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CHOOSING BETWEEN SCHOOL SYSTEMS

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Abstract

Hierarchical and comprehensive school systems are compared with respect to efficiency. At given ability, a student's probability of not completing school rises with increasing mean ability in class. Both school systems can yield identical average failure rates. Given that output losses in case of failure are stronger for more talented students, the comprehensive school system will generally lead to a higher total income.

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1 Introduction

One of the main points in the debate on the construction of school systems during the last decades has been whether or not comprehensive schools are preferable to a hierarchical school system. In a hierarchical school system students are sorted according to ability, while this does not happen in a comprehensive school system. Obviously the comprehensive school system offers equality of opportunity whereas the hierarchical system need not. On the other hand, promoters of a hierarchical school system stress that it allows a faster accumulation of human capital for more talented individuals. This paper investigates which school system is preferable with respect to efficiency, i.e. which school system maximizes income per capita.

To keep matters simple, the hierarchical school system consists of two schools, the lower school and the higher school. Individuals differ with respect to ability, which is not perfectly observable. At the end of primary education pupils are tested. The probability of a recommendation to attend the higher school increases with ability. Since the recommendation is binding, the other pupils are sent to the lower school. Human capital production depends on the student's ability, resources spent per capita in the respective school and average ability in the school. The latter impact is known as the peer group effect. However, teachers respond to a higher average ability by increasing the academic level in the courses. This yields a higher probability of failure at given level of ability. The government chooses the school system as to maximize total income. Should a hierarchical system be implemented, it also decides on allocating resources to the two schools.

The paper is related to the literature on optimal school systems in the presence of peer group effects. There is no clear evidence for which types of students peer group effects are particularly strong (Epple et al., 2000). While Summers and Wolfe (1977) find that peer group effects are significant only for low ability students, Henderson et al. (1978) conclude that peer group effects are similar for all students, and Argys et al. (1996) state that the impact of peer groups is strongest for high ability students. Obviously, the structure of peer group effects is a major factor in determining the optimal school system. Should, for example, peer group effects be stronger for weaker students, it may be concluded that mixed classes are preferable to a class structure in which pupils are sorted according to ability. However, Arnott and Rowse (1987) point out that the specific structure of the educational production function rather than the mere existence of peer group effects has to be considered when constructing an optimal class system. Peer group effects have also been discussed in models of internal migration, where they are one force creating segregation of communities (see, for example, de Bartolome (1990)). The analysis in the current contribution bears some similarity to the paper of Judson (1998) where the individuals' abilities are tested after having completed a given stage of education. In Judson's model the government allocates resources to the various stages of education and selects the cutoff levels of admission according to noisy test signals. The focus of her paper, however, is on the question whether or not universal enrollment for primary education is optimal. Effinger and Polborn (1999) study a hierarchical school system in which educational achievement depends on the student's ability, a peer group effect, and the academic level that is chosen in the higher school. Raising the academic level leads to a stronger increase in a higher school student's productivity if the mean ability in this school is higher. Effinger and Polborn show that under free choice too many students attend the higher school. The view taken in the current model is that the productivity impact of the academic level is already captured by the peer group effect while the disadvantage is given by higher failure rates. Hence, a higher academic level does not only increase output in case of successful completion of school. It also imposes a cost on all students by an increased risk of failure. A microfoundation for the probability of failure may look like the formulation in Costrell (1994) where individuals have differentiated preferences for leisure. A higher standard of teaching has the consequence that some students will choose not to satisfy the standard.

Obviously the aggregate failure rate is one decisive factor when determining the efficiency of the school system. The main results of this paper are as follows: if the probability of failure depends in a linear fashion on the difference between the individual's ability and average ability in school, the aggregate failure rate will not be contingent on the school system. Choosing a comprehensive school system rather than a hierarchical system reduces the failure rate of more talented individuals but increases the failure rate of less talented individuals. Given that the output loss caused by failure in school is higher for more talented individuals, the comprehensive school system is generally associated with a higher total output.

2 The model

We consider a continuum of individuals who are distinct with respect to their initial ability θ. Ability is distributed on the interval $[0,1]$ according to the density function $f(\theta)$ with $\bar{\theta}$ denoting mean ability. Under a hierarchical school system individuals are tested after having completed primary education. The test only imperfectly measures the individual's true ability. Attending the higher school is recommended with probability $h(\theta)$ where $h' > 0$. Thus, a more talented pupil exhibits an increased probability of being sent to the higher school. With probability $1 - h(\theta)$, the pupil will attend the lower school. Should a student successfully complete the higher school, her productivity will be $w(\theta, E_H, \bar{\theta}_H)$, where E_H denotes expenditure per higher school student and $\bar{\theta}_H$ is mean ability in the higher school. We assume that $\frac{\partial w}{\partial \theta} > 0$ and $\frac{\partial w}{\partial E} > 0$. Hence, talent matters and public expenditure is productive. Moreover, $\frac{\partial^2 w}{\partial E^2} < 0$ indicates diminishing returns to public expenditure. It is assumed that $\frac{\partial w}{\partial \bar{\theta}} \geq 0$. The possibility $\frac{\partial w}{\partial \bar{\theta}} > 0$ captures a peer group effect. An individual with ability θ faces a probability of failure $l(\bar{\theta}_H - \theta)$ in the higher school where $l' > 0$. This implies that more talented individuals show a higher propensity of completing the higher school. Conversely, a higher mean ability in class raises educational standards with the consequence of a higher risk of failure.

Note that even students with the highest ability may face a positive probability of failure. The risk of failure can be interpreted in two different ways. First, failure and success may be affected by idiosyncratic stochastic events at examination dates. Second, there may exist a distribution of preferences for leisure among individuals. In the latter case, students with a strong preference for leisure will not exert the effort level necessary to satisfy the educational standard. Both explanations suggest that the failure rate increases in ability in a strictly monotonous fashion.

Those who do not succeed will have the productivity w_0, where $w_0 \leq w(0,0,0)$ holds. In other words, completion of school always increases productivity. For simplicity, the productivity of a dropout is a constant.

At the lower school, expenditure per student is E_L and average talent is $\bar{\theta}_L$. Productivity after a completed term at the lower school is represented by $w(\theta, E_L, \bar{\theta}_L)$. The probability of failure is given by $l(\bar{\theta}_L - \theta)$. Since $\bar{\theta}_H > \bar{\theta}_L$

will always hold, the failure probability in the higher school exceeds the corresponding rate in the lower school for an individual with given ability level θ. Productivity in case of failure is w_0 again. The total number of students is normalized to unity. Hence, expected productivity under a hierarchical school system can be written as

$$\begin{aligned} V_h &= \int_0^1 f(\theta)h(\theta)\Big[[1-l(\bar{\theta}_H-\theta)]w(\theta,E_H,\bar{\theta}_H)+l(\bar{\theta}_H-\theta)w_0\Big]d\theta \quad (1)\\ &+\int_0^1 f(\theta)[1-h(\theta)]\\ &\cdot\Big[[1-l(\bar{\theta}_L-\theta)]w(\theta,E_L,\bar{\theta}_L)+l(\bar{\theta}_L-\theta)w_0\Big]d\theta. \end{aligned}$$

The budget equation is

$$E = E_H\int_0^1 f(\theta)h(\theta)d\theta + E_L\int_0^1 f(\theta)[1-h(\theta)]d\theta, \tag{2}$$

where E denotes total expenditure. Assuming that $\lim_{E\to 0}\frac{\partial w}{\partial E}=\infty$ guarantees an interior solution with respect to E_H and E_L. Substituting for E_L from (2), and optimizing V_h with respect to E_H, yields the first-order condition

$$\begin{aligned} \frac{\partial V_h}{\partial E_H} &= \int_0^1 f(\theta)h(\theta)[1-l(\bar{\theta}_H-\theta)]\frac{\partial w(\theta,E_H,\bar{\theta}_H)}{\partial E_H}d\theta \quad (3)\\ &-\frac{\int_0^1 f(\theta)h(\theta)d\theta}{\int_0^1 f(\theta)[1-h(\theta)]d\theta}\\ &\cdot\int_0^1 f(\theta)[1-h(\theta)][1-l(\bar{\theta}_L-\theta)]\frac{\partial w(\theta,E_L,\bar{\theta}_L)}{\partial E_L}d\theta\\ &= 0. \end{aligned}$$

This is equivalent to

$$\begin{aligned} &\frac{\int_0^1 f(\theta)h(\theta)[1-l(\bar{\theta}_H-\theta)]\frac{\partial w(\theta,E_H,\bar{\theta}_H)}{\partial E_H}d\theta}{\int_0^1 f(\theta)h(\theta)d\theta}\\ =\; &\frac{\int_0^1 f(\theta)[1-h(\theta)][1-l(\bar{\theta}_L-\theta)]\frac{\partial w(\theta,E_L,\bar{\theta}_L)}{\partial E_L}d\theta}{\int_0^1 f(\theta)[1-h(\theta)]d\theta}. \end{aligned}$$

The optimality condition states that spending one additional dollar in either school will lead to the same increase in total income. The question whether E_H will exceed or fall short of E_L depends both on properties of the human capital production function and on average failure rates in the respective schools. For example, if the function w exhibits additive separability, and if the average failure rate in the lower school is smaller than in the higher school, then E_L will exceed E_H. On the other hand, if average failure rates are identical, expenditure per pupil in the lower school will fall short of the corresponding amount in the higher school should $\frac{\partial^2 w}{\partial E \partial \theta} \geq 0$ and $\frac{\partial^2 w}{\partial E \partial \bar{\theta}} > 0$ hold.

Under a comprehensive school system, students with ability θ achieve the productivity $w(\theta, E, \bar{\theta})$ with success probability $1 - l(\bar{\theta} - \theta)$. In case of failure, a productivity of w_0 is the outcome again. Expected productivity can then be written as

$$V_c = \int_0^1 f(\theta)\Big[[1 - l(\bar{\theta} - \theta)]w(\theta, E, \bar{\theta}) + l(\bar{\theta} - \theta)w_0\Big]d\theta. \tag{4}$$

3 Failure rates and efficiency

Since dropouts can cause substantial losses in productivity, it is an interesting question which school system induces the higher total failure rate. Obviously, a student attending the higher school will face a higher risk of failure than under the comprehensive school system. If she has been sent to the lower school instead, the opposite holds.

Proposition 1 states that the two school systems generate an identical number of dropouts should the failure function l be linear.

Proposition 1 *If the failure function l is linear, the aggregate failure rates are identical under both school systems.*

Proof: Let g denote the difference between the aggregate failure rate under the comprehensive school system and the corresponding rate under the hierarchical system. Thus,

$$\begin{aligned} g \quad = \quad & \int_0^1 f(\theta)\Big[h(\theta)[l(\bar{\theta} - \theta) - l(\bar{\theta}_H - \theta)] \\ & +(1 - h(\theta))[l(\bar{\theta} - \theta) - l(\bar{\theta}_L - \theta)]\Big]d\theta. \end{aligned} \tag{5}$$

If the failure function is linear, then

$$l(\bar{\theta}-\theta)-l(\bar{\theta}_i-\theta)=l(\bar{\theta}-\bar{\theta}_i)-l(0)$$

with $i \in \{H, L\}$ holds. This implies

$$\begin{aligned} g &= l(\bar{\theta}-\bar{\theta}_H)\int_0^1 f(\theta)h(\theta)d\theta + l(\bar{\theta}-\bar{\theta}_L)\int_0^1 f(\theta)[1-h(\theta)]d\theta - l(0) \\ &= b\Big[\bar{\theta}-\Big[\bar{\theta}_H\int_0^1 f(\theta)h(\theta)d\theta+\bar{\theta}_L\int_0^1 f(\theta)[1-h(\theta)]d\theta\Big]\Big]=0 \end{aligned}$$

where $l(x) = a + bx$ with positive constant coefficients a and b. □

Should the failure function be strictly convex, it cannot be said which system produces the lower aggregate failure rate. The answer to this question mainly depends on the quality of the test. While individuals with low ability will generally benefit from the hierarchical system if they exclusively attend the lower school due to the lower academic level, losses from sending the wrong students to the higher school may be substantial and overcompensate the gains.

Consider the following simple example: Suppose there are only two ability levels, 0 and 1. Half of all pupils have low ability ($\theta = 0$), and half of all pupils are talented ($\theta = 1$). Mean ability is $1/2$, implying a failure rate of $\frac{1}{2}[l(\frac{1}{2}) + l(-\frac{1}{2})]$ under the comprehensive school system. Should the test under the hierarchical system be perfect, all pupils with high ability are sent to the higher school, while the others will attend the lower school. The failure rate will then be $l(0)$. This failure probability must fall short of the corresponding rate under the comprehensive system by the definition of a strictly convex function. Now consider the case where all talented individuals are sent to the higher school, while half of those with low ability are also selected for this school. Average ability in the higher school then amounts to $2/3$, implying an aggregate failure rate of $\frac{1}{4}l(0)+\frac{1}{4}l(\frac{2}{3})+\frac{1}{2}l(-\frac{1}{3})$. Choosing a sufficiently large value for $l(\frac{2}{3})$ ensures that this failure probability can be higher than under the comprehensive system.

Turning now to the problem which school system leads to a higher expected total income, we cannot expect that a unique answer exists. However, Proposition 2 indicates that the comprehensive school system has a clear advantage.

Proposition 2 *If there is no peer group effect, the failure function is linear, and the wage function exhibits additive separability, then a comprehensive school system yields a higher output than a hierarchical school system.*

Proof: Due to Proposition 1, a linear failure function implies identical total failure rates. Consider the difference between average failure rates for some type θ,

$$D(\theta) = h(\theta)l(\bar{\theta}_H - \theta) + (1 - h(\theta))l(\bar{\theta}_L - \theta) - l(\bar{\theta} - \theta) \tag{6}$$

Differentiating (6) with respect to θ yields

$$\frac{dD}{d\theta} = h'(\theta)[l(\bar{\theta}_L - \theta) - l(\bar{\theta}_H - \theta)] < 0. \tag{7}$$

All other terms cancel out since l' is a constant. It follows that a unique $\hat{\theta} \in (0, 1)$ exists, where the failure rate is higher in the comprehensive system for types $\theta \in [0, \hat{\theta})$, while it is higher for types $\theta \in (\hat{\theta}, 1]$ in the hierarchical system. A linear failure function implies that the average failure rate will always be equal to $l(0)$ in each school. Given that the wage function exhibits additive seperability in θ and E, the optimality condition (3) then requires $E_H = E_L = E$. Thus, expenditure per pupil is the same in each school. In the absence of peer group effects, net output losses due to switching to the hierarchical system amount to

$$\int_0^1 f(\theta)[h(\theta)l(\bar{\theta}_H - \theta) + (1 - h(\theta))l(\bar{\theta}_L - \theta) - l(\bar{\theta} - \theta)][w(\theta, E) - w_0]d\theta.$$

This expression is positive since

$$\begin{aligned} &- \int_0^{\hat{\theta}} f(\theta)[h(\theta)l(\bar{\theta}_H - \theta) - (1 - h(\theta))l(\bar{\theta}_L - \theta) - l(\bar{\theta} - \theta)]d\theta \\ = &\int_{\hat{\theta}}^1 f(\theta)[h(\theta)l(\bar{\theta}_H - \theta) - (1 - h(\theta))l(\bar{\theta}_L - \theta) - l(\bar{\theta} - \theta)]d\theta > 0 \end{aligned}$$

and $\frac{\partial w}{\partial \theta} > 0$ hold. □

The result is easily understood. Choosing the hierarchical school system rather than the comprehensive school system reduces failure rates of less talented individuals but increases the failure rates of more talented individuals. Since, however, productivity losses in case of failure are larger for more talented individuals, this change is associated with a lower output.

While Proposition 2 has been derived under somewhat restrictive conditions, it obviously also holds if only slight modifications are introduced. Therefore, even if a moderate peer group effect exists, working stronger for more talented individuals, the comprehensive school system can still lead to a higher output. Furthermore, even if expenditure is more productive when being spent on high-ability students, a comprehensive school system may still be preferable. Hence, the result will hold for a relatively wide range of parameters.

Of course, the rank-order of school systems can be reversed if either strong peer group effects or other types of strong complementarities in human capital production exist.

4 Concluding remarks

We have seen that one main point in favor of a comprehensive school system lies in the possibility of failure in school by pupils with a high talent. The gains that arise from lower failure rates of talented individuals may even be decisive in case of some moderate peer group effect. Choosing a comprehensive rather than a hierarchical school system can be interpreted as self-protection (Ehrlich and Becker (1972)) for talented individuals. The probability of an accident is reduced at the expense of a lower payoff in the better state. The analysis suggests that it may be worthwhile to supplement a hierarchical school system by a program aiming at reducing failure rates in schools with a higher academic level.

Obviously the formulation that all dropouts have the same productivity is crucial in the model. On the one hand, it can be taken for granted that school dropouts suffer from substantial disadvantages in the labor market. Being earmarked as a dropout does not only reduce the chances to obtain vocational training, but can also have a demotivating effect, resulting in high depreciation rates on human capital. Hence, the conception that talented individuals will lose more human capital in absolute terms should they fail does not seem implausible. On the other hand, it must be admitted that the problems in the labor market may often be only temporary in nature.

At least, if a dropout chooses to become self-employed, it can be expected that differences in ability will be translated into differences in productivity. Moreover, the analysis has ignored that dropouts of the higher school may nevertheless have a chance to obtain a lower school diploma. Incorporating this aspect would clearly change the results in favor of the hierarchical system.

Another limitation of the model is that academic standards are not treated as policy variables. Politicians usually have some influence on academic standards, for example, by prescribing centralized examination procedures. Taking into account that teachers can exert substantial discretion with respect to academic levels, the specification chosen in the model does not seem implausible, however.

The analysis has not taken into account the aspect of asymmetric information after completion of school. Brunello and Giannini (1999) stress that a hierarchical school system makes it easier for students to signal their true productivity to prospective future employers. It can be argued, however, that this problem may be mitigated by introducing uniform examination procedures in order to make grades more informative.

Last, the model could be extended by allowing for free choice of schools by parents. Yet, it is questionable whether parents have superior information with respect to their children's abilities. Therefore, there is no reason to expect fundamental changes in the results under such an alternative structure.

References

Argys, L. M., Rees, D. I., and Brewer, D. J. (1996): "Detracking America's Schools: Equity at Zero Cost?" *Journal of Policy Analysis and Management* 15: 623-645.

Arnott, R., and Rowse, J. (1987): "Peer Group Effects and Educational Attainment." *Journal of Public Economics* 32: 287-305.

Brunello, G., and Giannini, M. (1999): "Selective Schools." Bonn: IZA Discussion Paper No. 76.

Costrell, R. (1994): "A Simple Model of Educational Standards." *American Economic Review* 84: 956-971.

de Bartolome, C. (1990): "Equilibrium and Inefficiency in a Community Model with Peer Group Effects." *Journal of Political Economy* 98: 110-133.

Effinger, M. R., and Polborn, M. K. (1999): "A Model of Vertically Differentiated Education." *Journal of Economics* 69: 53-69.

Ehrlich, I., and Becker, G. S. (1972): "Market Insurance, Self-Insurance, and Self-Protection." *Journal of Political Economy* 80: 623-648.

Epple, D., Newlon, E., and Romano, R. (2000): "Ability Tracking, School Competition, and the Distribution of Educational Benefits." Cambridge: NBER Working Paper No. 7854.

Henderson, V., Mieszkowski, P., and Sauvageau, Y. (1978): "Peer Group Effects and Educational Production Functions." *Journal of Public Economics* 10: 97-106.

Judson, R. (1998): "Economic Growth and Investment in Education: How Allocation Matters." *Journal of Economic Growth* 3: 337-359.

Summers, A. A., and Wolfe, B. L. (1977): "Do Schools Make a Difference?" *American Economic Review* 67: 639-652.