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# The Dependence Structure between Carbon Emission Allowances and Financial Markets – A Copula Analysis


## Abstract

This paper applies different copulas in order to investigate the complex dependence structure between EU emission allowance (EUA) futures returns and those of other commodities, equity and energy indices. The analysis yields important insights into the relationship between carbon, commodities and financial markets. First of all, we find a significant relationship between EUA returns and those of the other considered variables that is most appropriately modeled by a Gaussian and Student-t copula. These results contradict some earlier studies that report no statistically significant or even negative correlations between returns of emission allowances and other financial variables. Secondly, considering time-varying copulas shows that the estimated copula parameters are not constant over time. We find in particular that the dependence is stronger during the period of the financial crisis. In a Value-at-Risk (VaR) analysis, finally, we further illustrate the advantages of copula methods. In particular the Student-t copula provides an appropriate quantification of VaR at different confidence levels while other models fail to specify the risk correctly. This analysis shows that ignoring the actual nature of dependence might lead to an underestimation of the risk for portfolios combining EUAs with commodities or equity investments.


JEL-Code: Q280, G130, C190.

Keywords: $CO_2$ emission trading, commodity markets, copula models, dependence structure.

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## 1 Introduction

Under the Kyoto Protocol the EU has committed to reducing greenhouse gas (GHG) emissions by 8% compared to the 1990 level by the years 2008-2012. In order to give a price to carbon emissions and to incentivize the reduction of the respective GHG, an EU-wide $CO_2$ emissions trading system (EU ETS) has been set up. Thus, the right to emit a particular amount of $CO_2$ has become a tradable commodity and is now a factor of production that is subject to stochastic price changes. This new market not only requires regulated emitters to run an adequate risk management, it also provides new business development opportunities for market intermediaries and service providers like brokers or marketeers. However, it is essential for carbon market players to learn about price dynamics in order to realize trading as well as risk strategies and investment decisions.

Since the beginning of the emission trading in 2005, a number of studies have analyzed the behavior of emission allowance prices. Benz and Trück (2009), Daskalakis et al. (2009) as well as Paolella and Taschini (2008) provide an econometric analysis of the behavior of allowance prices and investigate different models for the dynamics of short-term spot prices. Other studies have been investigating derivative products in EUA markets like convenience yields and the term structure of futures prices (Trück et al., 2006), as well as the effects of options trading on market volatility (Chevallier et al., 2009b). Böhringer and Lange (2005) and Schleich et al. (2006) conduct simulation studies on $CO_2$ market prices with respect to changes in different market design parameters.

The aim of this paper is to provide a thorough analysis of the dependence structure between EUA returns and those of other financial variables and commodities. As EUAs are a factor of production, it is plausible to assume that changes in emission allowance prices are related to the dynamics of other commodity markets. We contribute to the literature in the three dimensions. First of all, we apply different copula models in order to investigate the nature of dependence between EUA returns and those of other financial assets. Copulas are generally a very flexible method to model the relationship between different variables. Among the advantages are the possibility to account for different types of tail dependence of the return series under consideration. Thus, the application of copulas yields considerably closer insights than e.g. the application of linear correlation models. To our best knowledge, this paper is a pioneer study on copulas in the area of carbon market research. Secondly, we apply time-varying copulas in order to investigate whether the relationships under consideration are constant over time. This procedure allows us to study as to whether influencing factors on carbon prices changed over time and whether or not the financial crisis had an impact on the dependence between the considered variables. Finally, we conduct a risk management analysis in order to further illustrate the usefulness of the application of copulas. It is often argued that EUA prices are more strongly influenced by policy measures and regulatory changes than other commodities and could, in consequence, potentially be used for portfolio diversification, see e.g. Chevallier (2009). Therefore, we provide a risk analysis by comparing benchmark models including e.g. a standard variance-covariance approach to the estimated copula models with respect to the quantification of the risk. We show how a misspecification of the actual dependence structure might not only lead to an inappropriate specification of the portfolio return distribution, but also underestimate the risks from joint extreme returns.

The remainder of the paper is organized as follows. Section 2 provides a brief description of the market mechanism for $CO_2$ emission allowances, a classification of the assets as well as price drivers of the market. Section 3 provides a review of different copula models with respect to estimation, model testing, modeling the dependence structure and risk analysis. Section 4 describes the considered data and presents the empirical results of our study. Section 5 concludes and gives suggestions for future work.

## 2 The European Emission Trading Scheme

### *2.1 Regulatory Setting*

This section briefly discusses the regulatory setting of the EU ETS. The scheme affects combustion installations exceeding 20 MW including different kinds of industries like metal, cement, paper, glass as well as power generation and refineries. In total, the EU-ETS includes some 12500 installations, representing approximately 40% of EU's GHG emissions. From 2013 onwards the system will cover additional GHG emissions such as perfluorcarbons (PFCs) and dinitrogen monoxide ($N_2O$). After an initial pilot trading period (2005-2007), new national allocation plans (NAPs) have been issued for the second trading phase from 2008-2012. From 2013 a third trading period will run until 2020. In the third period the NAPs will be replaced by unified rules applying to all member states. Generally, allowances may either be allocated free of charge or auctioned. According to the European Commission the importance of auctioning will further increase over time. However, it is important to note that the annual quantity of allocated emission allowances is limited and already specified by the EU-Directive until 2020. Some regulatory settings are particularly important, as they shape compliance behavior and, thus, are likely to have price effects. Under the current system, banking - the storage of unused certificates - gives more leeway for complying parties and smoothes prices. A detailed analysis of banking and borrowing rules is provided by Alberola and Chevallier (2009). Another particularity of the current framework is a period of allocation overlap: allowances for the new compliance year are obtained in February, whereas the EUAs due for the previous year have to be submitted in April.

Generally, a lack of allowances requires a company to either buy a sufficient amount of EUAs or to invest in some plant-specific process improvements. A third option is the purchase of additional allowances and emission credits from Clean Development Mechanism (CDM) or Joint Implementation (JI) projects, the so-called Flexible Mechanisms under the Kyoto Protocol. Failure to submit a sufficient amount of allowances results in sanction payments of 100 Euro per missing ton of $CO_2$ allowance. In addition, companies have to surrender the missing allowance. As a consequence, participating companies face several risks specific to emissions trading. In particular, price risk of fluctuating allowance prices and volume risk, due to unexpected fluctuations in energy demand the emitters do not know ex ante their exact demand for EUAs, have to be considered. Naturally, market generic risks – like counterparty, operational, reputational, etc. – are also present. For a discussion see e.g. Bokenkamp et al. (2005).

### *2.2 Classifying Carbon Emission Allowances*

Having outlined some important regulatory issues, the nature of EUAs is now discussed in more detail. Due to a number of specific features, EUAs are different to traditional commodities. What is sold is essentially the lack or absence of $CO_2$. Therefore, emissions are said to become either an asset or a liability for the obligation to deliver allowances that cover those emissions (PointCarbon, 2004). Benz and Trück (2009) point out the differences between emission allowances and classical stocks. Demand and value of a stock is based on profit expectations of the underlying firm. Certainly, resource scarcity plays a role in this price formation process. The $CO_2$ allowance price, however, is much more directly determined by the expected market scarcity, which is externally set by the European commission.

Another insightful approach for specifying $CO_2$ emission allowances is their consideration as a factor of production (Fichtner, 2004). The shortage of emission allowances by reducing the emissions cap for the commitment periods classifies the assets as 'normal' factors of production - they can be 'exhausted' for the production of $CO_2$. After their redemption or at the end of the commitment period when they expire, they are then removed from the market. Accordingly, it seems more adequate to compare the right to emit $CO_2$ with other operating materials or commodities than with a traditional equity share.

In order to set up a comprehensive analysis, it is of great importance to identify the key price determinants of the $CO_2$ emission allowances. Following the investigation of $SO_2$ permit prices by Burtraw (1996), we categorize the principle driving factors of $CO_2$ allowance prices into (i) policy and regulatory issues and (ii) market fundamentals that directly concern the production of $CO_2$ and thus demand and supply of $CO_2$ allowances.

Regulatory settings, as in part (i), are likely to shape long-term level of prices. For our pricing model we are interested in the determinants of short-term price behavior. Policy changes may lead to sudden price changes if the market is suprised by decisions concerning the NAPs or a change of the European commitment to reduce 30% instead of 20% until 2020. Hence, the consequences of changes in such regulatory or policy issues may be sudden price jumps and phases of extreme volatility (Gronwald and Ketterer (2009), Sanin and Violante (2009)). Chevallier et al. (2009a) specifically investigate the EUA price drop in April 2007 and show that the market perception of risk changed substantially during that period.

Incorporating part (ii), allowance prices fundamentally depend on the emission level of $CO_2$ which is influenced by factors like economic growth or fuel prices. Some comprehensive research on determinants has been conducted by Alberola et al. (2008), Mansanet-Bataller et al. (2007) or Chesney and Taschini (2008). An important force is weather data, such as temperature, rain fall and wind speed. Hintermann (2010) detects a negative effect of availability of hydropower in Nordic countries during the first trading phase. Rickels et al. (2010) confirm this result for the second trading phase and find the same relationship with wind power: higher wind speeds in Germany lead to lower EUA prices. Mansanet-Bataller et al. (2007), Rickels et al. (2007) and Alberola et al. (2008) show that extremely hot or cold days have a positive effect on EUA prices.

Also energy variables have a clearly identified impact on the prices of emission allowances (Chevallier, 2009). For example, an electricity producer switching from 'cheap-but-dirty' coal to 'expensive-but-cleaner' gas can significantly reduce emissions per MWh of produced electricity. Therefore, fuel-switching from coal to gas implies less emissions to be covered with permits and the price of EUAs should be dependent on prices of gas and coal, see e.g. Fehr and Hinz (2006). With respect to the influence of energy prices on carbon prices, the literature discovers relatively robust patterns. Mansanet-Bataller et al. (2007) find positive effects of oil and gas prices on EUA forward prices in Phase I, while there is no significant influence of the coal price. The same results are given by Hintermann (2010). In a study by Rickels et al. (2007), coal shows up with a negative sign. Similarly, Alberola et al. (2008) reports a negative effect of coal on carbon prices and detect positive effects of gas, oil and electricity prices.

The dependence of carbon and energy prices is studied in bidirectional manner. On the one hand, electricity and commodity prices are identified as fundamental factors for carbon prices. On the other hand, the price effect of emissions trading on energy prices is traced. Kara et al. (2008) report that the EU emissions trading has a price increasing effect on electricity prices in Finland - but the authors do not consider daily or weekly data. For Germany, Hirschhausen and Zachmann (2008) show that carbon price changes are passed through to wholesale power prices. This effect, however, is asymmetric, as carbon price increases have a stronger impact. Daskalakis and Markellos (2009) confirm the asymmetry, but in their estimations a falling carbon price has a larger effect on electricity prices. Bunn and Fezzi (2007) investigate the economic impact of the EU ETS for carbon on wholesale electricity and gas prices in the UK. Using a structural co-integrated VAR model, they conclude that the prices of carbon and gas jointly influence the equilibrium price of electricity and estimate the transmission of shocks between gas, carbon and power prices. Nazifi and Milunovich (2010) apply a restricted VAR model in first differences to test for existence of causal relationship and long-run links between the price of carbon and the prices of energy fuels and electricity. They apply Granger-causality tests and generalized impulse response analysis and their results suggest that the dynamics of energy prices are rather independent from the price of carbon emissions permits for the considered time period. However, they find weak evidence of Granger-causality running from carbon futures prices to natural gas prices. Reinaud (2007) investigates the interaction between $CO_2$ allowance and electricity prices and the impact on the industry's electricity purchasing strategies in Europe. While the author concludes that there is no universal answer on how the EU ETS has affected electricity prices, at least some evidence for the $CO_2$ cost pass-through into electricity prices was provided during the abrupt fall of the $CO_2$ price in May 2006. The fall by ten Euros per tonne of $CO_2$ was immediately followed by a drop in wholesale electricity prices by five to ten Euros per MWh in several markets. Reinaud (2007) further argues that this electricity price adjustment is directly attributable to the $CO_2$ price fall, since it was not connected to other energy market movements that could also affect electricity prices. Generally, the results of the influence of carbon prices on other commodity prices are varied: so far there seems to be no common agreement whether the price of carbon emission allowances has a significant influence on energy markets or not.

The literature, however, paid less attention to the relationship of emission allowance prices with financial variables. Rising carbon prices, as a factor of production, could be related to additional costs and uncertainties for producers and consumers and might have an adverse effect on equity markets in general or equities of certain industries in particular. Kosobud et al. (2005) find no statistically significant correlations between monthly returns of $SO_2$ emission allowance prices in the US market and returns from various financial investments. Hintermann (2010) spots no influence of the British FTSE equity index during the first trading phase of the EU ETS. Oberndorfer (2009) examines the impact of stock returns of large electricity companies on carbon prices. He identifies a positive effect that varies across countries. Veith et al. (2009) employ a multifactor model and confirm this finding for the first trading phase. Daskalakis et al. (2009) detect negative correlations of EUA futures with equity market returns what may offer significant diversification opportunities to European equity investors. They argue that the factors determining stock and bond prices are substantially different from those affecting emission permits. In a study on the relationship between macroeconomic variables and carbon futures, Chevallier (2009) finds that stock and bond markets - as proxies for macroeconomic risk - have little influence on EUA futures. The author suggests that emission allowances are an too easily storable commodity and therefore not prone to react to macroeconomic shocks as much as stock markets. Chevallier (2009) therefore suggests that the use of emission allowance prices for diversification purposes should be further investigated.

To our best knowledge, so far there has been no empirical study concentrating mainly on the dependence structure between EUA returns and those of other financial variables or commodity markets. Next to standard approaches investigating linear dependence by correlation analysis in our study we also apply different copulas to model the complex dependence structure between the return series of carbon emission, commodity and equity markets.

## 3 Copula Models

Recently, the application of copulas became very popular in empirical finance. One reason for this is certainly that copulas are a flexible instrument for modeling the dependence structure between returns of financial time series. They allow for capturing different types of tail dependence between the variables under consideration. Thus, the application of copulas yields deeper insights into the dependence structure of financial variables. Moreover, there has been some general criticism towards the assumptions of multivariate normality for the joint distribution of asset returns and the use of a covariance matrix as the natural measure of dependence between financial assets. As shown in studies such as Jondeau and Rockinger (2006a), Junker et al. (2006), Luciano and Marena (2003) or McNeil et al. (2005), using correlations may not appropriately describe the dependence structure between financial assets and, in consequence, could lead to inadequate measurement of the risk of joint extreme price movements. The application of copula methods is suggested for modeling the dependence structure of the asset returns in order to overcome this problem.[1] For an excellent overview on copula methods in finance, see e.g. Cherubini et al. (2004), where the range of applications of copula methods includes various topics such as portfolio analysis, derivative pricing as well as credit risk analysis. With respect to analyzing the dependence structure between different financial assets, copula models, as alternative to the multivariate normal model, do not necessarily require assumptions of joint normality for the distributions. Instead a copula allows joining arbitrary marginal distributions into their one dimensional multivariate distribution. Therefore, a wide range of dependence structures can be captured by using different copulas. The multivariate joint distribution can be decomposed into marginal distributions and an appropriate functional form for the dependence between the asset returns.

This section provides a brief review on the estimation and goodness-of-fit tests for copulas that will be used in the subsequent empirical analysis. Since this can be considered as a pioneer study on applying and testing different copula models to emission allowance markets, we also briefly illustrate some basic concepts of copula families and the dependence measure Kendall's $\tau$.

### *3.1 Copula Functions*

A copula is a function that combines marginal distributions to form a joint multivariate distribution. The concept was initially introduced by Sklar (1959), but has only gained high popularity in modeling financial or economic variables in the last two decades. For an introduction to copulas see e.g. Nelsen (1999) or Joe (1997), for applications to various issues in financial economics and econometrics, see e.g. Cherubini et al. (2004), McNeil et al. (2005), Frey and McNeil (2003) and Hull and White (2004) to name just a few. Longin and Solnik (2001) empirically show that asset returns are more highly correlated during volatile markets and during market downturns. According to Dowd (2004) the strength of the copula stems from its feature that it does not require any assumption regarding the joint distributions among the financial assets in a portfolio. Overall, the use of copulas offers the advantage that the nature of dependence can be modeled in a more general as well as flexible setting than using only linear dependence that is captured by correlation.

A copula is the distribution function of a random vector in $\mathbb{R}^n$ with standard uniform marginals. Let $X = (X_1, \ldots, X_n)'$ be a random vector of real-valued random variables whose dependence structure is completely described by the joint distribution function

[1] Note, however, that the Gaussian copula with the assumption of normal marginals coincides with the multivariate normal distribution and, thus, is fully characterized by the correlation coefficient.

$$F(x_1, \ldots, x_n) = P(X_1 < x_1, \ldots, X_n < x_n). \tag{1}$$

Each random variable $X_i$ has a marginal distribution of $F_i$ that is assumed to be continuous for simplicity. The transformation of a continuous random variable $X$ with its own distribution function $F$ results in a random variable $F(X)$ which is uniformly distributed over $[0, 1]$. Thus transforming equation (1) component-wise yields

$$\begin{aligned} F(x_1, \ldots, x_n) &= P(X_1 < x_1, \ldots, X_n < x_n) \\ &= P[F_1(X_1) < F_1(x_1), \ldots, F_n(X_n) < F_n(x_n)] \\ &= C(F_1(x_1), \ldots, F_n(x_n)), \end{aligned} \tag{2}$$

where the function $C$ can be identified as a joint distribution function with standard uniform marginals — the copula of the random vector $X$. Equation (2) illustrates how the copula combines the marginals to the joint distribution. The copula framework can be generalized for any collection of marginal distributions and joint distributions. In our application we will mainly consider the bivariate case with a function $C(u, v)$ such that,

$$C(u, v) = C[F(x), G(y)]. \tag{3}$$

Then the function $C(u, v)$ is defined as a copula function which relates the marginal distribution functions $F(x)$ and $G(y)$ into their joint probability distribution. Moreover, if marginal distributions $F(x)$ and $G(y)$ are continuous, the copula function $C(u, v)$ is unique, see e.g. Sklar (1959).

### *3.2 Examples of copulas*

The literature reports a wide range of different copulas, see e.g. Joe (1997) or Nelsen (1999) for an overview of the most common parametric families of copulas. In the following we will describe four of the most commonly applied copulas: the Gaussian, Student-$t$, Clayton and Gumbel copula.

We will start with the multivariate Gaussian and Student-$t$ copula that belong to the class of elliptical copulas. The probably most intensively used copula in financial applications is the Gaussian copula. It is constructed from the multivariate normal distribution and is denoted by

$$C_\rho^N(u_1, \ldots, u_d) = \Phi_\Sigma^d(\Phi^{-1}(u_1), \ldots, \Phi^{-1}(u_d)) \tag{4}$$

Hereby, $\Phi$ denotes the the standard normal cumulative distribution function, $\Phi^{-1}$ the inverse of the standard normal cumulative distribution function and $\Phi_\Sigma^d$ the standard multivariate Normal distribution with correlation matrix $\Sigma$. Applying $C_\rho^N$ to two univariate standard normally distributed random variables results in a standard bivariate normal distribution with correlation coefficient $\rho$. Further note that, since the copula and the marginals can be arbitrarily combined, this (and any other) copula can be applied to any set of univariate random variables. The outcome will then surely not be multivariate normal, but the resulting multivariate distribution has inherited the dependence structure from the multivariate normal distribution. The multivariate normal copula correlates the random variables rather near the mean and, therefore, fails to incorporate dependence in the tail. The Student-$t$ copula, in contrast, is able to capture tail dependence. This copula is denoted by:

$$T_{\Sigma,v}(u_1, u_2, \ldots, u_d) = t_{\Sigma,v}(t_v^{-1}(u_1), t_v^{-1}(u_1), \ldots, t_v^{-1}(u_d)) \tag{5}$$

where $t_{\Sigma,v}$ is the multivariate Student-$t$ distribution with $v$ degrees of freedom and correlation matrix $\Sigma$. Depending on the degrees of freedom parameter, the Student-$t$ copula can also determine the strength of the tail dependence. Generally, low values of the parameter $v$ indicate strong tail dependence.

The elliptical copulas discussed have in common that they can be used to model symmetric tail dependence. It is, however, a common occurrence for economic and financial variables to exhibit tail-dependence in only one of the tails, either the upper right or lower left tail. For example, tail-dependence in the lower left tail indicates that the two variables show simultaneous extreme negative returns while high positive returns in one of the variables may not affect the other variable that much. To model asymmetric tail dependence, so-called Archimedean copulas can be used, see e.g. Cherubini et al. (2004). Two of the most prominent members of the family of Archimedean copulas are the Clayton and Gumbel copula that will be briefly described in the following. The Clayton copula is an asymmetric Archimedean copula, exhibiting greater dependence in the negative lower tail than in the positive upper one. The multivariate Clayton copula is denoted by:

$$C_\theta^{Cl}(u_1, ..., u_d) = \left[\sum_{i=1}^{d} u_i^{-\theta} - d + 1\right]^{1/\theta}, \tag{6}$$

For the Clayton copula, the parameter $\theta > 0$ can be interpreted as a measure of the degree of dependence. The greater $\theta$, the stronger is the dependence between the considered variables, in particular in the lower left tail. The Gumbel

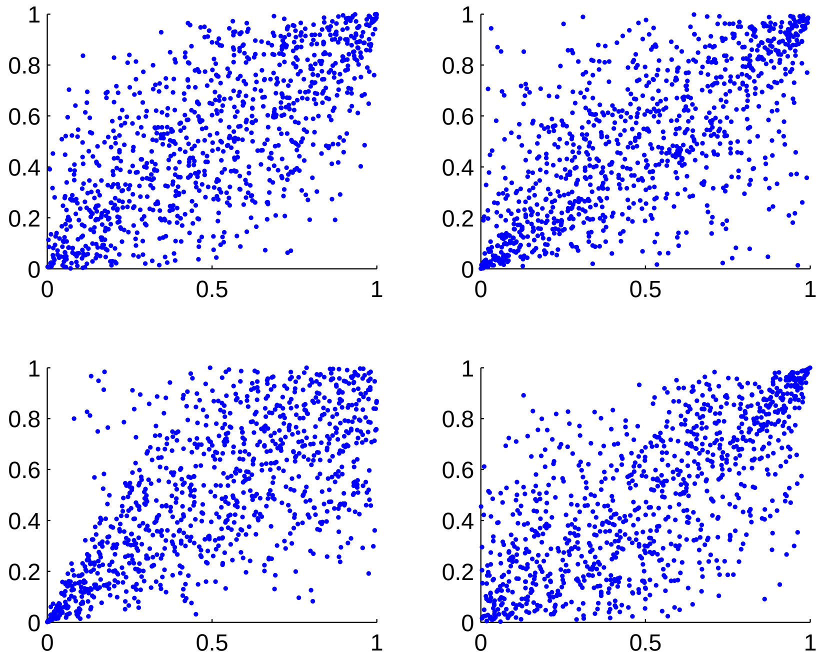

Fig. 1. Scatter plot of simulated dependence structure of ranks for different copulas with the same Kendal's $\tau = 0.5$. The graph illustrates the dependence between the ranks for the Gaussian (upper left panel), the Student-$t$ (upper right panel), Clayton (lower left panel) and Gumbel copula (lower right panel).

copula, in contrast, exhibits greater dependence in the upper right tail and is denoted by:

$$C_{\phi}^{Gu}(u_1, ..., u_d) = \exp\left[-\left\{\sum_{i=1}^{d}(-ln(u_i)^{\phi}\right\}^{1/\phi}\right], \tag{7}$$

where $\phi > 1$ indicates the dependence between the random variables $X_1, ...X_d$.

Often Kendall's $\tau$ is used for characterizing the dependence structure. Kendall's $\tau$ is a rank-based measure of dependence that provides consistent estimation of the true underlying copula as it is shown for example in Deheuvels (1979). [2] Values of $\tau$ range from $-1$ to $+1$, while in the case of independence $\tau$ will be 0, see e.g. Nelsen (1999). In the case of a bivariate one-parameter copula, Kendall's $\tau$ is an appropriate dependence measure, as there is a one-to-one relationship between the copula parameter and Kendall's $\tau$.

[2] Another rank-based measure of dependence is Spearman's $\rho$. Cherubini et al. (2004) explains there measures as well as their differences in greater detail.

For the purposes of illustrating the different copula models discussed above, Figure 1 shows scatter plots for four different copula functions based on the same Kendall's $\tau = 0.5$. The graph illustrates the symmetric dependence structure for the Gaussian and Student-$t$ copula, while the Student-$t$ copula exhibits more tail dependence in the lower left and upper right tail in comparison to the Gaussian one. Further the asymmetric Clayton copula exhibits greater dependence in the negative lower tail, while the Gumbel copula exhibits greater dependence in the positive upper tail as illustrated by the graph.

### *3.3 Estimation procedure*

As asserted above, copulas offer an alternative to the correlation coefficient as it comes to modeling the dependence structure. Different approaches to estimate copulas have been suggested in the literature. In this paper the copula parameters are estimated using the transforms from the empirical marginal distribution function $\hat{F}_i(x_i)$ by canonical maximum likelihood (CML) estimation (Bouye et al., 2000).[3] In this case the vector of parameters is estimated semi-parametrically by maximizing the loglikelihood for the copula density using the empirical marginals $\hat{F}_i(x_i)$.

Due to the conditional heteroscedasticity usually present in financial time series instead of modeling the unconditional return distribution we concentrate on the conditional returns and adopt the framework of semiparametric copula-based multivariate dynamic (SCOMDY) models suggested by Chen and Fan (2006). As the name suggests, this class of models arises from a combination of methods. In these models, the conditional mean and the conditional variance of a multivariate time series are specified parametrically while the joint distribution takes a semiparametric form using a parametric copula and nonparametric marginals. The method creates additional flexibility. The typical non-normal movements of financial time series may be captured more accurately. Still, the copula estimation remains low-dimensional and allows to represent various nonlinear and, asymmetric dependence structures (Linton and Yan, 2011).

[3] In the bivariate case, based on the estimated value of $\tau$, the dependence parameter for the chosen copula can be calculated as a function of $\tau$. For the Gaussian, Student-$t$, Clayton and Gumbel copula this is straightforward and as pointed out by Genest et al. (2009) under weak regularity conditions on the copula family, this yields a consistent estimator of the dependence parameter. Note, however, that for the Student-$t$ copula as indicated by equation (5) also the degrees of freedom parameter needs to be estimated. Due to its simplicity in comparison to other estimation techniques, copula estimation via rank transformation and Kendall's $\tau$ is often applied in practical applications. Unfortunately, it is limited to a bivariate setting because it makes inference on the dependence structure of the multivariate model from a chosen dependence coefficient.

Following the notation by Chen and Fan (2006), $(Y_t', X_t'), t = 1, ...n$ denotes a vector stochastic process, where $Y_t$ is a d-dimensional process of endogenous and $X_t$ is a vector of exogenous variables. A SCOMDY models is then defined as follows:

$$Y_t = \mu_t(\theta_1) + \sqrt{H_t(\theta)}\epsilon_t, \tag{8}$$

where the vector $\mu_t(\theta_1)$ denotes the true conditional mean parameters and the vector $H_t(\theta)$ the true conditional variance, both for given values of $Y_{t-1}, Y_{t-2}, ...$ and $X_t, X_{t-1}, ....$ The innovations in vector $\epsilon_t$ are i.i.d. with zero mean and unit variance and have the distribution function $F(\epsilon) = C(F_1(\epsilon_1)), ..., F_d(\epsilon_d)$ with $F_j(\cdot)$ as true but unknown continuous marginal and $C(u_1, ..., u_d)$ as true copula function.

Various non-linear models can be used for modeling the conditional mean and the conditional variance. Thus, in combination with the variety of available copula models, this approach allows a great extent of flexibility for the final model specification. For a more thorough description of the SCOMDY model class the reader is referred to the original paper by Chen and Fan (2006). The estimation procedure can be summarized the following way:

- Estimate all conditional mean and variance parameters in order to obtain standardized innovations.
- In a second step, the empirical distribution function of these standardized innovations, denoted as $\hat{F}_j(\mu_{j,t}(\theta))$, (j=1,...,d), is estimated nonparametrically. Section 4.3 describes this step for our dataset.
- Finally, the copula dependence parameter is derived by using the copula specification (as in (4) to (7)) and its density C($\hat{F}_1(\epsilon_{1,t}(\theta))$,..., $\hat{F}_d(\epsilon_{d,t}(\theta))$) for maximization of the loglikelihood (Trivedi and Zimmer, 2005).

### *3.4 Goodness-of-fit Tests*

One of the challenges is deciding on which copula provides the best fit to the actual dependence structure of the data. Berg and Bakken (2006) note that information criteria such as e.g. Akaike's Information Criterion (AIC) are generally not able to provide any understanding about the power of the decision rule employed. Instead, goodness-of-fit (GOF) approaches are more powerful in deciding whether to reject or accept parametric copulas, making them the preferred choice in empirical applications, see e.g. Genest et al. (2006, 2009), Panchenko (2005), Berg and Bakken (2006). Therefore, in our empirical analysis, for selecting the most appropriate among a set of copulas, we decided to use goodness-of-fit tests that investigate the distance between the estimated and the so-called empirical copula, see e.g. Genest et al. (2006, 2009). The empirical copula basically represents an observed frequency and is calculated from the empirical margins. The distance between estimated and empirical copula is then evaluated using the so-called Cramér-Von Mises test statistic. The parametric copula that is closest to the empirical copula is the most appropriate choice.

The following section describes the procedure in greater detail. Empirical copulas were introduced by Deheuvels (1979) under the name of empirical dependence functions. Let $(X_{1i}, ..., X_{ni})$ be $n$ observations of the random variable $X_i$. Then the empirical marginal cdf for a random variable $X_i$ is:

$$\hat{F}_i(x_i) = \frac{1}{n+1}\sum_{j=1}^{n} I(X_{ji} \leq x_i) \qquad i = 1, .., d \tag{9}$$

where $I(\cdot)$ denotes the indicator function returning the value of 1 if $X_{ji} \leq x_i$ and 0 otherwise. Further, in the denominator $n+1$ is used to keep the empirical cdf to be smaller than 1. Note that the empirical marginal distribution converges towards the actual distribution function for $n$ approaching infinity. Defining the empirical probability integral transforms $u_{ji} = \hat{F}_i(x_{ji})$ for $i = 1, .., d; \quad j = 1, .., n$, for the vector $u = (u_1, .., u_d)$, using the marginal cdf's, the empirical copula is given by

$$C^{emp}(u) = \frac{1}{n+1}\sum_{j=1}^{n} I(\hat{F}_1(x_{j1}) \leq u_1), ...., \hat{F}_d(x_{jd}) \leq u_d)) \tag{10}$$

$$= \frac{1}{n+1}\sum_{j=1}^{n} I(U_1 \leq u_1, ...., U_d \leq u_d) \tag{11}$$

According to e.g. Tsukahara (2005), the empirical copula is a consistent estimator of the true copula and, thus, is a well-accepted benchmark for copula goodness-of-fit tests.[4] The distance between the empirical and the estimated copula is measured using the Cramér-Von Mises statistic:

$$S_n = \sum_{i=1}^{n}[C^{emp}(U_i) - C_\theta(U_i)]^2$$

We concentrate on so-called 'blanket tests', where the implementation does

[4] Note that the empirical copula is not a copula according to the definition by Deheuvels (1979), but rather the observed frequency of $P(U_1 \leq u_1, ...., U_d \leq u_d)$.

not require an arbitrary categorization of the data or any strategic choice of smoothing parameters, weight functions or kernels. Genest et al. (2009) provide various options for such tests by conducting a large Monte Carlo experiment and report particularly good results for the blanket tests using ranks or the Rosenblatt transform. With respect to the chosen distance measure, the authors recommend the so-called Cramér-Von Mises statistic. Based on these results, we only describe tests based on ranks that use the Cramér-Von Mises for measuring the difference between the estimates and the empirical copula. [5]

For the suggested approach the test procedure for investigating whether the dependence structure of a multivariate distribution is well-represented by a specific parametric family of copulas can be roughly summarized as follows:

1. Based on the empirical cdfs for the marginal series, estimate the empirical copula $C^{emp}(U_i)$ and the parametric copula $C_\theta(U_i)$.

2. Using the Cramér-Von Mises statistic, calculate the distance between the empirical and estimated copula:

$$S_n = \sum_{i=1}^{n} [C^{emp}(U_i) - C_\theta(U_i)]^2$$

3. In a bootstrap procedure, for some large integer D, the following steps are repeated:

(1) Generate a random sample from $C_\theta$ and compute the associated rank vectors $(U_1^*, ..., U_n^*)$ as well as the empirical copula $C^{emp*}(u)$.
(2) Estimate the parametric copula $C_{\theta^*}$.

(3) Determine $S_n^* = \sum_{i=1}^{n} [C^{emp*}(U_i) - C_{\theta^*}(U_i)]^2$ for the generated sample.

4. From the D bootstrap samples, an approximate p-value, measuring the goodness-of-fit of the copula, can be calculated as the fraction of simulations where $S_n^* > S_n$.

If the considered copula provides a good fit to the actual dependence structure of the data, we should expect to get high p-values, while for a copula providing a bad fit to the actual data, we will expect the p-value to be low. In this case, depending on the level of confidence, the hypothesis that the dependence structure of a multivariate distribution is well-represented by a specific parametric family of copulas will be rejected. Note that for the case

[5] Berg and Bakken (2006) or Genest et al. (2009) describe various alternative tests.

where several copula families cannot be rejected by the goodness-of-fit tests, an alternative approach as specified in e.g. Chen and Fan (2006); Diks et al. (2010) needs to be implemented. These tests are particularly designed to compare competing copula models based on their in-sample (Chen and Fan, 2006) or out-of-sample (Diks et al., 2010) loglikelihood scores.

## 4 Empirical Analysis

### *4.1 The Data*

In this section we will investigate the dependence structure between daily returns from traded emission allowance contracts and various other financial variables during the time period January 2, 2008 to December 31, 2009. The existing literature discusses the factors which are most important for carbon prices. Based on this research, we examine a number of variables from commodity and financial markets. The literature identifies energy prices to exert a strong influence on carbon prices, due to fuel-switching in the power sector. Therefore, from commodity markets, we use gas and coal futures returns as well as 2010 oil futures returns. The gas and oil futures are obtained from the International Commodity Exchange (ICE). Data on coal futures as well as electricity futures (Phelix baseload futures) are taken from the European Energy Exchange (EEX) in Leipzig. Data on EUA prices are obtained from the London-based European Climate Exchange (ECX). As emission levels are related to economic activity, we take stock markets as a proxy for economic development. In addition to the broader European stock market index, the Eurostoxx 50, we consider the more energy-specific DJ Europe Energy Stock Index (E1ENE) and the European Renewable Energy Index (ERIXP). One may assume that the relationship between carbon prices and energy-related stocks is particularly strong. For our analysis we consider log-returns that are calculated as $r_t = ln(P_{t+1}/P_t)$ from the original price series.

### *4.2 Estimation results for the marginal series*

Following the SCOMDY approach described in section 3.3, in a first step we need to find an appropriate model for the marginals. Thus, we need to estimate the parameters for the conditional mean $\mu_{j,t}(\theta_1)$ and conditional variance $h_{j,t}(\theta)$ equations. We focus on different ARMA-GARCH specifications for each of the considered series and abstain from using additional exogenous variables. In order to avoid overfitting, the best model is chosen based on Akaike's Information Criterion (AIC) and Bayesian Information Criterion (BIC). Table

| Time Series | Suggested Model |
|---|---|
| EUA futures | ARMA(1,0)-GARCH(1,1) |
| Coal futures | ARMA(1,0)-GARCH(2,3) |
| Oil 2010 futures | ARMA(0,0)-GARCH(1,1) |
| Gas futures | ARMA(0,1)-GARCH(1,2) |
| EEX futures | ARMA(1,0)-GARCH(1,3) |
| Eurostoxx 50 Spot | ARMA(1,0)-GARCH(1,2) |
| E1ENE Spot | ARMA(1,0)-GARCH(1,2) |
| ERIXP Spot | ARMA(1,1)-GARCH(1,1) |

Table 1
Best model among the fitted ARMA-GARCH models for each of the considered time series. The choice of the best model is based on AIC and BIC model selection criteria.

1 summarizes the results for the considered series indicating for each series the model that provided the best results according to the considered model parsimony criteria. The outcome of this procedure are standardized residuals which will be used in the subsequent empirical analysis.

In order to test for i.i.d property of the standardized residuals, the BDS test for independence was applied to the standardized residuals. The BDS test is a portmanteau test for time based dependence in a series and can be used to examine whether the residuals are independent and identically distributed. We found that for none of the considered series, the null hypothesis of i.i.d could be rejected, such that in the following we assume that all standardized residuals exhibit the desired i.i.d. property necessary for copula estimation. [6] For ease of readability, we will henceforth adhere to the expression returns instead of using standardized residuals.

### *4.3 Estimation results for the copula functions*

In a next step we investigate the dependence structure between the returns from EUA and the other considered commodities and financial variables based on the fitted models for the marginal return series. As pointed out in section 3.3, applying SCOMDY models, the next step after estimating the parameters for the marginal series is to estimate the empirical distribution functions $\hat{F}_j(\mu_{j,t}(\theta))$. This has the advantage that the possibly unknown distribution for the returns is not required, since the empirical marginal cdf can be used. The

[6] The detailed results are available from the authors upon request.

| Asset | $\hat{\tau}$ | $\hat{\theta}$ | $\hat{\phi}$ | $\hat{\rho}_N$ | $\hat{\rho}_t$ |
|---|---|---|---|---|---|
| Coal futures | 0.2458** | 0.5250 | 1.2666 | 0.3680 | 0.3744 ($\hat{v} = 27.90$) |
| Oil 2010 futures | -0.0544 | 0.0000 | 1.0000 | -0.0696 | -0.0711 ($\hat{v} > 1000$) |
| Gas futures | 0.1140** | 0.2114 | 1.1175 | 0.1804 | 0.1857 ($\hat{v} = 14.96$) |
| EEX futures | 0.4135** | 0.9840 | 1.6010 | 0.5920 | 0.6008 ($\hat{v} = 14.19$) |
| Eurostoxx 50 Spot | 0.1818** | 0.3473 | 1.2055 | 0.2954 | 0.2984 ($\hat{v} = 21.47$) |
| E1ENE Spot | 0.2651** | 0.5732 | 1.3067 | 0.3937 | 0.4044 ($\hat{v} = 8.82$) |
| ERIXP Spot | 0.2005** | 0.4761 | 1.2115 | 0.3169 | 0.3203 ($\hat{v} = 12.31$) |

Table 2
Kendall's $\hat{\tau}$ and estimated copula dependence parameters $\hat{\theta}$ for the Clayton, $\hat{\phi}$ for the Gumbel, $\hat{\rho}_N$ for the Gaussian and $\hat{\rho}_t$ for the Student-$t$ copula for standardized residuals of 2010 EUA futures and the considered assets. For Kendall's $\tau$ we also report the results of a significance test with $H_0 : \tau = 0$. The asterix denotes significant rejection of the null hypothesis at the 1% ** and 5% * level.

CML method is then applied to the transforms from the empirical distribution function to estimate the dependence parameters $\hat{\theta}$ for the Clayton, $\hat{\phi}$ for the Gumbel and the copula correlation parameters $\hat{\rho}_G$ for the Gaussian and $\hat{\rho}_t$ for the Student-$t$ copula. Note that for the Student-$t$ copula also the degrees of freedom parameter $v$ needs to be estimated such that the results for the copula correlation parameters $\hat{\rho}_G$ and $\hat{\rho}_t$ are not necessarily identical.

We also estimate Kendall's $\hat{\tau}$ for each of the bivariate series and conduct a significance test for the dependence between the returns with $H_0 : \tau = 0$ versus $H_0 : \tau \neq 0$. The test is non-parametric, as it does not rely on any assumptions on the distributions of two variables $X$ and $Y$. Then under a null hypothesis of $X$ and $Y$ being independent, the sampling distribution of $\tau$ will have an expected value of zero. Note that the precise distribution cannot be characterized in terms of common distributions, however, it can be calculated exactly for small samples. For larger samples, commonly an approximation to the normal distribution, with zero mean and variance $2(2n + 5)/9n(n - 1)$ is used. For further details on the test we refer to Prokhorov (2001).

We find significant dependence at the 1% level between EUA returns and the other return series except for the oil futures where the estimated coefficient for Kendall's $\tau$ is not significant at the 1% or 5% level. The results are displayed in Table 2. We find that Kendall's $\tau$ ranges from approximately -0.05 to 0.41 for the different series while the Gaussian and Student-$t$ copula correlation parameters range from approximately -0.07 to 0.60. The highest dependence can be observed between the returns of 2010 EUA and electricity futures contracts while we observe the lowest rank dependence and correlation between 2010 EUA and oil futures contracts. Interestingly, here the estimated coefficients for Kendall's $\tau$, $\rho_G$ and $\rho_t$ are slightly negative. However, in 2008 and 2009 the oil futures behaved quite particular, dropping from a peak at 140 Dollars to a price remaining at around 80 Dollars. This may explain the weak correlation and the negative sign. There is not only a significant dependence between commodity and EUA futures contracts but also between EUAs and equity markets. In fact the returns of stock market indices like Eurostoxx 50 and the energy specific index E1ENE and the renewable energy index ERIXP seem to exhibit even a higher degree of dependence with EUA futures returns than for example oil and gas futures. Generally, our results contradict some of the earlier studies by Kosobud et al. (2005) and Daskalakis et al. (2009) on the dependence between emission allowance certificates and other financial assets. While the former found no statistically significant correlations between returns of $SO_2$ emission allowances and returns from other financial variables, the latter observed that EUA futures returns were negatively correlated with equity market returns during the pilot trading period.

Figure 2 provides a plot of the standardized residuals for daily EUA 2010 futures versus coal futures, the rank transforms of the standardized residuals for EUA 2010 futures versus coal futures, a 3d histogram of the rank transforms, and the fit of the Student-$t$ copula to the transforms. The analogous graphs are also provided for the series daily EUA 2010 futures versus E1ENE returns in Figure 3.

In order to investigate which of the copula describes the dependence structure best, we use the Cramér-Von Mises statistic to measure the distance between the empirical and estimated copulas. Since the distance between the estimated and empirical copula alone is not sufficient to determine whether any of the models really provides a good fit to the data, goodness-of-fit tests proposed by Genest et al. (2009) are conducted. Recall that for these tests, the null hypothesis is that the examined copula provides an appropriate fit to the data. Following the test procedure described in the previous section, for each of the copula families, we create $D = 1000$ bootstrap samples[7] and for each sample determine the distance between the empirical and estimated copula. The samples are then used to calculate p-values with respect to the null hypothesis. The p-value provides the level of significance at which the null hypothesis would be rejected and therefore a measure of how much evidence we have against the null hypothesis of an appropriate fit of the suggested copula. Results for the Cramér-Von Mises statistic as well as p-values for the considered copula families are presented in Table 3.

The results indicate that for the majority of the considered bivariate series the Student-$t$ copula yields the smallest distance between the estimated and

[7] This is the number of bootstrap samples that is also applied in Genest et al. (2009) providing good results for the considered goodness-of-fit tests.

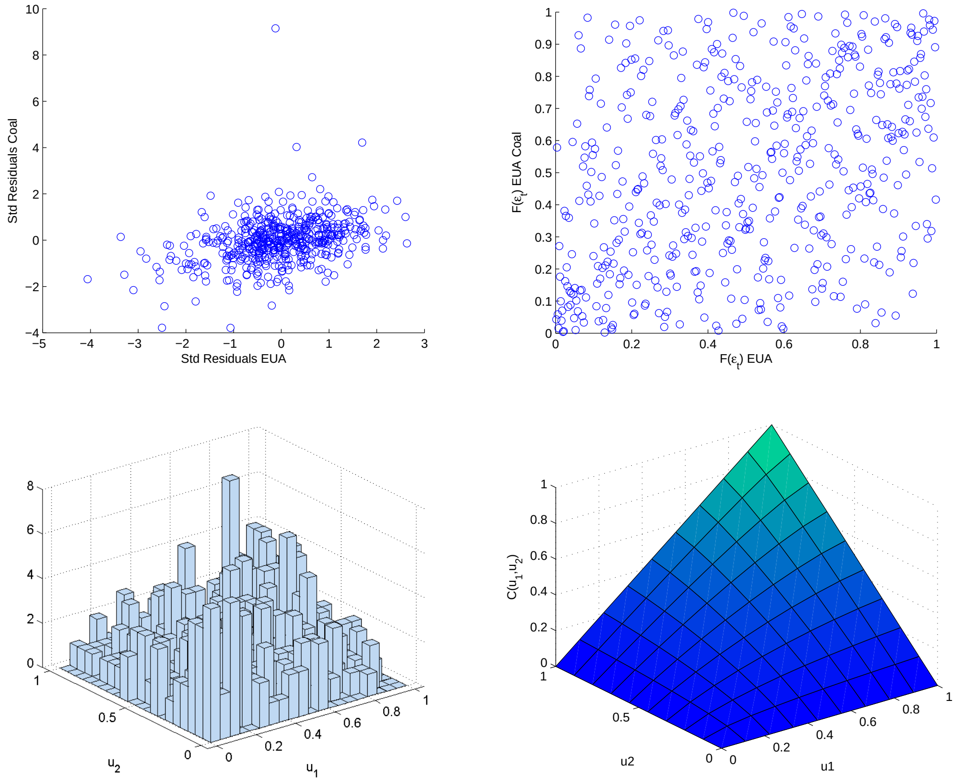


Fig. 2. Plot of standardized residuals for daily EUA 2010 futures versus coal futures (upper left panel), ranks for daily EUA 2010 Futures standardized residuals versus ranks for coal futures standardized residuals (upper right panel), 3d histogram of rank transforms for daily EUA 2010 futures versus coal futures (lower left panel) and fit of the Student-$t$ copula to the rank transforms (lower right panel).

the empirical copula. The distance is the smallest for five of the considered bivariate series, while it yields the second smallest distance for the other two pairs. Interestingly, also the Gaussian copula provides distances that are only slightly higher than those of the Student-$t$ copula and significantly smaller than those of the Clayton and Gumbel copula. Only for the relationship between EUA futures and Eurostoxx 50 spot returns, the Gumbel copula yields the smallest distance. For the relationship between EUA futures and ERIXP spot returns, the Clayton copula yields the smallest distance.

Our results are also confirmed by the conducted bootstrap goodness-of-fit tests. We find that the Student-$t$ and Gaussian copula perform best for most of the considered series. An appropriate fit of the Gaussian and Student-$t$ copula to the dependence structure cannot be rejected for any of the series at the 5% significance level. On the other hand, at the 5% significance level, the hypothesis of an appropriate fit of the Clayton and Gumbel copula is rejected for five out of seven series. At this level of significance only for the dependence

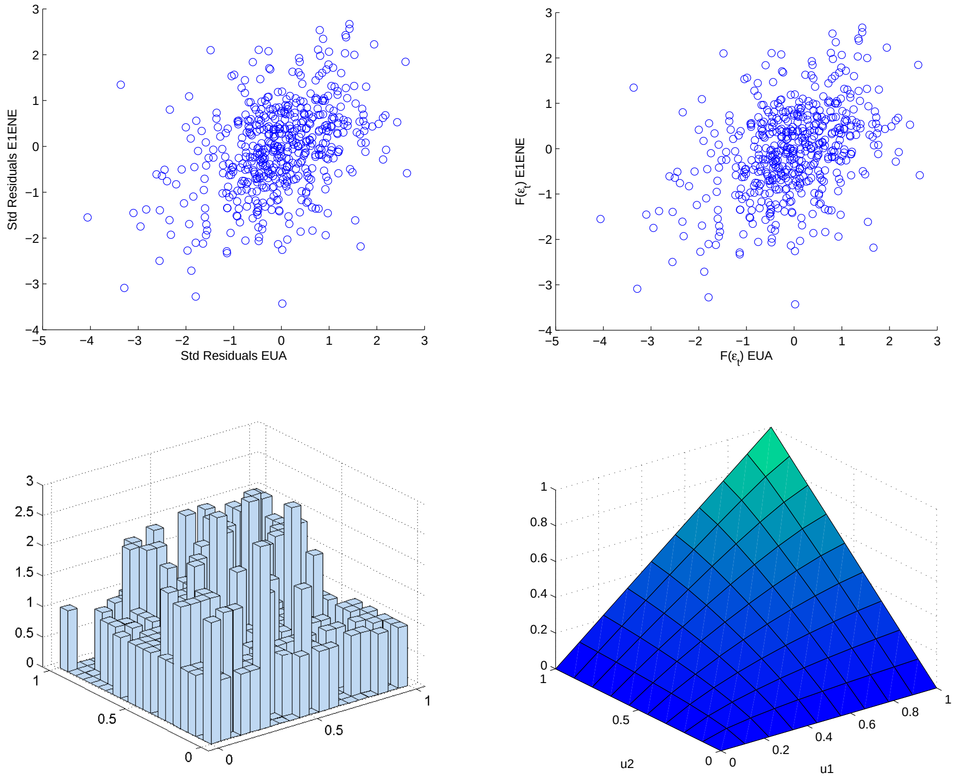


Fig. 3. Plot of standardized residuals for daily EUA 2010 futures versus E1ENE returns (upper left panel), ranks for daily EUA 2010 futures standardized residuals versus ranks of E1ENE standardized residuals (upper right panel), 3d histogram of ranks transforms for daily EUA 2010 futures versus E1ENE (lower left panel) and fit of the Student-$t$ copula to the rank transforms (lower right panel).

structure between EUA and gas futures (Clayton and Gumbel), EUA futures and ERIXP spot (Clayton) and EUA futures and Eurostoxx 50 (Gumbel) an appropriate fit of the copulas is not rejected. For the Gumbel copula an appropriate fit is even rejected at the 1% level for most of the series. Note that an appropriate fit of the Clayton copula to the dependence structure between 2010 EUA and oil futures returns cannot be rejected at the 1% level despite the large distance between the empirical and estimated copula. This confirms results by Genest et al. (2009) who state that the power of goodness-of-fit tests for copulas is often small when the dependence between the variables is low and a comparably small number of observations can be considered.

Overall, we find that the elliptical Gaussian and Student-$t$ copula provide an appropriate fit to all considered bivariate return series. Thus, given the rather symmetric dependence structure for most of the considered variables, the findings of Hirschhausen and Zachmann (2008) and Daskalakis and Markellos (2009) on an asymmetric relationship cannot be confirmed by our study. Note,

| Asset | Clayton | Gumbel | Gaussian | Student-$t$ |
|---|---|---|---|---|
| Coal futures | 0.0326 (0.036*) | 0.0557 (0.000**) | 0.0167 (0.536) | **0.0162** (0.601) |
| Oil 2010 futures | 0.0702 (0.014*) | 0.0702 (0.000**) | 0.0328 (0.064) | **0.0324** (0.062) |
| Gas futures | 0.0177 (0.471) | 0.0211 (0.158) | 0.0164 (0.585) | **0.0151** (0.654) |
| EEX futures | 0.1537 (0.000**) | 0.0521 (0.000**) | 0.0085 (0.980) | **0.0083** (0.938) |
| Eurostoxx 50 | 0.0519 (0.002**) | **0.0162** (0.478) | 0.0235 (0.242) | 0.0235 (0.205) |
| E1ENE | 0.0557 (0.003**) | 0.0471 (0.000**) | 0.0155 (0.635) | **0.0130** (0.797) |
| ERIXP | **0.0193** (0.333) | 0.0478 (0.000**) | 0.0239 (0.209) | 0.0220 (0.264) |

Table 3
Distance between estimated and empirical copula for the considered series. Consistently, either the Student-$t$ or the Clayton copula yields the lowest distance according to Cramér-Von Mises statistic. In brackets results for p-value based on bootstrap goodness-of-fit test (Genest et al, 2009). The asterix denote rejection of the copula model at the 1% ** and 5% * significance level.

however, that the conducted goodness-of-fit tests are not able to provide information on which copula provides the best fit to the data. The tests do neither reject the Gaussian nor the Student-$t$ at the 1% or 5% level for any of the series and for most of the considered return series provide p-values of a magnitude greater then 0.2. It should be pointed out that in order to decide which model is closer to the true model among a set of considered models that cannot be rejected, alternative tests as described in Chen and Fan (2006); Diks et al. (2010) would be required. As this pioneer application of copulas in the area of carbon pricing research is more interested in providing general insights rather than finding the ultimate model specification, we leave this investigation to future work. In this spirit, we proceed with the application of time-varying copulas in the next section and the investigation of the performance of the Gaussian and Student-$t$ copula in an empirical analysis on Value-at-Risk quantification for exemplary portfolios containing EUA futures contracts in Section 4.5.

### *4.4 Time-Varying Copulas*

To investigate the nature of the dependence through time, we further apply a time-varying estimation of the copula parameters for the bivariate series. Hereby, we decide to estimate the different copula parameters using a rolling window approach as it is applied e.g. in Giacomini et al. (2009); Grégoire et al. (2008). Again we consider a conditional approach such that in a first step we estimate ARMA-GARCH models for each return series and calculate the standardized residuals. Then in a second step the empirical distribution function is applied to the standardized residuals and the copula models are estimated based on the derived ranks. We decided to choose a window length of 126 trading days what corresponds to approximately six months. Thus, the first six month period considers returns from January 3, 2008 to June 30, 2008 while the last window uses data from July 6, 2009 to December 31, 2009. Note that more advanced approaches on the estimation of time-varying copulas, also with respect to the optimal choice of window length, have been suggested e.g. by Patton (2006); Rodriguez (2007); Giacomini et al. (2009) but our aim in this section is to provide a simple and rather descriptive analysis of the dependence structure through time. Figure 4 shows a plot of the estimated copula parameters for the Clayton, Gumbel and Student-$t$ copula for the relationship between EUA futures returns and coal, electricity and gas futures returns as well as Eurostoxx 50, E1ENE and ERIXP spot returns respectively. Note that for all series the estimated dependence parameter for the Gaussian copula was almost identical to the Student-$t$ copula parameter. Therefore, these parameters are not provided in the graphs.

For most of the considered series, we find that the estimated copula parameters exhibit time-variation. We generally find that the dependence between EUA futures and the considered commodity futures is increasing during the period of the financial crisis in the second half of 2008. On the other hand, the dependence between the return series seems to decrease to a lower level during 2009, in particular in the second half. This confirms general results on time-varying correlation or dependence suggesting that returns from financial markets exhibit higher dependence during periods of economic or market downturn.

The degree of time-variation, however, is considerably different for some of the relationships under investigation. The dependence structure between returns of EUAs and coal futures exhibits a particularly strong change: the copula parameters start to increase for samples beginning in the second half of 2008, e.g. the parameter of the Clayton copula parameter rises from approximately 0.4 to a value higher than 1. This indicates that joint downward movements of the two series occur considerably more often during this period of time. The parameters of the Student-$t$ and the Gumbel copula exhibit a similar behavior, but in a more retained manner. The relationship between EUA and electricity futures is generally found to be stronger for the entire time horizon. The relationship of EUA and gas futures seems to change only by the end of 2008. As for coal the estimated dependence parameters from summer 2009 onwards remained relatively constant.

Analyzing the relationship between EUA futures returns and the considered equity indices yields further interesting insights. While the dynamics of the dependence structure between EUA futures and the energy index E1ENE spot returns through time are quite similar to those of commodity markets, we

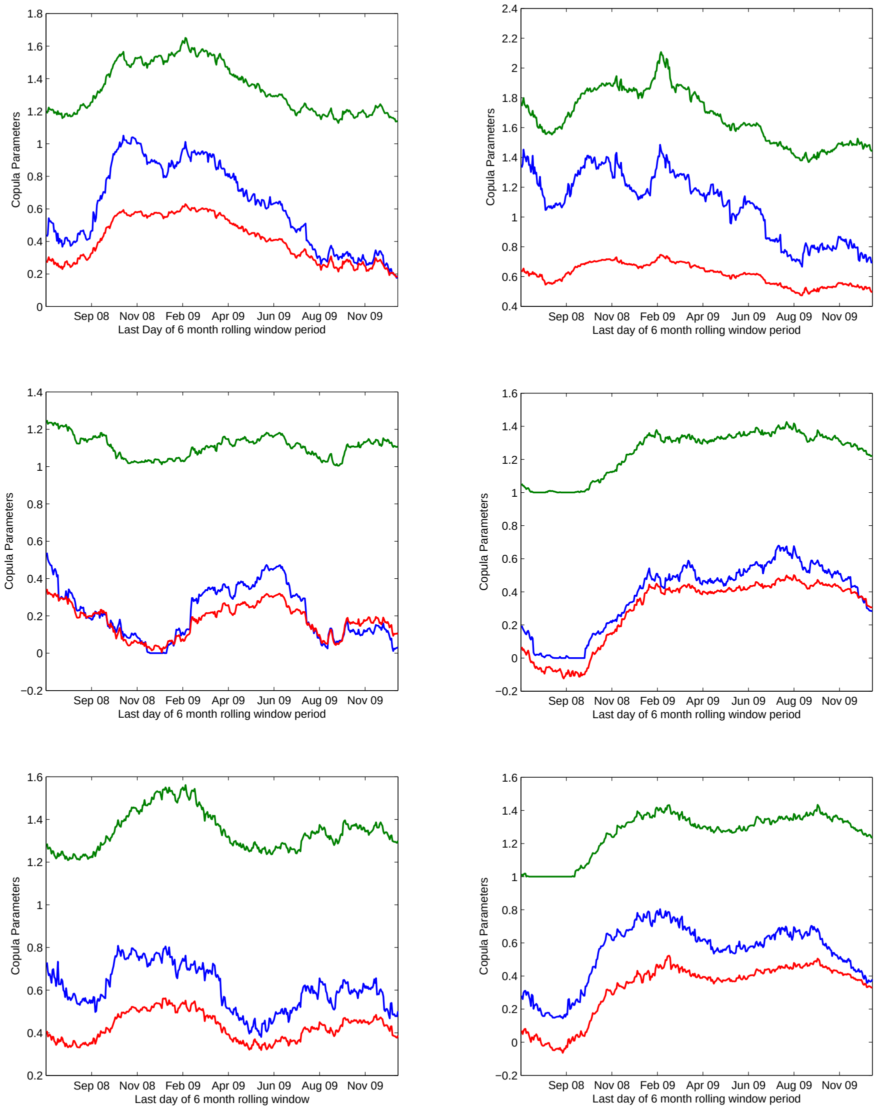


Fig. 4. Plot of estimated copula parameters for Clayton (blue), Gumbel (green) and Student-$t$ (red) copula for a six month rolling window period. The first window covers observations from January to June 2008, while the last period covers observations from July - December 2009. The graphs show the results for dependence structure between returns for daily EUA 2010 futures and coal futures (upper left panel), Electricity futures (upper right panel), Gas futures (middle left panel), Eurostoxx 50 spot contracts (middle right panel), E1ENE - DJ Europe Energy Stock Index spot contracts (lower left panel) and ERIXP - European Renewable Energy Index spot contracts (lower right panel).

find different results for the relationship between EUA futures and Eurostoxx 50 as well as ERIXP spot returns: here the dependence is very low during the first six months of 2008. The estimated parameters for the Clayton and Student-$t$ copula are close to zero while the parameter for the Gumbel copula is approximately one, indicating that the dependence is very weak during this period. However, three months later the dependence becomes stronger and the estimated parameters for all of the considered copulas start to increase. For the Eurostoxx 50, this increase continues until August 2009, while for the ERIXP the parameters rise significantly until February 2009. In absolute terms all copula parameters rise, in relative terms the increase is much higher in the Student-$t$ and the Clayton copula. This suggests that joint downward movements have been more pronounced during the financial crises. Towards the end of the investigated period we find a slightly decreasing dependence structure between EUAs and all of the considered equity indices. It is worth mentioning that conclusions as to whether there is a structural break or a significant change in the dependence structure during the considered period require further statistical tests as suggested by Patton (2006) or Giacomini et al. (2009). Note, however, that when investigating structural breaks related to the financial crises in October 2008, Chevallier (2010) finds no clear evidence of a fundamental change in EUA and related markets.

### *4.5 Risk Management Analysis*

As mentioned in Section 2, EUA prices are more likely to be influenced rather by policy measures and regulatory changes than conventional commodities. Therefore, one could assume that their price behavior This specific feature makes EUAs a potential asset for portfolio diversification. Therefore, we extend the present analysis to a risk management perspective and consider different exemplary portfolios with investments in several of the considered assets. We test the Gaussian and Student-$t$ copula models against two benchmark approaches: a standard (static) multivariate variance-covariance approach and a univariate AR-GARCH type model that is applied directly to the created return series of the constructed portfolios. The forecasting performance of the models is investigated by conducting an out-of-sample analysis comparing one-day-ahead VaR and distributional forecasts for the portfolios. We report the results for portfolios with equal weights in each of the assets, however, we would like to point out that conducting robustness checks with variation of the portfolio weights and included assets did not change the quality of the results. In the following results for four different portfolios will be reported:

- Portfolio 1 (PF1) with equal 25% weight for the following futures contracts: EUA, coal, oil and gas.
- Portfolio 2 (PF2) with equal 25% weiht for the following futures contracts: EUA, coal, gas and electricity.
- Portfolio 3 (PF3) with equal 25% weight for the following assets: EUA, electricity, Eurostoxx 50 and ERIXP.
- Portfolio 4 (PF4) with equal 25% weight for the following assets: EUA, Eurostoxx 50, E1ENE and ERIXP.

### 4.5.1 Value-at-Risk Analysis

In a first step, for each portfolio (PF1-PF4) we create the return series based on the assumed equal weights $w = 0.25$ for each asset. Then in an out-of-sample forecasting study the performance of the copula models is tested against a standard multivariate normal (variance-covariance) approach and a univariate AR(1)-GARCH(1,1) model for the portfolio return series. Note that the multivariate normal approach does not consider the conditional variance of the individual assets, so we would expect the forecasts to vary significantly less through time for this model. Therefore, one would also assume that the model will not be able to react to significant volatility changes in any of the assets and might underestimate the risk in particular during times of high volatility.

With respect to copula models, we decided to examine the forecasting performance using the Gaussian and Student-$t$ for the multivariate dependence structure between the returns of the individual assets. Note that while these copulas provided an appropriate fit to the dependence structure in the bivariate case, we cannot generally extrapolate these results to a multivariate setting. Therefore, before conducting our risk analysis, the fit of the Gaussian and Student-$t$ copula to the dependence structure between the individual assets of the portfolios was tested using the goodness-of-fit tests described in section 3.4 and 4.3. The results indicated that an appropriate fit both of the Gaussian and Student-$t$ to the multivariate data could not be rejected.[8]

Similar to section 4.4, also our risk analysis is conducted using a rolling window of $t = 126$ days length, corresponding roughly to six months of observations. For the univariate model, we derive the distributional forecast for the returns simply based on the fitted AR-GARCH model and the most recent forecast for the conditional volatility. For the benchmark variance-covariance approach, we simply assume that the return series and dependence structure can be described by a multivariate normal distribution. Under this assumption we simply need to estimate the variance-covariance matrix $\Sigma$ for the return series. Then using portfolio theory, based on the mean of the marginal return series, the given portfolio weights and the estimated variance-covariance matrix, we can calculate a distributional forecast of portfolio returns for the next day. For the copula approach, we apply the discussed SCOMDY model with an AR(1)-GARCH(1,1) process for the marginal series.[9] Therefore, for each time

[8] Detailed results for these tests are not reported here, however, they are available on request to the authors.

[9] Since the analysis was conducted in a rolling window setting, different AR-GARCH type models will provide the best fit to the data at different points in time. Since choosing the optimal model for each series at any time step based on a parsimonious model selection criteria would be tedious, we decided to stick to a simple AR(1)-GARCH(1,1) that generally provided a good fit to all of the series.

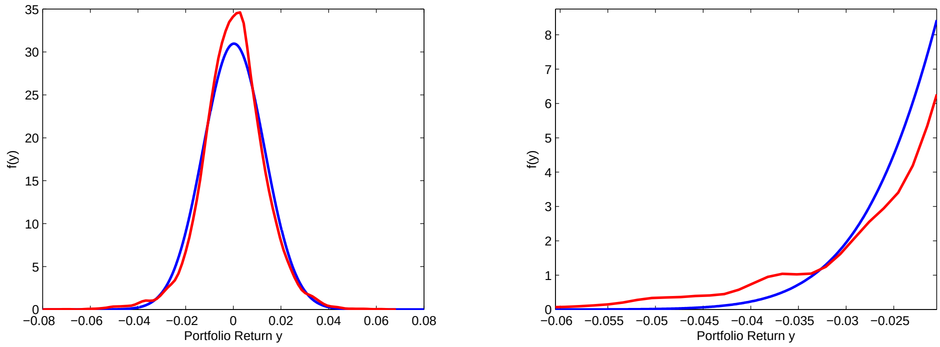


Fig. 5. Exemplary plot of return distribution forecast (left panel) and tail of return distribution forecast (right panel) for multivariate normal and Student-$t$ copula approach with ($v = 8.03$) for Portfolio 4. For both plots the blue line is the probability density for the multivariate normal approach, while the red line provides the simulated density for a model using the Student-$t$ copula to model the dependence structure between the rank transforms.

step, we initially fit an AR-GARCH model to the individual return series and calculate the standardized residuals. Then, using the transforms from the empirical distribution function for the standardized residuals, the Gaussian and Student-$t$ copula are fitted to the multivariate series. Thus, for each time step we estimate the multivariate Gaussian and Student-$t$ copula, hence the correlation matrix $\hat{C}_{Gaussian}$ and $\hat{C}_{Student}$ as well as the degrees of freedom parameter $\hat{v}$ for the Student-$t$ copula. Then we use the estimated copulas to simulate 10000 vectors of dependent uniformly distributed random variables $(u_1, u_2, u_3, u_4)$ from both copulas. In a next step, the inverse of the empirical distribution function and the conditional forecast for the volatility for the marginal series are used to calculate the simulated conditional asset returns for the series. Finally, using the portfolio weights we can then determine a simulated return distribution for the portfolio in $t + 1$.

An exemplary plot of the simulated return distribution for two of the methods and Portfolio 4 is provided in Figure 5. Here the distributional forecast for one of the time steps using the Student-$t$ copula model in comparison to a standard variance-covariance approach is plotted. Our results indicate that the standard variance-covariance approach provides a lower estimate for the risk in particular in the extreme tail of the distribution. Generally, for the model using the Student-$t$ copula, the simulated portfolio return distributions often exhibit some skewness and excess kurtosis.

We now report the results for the out-of-sample analysis comparing one-day-ahead VaR and distributional forecasts for the different portfolios. The first six months were chosen as calibration period such that forecasts for the time period July 1, 2008 to December 31, 2009 are compared. As mentioned above, the

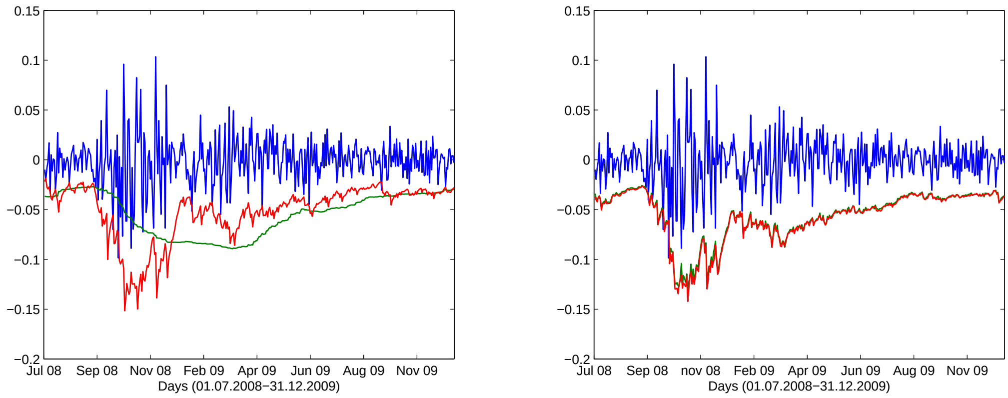


Fig. 6. Portfolio returns and 99%-VaR forecasts for Porfolio 4. The VaR forecasts are based on using a univariate GARCH model for the portfolio return series (red) and standard multivariate normal approach (green) *(left panel)* and for the conditional copula model using a Student-$t$ (red) and Gaussian (green) copula *(right panel)*.

forecasts are determined using a rolling window technique with re-estimation of the marginal distributions and dependence parameters after each time step. Thus, the length of the in-sample period is fixed with 126 trading days, while the start date and end date successively increase by one observation. Figure 6 provides a plot of the actual portfolio returns as well as the estimated 99%-VaR forecasts for Portfolio 4 using the univariate AR-GARCH model, a standard multivariate normal approach as well as the conditional copula models using a Student-$t$ and Gaussian copula. The left panel illustrates that since the multivariate normal approach does not take into account conditional volatility, there is significantly less variation in the VaR forecasts. Thus, during periods of extreme returns like in October - December 2008 the model continuously underestimates the risk. On the other hand, the second benchmark model, namely the univariate AR-GARCH model for the portfolio returns seems to provide reasonable forecasts for the 99%-VaR. The right panel shows that also the considered copula models seem to provide an appropriate quantification of the 99%-VaR with only a small number of VaR exceptions. From a firs glance, we also observe that there is only a minor difference with respect to VaR quantification between the Gaussian and Student-$t$ copula model. A more rigorous analysis based on VaR exceptions and distributional will be conducted in the following.

Given the estimated model parameters for the marginal distributions and dependence structure, we are able to calculate a model dependent confidence interval for the next observation of the portfolio return $y_{t+1}$. Following Kupiec (1995); Christoffersen (1998); Christoffersen and Diebold (2000); Hull (2007), we evaluate the quality of the VaR forecasts by comparing the nominal number of exceptions of the models to the true number of exceptions.

Since comparing the nominal and true coverage may be sensitive to the choice of the confidence level $\alpha$, we decided to investigate the coverage for three different values of $\alpha$. Thus, for each of the models we calculate the VaR for the 95%, 99% and 99.9% confidence level. If the model implied VaR forecasts are accurate, the percentage of exceedances should be approximately 5%, 1% and 0.1%, respectively. We further conduct a statistical test investigating whether a model provides an acceptable number of VaR exceptions. The test is based on the binomial distribution and simply investigates whether the number of exceedances is significantly higher than the expected number for $p = 0.05$, $p = 0.01$, and $p = 0.001$. The null hypothesis is that the model provides an adequate number of exceptions, such that rejection of the null indicates that the model significantly misspecifies VaR estimates.

With a total number of 385 days, the expected number of VaR exceptions is approximately 19.25 for the 95%, 3.85 for the 99% and 0.385 for the 99.9% confidence level. Table 4 reports the actual number and fraction of exceedances as well as the results for the significance test for the number of VaR exceptions. We find that for a vast majority of considered portfolios and confidence levels the copula models are superior to the benchmark models with respect to the difference between the actual and expected number of exceedances.

For the 95% confidence level all models provide a slightly higher number of exceedances than expected. However, in particular for the portfolios containing investments in commodities and equity (PF3 and PF4), the coverage is worse for the univariate GARCH and the multivariate normal model. For these portfolios, both copula approaches provide a better estimation of the risk quantile and yield a lower number of exceptions than the benchmark models. Also, the conducted tests for VaR exceptions indicate that for Portfolio 3 and 4 a correct specification of VaR levels is rejected at the 5% - often even at the 1% - significance level both for the multivariate normal and the univariate GARCH model. On the other hand, an appropriate specification of VaR for Portfolio 3 and 4 cannot be rejected for the Student-$t$ copula at any of the considered VaR confidence level.

For the 99% and 99.9% confidence levels the copula models seem to provide better VaR estimates. Here, the univariate GARCH and the multivariate normal approach do not yield appropriate VaR forecasts such that the observed number of exceptions for any of the considered portfolios consistently exceeds the expected number. Clearly better results are obtained for both copula models, where the nominal number of exceptions for the considered confidence levels is much closer to the theoretical number as can be seen in Table 4.

Overall, in terms of backtesting the VaR models the copula approach consistently outperforms the multivariate normal model. The univariate GARCH yields better results than the multivariate normal model, but still shows a

| | 95% VaR | | 99% VaR | | 99.9% VaR | |
|---|---|---|---|---|---|---|
| Portfolio | # Exc. | Fraction | # Exc. | Fraction | # Exc. | Fraction |
| PF1 Univariate | 31 | 8.05%** | 6 | 1.56% | 3 | 0.78%** |
| PF2 Univariate | 28 | 7.27%* | 8 | 2.08%* | 2 | 0.52%* |
| PF3 Univariate | 31 | 8.05%** | 8 | 2.08%* | 2 | 0.52%* |
| PF4 Univariate | 27 | 7.01%* | 9 | 2.34%** | 2 | 0.52%* |
| | 95% VaR | | 99% VaR | | 99.9% VaR | |
| Portfolio | # Exc. | Fraction | # Exc. | Fraction | # Exc. | Fraction |
| PF1 MVN | 25 | 6.49% | 12 | 3.12%** | 2 | 0.52%* |
| PF2 MVN | 26 | 6.75% | 9 | 2.34%** | 2 | 0.52%* |
| PF3 MVN | 28 | 7.67%* | 14 | 3.64%** | 6 | 1.56%** |
| PF4 MVN | 28 | 7.67%* | 11 | 2.86%** | 6 | 1.56%** |
| | 95% VaR | | 99% VaR | | 99.9% VaR | |
| Portfolio | # Exc. | Fraction | # Exc. | Fraction | # Exc. | Fraction |
| PF1 Gaussian | 26 | 6.75% | 6 | 1.56% | 0 | 0.00% |
| PF2 Gaussian | 24 | 6.23% | 4 | 1.04% | 1 | 0.26% |
| PF3 Gaussian | 21 | 5.45% | 5 | 1.30% | 3 | 0.78%** |
| PF4 Gaussian | 21 | 5.45% | 3 | 0.78% | 2 | 0.52%* |
| | 95% VaR | | 99% VaR | | 99.9% VaR | |
| Portfolio | # Exc. | Fraction | # Exc. | Fraction | # Exc. | Fraction |
| PF1 Student-$t$ | 26 | 6.75% | 6 | 1.56% | 0 | 0.00% |
| PF2 Student-$t$ | 24 | 6.23% | 4 | 1.04% | 0 | 0.00% |
| PF3 Student-$t$ | 21 | 5.45% | 6 | 1.56% | 1 | 0.26% |
| PF4 Student-$t$ | 21 | 5.45% | 3 | 0.78% | 1 | 0.26% |

Table 4
Number and fraction of exceedances for 95%-, 99%-, and 99.9%-VaR for the univariate GARCH model and the multivariate normal (MVN) as well as the Gaussian and Student-$t$ copula approach. The asterix denotes rejection of an appropriate VaR specification of the model for specific confidence level at 1% ** and 5% * significance (Hull, 2007).

higher number of exceptions than the copula models almost at all confidence levels. In comparison to the Gaussian copula, the Student-$t$ copula provides very similar results for the 95% and 99% confidence levels and slightly better results at the 99.9% confidence level. Furthermore, an appropriate VaR specification is rejected for almost all portfolios for the two benchmark models, while it is only rejected twice for the Gaussian copula and never for the Student-$t$ copula. So we conclude that the Student-$t$ copula model yields the best results for VaR specification.

### *4.5.2 Distributional Forecasts*

In a second step we investigate the ability of the models to provide accurate forecasts of the portfolio return distribution. Tests, being based on confidence intervals only, may be unstable in the sense that they are sensitive to the choice of the confidence level $\alpha$. Therefore, we also apply tests that investigate the complete distributional forecast, instead of a number of quantiles only. Evaluating the accuracy of the density forecasts we perform a distributional test following Crnkovic and Drachman (1996) and Diebold et al. (1998). We are interested in the distribution of the return $y_{t+1},\ t > 0$, which is forecasted at time $t$. Further, let $f(y_{t+1})$ be the probability density and

$$F(y_{t+1}) = \int_{-\infty}^{y_{t+1}} f(x)dx \tag{12}$$

be the associated distribution function of $y_{t+1}$. To conduct the test, we determine $\hat{F}(y_{t+1})$ by using the estimates for the marginal return distributions and copula or correlation parameters from the rolling window in-sample period. Based on this information we can calculate a rolling forecast of the portfolio return distribution for the next day. Rosenblatt (1952) shows that if $\hat{F}$ is the correct forecast for the distribution, the transformation of $y_t$, namely

$$u_{t+1} = \int_{-\infty}^{y_{t+1}} \hat{f}(x)dx = \hat{F}(y_{t+1}), \tag{13}$$

is i.i.d. uniformly on $[0, 1]$. Crnkovic and Drachman (1996) and Diebold et al. (1998) provide tests that can be used to investigate violations of either independence or uniformity in the forecasts.

Testing for uniformity, Crnkovic and Drachman (1996) suggest to use a test based on the distance between the empirical and the theoretical cumulative distribution function of the uniform distribution. This may be done using e.g. the Kuiper statistic $D_{Kuiper} = D^+ + D^-$ with $D^+ = sup\{F_n(u) - \hat{F}(u)\}$ and $D^- = sup\{\hat{F}(u) - F_n(u)\}$. Hereby $F_n(u)$ denotes the empirical distribution function for the probability integral transforms of the one-day ahead return forecasts and $\hat{F}(u)$ is the *cdf* of the uniform distribution. Table 5 presents the results for the conducted tests.

Again, we find that the Gaussian and Student-$t$ copula models generally provide better results than the multivariate normal model and the univariate GARCH model. Probability integral transforms of the one-day ahead return forecasts for the multivariate normal model are non-uniformly distributed. For Portfolio 1, 2 and 3, the test rejects the hypothesis of a uniform distribution even at the 1% level while for Portfolio 4 the uniformity assumption is rejected at the 5% level. In comparison to the univariate model, the Gaussian and Student-$t$ copula model perform better for Portfolio 1 and 2, while the univariate model provides the smallest distance to the uniform distribution for Portfolio 3 and 4. However, while the appropriateness of the three models is not rejected for Portfolio 3 and 4, the Student-$t$ copula model is the only one that cannot be rejected at the 1% level for Portfolio 1. For Portfolio 2, appropriate distributional forecasts are rejected for all considered models.

Furthermore, all models seem to provide better forecasts for PF3 and PF4 with a higher share in equity indices while they perform worse for PF1 and PF2 consisting of commodity futures only. Further, in terms of density forecasting, the Student-$t$ copula model clearly outperforms the multivariate normal model and seems to deliver slightly better results than the univarate GARCH and Gaussian copula approach.

Overall, our results suggest that copula models are particularly useful for risk management purposes and short-term forecasting of future return distributions for portfolios containing investments in emission allowances. These results could be important not only for risk management or hedging, but also for the purpose of portfolio optimization. Deviating from the standard variance-covariance approach could be of interest in particular when not only the mean and variance but also higher moments of the portfolio return distribution are considered or when risk-adjusted measures are used, see e.g. Jondeau and Rockinger (2006b); Jorion (2001); Keating and Shadwick (2002). Note that our results were also robust when alternative portfolio weights, combination of assets and different window sizes for the rolling estimation were considered.

| | PF1 | PF2 | PF3 | PF4 |
|---|---|---|---|---|
| Univariate GARCH | 0.1311** | 0.1367** | 0.0593 | 0.0577 |
| Multivariate Normal | 0.1183** | 0.1149** | 0.1121** | 0.0985* |
| Gaussian Copula | 0.1063** | 0.1055** | 0.0750 | 0.0710 |
| Student-$t$ Copula | 0.0966* | 0.1054** | 0.0807 | 0.707 |

Table 5
Results for Kuiper test statistics. The asterix denote rejection of the model at the 1% ** and 5% * significance level, for n=386 observations.

## 5 Conclusions

The aim of this paper is to deepen the understanding of the relationship between European carbon, commodity and financial markets. We apply different copulas in order to analyze the dependence structure between EUA futures returns and those of other financial assets and commodities during the Kyoto commitment period. Copulas offer great flexibility for modeling the relationship between different financial variables. The application of copulas also yields insights with respect to nonlinear dependence and tail dependence between the considered variables. Thus, we first investigate which copulas are most appropriate to model the dependence structure. Second, we deal with the question whether or not the dependence structure exhibits time-varying properties. The latter is motivated by examining whether the relationship between the considered variables has changed over time and whether or not the financial crisis had an influence on the dependence between EUA futures and other financial variables. The usefulness of copulas is further illustrated in a Value-at-Risk analysis. Within this part, we seize the argument that carbon emission allowances can be used for portfolio diversification purposes as regulatory decisions are said to be an important influence factor of EUA prices. We consider different portfolios combining investments in EUAs with several other assets and test the Student-$t$ as well as the Gaussian copula model against two benchmark models: a standard variance-covariance approach and a univariate AR-GARCH model that is applied directly to the portfolio returns. We conduct an out-of-sample analysis in which we compare one-day-ahead VaR and distributional forecasts for the constructed portfolios.

The following insights emerge from these efforts. First, a significant positive dependence structure is found between EUA futures and coal, gas and electricity futures returns as well as between EUA futures and equity spot returns. Only between EUA and oil futures we find the dependence to be rather insignificant. Our results at least somehow contradict earlier studies by Kosobud et al. (2005) and Daskalakis et al. (2009) suggesting no statistically significant or even negative correlations between emission allowances and other financial variables. On the other hand, we confirm results by Mansanet-Bataller et al. (2007) and Hintermann (2010) who find positive effects of several commodity prices on EUA forward prices. Regarding the nature of dependence, we find evidence of a symmetric dependence structure between EUAs and other financial assets. For the majority of the considered bivariate series, the Student-$t$ and Gaussian copula are most appropriate, significantly outperforming both the Clayton and Gumbel copula with respect to a goodness-of-fit test for the distance between the estimated and empirical copula.

Second, we obtain insightful results on time-variation of the estimated copula parameters. In particular we find a stronger dependence between EUA futures returns and most of the considered variables during the global financial crisis. This confirms general results on asset returns from financial markets exhibiting higher dependence during periods of extreme economic or market downturn.

Finally, our risk analysis illustrates that applying a standard variance-covariance approach to the multivariate series is likely to underestimate the kurtosis and in particular the tail risk of the portfolio return distribution. Also the application of an AR-GARCH model to the portfolio returns underestimates the risk in the lower extreme tail. Overall, with respect to both interval and density forecasts, the Student-$t$ copula model generally performs better than all the other considered models, including the implemented Gaussian copula model, what could be considered as indication for some tail dependence.

In a nutshell, our results recommend copulas as an appropriate tool for describing the dependence structure between returns from EUA contracts and those of other financial variables. The application of copulas might also be particularly useful for risk management purposes and short-term forecasting for investments in a portfolio containing emission allowances. Given the potential tail dependence, our findings are also relevant for investors or portfolio managers, in particular when higher moments of the portfolio return distribution or risk-adjusted measures are considered, see e.g. Jondeau and Rockinger (2006b); Jorion (2001); Keating and Shadwick (2002).

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