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ENDOGENOUS LOBBYING

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Abstract

In this paper we endogenize the number and characteristics of lobbies in a citizen-candidate model of representative democracy where citizens can lobby an elected policy-maker. We find that lobbying always matters. That is, lobbying always affects equilibrium policy outcomes. Moreover, only one policy outcome emerges in equilibrium. An "extremist" candidate is elected and implements a "centrist" policy that differs from the one most preferred by the median voter. These results are in contrast with the ones obtained in the context of a citizen-candidate model where lobbies are exogenous.

Keywords: Lobbying, citizen-candidate, representative democracy, electoral competition

JEL Classification: D72, D78

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1. Introduction

There is a long and glorious tradition in political economy that builds on the assumption that the main objective of politicians is to win an election (Downs 1957). Within this framework, known as the "downsian" paradigm, when competing for election political candidates shape their policy platforms to please the (policy-concerned) electorate so as to maximize their probability of winning. In other words, a building block of the downsian paradigm is that the preferences of political candidates differ from the preferences of the citizens, or equivalently, the (pre-specified) set of political candidates is not a subset of the citizenry.

Over the last twenty years, several authors have challenged this view by proposing alternative models of electoral competition where politicians are assumed to be not only office-motivated, but also policy-motivated (Alesina 1988, Hibbs 1977, Wittman 1977). Within this framework, known as the "partisan" paradigm, when competing for election, political candidates choose their policy platforms by trading-off their policy concerns with their desire to win the election. As in the downsian framework, however, the set of political candidates is exogenously specified.

Recently, Besley and Coate (1997) and Osborne and Slivinski (1996) have proposed an alternative approach to the study of political competition known as the "citizen-candidate" paradigm. This framework removes the artificial distinction between citizens and candidates that is prevalent in the other approaches. This is accomplished by assuming that politicians are selected by the people from those citizens who choose to become candidates in an election. Once in office, elected candidates implement their most preferred policies.

While ultimately implemented by elected representatives, policy-making is typically the outcome of a political process that also involves non elected political actors. In particular, lobbying is an important part of the policy-making process in representative democracies. This raises the question: To what extent does lobbying affect policy?

Several authors have analyzed this issue in the context of models of electoral competition where lobbies (or interest groups) are primitives of the model.[1] In particular, Besley and Coate (1999) consider a citizen-candidate model where exogenously given lobby groups compete to influence policy-makers by offering transfers conditional on policy. The main result of their analysis is that lobbying need have little or no effect on equilibrium policy outcomes.

In this paper, we extend the citizen-candidate framework to endogenize the number and characteristics of lobbies. Our main result is that lobbying always matters. That is, lobbying always affects equilibrium policy outcomes.

Like in Besley and Coate (1997), we model the political process as a multi-stage game that begins with the citizens' decisions to participate in the political process as candidates for public office. Given the set of candidates, all citizens have the right to vote in an election that selects a single representative to choose policy for one period. When casting their ballot, citizens are assumed to be strategic.[2]

We depart from this framework by assuming that after the electoral outcome is announced, citizens decide whether to participate in the policy-making process as lobbyists. Lobbies try to influence the policy choice of the elected candidate by offering him transfers in exchange for policy compromise. Contrary to most of the existing literature, we do not model lobbying as a "menu-auction" (Bernheim and Whinston 1986, Besley and Coate 1999, Dixit, Grossman, and Helpman 1997, Grossman and Helpman 1994, Grossman and Helpman 1996, Persson and Helpman 1998). Rather, we assume that given the set of lobbyists who enter the political process after an election, the elected candidate chooses the coalition of lobbyists he will bargain with over policy (in exchange for transfers). Thus, policy is the outcome of efficient bargaining between the elected policy-maker and a coalition of lobbyists selected by the policy-maker.[3]

[1]This literature originates from the work by Tullock (1967) on rent-seeking. For a partial account of the large literature on lobbying see, for example, Grossman and Helpman (1999).

[2]This assumption differentiates the citizen-candidate model of Besley and Coate (1997) from the one of Osborne and Slivinski (1996) where citizens are assumed to vote sincerely.

[3]Diermeier and Merlo (1999) use a similar framework to analyze the process of government formation in parliamentary democracies.

We find this framework appealing for at least two reasons. First, the main objective of the citizen-candidate approach is to endogenize the participation decisions of the actors involved in the political process. Thus, endogenizing the individual decision of entering politics as a lobbyist is a natural step forward within this approach. Second, casual observations suggest that while a number of lobby groups may be willing to offer favors to elected politicians in exchange for policy compromise, policy-makers have a choice as to whom to include in their bargaining coalition.

Our main results can be summarized as follows. First, as stated above, lobbying always influences policy. In other words, in any equilibrium of the endogenous lobbying game, the elected candidate never implements the policy outcome that would be implemented in the absence of lobbying (that is, the policy most preferred by some elected candidate). Policy is always the outcome of a compromise between the policy preferences of the elected candidate and those of the lobbyists who are included in the bargaining coalition. (Notice that the bargaining coalition is never empty in equilibrium).

This result is in contrast to one of the results obtained by Besley and Coate (1999) in the context of a citizen-candidate model where lobbies are exogenous. In their paper, the presence of (pre-specified) lobbies in the political process need have little or no effect on equilibrium policy outcomes. In particular, they show that it is possible to construct examples where the equilibrium set of policy outcomes of the game with lobbying is identical to the equilibrium set of the game without lobbying. The reason for the result is that voters can restrict the influence of lobbyists via strategic delegation by supporting candidates with offsetting policy preferences. In other words, in the game where lobbies are allowed to influence policy, voters can strategically elect a candidate who (after lobbying takes place) implements exactly the same policy that a different candidate would implement in the game where lobbying is ruled out.

In our model, the choice to become lobbyists is endogenous. This gives citizens an additional instrument to influence policy besides the electoral process. In particular, citizens can control the influence of lobbying directly, without the need to resort to strategic delegation. Lobbying takes the form of efficient bargaining between the elected candidate and a coalition of lobbyists of his choice. In equilibrium, not all lobbies operate for any elected candidate, and not all possible policies can be implemented. In particular, the equilibrium policy differs from the one that would be selected in the absence of lobbying.[4]

Second, endogenizing the number and characteristics of lobbies reduces the multiplicity of equilibria inherent in citizen-candidate models with strategic voting (Besley and Coate 1997). This multiplicity persists when lobbies are exogenously introduced into the analysis (Besley and Coate 1999). In our setting, only one policy outcome emerges in equilibrium. An "extremist" candidate is always elected, but the equilibrium policy is never extreme. In fact, policy is biased toward the center, even though in general it is not equal to the policy most preferred by the "median voter".

2. The Model

Each citizen $i \in \{1, ..., N\}$ has quasi-linear preferences over a one-dimensional policy outcome $x \in X = [0, 1]$ that has a public good nature and distributive benefits $y_i \in \mathbb{R}$ that have a private good nature. Citizens differ with respect to their policy preferences. For simplicity we restrict attention to the case where there are three types of citizens j, $j \in \{L, C, R\}$, indexed by their most preferred policy outcome z^j, $j = L, C, R$, and we assume that $z^L = 0$, $z^C = a \in (0, 0.5)$, and $z^R = 1$. We further assume that the number of citizens of type j, N^j, is large and such that $N^j < N/2$, $j = L, C, R$.[5]

The utility function of a citizen of type j, $j = L, C, R$, is given by

$$U_i(x, y_i, j) = u(x, j) + y_i \tag{1}$$

[4] In the context of a downsian model of political competition which embeds a menu-auction model of (exogenous) lobbies Grossman and Helpman (1996) also obtain that lobbying always influences policy. Our result may be interpreted as a way to provide a micro-foundation for reduced form "policy compromise" functions like, for example, the one used by Grossman and Helpman (1996).

[5] Like in Besley and Coate (1997) there is no incomplete information in our model. In particular, the type of each citizen is publicly observable.

where $u(x, j)$ is strictly concave, single-peaked at z^j and symmetric. For ease of exposition — in order to obtain closed-form solutions to the model — in what follows we take:[6]

$$u(x, j) = -(x - z^j)^2. \tag{2}$$

We normalize aggregate transfers to be zero (i.e., $\sum_{i \leq N} y_i = 0$). Also, we assume that any policy $x \in X$ is costless to implement.

As discussed in the Introduction our model modifies the framework of Besley and Coate (1997) by endogenizing the number and characteristics of lobbies.

We assume that the political process has four stages. In the first stage, all citizens choose whether to run for office. Given the set of candidates that have entered the electoral competition an election follows in the second stage. The election selects one candidate that is delegated the policy decision for one period. In the third stage, all non-elected citizens decide whether to become lobbyists and whether to subsidize lobbying activity. In the fourth and final stage, lobbying takes place and policy is chosen. We describe below the structure of each stage of the political process.

2.1. Entry of Candidates

Each citizen must decide simultaneously and independently whether or not to run for office. If a citizen enters the electoral competition as a candidate he has to pay a small monetary cost $\delta > 0$. The decision yields benefits to the citizens either directly from winning or indirectly by affecting the identity of the winner.

Let $s_i^j \in \{0, 1\}$ denote the decision by citizen i (of type $j \in \{L, C, R\}$) whether to become a candidate: $s_i^j = 1$ indicates citizen i's decision to enter the electoral competition.[7]

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[6]While the details of the derivation presented in the paper clearly depend on the quadratic form of the function $u(\cdot, \cdot)$, all of our results hold true for any strictly concave, single-peaked and symmetric function.

[7]In principle, we could allow candidates to randomize on their entry decision as in Besley and Coate (1997). However, as it will be clear below, in our setting allowing for mixed strategies does not expand the equilibrium set. In particular, the entry game has a whole set of pure strategy equilibria where the type and the policy choice (but not the identity) of the elected candidate are uniquely

In principle, it is possible that more than one citizen of the same type decide to run for office.[8] If this is the case, unlike in the standard citizen-candidate model, we assume that there exists a mechanism that selects only one of these citizens. In other words, at most one citizen of each type can be a candidate in a general election. For concreteness we label this selection mechanism "primaries".[9] We model primaries as an equal probability lottery over the set $\mathcal{S}^j$ of all type j citizens who decide to run for office: $\mathcal{S}^j = \{i \mid s_i^j = 1\}$. We denote $\sigma^j \in \{0, 1\}$, $j \in \{L, C, R\}$, the variable that indicates whether the set $\mathcal{S}^j$ is non-empty: $\sigma^j = 1$ if $\mathcal{S}^j \neq \emptyset$ and $\sigma^j = 0$ if $\mathcal{S}^j = \emptyset$. Thus, $\sigma = (\sigma^L, \sigma^C, \sigma^R)$ is the outcome of the entry-of-candidates subgame.

In Appendix B below, we characterize the set of equilibria of our political competition model in the absence of primaries. It is then possible to construct pathological equilibria that are based on the fact that the polity of a given type may split its votes among identical candidates in any possible way provided that each voter is not pivotal: that is, he cannot affect the electoral outcome by unilaterally switching his vote from one candidate to an identical one. Clearly the role of the primaries is to prevent these equilibria from arising.

We view this selection mechanism as a reasonable one on the ground that these equilibria are only based on the assumption that there exist multiple identical candidates of the same type. Clearly this assumption is not realistic. In particular, any uncertainty on the defining features of all the candidates of a given type will break the voters' indifference. Consider a model in which this uncertainty takes the form of a small amount of noise that affects the way in which the polity perceives each candidate. The mechanism through which a candidate of a given type is selected by the polity when the amount of noise converges to zero behaves like the primaries described above.

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determined.

[8] In fact, since, as in Besley and Coate (1999), lobbying creates rents to holding office, competition among a number of candidates with the same policy preferences will be the norm.

[9] Notice that in our model there are no political parties. The term primaries is only used here to describe a mechanism that selects one of possibly many identical candidates. For a citizen-candidate model with political parties see Riviere (1999).

For any given σ, let $\mathcal{C}(\sigma)$ represent the set of candidates with typical element e^j, $j = \{L, C, R\}$.

Finally, we assume that if no citizen runs for office a default policy $x_0 \in [0, 1]$ is implemented.

2.2. Voting

Elections are structured so that all citizens have one vote that, if used, must be cast for one of the candidates.

In particular, given a set of candidates $\mathcal{C}(\sigma)$, each citizen simultaneously and independently decides to vote for any candidate in $\mathcal{C}(\sigma)$ or abstains. Let v_i denote citizen i's choice: if $v_i = j$ then citizen i casts a vote for candidate $e^j \in \mathcal{C}(\sigma)$; while if $v_i = 0$ he abstains. A vector of voting decisions is denoted by $v = (v_1, \ldots, v_N)$.

The candidate who receives the most votes is elected, and in the event of ties, the winning candidate is chosen with equal probability from among the tying candidates. We denote $P^E(v) \in \mathcal{C}(\sigma)$, $E \in \{L, C, R\}$, the elected candidate.

We assume that citizens correctly anticipate the outcome of the lobbying stage that follows an election and vote strategically: each citizen i chooses his vote v_i so as to maximize his expected utility given the voting decision of every other citizen v_{-i}.

2.3. Entry of Lobbyists

For each type j, $j = L, C, R$, let l^j denote a potential lobby of type j.[10] After the election, all non-elected citizens of type j play a "subscription game" (Bagnoli and Lipman 1989) in which they pledge funds to lobby l^j. After all subscriptions are made, all non-elected citizens decide whether to become lobbyists. That is, each non-elected citizen must decide whether to become a member of the lobby of his type.[11]

[10] Without loss of generality we treat all potential lobbies of the same type as one lobby.
[11] The assumptions that citizens can only pledge funds to and enter the lobby of their type are made here to clarify lobbying activity.

Given the elected candidate P^E, the subscription game is structured as follows. Each citizen of type j simultaneously and independently decides whether to pledge a fixed amount $t^j_{P^E}$ to lobby l^j, as defined in (6) below. Denote $T^j_{P^E}$ the total amount of subscriptions made to lobby l^j. These subscriptions are contingent on at least one citizen entering lobby l^j. If no citizen of type j becomes a lobbyist, the funds pledged to lobby l^j are returned to the pledging citizens.[12]

Given $T^j_{P^E}$, $j = L, C, R$, each non-elected citizen simultaneously and independently decides whether to become a lobbyist. If a citizen chooses to become a lobbyist he has to pay a small monetary cost $\gamma > 0$. Each lobbyist of type j appropriates an equal share of the subscriptions $T^j_{P^E}$ made to lobby l^j.

Denote $\lambda_i \in \{0, 1\}$ the decision of citizen i (of type $j \in \{L, C, R\}$) whether to enter lobby l^j, and $\lambda = (\lambda_1, \ldots, \lambda_N)$ the vector of entry decisions by all citizens.[13] We take $\mathcal{L}_E(\lambda)$ to be the set of lobbies that are active when the elected candidate is P^E and the vector of entry decisions is λ. These are the lobbies that have at least one member.[14]

2.4. Lobbying

Each active lobby is assumed to be able to sign binding contracts on policy choices with the elected candidate P^E in exchange for transfers. These transfers are financed in equal share by the members of the lobby. Notice that the elected candidate P^E has the option of not signing any contract and implement his most preferred policy z^E.[15]

[12]Unlike Bagnoli and Lipman (1989) we exogenously fix the magnitude of the pledges in the subscription game. This simplifies the computation of each citizen's payoff.

[13]To simplify notation we omit the dependence of the vector of entry decisions on the subscriptions $(T^L_{P^E}, T^C_{P^E}, T^R_{P^E})$.

[14]For simplicity, we restrict attention to pure strategy equilibria of the entry-of-lobbyists subgame. As discussed below, while allowing citizens to randomize on their entry decision expands the equilibrium set, the qualitative features of our results remain valid.

[15]If the elected candidate chooses this option, then the model coincides with the original model of Besley and Coate (1997) where lobbying is not allowed.

Let Δ_E denote the power set of $\mathcal{L}_E(\lambda)$. We interpret this set as the collection of all possible coalitions of lobbies with whom the elected candidate P^E may choose to bargain over policy and transfers. For example, if a citizen P^L of type L is elected and $\mathcal{L}_L(\lambda) = \{l^C, l^R\}$ then the set of possible bargaining coalitions is $\Delta_L = \{\{\emptyset\}, \{l^C\}, \{l^R\}, \{l^C, l^R\}\}$.

We model lobbying as a two stage bargaining game. In the first stage, each possible coalition $\ell \in \Delta_E$ is associated with a willingness to pay, $W_\ell(x, z^E)$, for any policy $x \in X$ the elected candidate P^E may choose to implement instead of his most preferred policy z^E:

$$W_\ell(x, z^E) = \sum_{l^j \in \ell} w_j(x, z^E), \tag{3}$$

where $w_j(x, z^E)$ is the willingness to pay of lobby l^j measured in units of the private good and $W_\emptyset(x, z^E) \equiv 0$.

To determine the willingness to pay of a lobby, we take a noncooperative approach and allow the members of the lobby to free-ride on each other's willingness to pay for policy x. Given the public good nature of x, this approach implies that the amount a group of identical citizens (the members of each lobby) is willing to pay for any given policy is bounded above by the willingness to pay of a representative citizen of that group.[16] Based on these considerations, we take the willingness to pay of lobby l^j for any policy $x \in X$ to be the willingness to pay of a representative individual of type j.

In the second stage of the bargaining game, the elected candidate P^E first chooses an optimal policy $x_{P^E}(\ell)$ for any potential coalition $\ell \in \Delta_E$:

$$x_{P^E}(\ell) \in \arg\max_{x \in X} u(x, E) + W_\ell(x, z^E) \tag{4}$$

[16] This outcome would obtain if we were to explicitly model the way a lobby aggregates the preferences of its members through a voluntary contribution game. See, for example, Laffont (1988).

and then chooses a bargaining coalition ℓ_{P^E}:

$$\ell_{P^E} \in \arg\max_{\ell \in \Delta_E} u(x_{P^E}(\ell), E) + W_\ell(x_{P^E}(\ell), z^E) \tag{5}$$

Hence, an outcome of the bargaining game between the elected candidate P^E and a selected coalition ℓ_{P^E} is a policy choice $x_{P^E}(\ell_{P^E})$ and transfers $W_{\ell_{P^E}}(x_{P^E}(\ell_{P^E}), z^E)$.

Implicit in the statement of problems (4) and (5) is the assumption that the elected candidate appropriates the entire willingness to pay of the selected bargaining coalition. This is equivalent to assuming that at the lobbying stage the elected candidate has all the bargaining power.[17]

To complete the description of the lobbying subgame, we need to specify the size of the amounts $t^j_{P^E}$ citizens can pledge to lobby l^j. We take these amounts to be:

$$t^j_{P^E} = \frac{w_j(x_{P^E}, z^E)}{N^j}. \tag{6}$$

This assumption implies that, if all citizens of a given type j subscribe to lobby l^j and lobby l^j pays transfers to the elected candidate, then the only additional cost a lobbyist in l^j pays (over and above what every other citizen of his type pays) is the entry cost γ. As it will become clear in Section 3 below, this argument guarantees that every lobby that, if active, would be included in the bargaining coalition by the elected candidate, is indeed active in equilibrium. Using Bagnoli and Lipman (1989), the critical size of $t^j_{P^E}$ that would guarantee this result and is such that in the unique equilibrium of the subscription game all citizens choose to pledge, is lower than the one in (6) above. However, unlike (6), these critical values of $t^j_{P^E}$ for all types and all elected candidates depend in a rather cumbersome way on the set of lobbies that are active in each case. Hence, to simplify our analysis, we take $t^j_{P^E}$ to be equal to (6) and

[17]This assumption is not crucial for our results. The equilibrium characterization of the entry-of-lobbyists subgame as well as of the lobbying subgame remain the same if the gains from trade are shared between the elected candidate and the members of the coalition in any fixed proportion. As clarified below, in this case we would need to modify the amounts citizens can pledge to a lobby in the subscription game, that are defined in (6) below.

impose the additional assumption that in the event that not all type-j citizens choose to pledge t^j_{PE} in the subscription game, subscriptions are returned to the pledging citizens and $T^j_{PE} = 0$.

3. Results

We proceed backward to solve for the *subgame perfect equilibria* of the four-stage political game described in Section 2 above. We start from the last stage of the game: lobbying.

3.1. Equilibria of the Lobbying Subgame

Let P^E be the candidate elected in the voting subgame and $\mathcal{L}_E(\lambda)$ the set of active lobbies determined in the entry-of-lobbyists subgame.

We first compute the willingness to pay of each lobby for any policy choice x by the elected candidate. For any elected candidate P^E and any $\mathcal{L}_E(\lambda)$, the willingness to pay of an active lobby $l^j \in \mathcal{L}_E(\lambda)$ for any policy $x \in X$ the elected candidate P^E may choose to implement instead of his most preferred policy z^E is:

$$w_j(x, z^E) = u(x, j) - u(z^E, j) \tag{7}$$

This is the difference in utility with respect to the *status quo* that a citizen of type j obtains if the elected candidate P^E's policy choice is x. The *status quo* is here defined to be P^E's policy choice in the absence of any lobbying, z^E.

A direct implication of (7) is that for any policy $x \in [0, 1]$, the willingness to pay of a lobby of the same type E of the elected candidate is such that:

$$w_E(x, z^E) \leq 0. \tag{8}$$

From definition (3) and equation (7) we can obtain the total willingness to pay of coalition $\ell \in \Delta_E$ for a given policy choice x by the elected candidate P^E :

$$W_\ell(x, z^E) = \sum_{j \in \ell} \left[u(x, j) - u(z^E, j) \right]. \tag{9}$$

We can now provide a characterization of the elected candidate P^E's coalition choice. First notice that the elected candidate will never choose any coalition that contains a lobby of his own type. Indeed, from (8), such a lobby has a negative willingness to pay for any policy choice $x \neq z^E$. Therefore, the elected candidate is strictly better off by excluding from policy negotiations a lobby of his own type.

Lemma 1. *For any elected candidate P^E, if $l^E \in \mathcal{L}_E(\lambda)$, then $l^E \notin \ell_{P^E}$.*

Proof: The result follows from (7) and (8). ■

Without loss of generality, we therefore assume $l^E \notin \mathcal{L}_E(\lambda)$.

Second, notice that for any coalition $\ell \in \Delta_E$ the equilibrium policy choice that the lobbying process generates is uniquely determined.[18]

Lemma 2. *For any elected candidate P^E, any $\mathcal{L}_E(\lambda)$, and any coalition $\ell \in \Delta_E$, there exists a unique optimal policy choice $x_{P^E}(\ell)$ that solves problem (4):*

$$x_{P^E}(\ell) = \frac{1}{|\ell| + 1} \left(z^E + \sum_{i \in \ell} z^i \right). \tag{10}$$

Proof: From (2) the objective function in (4) is strictly concave. The first order conditions of problem (4) are:

$$(x - z^E) + \sum_{j \in \ell} (x - z^j) = 0. \tag{11}$$

[18]This result is similar to the one obtained by Diermeier and Merlo (1999) in the context of government coalition bargaining.

Then the unique solution of equation (11) is (10). ■

For any possible coalition ℓ, the outcome of the bargaining is a compromise between the policy most preferred by the elected candidate and the policy preferences of the lobbyists in the bargaining coalition. Given the quadratic specification of preferences we adopt, this policy compromise takes the form of a simple average of the most preferred policies of the parties involved in the negotiation.

We can now complete our characterization of the lobbying stage of the model by analyzing the elected candidate P^E's choice of the lobbying coalition ℓ_{P^E}. Let

$$\bar{a} = \frac{\sqrt{6}-2}{2}. \tag{12}$$

Lemma 3. *For any elected candidate P^E and any $\mathcal{L}_E(\lambda)$, the solution to problem (5) is:*

Case I: *If $\mathcal{L}_E(\lambda) = \{\emptyset\}$, then $\ell_{P^E} = \{\emptyset\}$;*

Case II: *If $\mathcal{L}_E(\lambda) = \{\emptyset, l^j\}$, $j \neq E$, then $\ell_{P^E} = \{l^j\}$;*

Case III: *If $\mathcal{L}_E(\lambda) = \{\emptyset, l^i, l^j\}$, $i, j \neq E$, then:*

(i) If $E = L$ then depending on the value of the parameter a we distinguish two cases:

(i′) If $a \leq \bar{a}$ then $\ell_{P^L} = \{l^R\}$.

(i″) If instead $a \geq \bar{a}$ then $\ell_{P^L} = \{l^C, l^R\}$.

(ii) If $E = C$ then $\ell_{P^C} = \{l^R\}$.

(iii) If $E = R$ then $\ell_{P^R} = \{l^L, l^C\}$.

The proof of Lemma 3 is presented in Appendix A. It follows from this lemma that if only one lobby is active then this lobby is always included in the bargaining coalition by every elected candidate. If two lobbies are active, then the elected candidate chooses the bargaining coalition ℓ that maximizes the total surplus from implementing policy $x_{P^E}(\ell)$.

We have now all the elements to present our first result. This result summarizes the outcome of the lobbying subgame for any possible elected candidate P^E and any possible set of active lobbies $\mathcal{L}_E(\lambda)$.

Proposition 1. *For any elected candidate P^E and any $\mathcal{L}_E(\lambda)$, the optimal policy and coalition choices x_{P^E} and ℓ_{P^E} are:*

If $\mathcal{L}_E(\lambda) = \{\emptyset\}$, then: $\ell_{P^E} = \{\emptyset\}$ and $x_{P^E} = z^E$ for $E \in \{L, C, R\}$.

If $\mathcal{L}_E(\lambda) = \{\emptyset, l^j\}$, $j \neq E$, then: $\ell_{P^E} = \{l^j\}$ and $x_{P^E} = \frac{1}{2}\left(z^E + z^j\right)$ for $E \in \{L, C, R\}$.

If $\mathcal{L}_E(\lambda) = \{\emptyset, l^i, l^j\}$, $i, j \neq E$, then:

- *If $E = L$ and $\mathcal{L}_L(\lambda) = \{\emptyset, l^C, l^R\}$ we distinguish two sub-cases:*
  - *If $a \leq \bar{a}$ then: $\ell_{P^L} = \{l^R\}$ and $x_{P^L} = \frac{1}{2}$.*
  - *If instead $a \geq \bar{a}$ then: $\ell_{P^L} = \{l^C, l^R\}$ and $x_{P^L} = \frac{1}{3}(1 + a)$.*
- *If $E = C$ and $\mathcal{L}_C(\lambda) = \{\emptyset, l^L, l^R\}$ then: $\ell_{P^C} = \{l^R\}$ and $x_{P^C} = \frac{1}{2}(1 + a)$.*
- *If $E = R$ and $\mathcal{L}_R(\lambda) = \{\emptyset, l^L, l^C\}$ then: $\ell_{P^R} = \{l^L, l^C\}$ and $x_{P^R} = \frac{1}{3}(1 + a)$.*

Proof: The proof follows directly from Lemma 2 and Lemma 3. ■

The characterization of the equilibrium of the lobbying subgame (Proposition 1) implies that the elected candidate P^E receives transfers $W_{\ell_{P^E}}(x_{P^E}, z^E)$ from coalition ℓ_{P^E}.

We can now move to the analysis of the entry-of-lobbyists stage of the model.

3.2. Equilibria of the Entry-of-Lobbyists Subgame

The key feature of this subgame is the fact that a lobbyist provides a public service to the citizens of his type. A citizen who decides to become a lobbyist pays the entry cost γ and may influence the elected candidate's policy choice in a way that is beneficial to all the citizens of his type by paying the elected candidate transfers. Therefore, when deciding whether to become a lobbyist, a citizen may have an incentive to free-ride on the activity of other lobbyists of his own type.

The subscription game described in Section 2.3 above allows citizens to subsidize the activities of the lobby of their type. Given our assumptions, the total amount of subscriptions made to lobby l^j is either equal to the transfers lobby l^j is willing to pay to the elected candidate P^E, $T^j_{PE} = u(x_{PE}, j) - u(z^E, j)$, or equal to zero, $T^j_{PE} = 0$.

The following proposition characterizes the pure strategy equilibria of the entry-of-lobbyists subgame.[19]

Proposition 2. *For any elected candidate P^E the equilibrium vectors of subscriptions $(T^L_{PE}, T^C_{PE}, T^R_{PE})$ and of entry decisions λ induce the following set of active lobbies $\mathcal{L}_E(\lambda)$:*

If $E = L$ there are two cases to consider:

- *If $a \leq \bar{a}$ then: $\mathcal{L}_L(\lambda) = \{l^R\}$.*
- *If instead $a \geq \bar{a}$ then: $\mathcal{L}_L(\lambda) = \{l^C, l^R\}$.*

If $E = C$ then: $\mathcal{L}_C(\lambda) = \{l^R\}$.

If $E = R$ then: $\mathcal{L}_R(\lambda) = \{l^L, l^C\}$.

Furthermore, each active lobby contains exactly one lobbyist.

[19] Of course, if no candidate is elected, the default policy x_0 is implemented and no lobby is active in equilibrium.

The proof of Proposition 2 is presented in Appendix A. Several observations are in order. First, the only lobbies that are active in equilibrium are the ones that influence policy. In other words, lobbies that would not be included in the bargaining coalition by the elected candidate do not operate. Second, the pure strategy equilibrium of the entry-of-lobbyists subgame characterized in Proposition 2 is unique up to the identity of the lobbyist in each active lobby. Third, this equilibrium solves the free-rider problem associated with citizens' participation in lobbying activities.[20]

It follows from our results that lobbies always influence the policy choice of any elected candidate.

Corollary 1. *Any elected candidate will never choose to implement his most preferred policy.*

Proof: The result follows from Propositions 1 and 2. ■

Another implication of our findings is that the policy most preferred by the median voter, that is policy a, would never be implemented.

Corollary 2. *Any elected candidate will never choose to implement the policy most preferred by the median voter.*

Proof: The result follows from Propositions 1 and 2. ■

To complete the characterization of the equilibrium of the entry-of-lobbyists subgame, the following corollary determines the total amounts of subscriptions lobbies receive in equilibrium.

Corollary 3. *For any elected candidate P^E, in the equilibrium of the entry-of-lobbyists subgame, all lobbies $l^j \notin \ell_{P^E}$, $j \in \{L, C, R\}$, receive subscriptions $T^j_{P^E} = 0$.*

[20] The entry-of-lobbyists subgame has also a mixed strategy equilibrium where there exists a positive probability that no citizen of any type will ever become a lobbyist. This is clearly a byproduct of the free-rider problem discussed above. While this free-rider problem may be of interest in itself, we find the properties of the pure strategy equilibrium more appealing.

If $E = L$ and $\bar{a} \leq a \leq 0.25$, lobby $l^C \in \ell_{PL}$ receives subscriptions $T^C_{PL} = 0$, while lobby $l^R \in \ell_{PL}$ receives $T^R_{PL} = u(x_{PL}, R) - u(z^L, R)$.

In all other cases every lobby $l^j \in \ell_{PE}$, $j \in \{L, C, R\}$, receives subscriptions $T^j_{PE} = u(x_{PE}, j) - u(z^E, j)$.

The proof of Corollary 3 is presented in Appendix A. It follows from this result that only lobbies that are included in the bargaining coalition by the elected candidate may receive positive subscriptions. In general, citizens need to pledge positive amounts to the lobby of their type to induce the entry of at least a lobbyist. There exists however a case where no subscriptions are made to an active lobby that affects the policy outcome. This happens when the costs to a citizen of moving the equilibrium policy closer to his ideal point by becoming a lobbyist are small compared to the costs of accepting the policy that would otherwise emerge if he chooses not to become a lobbyist.

3.3. Equilibria of the Voting Subgame

We have now all the elements to characterize the equilibria of the voting subgame. Recall from Propositions 1 and 2 that if elected, a candidate of type $j \in \{L, C, R\}$ implements the following policy:

$$\begin{aligned} x_{PL} &= \begin{cases} \frac{1}{2} & \text{if } a \leq \bar{a}, \\ \frac{1}{3}(1+a) & \text{if } a \geq \bar{a} \end{cases} \\ x_{PC} &= \frac{1}{2}(1+a), \\ x_{PR} &= \frac{1}{3}(1+a). \end{aligned} \tag{13}$$

We can then derive each citizen's payoff if a candidate of type $j \in \{L, C, R\}$ were to be elected.[21] First, consider a citizen of type L. His payoffs for all possible electoral

[21]These are the payoffs of a type j citizen who chooses not to be a lobbyist. The payoffs of a type j lobbyist are reduced by the amount of the entry cost γ.

outcomes are:

$$
\begin{aligned}
u(x_{P^L}, L) &= \begin{cases} -\frac{1}{4} & \text{if } a \leq \bar{a}, \\ -\frac{1}{9}(1+a)^2 & \text{if } a \geq \bar{a} \end{cases} \\
u(x_{P^C}, L) - t^L_{P^C} &= -\frac{1}{4}(1+a)^2 - \frac{a^2 - \frac{1}{4}(1+a)^2}{N^L}, \\
u(x_{P^R}, L) - t^L_{P^R} &= -\frac{1}{9}(1+a)^2 - \frac{1 - \frac{1}{9}(1+a)^2}{N^L}.
\end{aligned} \tag{14}
$$

Next, consider a citizen of type C. His payoffs for all possible electoral outcomes are:

$$
\begin{aligned}
u(x_{P^L}, C) - t^C_{P^L} &= \begin{cases} -\frac{1}{4}(1-2a)^2 - \frac{a^2 - \frac{1}{4}(1-2a)^2}{N^C} & \text{if } a \leq \bar{a}, \\ -\frac{1}{9}(1-2a)^2 & \text{if } \bar{a} \leq a \leq \frac{1}{4}, \\ -\frac{1}{9}(1-2a)^2 - \frac{a^2 - \frac{1}{9}(1-2a)^2}{N^C} & \text{if } a \geq \frac{1}{4} \end{cases} \\
u(x_{P^C}, C) &= -\frac{1}{4}(1-2a)^2, \\
u(x_{P^R}, C) - t^C_{P^R} &= -\frac{1}{9}(1-2a)^2 - \frac{(1-a)^2 - \frac{1}{9}(1-2a)^2}{N^C}.
\end{aligned} \tag{15}
$$

Finally, consider a citizen of type R. His payoffs for all possible electoral outcomes are:

$$
\begin{aligned}
u(x_{P^L}, R) - t^R_{P^L} &= \begin{cases} -\frac{1}{4} - \frac{1 - \frac{1}{4}}{N^R} & \text{if } a \leq \bar{a}, \\ -\frac{1}{9}(2-a)^2 - \frac{1 - \frac{1}{9}(2-a)^2}{N^R} & \text{if } a \geq \bar{a} \end{cases} \\
u(x_{P^C}, R) - t^R_{P^C} &= -\frac{1}{4}(1-a)^2 - \frac{(1-a)^2 - \frac{1}{4}(1-a)^2}{N^R}, \\
u(x_{P^R}, R) &= -\frac{1}{9}(2-a)^2.
\end{aligned} \tag{16}
$$

Using equations (14), (15), and (16), we can now rank all potential electoral outcomes according to the preferences of each citizen of type $j \in \{L, C, R\}$.[22]

First consider the case where $a \leq \bar{a}$. In this case, we have that

$$\begin{array}{ccccc} P^R & \succ_L & P^L & \succ_L & P^C \\ P^R & \succ_C & P^L & \succ_C & P^C \\ P^C & \succ_R & P^L & \succ_R & P^R \end{array} \tag{17}$$

where $\succ_j$ denotes the strict preference relation of a citizen of type j over potential winners of the electoral competition.[23]

Notice that when citizens of type C are relatively close to citizens of type L with respect to their policy preferences ($a \leq \bar{a}$), no citizen ever ranks a potential outcome where a candidate of his own type is elected at the top of his preference ordering. The intuition for this result follows from the fact that any elected candidate will always include in his bargaining coalition the lobby that is as far as possible from his most preferred policy. As illustrated in Proposition 1, this occurs because such a lobby has the highest willingness to pay for policy compromise. Therefore, the equilibrium policy choice that is closest to the most preferred policy of a type j citizen is never implemented by a type j candidate.

Consider now the case where $a \geq \bar{a}$. In this case, the preference orderings of all citizens over potential winners of the electoral competition can be described as follows.

$$\begin{array}{ccccc} P^L & \succ_L & P^R & \succ_L & P^C \\ P^L & \succ_C & P^R & \succ_C & P^C \\ P^C & \succ_R & P^R & \succ_R & P^L \end{array} \tag{18}$$

Notice that when citizens of type C are sufficiently different from citizens of type L with respect to their policy preferences ($a \geq \bar{a}$), type L citizens rank the electoral

[22]These preference orderings are derived under the maintained assumption that N^j, $j \in \{L, C, R\}$, is large.

[23]The same preference orderings hold for citizens who will decide to become lobbyists. This is so because, by assumption, the entry cost γ is small relative to the differences in utilities induced by the policy choices of potential winning candidates.

outcome where a type L candidate is the winner at the top of their preference ordering. The reason why the intuition provided above does not hold in this case is that two potential winners (namely P^L and P^R) would now implement the same equilibrium policy, if elected. This is the mean of the most preferred policies of all citizens and the most preferred equilibrium policy by type L citizens. Thus, the only difference between the two potential outcomes is the amount of transfers necessary to induce each winner to adopt such a policy. For a type L citizen the potential winner P^L is cheaper.[24]

A striking implication of the preference orderings in (17) and (18) is that there always exists an electoral outcome that is preferred by a majority of citizens to any other outcome. In particular, given three potential outcomes P^L, P^C, and P^R, there always exists a Condorcet winner. If $a \leq \bar{a}$, then the Condorcet winner is P^R. If instead $a \geq \bar{a}$, then the Condorcet winner is P^L.

We are now in a position to characterize the equilibrium of the voting subgame.

Proposition 3. *For any vector of candidate entry decisions σ that induces a set of candidates $\mathcal{C}(\sigma)$ there exists a unique electoral outcome.*

Suppose $a \leq \bar{a}$.

If $\mathcal{C}(\sigma) = \{e^j\}$ then $P^E = e^j$.

If $e^R \in \mathcal{C}(\sigma)$ then $P^E = e^R$.

If $\mathcal{C}(\sigma) = \{e^L, e^C\}$ then $P^E = e^L$.

Alternatively, suppose $a \geq \bar{a}$.

If $\mathcal{C}(\sigma) = \{e^j\}$ then $P^E = e^j$.

[24] Citizens who will decide to become lobbyists may have different preference relations between the two potential winners P^L and P^R who would implement the same policy because of the entry cost γ. However, this problem affects at most six citizens. Since N is large, we ignore this issue when characterizing the equilibrium of the voting subgame.

If $e^L \in \mathcal{C}(\sigma)$ *then* $P^E = e^L$.

If $\mathcal{C}(\sigma) = \{e^C, e^R\}$ *then* $P^E = e^R$.

Proof: The result is a direct implication of the preference orderings in (17) and (18). ■

We can now complete our characterization of the subgame perfect equilibrium of our model of political competition.

3.4. Equilibria of the Entry-of-Candidates Subgame

We start by proving a property of the citizens' decision whether to run for office.

Lemma 4. *All equilibria of the entry-of-candidates subgame are such that:*

If $a \leq \bar{a}$ *then* $e^R \in \mathcal{C}(\sigma)$.

If instead $a \geq \bar{a}$ *then* $e^L \in \mathcal{C}(\sigma)$.

Proof: Consider the case where $a \leq \bar{a}$. Assume by way of contradiction that $e^R \notin \mathcal{C}(\sigma)$. It follows from the preference orderings in (17) that by entering, a citizen of type R would be elected for sure. In this case, his payoff would be equal to

$$u(x_{PR}(\ell_{PR}), R) + W_{\ell_{PR}}(x_{PR}(\ell_{PR}), z^R) - \delta = \frac{(2-a)^2}{3} - \delta \tag{19}$$

which, for a small cost of entry δ, is positive. Notice that by not entering, the citizen's payoff would be negative regardless of the electoral outcome (see equations (16)). Thus, a citizen of type R would enter. This is a contradiction of the assumption $e^R \notin \mathcal{C}(\sigma)$.

The proof in the case $a \leq \bar{a}$ is analogous and therefore omitted. ■

Lemma 4 implies that for any location a of the policy most preferred by the median citizen-type C an extremist candidate — that is, either a type-L or a type-R citizen — always enters the electoral competition.

We have now all the elements to state and prove the main result of our paper.

Proposition 4. *All equilibria of the political competition game are such that:*

If $a \leq \bar{a}$ then in equilibrium $\mathcal{C}(\sigma) = \{e^R\}$, candidate e^R is elected and implements policy

$$x_{P^R} = \frac{1}{3}(1 + a). \tag{20}$$

If instead $a \geq \bar{a}$ then in equilibrium $\mathcal{C}(\sigma) = \{e^L\}$, candidate e^L is elected and implements policy

$$x_{P^L} = \frac{1}{3}(1 + a). \tag{21}$$

Proof: Let $a \leq \bar{a}$. By Lemma 4, $e^R \in \mathcal{C}(\sigma)$. By Proposition 3, $e^R \in \mathcal{C}(\sigma)$ implies that $P^E = e^R$. Thus, no candidate e^h, $h \neq R$, wants to enter the electoral competition since he cannot affect the outcome of the election and hence the policy outcome by running for office. Furthermore by not entering candidate e^h saves the entry cost δ.

Once again the analysis of the case $a \geq \bar{a}$ is analogous to the one presented above and therefore omitted. ■

We discuss the implications of Proposition 4 in detail in Section 4 below.

To complete the characterization of the equilibrium of our electoral competition model we still need to analyze each citizen's decision whether to enter the pool of identical citizens among which the primaries select a unique candidate. Since the electoral outcome is uniquely determined, it follows from Proposition 4 above that citizens of a type different from the one that will win the election have no incentive to enter the electoral competition. At the same time, citizens of the type that will indeed win the election have an incentive to compete for office. The entry decision entails a cost δ. Hence, identical citizens enter the primaries so as to compete away all the rents from running for office.[25]

[25] This result parallels the one on rent dissipation in Besley and Coate (1999). However, generalizing their analysis of the welfare cost of lobbying in representative democracies to a setting where lobbies are endogenous is beyond the scope of this paper.

Let $\bar{n}^R_\delta$ be the maximum integer such that:

$$\bar{n}^R_\delta \leq \frac{4(2-a)^2}{9\delta}; \tag{22}$$

and $\bar{n}^L_\delta$ be the maximum integer such that:

$$\bar{n}^L_\delta \leq \frac{4(1+a)^2}{9\delta}. \tag{23}$$

Notice that for δ small enough we have $\bar{n}^R_\delta > 1$ and $\bar{n}^L_\delta > 1$.

The following corollary identifies the type and number of citizens who compete in the primaries.[26]

Corollary 4. *If $a \leq \bar{a}$ then $\bar{n}^R_\delta$ citizens of type R pay the cost δ and enter the set S^R among which $e^R \in \mathcal{C}(\sigma)$ is selected.*

If instead $a \geq \bar{a}$ then $\bar{n}^L_\delta$ citizens of type L pay the cost δ and enter the set S^L among which $e^L \in \mathcal{C}(\sigma)$ is selected.

Proof: Let $a \leq \bar{a}$. Assume that $(m-1)$ type R citizens have decided to enter the electoral competition. Then a type R citizen, by entering will receive the expected payoff:

$$\pi_{in} = \left(\frac{1}{m}\right)\frac{(2-a)^2}{3} - \left(\frac{m-1}{m}\right)\frac{(2-a)^2}{9} - \delta. \tag{24}$$

If instead the citizen decides not to enter the electoral competition then his payoff is

$$\pi_{out} = -\frac{(2-a)^2}{9}. \tag{25}$$

From definition (22), if $m \leq \bar{n}^R_\delta$ and δ is small then $\pi_{in} \geq \pi_{out}$. There exists then a whole set of pure strategy equilibria in which exactly $\bar{n}^R_\delta$ type R citizens enter the electoral competition and are selected to run for office with probability $(1/\bar{n}^R_\delta)$. The identity of these potential candidates is, of course, indeterminate.

[26] For simplicity we restrict attention to the pure strategies equilibria of this entry game.

The proof in the case $a \leq \bar{a}$ is analogous and therefore omitted. ■

4. Discussion

To analyze the full set of implications of our model we begin by characterizing the set of equilibria of the benchmark model where lobbying is not allowed. This analysis is based on Besley and Coate (1997). For purpose of comparison, however, we assume the existence of primaries as in the model we described in Section 2 above. Moreover, we consider the case where the number of citizens of each type is the same, that is $N^j = N/3 = n$, $j = L, C, R$.

When lobbying is not allowed there exist only two types of equilibria of the electoral competition model. These are one-candidate or three-candidate equilibria.[27]

The one-candidate equilibria are such that a single citizen of the median type C runs unopposed and implements policy a. These equilibria parallel the median voter theorem for direct democracy.

The three-candidate equilibria are such that a citizen of each type runs for office. Each one of the three candidates is elected with equal probability. If elected, candidate e^j, $j \in \{L, C, R\}$, implements policy z^j. These equilibria have an "egalitarian" flavor in the sense that the most preferred policy of each type of citizen has an equal probability of being implemented.

We can now comment on the implications of our analysis.

Remark 1. Lobbying always matters.

The equilibrium policy outcome in the game with endogenous lobbies is not an equilibrium outcome in the game without lobbying. Notice that this result is in contrast with the one obtained by Besley and Coate (1999) in the context of a citizen-candidate model where lobbies are exogenously given. In particular Besley and Coate show that it is possible to construct equilibria of such a model where the policy choices

[27] Our assumptions on the preferences of type L, C and R citizens imply that two-candidate equilibria do not exist.

coincide with the ones that would emerge in the equilibria of the citizen-candidate model without lobbying. The equilibria of the two models are, however, different with respect to the identity of the elected candidate who implements such policies. In particular, in the model with exogenous lobbies citizens neutralize the influence of lobbies over policy by strategically electing a candidate with offsetting preferences. The features of their model that are critical to obtain this result are the freedom to choose the number, the size, and the location of the lobbies. While these features are available in a model in which lobbies are treated as primitives, in our model the number and characteristics of lobbies are endogenous.

To illustrate this point, consider the following example. Let $a = 0.2$ and suppose there exists only one lobby consisting of 4 citizens of type L. Following Besley and Coate (1999), the policy choice implemented by any elected candidate maximizes the combined surplus of the elected policy maker and the members of the lobby group. Hence, if the elected candidate is of type L, he implements policy $x_{PL} = 0$. If instead the elected candidate is of type C, he implements policy $x_{PC} = a^2 = 0.04$. Finally, if the elected candidate is of type R, he implements policy $x_{PR} = a = 0.2$. The one-candidate equilibria of this game with one exogenous lobby are such that a single citizen of type R runs unopposed and implements policy a. This is the same equilibrium policy outcome that would obtain in the absence of lobbying. However, the identity of the strategically elected candidate who implements such policy differs.

Remark 2. Lobbying reduces the set of equilibria of the citizen-candidate model of electoral competition.

Endogenizing the number and characteristics of lobbies increases the predictive ability of the citizen-candidate framework. As illustrated by Besley and Coate (1997), citizen-candidate models with strategic voting typically have a large number of equilibria. This multiplicity persists when exogenous lobbies are introduced into the analysis (Besley and Coate 1999).

Remark 3. The equilibrium policy is always biased toward the center of the policy space.

Notice that in all equilibria of our game one "extremist" candidate runs unopposed. However, the equilibrium policy is never extreme. In fact, regardless of his type, any elected candidate implements the same equilibrium policy. Such a policy is centrally located in the policy space, even though it is not equal to the policy most preferred by the median citizen-type C.

Remark 4. The equilibrium policy outcome is robust to changes in the electoral rule.

Osborne and Slivinski (1996) show that in a citizen-candidate framework with sincere voting the equilibrium predictions of the model are different under plurality rule and majority rule with a runoff. In our model, the equilibrium policy outcome is the same under either electoral rule. Also, our analysis implies that the characterization of the equilibrium policy choice above holds for any "Condorcet consistent" voting rule.[28]

In this paper, we have restricted attention to the simple case where there are only three types of citizens. While increasing the number of types significantly complicates the analysis, we conjecture that our main result (lobbying always matters) would generalize for any finite number of types.

[28] A voting rule is Condorcet consistent if it always selects a Condorcet winner whenever a Condorcet winner exists.

Appendix A

Proof of Lemma 3: Consider Case I first. In this case no lobby is active, hence $\mathcal{L}_E(\lambda) = \{\emptyset\}$ and P^E has no alternative but to choose his most preferred policy choice z^E. In this case the policy choice coincides with the one in Besley and Coate (1997).

Consider now Case II. If P^E chooses not to get involved in any lobbying then his utility function is $u(z^E, E) = 0$. Assume now that P^E chooses $\ell = \{l^j\}$. From (10) it follows that P^E's optimal policy choice is $x_{P^E}(\ell) = \frac{1}{2}\left(z^E + z^j\right)$. Therefore the elected candidate's utility is

$$u(x_{P^E}(\ell), E) + u(x_{P^E}(\ell), j) - u(z^E, j) = \frac{1}{2}\left(z^E - z^j\right)^2 > 0 \tag{A.1}$$

It follows from (A.1) that P^E chooses $\ell = \{l^j\}$.

Consider now Case III. We start from the first sub-case (i): $E = L$ and $\mathcal{L}_L(\lambda) = \{\emptyset, l^C, l^R\}$. Using (10) we obtain that P^L's utility if $\ell = \{\emptyset\}$ is

$$u(0, L) = 0. \tag{A.2}$$

Alternatively, if $\ell = \{l^R\}$ P^L's utility is

$$u\left(\frac{1}{2}, L\right) + u\left(\frac{1}{2}, R\right) - u(0, R) = \frac{1}{2}, \tag{A.3}$$

if instead $\ell = \{l^C\}$ P^L's utility is

$$u\left(\frac{1}{2}\,a, L\right) + u\left(\frac{1}{2}\,a, C\right) - u(0, C) = \frac{1}{2}\,a^2. \tag{A.4}$$

Finally if $\ell = \{l^C, l^R\}$ P^L's utility is:

$$u\left(\frac{1}{3}\,(1+a), L\right) + \sum_{j \in \{C,R\}} \left[u\left(\frac{1}{3}\,(1+a), j\right) - u(0, j)\right] = \frac{1}{3} + \frac{2}{3}\,a + \frac{1}{3}\,a^2. \tag{A.5}$$

Comparing (A.3) with (A.2) and (A.4), given that $a < 0.5$ we conclude that $\ell = \{l^R\}$ dominates both $\ell = \{\emptyset\}$ and $\ell = \{l^C\}$. Therefore the relevant comparison is the one between the choice of coalition $\ell = \{l^R\}$ and the choice of coalition $\ell = \{l^C, l^R\}$. Comparing (A.3) and (A.5) we conclude that if $a \leq \bar{a}$, where $\bar{a}$ is defined in (12) above, then the optimal coalition choice is $\ell = \{l^R\}$, if instead $a \geq \bar{a}$ then the optimal coalition choice is $\ell = \{l^C, l^R\}$.

The remaining two sub-cases (ii) and (iii) can be analyzed in an analogous way. Details are therefore omitted. ■

Lemma A.1. *For any elected candidate P^E, and any vector of entry choices λ_{-i}, if $T^j_{P^E} = 0$ the best reply of citizen i of type $j \in \{L, C, R\}$ in the entry-of-lobbyists subgame satisfies the following four properties:*

1) *A citizen of type j never enters a lobby l^j such that, if he decides not to enter, $l^j \in \mathcal{L}_E(\lambda)$.*

2) *A citizen of type j never enters a lobby $l^j \notin \ell_{P^E}$.*

3) *A citizen of type j never enters a lobby l^j such that, if he decides to enter, $\mathcal{L}_E(\lambda) = \{l^j\}$.*

4) *A citizen of type j does not enter a lobby l^j such that, if he decides not to enter, $l^j \notin \mathcal{L}_E(\lambda)$ and if he decides to enter, $l^j \in \ell_{P^E}$ except for the following two cases:*

- *if $E = L$ a citizen of type C enters lobby l^C if $\bar{a} \leq a \leq \frac{1}{4}$ and $l^R \in \mathcal{L}_L(\lambda)$,*
- *if $E = C$ a citizen of type R enters lobby l^R if $l^L \in \mathcal{L}_C(\lambda)$.*

Proof: We start from property 1). By assumption, the activity of a lobby does not depend on the number of lobbyists in it. By entering an active lobby when $T^j_{P^E} = 0$ a citizen reduces his payoff by an amount greater than or equal to the entry cost γ, depending on whether the lobby will be included in the bargaining coalition. Therefore, the choice of entering is strictly dominated.

Consider now property 2). By entering, a citizen of type j would not affect the elected candidate's policy choice and would not pay any transfers to the elected candidate. On the other hand, since $T^j_{P^E} = 0$ entering entails no transfers to the lobbyist and the payment of the entry cost γ. Therefore, a citizen is strictly better off by not entering in this case.

Consider now property 3). Recall that from Proposition 1 above if $\mathcal{L}_E(\lambda) = \{l^j\}$ then $\ell_{P^E} = \{l^j\}$ and $w_j(x_{P^E}(\ell_{P^E}), j) = u(x_{P^E}(\ell_{P^E}), j) - u(z^E, j)$. Therefore the payoff to a type-j citizen if he decides to enter the non-active lobby l^j is:

$$u(x_{P^E}(\ell_{P^E}), j) - \gamma - [u(x_{P^E}(\ell_{P^E}), j) - u(z^E, j)] = u(z^E, j) - \gamma. \tag{A.6}$$

If instead the citizen does not enter his payoff is

$$u(z^E, j). \tag{A.7}$$

Since $\gamma > 0$ the payoff in (A.7) dominates the payoff in (A.6).

Consider now property 4). Given property 3) we can restrict attention to the case where $\mathcal{L}_E(\lambda)$ contains two lobbies. We can distinguish three cases depending on the elected candidate E. Consider first the case in which $E = L$ and $a \geq \bar{a}$. A citizen of type C receives payoff

$$u(0, C) - \gamma = -a^2 - \gamma, \tag{A.8}$$

if he decides to enter the non-active lobby l^C, while his payoff is

$$u\left(\frac{1}{2}, C\right) = -\left(\frac{1-2a}{2}\right)^2, \tag{A.9}$$

if he does not enter lobby l^C. For γ small the payoff in (A.9) is greater than or equal to the payoff in (A.8) if and only if $a \geq \frac{1}{4}$. Therefore a citizen of type C enters lobby l^C if $\bar{a} \leq a \leq \frac{1}{4}$ and does not enter this lobby if $a \geq \frac{1}{4}$. Consider now the entry decision of a citizen of type R. If he decides to enter the lobby l^R his payoff is

$$u(0, R) - \gamma = -1 - \gamma, \tag{A.10}$$

while if he does not enter the lobby l^R his payoff is

$$u\left(\frac{a}{2}, R\right) = -\left(\frac{a-2}{2}\right)^2. \tag{A.11}$$

The payoff in (A.11) clearly dominates the payoff in (A.10). Consider now the case $a \leq \bar{a}$. By property 2) a citizen of type C does not enter lobby l^C since $l^C \notin \ell_{PL}$. This implies that in equilibrium it is not possible for both lobbies l^C and l^R to be active. Hence this case is irrelevant since by property 3) the only relevant cases entail two active lobbies.

Consider now the case $E = C$. A citizen of type R receives payoff

$$u(a, R) - \gamma = -(a-1)^2 - \gamma, \tag{A.12}$$

if he decides to enter the non-active lobby l^R, while his payoff is

$$u\left(\frac{a}{2}, R\right) = -\left(\frac{a-2}{2}\right)^2, \tag{A.13}$$

if he does not enter lobby l^R. For γ small the payoff in (A.12) is greater than the payoff in (A.13) implying that the citizen of type R enters the lobby l^R. Consider now the entry decision of a citizen of type L. In this case $\ell_{PC} = \{l^R\}$ whether this citizen decides to enter the non active lobby l^j or not. Hence property 2) applies and a citizen of type L does not enter lobby l^L.

The proof of the case $E = R$ is similar to the proof of the previous cases and therefore is omitted. ■

Lemma A.2. *For any elected candidate P^E, and any vector of entry choices λ_{-i}, if $T^j_{P^E} = u(x_{P^E}(\ell_{P^E}), j) - u(z^E, j)$ the best reply of citizen i of type $j \in \{L, C, R\}$ in the entry-of-lobbyists subgame satisfies the following three properties:*

$1'$) *A citizen of type j enters a lobby $l^j \notin \ell_{P^E}$ if and only if he expects m more citizens of type j to enter the same lobby, for every m such that $T^j_{P^E} \geq \gamma(m+1)$.*

$2'$) *A citizen of type j never enters a lobby l^j such that, if he decides not to enter, $l^j \in \ell_{P^E}$.*

$3'$) *A citizen of type j always enters a lobby l^j such that, if he decides not to enter, $l^j \notin \mathcal{L}_E(\lambda)$ and if he decides to enter, $l^j \in \ell_{P^E}$.*

Proof: We start from property $1'$). By assumption, $l^j \notin \ell_{P^E}$. Hence, no transfer is paid by lobby l^j to the elected candidate P^E. Therefore the difference in payoffs to a type-j citizen between entering or not entering lobby l^j is equal to $\frac{T^j_{P^E}}{m+1} - \gamma$ where m is the number of other type-j citizens who enter lobby l^j. Clearly a type-j citizen enters if and only if $\frac{T^j_{P^E}}{m+1} - \gamma \geq 0$.

Consider now property $2'$). By assumption, the activity of a lobby does not depend on the number of lobbyists in it. By entering a lobby l^j such that $l^j \in \ell_{P^E}$ a citizen i of type j reduces his payoff by the entry cost γ. This is because the share of subscriptions to lobby l^j, $T^j_{P^E}$, that citizen i appropriates by becoming a lobbyist is exactly equal to his share of the payment lobby l^j makes to the elected candidate P^E. Therefore, the choice of entering is strictly dominated.

Consider now property $3'$). Consider a citizen i of type j. For any λ_{-i} such that $l^j \in \ell_{P^E}$ if $\lambda_i = 1$, the payoff to citizen i if he enters l^j ($\lambda_i = 1$) is equal to

$$u(x_{P^E}(\ell_{P^E}), j) - \gamma - \frac{T^j_{P^E}}{N^j}. \tag{A.14}$$

Conversely, if citizen i does not enter ($\lambda_i = 0$) then $l^j \notin \mathcal{L}_E(\lambda)$ and his payoff is equal to

$$u(x_{P^E}(\hat{\ell}_{P^E}), j), \tag{A.15}$$

where $\hat{\ell}_{P^E}$ is the unique solution to problem (5) as characterized in Lemma 3 given λ_{-i} and $\lambda_i = 0$. Notice that $\hat{\ell}_{P^E}$ is either $\{\emptyset\}$ or $\{l^k\}$, where $k \neq j$. Comparing (A.14) and (A.15), it is optimal for citizen i to enter if and only if:

$$u(x_{P^E}(\ell_{P^E}), j) - u(x_{P^E}(\hat{\ell}_{P^E}), j) \geq \gamma + \frac{T^j_{P^E}}{N^j}. \tag{A.16}$$

Condition (A.16) is always satisfied as long as the entry cost γ is small and the number of type j citizens is large. This is the case because from Lemma 2 $u(x_{P^E}(\ell_{P^E}), j) > u(x_{P^E}(\hat{\ell}_{P^E}), j)$. ■

Lemma A.3. *For any elected candidate P^E and any lobby l^j the equilibrium of the subscription game is such that:*

If $l^j \notin \ell_{P^E}$, $j \in \{L, C, R\}$, then $T^j_{P^E} = 0$.

If $E = L$ and $\bar{a} \leq a \leq 0.25$, then $T^C_{P^L} = 0$ and $T^R_{P^L} = u(x_{P^L}, R) - u(z^L, R)$.

In every other case $T^j_{P^E} = u(x_{P^E}, j) - u(z^E, j)$.

Proof: The subscription game we consider is a special case of the one analyzed in Bagnoli and Lipman (1989). Their analysis guarantees that the unique undominated perfect equilibrium outcome of our subscription game is such that each citizen is pivotal when making his decision whether to pledge the amount $t^j_{P^E}$, as in (6) above, to lobby l^j. Hence whenever a citizen of type j benefits from having an active lobby of his type, and the lobby would not be active if $T^j_{P^E} = 0$, then this citizen will pledge the amount $t^j_{P^E}$. Notice that this happens only if $l^j \in \ell_{P^E}$, unless $j = C$, $E = L$ and $\bar{a} \leq a \leq 0.25$. For the details of the argument see Bagnoli and Lipman (1989). ■

Proof of Proposition 2: The proof follows from Lemma A.1, Lemma A.2 and Lemma A.3. ■

Proof of Corollary 3: The proof follows from Lemma A.3. ■

Appendix B

In this appendix we present the characterization of the set of equilibria of the electoral competition model described in Section 2 above when we remove the primaries from the extensive form. In other words we allow multiple citizens of the same type to run for office in the general election. For simplicity, we restrict attention to the case where the number of citizens of each type is the same, that is $N^j = N/3 = n$, $j = L, C, R$.

The characterization of the whole set of equilibria of the voting and entry of candidates subgames differs considerably from the one presented in Section 3 above. We start from the voting subgame.

In the analysis of this subgame, following Besley and Coate (1997), we rule out weakly dominated voting strategies.[29]

[29]See Besley and Coate (1997) for a definition of a weakly dominated voting strategy. Notice that in our framework this restriction implies that when a citizen is not pivotal (he cannot affect the outcome of the voting equilibrium by modifying his vote) he votes for the candidate that will implement the policy choice closest to the citizen's most preferred outcome.

Let s denote the vector of citizens' decisions s_i^j whether to run for office (in the framework without primaries such a vector is the outcome of the entry of candidates subgame, instead of σ). Then the set of candidates is $\mathcal{C}(s)$.

We start considering the case in which $a \leq \bar{a}$ and $\mathcal{C}(s)$ includes only one candidate of type R, e^R. Then, independently of how many other candidates e^h of type $h \neq R$ are included in $\mathcal{C}(s)$, candidate e^R wins the election. Correspondingly, if $a \geq \bar{a}$ and $\mathcal{C}(s)$ includes only one candidate e^L, of type L then, independently of how many other candidates e^k of type $k \neq L$ are present in $\mathcal{C}(s)$, candidate e^L wins the election.

Proposition A.1. *Let $a \leq \bar{a}$. For any vector of candidate entry decisions s that induces a set of candidates $\mathcal{C}(s)$ which contains only one e^R candidate and any number of e^h, $h \neq R$, candidates the unique electoral outcome is that e^R wins the election.*

Alternatively, let $a \geq \bar{a}$. For any vector of candidate entry decisions s that induces a set of candidates $\mathcal{C}(s)$ which contains only one e^L candidate and any number of e^k, $k \neq L$, candidates the unique electoral outcome is that e^L wins the election.

Proof: Let $a \leq \bar{a}$. The result follows from the observation that from the preference orderings in (17) it is a weakly dominant strategy for the type L and C citizens to vote for candidate e^R rather than for any candidate e^h, $h \neq R$. If instead $a \geq \bar{a}$ from the preference orderings in (18) it is a weakly dominant strategy for the type L and C citizens to vote for candidate e^L rather than for any candidate e^k, $k \neq L$. ■

Next, consider the case where all candidates in $\mathcal{C}(s)$ are of the same type. In this case, the outcome of the voting game is indeterminate. In fact, in equilibrium, the electorate can allocate votes among the candidates in any possible way. Notice that all the equilibria of this sort induce the same payoffs to all citizens other than the candidates.

Finally, consider the case where the set of candidates $\mathcal{C}(\sigma)$ contains two or more candidates of at least two types. Let $n(j)$ be the maximum number of citizens that vote for an individual candidate e^j of type $j \in \{L, C, R\}$.

Proposition A.2. *There exist a vector of candidate entry decisions s that induces a set of candidates $\mathcal{C}(s)$ which contains two or more candidates of at least two types and a vector of equilibrium voting decisions v such that $P^E = e^j$ for every $j \in \{L, C, R\}$.*

Proof: We proceed by construction. Consider the case where $a \leq \bar{a}$.

First, assume $\mathcal{C}(\sigma) = \{e^R, e^R, e^C, e^C\}$. Then, from the preference orderings in (17) it follows that it is a weakly dominant strategy for all citizens of types L and C to vote for one of the candidates e^R while it is a weakly dominant strategy for the type R citizens to vote for one of the e^C candidates. Further, the maximum number of citizens that will vote for one of the e^R candidates cannot fall below n: $n(R) \geq n$. If $n(R) > n$ then necessarily $P^E = e^R$. Indeed, no more than n type R citizens will vote for e^C. Consider then the case $n(R) = n$. If $n(C) < n$ then $P^E = e^R$. Therefore the only case left to consider is the case $n(R) = n$ and $n(C) = n$. In this case notice that each citizen of type L and C is pivotal: by switching his vote from one of the e^R candidates to the other a type L or C citizen will increase to one the probability that an e^R candidate wins the election increasing at the same time his own payoff. Therefore $n(R) = n$ and $n(C) = n$ cannot be an equilibrium of the voting subgame.

Next, assume $\mathcal{C}(\sigma) = \{e^L, e^L, e^C, e^C\}$. Then, from the preference orderings in (17) it is a weakly dominant strategy for all citizens of types L and C to vote for one of the e^L candidates, while it is a weakly dominant strategy for the type R citizens to vote for one of the e^C candidates. Then an argument symmetric to the one we used for the case $\mathcal{C}(\sigma) = \{e^R, e^R, e^C, e^C\}$ above shows that in every equilibrium of the voting subgame $P^E = e^L$.

Finally, assume $\mathcal{C}(\sigma) = \{e^R, e^R, e^R, e^C, e^C\}$ and v is such that one third of the citizens of types L and C vote for each candidate e^R and all citizens of type R vote for one of the e^C candidates. Hence, each e^R candidate receives $2n/3$ votes and one of the candidates e^C receives $n(C) \geq n$ votes and is therefore elected (that is, $P^E = e^C$). To show that this is an equilibrium of the voting subgame, notice that no citizen can profit from a deviation since no voter is pivotal.

The proof for the case where $a \geq \bar{a}$ is analogous. Details are therefore omitted. ■

We can now move to the characterization of the equilibria of the entry of candidates subgame. We first prove that in the model without primaries a result analogous to Lemma 4 above holds.

Lemma A.4. *All equilibria of the entry-of-candidates subgame are such that:*

If $a \leq \bar{a}$ then $e^R \in \mathcal{C}(s)$.

If instead $a \geq \bar{a}$ then $e^L \in \mathcal{C}(s)$.

Proof: Consider the case where $a \leq \bar{a}$. Assume by way of contradiction that $e^R \notin \mathcal{C}(s)$. It follows from the preference orderings in (17) that by entering, a citizen of type R would be elected for sure. In this case, his payoff would be equal to

$$\frac{(2-a)^2}{3} - \delta \tag{A.17}$$

which, for a small cost of entry δ, is positive. Notice that by not entering, the citizen's payoff would be negative regardless of the electoral outcome (see equations (16)). Thus, a citizen of type R would enter. This is a contradiction of the assumption $e^R \notin \mathcal{C}(\sigma)$.

The proof in the case $a \leq \bar{a}$ is analogous and therefore omitted. ■

We can now show that only one type of candidate with extremist preferences may win the elections in equilibrium. The type of this candidate will depend on the value of the parameter a.

Lemma A.5. *There does not exist an equilibrium of the electoral competition game such that, if $a \leq \bar{a}$ the elected candidate is e^L, while if $a \geq \bar{a}$ the elected candidate is e^R.*

Proof: Let $a \leq \bar{a}$. Assume by way of contradiction that such an equilibrium exists: $P^E = e^L$. For this to be an equilibrium e^L needs to collect the highest number of votes: $n(L) \geq n(k)$, for every $e^k \in \mathcal{C}(s)$, $k \neq L$. From Lemma A.4 above we have $e^R \in \mathcal{C}(s)$. Two alternatives are then possible: either $e^C \in \mathcal{C}(s)$ or $e^C \notin \mathcal{C}(s)$. Assume that $e^C \in \mathcal{C}(s)$. It is then a weakly dominant strategy for the type R citizens to vote for candidate e^C while it is a weakly dominant strategy for type L and C citizens to vote for candidate e^R. This implies that no citizen will vote for candidate e^L contradicting the hypothesis that e^L wins the election: $n(L) > n(k)$ for $k = C, R$. Consider now the case $e^C \notin \mathcal{C}(s)$. The equilibrium payoff of a type C citizen is then:

$$-\frac{1}{4}(1-2a)^2 - \frac{a^2 - \frac{1}{4}(1-2a)^2}{n}. \tag{A.18}$$

Consider now a deviation from this equilibrium in which a type C citizen decides to enter the electoral competition as candidate $\hat{e}^C$. Then it is a weakly dominant strategy for type R citizens to vote for candidate e^C while it is a weakly dominant strategy for type L and C citizens to vote for e^R: $n(L) = 0$. Therefore only two possible equilibria of the voting game can arise: candidate $\hat{e}^C$ wins or a candidate e^R wins. Assume that a candidate e^R wins. The payoff to candidate $\hat{e}^C$ is then

$$-\frac{1}{9}(1-2a)^2 - \frac{(1-a)^2 - \frac{1}{9}(1-2a)^2}{n} - \delta. \tag{A.19}$$

If instead candidate $\hat{e}^C$ wins his payoff is:

$$\frac{(1-a)^2}{2} - \delta \tag{A.20}$$

Notice that for a high n and a small δ both payoffs in (A.19) and (A.20) are strictly higher than the equilibrium payoff in (A.18). Hence entering the electoral competition is a profitable deviation for candidate $\hat{e}^C$ contradicting the hypothesis that there exist an equilibrium in which e^L wins and $e^C \notin \mathcal{C}(s)$.

The proof in the case $a \geq \bar{a}$ is analogous and therefore omitted. ■

Finally we show that when multiple candidates of a given type enter the electoral competition and win the election they must have equal probability of being elected. Let m^j denotes the number of e^j candidates in $\mathcal{C}(s)$.

Lemma A.6. *All equilibria of the model without primaries are such that:*

If $a \leq \bar{a}$ and a candidate e^R wins the election, then $n(R) = 2n/m^R$. If instead a candidate e^C wins the election then $n(C) = n/m^C$.

If instead $a \geq \bar{a}$ and a candidate e^L wins the election then $n(L) = 2n/m^L$. If instead a candidate e^C wins the election then $n(C) = n/m^C$.[30]

Proof: Let $a \leq \bar{a}$ and consider the case in which candidate e^R wins the election. Assume by way of contradiction that $n(R) > 2n/m^R$. Then there exists a candidate $\bar{e}^R \in \mathcal{C}(s)$ that will loose the election. His payoff is then:

$$-\frac{1}{9}(2-a)^2 - \delta. \tag{A.21}$$

Consider now a deviation in which candidate $\bar{e}^R$ decides not to enter the electoral competition. From Proposition A.2 above three equilibria of the voting game can occur: either an other e^R candidate wins the election or one of the e^C and e^L candidates wins the election. In the case an other e^R candidate wins the payoff to $\bar{e}^R$ is:

$$-\frac{1}{9}(2-a)^2. \tag{A.22}$$

If instead e^C wins the election the payoff to $\bar{e}^R$ is:

$$-\frac{1}{4}(1-a)^2 - \frac{(1-a)^2 - \frac{1}{4}(1-a)^2}{n}. \tag{A.23}$$

Finally if e^L wins the election the payoff to $\bar{e}^R$ is:

$$-\frac{1}{4} - \frac{1-\frac{1}{4}}{n}. \tag{A.24}$$

For a high n the payoffs in (A.22), (A.23) and (A.24) are strictly higher than the payoff in (A.21). This contradicts the hypothesis that there exist an equilibrium in which e^R wins the election and $n(R) > 2n/m^R$.

[30] The lemma is stated in the case in which more than one type of candidates is in $\mathcal{C}(s)$. If only one type of candidates, say e^R, is in $\mathcal{C}(s)$ then $n(R) = N(R)/m^R$, where $N(R)$ denotes the total number of citizens that cast their vote.

The proof in the case that e^C wins the election and in the case $a \geq \bar{a}$ is analogous and hence omitted. ■

We have now all the elements to characterize the set of equilibria of the model without primaries. We start by classifying these equilibria on the basis of the type of the elected candidate. In particular we can distinguish two types of equilibria that we label the 'extremist' equilibria and the 'centrist' equilibria, respectively.

If $a \leq \bar{a}$, the extremist equilibria are all such that candidate e^R is elected, while if $a \geq \bar{a}$ these equilibria are such that candidate e^L is elected. In all extremist equilibria the equilibrium policy choice is

$$x^* = \frac{(1+a)}{3}. \tag{A.25}$$

Regardless of the value of a, in all centrist equilibria candidate e^C is elected and implements the equilibrium policy

$$x^{**} = \frac{(1+a)}{2}. \tag{A.26}$$

We first provide a characterization of both types of equilibria and then prove that only these two types of equilibria exist. We start from the extremist equilibria.

Proposition A.3. *There exist two types of extremist equilibria: Equilibria with only one type of candidates and equilibria with multiple types of candidates. All extremist equilibria are such that the policy choice implemented is x^*, as in (A.25).*

The extremist equilibria with only one type of candidates are such that:

If $a \leq \bar{a}$ then $\mathcal{C}(s)$ contains m^R candidates e^R, such that $m^R = 1, \ldots, \bar{n}^R_\delta$, where $\bar{n}^R_\delta$ is defined in (22) above.

If instead $a \geq \bar{a}$ then $\mathcal{C}(s)$ contains m^L candidates e^L, such that $m^L = 1, \ldots, \bar{n}^L_\delta$, where $\bar{n}^L_\delta$ is defined in (23) above.

The extremist equilibria with multiple types of candidates are such that:

If $a \leq \bar{a}$ then $\mathcal{C}(s)$ contains m^R candidates e^R, such that $m^R = 3, \ldots, \bar{n}^R_\delta$ and m^h, $h \neq R$, candidates such that $2m^h > m^R$.

If instead $a \geq \bar{a}$ then $\mathcal{C}(s)$ contains m^L candidates e^L, such that $m^L = 1, \ldots, \bar{n}^L_\delta$ and m^k, $k \neq L$, candidates such that $2m^k > m^L$.

Proof: We start from the extremist equilibria with only one type of candidates. Let $a \leq \bar{a}$. From Lemma A.6 the polity will split its votes equally among the m^R candidates.

Consider now an $e^R \in \mathcal{C}(s)$ candidate. In equilibrium his expected payoff is:

$$\left(\frac{1}{m^R}\right)\frac{(2-a)^2}{3} - \left(\frac{m^R-1}{m^R}\right)\frac{(2-a)^2}{9} - \delta. \tag{A.27}$$

If instead the candidate e^R decides not to enter the electoral competition then his payoff is

$$-\frac{(2-a)^2}{9}. \tag{A.28}$$

From definition (22), if $m^R \leq \bar{n}^R_\delta$ and δ is small then the payoff in (A.27) is greater or equal to the payoff in (A.28). In other words, candidate e^R has no incentive to deviate.

Consider now a type $h \neq R$ citizen and assume that he deviates and decides to enter the electoral competition as candidate: $e^h \in \mathcal{C}(s)$. It is an equilibrium of the voting subgame for type L and C citizens to vote for one and only one of the e^R candidates. This implies that this e^R candidate wins the election. Therefore using, off the equilibrium path, this equilibrium of the voting subgame in which $e^h \in \mathcal{C}(s)$ and e^R wins, it is possible to construct a subgame perfect equilibrium of the entire game such that by entering candidate e^h does not change the policy but reduces his payoff of the entry cost δ. Hence e^h is strictly better off by not entering the electoral competition and has no incentive to deviate.

Finally, consider a type R citizen, different from each of the m^R candidates e^R, and assume that he deviates and decides to enter the electoral competition as candidate $\hat{e}^R \in \mathcal{C}(s)$. The set of candidate $\mathcal{C}(s)$ then contains $(m^R + 1)$ type R candidates. It is an equilibrium of the voting subgame for the polity to vote for only one of the original e^R candidates. This implies that this e^R candidate wins the election. Therefore using, off the equilibrium path, this equilibrium of the voting subgame in which $\hat{e}^R \in \mathcal{C}(s)$ and one of the e^R candidates wins, it is possible to construct a subgame perfect equilibrium of the entire game such that by entering candidate $\hat{e}^R$ does not change the policy outcome but reduces his payoff of the entry cost δ. Hence $\hat{e}^R$ is strictly better off by not entering the electoral competition and has no incentive to deviate. Therefore no citizens has an incentive to deviate from the prescribed strategies.

The proof in the case $a \geq \bar{a}$ is analogous and therefore omitted.

Consider now the extremist equilibria with multiple types of candidates. Let $a \leq \bar{a}$. Notice first that since by Lemma A.6 $n(R) = 2n/m^R$ and by definition $n(h) \geq n/m^h$ then provided that $2m^h \geq m^R$ or $(2n/m^R) \geq (n/m^h)$ it is an equilibrium of the voting game for the type L and C citizens to split equally their votes among the m^R candidates and $n(R) = (2n/m^R) > n(h)$ so that one of the e^R candidates wins the elections.

Consider now one of the m^R candidates $\hat{e}^R$. In equilibrium the expected payoff to $\hat{e}^R$ is the same as in (A.27). Further, it is an equilibrium of the voting subgame for all the type L and C citizens to vote for the same candidate $e^R \in \mathcal{C}(s)$. Therefore using, off-the-equilibrium-path, this equilibrium of the voting subgame it is possible to construct a subgame perfect equilibrium of the entire game such that if candidate $\hat{e}^R$ decides to deviate and not to enter the electoral competition an other e^R candidate will win the election. In this case the payoff to $\hat{e}^R$ is the same as in (A.28). From definition (22), if $m^R \leq \bar{n}^R_\delta$ and δ is small the payoff in (A.27) strictly dominates the payoff in (A.28). Therefore candidate $\hat{e}^R$ has no incentive to deviate.

Consider now one of the m^h, $h \neq R$, candidates $\hat{e}^h$. His equilibrium payoff is

$$-\frac{1}{9}(1+a)^2 - \frac{1-\frac{1}{9}(1+a)^2}{n} - \delta, \tag{A.29}$$

if $h = L$ and

$$-\frac{1}{9}(1-2a)^2 - \frac{(1-a)^2-\frac{1}{9}(1-2a)^2}{n} - \delta, \tag{A.30}$$

if $h = C$. Further, since $m^R > 2$ or $n > (2n/m^R)$ it is an equilibrium of the voting game for all the type R citizens to vote for only one of the e^h candidates and for this candidate to win the elections: $n(R) < n(h)$. Therefore using, off-the-equilibrium-path, this equilibrium of the voting subgame it is possible to construct a subgame perfect equilibrium of the entire game such that if candidate $\hat{e}^h$ decides to deviate and not to enter the electoral competition an other e^h candidate will win the election. In this case the payoff to $\hat{e}^h$ if a candidate e^L wins is $-\frac{1}{4}$ if $h = L$ and

$$-\frac{1}{4}(1-2a)^2 - \frac{a^2-\frac{1}{4}(1-2a)^2}{n}, \tag{A.31}$$

if $h = C$. If instead candidate e^C wins e^h's payoff is:

$$-\frac{1}{4}(1+a)^2 - \frac{a^2-\frac{1}{4}(1+a)^2}{n}, \tag{A.32}$$

if $h = L$ and

$$-\frac{1}{4}(1-2a)^2, \tag{A.33}$$

if $h = C$. Notice that if $h = L$ for a high n and a small δ the payoff in (A.29) strictly dominates both the payoff $-\frac{1}{4}$ and the payoff in (A.32); while if $h = C$ the payoff in (A.30) strictly dominates both payoffs in (A.31) and (A.33). Therefore candidate $\hat{e}^h$ has no incentive to deviate.

Finally, consider a type $j \in \{L, C, R\}$ citizen different from the m^R and m^h candidates in $\mathcal{C}(s)$ and assume that this citizen decides to enter the electoral competition as an additional candidate $\bar{e}^j \in \mathcal{C}(s)$. It is an equilibrium of the voting subgame for type L and C citizens to vote for only one of the original e^R candidates so as to win the election. This implies that the $\bar{e}^j$ candidate will not win the election. Therefore using, off-the-equilibrium-path, this equilibrium of the voting subgame it is possible to construct a subgame perfect equilibrium of the entire game such that, by entering, candidate $\bar{e}^j$ does not win the election and does not change the policy choice but only reduces his payoff of the entry cost δ. Hence $\bar{e}^j$ is strictly better off by not entering the electoral competition and has no incentive to deviate.

The proof in the case $a \geq \bar{a}$ is analogous and therefore omitted. ■

We can now move to the characterization of the centrist equilibria. Let $\bar{n}_\delta^C$ be the maximum integer such that:

$$\bar{n}_\delta^C \leq \frac{3(1-a)^2}{4\delta}. \tag{A.34}$$

Notice that for a small δ we get $\bar{n}_\delta^C > 1$.

Proposition A.4. *All centrist equilibria are such that each candidate e^C is elected with equal probability $(1/m^C)$ and implements policy x^{**} as in (A.26). These equilibria are such that:*

If $a \leq \bar{a}$, then $\mathcal{C}(s)$ contains $m^C = 1, \ldots, \bar{n}_\delta^C$ candidates e^C and $m^R \geq 2m^C + 1$ candidates e^R. Type R citizens split equally their votes among the e^C candidates, while type L and C citizens splits their votes among the e^R candidates in such a way that:

$$n(R) < \frac{n}{m^C}. \tag{A.35}$$

If instead $a \geq \bar{a}$, then $\mathcal{C}(s)$ contains $m^C = 1, \ldots, \bar{n}_\delta^C$ candidates e^C and $m^L \geq 2m^C + 1$ candidates e^L. Type R citizens split equally their votes among the e^C candidates, while type L and C citizens splits their votes among the e^L candidates in such a way that:

$$n(L) < \frac{n}{m^C}. \tag{A.36}$$

Proof: Let $a \leq \bar{a}$. Notice first that given condition (A.35) and Lemma A.6 above it is an equilibrium of the voting subgame for $n(C) = (n/m^C)$ and $n(R) < n(C)$.

Consider now one of the m^R candidates in $\mathcal{C}(s)$: $\tilde{e}^R$. Given condition (A.35), in equilibrium this candidate looses the election and his payoff is:

$$-\frac{1}{4}(1-a)^2 - \frac{(1-a)^2 - \frac{1}{4}(1-a)^2}{n} - \delta. \tag{A.37}$$

Assume, instead, that this candidate decides to deviate and does not enter the electoral competition. It is an equilibrium of the voting subgame for type L and C citizens to vote for one and only one of the other e^R candidates that in this way will win the election. The payoff to $\tilde{e}^R$ in this case is:

$$-\frac{1}{9}(2-a)^2. \tag{A.38}$$

For n large and δ small the payoff in (A.37) strictly dominates the payoff in (A.38) implying that candidate $\tilde{e}^R$ is better off by entering the electoral competition and therefore does not want to deviate.

Consider now one of the e^C candidates in $\mathcal{C}(s)$. By entering the electoral competition this candidates is elected with probability $(1/m^C)$ and obtains payoff:

$$\left(\frac{1}{m^C}\right)\frac{(1-a)^2}{2}-\left(\frac{m^C-1}{m^C}\right)\frac{(1-a)^2}{4}-\delta. \tag{A.39}$$

If instead e^C does not enter the electoral competition he obtains payoff:

$$-\frac{(1-a)^2}{4}. \tag{A.40}$$

From definition (A.34), if $m^C \leq \bar{n}^C_\delta$ and δ is small then the payoff in (A.39) strictly dominates the payoff in (A.40). In other words, candidate e^C has no incentive to deviate.

Finally, consider a type $j \in \{L, C, R\}$ citizen different from the m^R and m^C candidates in $\mathcal{C}(s)$ and assume that this citizen decides to enter the electoral competition as an additional candidate $\bar{e}^j \in \mathcal{C}(s)$. It is an equilibrium of the voting subgame for type L and C citizens to split their votes among the e^R candidates so as to satisfy condition (A.35), and for type R citizens to vote for only one of the original e^C candidates so that this candidate e^C wins. This implies that the $\bar{e}^j$ candidate will not win the election. Therefore using, off-the-equilibrium-path, this equilibrium of the voting subgame it is possible to construct a subgame perfect equilibrium of the entire game such that, by entering, candidate $\bar{e}^j$ does not win the election and does not change the policy choice but only reduces his payoff of the entry cost δ. Hence $\bar{e}^j$ is strictly better off by not entering the electoral competition and has no incentive to deviate.

The proof in the case $a \geq \bar{a}$ is analogous and therefore omitted. ■

A key feature of both these types of equilibria is that they are based on the fact that the citizens of each type can split their votes among identical candidates in any possible way provided that none of the voters is pivotal. We view this feature as pathological. As discussed in Section 2.1 above, it exclusively relies on the assumption that perfectly identical candidates may enter the electoral competition. This is the reason why we introduce primaries in the extensive form of the model.

We can now show that no other type of equilibria exist in the model with no primaries.

Proposition A.5. *The only two types of equilibria of the electoral competition model without primaries are extremist and centrist equilibria as defined in Propositions A.3 and A.4 above.*

Proof: Let $a \leq \bar{a}$. From Lemma A.4 we have $e^R \in \mathcal{C}(s)$. Further, from Lemma A.5 we have that there do not exist equilibria of the model without primaries such that a candidate e^L wins. There exist therefore only equilibria where a candidate e^R or a candidate e^C wins. Our proof is therefore complete if we show that there do not exist equilibria where a candidate e^R wins that differ from the extremist equilibria characterized in Proposition A.3 above; and there do not exist equilibria where a candidate e^C wins that differ from the centrist equilibria characterized in Proposition A.4 above. We proceed in three steps.

Step 1. There do not exist extremist equilibria with $m^h \geq 1$, $h \neq R$, candidates $e^h \in \mathcal{C}(s)$ and $2m^h \leq m^R$.

Assume, by way of contradiction that these equilibria exist. From Lemma A.6 above we have that $n(R) = (2n/m^R)$. Moreover by definition of $n(h)$ we have $n(h) \geq (n/m^h)$ while by assumption $(2n/m^R) \leq (n/m^h)$. We therefore conclude that

$$n(R) = \frac{2n}{m^R} \leq \frac{n}{m^h} \leq n(h).$$

For these extremist equilibria to exist we need $n(R) \geq n(h)$. Clearly this condition contradicts $n(R) < n(h)$, therefore the only alternative left is $n(R) = n(h)$. Notice however that in the case $n(R) = n(h)$ each type L and C citizen is pivotal since by switching his vote from one of the e^R candidate to an other each citizen can guarantee that an e^R candidate is elected with probability one improving in this way his payoff (as from the preferences in (17)). Therefore it is not an equilibrium of the voting subgame for $n(h) = n(R) = (2n/m^R)$.

Step 2. There do not exist centrist equilibria with $m^L \geq 1$ candidates $e^L \in \mathcal{C}(s)$.

Assume by way of contradiction that these equilibria exist. From Lemma A.5 we know that none of the e^L candidates can have a strictly positive probability of winning. Therefore the payoff to each e^L candidate is:

$$-\frac{1}{4}(1+a)^2 - \frac{a^2 - \frac{1}{4}(1+a)^2}{n} - \delta. \tag{A.41}$$

Assume now that one of the type L candidates $\tilde{e}^L$ does not enter the electoral competition. Since, from Proposition A.4, there are m^R candidates e^R in $\mathcal{C}(s)$ and m^C candidates e^C in $\mathcal{C}(s)$ it is a weakly dominant strategy for the type L and C citizens to vote for an e^R candidate and for the type R citizens to vote for an e^C candidates. Therefore only two outcomes of the voting subgame are possible: an e^R candidate wins or an e^C candidate wins. If an e^R candidate wins then $\tilde{e}^L$'s payoff is:

$$-\frac{1}{9}(1+a)^2 - \frac{1-\frac{1}{9}(1+a)^2}{n}. \tag{A.42}$$

If instead an e^C candidate wins then $\tilde{e}^L$'s payoff is:

$$-\frac{1}{4}(1+a)^2 - \frac{a^2-\frac{1}{4}(1+a)^2}{n}. \tag{A.43}$$

Both payoffs in (A.42) and in (A.43) strictly dominates the payoff in (A.41). Implying that it is a profitable deviation for a candidate $\tilde{e}^L$ not to enter the electoral competition.

Step 3. There do not exist centrist equilibria with m^R candidates e^R and m^C candidates e^C in $\mathcal{C}(s)$ such that $m^R \leq 2m^C$.

Assume, by way of contradiction that these equilibria exist. From Lemma A.6 above we have that $n(C) = (n/m^C)$. Moreover by definition of $n(R)$ we have $n(R) \geq (2n/m^R)$ while by assumption $(2n/m^R) \geq (n/m^C)$. We therefore conclude that

$$n(C) = \frac{n}{m^C} \leq \frac{2n}{m^R} \leq n(R).$$

For these centrist equilibria to exist from Proposition A.4 we need $n(C) \geq n(R)$. Clearly this condition contradicts $n(C) < n(R)$, therefore the only alternative left is $n(C) = n(R)$. The same argument presented in the proof of Step 1 above implies that there does not exist an equilibrium of the voting subgame such that $n(C) = n(R)$.

The proof in the case $a \geq \bar{a}$ is analogous and therefore omitted. ■

We conclude the appendix by observing that all one, two and three candidates equilibria of the model without primaries are extremist equilibria with only one type of candidates. In other words all the pathological equilibria of the model without primaries require at least four candidates in $\mathcal{C}(s)$.

Corollary A.1. *All one, two and three candidates equilibria of the model without primaries are such that if $a \leq \bar{a}$ then only e^R candidates are in $\mathcal{C}(s)$ while if $a \geq \bar{a}$ then only e^L candidates are in $\mathcal{C}(s)$.*

Proof: The proof follows from Propositions A.3, A.4 and A.5 above. ■

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