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Information Markets, Elections and Contracts

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Abstract

Politicians may pander to public opinion and may renounce undertaking beneficial long-term projects. To alleviate this problem, we introduce a triple mechanism involving political information markets, reelection threshold contracts, and democratic elections. An information market is used to predict the long-term performance of a policy, while threshold contracts stipulate a price level on the political information market that a politician must reach to have the right to stand for reelection. Reelection thresholds are offered by politicians during campaigns. We show that, on balance, the triple mechanism increases social welfare. Finally, we suggest several ways to avoid the manipulation of information markets and we discuss possible pitfalls of the mechanism.

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Keywords: democracy, elections, information markets, threshold contracts and triple mechanism.

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1 Introduction

Motivation

In democracies elections are the primary mechanisms for making politicians accountable. Holding reelections may induce incumbents to act in the public interest and enable the electorate to replace them with more promising candidates. However, at a particular election date citizens may sometimes lack the information required to decide wisely about whether an incumbent deserves to be re-elected. There may be various reasons for this lack of information. Voters may be rationally ignorant, since in a large electorate the likelihood of a single citizen affecting the outcome of an election is negligible. Alternatively, voters may have no access to information, e.g. in cases where policies have mainly long-term effects and precise information about the consequences of a project is not available at the election date.

A typical example of a long-term policy is unemployment.[1] Reforming the labor market is generally considered inevitable for remedying unemployment. However, introducing labor market reforms may initially cause disruptions and even higher unemployment, because some layoffs will occur immediately, while the creation of new jobs may take time. Thus in the short term it may be impossible for voters to judge the politician's performance in the field of labor market policy. A policy problem with a longer time horizon is also global warming. Due to the complex structure of the global warming problem it is difficult to assess how reducing greenhouse gases will affect the climate and the well-being of people in the future.[2]

Triple Mechanism

In this paper we suggest a triple mechanism involving political information markets, threshold incentive contracts, and democratic elections to solve this fundamental information problem. At the end of the first term, a political information market is held. Here investors can bet on whether the incumbent will be reelected at the end of the second

[1]A detailed description of unemployment in Europe can be found in Saint-Paul (2000), for instance.

[2]Most predictions suggest that the temperature associated with thermal equilibrium on earth will increase as a result of rapidly rising emissions of greenhouse gases (IPPC (2007)). Such temperature changes may have a sizable impact on the well-being of future generations (see e.g. Cline (1992), Fankhauser (1995), Nordhaus (2006), and Stern (2006)).

term and hence whether he has undertaken socially beneficial long-term policies. As it is uncertain whether the politician will be reelected for the first time at the end of period 1, this is a conditional information market. It aggregates the information on whether the incumbent has undertaken socially desirable long-term projects or whether the incumbent has merely pandered to current public opinion. A high price on the political information market indicates high probability that the incumbent will be elected a second time.

The second element of the triple mechanism involves reelection threshold contracts that competing politicians can offer before they start on their first term. The reelection threshold contract stipulates a critical price threshold the information market must reach or exceed for the incumbent to have the right to stand for first reelection. The critical price thresholds are offered competitively by politicians campaigning for their first term in office.

The third element of our mechanism are democratic elections that take place at three dates. In a first election, an office holder with a particular reelection threshold contract is elected by citizens. If the office holder fulfills his contract, he can stand for reelection at the second date. If he succeeds, he can try to get reelected a second time.

The main idea of our paper is as follows: Political information markets, price thresholds on these markets, and democratic elections increase the motivation of politicians to undertake long-term beneficial policies that may be unpopular at the time they are introduced. We develop this insight in the framework of a simple political agency model. We show that a carefully designed combination of political information markets and threshold contracts can – on balance – improve welfare. In section 5 we explore the robustness of the triple mechanism and we address several potential pitfalls such as attempts to manipulate information markets.

The Literature

Our model is most closely related to the suggestion to combine contracts and democratic elections introduced by Gersbach (2003) and extended by Gersbach (2006) and Gersbach and Liessem (2008). These papers show how the dual mechanism – contracts offered competitively during campaigns and elections – can improve political outcomes. All these papers rely on verifiable data by which contracts can be conditioned. As a contrast, we also analyze the case where the results from current policy can only be observed in a future period and may never be verifiable. We suggest a novel triple mechanism where a political information market produces verifiable information in the form of prices at a time when policy results are not observable.

Political information markets have attracted a lot of attention recently. Information markets have been suggested to improve public policy decisions. (See e.g. the recent surveys and discussions by Wolfers and Zitzewitz (2004), or Hanson (2003), who suggests to use information markets to select policies that are expected to raise GDP.) A comprehensive summary on this relatively new topic can be found, for example, in Hahn and Tetlock (2004). The basic idea behind information markets is the accumulation of scattered information in order to predict uncertain future events. Political information markets have turned out to be quite successful in predicting election results (see e.g. Berg, Forsythe and Rietz (1996) or Berlemann and Schmidt (2001)) and are already established in practice. We suggest a new type of information market. While standard markets predict the result of the next election, we use a market that predicts the result of the next but one election in order to obtain an approximation of the long-term effects of current policies. The idea is that the incumbent will only be reelected in the next but one election if the voters are satisfied with the long-term project results they learn about over time.

The information our prediction market aggregates could, in principle, also be provided by other sources, in particular by a free press. The information market has the advantage that it generates a verifiable signal in the form of a price on which reelection threshold contracts can be conditioned. This is not the case for information provision by the media, even if such provision were unbiased.

Our paper is broadly related to political agency and accountability theory. While this literature developed by Barro (1973), Ferejohn (1986), and Persson, Roland and Tabellini (1997) has established the advantages and drawbacks of democratic elections in making office-holders accountable, we present a new institutional framework to address the accountability of politicians. We would like to point out that our analysis is a theoretical exercise on how such a new institutional framework would function and how it might improve – on balance – existing electoral processes.

Organization of the Paper

The paper is organized as follows: In the next section we introduce the model. The results for elections only are analyzed in section 3. In section 4 we examine the triple mechanism involving political information markets, threshold incentive contracts, and democratic elections. In section 5 we look at some extensions to our basic model. Section 6 concludes. Appendix A contains the proofs. Appendices B and C describe the political information market in more detail. In Appendix D we provide a numerical example.

2 The Basic Model

Our basic model draws on Maskin and Tirole (2004) and Gersbach and Liessem (2008). There are three periods, denoted by $t = 1,2,3$.

2.1 The Election Framework

There is a continuum of identical voters of measure 1. We assume that there are two politicians denoted by $i = 1,2$. They compete for office before the first period starts. The elected politician has to take some kind of action during the first period. He can choose between action $a_1 = 1$ and action $a_1 = 0$. All voters have the same preference ranking for the two possible actions,[3] but they do not know their preferences when they decide about the office-holder for the first term. There are two possible states of the world $s_1 = 1$ and $s_1 = 0$, which are drawn randomly. State $s_1 = 1$ will occur with probability z, and state $s_1 = 0$ will occur with probability $1-z$. We assume that $\frac{1}{2} < z < 1$. The state of the world determines which action is optimal for the voters. If state $s_1 = 1$ is drawn, then the optimal action for the voters will be $a_1 = 1$. The optimal action for the voters will be $a_1 = 0$ in state $s_1 = 0$. If $a_1 = s_1$, voters get a payoff of 1, otherwise they get a payoff of 0. Voters are risk-neutral and want to maximize their expected utility. As $z > \frac{1}{2}$, we will refer to $a_1 = 1$ as the popular action and to $a_1 = 0$ as the unpopular action.

There are two types of politicians, either congruent or dissonant. Both politicians know their own type and the type of their opponent.[4] However, voters cannot observe the

[3]For the relevance of this assumption and for an outline of how to accommodate heterogeneous preferences of voters, see Maskin and Tirole (2004).

[4]The assumption that politicians have knowledge about each other's type may appear to be plausible

politicians' types. A politician is congruent with a probability of $\frac{1}{2}$. In this case he has the same preferences as the voters. A politician is dissonant with a probability of $\frac{1}{2}$, i.e. if $a_1 = 1$ is optimal for the voters, then $a_1 = 0$ is optimal for the dissonant politician and vice versa. The two political candidates may differ as to congruence or dissonance. In all other respects they are identical.

2.2 The Information Structure

At the beginning of the whole game, voters and politicians have a priori probabilities of z that state $s_1 = 1$ will occur and of $1 - z$ that state $s_1 = 0$ will occur. In the first period, the elected politician can learn precisely which state of the world has occurred, thus knowing with certainty which action is best for the voters and which action is best for himself.

We assume that voters are able to observe the action of the incumbent immediately and that the action is verifiable.[5] We also assume that, while it is impossible to verify which state of the world has occurred, the voters will be able to observe it. However, it is not clear when the voters will make this observation. We assume that before their first reelection decision voters will observe with probability μ which state of the world has realized, while the probability that they will observe the state in period 2 (i.e. after their first reelection decision) is $1 - \mu$. Further, we assume that $0 \leq \mu \leq \frac{1}{2}$ to analyze a situation where the possibility that the performance of a project is not observable in the short term is a serious problem.[6] Note that regardless of whether there is early observability or not, the project result will never be verifiable. Thus, the problem of non-verifiability is given in all cases.

We assume that the value of μ does not depend on the realized state of the world. This means that early observability is as likely in state $s_1 = 1$ as in state $s_1 = 0$. The incumbent has to undertake the action in the first period before he knows whether the voters will be able to observe the realized state in period 1.

Some remarks about our informational assumptions are in order here. We model a

because of their daily interaction. However, a candidate cannot use his knowledge about the type of his opponent in his election campaign, since he is not able to credibly communicate this information.

[5]Verification means that it can be proved in a court of law.

[6]The assumption that $\mu \leq \frac{1}{2}$ is not crucial for our qualitative results. It is only of importance for our quantitative welfare analysis in Appendix D.

situation where politicians obtain information earlier than voters. At the time the policy is undertaken, the incumbent can precisely identify the correct state of the world, while voters are still completely ignorant. Voters will observe the state of the world at a later point in time. If voters only observe the realized state in period 2, they do not know whether the incumbent has undertaken the socially optimal action at the time of their first reelection decision.

2.3 Reelection Schemes

Voters are able to observe the realized state in period 1 with a probability of μ. In this case they know whether the politician has undertaken the socially optimal action, and we assume that they will reelect the incumbent if $a_1 = s_1$, while they will deselect him if $a_1 \neq s_1$.[7] If voters are not able to observe the state of the world in period 1, which happens with a probability of $1-\mu$, they do not know whether the incumbent has acted congruently. Voters will reelect the politician if $a_1 = 1$, while they will deselect him if $a_1 = 0$, as $a_1 = 1$ is the action that is more likely to be correct.[8] We use r_1 to denote reelection probability for the incumbent after his first period in office. When politicians undertake their actions, their beliefs regarding reelection are given as

$$r_1 = \begin{cases} \mu + (1-\mu) = 1 & \text{if } a_1 = 1, s_1 = 1 \\ 0 & \text{if } a_1 = 0, s_1 = 1 \\ 1-\mu & \text{if } a_1 = 1, s_1 = 0 \\ \mu & \text{if } a_1 = 0, s_1 = 0 \end{cases} \tag{1}$$

We assume that reelection probability at the end of period 2 depends only on the outcomes realized in period 2 from the policy action undertaken in period 1. Further policy actions during the second term are assumed to be irrelevant for reelection chances at the end of period 2. This assumption greatly simplifies our analysis and can be justified in several ways. First, if the politician undertakes only long-term policies in the second period, then no new information may be available at the end of the second period when

[7]Note that voters are indifferent between reelection schemes, as the politician will undertake no further action during his second or third term in office. The retrospective voting scheme used in this paper is an optimal response of voters in our simple model and hence an equilibrium outcome. Retrospective voting is a particular resolution of the indifference of voters creating the highest possible disciplining device. The voting behavior can be further justified as a unique equilibrium outcome when we allow for an arbitrarily small amount of reciprocity. This justification has been developed by Hahn (2009). Of course, retrospective voting is a polar case and thus highlights the trade-offs the politician faces.

[8]Again, retrospective voting is a best response of voters.

the second reelection decision takes place. Second, the policy actions during his second term in office may be much less relevant than the first-period choices, so the performance of his policy depends only on his first-period action. Later we will extend our model to cover the case where the incumbent has to undertake further actions and discuss how this influences our result.

We use r_2 to denote the reelection probability for the incumbent at the end of period 2, and we assume that voters will reelect the incumbent if and only if he has acted congruently. This means that both types of politician are deselected with certainty after the second period at the latest if they behaved dissonantly in the first period, while both types of politicians are reelected with certainty at the end of the second period[9] if they behaved congruently in the first period. Thus, the beliefs of the politicians regarding reelection at the end of period 2 are given as:

$$r_2 = \begin{cases} 1 & \text{if } a_1 = 1 \text{ and } s_1 = 1 \text{ or if } a_1 = 0 \text{ and } s_1 = 0 \\ 0 & \text{if } a_1 = 1 \text{ and } s_1 = 0 \text{ or if } a_1 = 0 \text{ and } s_1 = 1 \end{cases} \tag{2}$$

2.4 Preferences of Politicians

The elected politician has personal benefits R from being in office. Furthermore, he obtains a private benefit or personal satisfaction G if he undertakes the action that is optimal for himself. This benefit G accrues to the politician in the period in which he performs the action.[10] We assume that the candidate receives no utility from the realization of his preferred action if another politician undertakes the action.[11] We use δ with $0 < \delta \leq 1$ to denote the discount factor for the politician. The utility of the politician in office is

[9]Note that it is possible that a politician who behaved congruently in his first term may be ousted from office by the voters when they make their first reelection decision.

[10]It may be useful to think that the action is irreversible, e.g. investment in public infrastructure, such that it cannot be overturned by a future office holder.

[11]We might also assume that the politician receives the same utility as an ordinary voter if his opponent performs the action. However, this assumption may be less plausible in the case of a dissonant politician. At all events, the results of our analysis are not affected as long as the value of G is sufficiently large in comparison to the utility of ordinary voters.

denoted by U^P and given by

$$U^P = R + r_1[\delta R + r_2\delta^2 R] + \begin{cases} G & \text{if a congruent politician acts congruently} \\ G & \text{if a dissonant politician acts dissonantly} \\ 0 & \text{if a congruent politician acts dissonantly} \\ 0 & \text{if a dissonant politician acts congruently} \end{cases} \tag{3}$$

where r_1 is given by equation (1) and r_2 is given by equation (2). Some examples will illustrate the point. An elected politician who is congruent has utility $R + (1-\mu)\delta R$ if he chooses $a_1 = 1$ in state $s_1 = 0$, while his utility is $R + G + \mu[\delta R + \delta^2 R]$ if he chooses $a_1 = 0$ in state $s_1 = 0$. A politician of the dissonant type has utility $R + G + (1-\mu)\delta R$ if he chooses $a_1 = 1$ in state $s_1 = 0$, while his utility is $R + [\delta R + \delta^2 R]$ if he chooses $a_1 = 1$ in state $s_1 = 1$.

We now need to examine the circumstances under which the elected politician will act congruently. Obviously, it is always optimal for the voters if the incumbent behaves congruently. We will use the following tie-breaking rule: If the elected politician is indifferent as to the two actions, he will undertake the action that is optimal for the voters.

2.5 Summary and Welfare Criterion

The timing of the whole game in its basic version is summarized in the following figure:

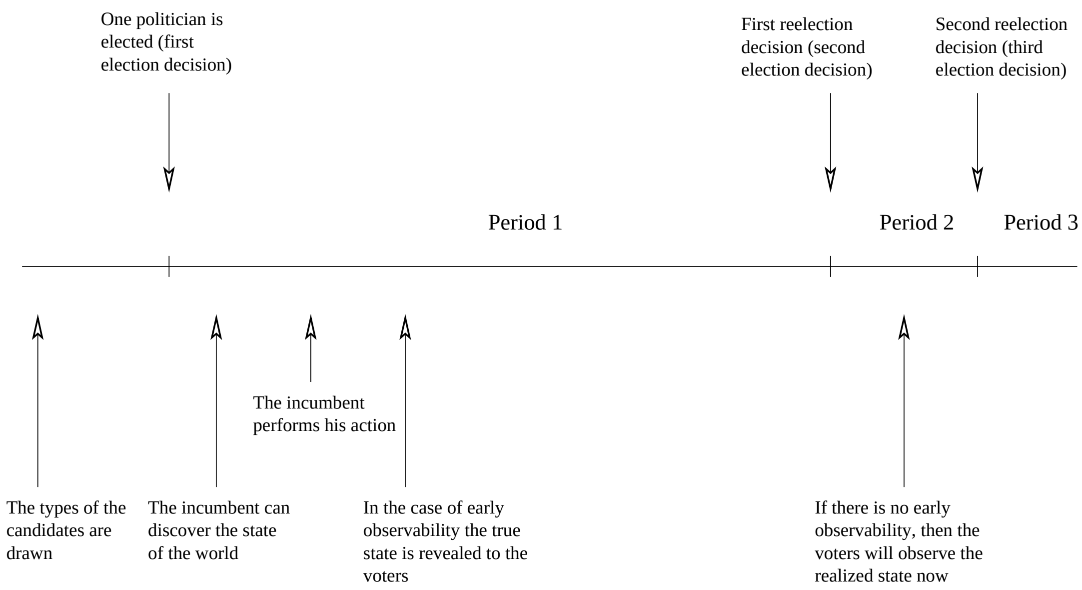

Figure 1

The welfare criterion we adopt is the expected utility of voters at the time when the first election starts. Maximization of voters' utility is equivalent to the maximization of the likelihood that the correct action is undertaken.[12]

3 Elections Only

In this section we consider the behavior of both types of politicians in the scenario without threshold contracts and information markets. Here elections are the only instrument used to discipline the incumbent.

3.1 Behavior of Dissonant Politicians

We first look at case $s_1 = 1$, where the popular action is optimal from the voters' point of view but the politician would prefer the unpopular action. The dissonant politician will only undertake the socially optimal action if

$$\begin{aligned} R + \delta R + \delta^2 R &\geq R + G \\ \Leftrightarrow \quad \delta R(1+\delta) &\geq G. \end{aligned} \tag{4}$$

Condition (4) will be violated if the personal gain from choosing the individually optimal action is sufficiently larger than the gains from holding office.

We next examine $s_1 = 0$. Here voters prefer the unpopular action while the politician prefers the popular action. The dissonant politician will only undertake the socially optimal action if

$$\begin{aligned} R + \mu(\delta R + \delta^2 R) &\geq R + G + (1-\mu)\delta R \\ \Leftrightarrow \quad \delta R(2\mu + \delta\mu - 1) &\geq G. \end{aligned} \tag{5}$$

This condition can only be fulfilled for certain values of δ and μ, as (5) cannot be satisfied if $(2\mu + \delta\mu - 1)$ is not positive. Note that $(2\mu + \delta\mu - 1)$ is monotonically increasing in δ. For $\delta = 1$ the condition $(2\mu + \delta\mu - 1) > 0$ is equivalent to $\mu > \frac{1}{3}$. This means that, even in the case of $\delta = 1$ (which is the value of δ that makes the condition most likely to be fulfilled), it is only possible to fulfill equation (5) for $\frac{1}{3} < \mu < \frac{1}{2}$. Hence, there are large

[12] As we have a continuum of voters, we neglect the utility of the politician in aggregate welfare.

parameter ranges where a dissonant politician cannot be motivated to perform the socially optimal action if the unpopular state has occurred. In particular, this will not be possible if the probability of early observation by voters is small, as reflected in a low value for μ. Further, it is obvious that condition (4) is easier to fulfill than condition (5).

Finally, we obtain the following intuitive results. If the parameters are such that condition (4) is fulfilled while condition (5) is not fulfilled, then there will be a distortion in favor of the popular action $a_1 = 1$. If neither condition (4) nor condition (5) are fulfilled, then there will be a distortion in favor of the unpopular action $a_1 = 0$.[13] It is useful to summarize the key observations in the following Proposition.

Proposition 1

Dissonant politicians will not choose the socially optimal action

(i) if $s_1 = 1$ *and* $\delta R(1+\delta) < G$ *or*

(ii) if $s_1 = 0$ *and* $\delta R(2\mu + \delta\mu - 1) < G$.

Three particularly interesting special cases of Proposition 1 are summarized in the following Corollary:

Corollary 1

Suppose $\delta = 1$. *A dissonant politician will not choose the socially optimal action,*

A) if $s_1 = 1$ *has occurred and* $G > 2R$ *or*

B) if $s_1 = 0$ *has occurred and* $G > \frac{1}{2}R$ *or*

C) if $s_1 = 0$ *has occurred and* $\mu < \frac{1}{3}$.

Note that $\delta = 1$ is most favorable for the public. If a dissonant incumbent cannot be motivated to act congruently in case $\delta = 1$, then it will never be possible.

[13] Note that $z > \frac{1}{2}$, so – under the assumption that neither (4) nor (5) are fulfilled – the probability that the incumbent will undertake $a_1 = 0$ in a situation where he should perform $a_1 = 1$ is higher than the probability for undertaking $a_1 = 1$ instead of the socially optimal action $a_1 = 0$.

3.2 Behavior of Congruent Politicians

The congruent politician will undertake the socially optimal action in state $s_1 = 1$ if

$$R + G + \delta R + \delta^2 R \geq R. \tag{6}$$

This condition is always fulfilled, which means that, in this state of the world, congruent politicians always undertake the socially optimal action, as both voters and the politician prefer the popular action.

We now look at case $s_1 = 0$, meaning that voters and the politician prefer the unpopular action. The congruent politician will only undertake the optimal action for the voters if

$$\begin{aligned} R + G + \mu(\delta R + \delta^2 R) &\geq R + (1-\mu)\delta R \\ \Leftrightarrow \quad G + \delta R(2\mu + \delta\mu - 1) &\geq 0. \end{aligned} \tag{7}$$

In contrast to the case of $s_1 = 1$, it may now be the case that even a congruent politician will not undertake the socially optimal policy, although he too would prefer this policy, since the socially optimal action is unpopular but the politician would like to be reelected. This condition resembles equation (5), but now G is on the left-hand side because a congruent politician receives personal benefits G by acting congruently, while a dissonant politician receives G by acting dissonantly. Hence, if condition (5) is fulfilled, then condition (7) will also hold. Obviously, if it is possible to motivate a dissonant politician to undertake the socially optimal action, then it is always possible to motivate a congruent politician to undertake the socially optimal action. Clearly, the reverse is not true. Furthermore, we have a distortion in favor of the popular action, given that it is possible for $a_1 = 1$ to be chosen too often, while the incumbent may not always carry out the unpopular action $a_1 = 0$ when he should. We summarize the results in the following Proposition:

Proposition 2

A politician of the congruent type will not undertake the socially optimal action if $s_1 = 0$ *and* $G + \delta R(2\mu + \delta\mu - 1) < 0$.

4 The Triple Mechanism

We now introduce reelection threshold contracts and analyze their effect on the behavior of politicians and on social welfare. We assume that there exists a political information market that yields a price predicting the reelection chances of the incumbent in the second reelection decision. Investors receive private signals about which state of the world has occurred, and information is aggregated in the information market.

In Appendix B we provide a detailed microfoundation of how prices are formed in this information market, and how the information market enters and affects the political process. The basic result is that the equilibrium price p^* in the information market will be higher if the incumbent undertakes the socially optimal action, as choosing the optimal action ensures his success in the second reelection decision. In Appendix B we prove the following result:

Proposition 3 (short version)

If the signals of investors are sufficiently informative, then the equilibrium price on the information market is larger than one-half if the incumbent undertakes the action that is socially optimal, while it is smaller than one-half if the incumbent chooses the socially undesirable action.

The detailed version of Proposition 3 and its proof can be found in Appendix B.

4.1 Reelection Thresholds

Before the first period starts, politician i can offer conditional reelection threshold contracts $C_i(p_i^1, p_i^0)$ with $0 \leq p_i^1 \leq 1$ and $0 \leq p_i^0 \leq 1$, which means that the incumbent will only be allowed to stand for reelection after the first period if the price p^* on a political information market fulfills the condition

$$p^* \geq \begin{cases} p_i^1 & \text{if } a_1 = 1 \\ p_i^0 & \text{if } a_1 = 0, \end{cases}$$

where p_i^1 is the threshold price if the incumbent undertakes $a_1 = 1$ and p_i^0 is the threshold price if he chooses $a_1 = 0$. As the action of the politician is observable and verifiable, politicians can condition the threshold prices on the action, therefore p_i^1 and p_i^0 may differ. Note that a contract with $p_i^1 = p_i^0 = 0$ is equivalent to offering no contract at all.

4.2 Reelection Schemes

Reelection schemes are given by equation (1) for the first reelection and by equation (2) for the second reelection.[14] Recall from equation (1) that the scheme for the first reelection is such that a politician will always be deselected if he acts dissonantly in state $s_1 = 1$. Thus, threshold contracts will have no effect in state $s_1 = 1$, as in this state the reelection scheme from equation (1) effectively deters the politician from acting dissonantly. Adding threshold contracts prohibiting a politician who has behaved dissonantly from running for reelection will not change the results, as the politician would not be reelected anyway. By contrast, threshold contracts will have a positive effect in state $s_1 = 0$. As a consequence, only the threshold price p_i^1 will impact on the behavior of the politician, as dissonant behavior in state $s_1 = 0$ means choosing $a_1 = 1$ and thus p_i^1 applies.

4.3 Summary

The timing of the whole game including threshold contracts and political information markets is summarized in the following figure:

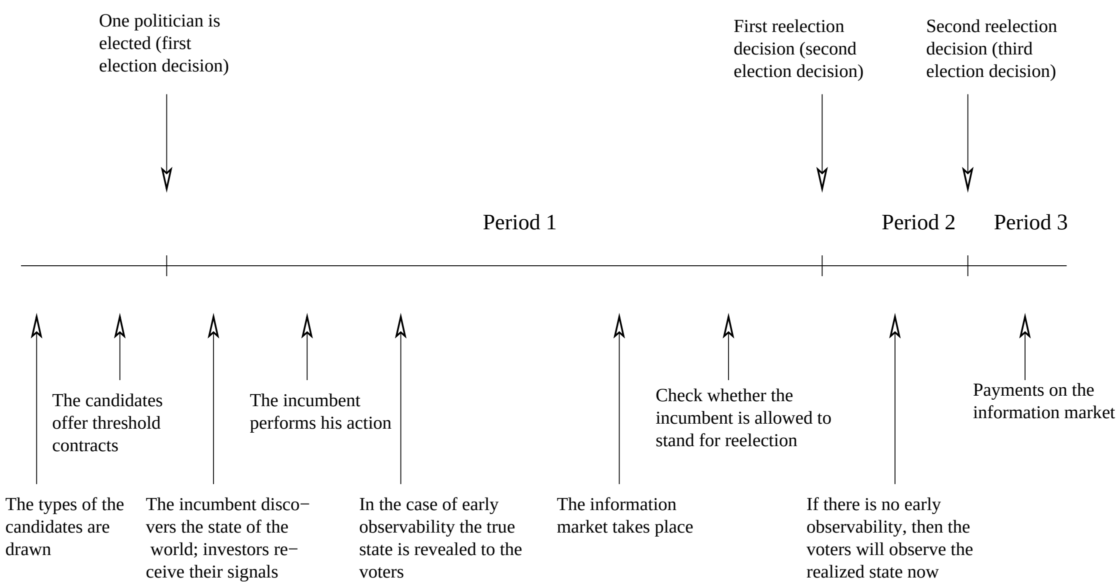

Figure 2

[14]If information markets are allowed and actually used, they might be taken into account by voters when making reelection decisions. Such feedback effects will be discussed in our extensions.

4.4 Robust Election Scheme

We assume that both politicians have to decide simultaneously about offering conditional threshold contracts. Moreover, we assume that voters use the following election scheme, where $e_1(p_1^1, p_1^0, p_2^1, p_2^0)$ denotes the probability of candidate 1 being elected at the first election decision:

$$e_1(p_1^1, p_1^0, p_2^1, p_2^0) = \begin{cases} 1 & \text{if } p_1^k \geq \frac{1}{2} \ \forall k \in \{0,1\} \text{ and } \exists l \in \{0,1\} : p_2^l < \frac{1}{2}, \\ 1 & \text{if } \exists k \in \{0,1\} : p_1^k \geq \frac{1}{2} \text{ and } p_2^l < \frac{1}{2} \ \forall l \in \{0,1\}, \\ 0 & \text{if } p_1^k < \frac{1}{2} \ \forall k \in \{0,1\} \text{ and } \exists l \in \{0,1\} : p_2^l \geq \frac{1}{2}, \\ 0 & \text{if } \exists k \in \{0,1\} : p_1^k < \frac{1}{2} \text{ and } p_2^l \geq \frac{1}{2} \ \forall l \in \{0,1\}, \\ \frac{1}{2} & \text{otherwise.} \end{cases} \tag{8}$$

We call this voting scheme *robust election scheme* (*RES*). The idea behind it is the following: Voters will elect a politician if and only if the threshold offers indicate that the politician will choose the socially optimal action, i.e. if $p \geq \frac{1}{2}$.[15] The precise values of p do not matter. Under *RES* a politician is elected with certainty if he offers prices for both actions that are equal to or above $\frac{1}{2}$ if the other politician does not do the same. If both candidates offer threshold values that are qualitatively similar with regard to the comparison to $\frac{1}{2}$, then both candidates will be elected with a probability of one-half. Later we will show that the assumptions of the voters in equilibrium are correct regarding the behavior of politicians. Accordingly, we call this an optimal voting scheme.

4.5 Equilibrium Notion

We are looking for perfect Bayesian equilibria of the game depicted in Figure 2 among politicians and investors. Voters are not highly sophisticated players. They vote according to *RES*, as described above, and to the reelection schemes given in equations (1) and (2). Henceforth a Bayesian equilibrium will simply be called "equilibrium". The entire game is solved by assuming *RES* and the property that a price equal to or above $\frac{1}{2}$ indicates that the politician has chosen the optimal action, while a price below $\frac{1}{2}$ indicates the opposite. The optimality of *RES* will be shown later. The property of the prices in the information market is established in Appendix B. There we show that sophisticated investors use their private signals and their updated beliefs from the signalling subgame when politicians

[15] Recall that the equilibrium price on the information market will be larger than one-half if and only if the incumbent undertakes the action that is socially optimal.

choose their action to trade on the information market. The equilibrium price indicates whether the office-holder has chosen the socially desirable action.

4.6 Equilibria

In this subsection we examine equilibria that involve robust election schemes. It is important to note that, when threshold contracts are offered, politicians do not know which state of the world will occur. We use the following plausible refinement:

Minimal Price Offer (MPO)

If a candidate is indifferent between two sets of prices for p_i^1 and p_i^0 given the contract choice of the other politician and RES, then he will choose the contract with the minimal values for p_i^1 and p_i^0 in the corresponding sets.

A formal description of MPO is as follows: Suppose a politician is indifferent between $C_i(p_i^1, p_i^0)$ and $\tilde{C}_i(\tilde{p}_i^1, \tilde{p}_i^0)$. Then he will choose $C_i(p_i^1, p_i^0)$ if $p_i^k \leq \tilde{p}_i^k \ \forall k \in \{0,1\}$ and $\exists l \in \{0,1\} : p_i^l < \tilde{p}_i^l$, but $\tilde{C}_i(\tilde{p}_i^1, \tilde{p}_i^0)$ if $p_i^k \geq \tilde{p}_i^k \ \forall k \in \{0,1\}$ and $\exists l \in \{0,1\} : p_i^l > \tilde{p}_i^l$.

The refinement can be justified by the fact that the likelihood of fulfilling a given threshold is non-increasing in the value of the prices.[16] Through the observation that the utility of an elected politician weakly decreases in his price offers we obtain the following Lemma:

Lemma 1

Under MPO and RES, equilibrium price offers satisfy $p_i^k \leq \frac{1}{2} \ \forall k \in \{0,1\}, i = 1,2$.

Next, as a consequence of Lemma 1 and RES, we can restrict ourselves to four cases:

(i) $p_i^1 = \frac{1}{2}$, $p_i^0 = \frac{1}{2}$

(ii) $p_i^1 = \frac{1}{2}$, $p_i^0 < \frac{1}{2}$

(iii) $p_i^1 < \frac{1}{2}$, $p_i^0 = \frac{1}{2}$

[16] MPO can be justified by arbitrarily small errors of investors. Suppose there is a possibility of such errors by investors. Then, the probability of fulfilling a given threshold is strictly decreasing in prices. Without MPO, other prices than $\frac{1}{2}$ in threshold contracts can emerge in equilibrium in Proposition 4. The implications for the behavior of office holders, however, are the same.

(iv) $p_i^1 < \frac{1}{2}, p_i^0 < \frac{1}{2}$

As only the threshold price p_i^1 will impact on the behavior of the incumbent, we thus obtain the following Lemma:

Lemma 2

A) Cases (i) and (ii) induce the same behavior by an elected politician.

B) Cases (iii) and (iv) induce the same behavior by an elected politician.

In Appendix A we show

Proposition 4

Both politicians will offer threshold contracts $C_i(p_i^1 = p_i^0 = \frac{1}{2})$ under election scheme RES, irrespective of their own type and irrespective of the type of their opponent.

Given this result of Proposition 4, we next show that the voting behavior of the *RES* is indeed optimal:

Proposition 5

The robust election scheme (RES) is optimal for voters.

The proof is given in Appendix A. The strength of *RES* is that voters do not need to have specific information regarding the parameters of projects or the signals of investors in the information market. They simply judge whether politicians are willing to compete against a fair coin. The next Proposition is our main result.

Proposition 6

The conditions under which politicians in state $s_1 = 0$ behave congruently with threshold contracts are less strict, and dissonant behavior is less attractive, than without threshold contracts. This holds for both types of politicians. In particular, with the triple mechanism we obtain:

(i) A dissonant politician acts congruently in state $s_1 = 1$ if and only if $\delta R(1+\delta) \geq G$.

(ii) A dissonant politician acts congruently in state $s_1 = 0$ if and only if $\delta R\mu(1+\delta) \geq G$.

(iii) A congruent politician always behaves congruently in both states.

The proof is given in Appendix A. The intuition is as follows: Given equilibrium threshold contracts $C_i(p_i^1 = \frac{1}{2})$, politicians have no chance of being reelected in state $s_1 = 0$ if they behave dissonantly, i.e. if they undertake $a_1 = 1$. If they behave congruently, their reelection chances are given by probability μ. If threshold contracts are absent, a politician who behaves dissonantly still has a chance to get reelected, while congruent behavior does not yield higher reelection probabilities than μ. Hence, threshold contracts make dissonant behavior in state $s_1 = 0$ less attractive than congruent behavior.

In Appendix D we provide a brief example of the welfare gains that can be achieved with the triple mechanism. The example illustrates, among other things, that threshold contracts have the largest effect on welfare when R is larger than G and when the probability of the unpopular state $s_1 = 0$ is rather high (i.e. z close to $\frac{1}{2}$).

5 Extensions, Robustness and Pitfalls

In the following we discuss several extensions of the model, thereby exploring the robustness and potential pitfalls of the triple mechanism.

5.1 Monotonic Election Scheme and Overpromising

As we showed in Proposition 5 the robust election scheme used in the last section is optimal for voters. However, it is not clear whether the scheme is unique. In this section we consider further candidates for election schemes. We start with a simple and intuitive scheme that we call the *monotonic election scheme* (*MES*):

$$e_1(p_1^1, p_1^0, p_2^1, p_2^0) = \begin{cases} 1 & \text{if } p_1^k \geq p_2^k \ \forall k \in \{0,1\} \text{ and } \exists k \in \{0,1\} : p_1^k > p_2^k, \\ 0 & \text{if } p_1^k \leq p_2^k \ \forall k \in \{0,1\} \text{ and } \exists k \in \{0,1\} : p_1^k < p_2^k, \\ \frac{1}{2} & \text{otherwise.} \end{cases}$$

The *MES* is intuitive in the sense that voters simply elect the candidate who offers tighter constraints on his reelection thresholds. The problem is, however, that *overpromising* may occur under *MES*. We call a threshold contract with prices p^1, p^0 overpromising if at the date of the offer the politician already knows that at least one of the thresholds can never be reached. Such overpromising may occur if it is more profitable for a politician to be elected with certainty in the first election and to be certainly not reelected in the next election, in comparison to being elected with probability $\frac{1}{2}$ in the first election and having a positive reelection probability. In Appendix A we show

Proposition 7

Under the monotonic election scheme, overpromising may occur.

Overpromising invites extreme short-termism, where both types of politicians simply behave in accordance with their first-period preferences and maximize their first period utility. In the case of overpromising, dissonant politicians will always behave dissonantly, while a congruent politician will behave congruently.[17] Hence, the monotonic election scheme is not optimal and thus is not an equilibrium response by voters.

5.2 Sophisticated Election Scheme

We next examine a voting scheme which we call the *sophisticated election scheme* (*SES*):

$$e_1(p_1^1, p_1^0, p_2^1, p_2^0) = \begin{cases} 1 & \text{if } p_1^1 \geq z, \ \ p_1^0 \geq 1-z, \ \text{ and } \ p_2^1 < z \ \text{ or } \ p_2^0 < 1-z, \\ 1 & \text{if } p_2^1 < z, \ \ p_2^0 < 1-z, \ \text{ and } \ p_1^1 \geq z \ \text{ or } \ p_1^0 \geq 1-z, \\ 0 & \text{if } p_1^1 < z, \ \ p_1^0 < 1-z, \ \text{ and } \ p_2^1 \geq z \ \text{ or } \ p_2^0 \geq 1-z, \\ 0 & \text{if } p_2^1 \geq z, \ \ p_2^0 \geq 1-z, \ \text{ and } \ p_1^1 < z \ \text{ or } \ p_1^0 < 1-z, \\ \frac{1}{2} & \text{otherwise.} \end{cases} \tag{9}$$

This voting scheme is similar to *RES* but the critical values are not $\frac{1}{2}$ but z for action $a_1 = 1$ and $1 - z$ for action $a_1 = 0$. This reflects the fact that the a priori probability for $s_1 = 1$ (where $a_1 = 1$ is socially optimal) is z and for $s_1 = 0$ (where $a_1 = 0$ is socially optimal) is $1 - z$. With a sophisticated election scheme, voters demand that the prices on the information market at least reach the a priori probabilities. In this voting scheme voters use the following result:

[17] It is obvious that overpromising is socially detrimental in the case of dissonant politicians. If the incumbent is congruent, there will be no immediate negative effect on social welfare. However, as a congruent incumbent who overpromises will be replaced by a new politician who can either be congruent or dissonant, overpromising by congruent politicians would have negative effects on social welfare in an extended version of the model, where the incumbent would undertake further action in periods 2 or 3.

Proposition 8 (short version)

If the signals of investors in the information market are sufficiently informative, then the following holds:

(i) If the incumbent undertakes $a_1 = 1$, then the price on the information market will be larger than z if $s_1 = 1$ and smaller than z if $s_1 = 0$.

(ii) If the incumbent chooses $a_1 = 0$, then the price on the information market will exceed $1-z$ if $s_1 = 0$, while the price will be below $1-z$ if $s_1 = 1$.

The detailed version of Proposition 8 and its proof can be found in Appendix B. Furthermore, one can show that under *SES* both politicians will offer $C_i(p_i^1 = z, p_i^0 = 1-z)$ and that *SES* is also optimal for voters. The proof follows the same lines as the proof of Propositions 4 and 5 and is therefore omitted here. Note that, with a sophisticated election scheme, voters anticipate that the market price will be higher under congruent behavior of the politician in state $s_1 = 1$ than under congruent behavior of the politician in state $s_1 = 0$. While both *RES* and *SES* are optimal for voters, we will show in Corollary 2 of Appendix B that the conditions for Proposition 8 to hold are weaker than the conditions required for Proposition 3 to hold.

5.3 Market-Based Voting

In our basic model we have assumed that the price on the information market has no influence on reelection probability. Now we assume the other polar case, where voters only use the price on the information market as a basis for their first reelection decision. In this case, threshold contracts are without effect (either positive or negative). The existence of a political information market that predicts the reelection chances after the next term is sufficient to generate all efficiency gains when voters solely use information markets as their forward-looking reelection scheme.[18] The reason is that the price on the information market is the best predictor regarding the quality of the decisions of the politician. Purely market-based voting is a polar case. It is likely that actual voting will be between both polar cases (no market-based voting and purely market-based voting). Then, reelection threshold contracts will continue to have beneficial welfare effects.

[18]The same would occur if there existed other means that perfectly aggregate the information of investors.

5.4 Repeated Action

Another potential extension is to examine repeated actions by the politician. Suppose that the incumbent stays in office as long as he gets reelected, that he undertakes an action a_t in each period t in office,[19] and that the candidates are allowed to offer threshold contracts before each election. In the precursor of this paper (Gersbach and Müller (2006)), we have shown that the results of the two-period case still hold when actions are repeated. In particular, threshold contracts are always socially advantageous compared to elections alone, since the probability of a politician behaving congruently is higher when threshold contracts are used.

5.5 More Candidates

Our analysis can be extended to more than two candidates running for office in the first election. The election probability of a candidate would decrease accordingly and would amount to $\frac{1}{n}$ where n is the number of candidates. Equilibrium threshold contract offers and behavior of office holders, as well as the price behavior in the information market, would remain the same.

5.6 Manipulations

A serious concern is manipulation. The incumbent might try to push the equilibrium price above the price in his threshold contract via trading in such markets.[20] The most obvious way to prevent manipulation is to prohibit trading by politicians and to punish the use of stooges. However, such prohibitions may be not sufficient to prevent manipulations. A robust possibility to ensure that the incumbent is not interested in manipulating the market is to use an average price calculated over a longer time-span. One can use the entire time span between the action of the incumbent and the end of the first term to operate the information market. An incumbent who wants to raise the average market price above his threshold price via manipulation would be forced to manipulate the price

[19] We assume that the politician will undertake no action in the last two periods, which corresponds to our assumption in the basic model where the politician does not take any action in the second and third periods.

[20] Rhode and Strumpf (2004) discuss several historical manipulations episodes and provide important insights how these can be engineered.

in the information market every day over several years, which would become very costly over time.

6 Conclusion

In this paper we have suggested a triple mechanism for improving the functioning of democracies when information is not observable or not verifiable. The results seem to be quite robust for various extensions. Moreover, the idea of the triple mechanism could be extended to multi-task settings, where the politician decides on many issues in his first term. As the threshold contract depends on the average long-term performance of the politician, the standard problem may aggravate distortions in favor of tasks with better observability.

Political information markets are an instrument for solving the problem of short-term unobservability coupled with long-term non-verifiability. Hence threshold contracts combined with information markets can be used successfully when projects have long-term effects and information on project results is not available in the short term. Of course, any suggestion of a new institution such as the one made in this paper has to be subjected to further scrutiny.[21] Such scrutiny will be undertaken in our future research work.

[21] One might, for example, wonder how the triple mechanism could be introduced. The best way to try to implement the triple mechanism is political competition. If one candidate proposes the idea, then competing candidates are forced to offer the same in order to avoid losing votes, as the triple mechanism is welfare-improving.

A Appendix A: Proofs

Proof of Proposition 4

Suppose that voters use the robust election scheme RES. Both candidates decide simultaneously about their threshold contracts. We show that $C_i(p_i^1 = p_i^0 = \frac{1}{2})$ for $i = 1,2$ is the unique equilibrium of the politician's contract choice, given that voters will use RES.

Step 1: Given that candidate $g \in \{1,2\}$ offers $C_g(p_g^1 = p_g^0 = \frac{1}{2})$, politician $h \neq g$, $h \in \{1,2\}$ will not offer $p_h^k < \frac{1}{2}$ for any $k \in \{0,1\}$, since he would have no chance of winning the election. Furthermore, he has no incentive to offer $p_h^k > \frac{1}{2}$ for any $k \in \{0,1\}$, since this does not increase his chances of winning the election. Thus, given that candidate g offers $C_g(p_g^1 = p_g^0 = \frac{1}{2})$, a best response for candidate h is to offer $C_h(p_h^1 = p_h^0 = \frac{1}{2})$, independently of his type. Hence, offering $C_i(p_i^1 = p_i^0 = \frac{1}{2})$ $\forall i \in \{1,2\}$ is an equilibrium. In the next steps we show that it is unique.

Step 2: We know from Lemma 1 that $p_i^k \leq \frac{1}{2}$ $\forall k \in \{0,1\}, i = 1,2$, so we only have to examine whether there may exist other equilibria with threshold offers below $\frac{1}{2}$. Suppose that candidate g offers a contract with $p_g^k \leq \frac{1}{2} \forall k \in \{0,1\}$ and $p_g^k < \frac{1}{2}$ for at least one $k \in \{0,1\}$. We distinguish three cases:

Step 2a: First, consider a constellation with candidates g and h offering contracts $C_g(p_g^1 < \frac{1}{2}, p_g^0 < \frac{1}{2})$ and $C_h(p_h^1 < \frac{1}{2}, p_h^0 < \frac{1}{2})$. Then candidate h has an incentive to deviate by offering $C_h(p_h^1 < \frac{1}{2}, p_h^0 = \frac{1}{2})$, as his election chances are higher with offer $C_h(p_h^1 < \frac{1}{2}, p_h^0 = \frac{1}{2})$ and because offering $p_h^0 = \frac{1}{2}$ does not reduce the reelection chances of h, whether he behaves congruently or dissonantly.[22]

Step 2b: Consider next a constellation with candidates g and h offering contracts $C_g(p_g^1 = \frac{1}{2}, p_g^0 < \frac{1}{2})$ and $C_h(p_h^1 = \frac{1}{2}, p_h^0 < \frac{1}{2})$. Then candidate h can profitably deviate by offering $C_h(p_h^1 = \frac{1}{2}, p_h^0 = \frac{1}{2})$ as – according to Lemma 2 – this induces the same behavior and gives the same reelection chances, while increasing the election chances from $\frac{1}{2}$ to 1.

Step 2c: We are left with the optimal response of politician $h \neq g$ if candidate g offers $C_g(p_g^1 < \frac{1}{2}, p_g^0 = \frac{1}{2})$. There are two possibilities for optimal responses: $C_h(p_h^1 = p_h^0 = \frac{1}{2})$ and $C_h(p_h^1 < \frac{1}{2}, p_h^0 = \frac{1}{2})$. We show now that both types of politicians will prefer to offer $C_h(p_h^1 = p_h^0 = \frac{1}{2})$ rather than $C_h(p_h^1 < \frac{1}{2}, p_h^0 = \frac{1}{2})$ in response to a contract $C_g(p_g^1 < \frac{1}{2}, p_g^0 = \frac{1}{2})$. Suppose that a candidate, say candidate 2, offers $C_2(p_2^1 < \frac{1}{2}, p_2^0 = \frac{1}{2})$.

We consider candidate 1 and assume first that he is of the congruent type. If a congruent politician offers a contract with $p_1^1 = p_1^0 = \frac{1}{2}$ and gets elected, then he will always behave congruently.[23] If a congruent politician offers a contract with $p_1^1 < \frac{1}{2}$ and $p_1^0 = \frac{1}{2}$ and gets elected,[24] then his behavior in state $s_1 = 0$ will depend on whether

[22] Recall that only threshold p_i^1 can affect the reelection chances of the incumbent.

[23] This is obvious in state $s_1 = 1$. In state $s_1 = 0$, the politician has utility $R + G + \mu[\delta R + \delta^2 R]$ when he behaves congruently and utility R when be behaves dissonantly. Hence, the politician will always behave congruently. Closer reasoning will be given in Proposition 6.

[24] Note that in this case the election probability is only $\frac{1}{2}$.

$R+G+\mu[\delta R+\delta^2 R]$ is larger or smaller than $(R+(1-\mu)\delta R)$. Candidate 1 is better off by choosing $p_1^1 = p_1^0 = \frac{1}{2}$ if

$$\begin{array}{rcl} z[R+G+\delta R+\delta^2 R] & + & (1-z)\{R+G+\mu[\delta R+\delta^2 R]\} \\ & \geq & \\ \frac{1}{2}z[R+G+\delta R+\delta^2 R] & + & \frac{1}{2}(1-z)\max\{R+G+\mu[\delta R+\delta^2 R]; R+(1-\mu)\delta R\}. \end{array} \tag{10}$$

To analyze this inequality, we consider the two possible cases, starting with $R+G+\mu[\delta R+\delta^2 R] \geq (R+(1-\mu)\delta R)$. In this case, inequality (10) simplifies to $1 \geq \frac{1}{2}$ and thus holds. Next we look at $R+G+\mu[\delta R+\delta^2 R] < (R+(1-\mu)\delta R)$. Then inequality (10) can be simplified to

$$\begin{array}{l} \frac{1}{2}z[R+G+\delta R+\delta^2 R]+(1-z)\{R+G+\mu[\delta R+\delta^2 R]\} \\ \qquad \geq \frac{1}{2}(1-z)[R+(1-\mu)\delta R]. \end{array}$$

This condition is always fulfilled because $\frac{1}{2}z[R+\delta R] > \frac{1}{2}(1-z)[R+(1-\mu)\delta R]$ and the other terms on the left hand side of the condition are positive. Thus, a congruent politician 1 will offer a contract with $p_1^1 = p_1^0 = \frac{1}{2}$ in response to $C_2(p_2^1 < \frac{1}{2}, p_2^0 = \frac{1}{2})$.

Next we analyze the behavior of politician 1 if he is dissonant and candidate 2 offers $C_2(p_2^1 < \frac{1}{2}, p_2^0 = \frac{1}{2})$. In contrast to our considerations for congruent politicians above, it is no longer clear this time whether politician 1 will behave congruently or dissonantly. Nevertheless, it still holds that he will offer a contract $C_1(p_1^1 = p_1^0 = \frac{1}{2})$. To substantiate this claim we distinguish four cases:

(i) Suppose candidate 1 is elected and behaves in a dissonant manner regardless of the threshold contract he has offered.[25] Then we obtain

$$EU^1\left(p_1^1 = p_1^0 = \frac{1}{2}\right) = z(R+G)+(1-z)(R+G) = R+G \tag{11}$$

and

$$\begin{array}{rcl} EU^1\left(p_1^1 < \frac{1}{2}, p_1^0 = \frac{1}{2}\right) & = & \frac{1}{2}\{z[R+G]+(1-z)[R+G+(1-\mu)\delta R]\} \\ & = & \frac{1}{2}[R+G+(1-z)(1-\mu)\delta R] < R+\frac{G}{2}, \end{array} \tag{12}$$

where EU^1 denotes the expected utility of politician 1 depending on the contract he has offered. Hence, expected utility will be larger if he offers $p_1^1 = p_1^0 = \frac{1}{2}$.

(ii) Suppose candidate 1 is elected and behaves in a congruent manner, regardless of the threshold contract he has offered.[26] For such circumstances we obtain

$$\begin{array}{rcl} EU^1\left(p_1^1 = p_1^0 = \frac{1}{2}\right) & = & z[R+\delta R+\delta^2 R]+(1-z)[R+\mu(\delta R+\delta^2 R)] \\ & = & R+[z+(1-z)\mu](1+\delta)\delta R \end{array} \tag{13}$$

[25] Intuitively, this will occur if the value of G is sufficiently large.
[26] This will occur if the value of G is sufficiently small.

and

$$EU^1\left(p_1^1 < \frac{1}{2}, p_1^0 = \frac{1}{2}\right) = \frac{1}{2}\{z[R+\delta R+\delta^2 R]+(1-z)[R+\mu(\delta R+\delta^2 R)]\}$$
$$= \frac{1}{2}\{R+[z+(1-z)\mu](1+\delta)\delta R\}. \quad (14)$$

As the expression (13) is larger than the expression in (14), candidate 1 is better off by offering $p_1^1 = p_1^0 = \frac{1}{2}$.

(iii) Suppose candidate 1 is elected and behaves dissonantly with a contract $C_1(p_1^1 = p_1^0 = \frac{1}{2})$ and congruently with a contract $C_1(p_1^1 < \frac{1}{2}, p_1^0 = \frac{1}{2})$. According to equations (12) and (14) acting congruently after having offered $p_1^1 < \frac{1}{2}$ is only optimal if $G < [z+(1-z)\mu](1+\delta)\delta R$. However, for $G < [z+(1-z)\mu](1+\delta)\delta R$ the politician would act congruently after having offered $p_1^1 = \frac{1}{2}$ according to equations (11) and (13). This is a contradiction. Hence, case (iii) cannot occur.

(iv) Suppose candidate 1 is elected and behaves congruently with the contract $C_1(p_1^1 = p_1^0 = \frac{1}{2})$ while behaving dissonantly with $C_1(p_1^1 < \frac{1}{2}, p_1^0 = \frac{1}{2})$. The utility of acting dissonantly with contract $C_1(p_1^1 < \frac{1}{2}, p_1^0 = \frac{1}{2})$ is smaller than the utility of acting dissonantly with contract $C_1(p_1^1 = p_1^0 = \frac{1}{2})$. As we have assumed that the candidate behaves congruently under $C_1(p_1^1 = p_1^0 = \frac{1}{2})$ and thus achieves higher or equal utility than by acting dissonantly, the utility of acting dissonantly with $C_1(p_1^1 < \frac{1}{2}, p_1^0 = \frac{1}{2})$ is smaller than the utility of behaving congruently with contract $C_1(p_1^1 = p_1^0 = \frac{1}{2})$.

Hence, we can conclude that if politician 1 is of the dissonant type, he will always offer a contract $C_1(p_1^1 = p_1^0 = \frac{1}{2})$ given that candidate 2 offers a contract $C_2(p_2^1 < \frac{1}{2}, p_2^0 = \frac{1}{2})$.

To sum up, $C_i(p_i^1 = p_i^0 = \frac{1}{2})$ $\forall i \in \{1,2\}$ is the unique equilibrium under the election scheme *RES*.

■

Proof of Proposition 5

In Proposition 4 we have shown that both politicians will offer $C_i(p_i^1 = p_i^0 = \frac{1}{2})$ if they believe that voters will use *RES*. Now we show that *RES* is optimal for voters.

Proposition 3 in Appendix B shows that the equilibrium price on the information market will be larger than $\frac{1}{2}$ if the incumbent chooses the socially optimal action, while it will be smaller than $\frac{1}{2}$ if the incumbent chooses the socially undesirable action. So *RES* is optimal, as it induces the socially optimal action. Specifically, under *RES* a politician (say $i = 2$) who offers a contract with a price smaller than $\frac{1}{2}$ will never generate a higher utility than a politician who offers thresholds p_1^1 and p_1^0 equal to $\frac{1}{2}$. Thus in this case electing politician 1 can never be worse than electing politician 2. Finally, we note that under *RES* a politician (say $i = 2$) who offers a contract with a threshold strictly larger than $\frac{1}{2}$ will never generate a higher utility than a politician who offers thresholds p_1^1 and p_1^0 equal to $\frac{1}{2}$. In this case, electing politician 1 can never be worse than electing politician 2. This completes the proof.

■

Proof of Proposition 6

We start with dissonant politicians and look first at the case $s_1 = 1$ where the popular action is optimal from the voters' point of view. The politician, however, would prefer the unpopular action. In the scenario with threshold contracts, the dissonant politician will undertake the socially optimal action if and only if

$$\begin{aligned} R + \delta R + \delta^2 R &\geq R + G \\ \Leftrightarrow \quad \delta R(1+\delta) &\geq G. \end{aligned} \tag{15}$$

Comparison with the condition when threshold contracts are absent shows that condition (15) is identical to condition (4). The reason is that threshold contracts have no impact in state $s_1 = 1$.

We next consider the case $s_1 = 0$. In this state, voters prefer the unpopular action, while the dissonant politician prefers the popular action. The dissonant politician will only undertake the socially optimal action if

$$\begin{aligned} R + \mu(\delta R + \delta^2 R) &\geq R + G \\ \Leftrightarrow \quad \delta R \mu(1+\delta) &\geq G. \end{aligned} \tag{16}$$

Comparison with the condition in the scenario without threshold incentive contracts shows that condition (5) is tighter than condition (16), i.e. the set of parameter values fulfilling (16) is larger than the corresponding set for condition (5). For instance, equation (16) is always fulfilled if R is sufficiently high, which is not true in general under condition (5).

Next consider congruent politicians. In case $s_1 = 1$, a congruent politician will undertake the socially optimal action if

$$R + G + \delta R + \delta^2 R \geq R. \tag{17}$$

This condition is always fulfilled. In case $s_1 = 0$, a congruent politician will undertake the optimal action if

$$R + G + \mu(\delta R + \delta^2 R) \geq R. \tag{18}$$

Again, this condition is always fulfilled. Hence, in both states of the world, the politician will always pursue the policy optimal for the voters if he has offered a threshold contract with $p_i^1 = p_i^0 = \frac{1}{2}$. As showed above in equation (7), this is not necessarily true for congruent politicians in the scenario without threshold contracts.

■

Proof of Proposition 7

Suppose that G is sufficiently large relative to R, such that congruent politicians will always act congruently and dissonant politicians will always act dissonantly, irrespective of the threshold contracts they have offered. In Appendix B we show that, for G sufficiently large relative to R, the equilibrium price will be smaller than 1, even if politicians act in a socially optimal way. Thus, if both candidates offered contracts $p_i^1 = p_i^0 = 1$, neither of them would ever be able to fulfill their contract. This is an example of overpromising.

Suppose next that both candidates are of the congruent type. Then no candidate will deviate from a Nash equilibrium $p_i^1 = p_i^0 = 1$, as a deviating candidate would never be elected.

Next we show that the Nash equilibrium $p_i^1 = p_i^0 = 1$ is unique for certain parameters. Suppose both candidates offer threshold contracts with $p_1^1 = p_2^1 < 1$ and $p_1^0 = p_2^0 < 1$. Politicians face the trade-off between offering the largest thresholds that can be reached by acting congruently and deviating from this offer to higher values, thereby increasing election chances to 1. Deviation to higher threshold values is profitable if

$$\begin{aligned} \frac{1}{2}\{z[R+G+\delta R+\delta^2 R]+(1-z)[R+G+\mu(\delta R+\delta^2 R)]\} &< (R+G) \\ \Leftrightarrow R\{[z+\mu(1-z)](\delta+\delta^2)-1\} &< G. \end{aligned} \tag{19}$$

We see that this condition will always be fulfilled if G is sufficiently large relative to R.[27]

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B Appendix B: Political Information Market

In this Appendix we describe in detail the functioning of the political information market. we first describe the assets and the investors. As investors receive information from two sources – the private signals and actions of politicians – we have to examine step by step how both sources of information jointly determine the beliefs of the investors. Finally, we determine the equilibrium price in the market.

B.1 Assets

We assume that a political information market is organized during the first period after politicians have chosen their actions.

There are two assets, D and E, in which investors can trade. If the politician is reelected after the second period, the owners of asset D receive one monetary unit for a single unit of D. If the politician stands for reelection but is not reelected after the second

[27]There exist other constellations where overpromising occurs. Details are available on request.

period, the owners of asset E receive one monetary unit for a single unit of E. If the politician is not able to run for second reelection, e.g. if he was already deselected at the first reelection or if he does not want to stand for reelection, then all transactions that have occurred will be neutralized, i.e. each investor will be paid back the money he has invested. [28]

The information market works as follows: A bank or an issuer offers an equal amount of assets D and E. On the secondary market, traders can buy assets D or E.[29] Trading in the secondary market results in price p for one unit of asset D. As buying one unit of D and one unit of E pays one monetary unit with certainty, the price of asset E must be $1-p$, otherwise either traders or the issuer could make riskless profits. An equilibrium on the information market is a price p^* such that traders demand an equal amount of assets D and E.[30]

It is useful to look more closely at the event tree associated with the assets. If, for example, an investor buys one unit of asset D at price p, then the event tree and the payoffs for the information market are given as:

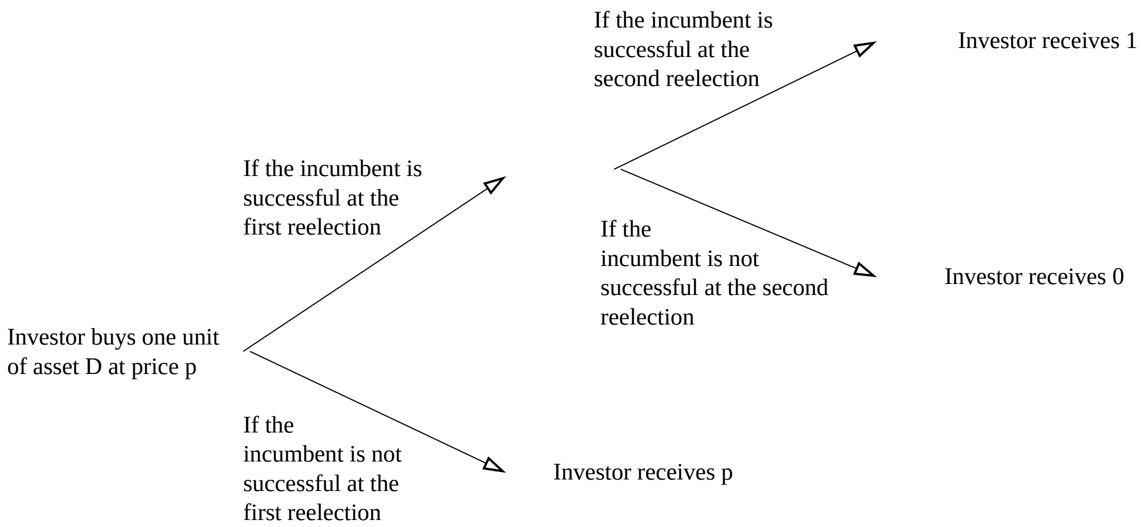

Figure 3

In this paper we specifically design information markets to allow for the design of reelection threshold contracts. If threshold contracts are offered, then the event tree and the payoffs for the information market have to be modified in the following way:

[28] See Berg and Rietz (2003) for alternative ways to implement conditional prediction markets in practice.

[29] We could allow for short-selling, but this is immaterial to our analysis.

[30] This is equivalent to an information market with only asset D where traders can buy or sell D and an equilibrium is obtained when supply equals demand.

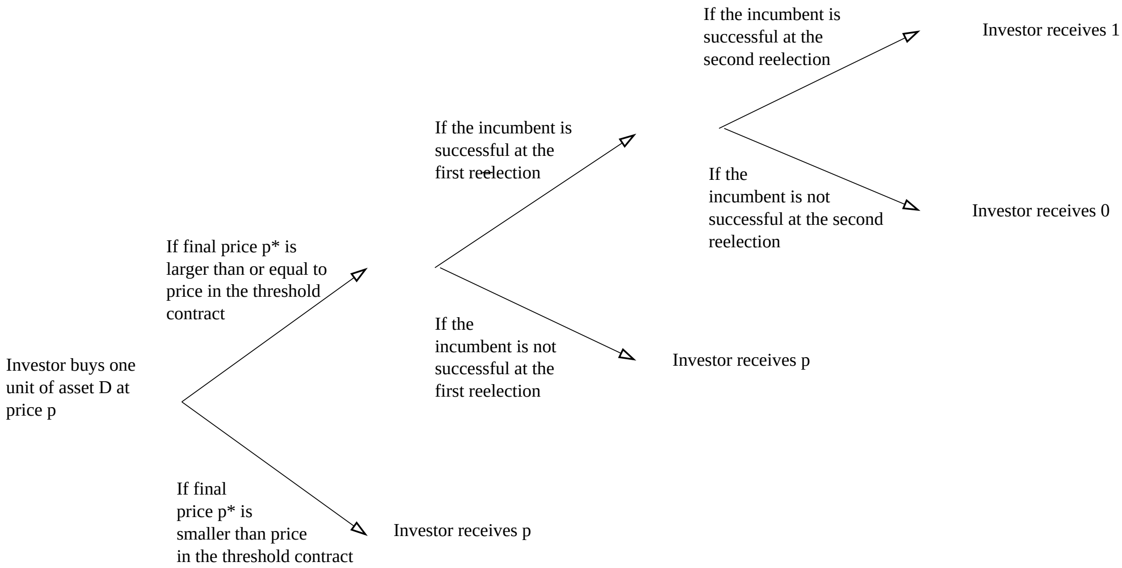

Figure 4

Finally, note that with probability μ there is complete information in period 1. Then the price in the information market will be either 1 or 0, depending on whether the politician undertook the socially optimal action or not.

B.2 Investors

There are N potential investors.[31] Investors are a subgroup of voters. We assume that there are many investors in the market. However, compared to the total number of voters investors constitute a minority and can not influence the voting outcome.

We assume that investors have log utility with

$$U_j(Y_j + W_j) = \ln(Y_j + W_j), \tag{20}$$

where W_j is the investor's wealth and Y_j is gain or loss in the information market.[32] Each investor j obtains a signal $\sigma_j \in \{0;1\}$ about the state of the world at the point in time when the politician in office discovers the state of the world.[33] The probability that investor j receives a correct signal, i.e. that $\sigma_j = s_1$, is given by $h_j \in (\frac{1}{2}, 1)$, where each investor j knows his personal signal quality h_j. Our assumption $h_j > \frac{1}{2}$ implies that the signals are not completely uninformative.[34] We assume that h_j does not depend on the state that has

[31] It is sensible for only individuals to be allowed to trade in such information markets and for the trading volume per person to be limited so as to avoid large-scale manipulation attempts.

[32] Note that we neglect utility from the action of the politician in the utility function of investors, as policy outcomes have no influence on the trading behavior of investors.

[33] There are several justifications why investors may be better informed than voters. It is fair to assume that investors spend time collecting information concerning the state of the world and thus have more knowledge than ordinary voters.

[34] We could also allow for poor signal qualities, i.e. $h_j \in (0, \frac{1}{2})$. As investor j knows h_j, a value of h_j near to 0 is as informative as a value near to 1. The lowest information gain is received by a signal which is correct with a probability of $\frac{1}{2}$. Nevertheless, we restrict the signal quality to $h_j \in (\frac{1}{2}, 1)$ in order to avoid additional case differentiations.

occurred.[35]

We first calculate the investors' posterior probability estimations of the state after they have received their signals. We obtain

$$Prob(s_1 = 1|\sigma_j = 1) = \frac{zh_j}{zh_j + (1-z)(1-h_j)}, \tag{21}$$

$$Prob(s_1 = 1|\sigma_j = 0) = \frac{z(1-h_j)}{(1-z)h_j + (1-h_j)z}, \tag{22}$$

$$Prob(s_1 = 0|\sigma_j = 1) = \frac{(1-z)(1-h_j)}{zh_j + (1-z)(1-h_j)}, \tag{23}$$

$$Prob(s_1 = 0|\sigma_j = 0) = \frac{(1-z)h_j}{(1-z)h_j + (1-h_j)z}. \tag{24}$$

B.3 Information from the Politician's Choice

Investors may receive additional information about the state by observing the action of the incumbent. Recall that a politician of the congruent type will always behave congruently in equilibrium when threshold contracts are used. The behavior of a dissonant incumbent depends on the parameters R, G, δ, and μ, which are common knowledge among investors. Three cases may occur: First, the value of G may be sufficiently low relative to R. Then dissonant politicians will behave congruently. Second, the value of G is at an intermediate level, and dissonant politicians will behave congruently in the popular state $s_1 = 1$, while they will behave dissonantly in the unpopular state $s_1 = 0$. Third, the value of G may be rather high relative to the benefits from holding office. Then dissonant politicians will behave dissonantly in both states of the world. We summarize the three cases in the following table, where a_1^c denotes the action of a congruent politician, while a_1^d denotes the action of a dissonant politician.

Case	Condition	a_1^c if $s_1 = 1$	a_1^c if $s_1 = 0$	a_1^d if $s_1 = 1$	a_1^d if $s_1 = 0$
1	$G \leq \mu\delta R(1+\delta)$	1	0	1	0
2	$\mu\delta R(1+\delta)$ $< G$ $\leq \delta R(1+\delta)$	1	0	1	1
3	$\delta R(1+\delta) < G$	1	0	0	1

Table 1

[35]The extension to state contingent values of h_j does not change the qualitative results of our model.

In the following we use $c \in \{1,2,3\}$ to denote the cases. In the next step we calculate the conditional probabilities $Prob_c(s_1 = 1|a_1 = 1)$ and $Prob_c(s_1 = 0|a_1 = 0)$ for an individual investor without private signals updating his beliefs in the signalling game with politicians choosing their action. For example, we obtain $Prob_2(s_1 = 1|a_1 = 1)$ as $\frac{z}{z+\frac{1}{2}(1-z)} = \frac{2z}{z+1}$ for $c = 2$. We summarize the conditional probabilities in the following table:

Case	Condition	$Prob_c(s_1 = 1\|a_1 = 1)$	$Prob_c(s_1 = 0\|a_1 = 0)$
1	$G \leq \mu\delta R(1+\delta)$	1	1
2	$\mu\delta R(1+\delta) < G \leq \delta R(1+\delta)$	$\frac{2z}{z+1}$	1
3	$\delta R(1+\delta) < G$	z	$1-z$

Table 2

B.4 Private Signals and Information from Politicians

Finally, we calculate the conditional probabilities $Prob_c(s_1|\sigma_j, a_1)$ for $c \in \{1,2,3\}$ when voters have received their private signals σ_j and draw inferences from the signalling games among politicians.

Case $c = 1$

Suppose $c = 1$. Then investors will learn the state with certainty by observing the action of the incumbent and can disregard their signals σ_j. We obtain

$$Prob_1(s_1 = 1|\sigma_j = 1, a_1 = 1) = Prob_1(s_1 = 1|\sigma_j = 0, a_1 = 1) = 1,$$

$$Prob_1(s_1 = 1|\sigma_j = 1, a_1 = 0) = Prob_1(s_1 = 1|\sigma_j = 0, a_1 = 0) = 0,$$

$$Prob_1(s_1 = 0|\sigma_j = 1, a_1 = 0) = Prob_1(s_1 = 0|\sigma_j = 0, a_1 = 0) = 1,$$

$$Prob_1(s_1 = 0|\sigma_j = 1, a_1 = 1) = Prob_1(s_1 = 0|\sigma_j = 0, a_1 = 1) = 0.$$

Case $c = 2$

In case 2, investors know with certainty that the true state of the world is $s_1 = 0$ when they observe $a_1 = 0$, i.e.

$$Prob_2(s_1 = 1|\sigma_j = 1, a_1 = 0) = Prob_2(s_1 = 1|\sigma_j = 0, a_1 = 0) = 0,$$

$$Prob_2(s_1 = 0|\sigma_j = 1, a_1 = 0) = Prob_2(s_1 = 0|\sigma_j = 0, a_1 = 0) = 1.$$

If investors observe $a_1 = 1$, then the signalling game reveals that the probability of $s_1 = 1$ after observing $a_1 = 1$ is equal to $\frac{2z}{z+1}$. Using the additional information from signal σ_j, investor j forms the following a posteriori belief:[36]

$$Prob_2(s_1 = 1|\sigma_j = 1, a_1 = 1) = \frac{\frac{2z}{z+1}h_j}{\frac{2z}{z+1}h_j + \frac{1-z}{z+1}(1-h_j)} = \frac{2zh_j}{2zh_j + (1-z)(1-h_j)}.$$

In a similar way we obtain

$$Prob_2(s_1 = 1|\sigma_j = 0, a_1 = 1) = \frac{2z(1-h_j)}{2z(1-h_j) + (1-z)h_j},$$

$$Prob_2(s_1 = 0|\sigma_j = 1, a_1 = 1) = \frac{(1-z)(1-h_j)}{2zh_j + (1-z)(1-h_j)},$$

$$Prob_2(s_1 = 0|\sigma_j = 0, a_1 = 1) = \frac{(1-z)h_j}{2z(1-h_j) + (1-z)h_j}.$$

Case $c = 3$
In case 3, the investors do not gain any information from the politician's action, as there is complete pooling. A congruent politicians behaves congruently, while all dissonant politicians behave dissonantly, and the probability for both types of politician equals $\frac{1}{2}$. Hence,

$$Prob_3(s_1 = 1|\sigma_j = 1, a_1 = 1) = Prob_3(s_1 = 1|\sigma_j = 1, a_1 = 0) = \frac{zh_j}{zh_j + (1-z)(1-h_j)},$$

$$Prob_3(s_1 = 1|\sigma_j = 0, a_1 = 1) = Prob_3(s_1 = 1|\sigma_j = 0, a_1 = 0) = \frac{z(1-h_j)}{(1-z)h_j + z(1-h_j)},$$

$$Prob_3(s_1 = 0|\sigma_j = 1, a_1 = 1) = Prob_3(s_1 = 0|\sigma_j = 1, a_1 = 0) = \frac{(1-z)(1-h_j)}{zh_j + (1-z)(1-h_j)},$$

$$Prob_3(s_1 = 0|\sigma_j = 0, a_1 = 1) = Prob_3(s_1 = 0|\sigma_j = 0, a_1 = 0) = \frac{(1-z)h_j}{(1-z)h_j + z(1-h_j)}.$$

B.5 Price Formation Process

For ease of exposition, we assume that all investors are homogeneous concerning the quality of their signals σ_j, i.e. we assume that $h_j = h \ \forall j$.[37] Thus, investors only differ

[36] Alternatively, one could calculate the a posteriori belief of investor j in the following way:

$$Prob_2(s_1 = 1|\sigma_j = 1, a_1 = 1) = \frac{2Prob(s_1 = 1|\sigma_j = 1)}{Prob(s_1 = 1|\sigma_j = 1) + 1} = \frac{2zh_j}{2zh_j + (1-z)(1-h_j)}.$$

Both methods lead to the same result.

[37] Further, we assume that investors are homogeneous concerning their wealth and their subjective confidence in their own signals. In Appendix C we will derive some general results for heterogeneous investors. Using the notation of Appendix C we assume $W_j = W \ \forall j$ and $b_j = b \ \forall j$ in this section. At the cost of additional notational complexity, the results can be extended to heterogeneous investors by using the formulas derived in Appendix C.

as to whether they receive signal $\sigma_j = 1$ or $\sigma_j = 0$. When the number of investors is sufficiently large a fraction h of the investors will receive the correct signal, i.e. they receive $\sigma_j = 1$ if $s_1 = 1$ or $\sigma_j = 0$ if $s_1 = 0$, respectively.[38] A fraction $1-h$ will receive a misleading signal, i.e. they receive $\sigma_j = 1$ if $s_1 = 0$ or $\sigma_j = 0$ if $s_1 = 1$.

From Corollary 3 in Appendix C we know that the price in the information market will be a weighted average of the prices that would arise in the two subgroups of investors. This means that the price will be h times the price that would arise in a market where all investors receive a correct signal plus (1-h) times the price in a market where investors only receive incorrect signals. Again, we go through all three cases.

Case $c = 1$
We start with case $c = 1$. In this scenario, the action of the incumbent will perfectly reveal the state of the world. Thus, we obtain

$$p^{*1}_{1,1} = p^{*1}_{0,0} = 1 \tag{25}$$

and

$$p^{*1}_{1,0} = p^{*1}_{0,1} = 0, \tag{26}$$

where $p^{*c}_{a_1,s_1}$ denotes the equilibrium price in case c given action a_1 and state s_1. The equilibrium price will equal one if the incumbent chooses the socially optimal action, while the price will be zero if the politician chooses the non-optimal action.

Case $c = 2$
In case 2, we obtain

$$p^{*2}_{0,0} = 1 \tag{27}$$

and

$$p^{*2}_{0,1} = 0, \tag{28}$$

which reflects the fact that the equilibrium price will be equal to zero or one upon observing $a_1 = 0$, as this action reveals the true state of the world with certainty. If the incumbent undertakes $a_1 = 1$ in case $c = 2$, then we obtain

$$p^{*2}_{1,1} = 1 - \frac{(1-z^2)h(1-h)}{[2zh+(1-z)(1-h)][2z(1-h)+(1-z)h]} \tag{29}$$

and

$$p^{*2}_{1,0} = \frac{2z(1+z)h(1-h)}{[2zh+(1-z)(1-h)][2z(1-h)+(1-z)h]}. \tag{30}$$

Case $c = 3$
In case 3 we obtain

$$p^{*3}_{1,1} = 1 - \frac{(1-z)h(1-h)}{[zh+(1-z)(1-h)][z(1-h)+(1-z)h]}, \tag{31}$$

[38] For a finite number of investors the variance of the fraction of investors receiving the correct signal is not zero. However, for a sufficiently large number of investors the variance becomes arbitrarily small. For instance, for $N = 10000$ the probability that the share of investors with a correct signal is in $[0.89, 0.91]$ for $h = 0.9$ is larger than 99.9%.

$$p^{*3}_{1,0} = \frac{zh(1-h)}{[zh+(1-z)(1-h)][z(1-h)+(1-z)h]}, \quad (32)$$

$$p^{*3}_{0,0} = 1 - \frac{zh(1-h)}{[zh+(1-z)(1-h)][z(1-h)+(1-z)h]}, \quad (33)$$

$$p^{*3}_{0,1} = \frac{(1-z)h(1-h)}{[zh+(1-z)(1-h)][z(1-h)+(1-z)h]}. \quad (34)$$

We observe that $p^{*3}_{1,1} = 1 - p^{*3}_{0,1}$ and $p^{*3}_{0,0} = 1 - p^{*3}_{1,0}$. The next Proposition is the main result of Appendix B.

Proposition 3 (detailed version)
Suppose that $h > \hat{h}(z)$ *with*

$$\hat{h}(z) = \frac{1 + \sqrt{\frac{3z^2+2z-1}{-5z^2+10z-1}}}{2} < 1. \quad (35)$$

Then the equilibrium price in the information market fulfills the following conditions:

$$p^{*c}_{1,1} > \frac{1}{2} \ \forall c,$$

$$p^{*c}_{0,0} > \frac{1}{2} \ \forall c,$$

$$p^{*c}_{1,0} < \frac{1}{2} \ \forall c$$

and

$$p^{*c}_{0,1} < \frac{1}{2} \ \forall c.$$

Proposition 3 shows that for $h > \hat{h}(z)$ the equilibrium price in all circumstances will be larger than one-half if the incumbent behaves congruently, while the equilibrium price will be smaller than one-half if the politician behaves dissonantly. Note that $\hat{h}(z)$ is increasing in z for $z \in (\frac{1}{2}, 1)$ and that $\hat{h}(z) \in (\frac{1}{2} + \sqrt{\frac{3}{44}}, 1)$ for $z \in (\frac{1}{2}, 1)$. The intuition that $\hat{h}(z)$ must be larger than $\frac{1}{2}$ runs as follows: The signal must be sufficiently informative in order to detect dissonant behavior of a politician in the unpopular state $s_1 = 0$ in case $c = 2$, where $Prob(s_1 = 0|a_1 = 1)$ is rather low. A formal derivation and explanation for condition (35) is given in the following proof of Proposition 3:

Proof of Proposition 3
First, it is obvious that $p^{*1}_{1,1} > \frac{1}{2}$, $p^{*1}_{0,0} > \frac{1}{2}$, $p^{*1}_{1,0} < \frac{1}{2}$, $p^{*1}_{0,1} < \frac{1}{2}$, $p^{*2}_{0,0} > \frac{1}{2}$ and $p^{*2}_{0,1} < \frac{1}{2}$ for any values of $h \in (\frac{1}{2}, 1)$.

The condition $p^{*2}_{1,1} > \frac{1}{2}$ is equivalent to

$$\frac{(1-z^2)h(1-h)}{[2zh+(1-z)(1-h)][2z(1-h)+(1-z)h]} < \frac{1}{2}. \quad (36)$$

After some manipulations we obtain the condition

$$2z(1-z) + h(1-h)(11z^2 - 6z - 1) > 0. \quad (37)$$

We note that $h(1-h) < \frac{1}{4}\ \forall h \in (\frac{1}{2}, 1)$ and that $2z(1-z) > -\frac{1}{4}(11z^2 - 6z - 1)\ \forall z \in (\frac{1}{2}, 1)$. Thus, condition (37) is always fulfilled.

Next we examine $p^{*2}_{1,0} < \frac{1}{2}$, which is equivalent to

$$\frac{2z(1+z)h(1-h)}{[2zh + (1-z)(1-h)][2z(1-h) + (1-z)h]} < \frac{1}{2}. \tag{38}$$

Rearranging terms yields

$$h(1-h) < \frac{2z(1-z)}{-5z^2 + 10z - 1}. \tag{39}$$

Solving for h leads to

$$h > \frac{1 + \sqrt{\frac{3z^2+2z-1}{-5z^2+10z-1}}}{2}. \tag{40}$$

The next condition $p^{*3}_{1,1} > \frac{1}{2}$ is equivalent to

$$\frac{(1-z)h(1-h)}{[zh + (1-z)(1-h)][z(1-h) + (1-z)h]} < \frac{1}{2}. \tag{41}$$

This condition can be transformed to

$$z(1-z) + h(1-h)(4z^2 - 2z - 1) > 0. \tag{42}$$

We note that $h(1-h) < \frac{1}{4}\ \forall h \in (\frac{1}{2}, 1)$ and that $z(1-z) > -\frac{1}{4}(4z^2 - 2z - 1)\ \forall z \in (\frac{1}{2}, 1)$. Thus, condition (42) is always fulfilled.

The condition $p^{*3}_{1,0} < \frac{1}{2}$ is equivalent to

$$\frac{zh(1-h)}{[zh + (1-z)(1-h)][z(1-h) + (1-z)h]} < \frac{1}{2}. \tag{43}$$

After some manipulations we obtain

$$h(1-h) < \frac{z(1-z)}{-4z^2 + 6z - 1}, \tag{44}$$

which yields

$$h > \frac{1 + \sqrt{\frac{2z-1}{-4z^2+6z-1}}}{2}. \tag{45}$$

Condition (45) is a weaker condition than condition (40) as the following inequality holds for all $z \in (\frac{1}{2}, 1)$:

$$\sqrt{\frac{2z-1}{-4z^2+6z-1}} < \sqrt{\frac{3z^2+2z-1}{-5z^2+10z-1}}. \tag{46}$$

Hence, if $h > \frac{1+\sqrt{\frac{3z^2+2z-1}{-5z^2+10z-1}}}{2}$, then condition (45) will always be fulfilled.

Next we investigate $p^{*3}_{0,0} > \frac{1}{2}$, which leads to

$$\frac{zh(1-h)}{[zh + (1-z)(1-h)][z(1-h) + (1-z)h]} < \frac{1}{2}. \tag{47}$$

This condition is the same as (43) and thus always fulfilled for $h > \frac{1+\sqrt{\frac{3z^2+2z-1}{-5z^2+10z-1}}}{2}$.

Finally, we consider $p^{*3}_{0,1} < \frac{1}{2}$, which yields

$$\frac{(1-z)h(1-h)}{[zh+(1-z)(1-h)][z(1-h)+(1-z)h]} < \frac{1}{2}. \tag{48}$$

This is identical to condition (41), which always holds as shown above.

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B.6 Sophisticated Election Scheme

In this subsection we prove Proposition 8 by proving the following detailed version of Proposition 8:

Proposition 8 (detailed version)
Suppose that $h > \hat{\hat{h}}$ with

$$\hat{\hat{h}} = \frac{1+\sqrt{\frac{z+1}{9z+1}}}{2} < \frac{1}{2} + \sqrt{\frac{3}{44}} \approx 0.761. \tag{49}$$

Then the equilibrium price in the information market fulfills the following conditions:

$$p^{*c}_{1,1} > z\ \forall c,$$

$$p^{*c}_{0,0} > 1-z\ \forall c,$$

$$p^{*c}_{1,0} < z\ \forall c$$

and

$$p^{*c}_{0,1} < 1-z\ \forall c.$$

Proof of Proposition 8
The proof follows the same lines as the proof of Proposition 3. First, it is obvious that $p^{*1}_{1,1} > z$, $p^{*1}_{0,0} > 1-z$, $p^{*1}_{1,0} < z$, $p^{*1}_{0,1} < 1-z$, $p^{*2}_{0,0} > 1-z$ and $p^{*2}_{0,1} < 1-z$.

We explore the condition $p^{*2}_{1,1} > z$, which is equivalent to

$$\frac{(1-z^2)h(1-h)}{[2zh+(1-z)(1-h)][2z(1-h)+(1-z)h]} < 1-z. \tag{50}$$

After some manipulations we obtain

$$2(1-z)^2 + h(1-h)(-9z^2+16z-7) > 0. \tag{51}$$

Using $h(1-h) < \frac{1}{4}\ \forall h \in (\frac{1}{2}, 1)$ and $2(1-z)^2 > \frac{1}{4}(9z^2-16z+7)\ \forall z \in (\frac{1}{2}, 1)$ shows that condition (51) is fulfilled for all $z \in \{\frac{1}{2}, 1\}$.

Next we examine $p^{*2}_{1,0} < z$, which yields

$$\frac{2(1+z)h(1-h)}{[2zh+(1-z)(1-h)][2z(1-h)+(1-z)h]} < 1. \tag{52}$$

Rearranging terms leads to

$$h(1-h) < \frac{2z(1-z)}{-9z^2+8z+1} = \frac{2z}{9z+1}, \tag{53}$$

which implies

$$h > \frac{1+\sqrt{\frac{z+1}{9z+1}}}{2}. \tag{54}$$

Finally, conditions $p^{*3}_{1,1} > z$, $p^{*3}_{1,0} < z$, $p^{*3}_{0,0} > 1-z$ and $p^{*3}_{0,1} < 1-z$ all result in

$$\frac{h(1-h)}{[zh+(1-z)(1-h)][z(1-h)+(1-z)h]} < 1, \tag{55}$$

which is equivalent to

$$h(1-h) < \frac{1}{4}. \tag{56}$$

As $h \geq \frac{1}{2}$ condition 56 is always fulfilled.

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By comparing $\hat{h}$ and $\hat{\hat{h}}$ we obtain the following Corollary:

Corollary 2
$\hat{\hat{h}} < \hat{h}$ for all z with $\frac{1}{2} < z < 1$.

Hence, for all values $z \in (\frac{1}{2}, 1)$ condition (49) is easier to fulfill than condition (35).[39] As a consequence, *SES*, which uses the results from Proposition 8, is applicable for signals with lower information content than *RES*. Note that Corollary 2 follows directly from comparing $\hat{h}$ and $\hat{\hat{h}}$. The claim $\hat{\hat{h}} < \hat{h}$ can be transformed to $2z^2 + z > 1$, which proves the Corollary.

C Appendix C: General Price Formation Process

In this Appendix we determine a general formula for an information market with heterogeneous agents. Suppose, without loss of generality, that politician 1 has been elected after offering a contract $C_1(p^1_1, p^0_1)$, that the politician undertakes $a_1 = 1$, and hence that p^1_1 applies.

For a price $p < p^1_1$, no investor will have a strict incentive to buy assets, as he will be paid back p. Suppose $p \geq p^1_1$. An investor j with signal σ_j has to weigh up the state of

[39]For $z = \frac{1}{2}$ equation (35) would be identical to condition (49).

his information and the information the market price will reveal.[40] One way of modeling the information aggregation process is as follows:

$$Prob_j(RE|p) = b_j \; Prob_j(RE) + (1 - b_j) \; p, \tag{57}$$

where $Prob_j(RE|p)$ is the probability assessment of investor j that the incumbent will be reelected, taking into account the information inferred from the market price. The term $Prob_j(RE)$ is given as the individual reelection probability estimation of an investor and depends on his signal σ_j, the signal quality h_j, the action a_1, and the case c. If, e.g., $c = 3$, $a_1 = 1$, and $\sigma_j = 1$, then $Prob_j(RE) = \frac{zh_j}{zh_j+(1-z)(1-h_j)}$, where we assume that z and h_j are known to investor j. The weight b_j (with $0 < b_j \leq 1$) describes self-assessed confidence, i.e. the subjective confidence of an investor in his estimation $Prob_j(RE)$ relative to the market belief expressed by the price p.[41] The information aggregation formula (57) is flexible. It captures the case $b_j = 1$ when investors rely only on their own signal, which would occur if they can only submit a quantity (and not an entire demand/supply schedule depending on the price) to the market. For small values of b_j, investors rely mainly on the information aggregated by the market.[42]

Given price p and signal $Prob_j(RE)$, an investor j maximizes

$$\max_{d_j} \; EU_j = Prob_j(RE|p) \; ln(W_j + d_j(1-p)) + (1 - Prob_j(RE|p)) \; ln(W_j - d_j p), \tag{58}$$

where d_j is the demand. If d_j is positive, investor j will want to buy d_j units of asset D. If d_j is negative, investor j will want to buy d_j units of asset E. The solution of the investor's problem yields

$$\begin{aligned} d_j^* &= W_j \, \frac{b_j \; Prob_j(RE) + (1-b_j)p - p}{p(1-p)} \\ \Leftrightarrow \; d_j^* &= W_j \, \frac{b_j \; Prob_j(RE) - p \; b_j}{p(1-p)}. \end{aligned} \tag{59}$$

We thus obtain

Proposition 9
There is a unique equilibrium in the information market given by

$$p^* = \sum_{j=1}^{N} Prob_j(RE) \; \frac{W_j \; b_j}{\sum_{k=1}^{N} W_k \; b_k} \; . \tag{60}$$

[40] Note that investors learn nothing from the threshold contract offers of the candidates because in equilibrium both types of politicians will offer the same contract, as we will show later.

[41] For a statistical foundation, see Morris (1983) and Rosenblueth and Ordaz (1992). Wolfers and Zitzewitz (2006) have independently suggested a similar procedure.

[42] Note that it can never be rational to set $b_j = 0 \;\; \forall j$ as the price would contain no information contradicting the assumption of investors to rely only on the information inferred from the market price. This is the information paradox addressed by Grossman and Stiglitz (1980).

Proof of Proposition 9

Equilibrium in the information market requires that condition $\sum_{j=1}^{N} d_j^* = 0$ be fulfilled, which implies $\sum_{j=1}^{N} W_j\, b_j\, Prob_j(RE) \; - p \sum_{j=1}^{N} W_j\, b_j \; = 0$. The assertion follows from that.

■

The market price is a wealth- and confidence-weighted average belief on the part of investors. We note that the market price is equal to the simple average belief of investors if traders are homogeneous with respect to wealth and confidence in their own belief. If confidence levels are homogeneous, the market price is a wealth-weighted average belief on the part of traders. We summarize both cases in the following Corollary:

Corollary 3

(i) Suppose $W_j = W \;\; \forall j$ *and* $b_j = b \;\; \forall j$. *Then* $p^* = \frac{1}{N} \sum_{j=1}^{N} Prob_j(RE)$.

(ii) Suppose $b_j = b \;\; \forall j$. *Then* $p^* = \sum_{j=1}^{N} Prob_j(RE) \frac{W_j}{\sum_{k=1}^{N} W_k}$.

D Appendix D: Welfare Gains

Here we provide an example of the welfare gains that can be achieved with the triple mechanism. Suppose that, at a time when this institution is introduced, it is only known that δ is equal to 1 and that μ is uniformly distributed in $[0, \frac{1}{2}]$. Since only the proportion of R and G is important for our analysis, we write $G = \alpha R$ with $0 \;\leq\; \alpha \;<\; \infty$. In the following, we calculate the values of μ that enable congruent behavior by the incumbent. We use *eo* to denote the case with elections only and *tm* to denote the scenario with the triple mechanism. From condition (6) we conclude that, in the case of elections alone, a congruent politician will only behave congruently in state $s_1 = 1$ if

$$\alpha R + 3R \geq R.$$

This condition is equivalent to $\alpha \geq -2$. In the same way we obtain the other conditions summarized in the following table:

	Congruent Politician		Dissonant Politician	
	$s_1 = 1$	$s_1 = 0$	$s_1 = 1$	$s_1 = 0$
Elections Only	$\alpha \geq -2$	$\mu \geq \frac{1-\alpha}{3}$	$\alpha \leq 2$	$\mu \geq \frac{1+\alpha}{3}$
Triple Mechanism	$\alpha \geq -2$	$\mu \geq -\frac{\alpha}{2}$	$\alpha \leq 2$	$\mu \geq \frac{\alpha}{2}$

Table 3

Note that congruent politicians will always behave congruently in the scenario with the triple mechanism, as conditions $\alpha \geq -2$ and $\mu \geq -\frac{\alpha}{2}$ are always fulfilled. Furthermore, if $\alpha \geq 1$ congruent politicians will always behave congruently in the scenario with elections only. Finally, it is apparent that a dissonant politician will never act congruently for $\alpha > 2$, which clearly derives from Corollary 1 and Proposition 6. In the next stage, we calculate expected utilities, starting with the triple mechanism scenario:

$$EU^{tm} = \frac{1}{2} + \frac{1}{2}z \begin{cases} \int_0^{\frac{1}{2}} 2d\mu & \text{if } \alpha \leq 2 \\ \int_{\frac{1}{2}}^{\frac{1}{2}} 2d\mu & \text{if } \alpha > 2 \end{cases} + \frac{1}{2}(1-z) \begin{cases} \int_{\frac{\alpha}{2}}^{\frac{1}{2}} 2d\mu & \text{if } \alpha \leq 1 \\ \int_{\frac{1}{2}}^{\frac{1}{2}} 2d\mu & \text{if } \alpha > 1 \end{cases}$$

The reasoning for the above expression is as follows: A politician is of the congruent type with probability $\frac{1}{2}$. He always behaves congruently and thus generates a voter utility of 1. The probability that a politician is of the dissonant type and that state $s_1 = 1$ occurs is given by $\frac{1}{2}z$. In this case, the politician generates a utility of 1 for all feasible values of μ, as long as α is not larger than 2. Finally, the probability that a politician is of the dissonant type and that state $s_1 = 0$ occurs is given by $\frac{1}{2}(1-z)$. In this case, the politician generates a utility of 1 for all values of μ with $\mu \geq \frac{\alpha}{2}$, as long as α is not larger than 1.[43] The calculation in the scenario with elections alone is similar and yields

$$\begin{aligned} EU^{eo} &= \frac{1}{2}z + \frac{1}{2}(1-z) \begin{cases} \int_{\frac{1-\alpha}{3}}^{\frac{1}{2}} 2d\mu & \text{if } \alpha \leq 1 \\ \int_0^{\frac{1}{2}} 2d\mu & \text{if } \alpha > 1 \end{cases} \\ &+ \frac{1}{2}z \begin{cases} \int_0^{\frac{1}{2}} 2d\mu & \text{if } \alpha \leq 2 \\ \int_{\frac{1}{2}}^{\frac{1}{2}} 2d\mu & \text{if } \alpha > 2 \end{cases} + \frac{1}{2}(1-z) \begin{cases} \int_{\frac{1+\alpha}{3}}^{\frac{1}{2}} 2d\mu & \text{if } \alpha \leq \frac{1}{2} \\ \int_{\frac{1}{2}}^{\frac{1}{2}} 2d\mu & \text{if } \alpha > \frac{1}{2} \end{cases} \end{aligned}$$

[43] Note that we have assumed that μ is uniformly distributed in $[0, \frac{1}{2}]$.

These expressions can be simplified to

$$EU^{tm} = \begin{cases} \frac{1}{2}+\frac{1}{2}[1-\alpha(1-z)] & \text{if } \alpha \leq 1 \\ \frac{1}{2}+\frac{1}{2}z & \text{if } 1 < \alpha \leq 2 \\ \frac{1}{2} & \text{if } \alpha > 2 \end{cases} \tag{61}$$

and

$$EU^{eo} = \begin{cases} z+\dfrac{1}{3}(1-z) & \text{if } \alpha \leq \frac{1}{2} \\ z+(1-z)\dfrac{(1+2\alpha)}{6} & \text{if } \frac{1}{2} < \alpha \leq 1 \\ \frac{1}{2}+\frac{1}{2}z & \text{if } 1 < \alpha \leq 2 \\ \frac{1}{2} & \text{if } \alpha > 2. \end{cases} \tag{62}$$

We illustrate the relationships by calculating the utilities for four different values of α. We choose one value of α that is smaller than 1, one value larger than 1, and α equal to 1. These values correspond to the cases where, for the politician, utility G is lower/higher than or equal to utility R. Furthermore, we add the special case $\alpha = 0$, where the politician has no private benefits G. The expected utilities in these four cases are summarized in the following table:

	$\alpha = 3$	$\alpha = 1$	$\alpha = 0.1$	$\alpha = 0$
EU^{eo}	$\frac{1}{2}$	$\frac{1+z}{2}$	$\frac{1+2z}{3}$	$\frac{1+2z}{3}$
EU^{tm}	$\frac{1}{2}$	$\frac{1+z}{2}$	$\frac{19+z}{20}$	1
$EU^{tm} - EU^{eo}$	0	0	$\frac{37(1-z)}{60}$	$\frac{2(1-z)}{3}$
$\Delta_{EU} = \frac{EU^{tm} - EU^{eo}}{EU^{eo}}$	0	0	$\frac{37(1-z)}{20+40z}$	$\frac{2(1-z)}{1+2z}$

Table 4

Note that in all cases we have $EU^{tm} \geq EU^{eo}$. Further, we see that EU^{tm} is strictly larger than EU^{eo} if $z < 1$ and $\alpha < 1$. The difference between EU^{tm} and EU^{eo} depends on z for $0 < \alpha < 1$. The last row in the table shows the relative welfare gains (Δ_{EU}). Δ_{EU} is maximum for $\alpha = 0$. The example illustrates the following insights:

(i) Threshold contracts have the highest effect in the case $\alpha = 0$, i.e. if the politicians are only motivated by benefits R acquired from holding office. Note that threshold contracts may reduce the reelection chances of the incumbent. Thus, threshold contracts will be more effective if politicians are mainly interested in getting reelected, which is expressed in a low value of α.

(ii) If α is at least equal to 1, i.e. if politicians are at least as motivated by G as by R, then there is no effect from threshold contracts. This is due to the fact that in state $s_1 = 0$ congruent politicians always behave congruently, while dissonant candidates always behave dissonantly. The conditions for congruent behavior in state $s_1 = 1$ are the same in the scenarios with or without threshold contracts.
If α is at least equal to 2, then congruent politicians will always behave congruently, while dissonant candidates will always behave dissonantly. Thus, the expected utility is equal to $\frac{1}{2}$.

(iii) Finally, for a given value of α we discover that Δ_{EU} is (weakly) increasing when z decreases. Thus, the higher the probability of the unpopular state $s_1 = 0$, the larger is the effect of threshold contracts.

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