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CESifo Working Papers

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CESifo Working Paper No. 620

December 2001

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ISSN 1617-9595

CESifo *Working Paper No. 620*
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Abstract

Tullock's analysis of rent-seeking is reconsidered from an evolutionary point of view. We show that evolutionarily stable behavior in a rent-seeking contest differs from efficient rent-seeking behavior in a Nash equilibrium. We explore that implications of evolutionary stability for rent-seeking behavior and relate them to the well examined Nash equilibrium behavior. A most interesting result is an overdissipation law, which holds in evolutionary equilibrium.

JEL Classification: C79, D72.

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1 Introduction

Alan Lockard and Gordon Tullock (2001) have just edited a volume on "Efficient Rent-Seeking" as a "chronicle of an intellectual quagmire", which documents Tullock's rent-seeking paradox and the profession's response to it. The volume represents an attempt to revive interest in the still unresolved parts of the otherwise well dissected structure of the paradox, which have implications for the functionings of markets far beyond the realm of rent-seeking (Lockard and Tullock, Introduction, p.1). The present contribution is directly inspired by this volume and aims to retell the rent-seeking tale – as exemplified by Tullock's (1980) original game – from a different theoretical vantage point: it is an evolutionary tale that encompasses as a limiting case the efficient tale of the chronicle.

Efficient rent-seeking is based on Nash equilibrium as a solution concept; for the individual decision maker this implies that his decisions in a contest are based on rational expectations.[1] Evolutionary rent-seeking – as presented in this paper – is based on the notion of an evolutionary stable strategy (ESS) as a solution concept, which focusses on behavior (and its diffusion) itself rather than on choice of behavior. An ESS is the simplest solution concept from evolutionary game theory, which has been used to justify rationalistic Nash equilibrium. However, the popular view that an ESS represents a *refinement* of Nash equilibrium is only correct, if one applies the evolutionary argument – often implicitly, but not explicitly assumed – to an *infinite* population of players. For finite populations, a staple in the rent-seeking literature, an ESS *may differ* from Nash equilibrium (Schaffer, 1988). It indeed does so in the case of rent-seeking and we explore the implications of these differences in the rent-seeking context in detail.

This difference is of interest from a principal point of view: Alchian (1950) argued in his classic essay on evolutionary economics that the postulate of maximization (as in Nash equilibrium) may be false but that its use is justified by the tenets of "survival of the fittest" under competition. Economists usually read "competition" as "perfect competition" (or "price-taking" behavior), which implicitly requires a large ("almost" infinite) population. In more general terms, however, competition means a contest or fight between several claimants, who aim to "beat" each other. Evolutionary behavior precisely corresponds to this second meaning and is therefore

[1] Following Tullock (1980) and the subsequent literature we refer to Nash equilibrium behavior in a rent-seeking contest as "efficient" rent-seeking, although it is not efficient in the usual economic sense of the word.

highly relevant for contest theory[2]. A reading of our results, that is faithful to Alchian (1950), then is that evolution justifies a maximization postulate, which is *different* from the one embodied in efficient rent-seeking.

Section II recollects Tullock's analysis of efficient rent-seeking. Section III motivates our new solution concept and shows that efficient rent-seeking is not evolutionarily stable. Section IV proves existence of an evolutionary stable strategy and discusses its properties. Most remarkably, an "overdissipation postulate" can be proven: in the presence of increasing returns to scale of the rent-seeking technology evolutionarily stable behavior implies *overdissipation* of the rent. Section V accounts for the differences between evolutionary and efficient rent-seeking in a systematic way and shows how efficient rent-seeking emerges as a limit of evolutionary rent-seeking. Section VI concludes.

2 Efficient rent-seeking

Recall that Tullock (1980) proposed a model, in which n rent-seekers compete for a rent of size V. If the contestants expend $x = (x_1, \ldots, x_n), x_i \geq 0$, the probability of success for player $i, i = 1, \ldots, n$ is given by

$$p_i(x_1, \ldots, x_n) = \frac{x_i^r}{\sum_{j=1}^n x_j^r}$$

and expected profit for player i is given by

$$\Pi_i(x_1, \ldots, x_n) = p_i(x_1, \ldots, x_n) \cdot V - x_i = \frac{x_i^r}{\sum_{j=1}^n x_j^r} \cdot V - x_i.$$

One can show that for $r \leq \frac{n}{n-1}$ a unique Nash equilibrium in pure strategies exists in this game, in which each player maximizes expected payoff by bidding

$$x^* = \frac{n-1}{n^2} \cdot r \cdot V.$$

Aggregate rent-dissipation then amounts to

$$n \cdot x^* = \frac{n-1}{n} \cdot r \cdot V.$$

[2]Possajennikov (2001) relates both meanings to each other by identifying perfect competition with "aggregate-taking behavior".

Equilibrium expenditures never exceed V, the value of the rent, but may be strictly less than V. The "full rent dissipation"-hypothesis does not hold; yet overdissipation is incompatible with individually rational payoff maximization as it would imply, that at least one contestant has a negative payoff in equilibrium (and would therefore be better off by non-participating or bidding zero).

Important variations of this basic model include Corcoran and Karels (1985), Higgins et al. (1987), Hillman and Samet (1987), Leininger (1993), Leininger and Yang (1994) and Baye et al. (1994), in which the analysis is extended to mixed strategies and extensive form, dynamic games.

3 Evolutionarily stable Strategies

An alternative to the rational-expectations approach of the previous section is to resort to a more biologically inspired view of human behavior; namely that economic behavior diffuses, stabilizes, mutates and disappears along an evolutionary path that leads to the survival of best or best adapted strategies or standards of behavior. A particularly useful concept in this respect is the notion of an evolutionarily stable strategy (ESS) as defined by Maynard Smith (Maynard Smith and Price (1973), Maynard Smith (1974, 1982)), because it allows to say something about the stable dynamic properties of an evolutionary system without the need to commit oneself to specific dynamics. We follow Schaffer (1988), who adapted the ESS notion to finite populations of interacting agents.

A strategy is evolutionarily stable, if a whole population using that strategy cannot be invaded by a sufficiently small group of "mutants" using another strategy (as a so-called "mutant genotype"). Similarly, a standard of behavior in an economic contest is evolutionarily stable, if – upon being adopted by all participants in the contest – no small subgroup of individuals using a different standard of behavior can invade and "take over". Obviously, in the context of finite populations the smallest meaningful number of mutants is one. The emphasis of the evolutionary approach is not on explaining actions (as a result of particular choice or otherwise), but on the diffusion of forms of behavior in groups (as a result of learning, imitation, reproduction or otherwise).

The definition of invadability is all important:

Definition:

i) Let a strategy (standard of behavior) x be adapted by all players $i, i = 1, \dots, n$. A mutant strategy $\bar{x} \neq x$ can invade x, if the payoff for a single player using $\bar{x}$ (against x of the $(n-1)$ other players) is strictly higher than the payoff of a player using x (against $(n-2)$ other players using x and the mutant using $\bar{x}$).

ii) A strategy x^{ESS} is evolutionarily stable, if it cannot be invaded by any other strategy.

Roughly speaking, an ESS is such that, if almost all members of a group adopt it, there is no other strategy that could give a higher relative payoff, if used by a group member. The dynamic justification for this notion of equilibrium is, that more successful strategies diffuse or "reproduce" faster than less successful ones and ultimately extinct the latter.

We now formalize ESS in the context of Tullock's rent-seeking game and search for an expenditure level x which qualifies as an ESS.

Denote by x the expenditure profile $(x_1, \dots, x_n) = (x, \dots, x); x$ can be invaded, if there exists $x_1 = \bar{x}$, say, such that

$$\Pi_1(\bar{x}, x, \dots, x) > \Pi_i(\bar{x}, x, \dots, x) \qquad \text{for} \quad i = 2, \dots, n.$$

Consequently, a strategy x^{ESS} is an ESS if and only if

$$\Pi_1(\bar{x}, x^{ESS}, \dots, x^{ESS}) < \Pi_i(\bar{x}, x^{ESS}, \dots, x^{ESS})$$

$$\text{for} \quad i = 2, \dots, n \text{ and for all} \quad \bar{x} \neq x^{ESS}.$$

Our first result may come as a surprise to those who have become accustomed to the view that the equilibrium concept of an ESS leads to a *refinement* of the Nash equilibrium concept (which *is* true for infinitely large populations):

Theorem 1: Efficient rent-seeking is not evolutionarily stable.

Proof: We claim that $x^* = \frac{n-1}{n^2} \cdot r \cdot V$ – the Nash standard of behavior – is not an ESS!

We prove this claim by showing that a mutant strategy $\bar{x} = (1 + \varepsilon)x^*$, $\varepsilon > 0$, sufficiently small, can invade:

So let player 1 bid $\bar{x} = (1+\varepsilon)x^*$ and players 2 to n bid x^*; we then have

$$\Pi_1((1+\varepsilon)x^*, x^*, \ldots, x^*) = \frac{(1+\varepsilon)^r \cdot x^{*^r}}{n \cdot x^{*^r} + A \cdot x^{*^r}} \cdot V - x^* - \varepsilon \cdot x^*$$

with $A = (1+\varepsilon)^r - 1$
and

$$\Pi_i((1+\varepsilon)x^*, x^*, \ldots, x^*) = \frac{x^{*^r}}{n \cdot x^{*^r} + A \cdot x^{*^r}} \cdot V - x^*.$$

Clearly,

$$\Pi_1 - \Pi_i > 0 \iff \frac{A}{n+A} \cdot V - \varepsilon \cdot x^* > 0.$$

With $x^* = \frac{n-1}{n^2} \cdot r \cdot V$ this amounts to

$$\frac{A}{n+A} \cdot V > \varepsilon \cdot \frac{n-1}{n^2} \cdot r \cdot V$$

or – equivalently –

$$\frac{A}{\varepsilon} > \frac{n-1}{n^2} \cdot r(n+A). \tag{1}$$

But the last inequality always holds for ε small enough:

> The right-hand side approaches $\frac{n-1}{n} \cdot r$ monotonically as $\varepsilon \to 0$ since $A = (1+\varepsilon)^r - 1$ goes to zero.
> The left-hand side approaches r as $\varepsilon \to 0$, since by l'Hospital's rule we have
> $$\lim_{\varepsilon \to 0} \frac{(1+\varepsilon)^r - 1}{\varepsilon} = \lim_{\varepsilon \to 0} \frac{r \cdot (1+\varepsilon)^{r-1}}{1} = r.$$
> Thus for ε small (1) holds and the claim is proven.

The proof shows that more aggressive behavior than shown by a (rational) Nash-strategist does better in *relative* terms! The more aggressive mutant does – of course – not better in absolute terms, because he does not play a *best response* (that would be x^*). This "loss" in absolute terms is more than offset by a gain in *relative* terms, he now has the advantage of the highest payoff realized among all the contestants, because his higher aggressiveness lowers the opponents payoffs by more than it lowers his own! This kind of behavior has been called "spiteful" (Hamilton, 1971) in the sense that an ESS-strategist pursues not only a larger payoff for himself but also a lower payoff for his competitors. Consequently, in a stable group of all ESS-strategists and no mutants an ESS-strategist is not in general maximizing his payoff or fitness, but the difference between his own payoff and the average payoff of the other players (Schaffer, 1988).

Relative considerations are undoubtedly important in a rent-seeker's calculus. The contest success function of the Tullock model is homogeneous of degree zero; i.e. $p_i(\lambda x_1, \lambda x_2, \ldots, \lambda x_n) = p_i(x_1, \ldots, x_n)$ for all $i = 1, \ldots, n$, and thus only depends on the *relative* size of bids: e.g. set $\lambda = \frac{1}{x_1}$ to get $p_i = p(1, \frac{x_2}{x_1}, \ldots, \frac{x_n}{x_1})$. Axiomatic characterisations of contest success functions (e.g. Kooreman and Schoonbeek, 1997) precisely use zero-homogeneity as an axiom to account for the importance of *relative* bids in contests. Hence, already in Tullock's original model a sizeable part of the absolute payoff function is determined by the *relative* size of bids.
It therefore makes sense to explore the implications of evolutionary equilibrium, which can be thought of as rent seeking behavior by contestants, who not only care about relative bids but also about relative payoffs.

4 Evolutionary Rent-seeking

We now show existence of an ESS for the Tullock model and discuss its properties.

Theorem 2: An ESS exists whenever $r \leq \frac{n}{n-1}$. It is given by

$$x^{ESS} = (\frac{r}{n} \cdot V, \frac{r}{n} \cdot V, \ldots, \frac{r}{n} \cdot V).$$

Proof: The argument in the proof of Theorem 1 shows, that a strategy x^{ESS} constitutes an ESS, if it solves the maximization problem (Schaffer, 1988)

$$max_x \Pi_1(x, x^{ESS}, \ldots, x^{ESS}) - \Pi_i(x, x^{ESS}, \ldots, x^{ESS}), \;\; i = 2, \ldots, n.$$

So let x be the mutant strategy and $\bar{x}$ be the candidate strategy for an ESS.

$$\Pi_1(x, \bar{x}, \ldots, \bar{x}) - \Pi_i(x, \bar{x}, \ldots, \bar{x}) = \frac{x^r - \bar{x}^r}{x^r + (n-1) \cdot \bar{x}^r} \cdot V - x + \bar{x}$$

The first-order condition w.r.t. x then reads (see Appendix)

$$\frac{n(x \cdot \bar{x})^r}{(x^r + \bar{x}^r \cdot n - \bar{x}^r)^2 \cdot x} \cdot r \cdot V = 1$$

Since we look for a symmetric solution, setting $x = \bar{x}$ reduces the first-order condition to

$$n \cdot \bar{x}^{2r} \cdot r \cdot V = n^2 \cdot \bar{x}^{2r+1}$$

solving for $\bar{x}$ yields $\bar{x} = \frac{r}{n} \cdot V$.
This is indeed a global maximum of the relative payoff function, if participation

in the contest with $\bar{x} = \frac{r}{n} \cdot V$ is at least as good as non-participation with $x = 0$ (see Appendix).
Obviously, $\Pi_1(\bar{x}, \bar{x}, \ldots, \bar{x}) - \Pi_i(\bar{x}, \bar{x}, \ldots, \bar{x}) = 0$
whereas $\Pi_1(0, \bar{x}, \ldots, \bar{x}) - \Pi_i(0, \bar{x}, \ldots, \bar{x}) = \frac{n(r-1)-r}{n(n-1)} \cdot V$.
The latter is less or equal to zero, if $r \leq \frac{n}{n-1}$.

Note that evolutionary stability can be thought of *as if* rent-seekers tried to maximize *relative* payoff. Hence it is relative, not absolute payoff maximization that follows from Alchian's (1950) evolutionary position on competition. The evolutionarily successful strategy $\bar{x} = \frac{r}{n} \cdot V$ could even entail a negative absolute payoff; if evaluated with the – irrelevant – absolute payoff function: $\Pi(\bar{x}, \bar{x}, \ldots, \bar{x}) = \frac{1}{n} \cdot V - \frac{r}{n} \cdot V = (1-r)\frac{V}{n} < 0$, if $r > 1$! The payoff in a Nash-equilibrium in contrast is $(1 - \frac{n-1}{n} \cdot r) \cdot \frac{V}{n}$ and cannot be negative for $r \leq \frac{n}{n-1}$. But as Theorem 1 tells us this property comes at a price: absolute payoff maximization does not guarantee survival in a finite population, relative payoff maximization does.

We summarize our findings in

Theorem 3:

i) Individual expenditures and aggregate rent-dissipation in an evolutionary equilibrium $\bar{x} = (\frac{r}{n} \cdot V, \ldots, \frac{r}{n} \cdot V)$ are always higher than in an efficient (Nash) equilibrium $x^* = (\frac{n-1}{n^2} \cdot r \cdot V, \ldots, \frac{n-1}{n^2} \cdot r \cdot V)$

ii) Aggregate rent-dissipation in an evolutionary equilibrium is independent from the number of contestants, it is solely determined by the rent-seeking technology (contest success function) and the value of the rent:

$$n \cdot \frac{r}{n} \cdot V = r \cdot V$$

iii) For $r > 1$ there is overdissipation of the rent in an evolutionary equilibrium; for $r = 1$ there is full dissipation of the rent and for $r < 1$ there is underdissipation of the rent.

Theorem 3 deserves some further comments: as the next section will show, parts i) and ii) are a consequence of the fact that the entire group (or population) of rent-seekers takes part in the rent-seeking contest. This "playing-the-field" context leads to maximal exertion of evolutionary pressure, which only remains constrained by the parametrically given environment, i.e. the size of the rent and the success function.

Part iii) is truly remarkable: overdissipation of the rent in a *pure-strategy* evolutionary equilibrium is the consequence of "spite" in the presence of increasing returns to expenditures ($r > 1$). This overdissipation result in evolutionary equilibrium would persist even for $r > \frac{n}{n-1}$, if we allowed a mutant to employ a mixed strategy when applying the stability criterion. Our conjecture (proven so far for $n = 2$) is that we would then find a mixed ESS, an analysis that is beyond the scope of this paper. The important insight, namely that evolutionary stable behavior *necessitates* spite and spiteful behavior coupled with increasing returns leads to overdissipation, is summarized in

Corollary 4: ("Overdissipation Law")

> An evolutionary rent-seeking equilibrium in the presence of increasing returns requires overdissipation of the rent.

This Corollary vindicates Tullock's "overdissipation postulate", a target of much (justified) criticism, in the confines of his very own model. And it does so in an exceedingly elegant way: much of Tullock's "intellectual swamp" (Tullock, 1985) is avoided by concentrating on stable states of an – unmodelled – *dynamic* process of selection and mutation (some of it, admittedly, may reappear if this process *is* modelled; e.g. by replicator dynamics or otherwise). The evolutionary argument also obviates the prescription of Tylenol as recommended by Tullock (1985): Theorem 1 and 3 precisely answer Tullock's puzzling questions from "Back to the bog" where he notes that "in the particular case of efficient rent-seeking, it is in general, more profitable .... if you do not make the same bid as your colleagues. If one deviates from the pattern, he makes a profit, if all follow his example, they lose. The sensible behavior for an individual is not sensible unless the other people are doing something else." The evolutionary perspective offers a *consistent* and *sensible* account of such behavior. Unfortunately, it has a new unpleasant implication: it raises rent-seeking cost above the efficient level and even – in some circumstances – above the rent. Evolutionary stability – and hence survival – needs "waste" (from the efficiency perspective) to safeguard against mutant invasion and extinction.

5 Evolutionary vs. Efficient Rent-Seeking

Theorem 3i) shows, that individual (and consequently aggregate) rent-seeking expenditures are higher in an evolutionary equilibrium than in a Nash equilibrium. Since an ESS-strategist maximizes the relative payoff while a Nash-strategist maximizes absolute payoff, it is easily seen that if one of the n players devotes an additional unit to rent-seeking as an ESS-strategist he appropriates not only a marginal

gain through a higher probability of success (like a Nash-strategist), but in addition one from lowering the competitors probability of winning. This suggests that – ceteris paribus – evolutionary rent-seeking will involve a larger aggregate expenditure than efficient rent-seeking.
Put differently, one and the same amount of rent dissipation (for a given r) results from different contest sizes in the evolutionary and efficient rent-seeking models. For $r = 1$ evolutionary rent-seeking leads to full rent dissipation for any number of participants, n. This result only holds for infinitely many participants in the efficient rent-seeking model.
This section aims to account for those differences in a systematic way and establishes the efficient rent-seeking model as a limit of evolutionary rent-seeking models.

Consider a group of n potential rent-seekers who may engage in a rent-seeking contest of size $\bar{n} < n$; i.e. only $\bar{n}$ out of n candidates qualify for participation in the contests. It does not matter for our purposes here what causes this entry restriction (for a discussion of this issue see e.g. Gradstein and Konrad (1994) or Amegashi (1998)) and for simplicity (and symmetry) we assume that players are chosen randomly and with equal probabilities for participation in the contest.
Clearly, if all n potential players adhere to the same standard of behavior – the strategy x, say – then all players chosen for the contest will use this strategy. Now suppose – as before – that one player out of the n potential players – player 1 – subscribes to a mutant strategy $\bar{x}$. Could such a mutant invade the n-player population? This, again, is decided by a comparison of payoffs for mutant and average population member of the $(n-1)$ non-mutants.

First, consider the expected payoff for the mutant in a contest: since he necessarily faces $(n-1)$ players playing x he expects

$$\Pi_1(\bar{x}, x, \ldots, x) = \frac{\bar{x}^r}{(\bar{n}-1) \cdot x^r + \bar{x}^r} \cdot V - \bar{x}.$$

A regular player $i \in \{2, \ldots, n\}$ chosen for the contest expects

$$\bar{\Pi}_i = \left(1 - \frac{\bar{n}-1}{n-1}\right) \cdot \Pi_i(x, x, \ldots, x) + \frac{\bar{n}-1}{n-1} \cdot \Pi_i(\bar{x}, x, \ldots, x)$$

as the probability that a chosen player i will also face player 1 from the remaining $(n-1)$ potential players is $\frac{\bar{n}-1}{n-1}$ (there are $\bar{n}-1$ contest spots left to be filled).

Consequently, an ESS strategy x^{ESS} must now solve the maximization problem (Schaffer, 1988)

$$max_x \Pi_1(x, x^{ESS}, \ldots, x^{ESS}) - \left(1 - \frac{\bar{n}-1}{n-1}\right) \cdot \Pi_i(x^{ESS}, \ldots, x^{ESS}) - \frac{\bar{n}-1}{n-1} \cdot \Pi_i(x, x^{ESS}, \ldots, x^{ESS})$$

or

$$max_x \Pi_1(x, x^{ESS}, \ldots, x^{ESS}) - \frac{\bar{n}-1}{n-1} \cdot \Pi_i(x, x^{ESS}, \ldots, x^{ESS}) \qquad (2)$$

as $\Pi_i(x^{ESS}, \ldots, x^{ESS})$ is a constant. If x^{ESS} solves (2) then $\Pi_1(x, x^{ESS}, \ldots, x^{ESS}) \leq \bar{\Pi}_i$ always holds (and invasion is not possible).

So let $\bar{x}$ be the candidate for an ESS:

$$max_x \frac{x^r}{(\bar{n}-1)\bar{x}^r + x^r} \cdot V - x - \frac{\bar{n}-1}{n-1} \cdot \frac{\bar{x}^r}{(\bar{n}-1)\bar{x}^r + x^r} \cdot V + \frac{\bar{n}-1}{n-1} \cdot \bar{x}$$

leads to the first-order condition

$$\frac{(\bar{n}-1)x^r \cdot \bar{x}^r + \frac{\bar{n}-1}{n-1} \cdot x^r \cdot \bar{x}^r}{x \cdot [x^r + (\bar{n}-1)\bar{x}^r]^2} \cdot r \cdot V - 1 = 0$$

which has the unique symmetric solution

$$x^{ESS} = \frac{(\bar{n}-1)n}{(n-1) \cdot \bar{n}^2} \cdot r \cdot V.$$

Aggregate expenditures are $\frac{\bar{n}-1}{n-1} \cdot \frac{n}{\bar{n}} \cdot r \cdot V$, they depend on n in negative way. Note, that $\lim_{n \to \infty} x^{ESS} = \frac{\bar{n}-1}{\bar{n}^2} \cdot r \cdot V$, this is the *efficient* rent-seeking solution for a contest of fixed size $\bar{n}$. We note

Theorem 5: Efficient rent-seeking with n participants results from evolutionary rent-seeking of an infinite population, that engages in contests of size n.

Proof: (2) already indicates that for $n \to \infty$ the evolutionary equilibrium condition reduces to

$$max_x \Pi_1(x, x^{ESS}, \ldots, x^{ESS})$$

i.e. x^{ESS} has to be a best response to itself and thus constitutes a Nash equilibrium strategy.

The rent-seekers of Theorem 5 do not "play the field "; they only play a subfield, which is randomly chosen from the field. This lowers evolutionary pressure: the larger the field, the smaller the probability that a mutant is chosen as participant of the subfield contest. An evolutionary stable standard of behavior now maximizes the difference between own payoff and the payoff of a typical other player *weighted* by the probability of the occurence of a mutant in the contest. In the previous "playing the field"-scenario this probability always equalled one. Theorem 5 also indicates that the smaller the population size n the more pronounced is "spiteful" behavior in an evolutionary equilibrium. Accordingly, overdissipation is most severe in these circumstances. Two-player contests, which are ubiquitous, therefore elicit the highest efforts.

6 Conclusion

We have examined rent-seeking behavior in Tullock's classic model from an evolutionary point of view. Our main findings, however, extend far beyond this particular model. They are:
i) evolutionary rent-seeking leads to higher rent-dissipation than efficient rent-seeking. In an evolutionarily stable strategy equilibrium contestants behave *as if* they are maximizing a *relative* payoff function consisting of the difference of own payoff and the average payoff of the competitors. This goal is not only furthered by gaining own utility but also by lowering others' utility; this additional motive justifies additional ("spiteful") investments in the rent-seeking contest.
ii) Rent dissipation (as a fraction of the rent) in an evolutionary equilibrium is independent of the number of contestants and only determined by the contest success function.
iii) If the contest success function exhibits increasing (decreasing) returns the evolutionary equilibrium is characterized by *overdissipation* (underdissipation) of the rent. With constant returns full rent dissipation applies.
iv) Efficient rent-seeking behavior results from evolutionary rent-seeking behavior of an infinite population of rent-seekers, who engage in contests of finite size.

7 Appendix

Proof of Theorem 2:

Consider the maximization problem

$$\max_x \frac{x^r - \bar{x}^r}{x^r + (n-1)\bar{x}^r} \cdot V - x + \bar{x} \ ,$$

whose first-order condition is

$$\frac{r \cdot x^{r-1}[x^r + (n-1) \cdot \bar{x}^r] \cdot V}{[x^r + (n-1)\bar{x}^r]^2} - \frac{(x^r - \bar{x}^r) \cdot r \cdot x^{r-1} \cdot V}{[x^r + (n-1)\bar{x}^r]^2} = 1$$

equivalently,

$$\frac{(x^{2r} + x^r \cdot \bar{x}^r \cdot n - x^{2r}) \cdot r \cdot V}{x[x^r + n\bar{x}^r]^2} = 1$$

which simplifies to

$$\frac{n \cdot x^r \cdot \bar{x}^r}{x[x^r + \bar{x}^r \cdot n - \bar{x}^r]^2} \cdot r \cdot V = 1$$

as used in the proof of Theorem 2. Moreover, the second-order condition for a maximum becomes – upon differentiating the above expression –

$$\frac{[r \cdot x^{2r} \cdot \bar{x}^r + r \cdot x^r \bar{x}^{2r} \cdot n - r \cdot x^r \cdot \bar{x}^{2r} - 2r \cdot x^{2r} \cdot \bar{x}^r}{x^2[x^r + \bar{x}^r \cdot n - \bar{x}^r]^3}$$

$$+\frac{-x^{2r} \cdot \bar{x}^r - x^r \cdot \bar{x}^{2r} \cdot n + x^r \bar{x}^{2r}] \cdot r \cdot V \cdot n}{x^2[x^r + \bar{x}^r \cdot n - \bar{x}^r]^3} < 0.$$

Simplification leads to

$$\frac{x^r \cdot \bar{x}^r[(r-1)(n-1)\bar{x}^r - x^r(r+1)] \cdot r \cdot V \cdot n}{x^2[x^r + n \cdot \bar{x}^r - \bar{x}^r]^3} < 0 \ ; \qquad (3)$$

this holds in the symmetric solution $\bar{x} = x = \frac{r \cdot V}{n}$, if

$$(r-1)(n-1)(\frac{r \cdot V}{n})^r - (\frac{r \cdot V}{n})^r(r+1) < 0$$

or, equivalently, $r \cdot n - 2r - n < 0$. Thus, the second-order condition for local maximization holds for

$$r < \frac{n}{n-2}.$$

Also note that (3) implies global concavity of the relative payoff function, if $r \leq 1$ holds: The bracketed term in the nominator is always negative, the one in the denominator always positive. Hence, the local optimum is a global one for all $r \leq 1$. If $r > 1$, then (3) shows that the relative payoff function is locally convex for sufficiently small x: substituting

$$\bar{x} = \frac{r \cdot V}{n}$$

into (3) yields

$$\frac{x^r \left(\frac{r \cdot V}{n}\right)^r \left[(r-1)(n-1)\left(\frac{r \cdot V}{n}\right)^r - x^r(r+1)\right] \cdot r \cdot V \cdot n}{x^2 \left[x^r + \left(\frac{r \cdot V}{n}\right)^r \cdot n - \left(\frac{r \cdot V}{n}\right)^r\right]^3}.$$

This expression is positive for all $0 < x < \hat{x}$ and negative for all $x > \hat{x}$, where $\hat{x}$ solves the equation

$$(r-1)(n-1)\left(\frac{r \cdot V}{n}\right)^r - x^r(r+1) = 0$$

i.e.

$$\bar{x} = \left(\frac{r-1}{r+1} \cdot (n-1)\left(\frac{r \cdot V}{n}\right)^r\right)^{\frac{1}{r}} = \left(\frac{r-1}{r+1}\right)^{\frac{1}{r}} \cdot (n-1)^{\frac{1}{r}} \cdot \frac{r \cdot V}{n}.$$

As a consequence, the relative payoff function is convex on $(0, \hat{x})$ and concave on $(\hat{x}, \infty)$. Moreover, we have

$$\hat{x} < \frac{r \cdot V}{n} = x^{ESS}$$

if and only if

$$\left(\frac{r-1}{r+1}\right)^{\frac{1}{r}} \cdot (n-1)^{\frac{1}{r}} < 1.$$

The latter is equivalent to $r < \frac{n}{n-2}$. Hence, in this range the only candidates for a global maximum of the relative payoff function are 0 and $\frac{r \cdot V}{n}$. This completes the proof of Theorem 2 as $\frac{n}{n-1} < \frac{n}{n-2}$.

References

[1] Alchian, A., 1950, Uncertainty, Evolution and Economic Theory, Journal of Political Economy 58, 211-221

[2] Amegashi, J., 1998, The Design of Rent-Seeking Competitions, Public Choice 99, 63-76

[3] Baye, M., Kovenock, D. and C. de Vries, 1994, The Solution to the Tullock Rent-seeking Game when $R > 2$, Public Choice 81, 363-380

[4] Corcoran, W. and G. Karels, 1985, Rent-Seeking Behavior in the long-run, Public Choice 46, 227-247

[5] Gradstein, M. and K. Konrad, 1994, Orchestrating Rent-Seeking Contests, The Economic Journal 109, 34-48

[6] Hamilton, W., 1971, Selfish and spiteful behavior in an evolutionary model, Nature 228, 1218-1220

[7] Higgins, R., Shugart, W. and R. Tollison, 1985, Free entry and efficient Rent-seeking, Public Choice 46, 247-258

[8] Hillman, A. and D. Samet, 1987, Dissipation of contestable rents by small numbers of contenders, Public Choice 54, 63-82

[9] Kooreman, P. and L. Schoonbeek, 1997, The Specification of the Probability Functions in Tullock's rent-seeking contest, Economics Letters 56, 59-61

[10] Leininger, W., 1993, More efficient Rent-seeking, Public Choice 75, 43-62

[11] Leininger, W. and C. Yang, 1994, Dynamic Rent-seeking Games, Games and Economic Behavior 7, 406-422

[12] Lockard,A. and G. Tullock (eds.), 2001, Efficient Rent-seeking: chronicle of an Intellectual Quagmire, Kluwer Academic Publishers

[13] Maynard Smith, J. and G. Price, 1973, The Logic of animal conflict, Nature 246, 15-18

[14] Maynard Smith, J., 1974, The Theory of Games and the Evolution of Animal Conflicts, Journal of Theoretical Biology 47, 209-221

[15] Maynard Smith, J., 1982, Evolution and the Theory of Games, Cambridge University Press

[16] Possajennikov, A., 2001, Evolutionary Foundations of Aggregate-Taking Behavior, Discussion Paper 01-10, University of Dortmund

[17] Schaffer, M., 1988, Evolutionarily Stable Strategies for a finite Population and a variable contest size, Journal of Theoretical Biology 132, 469-478

[18] Tullock, G., 1980, Efficient Rent-Seeking, in: Buchanan, Tollison and Tullock (eds.), Toward a Theory of the Rent-seeking Society, Texas A & M University Press, 3-15

[19] Tullock, G., 1985, Back to the Bog, Public Choice 46, 259-263