analyze the time series at stake first by means of a general analysis of variance to quantitatively assess their relative volatility and contribution to the constitution of some aggregate series and second to investigate their cyclical properties in the frequency domain by means of spectral analysis. Given that the latter method seems an ideal tool for studying cyclical phenomena, one might wonder why it has not been used more in economics. In my opinion the answer lies in four problems that spectral analysis faces when confronted with economic data:

First, it can be applied only to stationary time series. Because most of economic variables contain a trend component, this must be effectively removed. Failure to do this leads to the "typical spectral shape," reported by Granger (1966), where most of the mass of the spectrum is skewed in a "hump-shaped" fashion to the low frequency range. However, the technical appendix A to the present study suggests adequate strategies as well as recently established detrending technologies to overcome this problem.

The second problem is that many economic series are so short that classical non-parametric methods of spectral analysis cannot be successfully used. Although, this circumstance led to the development of several modified standard methods, like window-smoothing of spectra, they continued to fail to produce reasonable estimates: Both, the standard periodogram and window-smoothed versions of it lack a satisfying degree of resolution in the case of short time series. They usually show many discrete spikes in the estimated spectral densities which often lack a clear shape and make it very difficult, if not impossible, to discriminate between different peaks. These drawbacks can result in misleading interpretations in the sense of spurious cycles. Some authors even claim that this difficulty in interpreting estimates of spectra in an "economically meaningful" way has been the primary obstacle to a wider use of spectral analysis in economics, cf. the argumentation in Zarnowitz (1985 and 1996, chapter 8). For a detailed examination and

[1] Cf. also the discussion in Muellbauer (1997), whether Great Britain really should participate in Delors II from a strategic (business cycle) policy point of view.

demonstration of the disadvantages of standard techniques see Woitek (1997, chapter 2).

A third reason for the relatively low spread of spectral analytical applications in economics, might be their emphasis on description rather than testing, though they are recently experiencing a kind of renaissance.[2] The request for standard errors and measures of fit of the model is a request with which the business cycle analyst, who applies spectral methods, is not seldomly confronted with. The duality-relationship between the parametric approach and AR/VAR processes is a feature that also overcomes this objection. In the technical appendix to the present study, it is shown that ascribing standard errors to the central AR spectral estimates is possible without relying on inadequate bootstrapping or similar practices.

Finally, methods from the (putatively) less complex time domain, like cointegration analysis, seem to (still) play the predominate role in applied business cycle analysis. However, standard time domain estimates, like cointegration measures, are often binary and fail in contrast to multivariate spectral measures to establish different degrees of association. In general, spectral measures can be quite more informative and quantitative in nature in comparison to their corresponding time domain measures. See in the multivariate case (informational content of coherence vs. cointegration measures) Croux, Forni and Reichlin (2001) or in the univariate case (informational content of autocovariance functions vs. spectra) Howrey (1980).

The paper is divided into three main parts: A first non-technical part of the paper describes the analyzed time series and data sources and presents a detailed interpretation of the results. It is followed by a tabular part displaying, in detail, the estimation results and figures. Finally, the technical appendices A to C outline the applied methodology in technical terms. The text is organized as follows: The proceeding section gives an overview of the data sources and macro-aggregates under investigation. It formulates the addressed questions and briefly describes the central calculated volatility and spectral measures. Section 3 then documents the national uni- and bivariate business cycle stylized facts, both with regard to the respective aggregate's contribution and to country specific characteristics. Stylized facts that are salient over all three applied detrending methods are considered to represent robust stylized facts. They are reported for uni- and bivariate estimations in the last two subsections of section 3, respectively. Section 4 documents the contribution and respective bivariate relationship of the different aggregates and economies to the cyclic dynamics of the G7 and EURO15 supranational economies. Finally, section 5 concludes.

2. Data and Strategy

The central data source for the analysis of the present study is the national income and productivity accounts (NIPA) annual statistics database maintained by EUROSTAT, statistical office of the European Commission. As supplementary sources the OECD Statistical Compendium (on CD-ROM) and the International Financial Statistics (IFS on CD-ROM) provided by the IMF are considered.[3] The series are in annual frequency for the

[2] See *inter alia* the recent contributions of Forni and Lippi (1997), Croux, Forni and Reichlin (1999), Forni and Reichlin (1999) and Forni et al. (2001).

[3] This reverting to OECD and IMF data sources concerns in detail: the private consumption (CON) series for Canada for the years from 1961 to 1963 and the observation for 2000. The GDP series for Canada from 1961 to 1964, the total private investment (TPI) series for Canada for the years 1961 to 1963, 1993,

period 1960 to 2000, making a total of 40 observations. Throughout, they are provided in deflated form on the base year 1990.[4] Furthermore, the NIPA classification scheme of the US Bureau of Economic Analysis (BEA) is widely adopted and corresponding additivity guaranteed. Accordingly, the quantities add to GDP as displayed in the following:

Gross Domestic Product (GDP)
1. Personal consumption expenditures (Private consumption: CON)[5]
2. Gross private domestic investment (Total private investment: TPI)
   - 2a. Fixed investment (Gross fixed capital formation: GFCF)
   - 2b. Change in private inventories (Inventory investment: II)
3. Net exports of goods and services (Primary current account: PCA)
   - 3a. Exports (EXP)
   - 3b. Imports (IM)
4. Government consumption and gross investment (Government spending: GOV)

By considering explicitly the two investment components, GFCF and II, the present study places in the investment cycle tradition (see Hillinger (1992), p. 5-46) special emphasis on these two variables. By means of the instruments described in detail in the technical appendices the following questions are addressed:

1. How volatile is the respective series and how much does the series, as a constituting component, contribute in magnitude and volatility terms to a respective aggregate of higher statistical dimension?
2. Are there regular cycles in the series? What are the period lengths of these cycles?
3. Is the cyclical structure significant and how prominent is it in the overall dynamics, i.e. how strongly and significantly is it damped?
4. If the series show profound and significant cyclical structure, how many cycles can be identified?
5. How important are they, i.e. how large is the part of the variance explained by one or more complex roots?
6. How robust are the results for different detrending procedures?
7. What can be said about the lead-lag relationship between the constituting cycles and the respective national and supranational aggregate?

The following paragraphs give a short overview of the central calculated volatility and spectral measures of the analysis, including how to quickly interpret them.

Relative trend level (RTL), *standardized standard deviation* (SSD)
& *contribution to standard deviation* (CTS):

1994 and 1997 to 2000, as well as for the US from 1996 to 2000. The data for gross fixed capital formation (GFCF) and inventory investment (II) series stem for all analyzed economies from OECD databases.

[4] The quantities are throughout billion $US in prices and exchange rates from 1990.

[5] In parantheses are the (eventually) more familiar notations, mainly, though not uniformly, used by the central databases of EUROSTAT, OECD and IMF.

The relative trend level quantifies the relative magnitude or share of the constituent series' trend level to the corresponding aggregate series trend level; see equation (B.1), technical appendix B. The SSD measure assesses the relative volatility of a variable. It is defined as the standard deviation of the detrended series of a variable at stake standardized on its mid trend level value; see equation (B.2), technical appendix B. CTS gives the contribution to standard deviation measured in standard deviation terms of the respective aggregate, i.e. in percent of the total aggregate's standard deviation; see equation (B.3), technical appendix B. A positive CTS-value identifies a destabilizing constituent component, a negative CTS-value identifies a stabilizing constituent component.

Period length (period) & *modulus:*
Period denotes the period length (in years) of the contained cyclic structure in the series according to the AR-spectral estimate. For the multivariate estimates it denotes the period length (in years) at maximum of squared coherency (*sc*). The modulus measures the damping rate of a cyclicality contained in a series. For a starting amplitude-value Θ, a modulus of 0.5 implies that the cyclic component of the series has a remaining amplitude-value of $\frac{1}{2}\Theta$ after one year, $(0.5)^2\,\Theta$ after two years, etc.

Squared coherency (sc) & *phase shift* (phase):
Suppose that the spectra of two series exhibit peaks at a certain frequency ω^*. *sc* then measures the extent of the linear relationship between the cycles in the two series. It can be interpreted similarly to a correlation coefficient. Phase measures the lead-lag relationship between two cyclicalities of the same frequency ω^* in the two series at stake, i.e. the distance between the turning points of contained cyclicalities with the same frequency. A value of phase > 0 indicates a phase lead (in years), a value < 0 a phase lag (in years), see equations (B.6) and (B.7), technical appendix B.

3. National Uni- and Bivariate Stylized Facts

The results of the preliminary analysis of volatility, relative magnitude and contribution to aggregate volatility are presented in Table 1. The univariate spectral estimations are displayed in Tables 2 to 4 for the three detrending procedures outlined in technical appendix A and Figures 1 to 3 for the modified Baxter/King (MBK)-filtered series[6] over the eight NIPA aggregates GDP, CON, TPI, GFCF, II, EXP, IM and GOV. It should be noted that in the case of two contained periodicities in the national series, the respectively longer cycles and corresponding moduli are displayed throughout in the first row of the respective NIPA aggregate's column, the respectively shorter cycle lengths and corresponding moduli in the second row. The reported period lengths are given in years and the adjusted R^2s correspond to the respectively underlying AR model estimation. Furthermore, it should be noted that a visual inspection of the raw inventory investment series leads to the conclusion that most of these series show stationary behavior. Log-differencing these series leads to a destruction of any potentially contained cyclic dynamics. Therefore Table 4 displays estimation results for the unfiltered inventory investment series.

The results of the first set of bivariate spectral estimations, i.e. every NIPA aggregate relative to its respective national GDP aggregate, are presented in Tables 5 to 7. Again,

[6] See technical appendix A.

in the case of two complex roots implied by the adjusted VAR system, the respectively longer cycles and corresponding bivariate measures are displayed first.

3.1 *General salient facts*

All of the following spectral estimations have to be seen in the light of the relative shares of contribution of the analyzed aggregates to the respective national GDP. This needs to be taken into account in both respects, first with regard to their relative trend levels (NIPA constituent series to national GDP) and relative standard deviation (of NIPA constituent series to mid trend level value of the constituent series), see equations (B.1) and (B.2), technical appendix B. This information is summarized in Table 1. The first measure, constituent trend level value relative to respective GDP trend level value, is given at three distinct points of the estimated MBK-trend functions: a) at the starting point, i.e. corresponding to the year 1963,[7] b) at the mid point, i.e. corresponding to the year 1980, and finally c) at the end point, i.e. corresponding to the year 1997. There are several things to note, regarding the results given in Table 1: For all economies the majority of results shows quite constant shares of their NIPA constituents to national GDP over the three selected points of observation - with inventory investment generally showing the smallest, private consumption the largest fraction. The exception is given by the two PCA constituents that display the least stable fractions over time, ranging from 4 to 5% for the US at the starting point up to equal or close to 40% for Canada at the mid point of the observation's horizon. Concerning the SSD measures, changes in private inventories, i.e. the II series, show throughout the highest volatility for all NIPA constituent series. This volatility of inventory dynamics is, in comparison to the volatility of the respective national GDP data, the highest for Germany, the U.K. and especially Canada (making up to about 50-times the volatility of Canadian GDP), the lowest it is for the Japanese economy (making up a little less than 5-times the volatility of Japanese GDP). This suggests the interpretation that the extensive implementation of innovative inventory management systems like just-in-time production, the Kanban-system, etc. during the last decades has led to a remarkable reduction of the volatility in the II series of the Japanese economy in an international comparison. Nevertheless, Japan shows the highest relative variance in GDP. A fact that might at least partially be induced by the recent and severe breakdown of the financial system in Japan, known as the Asian crisis. With the exception of the Japanese economy, the second most volatile series for all other G7 economies are the two remaining investment series under investigation, namely TPI and GFCF. Furthermore, the US economy shows a most distinct characteristics: The high SSD measures of TPI and GFCF, compared to the US GDP series,[8] can be seen as an indication that the United States play the role of an "investment dynamics dominated" economy. An indication which is further underpinned in the course of the present study. Throughout the G7 economies the least volatile series are the EXP and CON series, while IM and GOV series seem to be intermediary volatile. Let us turn to the remaining CTS measures, presented in Table 1. As argued above, a CTS-value > 0 characterizes a destabilizing component, a CTS-value < 0 a stabilizing constituent variable; see equation (B.3), technical appendix B. As widely agreed on, the calculated

[7]Note the MBK-filter shortens the series by three datapoints at the beginning and end of the series, respectively.

[8]Even outnumbering the SSD of US GDP by taking on a value more than 5- to 6-times that of GDP.

figures of Table 1 classify for all G7 countries the IM series as a stabilizing variable. Technically, it should be noted that for a substractively contributing variable, like IM (since GDP = CON + TPI + GOV + PCA, where PCA is defined as EXP − IM), the construction of the CTS-measure is modified in that the nominator of equation (B.3), technical appendix B, becomes $\sigma(y^a) - \sigma(y^a + y_i^c)$. There are three more negatively valued CTS measures displayed in Table 1, but their negligible absolute sizes suggest that these series hardly contribute to the respective GDP series' standard deviation at all. While for all other G7 economies, the pattern is such that the CON series contributes about double the value (on average 57% of standard deviation terms of the aggregate) to the respective national product in contrast to the investment components TPI, GFCF and II (for example TPI on average 24%), the US stand out: For the US economy the TPI series contributes 57% to the standard deviation in US GDP and furthermore it shows, in comparison to the CON series, a close to 1.6-times higher CTS value (37% vs. 57%). Even the isolated GFCF series still contributes 39%, and therefore more than US CON, to the standard deviation of the US GDP series. This can be seen as further evidence pointing in the direction that the US product's business cycle component extracted by means of the MBK-filter is dominated by its constituting investment variables.

Table 1. *G7 Economies*: GDP MBK-trend level share, CTS and standardized std. dev.

			CON	TPI	GFCF	II	EXP	IM	GOV
Canada	RTL	starting	0.61	0.23	0.22	0.01	0.18	0.18	0.14
		mid	0.59	0.18	0.18	0.01	0.40	0.38	0.33
		end	0.55	0.23	0.23	0.01	0.26	0.25	0.33
	CTS		0.43	0.27	0.27	0.01	0.11	-.23	0.27
	SSD	(GDP: 0.03)	0.02	0.06	0.05	1.44	0.04	0.04	0.04
France	RTL	starting	0.61	0.24	0.23	0.01	0.13	0.12	0.13
		mid	0.60	0.17	0.18	0.01	0.25	0.22	0.19
		end	0.58	0.23	0.22	0.01	0.21	0.21	0.17
	CTS		0.57	0.24	0.23	0.02	0.18	-.19	0.16
	SSD	(GDP: 0.06)	0.06	0.08	0.07	0.55	0.06	0.07	0.06
Germany	RTL	starting	0.59	0.27	0.25	0.01	0.17	0.16	0.15
		mid	0.65	0.21	0.21	0.01	0.25	0.24	0.11
		end	0.62	0.21	0.21	0.01	0.25	0.24	0.14
	CTS		0.61	0.24	0.18	0.04	0.17	-.22	0.13
	SSD	(GDP: 0.08)	0.08	0.11	0.09	2.19	0.09	0.09	0.08
Italy	RTL	starting	0.62	0.23	0.27	0.01	0.14	0.14	0.13
		mid	0.61	0.17	0.18	0.01	0.26	0.22	0.16
		end	0.60	0.25	0.23	0.01	0.23	0.23	0.14
	CTS		0.59	0.25	0.24	0.02	0.15	-.21	0.15
	SSD	(GDP: 0.07)	0.07	0.10	0.08	0.66	0.05	0.07	0.07
Japan	RTL	starting	0.58	0.34	0.31	0.02	0.09	0.09	0.08
		mid	0.60	0.28	0.28	0.01	0.10	0.08	0.09
		end	0.58	0.31	0.30	0.01	0.12	0.12	0.09
	CTS		0.60	0.26	0.26	0.00	0.04	-.02	0.10
	SSD	(GDP: 0.14)	0.14	0.13	0.13	0.67	0.06	0.11	0.15

Table 1 (continued). *G7 Economies*: GDP MBK-trend share, CTS and std. std. dev.

			CON	TPI	GFCF	II	EXP	IM	GOV
U.K.	RTL	starting	0.65	0.18	0.17	0.01	0.19	0.20	0.17
		mid	0.64	0.16	0.17	0.01	0.28	0.29	0.18
		end	0.59	0.18	0.17	0.01	0.27	0.26	0.21
	CTS		0.62	0.18	0.19	-.01	0.19	-.24	0.18
	SSD	(GDP: 0.06)	0.07	0.10	0.09	3.23	0.05	0.08	0.06
USA	RTL	starting	0.62	0.19	0.13	0.06	0.05	0.04	0.17
		mid	0.70	0.18	0.18	0.01	0.11	0.13	0.15
		end	0.63	0.20	0.14	0.06	0.09	0.09	0.16
	CTS		0.37	0.57	0.39	0.16	-.01	-.22	-.02
	SSD	(GDP: 0.01)	0.01	0.05	0.06	0.07	0.05	0.04	0.01

General salient facts, regarding the spectral estimation results reported in Tables 2 to 4 and the spectra in Figures 1 to 3 are: As discussed in somehow greater detail in technical appendix A, the HP(100)-filter is most obviously biased in favor of the lower frequencies, i.e. higher period cyclicalities. Furthermore, as is the case for several national series, the HP-filter does not sustain potential second periodicities. As expected the logD-filter destroys a lot of contained periodicity and is biased towards relatively short cyclicalities, cf. the results for the Japanese economy in Table 4.

The graphics of the spectral densities displayed in Figures 1 to 3 are inspired by Sargent (1987, chapter XI, section 10). The central features that should become obvious are a) the shape of the spectrum and b) its position (left or right) towards the median peak frequency (below or above) of the G7 economies, as indicated by the parallel straight line to the ordinate. Concerning a) on the one hand it is interesting whether one or more periodicities are contained in the series, i.e. whether the power spectra are single or double peaked. Secondly, the sharpness of the spectrum is relevant, i.e. the less damped the roots are, the more regular are the cycles in the series, and the higher and sharper is the peak in the spectrum, cf. the remarkably low bandwidth of the spectral density for the Japanese GDP series, for example in comparison to the relatively flat spectrum of the Canadian TPI series. Concerning b) the median period length is matched for GDP and CON by Italy, for TPI, GFCF and II by Japan, for EXP and GOV by the US and finally for the IM series by the U.K. economy. A final feature of the spectral graphics displayed in Figures 1 to 3 is the dashed line that represents the integrated spectrum. The closer this line is to an imaginary 45 degree line, the more the dynamics of the series are driven by noise. The integrated spectrum of a white noise process would exactly equal the 45 degree line.[9]

Disregarding the two primary current account components, the clearest double peaking spectral density is given by the German economy. Except for the two private investment components the peak in the higher frequency range dominates the long cycle in the Ger-

[9]Note, there exist several tests that avail this feature of the integrated spectrum to test on cyclicality. These include the Fisher Test, Bartlett's Test, the Durbin-Test and the Kolmogorov-Smirnov-Test, see for example Brockwell and Davis (1991, § 10.2). Unfortunately, they are throughout of weak power and based on the (nonparametric) periodogram method and therefore unsuited for the short time series under analysis in the present study.

man case. For the German investment components (GFCF and II, as well as TPI, i.e. the sum of both,) both cycles concentrate a reasonable amount of spectral power, while the longer cycle bears about 20 to 50% more spectral mass.

Another important and interesting piece of information is given by the median values of estimated univariate spectral measures. These are displayed in the respective last row of Table 2 to 4. As a technical remark, note that only in cases where ≥ 2 periodicities and corresponding measures are estimated over the 7 economies a median value is calculated. Furthermore, in cases when the underlying distribution over the G7 economies is of an even number of results, the median values are 'artificial' figures corresponding to the averages of the two middle-valued estimation results. This is the case for the variables CON, GFCF and II in Table 2, for GDP and IM in Table 4. Comparing these median values for the different detrending filters (Table 2 to 4), it becomes obvious that in general the highest importance, in the sense of explaining variance of the detrended series (adjusted R^2 values), of the contained cyclicalities is estimated for the HP-, followed by the MBK-filter. In both cases the cycles explain from about one third to more than 60% of the variance of the respective series (in median terms). For the MBK-filter results, the most variance is explained in median terms by the cycle in GFCF, followed by the one contained in the GDP and GOV series, respectively. For the HP-filter results the corresponding top three important cycles are to be found in CON, GDP and GFCF - throughout explaining more than half of the variance contained in the respective series. For the logD-filter these are II, CON and TPI. Concerning the prominence of the cycles, as expressed by the median values of the moduli, the following patterns are striking: For the MBK-filtered series, the cyclicalities of the TPI, II and GFCF series are the most prominent, showing reasonably high moduli > 0.85. Similarly, for the HP-filtered series, the cycles contained in the CON, GFCF, GDP, II and GOV series imply moduli > 0.8. Finally, for the logD-detrending procedure, the inherent cycles in CON, TPI and II, though over all rather weakly significant (in median R^2 terms), still show moduli > 0.65. Nearly all median values of estimated period lengths (in years) lie in the frequency range of classical (Juglar) business cycles with periodicities ranging from about 5-6 years up to approximately 9 years. The only exception, in the sense of an outlier, is given for the logD-filtered GDP series' result (12.42 years). As expected the median values of period lengths for the II series reflect the shortest cyclicalities over the 8 analyzed aggregates, taking on values from about 3 to 6 years. Broadly interpreted, this is (weak) evidence for a Kitchin periodicity in the G7 II series.

3.2 *Robust univariate stylized facts: National cycles*

Germany most clearly shows a "two cycle"- or "M/W-cyclical" dynamic structure over all NIPA aggregates. With the exception of the two PCA series this M/W-shape is confirmed over all three detrending procedures and might therefore be seen as a robust stylized fact. As indicated by the relatively higher moduli and lower standard errors, the shorter cyclic pattern dominates the German economy, cf. also the argumentation above and the spectra in Figures 1 to 3. As we will see this fact will be confirmed by the national and supranational bivariate spectral estimations, too. In contrast to this finding, for the French economy, only in the case of series detrended by the MBK-filter, the GFCF and II series show a long as well as a short business cycle periodicity at the classical frequencies. Japan shows a remarkably prominent long cycle of about 7 years length in its NIPA aggregates. Taking the results for the HP- and logD-filter into account, Japan's TPI and GFCF series seem to also bear this period length, while Japanese inventory investment series show a weakly estimated and more strongly damped classical Kitchin periodicity of about 4 to 5 years length. The latter seems to hold also for the U.K. economy. Although the shorter and more strongly damped cycle seems to stem from an aliasing[10] of a double-cycle inventory dynamics. In line with earlier findings, cf. Woitek (1996), the United States seem (in contrast to the other G7 economies) be predominated by a periodicity of an intermediary length of about 5 to 6.5 years length in GDP and TPI. This can clearly be seen also from Figure 1, where the US spectra nearly throughout peak right from, i.e. at a higher frequency than the median G7 frequency, as indicated by the parallel line to the ordinate. The exceptions are the US government spending and export series that match the G7 median frequencies.

3.3 *Robust bivariate stylized facts: National cycles*

In this subsection I will first describe and interpret the bivariate spectral findings country by country (for all three filters) and then illuminate the salient bivariate features of the NIPA business cycle components in general by means of the mean bivariate spectra (for the MBK-filter), as displayed, constituent by constituent, in Figure 4.

Concerning *Canada*: Obviously, the Canadian exports series contribute about 5 to 10% to the standard deviation of Canadian GDP and lag the Canadian GDP cycle, according to the MBK- and HP-filter results of Table 5 and 6, by 1 to 2 years. Most probably this also holds for the Canadian imports (a stabilizer, reducing GDP volatility by about 20%), though by slightly less time, i.e. by about 0.5 to 1 year. Both PCA components fluctuate less coherent with Canadian GDP than the other NIPA constituents that behave remarkably coherent. According to the MBK- and logD-filter results, the cycle in Canadian inventories obviously lags the national product cycle by approximately 2.5 to 3 years, though contributing negligibly to the variance of the Canadian product cycle.

Concerning *France*: One can hardly make any real robust statement on the bivariate dynamics of the French NIPA series with its GDP cycle. Nevertheless, it can be stated that: Firstly the French PCA components seem to move quite coherent with the French GDP cycle at a single periodicity of about 6.5 to 10 years period length. And secondly, according to the logD- and HP-filtered series' results, the inventory cycle potentially bears an unconventionally long period length of about 9 to 10 years length and lags the GDP cycle by about one year.

Concerning *Germany*: All German constituent series are highly coherent with the national GDP cycle, while in growth rates (logD-filter) only CON and the two investment components (GFCF and II) show coherent cyclic dynamics with regard to the German GDP cycle. Most of the German constituent NIPA aggregate cycles also fluctuate quite in phase with the cyclic pattern in GDP. Comparing the MBK- and HP-filter results, it remains unclear whether German exports lead or lag the GDP cycle by about one year.

Concerning *Italy*: According to the national GDP bivariate estimation results, the Italian economy is especially characterized by a traditional and approximately in GDP cycle phase TPI cycle with a traditional length of about 7.5 to 10 years, see Table 5 and

[10] Aliasing in the context of spectral analysis denotes the case when two frequencies close in value "fuse" due to their closeness to one single frequency in the spectrum; see, for example, Marple (1987, section 2.6).

6. Furthermore, Italy seems to bear a robust (i.e. over all filtering methods consistently estimated) inventory cycle in the constitution of its national GDP with a rather unconventional length of about 6 to 10 years. This inconvenient II cycle fluctuates, as in the case of the French economy, only weakly coherent to the GDP cycle and lags it by up to about a half year.

Concerning *Japan*: The Japanese economy is robustly characterized by its highly GDP coherent investment (including inventory investment) cycle dynamics and the cyclicalities inherent in its import series. The former[11] reflects the "central role" that "investment has played ... in both the growth and fluctuations in the Japanese economy." This feature as well as the questions why "business cycles in Japan were basically investment-led" and "why Japanese firms invest so much," are extensively outlined in a historical perspective of the Japanese post world war II economy by Yoshikawa (1995), chapter 7, p. 244-291. For the Japanese inventory series, the in GDP cycle phase periodicity shows the classical short cycle period length of 4 to 5 years, although it is certainly estimated less coherent and contributing marginally in standard deviation terms (for the aforementioned reasons) to the longer Japanese GDP cycle periodicity, cf. Figure 1. The GFCF series contributes about 27% to the product series' standard deviation. It shows its highest coherency with the GDP cycle at period lengths of about 11 to 12 and 7 years. These periodicities might be aliased to a single periodicity of 10.75 years length at the maximum of squared coherency in the HP-filter case. A further indication for an aliasing problem in this context is that for both logD- and MBK-filter results, the one cycle slightly leads, the other slightly lags the GDP cycle. Similarly, the two periodicities (of about 12 and 2 to 7 years length) at maximum of squared coherency for logD- and HP-filtered TPI series might be aliased to a single period length of 6.5 years, shared with the GDP cycle, in the case of the MBK-filtered TPI series. On the contrary, there is a robust double period coherency pattern for the Japanese import series, though stabilizing the Japanese GDP cycle only marginally ($< 10\%$). Most robustly, the Japanese imports fluctuate a) quite regularly, cf. the narrow bandwidth of the spectrum displayed in Figure 2, b) highly and the most coherent with the Japanese GDP cycle at a period length of about 7 to 7.5 years[12], and c) with a phase lead of approximately 1.5 years relative to the national GDP cycle.

Concerning the *United Kingdom*: As expected on base of the univariate spectral estimations and since the U.K. shows the longest of all G7 GDP cycles, see Figure 1, the highest coherency of U.K. NIPA constituents with the national GDP cycle is estimated at a period length of, on average, 8 years. Especially the TPI and GOV series show a high coherency at a traditional long cycle frequency corresponding to 8 to 9 years and a relatively lower coherent classical short cycle at a period length of approximately 3 to 4 years. Robustly, the TPI cycles slightly lag, while the GOV cycles slightly lead the cyclicalities contained in the U.K. GDP series. Concerning the TPI cycle the long one most probably stems from a traditional long and GDP coherent common cycle in the GFCF series (< 8 years) in combination with an approximately 8 years cycle, contained in the II component and lagging the GDP cycle by about 2 years. The short lagging cycle, also inherent in the TPI series, may be constituted from the short cyclicality of about

[11] Contributing roughly 30% to the standard deviation of the Japanese GDP cycle component.

[12] Although there is another shorter period cycle of about 3 to 4 years length that less coherently fluctuates in line with the Japanese GDP cycle.

4 years contained as well in the II series. This is at least indicated for the MBK- and logD-filtered series' results. Furthermore, it should be noted that the CON series of the U.K. fluctuates robustly both strongly coherent to and in phase with the national GDP series at a somehow shorter than the univariate GDP cycle length of about 7 to 8 years.

Concerning the *United States*: As can be seen from the CTS measures in Table 5, the US TPI and GFCF series highly contribute to the standard deviation of the MBK-filtered GDP series (in both cases even more than does the MBK-filtered private consumption series). It is noteworthy that, as in the case of the Japanese economy, the US economy is robustly characterized by its highly GDP coherent investment (including inventory investment) cycle dynamics[13] and the cyclicalities inherent in both national PCA components. There is a "small but growing" body of empirical literature that investigates the business cycle nexus between Japan and the United States as well as generally on international business cycle interdependencies. It is surveyed *inter alia* in Selover (1997), Artis and Zhang (1999) and Imbs (1999). Selover (1997) also finds that despite the popular saying in Japan that 'if the United States sneezes, Japan catches a cold', "the US does not 'drive' the Japanese economy, rather it transmit shocks which partially synchronize the two economies, perhaps through the 'mode-lock' phenomenon," p. 385. Besides its cyclic dynamic analogy to the Japanese economy, the US exports quite robustly fluctuate the most coherent with national GDP at a periodicity of 6.5 to 8 years length and lag the GDP cycle by about 1.5 to 2 years. On the contrary, US imports slightly lead the GDP cycle and show an intermediate strength of coherency at a period length of approximately 6 to 6.5 years.

Let me now turn to document the salient bivariate features of the NIPA business cycle components in general, i.e. constituent by constituent series. I will do so on the base of the mean bivariate spectra (for the MBK-filter), namely the mean densities of the respective coherence spectra, see equation (B.7), and phase spectra, see equation (B.6), technical appendix B. The idea to construct the mean bivariate spectral densities over a population of units of investigation (in the present case economies) is adopted from Forni and Lippi (1997, Fig. 14.4) and Forni and Reichlin (1998, Fig. 7). These authors proceed similarly in constructing the sum of cospectra[14] over sectoral estimation results. The essential findings can be summarized in the following: The respective CON series fluctuates on average highly coherent to the respective GDP series of the G7 economies. Both bivariate spectra show a clear and smooth shape. The phase spectrum suggests that the fluctuations contained in the CON series, on average, represent a leading indicator for the GDP cycles with a phase lead of about one year, as measured at the maximum of *sc*. The TPI cycles fluctuate also highly coherent to the GDP cyclicalities - maximally at frequencies corresponding to periodicities of approximately 9 and 4.5 years. It should be noted that the first peak of the coherence spectrum is rather broad, suggesting that for the first periodicity high coherence values are also obtained for classical cyclicalities with period lengths starting from 6 up to 9 years. These first shared cyclicalities nearly coincide with the GDP cyclicalities, while the second shorter ones obviously lead it by

[13]These, although sometimes showing two periodicities at their respective maximum of squared coherency, are dominated by a 5 to 5.5 years cycle that manifests itself and is reflected in the US GDP cycle - at least for the MBK-filtering routine.

[14]I.e. the real parts of the cross-spectra, implied in squared absolute expressions in the nominator of the coherence spectrum's definition, see equation (B.7), technical apendix B.

about two years. The GFCF cycles are on average as well estimated strongly coherent to the GDP cyclicalities, the strongest at a period length of 7.6 years with a marginal phase lag. On the other hand, the II series fluctuate on average less coherent to the GDP cycles, reflected in a maximum peak of the mean coherence spectrum at a *sc*-value of about 0.6. The mean phase spectral density has a quite jagged shape and indicates a coincidence relationship to the GDP cycle at a period length of about 6 to 7 years. Both PCA components show on average a quite high *sc* with GDP cycles, taking on a maximal value at the classical frequencies, corresponding to period lengths of about 7.5 to 7.8 years. Both represent rather coinciding GDP indicators. Finally, the coherence spectra of the GOV series show on average their highest coherence with GDP cycles at a periodicity of about 10 years, though the peak is, as in the case of TPI quite broad, implying also periodicities with higher frequencies (shorter period cycles) to stand in a relatively high coherence relation to the GDP cycles.

4. Supranational Bivariate Stylized Facts

Hitherto our investigations have shown that the MBK-filter provides reasonably intermediary results concerning the spectral estimation results of all three applied filtering procedures. Furthermore, it is the latest detrending technology and in several respects superior to the conventional BK bandpass filter, see Woitek (1998). I therefore decided to concentrate on spectral estimation results based on MBK-filtered series. In Figure 5, the univariate GDP cycle estimates for the (artificial) supranational economies G7 and of the so-called Euro15-zone (EURO15), including the economies of Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Netherlands, Portugal, Spain, Sweden and Great Britain) are displayed. Both show a double peaking spectrum at approximately similar periodicities of about 8 and 4 years. In contrast to the G7 GDP cycle, the EURO15 GDP cycle is characterized by a quite stronger prominence of its contained shorter classical cycle. In analogy to the bivariate stylized facts of national cycles, I will first make some brief general statements and then go through (only) the most salient supranational bivariate stylized facts country by country.

In general it can be noted that, for the above discussed reasons, Japan contributes the most in standard deviation terms to the total G7 GDP fluctuations, followed by the German, French and U.K. economy. The least contribute the Canadian and US economy. The most contribute the CON series, followed by the investment components (with the exception of II, contributing at maximum 1% in CTS terms), while both PCA components contribute throughout marginally to the amplitudes in supranational product series. Regarding the signs of CTS-values, the IM series play the role of a stabilizer. Germany, Italy and France contribute more or less in equal shares of about 18% to EURO15 GDP in standard deviation terms. In contrast, the U.K. contributes a relatively less figure of about 12%. *Prima facie*, the German economy seems to be responsible for the profound short cycle in the EURO15 GDP series and maybe also for the rather less profound one in the G7 GDP series. Most national GDP cycles close to coincide with the respective supranational ones. The period lengths at maximum of *sc* throughout match with classical frequencies, for Germany mainly with a short Kitchin periodicity of about 4 years, for the other 6 economies mainly with a traditional Juglar frequency of about 6.5 to 8 years length.

Concerning *Canada*: For Canada only the cyclicality inherent in the GFCF and CON series fluctuates - at a period length of slightly more than 7 years - profoundly coherent with and leads the G7 cycle by about 1.5 and 2 years, respectively.

Concerning *France*: Obviously, the French economy contributes to the classic long investment cycle of about 6 to 9 years found in both supranational GDP cycles. French inventory investments clearly lag the supranational product cycles and fluctuate only weakly coherent to and in phase with the short cycle frequency contained in both of them, cf. Figure 5. The most in phase with a supranational GDP cycle is French TPI, namely with the EURO15 GDP cycle.

Concerning *Germany*: Looking at the period lengths that a VAR system with German NIPA and supranational GDP series bears, it becomes quickly obvious that, despite the German GFCF series and partly also the German EXP series, all other German NIPA series at stake contribute clearly and, as reflected in relatively high *sc* values, profoundly to the secondary short periodic regularity found in both analyzed supranational series. Most constituent series, as well as the German GDP aggregate, are at this frequency quite in phase with both supranational series. Only the German GFCF series seems to slightly lag the supranational cycles and German exports lead the G7 product cycle by about one year.[15] It is noteworthy that the German economy shows the highest CTS to the EURO15 GDP values with regard to the other European economies.

Concerning *Italy*: The Italian NIPA series share an "in between"-frequency with the G7 product cycle, i.e. in between of the two frequencies found in the G7 GDP series. This might be due to the fact that the spectra for Italian series are mainly one-time peaking, see Figures 1 to 3. The Italian series seem to be closely related to the short European GDP periodicity, indicated by the shared period lengths, relatively high *sc* values (with the exception of the fluctuations in the II series) and a phase position quite in phase (< 1 year) with both supranational product cycles. This may be interpreted as an indication in favor of the hypothesis put forward by Artis and Zhang (1999). Accordingly, the Italian cycle had a "strong idiosyncratic element" in the pre-1975 period, while for the post-1975 period EU countries in general had become "more synchronized with the German cycle," p. 120-122.

Concerning *Japan*: The outstanding volatility of the Japanese business activity is reflected in the highest destabilizing CTS measures of all analyzed economies. This fact is, as already mentioned, not that surprising if one considers the development of the Japanese economy in the last 15 years. For example Kiyotaki and West (1996) note: "In the last decade, the Japanese economy has gone through both its strongest expansion of the last twenty years and its most severe recession of the last forty years. During this decade, business fixed investment was unusually volatile, and ... seemed to be a dominant factor in both the recent 1986-1991 boom and the post-1991 bust," p. 277. As in the Italian case, Japanese NIPA series are characterized by sharing "in between"-frequencies with the G7 product cycle. With the exception of the Japanese inventory fluctuations, they move quite coherent and, with the exception of the bivariate VAR for Japanese imports and supranational product series, quite in phase with the G7 GDP cycle.

Concerning the *United Kingdom*: In contrast to German macro aggregates, the ones of the U.K. behave first and foremostly in line with the long traditional cycle, found in both supranational GDP series. The constituent and national GDP series move quite

[15]Regarding the national stylized facts above, the reader may be warned that the latter result may be dependent on the applied detrending procedure.

coherent and in phase with the supranational product cycles. The obvious exception, as in the case of the French economy, is the II series that shares an intermediary periodicity in combination with a weak coherence with and a phase lag relationship to the supranational product cycles.

Concerning the *United States*: As already stated above and indicated in the results displayed in Table 1, the US economy's cyclical structure is coined by investment cycle dynamics. A fact that is also reflected in the bivariate spectral estimations for its investment components and the G7 GDP series. In contrast to the other US NIPA series, there is a strong coherency between the G7 GDP cycle and the US investment NIPA series, TPI and GFCF. It is estimated at a classical business cycle frequency corresponding to an approximate period length of 7.5 to 8 years. The US TPI and GFCF cycles are lagging the G7 GDP cycle by a little less than one year. On the contrary, US private consumption and exports are clearly leading indicators for the G7 GDP cycle with a phase lead of 2.1 and 1.2 years, respectively.

5. Concluding Remarks and Summary of Results

The study reveals evidence of classical business cycles with frequencies corresponding to about two to four and about seven to ten years period lengths that are both also reflected in the spectra of the two supranational economies. Close to all univariate spectral measures are estimated significantly. We may summarize:

(i) The central contribution-to-variance, GDP share and volatility characteristics are the following: As generally agreed on, imports act as stabilizing quantities in the constitution of the national product series. The most destabilizing variable is private consumption with the exception of the US economy for which the total private investment series is the most contributing to volatility and standard deviation terms of the GDP series (about 60% percent to the standard deviation of the GDP business cycle component). For all analyzed economies the majority of results shows quite constant shares of their NIPA constituents to national GDP over different points of the observation horizon - with inventory investment generally showing the smallest, private consumption the largest fraction. As expected, the inventory investment series show throughout the highest volatility of all NIPA constituent variables. This volatility of inventory dynamics is the highest for Germany, the U.K. and Canada, the lowest for the Japanese economy. Nevertheless, Japan shows the highest relative variance in its national product series. With regard to the supra-economic dynamics, Japan contributes the most in standard deviation terms to the total G7 GDP fluctuations, followed by the German, French and U.K. economy. Germany, Italy and France contribute more or less in equal shares of about 18% to the EURO15 GDP series in standard deviation terms. In contrast, the U.K. contributes a relatively less figure of about 12%. The most contribute the private consumption series, followed by the investment components, while both PCA components and government spending contribute throughout only marginally to the amplitudes in the supranational product series.

(ii) The most variance is, in median terms, explained by the cycle in gross fixed capital formation, followed by the one contained in the GDP series. Throughout these cyclicalities explain more than half of the variance contained in the respective series. On average, the most prominent cycles are found in the investment components, i.e. in total private investment, inventory investment and fixed investment. The estimated periodicities correspond to classical business cycle lengths of about two to four and about seven to ten years length. Two economies stand out: First, Germany most clearly shows a double-cyclic dynamic structure over the majority of NIPA aggregates. Second, in line with earlier findings, the US seem to be predominated by a single periodicity of an intermediary length of about 5 to 6.5 years length in GDP and total private investment.

(iii) On average the lead/lag and coherence relationships are characterized by the following features: The national private consumption series fluctuate on average highly coherent to the respective GDP series of the G7 economies. The phase spectra suggest that the fluctuations contained in the consumption series, on average, represent a leading indicator for the national product cycles with a phase lead of about one year. The cycles in the total private investment series also fluctuate highly coherent to the GDP cyclicalities - on average, maximally at frequencies corresponding to periodicities of about 9 and 4.5 years. They close to coincide with the GDP cycles. The same holds true for the fixed investment series at a period length of about 7.5 years, while inventories fluctuate less coherent to the national product cycles. The PCA components show on average a quite high coherency with the product cycles, taking on a maximal value at a classical business cycle frequency, corresponding to a period length of about 7.5 years. Both represent rather coinciding GDP indicators.

DETAILED TABLES AND FIGURES

Table 2. *G7 Economies*: MBK-filtered series, univariate spectral estimations

		GDP	CON	TPI	GFCF	II	EXP	IM	GOV
Canada	Period[a]	7.206 (0.102)	7.923 (0.170)	5.025 (0.202)	7.447 (0.087)	4.787 (0.014)	5.608 (0.324)	5.614 (0.050)	5.253 (0.057)
	Modulus[a]	0.777 (0.002)	0.760 (0.002)	0.436 (0.006)	0.812 (0.001)	0.817 (0.001)	0.822 (0.001)	0.760 (0.002)	0.721 (0.002)
	adj. R-squ.	0.305	0.316	0.018	0.398	0.199	0.324	0.158	0.166
	AR order	3	3	2	3	3	3	3	2
France	Period	6.202 (0.062)	6.073 (0.059)	8.381 (3.843)	6.499 (0.069)	6.839 (0.043)	6.305 (0.054)	6.120 (0.056)	5.190 (0.084)
				3.864 (0.850)		3.218 (0.003)			
	Modulus	0.767 (0.002)	0.770 (0.002)	0.872 (0.003)	0.782 (0.002)	0.947 (0.001)	0.798 (0.001)	0.781 (0.002)	0.541 (0.003)
				0.809 (0.023)		0.869 (0.002)			
	adj. R-squ.	0.167	0.166	0.245	0.224	0.180	0.294	0.239	0.097
	AR order	3	3	5	3	5	3	3	2
Germany	Period	9.683 (2.222)	9.933 (2.457)	9.200 (1.703)	8.282 (2.660)	5.664 (1.878)	5.919 (0.037)	4.942 (0.016)	9.268 (2.237)
		4.113 (0.313)	4.180 (0.303)	3.887 (0.383)	3.797 (1.258)	3.031 (2.162)			4.124 (0.371)
	Modulus	0.820 (0.009)	0.839 (0.013)	0.838 (0.038)	0.859 (0.072)	0.899 (0.001)	0.791 (0.001)	0.774 (0.001)	0.838 (0.028)
		0.953 (0.009)	0.981 (0.010)	0.925 (0.001)	0.876 (0.001)	0.857 (0.001)			0.954 (0.007)
	adj. R-squ.	0.408	0.520	0.413	0.268	0.436	0.133	0.359	0.485
	AR order	5	5	5	5	5	3	2	5
Italy	Period	6.389 (0.054)	6.313 (0.048)	8.064 (2.568)	6.342 (0.045)	7.279 (7.698)	5.685 (0.045)	6.026 (0.035)	8.141 (2.541)
				3.676 (0.508)		3.324 (0.573)			3.989 (0.769)
	Modulus	0.789 (0.001)	0.793 (0.001)	0.871 (0.001)	0.807 (0.001)	0.775 (0.023)	0.703 (0.001)	0.817 (0.001)	0.889 (0.007)
				0.824 (0.029)		0.875 (0.007)			0.769 (0.009)
	adj. R-squ.	0.234	0.225	0.190	0.327	0.230	0.327	0.303	0.330
	AR order	3	3	5	3	5	2	3	5

Table 2 (continued). *G7 Economies*: MBK-filtered series, univ. spectral estimations

		GDP	CON	TPI	GFCF	II	EXP	IM	GOV
Japan	Period[a]	7.069 (0.028)	7.156 (0.019)	6.517 (0.043)	6.645 (0.034)	5.350 (0.028)	7.845 (0.156)	6.689 (0.025)	7.131 (0.020)
		2.515 (0.024)	3.418 (0.058)			2.772 (0.004)	3.289 (0.016)	3.208 (0.009)	3.304 (0.022)
	Modulus[a]	0.991 (0.001)	0.990 (0.001)	0.895 (0.002)	0.939 (0.001)	0.872 (0.001)	0.990 (0.003)	0.994 (0.001)	0.993 (0.001)
		0.537 (0.007)	0.531 (0.004)			0.641 (0.004)	0.865 (0.014)	0.561 (0.005)	0.605 (0.001)
	adj. R-squ.	0.446	0.510	0.309	0.383	0.197	0.252	0.700	0.502
	AR order	4	5	3	3	4	5	4	5
U.K.	Period	7.861 (0.988)	7.876 (1.152)	8.381 (3.843)	8.237 (2.671)	5.655 (0.037)	6.821 (0.050)	5.971 (0.031)	7.331 (0.990)
		3.606 (0.704)	3.651 (0.794)	3.864 (0.850)	4.294 (1.824)	2.988 (0.005)	2.945 (0.005)		3.745 (0.691)
	Modulus	0.951 (0.002)	0.955 (0.010)	0.872 (0.003)	0.900 (0.016)	0.867 (0.001)	0.880 (0.001)	0.832 (0.001)	0.913 (0.001)
		0.762 (0.050)	0.751 (0.034)	0.809 (0.023)	0.746 (0.009)	0.867 (0.001)	0.614 (0.004)		0.780 (0.033)
	adj. R-squ.	0.483	0.505	0.245	0.486	0.365	0.351	0.341	0.377
	AR order	5	5	5	5	4	4	3	5
USA	Period	5.229 (0.025)	5.385 (0.191)	6.171 (0.037)	5.438 (0.014)	6.197 (3.809)	6.174 (0.040)	4.739 (0.044)	5.583 (0.067)
				2.895 (0.004)		2.824 (3.809)			
	Modulus	0.737 (0.009)	0.704 (0.002)	0.853 (0.001)	0.918 (0.001)	0.787 (0.002)	0.828 (0.001)	0.547 (0.003)	0.625 (0.002)
				0.656 (0.003)		0.810 (0.001)			
	adj. R-squ.	0.334	0.191	0.261	0.439	0.128	0.520	0.062	0.233
	AR order	2	2	4	4	4	2	2	2
median	Period	7.069	7.256	8.064	6.517	5.664	6.174	5.971	7.131
		3.606	3.651	3.864		3.103			3.867
	Modulus	0.789	0.793	0.871	0.859	0.867	0.822	0.781	0.838
		0.762	0.751	0.809		0.862			0.774
	adj. R-squ.	0.334	0.316	0.245	0.383	0.199	0.324	0.303	0.330

Note: a. standard errors in brackets

Table 3. *G7 Economies*: HP-filtered series, univariate spectral estimations

		GDP	CON	TPI	GFCF	II	EXP	IM	GOV
Canada	Period[a]	10.09	11.40	9.140	9.747	6.053	6.181	8.020	7.316
		(0.29)	(0.327)	(0.293)	(0.142)	(0.043)	(0.126)	(0.170)	(0.446)
	Modulus[a]	0.823	0.871	0.762	0.901	0.836	0.611	0.747	0.607
		(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.002)	(0.002)	(0.002)
	adj. R-squ.	0.629	0.724	0.402	0.694	0.162	0.285	0.297	0.397
	AR order	3	3	3	3	3	2	3	2
France	Period	9.337	9.096	8.620	8.941	—	8.733	8.230	10.66
		(1.837)	(1.381)	(1.518)	(3.025)		(0.913)	(0.947)	(–)
	Modulus	0.559	0.554	0.621	0.602	—	0.669	0.625	0.504
		(0.002)	(0.003)	(0.002)	(0.002)		(0.001)	(0.002)	(0.069)
	adj. R-squ.	0.457	0.438	0.502	0.498	—	0.541	0.476	0.434
	AR order	2	2	2	2	—	2	2	2
Germany	Period	12.70	12.96	12.18	10.78	5.766	7.934	10.76	12.35
		(3.92)	(4.486)	(1.63)	(2.14)	(3.312)	(0.137)	(2.204)	(8.59)
		3.966	4.006	3.832	3.248	2.910		3.995	3.949
		(0.628)	(0.620)	(0.003)	(2.47)	(0.067)		(0.663)	(0.012)
	Modulus	0.888	0.910	0.879	0.849	0.623	0.768	0.814	0.873
		(0.038)	(0.063)	(0.012)	(0.001)	(0.005)	(0.008)	(0.038)	(0.023)
		0.940	0.956	0.917	0.797	0.868		0.880	0.922
		(0.001)	(0.001)	(0.003)	(0.001)	(0.003)		(0.001)	(0.013)
	adj. R-squ.	0.568	0.635	0.606	0.463	0.225	0.309	0.477	0.629
	AR order	5	5	5	4	5	3	5	5
Italy	Period	8.951	8.845	7.777	7.940	—	8.718	7.565	8.753
		(0.217)	(0.215)	(1.383)	(0.339)		(0.635)	(0.371)	(0.126)
	Modulus	0.787	0.780	0.565	0.682	—	0.693	0.643	0.830
		(0.002)	(0.002)	(0.002)	(0.001)		(0.001)	(0.002)	(0.001)
	adj. R-squ.	0.514	0.474	0.387	0.529	—	0.579	0.453	0.523
	AR order	3	3	2	2	—	2	2	3

Table 3 (continued). *G7 Economies*: HP-filtered series, univ. spectral estimations

		GDP	CON	TPI	GFCF	II	EXP	IM	GOV
Japan	Period[a]	7.052 (0.047)	6.969 (0.035)	7.574 (0.102)	7.701 (0.143)	4.257 (0.019)	7.030 (0.109)	7.119 (0.037)	7.337 (0.027)
	Modulus[a]	0.906 (0.001)	0.918 (0.001)	0.723 (0.001)	0.753 (0.001)	0.558 (0.002)	0.848 (0.001)	0.877 (0.001)	0.919 (0.001)
	adj. R-squ.	0.603	0.624	0.544	0.590	0.051	0.302	0.555	0.568
	AR order	4	4	2	2	2	3	3	3
U.K.	Period	7.931 (0.037)	7.999 (0.037)	7.870 (0.078)	8.348 (0.064)	6.534 (0.063)	6.969 (0.109)	6.899 (0.119)	7.261 (0.052)
						3.019 (0.003)			
	Modulus	0.905 (0.001)	0.908 (0.001)	0.833 (0.001)	0.884 (0.001)	0.841 (0.001)	0.713 (0.001)	0.694 (0.001)	0.837 (0.001)
						0.670 (0.003)			
	adj. R-squ.	0.623	0.640	0.543	0.647	0.251	0.485	0.458	0.460
	AR order	3	3	3	3	4	2	2	3
USA	Period	6.679 (0.086)	10.59 (1.09)	6.006 (0.102)	10.52 (3.07)	8.874 (0.169)	7.918 (0.114)	7.089 (7.118)	8.908 (2.410)
			4.071 (1.221)		4.724 (0.193)	2.857 (0.001)			
	Modulus	0.715 (0.001)	0.953 (0.002)	0.610 (0.002)	0.829 (0.002)	0.804 (0.001)	0.806 (0.001)	0.456 (0.005)	0.615 (0.002)
			0.802 (0.018)		0.894 (0.035)	0.810 (0.001)			
	adj. R-squ.	0.466	0.597	0.272	0.498	0.125	0.654	0.218	0.512
	AR order	2	5	2	5	4	2	2	2
median	Period	8.951	9.069	7.870	8.941	6.053	7.934	7.565	8.753
						2.910			
	Modulus	0.823	0.908	0.723	0.829	0.804	0.713	0.694	0.830
						0.810			
	adj. R-squ.	0.568	0.624	0.502	0.529	0.162	0.485	0.458	0.512

Note: a. standard errors in brackets

Table 4. *G7 Economies*: logD-filtered series, univariate spectral estimations

		GDP	CON	TPI	GFCF	II[b]	EXP	IM	GOV
Canada	Period[a]	—	—	—	—	5.309 (0.045)	—	—	—
	Modulus[a]	—	—	—	—	0.814 (0.002)	—	—	—
	adj. R-squ.	—	—	—	—	0.160	—	—	—
	AR order	—	—	—	—	4	—	—	—
France	Period	—	—	8.708 (−)	16.37 (−)	—	—	—	—
	Modulus	—	—	0.279 (0.021)	0.233 (0.017)	—	—	—	—
	adj. R-squ.	—	—	0.100	0.100	—	—	—	—
	AR order	—	—	2	2	—	—	—	—
Germany	Period	17.77 (8.57)	19.53 (8.80)	11.95 (9.35)	11.68 (8.51)	6.045 (8.997)	—	9.805 (−)	4.026 (0.010)
		4.012 (0.334)	4.109 (0.309)	3.798 (0.374)	3.654 (0.704)	2.929 (0.075)			
	Modulus	0.725 (0.001)	0.736 (0.001)	0.790 (0.010)	0.753 (0.010)	0.774 (0.008)	—	0.409 (0.045)	0.688 (0.001)
		0.834 (0.005)	0.856 (0.006)	0.852 (0.001)	0.757 (0.001)	0.848 (0.008)			
	adj. R-squ.	0.117	0.214	0.163	0.006	0.212	—	0.059	0.222
	AR order	5	5	5	5	5	—	2	3
Italy	Period	16.64 (−)	—	7.026 (0.144)	—	12.51 (12.14)	—	7.599 (−)	—
						3.387 (0.521)			
	Modulus	0.198 (0.014)	—	0.671 (0.003)	—	0.831 (0.005)	—	0.380 (0.039)	—
						0.835 (0.005)			
	adj. R-squ.	0.077	—	0.058	—	0.370	—	0.048	—
	AR order	2	—	3	—	5	—	2	—

Table 4 (continued). *G7 Economies*: logD-filtered series, univ. spectral estimations

		GDP	CON	TPI	GFCF	II[b]	EXP	IM	GOV
Japan	Period[a]	2.761 (0.747)	2.690 (1.023)	5.834 (–)	6.694 (–)	—	—	8.385 (–)	—
	Modulus[a]	0.401 (0.002)	0.218 (0.021)	0.290 (0.023)	0.290 (0.023)	—	—	0.238 (0.016)	—
	adj. R-squ.	0.001	0.001	0.017	0.054	—	—	0.052	—
	AR order	4	4	2	2	—	—	2	—
U.K.	Period	8.208 (0.135) 3.304 (0.042)	8.201 (0.142) 3.378 (0.051)	8.539 (0.257) 3.393 (0.054)	8.949 (2.839) 3.691 (0.633)	7.671 (1.620)	5.842 (1.121)	5.428 (0.221)	6.124 (0.054)
	Modulus	0.902 (0.001) 0.634 (0.014)	0.907 (0.001) 0.612 (0.017)	0.879 (0.002) 0.681 (0.015)	0.908 (0.001) 0.705 (0.007)	0.581 (0.003)	0.427 (0.005)	0.468 (0.004)	0.727 (0.002)
	adj. R-squ.	0.001	0.271	0.177	0.306	0.365	0.051	0.054	0.081
	AR order	5	5	5	5	2	2	2	3
USA	Period	—	—	—	7.793 (6.708)	2.924 (0.002)	11.53 (–)	—	—
	Modulus	—	—	—	0.540 (0.025)	0.757 (0.001)	0.446 (0.053)	—	—
	adj. R-squ.	—	—	—	0.001	0.804	0.069	—	—
	AR order	—	—	—	3	4	2	—	—
median	Period	12.42	8.201	8.539	8.949	6.045		7.992	
	Modulus	0.563	0.736	0.671	0.540	0.774		0.395	
	adj. R-squ.	0.039	0.214	0.100	0.054	0.365		0.050	

Note: a. standard errors in brackets, b. not filtered

Figure 1. GDP CON TPI

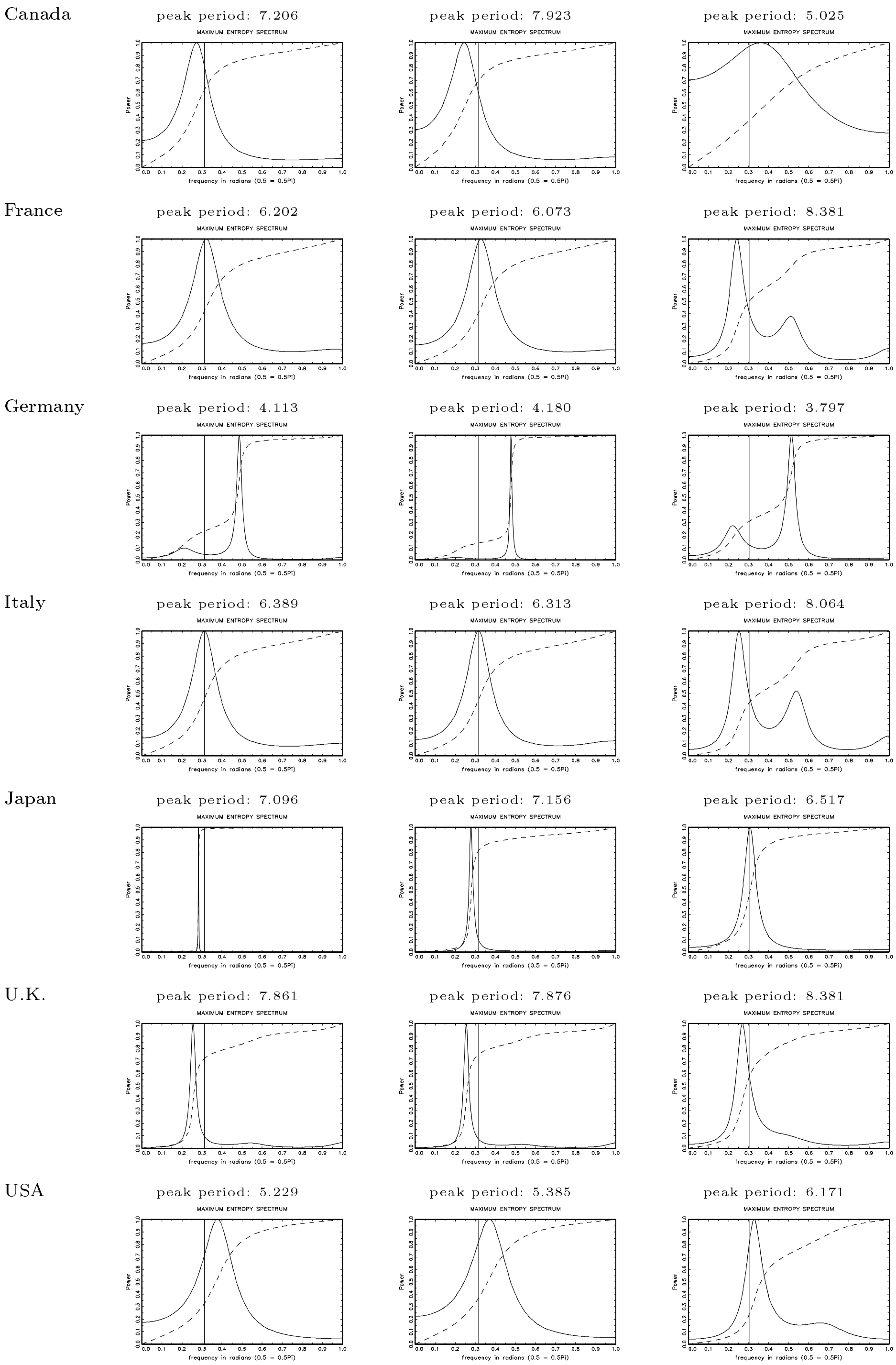
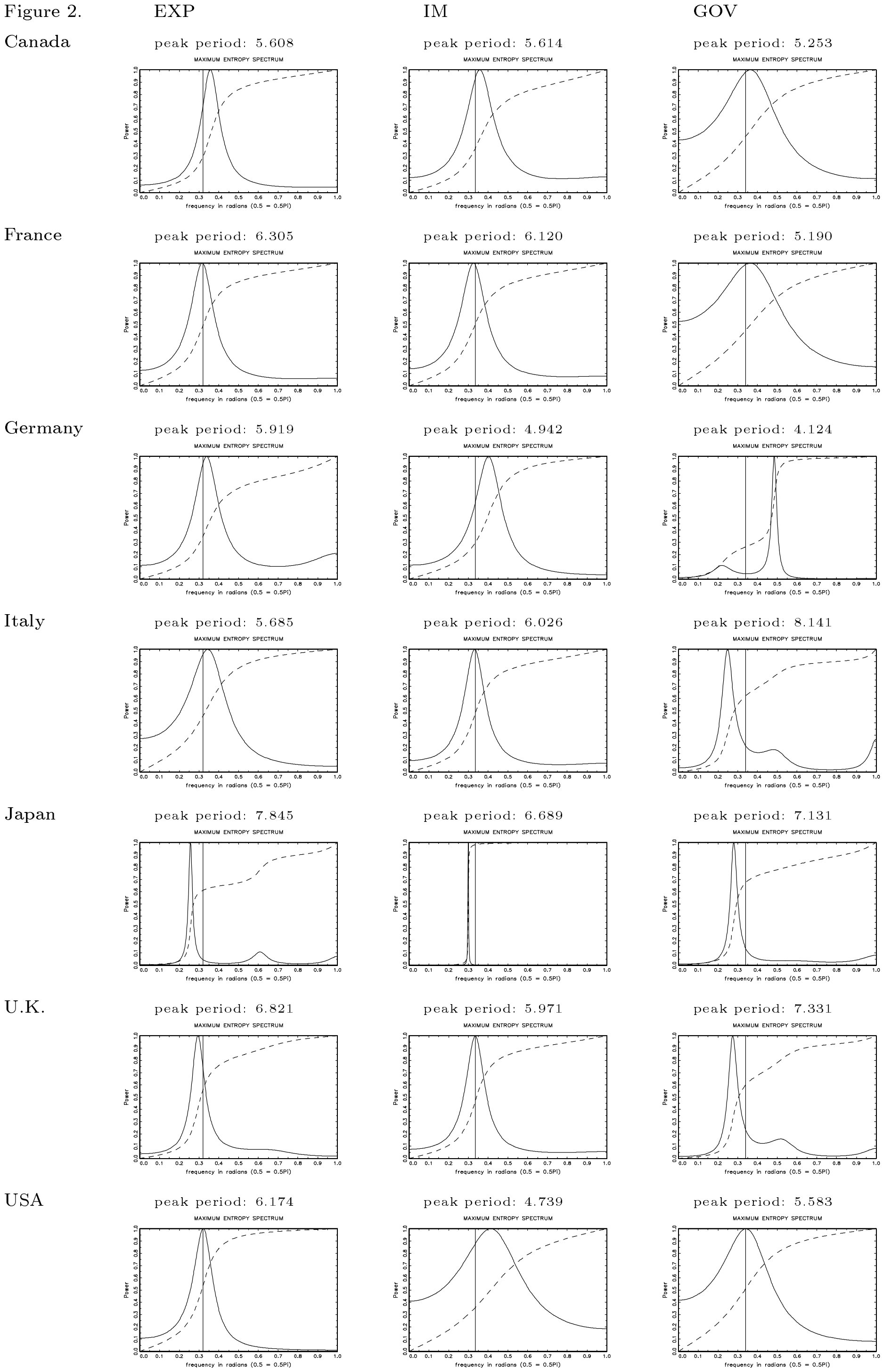


Figure 2.

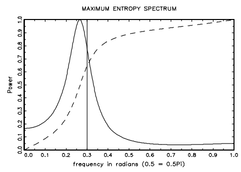


EXP
IM
GOV
Canada
peak period: 5.608
peak period: 5.614
peak period: 5.253
France
peak period: 6.305
peak period: 6.120
peak period: 5.190
Germany
peak period: 5.919
peak period: 4.942
peak period: 4.124
Italy
peak period: 5.685
peak period: 6.026
peak period: 8.141
Japan
peak period: 7.845
peak period: 6.689
peak period: 7.131
U.K.
peak period: 6.821
peak period: 5.971
peak period: 7.331
USA
peak period: 6.174
peak period: 4.739
peak period: 5.583
MAXIMUM ENTROPY SPECTRUM
Power
frequency in radians (0.5 = 0.5PI)

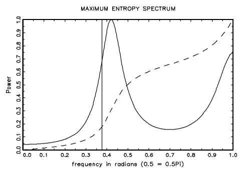
Figure 4.3. GFCF II

Canada

peak period: 7.447

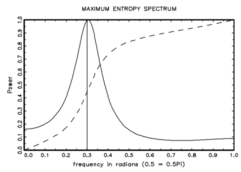


peak period: 4.787

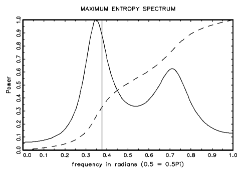


France

peak period: 6.499

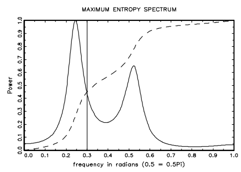


peak period: 6.839

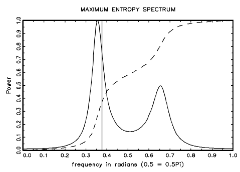


Germany

peak period: 8.282

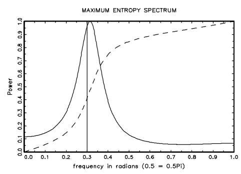


peak period: 5.664

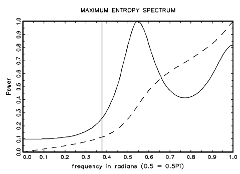


Italy

peak period: 6.342

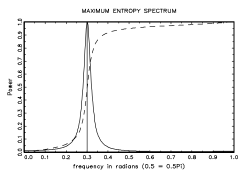


peak period: 3.324

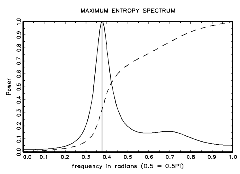


Japan

peak period: 6.645

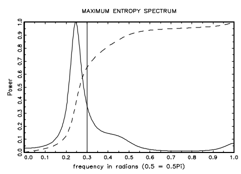


peak period: 5.350

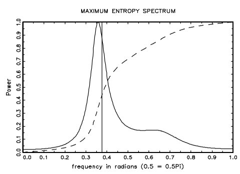


U.K.

peak period: 8.237

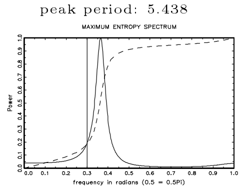


peak period: 5.655



USA

peak period: 5.438



peak period: 2.824

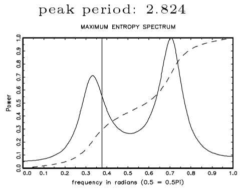


Table 5. *G7 Economies*: MBK-filtered series, national bivariate spectral estimations

	GDP →	CON	TPI	GFCF	II	EXP	IM	GOV
Canada	CTS	0.434	0.275	0.278	0.004	0.113	-.237	0.275
	period	8.62/3.06	8.62/3.23	8.85/3.04	8.62/2.31	8.57/3.67	8.26/3.27	9.00
	sc	0.98/0.82	0.90/0.73	0.93/0.60	0.95/0.85	0.75/0.67	0.64/0.67	0.95
	phase	0.29/-.18	-.47/0.18	-.29/0.11	-3.2/0.20	-1.4/0.28	-1.1/0.25	0.40
	VAR order	2	3	3	2	2	2	5
France	CTS	0.578	0.248	0.232	0.026	0.182	-.199	0.166
	period	6.15	—	—	—	7.09	6.36	—
	sc	0.99	—	—	—	0.93	0.93	—
	phase	0.07	—	—	—	0.15	0.33	—
	VAR order	2	—	—	—	2	2	—
Germany	CTS	0.612	0.242	0.188	0.043	0.179	-.225	0.130
	period	10.9/4.23	12.5/4.14	9.34/4.44	4.21/2.89	7.51/4.29	6.53/3.98	7.40/4.23
	sc	0.99/0.99	0.97/0.99	0.99/0.99	0.99/0.97	0.97/0.99	0.71/0.88	0.99/0.99
	phase	0.22/0.09	0.33/0.02	0.16/-.30	0.30/-.73	1.03/-.40	0.53/-.14	-.23/0.13
	VAR order	5	5	5	5	5	2	5
Italy	CTS	0.596	0.258	0.242	0.020	0.152	-.217	0.157
	period	9.80/4.36	7.51	7.56	6.32/2.75	19.6	6.32	—
	sc	0.99/0.99	0.93	0.96	0.40/0.08	0.83	0.89	—
	phase	0.06/0.06	0.06	0.21	-.38/0.93	-.42	0.28	—
	VAR order	5	2	3	2	1	2	—
Japan	CTS	0.602	0.262	0.267	-.003	0.043	-.026	0.102
	period	6.53	6.53	11.4/7.44	6.75/4.08	7.04	7.57/3.57	6.75/2.35
	sc	0.99	0.96	0.95/0.99	0.95/0.64	0.99	0.99/0.79	0.99/0.99
	phase	-.01	0.32	-.14/0.20	2.18/0.10	0.67	1.52/0.63	-.06/0.02
	VAR order	3	1	4	4	3	4	4

Table 5 (continued). *G7 Economies*: MBK-filtered series, national bivariate spectra

	GDP →	CON	TPI	GFCF	II	EXP	IM	GOV
U.K.	CTS	0.627	0.183	0.198	-.011	0.190	-.245	0.188
	period	7.24	8.00/3.98	7.81/3.23	8.00/4.92	8.13/3.96	8.00/4.23	7.46/4.85
	sc	0.99	0.98/0.65	0.98/0.87	0.91/0.58	0.98/0.75	0.95/0.68	0.98/0.90
	phase	0.06	-.16/-.27	0.07/0.20	-2.0/-1.6	-.30/-.13	-.04/-.31	0.37/0.25
	VAR order	3	5	5	5	5	5	3
USA	CTS	0.375	0.571	0.399	0.169	-.015	-.225	-.020
	period	5.68	5.34/2.84	7.51/5.05	5.52	6.41/3.33	5.65	5.49/2.81
	sc	0.87	0.99/0.85	0.87/0.99	0.67	0.85/0.23	0.80	0.68/0.67
	phase	-.21	-.02/0.09	-.65/0.21	-.20	1.57/-.27	0.38	2.27/1.27
	VAR order	2	4	4	2	2	2	5

Table 6. *G7 Economies*: HP-filtered series, national bivariate spectral estimations

	GDP →	CON	TPI	GFCF	II	EXP	IM	GOV
Canada	CTS	0.526	0.313	0.322	-.021	0.046	-.203	0.267
	period	14.4/8.47	11.3/2.36	9.17/2.47	4.76/2.25	9.52/3.10	10.5	10.5
	sc	0.99/0.98	0.93/0.74	0.93/0.60	0.95/0.90	0.49/0.51	0.64	0.78
	phase	-.01/0.37	-.30/0.14	-.37/0.16	1.53/0.30	-2.1/0.01	-.54	0.54
	VAR order	5	3	3	5	2	3	2
France	CTS	0.572	0.250	0.227	0.023	0.195	-.204	0.172
	period	15.3	13.6/8.33	8.85/13.5	9.52	10.6	9.90	14.0/4.40
	sc	0.99	0.98/0.97	0.99/0.99	0.59	0.97	0.96	0.99/0.98
	phase	0.12	-.54/0.36	0.47/-.44	-.73	0.28	0.44	0.41/0.12
	VAR order	1	5	5	2	2	2	5
Germany	CTS	0.612	0.245	0.205	0.035	0.158	-.208	0.130
	period	13.6/4.13	13.1/4.08	13.5/4.18	13.5/2.82	14.2/8.26	13.1/4.03	13.6/4.11
	sc	0.99/0.99	0.99/0.98	0.99/0.98	0.99/0.98	0.99/0.99	0.99/0.98	0.99/0.99
	phase	0.27/0.08	0.10/-.01	0.21/-.27	-.50/-.85	-1.2/0.81	0.09/0.01	0.24/0.13
	VAR order	5	5	5	5	5	5	5
Italy	CTS	0.589	0.278	0.245	0.036	0.152	-.212	0.157
	period	—	10.8	—	10.0	6.94/15.1	9.00	10.7
	sc	—	0.93	—	0.48	0.95/0.88	0.93	0.96
	phase	—	0.07	—	-.41	-.37/1.48	0.50	0.16
	VAR order	—	2	—	2	4	2	2
Japan	CTS	0.577	0.287	0.284	0.005	0.048	-.040	0.096
	period	7.63	12.6/6.99	12.1/7.09	11.1/5.29	7.57	7.14/4.40	6.94
	sc	0.99	0.97/0.99	0.98/0.99	0.62/0.76	0.98	0.99/0.91	0.99
	phase	0.03	-.36/0.23	-.33/0.17	-.2.2/0.87	0.55	1.56/0.43	-.07
	VAR order	3	4	4	4	2	5	3

Table 6 (continued). *G7 Economies*: HP-filtered series, national bivariate spectra

	GDP →	CON	TPI	GFCF	II	EXP	IM	GOV
U.K.	CTS	0.627	0.206	0.219	-.013	0.181	-.239	0.176
	period	7.93	9.17/4.40	11.7/7.69	7.81	7.93	8.00	8.19/3.95
	sc	0.99	0.97/0.62	0.97/0.99	0.64	0.95	0.88	0.98/0.83
	phase	0.04	-.45/-.57	-.95/0.29	-2.2	-.23	-.12	0.37/0.11
	VAR order	3	5	5	3	3	3	3
USA	CTS	0.468	0.397	0.317	0.173	-.032	-.248	0.040
	period	7.93	11.9/5.98	7.40/4.60	9.25/2.81	7.93	6.71	11.3/4.29
	sc	0.87	0.95/0.98	0.92/0.99	0.73/0.90	0.54	0.76	0.45/0.18
	phase	-.08	-2.1/-.08	-.21/-.10	0.42/0.08	1.91	0.15	0.32/1.89
	VAR order	2	4	5	5	2	2	2

Table 7. *G7 Economies*: logD-filtered series, national bivariate spectral estimations

	GDP →	CON	TPI	GFCF	II[a]	EXP	IM	GOV
Canada	CTS	0.572	-.619	-.210	-.010	-.213	-1.27	-1.47
	period	—	—	—	8.13	—	—	—
	sc	—	—	—	0.39	—	—	—
	phase	—	—	—	-2.5	—	—	—
	VAR order	—	—	—	2	—	—	—
France	CTS	0.879	0.461	0.643	0.025	0.498	-.935	0.746
	period	—	—	—	10.3	—	—	—
	sc	—	—	—	0.27	—	—	—
	phase	—	—	—	-1.2	—	—	—
	VAR order	1	—	—	1	—	—	—
Germany	CTS	0.825	0.418	0.526	0.055	0.363	-.899	0.726
	period	8.19	—	12.0/3.95	4.16/2.80	—	—	—
	sc	0.98	—	0.96/0.95	0.99/0.98	—	—	—
	phase	0.14	—	0.17/-.05	0.25/-.75	—	—	—
	VAR order	1	—	5	5	—	—	—
Italy	CTS	0.874	0.262	0.517	0.024	0.451	-1.00	0.661
	period	—	—	—	5.95	—	—	—
	sc	—	—	—	0.21	—	—	—
	phase	—	—	—	-.05	—	—	—
	VAR order	—	—	—	2	—	—	—
Japan	CTS	0.826	0.539	0.645	-.005	0.085	-.734	0.671
	period	—	12.1/2.35	10.75	7.14/3.96	—	6.71/2.77	—
	sc	—	0.95/0.95	0.95	0.91/0.36	—	0.96/0.66	—
	phase	—	-.11/0.01	0.18	3.29/0.09	—	1.45/0.91	—
	VAR order	—	4	1	4	—	5	—

Table 7 (continued). *G7 Economies*: logD-filtered series, national bivariate spectra

	GDP →	CON	TPI	GFCF	II[a]	EXP	IM	GOV
U.K.	CTS	0.712	0.132	0.320	-.014	0.358	-.936	0.588
	period	6.99	8.69/3.62	—	8.00/4.71	—	—	8.06/2.43
	sc	0.99	0.96/0.65	—	0.88/0.53	—	—	0.98/0.94
	phase	0.04	-.26/-.07	—	-2.0/-1.3	—	—	0.34/0.01
	VAR order	3	5	—	5	—	—	4
USA	CTS	0.432	-2.14	-2.06	-.195	-2.46	-3.17	-.549
	period	—	5.12/2.44	4.71	5.49/2.84	6.49	5.71	—
	sc	—	0.98/0.88	0.88	0.79/0.84	0.54	0.51	—
	phase	—	-.03/0.13	-.02	-.25/0.09	1.40	0.59	—
	VAR order	—	5	2	4	1	1	—

Note: a. not filtered

Figure 4. *G7 Economies*: Mean bivariate spectra (filter: MBK)

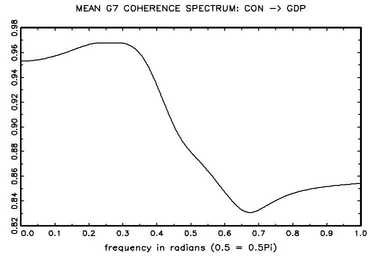
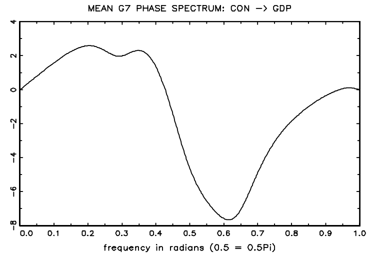
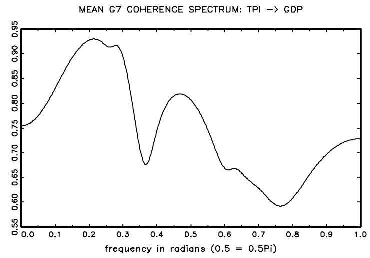
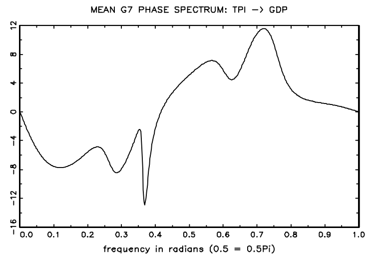
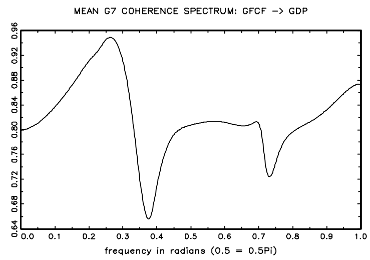
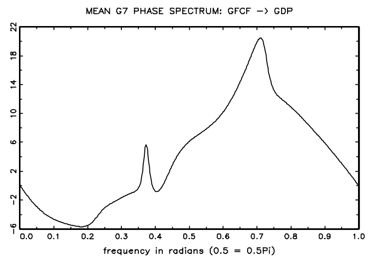
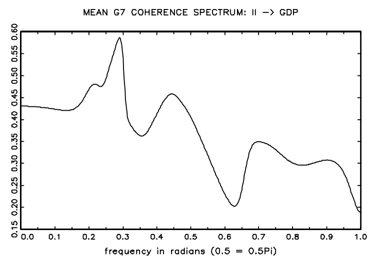
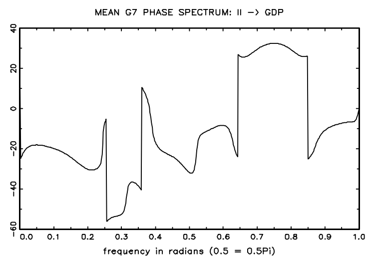
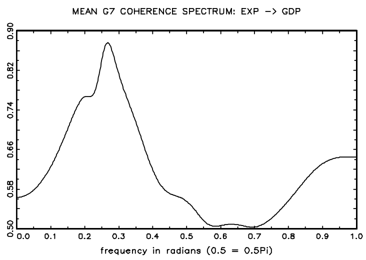
	Mean G7 Coherence Spectra (w/ GDP)	Mean G7 Phase Spectra (w/ GDP)
CON	 max. at period (yrs.): 6.99	 *ps* at max. of *sc* (yrs.): 1.01
TPI	 max. at period (yrs.): 9.25 4.44	 *ps* at max. of *sc* (yrs.): -.39 1.95
GFCF	 max. at period (yrs.): 7.63	 *ps* at max. of *sc* (yrs.): -.74
II	 max. at period (yrs.): 6.89 4.54	 *ps* at max. of *sc* (yrs.): -.03 - 10

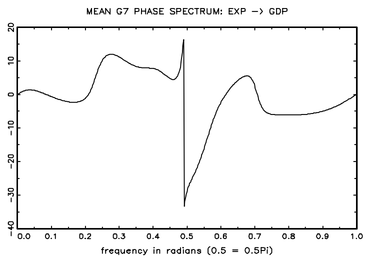
Figure 4 (continued). *G7 Economies*: Mean bivariate spectra (filter: MBK)

Mean G7 coherence spectra (w/ GDP) — Mean G7 phase spectra (w/ GDP)

EXP

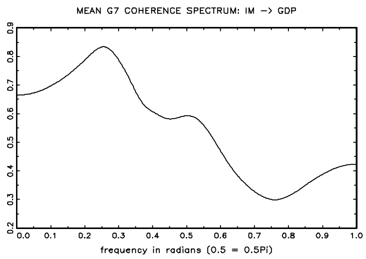


max. at period (yrs.): 7.46

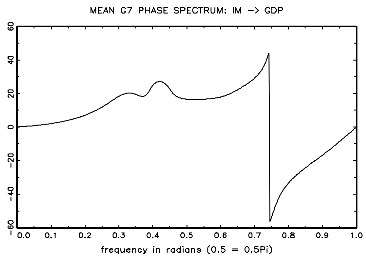


ps at max. of *sc* (yrs.): 0.16

IM

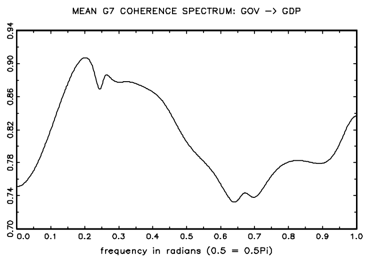


max. at period (yrs.): 7.81

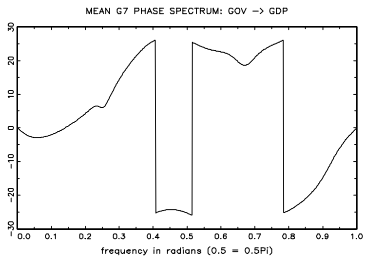


ps at max. of *sc* (yrs.): 0.15

GOV



max. at period (yrs.): 9.90



ps at max. of *sc* (yrs.): 0.45

Table 8. *G7 Economies*: Supranational bivariate spectral estimations (filter: MBK)

Canada	GDP	CON	TPI	GFCF	II	EXP	IM	GOV
CTS(G7-GDP)	0.003	0.001	0.002	0.002	0.001	0.006	-.004	0.001
period(G7-GDP)[a]	7.576	7.246	6.667	7.299	8.197	6.494	7.752	7.194
sc(G7-GDP)	0.672	0.980	0.535	0.987	0.491	0.843	0.563	0.944
phase(G7-GDP)[a]	1.705	2.471	1.040	1.489	-1.28	0.323	0.461	2.170
VAR order	1	5	2	5	2	3	2	1
France								
CTS(G7-GDP)	0.163	0.094	0.044	0.038	0.007	0.030	-.030	0.027
CTS(EU-GDP)	0.179	0.103	0.048	0.043	0.007	0.035	-.037	0.030
period(G7-GDP)[a]	6.667	6.623	6.849	6.579	5.780	8.403	6.579	6.667
period(EU-GDP)[a]	5.814	—	7.692	7.874	7.874	8.621	6.369	4.219
sc(G7-GDP)	0.930	0.923	0.901	0.871	0.452	0.980	0.900	0.890
sc(EU-GDP)	0.967	—	0.984	0.974	0.429	0.989	0.920	0.987
phase(G7-GDP)[a]	0.239	0.280	0.354	0.371	-.376	0.350	0.649	0.219
phase(EU-GDP)[a]	-.073	—	0.040	0.164	-1.20	0.139	0.267	0.098
VAR order(G7)	2	2	2	2	2	5	2	2
VAR order(EU)	2	—	3	3	1	5	2	5
Germany								
CTS(G7-GDP)	0.173	0.105	0.043	0.041	0.005	0.038	-.036	0.023
CTS(EU-GDP)	0.184	0.113	0.047	0.038	0.011	0.046	-.046	0.024
period(G7-GDP)[a]	4.202	4.255	4.032	6.803	3.953	7.042	4.049	4.255
period(EU-GDP)[a]	4.098	4.149	4.000	6.897	4.132	4.367	4.975	4.132
sc(G7-GDP)	0.998	0.999	0.990	0.976	0.924	0.999	0.978	0.997
sc(EU-GDP)	0.996	0.996	0.990	0.999	0.922	0.980	0.839	0.999
phase(G7-GDP)[a]	0.045	0.128	0.039	-.407	0.209	0.950	0.044	0.127
phase(EU-GDP)[a]	-.018	0.034	0.029	-.429	0.265	-.385	0.070	0.101
VAR order(G7)	5	5	5	5	5	5	5	5
VAR order(EU)	5	5	5	5	5	5	2	5

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Table 8 (continued). *G7 Economies*: Supranational bivariate spectra (filter: MBK)

Italy	GDP	CON	TPI	GFCF	II	EXP	IM	GOV
CTS(G7-GDP)	0.141	0.086	0.040	0.036	0.007	0.029	-.033	0.023
CTS(EU-GDP)	0.183	0.112	0.051	0.046	0.008	0.033	-.041	0.029
period(G7-GDP)	6.757	6.667	6.061	4.049	5.882	6.667	6.329	2.222
period(EU-GDP)	4.065	4.115	—	2.128	5.952	6.250	6.289	4.202
sc(G7-GDP)	0.988	0.993	0.686	0.945	0.195	0.876	0.839	0.969
sc(EU-GDP)	0.997	0.998	—	0.966	0.195	0.937	0.882	0.949
phase(G7-GDP)	0.673	0.799	0.426	0.112	0.049	0.289	0.642	-.305
phase(EU-GDP)	0.039	0.079	—	-.070	-.305	-.157	0.259	0.037
VAR order(G7)	3	3	3	5	2	2	2	5
VAR order(EU)	5	5	—	5	2	2	2	5
Japan								
CTS(G7-GDP)	0.267	0.172	0.103	0.101	0.003	0.015	-.019	0.031
period(G7-GDP)	6.757	2.212	6.579	6.623	5.319	2.217	8.333	2.208
sc(G7-GDP)	0.996	0.966	0.952	0.997	0.267	0.940	0.991	0.971
phase(G7-GDP)	-.311	0.331	-.090	-.203	0.717	0.340	1.230	0.349
VAR order	3	3	3	3	2	5	5	5
U.K.								
CTS(G7-GDP)	0.108	0.067	0.026	0.024	0.003	0.025	-.030	0.020
CTS(EU-GDP)	0.125	0.076	0.024	0.025	0.001	-.001	-.032	0.026
period(G7-GDP)	8.000	8.065	7.353	6.897	6.711	6.494	6.289	8.197
period(EU-GDP)	7.813	7.874	—	7.194	6.329	7.246	7.246	7.407
sc(G7-GDP)	0.995	0.999	0.659	0.928	0.393	0.893	0.820	0.965
sc(EU-GDP)	0.979	0.982	—	0.888	0.492	0.875	0.711	0.984
phase(G7-GDP)	0.637	0.745	0.477	0.438	-1.27	0.493	0.572	0.832
phase(EU-GDP)	0.152	0.175	—	0.059	-1.70	-.011	0.075	0.448
VAR order(G7)	5	5	2	3	2	2	2	5
VAR order(EU)	4	5	—	3	2	2	2	5

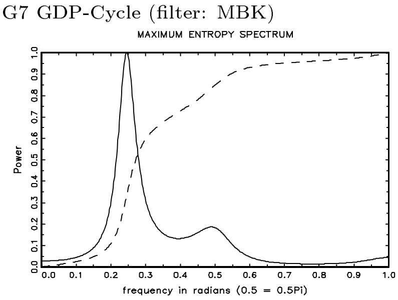
Table 8 (continued). *G7 Economies*: Supranational bivariate spectra (filter: MBK)

USA	GDP	CON	TPI	GFCF	II	EXP	IM	GOV
CTS(G7-GDP)	0.043	0.009	0.018	0.024	-.039	0.017	-.022	0.001
period(G7-GDP)	6.803	4.292	7.576	7.752	—	5.988	6.711	2.299
sc(G7-GDP)	0.640	0.822	0.955	0.952	—	0.969	0.531	0.896
phase(G7-GDP)	-.875	2.100	-.892	-.923	—	1.212	-.131	-.595
VAR order	2	2	2	5	—	2	2	5

Note: a. throughout only the measures for the contained cyclicality with highest sc-value are displayed.

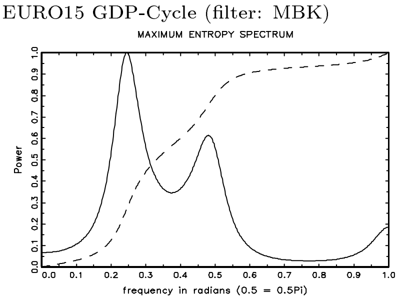
Figure 5. Supranational GDP series' spectra (filter: MBK)

G7 GDP-Cycle (filter: MBK)



Period[a]	Modulus[a]	adj. R-squ.	AR order
8.206 (3.203)	0.906 (0.001)	0.389	5
3.963 (1.084)	0.804 (0.027)		

EURO15 GDP-Cycle (filter: MBK)



Period[a]	Modulus[a]	adj. R-squ.	AR order
8.324 (1.048)	0.854 (0.012)	0.307	5
4.076 (1.259)	0.833 (0.014)		

Note: a. standard errors in brackets

TECHNICAL APPENDICES

Technical Appendix A. Detrending

The raw time series under analysis in the paper are not stationary and need to be transformed to apply the spectral methods suggested in the following. A large literature on detrending exists which shows that an inappropriate detrending method distorts the spectrum of the residuals and may produce spurious cyclical behavior, see *inter alia* Canova (1994), Harvey and Jaeger (1993), Canova (1998) and more recently Schenk-Hoppé (2000). Much effort has been devoted to developing tests to determine the correct detrending procedure for a given time series. *Inter alia* these unit root (UR) tests imply the Dickey-Fuller, Augmented Dickey-Fuller and Phillips-Perron tests. Unfortunately, these tests are rather weak, cf. the comprehensive outline in Banerjee et al. (1993).[16] An alternative to testing, is to use several filters, including ones that tend to distort the spectrum in opposite directions, cf. Reiter (1995, Section 2.3), Woitek (1997, Section 4.2 and Appendix A). Features of the spectrum which survive different filtering procedures are considered to be robust. Such a strategy has also been advocated by Canova (1998). The following set of widely used filtering methods is chosen in the present study: first log-differences (logD), i.e. growth rates in a first approximation, the Hodrick-Prescott (HP) filter and a modified version of the Baxter-King (MBK) filter as proposed by Woitek (1998). Their mathematical objectives and determinations are given by the following set of equations:[17]

$$\text{logD:} \qquad \widetilde{y}_t - \widetilde{y}_{t-1} = \ln(y_t) - \ln(y_{t-1}) \approx \tfrac{y_t - y_{t-1}}{y_{t-1}} \tag{A.1}$$

$$\begin{aligned} \text{HP(100):} \quad & \min_{\{\tau_t\}_t^T} \textstyle\sum_{t=1}^{T} (\widetilde{y}_t - \tau_t)^2 \quad \text{s.t.} \\ & \textstyle\sum_{t=2}^{T-1} \left[(\tau_{t+1} - \tau_t) - (\tau_t - \tau_{t-1})\right]^2 \leq \mu, \text{ where} \\ & \tau_t \equiv \text{trend}, \ \mu \equiv \text{smoothing (Lagrangian} = 100 \text{ for annual data)} \end{aligned} \tag{A.2}$$

$$\begin{aligned} \text{(M)BK:} \quad & \min_{a_j} \textstyle\int_{-\pi}^{\pi} |\beta(\omega) - \alpha(\omega)|^2 \, d\omega \quad \text{s.t. } \alpha(0) = 0, \text{ where} \\ & \alpha(\omega) \equiv \text{Fourier transform of } a_j \ (K = 3 \text{ for annual data}) \\ & \beta(\omega) \equiv \text{“ideal” filter gain} \end{aligned} \tag{A.3}$$

According to (A.1) to (A.3) the outcome of log differencing is given by $y_t|_{\text{logD}} = \widetilde{y}_t - \widetilde{y}_{t-1} = \ln(y_t) - \ln(y_{t-1})$, of the HP-filtering routine by $y_t|_{\text{HP}} = \widetilde{y}_t - \tau_t = \ln(y_t) - \tau_t$, of

[16] Another recent reference in the unit root discussion is Diebold and Senhadji (1996) who note: “There is no doubt that unit-root tests do suffer from low power in many situations of interest ... Our results suggest that U.S. aggregate output is not likely to be difference-stationary: the dominant autoregressive root is close to, but less than, unity ... There is simply no substitute for serious, case-by-case, analysis,” p. 1297.

[17] In my opinion the most intuitive outline of the HP-filter's logic is given in Prescott (1986). A survey of the statistical justifications of the HP-filter is provided by Reeves et al. (1996). For a detailed specification of the ‘digital’ or ‘bandpass’ filters, BK- and MBK-filter, the exact differences in their derivation and their effects on series with different frequencies (i.e. annual, quarterly and monthly), the reader is referred to the thorough presentation and discussion in Woitek (1998). A comparison of unfiltered, HP- and BK-filtered series, having a peak in their spectrum at business cycle frequencies, can be found in Guay and St-Amant (1997).

the MBK-filter by its moving average (MA) representation, i.e. $y_t \mid_{\text{MBK}} = \sum_{j=-K}^{K} a_j L^j y_t$ of order K, respectively.

As widely known, the logD-filter tends to amplify the high frequencies, the HP-filter to amplify the low frequencies and for the MBK-filter the gain function is relatively flat over the relevant business cycle frequencies. As can be seen from equations (A.1) to (A.3), the logD-procedure shortens the time series by one datapoint, the MBK-filter by six (three at the beginning, three at the end of the respective series) and the HP-filter leaves the length of the series unchanged. In this context, it is noteworthy that Den Haan and Sumner (2001) recently provide a proof that band-pass filters like the MBK-filter isolate the desired set of frequencies even in the case when the raw series are $I(1)$ or $I(2)$, i.e. actually difference stationary or follow a linear and/or quadratic trend. Their benchmarking contribution gives a further justification to refrain from UR tests.

Technical Appendix B. Volatility and Spectral Analysis

Contribution-to-Variance Analysis. To quantitatively assess the relative[18] fluctuations of a time series under investigation or, generally speaking, its relative volatility and contribution to the constitution process to an aggregate series of higher dimension, the following three measures are suggested: relative trend level (RTL), standardized standard deviation (SSD) and contribution to standard deviation (CTS).

The first of these measures is a straightforward method to quantify the relative magnitude or share of the ith constituent series' trend level $\tau_t(y_i)$ to the corresponding aggregate series' trend level $\tau_t\left(\sum_i^N y_i\right)$, where $i = 1, ..., N$. It is calculated according to

$$\text{RTL} = \frac{\tau_t(y_i)}{\tau_t\left(\sum_i^N y_i\right)} \tag{B.1}$$

for all i constituent components at several distinct points of the observation period $t = 1, ..., T$. Actually, as reported in the text, starting point $(t = 1)$, mid point $(t = T_m)$ and end point $(t = T)$ of τ_t are considered.

The standardized standard deviation (std. std. dev.), defined as the standard deviation of the detrended series of a variable at stake standardized on its mid trend level value, provides an adequate measure of the relative volatility of a variable:

$$\text{SSD} = \frac{\sigma\left[\left(y_{i,t} - \tau_t(y_i)\right)\right]}{\tau_{T_m}}. \tag{B.2}$$

This measure is similar, though preferable, to the volatility measure of a variable suggested by Christiano (1988), whose ratio employs the mean of absolute changes of the undetrended time series of a variable in the nominator, the mean of the undetrended series in the denominator, cf. also Fitzgerald (1997).[19] The std. std. dev. measure, as given by (B.2), is superior to Christiano's measure, because the latter implies arithmetic means which tend to weight a deviation proportional to the magnitude of the deviation. This undesirable feature is overcome by the std. std. dev. measure (B.2).

[18] I.e. relative to its median trend level value or to the fluctuating properties of a higher-dimensional aggregate series.

[19] A survey of similar volatility measures is given by McKenzie (1999).

Consider the aggregate y^a of $i = 1, ..., N$ business cycle components y_i^c, as extracted by an adequate detrending method from the original series. The standard deviation of y^a is then theoretically composed by the following relationship

$$\sigma\left(y^a\right) = \sqrt{\sigma^2\left(\sum_i y_i^c\right)} = \left[\sum_i \sigma^2\left(y_i^c\right) + 2\sum_i\sum_k \operatorname{cov}\left(y_i^c, y_k^c\right)\right]^{\frac{1}{2}},$$

where $i = 1, ..., N$ and $k = i + 1, ..., N$. Obviously, the respective variances $\sigma^2\left(y_i^c\right)$ contribute positively to $\sigma\left(y^a\right)$, but given a constituent y_i^c is mainly correlated negatively with the other constituent series, there might be a damping impact on $\sigma\left(y^a\right)$ from including y_i^c in the constitution of y^a. Therefore the contribution of the ith constituent component to the aggregate series' standard deviation is adequately captured by

$$\text{CTS} = 1 - \frac{\sigma\left(y^a - y_i^c\right)}{\sigma\left(y^a\right)} = \frac{\sigma\left(y^a\right) - \sigma\left(y^a - y_i^c\right)}{\sigma\left(y^a\right)}, \tag{B.3}$$

where $\sigma\left(y^a - y_i^c\right)$ denotes the reduction in standard deviation of the aggregate, when y_i^c is dropped in the constitution process. It is theoretically given by

$$\sigma\left(y^a - y_i^c\right) = \sqrt{\sigma^2\left(y^a\right) - \sigma^2\left(y_i^c\right) - 2\sum_k \operatorname{cov}\left(y_i^c, y_k^c\right)}, \text{ where } k \neq i.$$

Therefore, CTS, as given by (B.3), is the contribution to standard deviation measured in standard deviation terms of the aggregate, i.e. in percent of the total aggregate's standard deviation. Following the argumentation above, a CTS-value > 0 characterizes a destabilizing variable, whereas a CTS-value < 0 characterizes a stabilizing constituent variable.

(V)AR Spectral Analysis. For the general multivariate case, the spectral density matrix $\mathbf{F}(\omega)$ of an n-dimensional stochastic process is given by the Fourier transform of the covariance function of the process:

$$\mathbf{F}(\omega) = \frac{1}{2\pi}\sum_{\tau=-\infty}^{+\infty} \boldsymbol{\Gamma}(\tau)\, e^{-i\omega\tau}, \quad -\pi \leq \omega \leq \pi, \tag{B.4}$$

with

$$\boldsymbol{\Gamma}(\omega) = \begin{pmatrix} \gamma_{11}(\omega) & \cdots & \gamma_{1n}(\omega) \\ \vdots & \ddots & \vdots \\ \gamma_{n1}(\omega) & \cdots & \gamma_{nn}(\omega) \end{pmatrix} \text{ and } \mathbf{F}(\omega) = \begin{pmatrix} f_{11}(\omega) & \cdots & f_{1n}(\omega) \\ \vdots & \ddots & \vdots \\ f_{n1}(\omega) & \cdots & f_{nn}(\omega) \end{pmatrix}.$$

Because $\mathbf{F}(\omega)$ is an even function, it is sufficient to examine it in the interval $[0, \pi]$.

The univariate case: The diagonal elements $f_{11}(\omega), ..., f_{nn}(\omega)$ are the real-valued *autospectra* or *power spectra* (in the text referred to as univariate AR spectra). The underlying estimation model can be interpreted as a standard AR model of order p. In general defined as

$$x_t = a_1 x_{t-1} + a_2 x_{t-2} + ... + a_p x_{t-p} + \varepsilon_t = \sum_{j=1}^{p} a_j x_{t-j} + \varepsilon_t,$$

where ε_t follows a white noise process with mean 0 and variance σ^2. Applying the notation of a standard lag operator $a(L)\,x_t = \varepsilon_t$, where $a(L) = 1 - a_1 L - a_2 L^2 - ... - a_p L^p$and $L^n x_t = x_{t-n}$ with $n = 1, 2, ...$, the ε_t can be expressed as the output of a linear filter $a(L)$ applied to the AR process x_t: $\varepsilon_t = a(L)\,x_t$. If the process x_t is a stationary process,e.g. achieved by extraction from the raw series via an adequate detrending procedure, the process x_t can be *vice versa* represented as output of the linear filter $a(L)^{-1}$ applied to the irregular component ε_t: $x_t = a(L)^{-1}\varepsilon_t$. With regard to the constitution of macroeconomic time series, Granger and Newbold (1986) note: "Two situations where series are added are of particular interpretational importance. The first is where series are aggregated to form some total, and the second is where the observed series is the sum of the true process plus observational error, corresponding to the classical 'signal plus noise' situation," p. 30. Most macroeconomic series, such as GDP, or in general NIPA aggregates, as analyzed in the present study, are such aggregated series. In this context, Granger and Newbold (1986) derive a central theorem that implies that "[i]f the observed series is the sum of a 'true' series that is AR(p) plus a white noise observation error, then an ARMA(p, q) series results," p. 31, cf. also Forni and Lippi (1997, chapter 10 and appendix A.7). For ARMA(p, q) models a series x_t is supposed to be generated by[20]

$$x_t = \sum_{j=1}^{p} a_j x_{t-j} + \sum_{j=0}^{q} b_j \varepsilon_{t-j},$$

where ε_t is a zero-mean white noise and $b_0 = 1$. Applying for x_t the lag operator L, leads to

$$a(L)\,x_t = b(L)\,\varepsilon_t$$

so that the corresponding generating function form is

$$a(z)\,x(z) = b(z)\,\varepsilon(z)\,, \text{ where } a(z) = 1 - \sum_{j=1}^{p} a_j z^j \text{ and } b(z) = \sum_{j=0}^{q} b_j z^j.$$

From this representation the following three implications can be proven to hold in general (see Granger and Newbold (1986), p. 25-26):

(i) The ARMA process is stationary if the roots of $a(z) = 0$ all lie outside the unit circle $|z| = 1$;

(ii) if the process is stationary, then there is an equivalent MA(∞) process

$$x_t = \sum_{j=0}^{\infty} c_j \varepsilon_{t-j}, \text{ where } c_0 = 1 \text{ and } c(z) = \sum_{j=0}^{\infty} c_j z^j = \frac{b(z)}{a(z)};$$

[20]The following argumentation widely summarizes Granger and Newbold (1986, section 1.5 - 1.7).

(iii) there is an equivalent AR(∞) process

$$x_t = \sum_{j=1}^{\infty} d_j x_{t-j}, \text{ where } d(z) = 1 - \sum_{j=1}^{\infty} d_j z^j = \frac{a(z)}{b(z)},$$

provided the roots of $b(z) = 0$ lie outside the unit circle $|z| = 1$, that is, provided the so-called invertibility condition holds.

From (i) to (iii) it follows that a stationary process "can always be well approximated" by an MA or AR process of sufficiently high order, cf. Granger and Newbold (1986), p. 26. To determine the right order several information criteria are suggested in the literature, including the final prediction error criterion (FPE), Akaike's information criterion (AIC), the Hannan-Quinn criterion (HQ) and the Schwarz criterion (SC); see e.g. Lütkepohl (1991). Recently, several Bayesian information criteria are refined and suggested in the literature, e.g. the BAC criterion by Heintel (1998) has been shown in a series of simulation experiments to be first best in determining the right order of a (V)AR model. As in the case of choosing the adequate detrending procedure, I follow a serious, case-by-case, strategy. I rely primarily on the latest order determination technology, i.e., the BAC suggested by Heintel (1998), and consider a visual analysis of respective orders and implied spectra for cases in which the standard information criteria FPE, AIC, HQ and SC diverge on the majority from the order suggested by the BAC criterion. Since in a general AR(p) model ε_t is assumed i.i.d., s.t. $E(\varepsilon_t) = 0$, applying an expectation operator, renders the deterministic part of the process, or in other words, the homogenous part of a difference equation of order p. The deterministic system can always be reformulated in a first order lag form $\mathbf{w}_t = \mathbf{A}\mathbf{w}_{t-1}$, where

$$\mathbf{w}_t = \begin{pmatrix} x_t \\ Lx_t \\ L^2 x_t \\ \vdots \\ L^p x_t \end{pmatrix}, \; \mathbf{A} = \begin{pmatrix} a_1 & a_2 & a_3 & \cdots & a_p \\ 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \text{ and } \mathbf{w}_{t-1} = \begin{pmatrix} Lx_t \\ L^2 x_t \\ L^3 x_t \\ \vdots \\ L^{p+1} x_t \end{pmatrix}.$$

The eigensystem of the above system is given by $\boldsymbol{\lambda}\mathbf{y} = \mathbf{A}\mathbf{y}$, where $\boldsymbol{\lambda}$ denotes the eigenvalues of the quadratic matrix $\mathbf{A}$ and $\mathbf{y} \neq 0$ expresses the eigenvector of the system. For $\mathbf{y} \neq 0$ the characteristic polynomial is found via $|\mathbf{A} - \boldsymbol{\lambda}\mathbf{I}| = 0$, where $\mathbf{I}$ denotes an identity matrix of order p. Solving the characteristic polynomial renders a real part c_1 and a complex part c_2 for one or more (potentially) complex solutions in terms of the AR coefficients $a_1, ..., a_p$. The (angular) frequency ω for the respective (potentially) complex solution is defined as

$$\omega = \arctan\left(\frac{c_2}{c_1}\right) = \frac{2\pi}{P},$$

where P denotes the period length for a cyclic solution, as given in the case of complex (i.e. negative) roots in the solution of the characteristic polynomial.

The modulus of the respective complex solution is given by

$$\text{mod} = \sqrt{c_1^2 + c_2^2}.$$

A mod close to value 1 characterizes the fact that the complex roots of the deterministic part of the process are almost undamped and therefore the spectral density function is peaking at the frequency corresponding to an implied period length P of the respective complex roots, as defined above. The closer mod approaches a zero value, the more damped is a contained cyclicality of a series. In general, the less damped the roots are, the more regular is (are) the cycle(s) contained in a series under investigation and the higher and sharper is (are) the peak(s) in the spectrum.

The multivariate case: The off-diagonal elements of the VAR system (B.4) reflect cross spectra $f_{jk}(\omega) = c_{jk}(\omega) - iq_{jk}(\omega)$, consisting of $c_{jk}(\omega)$ cospectra and $q_{jk}(\omega)$ quadrature spectra. Uni- and bivariate spectral measures are derived by the estimated parameter matrix $\mathbf{A}$ of a general VAR(p) process adjusted to a respective pair of series:

$$\mathbf{F}(\omega) = \frac{1}{2\pi}\mathbf{A}(\omega)^{-1}\sum\mathbf{A}(\omega)^{-*}, \tag{B.5}$$

where $\sum$ denotes the error variance-covariance matrix, $\mathbf{A}(\omega)$ the Fourier transform of the matrix lag polynomial $\mathbf{A}(L) = I - A_1 L - \ldots A_1 L^p$ and superscript "*" complex conjugate transpose. The off-diagonal elements of (B.4), estimated via (B.5) allow the computation of the phase shift measure by means of the following simple manipulation

$$ps(\omega) = \arctan\frac{-q_{jk}(\omega)}{c_{jk}(\omega)}. \tag{B.6}$$

The phase shift (ps) measures the phase lead or lag of a series j over the series k at a certain frequency ω. The respective ps measure is computed at the maximum of squared coherency sc, i.e. at that frequency ω, where the cyclic components contained in the two series at stake show the highest degree of linear relationship, as defined by

$$sc = \kappa_{jk}^2(\omega) = \frac{|f_{jk}(\omega)|^2}{f_{jj}(\omega)f_{kk}(\omega)}. \tag{B.7}$$

The sc measure takes on values between 0 and 1. Precisely, it indicates the proportion of the variance of the component of frequency ω of either series that can be explained by its linear regression on the other series; see Koopmans (1995), p. 142. For example, if the series are statistically independent from each other, sc equals zero; if one process is a linear transformation of the other one, sc would consequently take on a value of one. Both spectral parameters ps and sc can certainly also be calculated and displayed for a range of different frequencies. These illustrations are called phase and coherence spectrum, respectively. For $sc < 1$, the relationship between the two business cycle components under investigation is not strict, but the phase spectrum can be interpreted as displaying the expected values of respective phase shifts for different frequencies (cf. Koopmans (1995), p. 138). Reiter (1995) accurately summarizes: "The bivariate spectral parameters are ideal tools for investigating the comovements of series over the business cycle. For example, a good leading indicator is a series having a positive lead relative to the reference series, which can be seen from the phase spectrum, and a high coherency," p. 25.

Technical Appendix C. Ascribing Standard Errors to mod and period

The method that will be presented in the following is superior to conventional bootstrap strategies since, as Kiefer (1996) notes, "the bootstrap method scrambles the order of the observations," p. 50, by randomly sampling from data and therefore violating the autoregressivity of the series at stake. It stems from an approximation strategy adopted from continuous time econometrics which is called *sensitivity analysis*.

Following Wymer (1993, Section 3.3), the general continuous time model (of sufficient order to imply a complex root) can be written as

$$Dy\left(t\right) = A\left(\phi\right)y\left(t\right) + B\left(\phi\right)z\left(t\right) + u\left(t\right), \tag{C.1}$$

where $Dy\left(t\right) = \frac{dy(t)}{dt}$ and ϕ is the vector of all parameters to be calculated and the elements of A and B are any differentiable functions of ϕ incorporating in the simplest case the restrictions derived from linearization of a more general model. The second term on the right-hand side, $B\left(\phi\right)z\left(t\right)$, can be interpreted as the inhomogeneous part of the differential equation system (C.1) and is for reasons of simplicity left out of the further derivation:

$$Dy\left(t\right) = A\left(\phi\right)y\left(t\right) + u\left(t\right). \tag{C.2}$$

As Wymer (1993) notes, this model can be represented by a stochastically equivalent difference equation system such that this exact discrete model will be satisfied by any set of equispaced observations generated by the system of differential equations and can be estimated from a sample of such observations. For any such linear difference equation system the eigensystem is given by

$$\Lambda = H^{-1}AH, \tag{C.3}$$

where, providing that the eigenvalues of A are distinct, H is the matrix of eigenvectors and Λ is a diagonal matrix of eigenvalues λ_i. Complex conjugate eigenvalues of the form $c_1 \pm c_2 i = \nu \pm i\omega$ (where c_1 and c_2 denote real and imaginary values of the complex root, respectively) correspond to a cycle with an amplitude proportional to $e^{\nu t}$, a period of $|2\pi/\omega|$ length and an implied modulus or damping rate $\sqrt{c_1^2 + c_2^2}$; see Wymer (1993, Section 3.4), or Gandolfo (1981, Section 2.2.4). The task of the present subsection is to determine and assign standard errors to periodicities and moduli implied by a discrete analogue of model (C.2), i.e. an AR(p) process of an arbitrary order $p \geqslant 2$ for which an eigensystem of the form (C.3) can be calculated, given parameter estimates of the model. The applied concept will be the concept of *sensitivity analysis*, as suggested by Gandolfo (1981), Wymer (1993) and Reiter (1995).

At the heart of sensitivity analysis stands the evaluation of the partial derivatives of the eigenvalues λ_i with respect to the elements a_{jk} of the parameter matrix A of the system given in (C.3). These partial derivatives are also called *sensitivity coefficients*. The columns i of the sensitivity coefficient Jacobian matrix with respect to the elements of the difference equation system $J_{A\ i}$ is given by

$$J_{A\ i} = \frac{\partial \lambda_i}{\partial A} = \frac{\partial \lambda_i}{\partial a_{jk}}, \text{ where } A = \left(a_{jk}\right). \tag{C.4}$$

And since the elements a_{jk} of A are differentiable functions of the parameters ϕ_p of the system, where p equals the order of the adjusted AR(p) process, the sensitivity coefficient matrix with respect to the estimated AR parameters ϕ_p can be constructed in the following way

$$J_{\phi_p\ i} = \sum_{j,k} \frac{\partial \lambda_i}{\partial a_{jk}} \frac{\partial a_{jk}}{\partial \phi_p} \text{ for all } i, p. \tag{C.5}$$

For a numerically specified system matrix A and estimates of the parameters ϕ_p the calculation of (C.4) is straightforward and can be conducted numerically. Gandolfo (1981) and Wymer (1993) suggest the following general way to obtain J_A and therefrom J_{ϕ_p} via the relationship given in (C.5): Accordingly, J_A can be constructed by letting h_i be the i^{th} column of the matrix of eigenvectors H in (C.3), h_i^* the column vector corresponding to the i^{th} row of the inverse matrix H^{-1}, and λ_i the i^{th} eigenvalue of Λ. Equation (C.4) can then be expressed in terms of h_i and h_i^*:

$$J_{A\ i} = \frac{\partial \lambda_i}{\partial A} = \frac{\partial \lambda_i}{\partial a_{jk}} = h_i^* h_i^{'} \tag{C.6}$$

and J_{ϕ_p} is obtained by (C.5). A formal proof for this procedure is given in Gandolfo (1981), Section 2.2.4.2.

Following Reiter (1995, Section 13.2), the computation of J_{ϕ_p} needs to be computed separately for the real and imaginary parts of Λ (with column elements λ_i^r and λ_i^i, respectively, where superscript "r" and "i" denote real and imaginary parts of the respective column λ_i of matrix Λ). The standard errors of the real c_1 and imaginary parts c_2 of the complex roots of system (C.2) are therefore given as the square roots of the diagonal of the following matrix product, involving the estimated variance-covariance matrix $\widehat{\sum}$ of the regression coefficients ϕ_p (with dimension $p \times p$):

$$\widehat{\sigma}_{c_1}^2 = \text{diag}\left[J_{\phi_p}^r \widehat{\sum} \left(J_{\phi_p}^r\right)^{'}\right], \tag{C.7}$$

$$\widehat{\sigma}_{c_2}^2 = \text{diag}\left[J_{\phi_p}^i \widehat{\sum} \left(J_{\phi_p}^i\right)^{'}\right]. \tag{C.8}$$

Analogously, it is possible to ascribe standard errors to moduli mod_i implied by fully numerically specified systems, where I numerically construct the Jacobian of the moduli with respect to the regression coefficients:

$$J_{\phi_p}^{\text{mod}} = \frac{\partial\, \text{mod}_i}{\partial \phi_p}, \tag{C.9}$$

and apply the analogue strategy as above, in (C.7) and (C.8):

$$\widehat{\sigma}_{\text{mod}} = \text{diag}\left[J_{\phi_p}^{\text{mod}} \widehat{\sum} \left(J_{\phi_p}^{\text{mod}}\right)^{'}\right]^{0.5}. \tag{C.10}$$

I proceed in the same way for the implied period length(s). *Nota bene*: Though the ratio of estimated measure and standard error, determined according to (C.9) and (C.10), can be interpreted in the spirit of a Student's t-value for the mod measures,[21] this interpretation

[21] The corresponding null would be a mod = 0, i.e. a fully out-damped cyclic component, contained in the series under investigation.

is not possible in the case of estimated period lengths. In the latter case a confidence interval interpretation is quite suggestive, providing an additional piece of information.

References

Artis, M. J. and W. Zhang (1999), "Further Evidence on the International Business Cycle and the ERM: Is there a European Business Cycle?," *Oxford Economic Papers*, Vol. 51, 120-132.

Banerjee, A., Dolado, J., Galbraith, J. and D. Hendry (1993), *Cointegration, Error-Correction and the Econometric Analysis of Non-Stationary Data*, Oxford University Press, Oxford.

Brockwell, P. J. and R. A. Davis (1991), *Time Series: Theory and Methods*, 2nd ed., Springer Series in Statistics, Springer, Berlin, Heidelberg, New York.

Canova, F. (1994), "Detrending and Turning Points," *European Economic Review*, Vol. 38, 614-623.

Canova, F. (1998), "Detrending and Business Cycle Stylized Facts," *Journal of Monetary Economics*, Vol. 41, 475-512.

Christiano, L. J. (1988), "Why Does Inventory Investment Fluctuate So Much?," *Journal of Monetary Economics*, Vol. 21, 247-280.

Christiano, L. J. and T. J. Fitzgerald (1998), "The Business Cycle: It's Still a Puzzle," *Federal Reserve Bank of Chicago: Economic Perspectives*, 4th quarter, 56-83.

Croux, C., M. Forni and L. Reichlin (2001), "A Measure of Comovement for Economic Variables," *Review of Economics and Statistics*, Vol. 83, 232-241.

Den Haan, W. J. and S. Sumner (2001), "The Comovements between Real Activity and Prices in the G7," *NBER Working Paper*, No. 8195.

Diebold, F. X. and A. S. Senhadji (1996), "The Uncertain Unit Root in Real GNP: Comment," *American Economic Review*, Vol. 86, 1291-1298.

Dueker, M. and K. Wesche (1999), "European Business Cycles: New Indices and Analysis of their Synchronization," *Federal Bank of St. Louis Working Paper*, No. 99-019.

Fitzgerald, T. J. (1997), "Inventories and the Business Cycle: An Overview," *Economic Review of the Federal Bank of Cleveland*, 11-22.

Forni, M., M. Hallin, M. Lippi and L. Reichlin (2001), "Coincident and Leading Indicators for the EURO Area," *Economic Journal*, Vol. 111, C61-C85.

Forni, M. and M. Lippi (1997), *Aggregation and the Microfoundations of Dynamic Macroeconomics*, Clarendon Press, Oxford.

Forni, M. and L. Reichlin (1999), "Federal Policies and Local Economies: Europe and the US," forthcoming in *European Economic Review.*

Gallegati, M. and M. Gallegati (2000), "Business Cycle Fluctuations in Europe (1960-1998)," mimeo, University of Ancona and University of Teramo.

Gandolfo, G. (1981), *Qualitative Analysis and Econometric Estimation of Continuous Time Dynamic Models*, North-Holland, Amsterdam.

Granger, C. W. J. (1966), "The Typical Spectral Shape of an Economic Variable," *Econometrica*, Vol. 34, 150-161.

Granger, C. W. J. and P. Newbold (1986), *Forecasting Economic Time Series*, 2nd ed., Academic Press, Orlando.

Guay, A. and P. St-Amant (1997), "Do the Hodrick-Prescott and Baxter-King Filters Provide a Good Approximation of Business Cycles?," *CREFE Working Paper (Cahier de Recherche) - Université du Québec à Montréal*, No. 53.

Harvey, A. C. and A. Jaeger (1993), "Detrending, Stylized Facts and the Business Cycle," *Journal of Applied Econometrics*, Vol. 8, 231-247.

Heintel, M. (1998), "A Note on a Bayesian Order Determination Procedure for Vector Autoregressive Processes," *Statistical Papers*, Vol. 39, 213-221.

Hillinger, C. (1992), "Paradigm Change and Scientific Method in the Study of Economic Fluctuations," in C. Hillinger (ed.), *Cyclical Growth in Market and Planned Economies*, Clarendon Press, Oxford, 5-46.

Howrey, P. E. (1980), "The Role of Time Series Analysis in Econometric Model Evaluation," in J. Kmenta and J. B. Ramsey (eds.), *Evaluation of Econometric Models*, Academic Press, New York, London, 275-307.

Imbs, J. (1999), "Co-Fluctuations," *CEPR Discussion Paper*, No. 2267.

Kiefer, D. (1996), "Searching for Endogenous Business Cycles in the U.S. Postwar Economy," *Metroeconomica*, Vol. 47, 34-56.

Kiyotaki, N. and K. D. West (1996), "Business Fixed Investment and the Recent Business Cycle in Japan," in B. S. Bernanke and J. J. Rotemberg (eds.), *NBER Macroeconomics Annual 1996*, MIT Press, Cambridge, London, 277-323.

Koopmans, L. H. (1995), *The Spectral Analysis of Time Series,* 2nd ed., Academic Press, San Diego.

Lütkepohl, H. (1991), *Introduction to Multiple Time Series Analysis*, Springer, Berlin, Heidelberg, New York.

Marple, L. S. Jr. (1987), *Digital Spectral Analysis with Applications*, Prentice Hall, Englewood Cliffs, New Jersey.

McKenzie, M. D. (1999), "The Impact of Exchange Rate Volatility on International Trade Flows," *Journal of Economic Surveys*, Vol. 13, 71-106.

Muellbauer, J. (1997), "The Assessment: Business Cycles," *Oxford Review of Economic Policy*, Vol. 13, 1-18.

Prescott, E. C. (1986), "Theory Ahead of Business Cycle Measurement," *Federal Reserve Bank of Minneapolis Quarterly Review*, 9-22; reprinted in B. Snowdon and H. R. Vane (eds.), *A Macroeconomics Reader*, Routledge, London, 1997, 366-388.

Reeves, J. J., C. A. Blyth, C. M. Triggs and J. P. Small (1996), "The Hodrick Prescott Filter, a Generalisation, and a New Procedure for Extracting an Empirical Cycle from a Series," *University of Auckland - Department of Economics Working Paper*, No. 160.

Reiter, M. (1995), *The Dynamics of Business Cycles: Stylized Facts, Economic Theory, Econometric Methodology and Applications*, Physica (Springer), Heidelberg.

Sargent, T. J. (1987), *Macroeconomic Theory*, 2nd ed., Academic Press, San Diego.

Schenk-Hoppé, K. R. (2000), "Economic Growth and Business Cycles: A Critical Comment on Detrending Time Series," *Institute for Empirical Research in Economics, University of Zurich, Working Paper*, No. 54.

Selover, D. D. (1997), "Business Cycle Transmission between the United States and Japan: A Vector Error Correction Approach," *Japan and the World Economy*, Vol. 9, 385-411.

Woitek, U. (1996), "The G7 Countries: A Multivariate Description of the Business Cycle Stylized Facts," in W. A. Barnett, G. Gandolfo and C. Hillinger (eds.), *Dynamic Disequilibrium Modelling*, Cambridge University Press, Cambridge, 283-309.

Woitek, U. (1997), *Business Cycles: An International Comparison of Stylized Facts in a Historical Perspective*, Physica (Springer), Heidelberg.

Woitek, U. (1998), "A Note on the Baxter-King Filter," *University of Glasgow - Discussion Papers in Economics*, No. 9813.

Wymer, C. R. (1993), "Continuous-time Models in Macroeconomics: Specification and Estimation," in G. Gandolfo (ed.), *Continuous Time Econometrics - Theory and Applications*, Chapman and Hall, London, 35-79.

Yoshikawa, H. (1995), *Macroeconomics and the Japanese Economy*, Oxford University Press, Oxford.

Zarnowitz, V. (1985), "Recent Work on Business Cycles in a Historical Perspective: A Review of Theory and Evidence," *Journal of Economic Literature*, Vol. 23, 523-580.

Zarnowitz, V. (1996), *Business Cycles: Theory, History, Indicators, and Forecasting*, University of Chicago Press, Chicago.