



Endogenous Public Information and Welfare

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CESIFO WORKING PAPER NO. 3492

CATEGORY 11: INDUSTRIAL ORGANISATION

ORIGINAL VERSION: JUNE 2011

THIS VERSION: APRIL 2012

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Abstract

This paper performs a welfare analysis of economies with private information when public information is endogenously generated and agents can condition on noisy public statistics in the rational expectations tradition. Equilibrium is not (restricted) efficient even when feasible allocations share similar properties to the market context (e.g., linear in information). The reason is that the market in general does not internalize the informational externality when public statistics (e.g., prices) convey information and does not balance optimally non-fundamental volatility and the dispersion of actions. Under strategic substitutability, equilibrium prices will tend to convey too *little* information when the “informational” role of prices prevails over its “index of scarcity” role and too *much* information in the opposite case. Under strategic complementarity, prices always convey too little information. The welfare loss at the market solution may be increasing in the precision of private information. These results extend to the internal efficiency benchmark (accounting only for the collective welfare of the active players). Received results—on the relative weights placed by agents on private and public information, when the latter is exogenous—may be overturned.

JEL-Code: D820, D830.

Keywords: information externality, strategic complementarity and substitutability, asymmetric information, excess volatility, team solution, rational expectations, behavioral traders.

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Revised April 2012

I am grateful to participants at seminars at Booth (U. of Chicago), NHH (Bergen), UCLA, and to Anna Bayona, Matthew Gentzkow, Jean-Paul L’Huillier, and Carolina Manzano for providing very useful comments, and to Rodrigo Escudero and Jorge Paz for their very able research assistance. The research leading to these results received funding from the European Research Council under the European Advanced Grants scheme, Grant Agreement no. 230254, and from project ECO2008-05155 of the Spanish Ministry of Education at the Public-Private Sector Research Center at IESE.

1. Introduction

There has been a recent surge of interest in the welfare analysis of economies with private information and in particular on the role of public information in such economies (see, e.g., Morris and Shin 2002; Angeletos and Pavan 2007; Amador and Weill 2010). Agents may fail to place welfare-optimal weights on private and public information owing to payoff and information externalities. In this paper we examine the issue in a context where public information is endogenously generated and agents can condition on public statistics when making their choices. In the rational expectations tradition, agents learn from prices and from public statistics in general, which are themselves the aggregate outcome of individual decisions.

Endogenous public information is relevant for a broad array of markets and situations. In financial markets, prices are noisy statistics that arise from the decisions of traders. In goods markets, prices aggregate information on the preferences of consumers and the quality of the products. In the overall economy, the release of GDP data is a noisy public signal that is the outcome of actions taken by economic agents.¹

Any welfare analysis of rational expectations equilibria faces several difficulties. First of all, it must employ a model capable of dealing in a tractable way with the dual role of prices as conveyors of information and determinants of traders' budget constraints. Grossman and Stiglitz (1980) were pioneers in this respect with their CARA-normal model. Second, we require a welfare benchmark against which to test market equilibria in a world with asymmetric information. The appropriate benchmark for measuring inefficiency at the market equilibrium is the team solution in which agents internalize collective welfare but must still rely on private information when making their own decisions (Radner 1979; Vives 1988; Angeletos and Pavan 2007). This is in the spirit of Hayek (1945), where the private signals of agents cannot be communicated to a center. The team-efficient solution internalizes the payoff and information externalities associated with the actions of agents in the market. Collective welfare may refer to the surplus of all market participants, active or passive, or may be restricted to the internal welfare of the active agents. The third challenge for such welfare analysis is dealing with the interaction of payoff and informational

¹ See, for example, Rodríguez-Mora and Schulstad (2007).

externalities. If we take as a benchmark a pure prediction model with no payoff externalities, then agents will typically rely too much on public information. The reason is that agents do not take into account that their reaction to private information affects the informativeness of public statistics and general welfare. In other words, agents do not internalize an information externality. Pure information externalities will make agents insufficiently responsive to their private information (Vives 1993, 1997; Amador and Weill 2011). We will see that payoff externalities complicate welfare analysis and may rebalance weightings in the opposite direction.

In this paper we consider a tractable linear-quadratic-Gaussian model that allows us to address the three challenges just described when public information is endogenously generated and influenced by the actions of agents. There is uncertainty about a common valuation parameter about which agents have private information, and the endogenous public statistic or “price” is noisy. We use a model with a rational expectations flavor but in the context of a well-specified game, where a continuum of agents compete in schedules, and allow actions to be strategic substitutes or complements. We focus our attention on linear Bayesian equilibria. The model is flexible and admits several interpretations in terms of firms competing in a homogenous product market, investment complementarities, monopolistic competition, traders (both rational and “behavioral”) in a financial market, and asset auctions.

We show that agents correct the slope of their strategy according to what they learn from the public statistic and the character of competition. Under strategic substitutes competition the price’s informational and index-of-scarcity roles conflict. With strategic substitutes and private information, a high price is bad news and the equilibrium schedule is steeper than with full information. In fact, in equilibrium schedules may slope the “wrong” way (e.g., downward for a supply schedule) when the informational role of prices dominates their index-of-scarcity role. This will occur when there is little noise in the public statistic. With strategic complements there is no conflict: a high price is good news, and the equilibrium schedule is flatter than with full information.

It is interesting that the impact on the slope of the equilibrium schedule of a change in the exogenous (prior) precision of public information is opposite to the change in the precision of the noise in the endogenous public signal; consequently, market depth is increasing in the former and decreasing in the latter. The reason is that an increase in the *exogenous* precision of public information decreases the informational component of the public statistic whereas an increase in the *endogenous* precision increases it. Furthermore, an increase in the degree of the game's complementarity will increase the response to private information and the dispersion of actions under strategic complements. The opposite results obtain under strategic substitutes.

Consider the collective welfare benchmark and an economy in which not only the full information equilibrium is efficient but also the equilibrium with private information when public information is exogenous (this is as in Vives 1988 or Section 5.3 in Angeletos and Pavan 2007). We show that market equilibria will not be team-efficient even when the allowed allocations have properties (e.g., being linear in information) similar to those of the market equilibrium. This is because the market in general does not internalize the informational externality that results from public statistics (e.g., prices) conveying information. Indeed, a competitive agent is an information taker while the precision of the public statistic is endogenous. The market equilibrium is characterized by the *privately* efficient use of private information. Team efficiency instead makes *socially* efficient use of private information. Market equilibria will be team-efficient only in exceptional circumstances (as when the information externality vanishes). This occurs, for example, when public information is exogenous. We find that, under strategic substitutability, equilibrium prices will tend to convey too little information when the informational role of prices prevails and too much information when its index-of-scarcity role prevails. At the boundary of those situations there is a knife-edge case where parameters are such that agents use vertical schedules (as in a Cournot game), non contingent on the price (public statistic), and therefore the information externality disappears. In this particular case constrained efficiency is restored. Under strategic complementarity, prices always convey too little information.

The intuition of the results is as follows. Consider a homogenous product market with random demand and a continuum of firms competing in supply schedules with increasing and symmetric marginal costs with uncertain intercept. Each firm receives

a private signal on the marginal cost intercept and this induces both allocative and productive inefficiency. Allocative inefficiency refers to a distorted total output and productive inefficiency refers to a distorted distribution of a given total output. The equilibrium in the complete information economy is efficient since it is competitive. In this equilibrium all firms produce the same amount since they all have full information on costs, which are symmetric. The team-efficient solution in an economy with asymmetric information optimally trades off the tension between the two sources of welfare loss, allocative and productive inefficiency, when firms respond to private information. Allocative inefficiency is proportional to non-fundamental price volatility and productive inefficiency to the dispersion of individual actions. We can see, therefore, the team-efficient solution trading off both sources of welfare loss. We have that a higher response to private information makes prices more informative and reduces allocative inefficiency (since the total quantity is closer to the full information first best), as well as non-fundamental price volatility, but at the same time the dispersion of quantities increases and with it productive inefficiency. The somewhat surprising possibility that prices are too informative arises then since at the market solution firms may respond excessively to private information generating too much productive inefficiency. In this case there is too little non-fundamental price volatility. This happens under strategic substitutability, when the dual role of prices conflict, if the index of scarcity role of prices dominates the information role. When this does not happen and prices convey too little information, which is always the case with strategic complementarity, then there is excessive volatility at the market solution.

More precise information, be it public or private, reduces the welfare loss at the team-efficient solution. The reason is that the direct impact of the increased precisions is to decrease the welfare loss and this is the whole effect since at the team-efficient solution the response to private and public information are already (socially) optimized. In contrast, at the market solution an increase in, say, the precision of private information will increase the response of an agent to his private signal and this will tend to increase the welfare loss when the market calls already for a too large response to private information. If this indirect effect is strong enough the welfare loss may be increasing with the precision of private information. In principle the same

effect could happen with the precision of public information but we can show that the indirect effect of changes in both the exogenous public precision of information and the precision of the noise in the endogenous public signal are always dominated by the direct effect. The result is that the welfare loss at the market solution is always decreasing with the precisions of public information.

Recent literature has examined the circumstances under which more public information actually reduces welfare (as in Burguet and Vives 2000; Morris and Shin 2002; Angeletos and Pavan 2007; Amador and Weill 2010, 2011). In Burguet and Vives (2000) a higher (exogenous) public precision may discourage private information acquisition and lead to a higher welfare loss in a purely informational externality model. In Morris and Shin (2002) the result is driven by a socially excessive incentive to coordinate by agents. Angeletos and Pavan (2007) qualify this result and relate it to the payoff externalities present in a more general model. In Amador and Weill (2010) a public release of information reduces the informational efficiency of prices and this effect may dominate the direct information provision effect. Their model is purely driven by information externalities in the presence of strategic complementarities in terms of responses to private information.² In our model more public information is not damaging welfare but more private precision may be. This happens when at the market solution there is already too much dispersion of actions and an increase in private precision exacerbates the problem.

The results can be extended to the internal team-efficient benchmark (where only the collective welfare of the players is taken into account, for example, ignoring passive consumers). In this case also, endogenous public information may overturn conclusions reached using exogenous information models (e.g., Angeletos and Pavan 2007) when the informational role of the price is in conflict and dominates its index of scarcity role.

The plan of the paper is as follows. Section 2 presents the model and the leading interpretation of firms competing in a homogenous product market. Section 3 characterizes the equilibrium and Section 4 its comparative statics properties and the

² Ganguli and Yang (2009) develop the implications of strategic complementarities for information acquisition in noisy rational expectations models.

value of information. Section 5 performs a welfare analysis, and Section 6 studies the internal team-efficient benchmark. Section 7 presents alternative interpretations of the model. Concluding remarks are given in Section 8. Proofs are gathered in the Appendix.

2. The model

Consider a quadratic payoff game with a continuum of players indexed within the interval $[0,1]$. Player i has the payoff function

$$\pi(x_i, \tilde{x}) = (\alpha - \theta + u)x_i - \beta \tilde{x}x_i - \frac{\lambda}{2}x_i^2,$$

where x_i is the individual action of the player, $\tilde{x} = \int_0^1 x_i di$ is the aggregate action, θ and u are parameters that, for the moment, are simply given, and α, λ are positive parameters. Then $\partial^2 \pi / (\partial x_i)^2 = -\lambda < 0$ and $\partial^2 \pi / \partial x_i \partial \tilde{x} = -\beta$, and the slope of the best reply of a player is $m \equiv (\partial^2 \pi / \partial x_i \partial \tilde{x}) / (-\partial^2 \pi / (\partial x_i)^2) = -\beta / \lambda$. Thus we have strategic substitutability (complementarity) for $\beta > 0$ (for $\beta < 0$), and m can be understood as the degree of complementarity in the payoffs. (In the rest of this paper, when discussing strategic substitutability or complementarity we refer to this meaning in the context of this certainty game). We assume that $m < 1/2$ or $2\beta + \lambda > 0$, limiting the extent of strategic complementarity. The condition $2\beta + \lambda > 0$ guarantees that $\pi(x, x)$ is strictly concave in x ($\partial^2 \pi / (\partial x)^2 = -(2\beta + \lambda) < 0$). Observe that there are no payoff externalities among players when $\beta = 0$.

Consider now a game with uncertainty and in which θ and u are random. The parameter θ is uncertain; it has prior Gaussian distribution with mean $\bar{\theta}$ and variance σ_θ^2 (we write $\theta \sim N(\bar{\theta}, \sigma_\theta^2)$ and, to ease notation, set $\bar{\theta} = 0$). Player i receives a signal $s_i = \theta + \varepsilon_i$ with $\varepsilon_i \sim N(0, \sigma_\varepsilon^2)$. Error terms are uncorrelated across players, and the random variables $\{\theta, \varepsilon_i, u\}$ are mutually independent. We establish the convention that error terms cancel in the aggregate: $\int_0^1 \varepsilon_i di = 0$ almost surely (a.s.).

Then the aggregation of all individual signals will reveal the underlying uncertainty: $\int_0^1 s_i di = \theta + \int_0^1 \varepsilon_i = \theta$.³

Players have access to the (endogenous) public statistic $p = \alpha + u - \beta \tilde{x}$, where $u \sim N(0, \sigma_u^2)$; this can be interpreted as the marginal benefit of taking action level x_i , which has cost $\theta x_i + (\lambda/2)x_i^2$.⁴ When $\beta = 0$, there are no informational externalities among players.

The payoff to player i can be written as

$$\pi_i = px_i - \theta x_i - \frac{\lambda}{2} x_i^2$$

where p is the public statistic, and the dual role of β as both a parameter in the payoff function and in the public statistic should be noted. This situation arises naturally in the applications.

The timing of the game is as follows. At $t = 0$, the random variables θ and u are drawn but not observed. At $t = 1$, each player observes his own private signal s_i and submits a schedule $X_i(s_i, \cdot)$ with $x_i = X_i(s_i, p)$, where p is the public statistic. The strategy of a player is a map from the signal space to the space of schedules. Finally, the public statistic is formed (the “market clears”) by finding a p that solves $p = \alpha + u - \beta \left(\int_0^1 X_j(s_j, p) dj \right)$, and payoffs are collected at $t = 1$.

³ That is, I assume that the strong law of large numbers (SLLN) holds for a continuum of independent random variables with uniformly bounded variances. Suppose that $(q_i)_{i \in [0,1]}$ is a process of independent random variables with means $E[q_i]$ and uniformly bounded variances $\text{var}[q_i]$. Then we let $\int_0^1 q_i di = \int_0^1 E[q_i] di$ a.s. This convention will be used while taking as given the usual linearity property of integrals. Equality of random variables must be assumed to hold almost surely. It can be checked that the results obtained in the continuum economy are the limit of finite economies under the usual SLLN.

⁴ Normality of random variables means that prices and quantities can be negative with positive probability. The probability of this event can be controlled, if necessary, by an appropriate choice of means and variances. Furthermore, for this analysis the key property of Gaussian distributions is that conditional expectations are linear. Other prior-likelihood conjugate pairs (e.g., beta-binomial and gamma-Poisson) share this linearity property and can display bounded supports.

Let us assume that there is a unique public statistic $\hat{p}\left(\left(X_j(s_j, \cdot)\right)_{j \in [0,1]}\right)$ for any realization of the signals.⁵ Then, for a given profile $\left(X_j(s_j, \cdot)\right)_{j \in [0,1]}$ of players' schedules and realization of the signals, the profits for player i are given by

$$\pi_i = (p - \theta)x_i - \frac{\lambda}{2}x_i^2,$$

where $x_i = X_i(s_i, p)$, $\tilde{x} = \int_0^1 X_j(s_j, p) dj$, and $p = \hat{p}\left(\left(X_j(s_j, \cdot)\right)_{j \in [0,1]}\right)$. This formulation has a rational expectations flavor but in the context of a well-specified schedule game. We will restrict our attention to linear Bayesian equilibria of the schedule game. The model admits several interpretations and we present below the leading one linking supply function competition and rational expectations (see Section 6 for the other interpretations).⁶

Firms competing in a homogenous product market with quadratic production costs.

In this case, $p = \alpha + u - \beta\tilde{x}$ is the inverse demand for the homogenous product, x_i is the output of firm i , and the cost function of firm i is given by $C(x_i) = \theta x_i + (\lambda/2)x_i^2$. Firms use supply functions as strategies, and markets clear: $p = \alpha + u - \beta\left(\int_0^1 X_i(s_i, p) di\right)$. Costs are random and firm i has a noisy estimate of the intercept of marginal cost $s_i = \theta + \varepsilon_i$ at the time of submitting the supply function. If $\beta > 0$, then demand is downward sloping and we have strategic substitutability in the usual partial equilibrium market. If $\beta < 0$, we have strategic complementarity and demand is upward sloping. The latter situation may arise in the case of a network good with compatibility.

We will maintain a supply interpretation of the model up to Section 6. We let $p \equiv \alpha + u - \beta\tilde{x}$ be the marginal benefit or “price” of taking an action and let $MC(x_i) \equiv \theta + \lambda x_i$ be the marginal cost.

⁵ We assign zero payoffs to the players if there is no p that solves the fixed point problem. If there are multiple solutions, then the one that maximizes volume is chosen.

⁶ See Chapter 3 in Vives (2008) for an overview of the connection between supply function competition and rational expectations models, as well as examples.

3. Equilibrium

We are interested in a linear (Bayesian) equilibrium—equilibrium, for short—of the schedule game for which the public statistic functional is of type $P(\theta, u)$. Since the payoffs and the information structure are symmetrical and since payoffs are strictly concave, there is no loss of generality in restricting our attention to symmetric equilibria. Indeed, the solution to the problem of player i ,

$$\max_{x_i} E \left[\left(p - \theta - \frac{\lambda}{2} x_i \right) x_i \mid s_i, p \right],$$

is both unique (given strict concavity of profits) and symmetric across players (since the cost function and signal structure are symmetric across firms):

$$X(s_i, p) = \lambda^{-1} (p - E[\theta \mid s_i, p]),$$

where $p = P(\theta, u)$. A strategy for player i may be written as

$$x_i = \hat{b} + \hat{c}p - as_i,$$

in which case the aggregate action is given by

$$\tilde{x} = \int_0^1 x_i di = \hat{b} + \hat{c}p - a\theta.$$

It then follows from $p = \alpha + u - \beta\tilde{x}$ that, provided $\hat{c} \neq -\beta^{-1}$,

$$p = P(\theta, u) = (1 + \beta\hat{c})^{-1} (\alpha - \beta\hat{b} + z);$$

here the random variable $z = \beta a\theta + u$ is informationally equivalent to the “price” or public statistic p . Because u is random, z (and the public statistic) will typically generate a noisy signal of the unknown parameter θ .

Market depth—that is, the inverse of how much the price moves to accommodate a unit increase in u —is given by $(\partial P / \partial u)^{-1} = 1 + \beta\hat{c}$.⁷ Excess demand is given by

$$\Xi(p) \equiv \beta^{-1} (\alpha + u - p) - \hat{b} + a\theta - \hat{c}p.$$

The information available to player i is $\{s_i, p\}$ or, equivalently, $\{s_i, z\}$. Since

$E[\theta \mid s_i, p] = E[\theta \mid s_i, z]$, we can posit strategies of the form

⁷ See, for example, Kyle (1985).

$$X(s_i, z) = b - as_i + cz$$

and obtain that $p = \alpha - \beta b + (1 - \beta c)z$. If $1 + \beta \hat{c} > 0$ then $1 - \beta c > 0$ (since $\hat{c} = (c^{-1} - \beta)^{-1}$ and $1 + \beta \hat{c} = (1 - \beta c)^{-1}$) and so p and z will move together. The strategy of player i is then given by

$$X(s_i, z) = \lambda^{-1}(\alpha - \beta b + (1 - \beta c)z - E[\theta | s_i, z]).$$

We can solve for the LE in the usual way: identifying coefficients with the candidate linear strategy $x_i = b - as_i + cz$ by calculating $E[\theta | s_i, z]$ and using the supply function of a player.

The following proposition characterizes the equilibrium.

Proposition 1. *Let $\tau_\varepsilon \geq 0$ and $\tau_u \geq 0$. Then there is a unique (and symmetric) equilibrium*

$$X(s_i, p) = \lambda^{-1}(p - E[\theta | s_i, p]) = \hat{b} - as_i + \hat{c}p,$$

where a is the unique (real) solution of the equation $a = \tau_\varepsilon \lambda^{-1}(\tau_\varepsilon + \tau_\theta + \tau_u \beta^2 a^2)^{-1}$,

$\hat{c} = ((\beta + \lambda)(1 - \beta \lambda \tau_u a^2 \tau_\varepsilon^{-1})^{-1} - \beta)^{-1}$, and $\hat{b} = \alpha(1 - \lambda \hat{c})/(\beta + \lambda)$. In equilibrium,

$a \in (0, \tau_\varepsilon \lambda^{-1}(\tau_\theta + \tau_\varepsilon)^{-1})$ and $1 + \beta \hat{c} > 0$.

Remark 1. We have examined linear equilibria of the schedule game for which the public statistic function is of type $P(\theta, u)$. In fact, these are the equilibria in strategies with bounded means and with uniformly (across players) bounded variances. (See Claim 1 in the Appendix.)

Remark 2. We can show that the equilibrium in the continuum economy is the limit of equilibria in replica economies that approach the limit economy. Take the homogenous market interpretation with a finite number of firms n and inverse demand $p_n = \alpha + u - \beta \tilde{x}_n$, where \tilde{x}_n is the average output per firm, and with the same informational assumptions. In this case, given the results in Section 5.2 of Vives

(2011), the supply function equilibrium of the finite n -replica market converges to the equilibrium in Proposition 1.

The public statistic or price serves a dual role as index of scarcity and conveyor of information. Indeed, a high price has the direct effect of increasing an agent's competitive supply, but it also conveys news about costs—namely, that costs are high (low) if $\beta > 0$ ($\beta < 0$). In equilibrium, the “price impact” (or inverse of the depth of the market) is always positive, $\partial P / \partial u = (1 + \beta \hat{c})^{-1} > 0$, and excess demand is downward or upward sloping depending on β : $\Xi' = -(\beta^{-1} + \hat{c})$ or $\text{sgn}\{\Xi'\} = \text{sgn}\{-\beta\}$. That is, the slope's direction depends on whether the competition is in strategic substitutes or in strategic complements.

In equilibrium, agents take public information z , with precision $\tau \equiv (\text{var}[\theta|z])^{-1}$, as given and use it to form probabilistic beliefs about the underlying uncertain parameter θ . We have that $E[\theta/s_i, z] = \gamma s_i + (1 - \gamma)E[\theta/z]$ with $\gamma = \tau_\varepsilon (\tau_\varepsilon + \tau)^{-1}$. Revised beliefs and optimization, in turn, determine the coefficients a and c for private and public information, respectively. In equilibrium, the informativeness of public information z depends on the sensitivity of strategies to private information a : $\tau = \tau_\theta + \tau_u \beta^2 a^2$. Agents behave as information takers and so, from the perspective of an individual agent, public information is exogenous. This fact is at the root of the equilibrium's *informational externality*. That is, agents fail to account for the impact of their own actions on public information and hence on other agents.

Consider as a benchmark the full information case with perfectly informative signals ($\tau_\varepsilon = \infty$). This puts us in a full information competitive equilibrium and we have $c = (\beta + \lambda)^{-1}$, $a = \hat{c} = \lambda^{-1}$, and $X(\theta, p) = \lambda^{-1}(p - \theta)$. In this case, agents have nothing to learn from the price. If signals become noisy ($\tau_\varepsilon < \infty$) then $a < \lambda^{-1}$ and $\hat{c} < \lambda^{-1}$ for $\beta > 0$, with supply functions becoming steeper (lower \hat{c}) as agents protect themselves from adverse selection. The opposite happens ($\hat{c} > \lambda^{-1}$ and flatter

supply functions) when $\beta < 0$, since then a high price is good news (entailing lower costs).⁸ There is then “favorable” selection.

Two other cases in which $\hat{c} = \lambda^{-1}$ and there is no learning from the price are when signals are uninformative about the common parameter θ ($\tau_\varepsilon = 0$) and when the public statistic is extremely noisy ($\tau_u = 0$). In the first case, the price has no information to convey; $a = 0$ and $X(s_i, p) = \lambda^{-1}(p - \bar{\theta})$. In the second case, public information is pure noise, $a = \lambda^{-1}\tau_\varepsilon(\tau_\theta + \tau_\varepsilon)^{-1}$, with $X(s_i, p) = \lambda^{-1}(p - E[\theta|s_i])$.⁹ In all three cases, there is no information externality via the public statistic.

As τ_u tends to ∞ , the precision of prices τ also tends to ∞ , the weight given to private information a tends to 0, and the equilibrium collapses (with $1 + \beta\hat{c} \rightarrow 0$). Indeed, the equilibrium becomes fully revealing and is not implementable.

4. Comparative statics and the value of information

This section studies the comparative statics properties of the equilibrium and how the weights and the responses to public and private information vary with underlying parameters. The following proposition presents a first set of results. The effects of changes in the degree of complementarity are dealt with afterwards.

Proposition 2. *Let $\tau_\varepsilon > 0$ and $\tau_u > 0$. In equilibrium, the following statements hold.*

- (i) *Responsiveness to private information a decreases from $\lambda^{-1}\tau_\varepsilon(\tau_\theta + \tau_\varepsilon)^{-1}$ to 0 as τ_u ranges from 0 to ∞ , decreases with τ_θ , $|\beta|$ and λ , and increases with τ_ε .*

⁸ This follows because, with upward-sloping demand, we assume that $2\beta + \lambda > 0$ and therefore $\lambda > -\beta$.

⁹ The same happens when $\beta = 0$ (in which case there is no payoff externality, either).

- (ii) *Responsiveness to the public statistic \hat{c} goes from λ^{-1} to $-\beta^{-1}$ as τ_u ranges from 0 to ∞ . Furthermore, $\text{sgn}\{\partial\hat{c}/\partial\tau_u\} = \text{sgn}\{-\partial\hat{c}/\partial\tau_\theta\} = \text{sgn}\{-\beta\}$ and $\text{sgn}\{\partial\hat{c}/\partial\tau_\varepsilon\} = \text{sgn}\left\{\beta\left(\beta^2\tau_u\tau_\varepsilon^2 + 4\lambda^2\tau_\theta^2(\tau_\varepsilon - \tau_\theta)\right)\right\}$. Market depth $1 + \beta\hat{c}$ is decreasing in τ_u and increasing in τ_θ .*
- (iii) *Price informativeness τ is increasing in $|\beta|$, τ_u , τ_θ and τ_ε , and decreasing in λ .*
- (iv) *Dispersion $E\left[(x_i - \tilde{x})^2\right]$ decreases with τ_u , τ_θ , $|\beta|$ and λ .*

How the equilibrium weights to private and public information vary with the deep parameters of the model help to explain the results. We have that $E[\theta/s_i, z] = \gamma s_i + hz$ where $h = \beta a \tau_u (\tau_\varepsilon + \tau)^{-1}$. Identify the informational component of the price with the weight $|h|$ on public information z , with $\text{sgn}\{h\} = \text{sgn}\{\beta\}$. When $\beta > 0$ there is adverse selection (a high price is bad news about costs) and $h > 0$ while when $\beta < 0$, $h < 0$ and there is favorable selection (a high price is good news). We have that $\text{sgn}\{\partial|h|/\partial\beta\} = \text{sgn}\{\beta\}$. As β is decreased from $\beta > 0$ adverse selection is lessened, and when $\beta < 0$ we have favorable selection with $h < 0$ and $\partial|h|/\partial\beta < 0$. The result is that an increase in $|\beta|$ increases the public precision¹⁰ τ and decreases the response to private information. We have also that increasing the precision of the prior decreases the informational component of the price, $\partial|h|/\partial\tau_\theta < 0$, while that increasing the precision of the noise in the price increases it, $\partial|h|/\partial\tau_u > 0$. (See Claim 2 in the Appendix.) The effect of τ_ε is ambiguous.

In order to gain further intuition from these results, we first consider the case $\beta > 0$. As τ_u increases from 0, \hat{c} decreases from λ^{-1} (and the slope of supply increases) because of the price's increased informational component $h > 0$. Agents are more

¹⁰ An increase in $|\beta|$ has a direct positive effect on τ and an indirect negative effect via the induced change in a . The direct effect prevails. Note that changing β modifies not only the public statistic p but also the degree of complementarity in the payoff.

cautious when seeing a high price because it may mean higher costs. As τ_u increases more, \hat{c} becomes zero at some point and then turns negative; as τ_u tends to ∞ , \hat{c} tends to $-\beta^{-1}$.¹¹ At the point where the scarcity and informational effects balance, agents place zero weight ($\hat{c} = 0$) on the public statistic. In this case, agents do not condition on the price and the model reduces to a quantity-setting model à la Cournot (however, not reacting to the price is optimal). If τ_θ increases then the informational component of the price diminishes since the agents are now endowed with better prior information, and induces a higher \hat{c} (and a more elastic supply). An increase in the precision of private information τ_ε always increases responsiveness to the private signal but has an ambiguous effect on the slope of supply. The parameter \hat{c} is U-shaped with respect to τ_ε . Observe that $\hat{c} = \lambda^{-1}$ not only when $\tau_\varepsilon = \infty$ but also when $\tau_\varepsilon = 0$ and that $\hat{c} < \lambda^{-1}$ for $\tau_\varepsilon \in (0, \infty)$. If τ_ε is high, then a further increase in τ_ε (less noise in the signals) lowers adverse selection (and h) and increases \hat{c} . If τ_ε is low then the price is relatively uninformative, and an increase in τ_ε increases adverse selection (and h) while lowering \hat{c} .

If $\beta < 0$ then a high price conveys goods news in terms of both scarcity effects and informational effects, so supply is always upward sloping in this case. Indeed, when $\beta < 0$ we have $\hat{c} > \lambda^{-1}$. A high price conveys the good news that average quantity tends to be high and that costs therefore tend to be low ($h < 0$). In this case, increasing τ_u , which reinforces the informational component of the price, increases \hat{c} —the opposite of what happens when τ_θ increases. An increase in the precision of private information τ_ε increases responsiveness to the private signal but, as before, has an ambiguous effect on the slope of supply. Now the parameter \hat{c} is hump-shaped with respect to τ_ε because $\hat{c} > \lambda^{-1}$ for $\tau_\varepsilon \in (0, \infty)$ and $\hat{c} = \lambda^{-1}$ in the extremes of the interval $(0, \infty)$.

¹¹ See Wilson (1979) for a model in which adverse selection makes demand schedules upward sloping.

In either case ($\beta > 0$ or $\beta < 0$) market depth $(\partial P/\partial u)^{-1} = 1 + \beta\hat{c}$ is decreasing in τ_u and increasing in τ_θ .¹²

Table 1 summarizes the comparative statics results on the equilibrium strategy.

Table 1: Comparative Statics on the Equilibrium Strategy

sgn	$\partial\tau_u$	$\partial\tau_\theta$	$\partial\tau_\varepsilon$
∂a	-	-	+
$\partial\hat{c}$	$-\beta$	β	$\beta(\beta^2\tau_u\tau_\varepsilon^2 + 4\lambda^2\tau_\theta(\tau_\varepsilon - \tau_\theta))$

The degree of complementarity $m \equiv -\beta/\lambda$ depends on λ for a fixed β (it makes sense to keep β fixed since β also affects the public statistic $p = \alpha + u - \beta\tilde{x}$). For fixed β we have that $\text{sgn}\{\partial m/\partial\lambda\} = \text{sgn}\{\beta\}$. From Proposition 2 we have then that $\text{sgn}\{\partial a/\partial m\} = \text{sgn}\{-\beta\}$, $\text{sgn}\{\partial\gamma/\partial m\} = \text{sgn}\{\beta\}$, and $\text{sgn}\{\partial\tau/\partial m\} = \text{sgn}\left\{\partial E\left[(x_i - \tilde{x})^2\right]/\partial m\right\} = \text{sgn}\{-\beta\}$. The results are summarized in Table 2.

Table 2: Effects of a Change in the Degree of Complementarity (β fixed)

sgn	∂a	$\partial\gamma$	$\partial\tau$	$E\left[(x_i - \tilde{x})^2\right]$
∂m	$-\beta$	β	$-\beta$	$-\beta$

Increased reliance on public information as complementarity increases is a general theme in the work of Morris and Shin (2002) and Angeletos and Pavan (2007) when public signals are exogenous. In stylized environments more complementarity increases the value of public information in forecasting aggregate behavior and decreases the dispersion of actions (e.g., Cor. 1 in Angeletos and Pavan 2007). In our

¹² It can also be checked that when $\beta < 0$ market depth is increasing with β .

model this happens in the strategic substitutes case ($\beta > 0$). With strategic complements ($\beta < 0$) an increase in m (a lower λ) makes agents rely less on private information (γ decreases) but respond more to private information (a increases), and increases dispersion as well as increases the precision of public information. (See Table 2.)

5. Welfare analysis

Consider the homogeneous product market with quadratic production costs. The inverse demand $p = \alpha + u - \beta \tilde{x}$ arises from a benefit or surplus function $(\alpha + u - (\beta/2)\tilde{x})\tilde{x}$, and the welfare criterion is total surplus:

$$\text{TS} = \left(\alpha + u - \beta \frac{\tilde{x}}{2} \right) \tilde{x} - \int_0^1 \left(\theta x_i + \frac{\lambda}{2} x_i^2 \right) di.$$

Under our assumptions, $\beta + \lambda > 0$ and the TS function is strictly concave for symmetric solutions.

The equilibrium is partially revealing (with $0 < \tau_u < \infty$ and $0 < \tau_\varepsilon < \infty$), so expected total surplus should be strictly greater in the first-best allocation (full information) $x^o = (\lambda + \beta)^{-1}(\alpha + u - \theta)$, which is just the market solution with full information, than at the LE. The reason is that suppliers produce under uncertainty and rely on imperfect idiosyncratic estimation of the common cost component; hence they end up producing different amounts even though costs are identical and strictly convex. However, since producers are competitive they produce in expected value the right amount at the equilibrium: $E[\tilde{x}] = E[x^o] = \alpha(\lambda + \beta)^{-1}$.

The welfare benchmark that we use is the team solution maximizing expected total surplus subject to employing linear decentralized strategies (as in Vives 1988; Angeletos and Pavan 2007). This team-efficient solution internalizes the information externalities of the actions of agents, and it is restricted to using the same type of strategies (decentralized and linear) that the market employs. Indeed, when reacting to information, an agent in the market does not take into account the influence her own actions have on public statistics.

It is worth noting that in the economy considered if firms would not condition on prices, i.e. if each firm would set quantities conditioning only on its private information, then the market solution would be team-efficient (Vives 1988). This will not be the case in general when public information is endogenous because of information externalities.

At the team-efficient solution, expected total surplus $E[\text{TS}]$ is maximized under the constraint that firms use decentralized linear production strategies. That is,

$$\begin{aligned} & \max_{a,b,c} E[\text{TS}] \\ \text{subject to } & x_i = b - as_i + cz, \quad \tilde{x} = b - a\theta + cz, \quad \text{and } z = u + \beta a\theta. \end{aligned}$$

Equivalently, the team-efficient solution minimizes, over the restricted strategies, the expected welfare loss WL with respect to the full information first best. It is possible to show that

$$\text{WL} = \left((\beta + \lambda) E\left[(\tilde{x} - x^o)^2 \right] + \lambda E\left[(x_i - \tilde{x})^2 \right] \right) / 2,$$

where the first term in the sum corresponds to allocative inefficiency (how distorted is the average quantity \tilde{x} while producing in a cost-minimizing way), which is proportional to $E\left[(\tilde{x} - x^o)^2 \right]$, and the second term to productive inefficiency (how distorted is the distribution of production of a given average quantity \tilde{x}), which is proportional to the dispersion of outputs $E\left[(x_i - \tilde{x})^2 \right]$. Let p^o be the full information first best price. Note that the non-fundamental price volatility is given by $E\left[(p - p^o)^2 \right] = \beta^2 E\left[(\tilde{x} - x^o)^2 \right]$ and therefore it is proportional to allocative inefficiency.

It is easily seen that the form of the optimal team strategy is $x_i = \lambda^{-1} \left[p - (\gamma s_i + (1 - \gamma) E[\theta | z]) \right]$ where the weight to private information $\gamma = \lambda a$ may differ from the market weight. Note that both in the market and the team solutions we have that $\gamma = \lambda a$. It follows then that the welfare loss at any candidate

team solution will depend only on the response to private information a since we have $E\left[(\tilde{x} - x^o)^2\right] = (1 - \lambda a)^2 / \left(\tau(\beta + \lambda)^2\right)$, $\tau = \tau_\theta + \tau_u \beta^2 a^2$, and $E\left[(x_i - \tilde{x})^2\right] = a^2 / \tau_\varepsilon$. This yields a strictly convex WL as a function of a . Changing a has opposite effects on both sources of the welfare loss since allocative inefficiency decreases with a , as price informativeness τ increases and the average quantity gets close to the full information allocation, but productive inefficiency increases with a as dispersion increases. Note that a more informative price reduces allocative inefficiency and non-fundamental price volatility but increases productive inefficiency. The team solution optimally trades them off among decentralized strategies.

If there was no information externality a would not affect τ (which would be exogenous). In this case it is easy to see that the team and the market solution coincide with $a = \lambda^{-1} \tau_\varepsilon (\tau_\theta + \tau_\varepsilon)^{-1}$. Otherwise there is an information externality and the market is inefficient.

The sign of the information externality can be found easily by breaking down the impact of the sensitivity to private information a on $E[\text{TS}]$ between the market effect, where the public statistic z is taken as given, and the information externality effect (IE), where the impact on z is taken into account.

$$\frac{\partial E[\text{TS}]}{\partial a} = E \left[\underbrace{(p - \text{MC}(x_i)) \left(\frac{\partial x_i}{\partial a} \right)_{z \text{ ct.}}}_{\text{Market}} \right] + E \left[\underbrace{(p - \text{MC}(x_i)) \left(\frac{\partial x_i}{\partial z} \frac{\partial z}{\partial a} \right)}_{\text{IE}} \right].$$

The market term is null at the market solution (denoted $*$) and the IE term can be evaluated as follows:

$$\text{sgn} \left\{ \left. \frac{\partial E[\text{TS}]}{\partial a} \right|_{a=a^*} \right\} = \text{sgn} \{ \text{IE} \} = \text{sgn} \{ -\beta c^* \}.$$

The sign of the informational externality depends on whether we have strategic substitutes or complements competition and on whether supply slopes upwards or downwards. If $\beta > 0$ there is adverse selection and a high price indicates high costs. If supply is upward sloping ($c^* > 0$) and, say, costs are high ($\theta - \bar{\theta} > 0$) then an increase in a will increase x_i ($\frac{\partial x_i}{\partial z} \frac{\partial z}{\partial a} = c\beta(\theta - \bar{\theta}) > 0$) while $(p - MC(x_i))$ will tend to be low (since at the market solution $E[(p - MC(x_i))(\theta - \bar{\theta})] < 0$). This means that $IE < 0$ and that a must be reduced. If supply is downward sloping ($c^* < 0$) in the same situation an increase in a will decrease x_i , which is welfare enhancing. The same will happen if $\beta < 0$ since then $c^* > 0$ and an increase in a will decrease x_i . In the last two situations $IE > 0$ and a must be increased.

The following proposition characterizes the response to private information at the team solution (superscript T) and compares it with the equilibrium solution (superscript $*$).

Proposition 3. *Let $\tau_\varepsilon > 0$. Then the team problem has a unique solution with $\lambda^{-1} > a^T > 0$, and $\text{sgn}\{a^* - a^T\} = \text{sgn}\{\beta c^*\}$.*

If $\beta = 0$ then there no informational externality, and the team and market solutions coincide. For $\beta \neq 0$, $\tau_\varepsilon > 0$, and $\tau_u > 0$, the solutions coincide only if $c^* = 0$. This occurs only at the equilibrium $a = \tau_\varepsilon / (\lambda(\tau_\varepsilon + \tau_\theta) + \beta\tau_\varepsilon)$ (with $\beta > 0$). When firms do not respond to the price ($c = 0$), the model reduces to a quantity-setting model with private information. This is consistent with Vives (1988), where it is shown that a Cournot market with private information and a continuum of suppliers solves a team problem whose objective function is expected total surplus. If $c^* < 0$ then a should be increased, and the contrary holds for $c^* > 0$.

At the equilibrium with strategic substitutability, for which $\beta > 0$, and since then c^* is decreasing in τ_u , there is too much (not enough) weight given to private

information whenever τ_u is small (large) and supply functions are increasing (decreasing). In the second case the market displays too much allocative inefficiency (the price contains too little information); in the first too little, the price is too informative, and there is too much productive inefficiency. With strategic complementarity ($\beta < 0$) we have that both $c^* > 0$ and $\text{sgn}\{\partial E[\text{TS}]/\partial a\} = \text{sgn}\{-\beta c^*\} > 0$ always, agents give insufficient weight to private information and the market displays too much allocative inefficiency.

There is no information externality when firms have perfect information ($\tau_\varepsilon = \infty$) and the full information, first-best outcome (price equal to marginal cost) is obtained; when the price contains no information ($\tau_u = 0$); or when signals are uninformative ($\tau_\varepsilon = 0$). In each of these cases, the team and the market solution coincide in terms of $E[\text{TS}]$. For both the team and the market solutions $c = 1/(\beta + \lambda)$, if $\tau_u = 0$ then $E[\text{TS}]$ is infinite; if $\tau_\varepsilon = 0$, then $a = 0$. The case $\tau_u = 0$ is akin to the case with exogenous public information where the market allocation is constrained efficient (Vives (1988), Angeletos and Pavan (2007)). Constrained efficiency no longer holds when the information externality is present.

The conclusion is that, with strategic substitutability, team efficiency requires a decrease (increase) in c when c^* is negative (positive). When $c^* < 0$, the informational role of the price dominates and the price reveals too little information. In this case, more weight should be given to private signals so that public information becomes more revealing to reduce allocative inefficiency. Conversely, when the price is mainly an index of scarcity, $c^* > 0$, it reveals too much information and a should be decreased to reduce excessive dispersion. Only in the knife-edge (Cournot) case, where $c^* = 0$, is the equilibrium team-efficient. With strategic complementarity, agents place too little weight on private information. When $\beta < 0$, the informational externality is aligned with the price scarcity effect; in this case, it is always preferable to induce agents to rely more on their private information to reduce allocative inefficiency.

Remark 3. If the signals of agents can be communicated to a center, then questions arise concerning the incentives to reveal information and how welfare allocations may be modified. This issue is analyzed in a related model by Messner and Vives (2006), who use a mechanism design approach along the lines of Laffont (1985).

The question arises as of how the welfare loss WL at the market solution depends on information precisions τ_ε, τ_u and τ_θ . We know that WL at a linear allocation as a function of a is given by the strictly convex function

$$WL(a) = \frac{1}{2} \left(\frac{(1 - \lambda a)^2}{(\tau_\theta + \tau_u \beta^2 a^2)(\beta + \lambda)} + \frac{\lambda a^2}{\tau_\varepsilon} \right).$$

It is immediate then that at the team-efficient solution $WL(a^T)$ is decreasing in τ_ε, τ_u and τ_θ . This is so since WL is decreasing in τ_ε, τ_u and τ_θ for a given a and $WL'(a^T) = 0$. Things are potentially different at the market solution a^* since then $WL'(a^*) > 0$ or $WL'(a^*) < 0$ depending on whether $a^* > a^T$ or $a^* < a^T$. Since a^* is decreasing in τ_u and τ_θ , and increasing in τ_ε we have thus that $WL(a^*)$ is decreasing in τ_u and τ_θ when $a^* > a^T$ and in τ_ε when $a^* < a^T$. It is possible in principle that increasing precisions of public information τ_u and τ_θ increases the welfare loss when $a^* < a^T$ when the direct effect of the increase of τ_u or τ_θ is dominated by the indirect effect via the induced decrease in a^* (and similarly for an increase in τ_ε when $a^* > a^T$). We can check, however, that $WL(a^*)$ is always decreasing in τ_θ and τ_u because the direct effect always dominates the indirect effect. This need not be the case when changing τ_ε .

Proposition 4. The welfare loss at the team-efficient solution is decreasing in τ_ε, τ_u and τ_θ . The welfare loss at the market solution is also decreasing in τ_θ and τ_u and it may be decreasing or increasing in τ_ε (it will be increasing for $\beta > \lambda$ and $\tau_\varepsilon/\tau_\theta$ or τ_u small enough).

5. Internal welfare benchmark

A different benchmark is provided by the collective welfare of the players, the producers in our example. At the internal team-efficient solution, expected average profit $E[\tilde{\pi}]$ (where $\tilde{\pi} = \int_0^1 \pi_i di$ and $\pi_i = (\alpha + u - \beta\tilde{x} - \theta)x_i - (\lambda/2)x_i^2$) is maximized under the constraint that agents use decentralized linear strategies. Since the solution is symmetric we have that $E[\tilde{\pi}] = E[\pi_i]$. This is the cooperative solution from the players' perspective. That is,

$$\begin{aligned} & \max_{a,b,c} E[\pi_i] \\ \text{subject to } & x_i = b - as_i + cz, \quad \tilde{x} = b - a\theta + cz, \quad \text{and} \quad z = u + \beta a\theta. \end{aligned}$$

It should be clear that the market solution, not even with complete information, will attain the full information cooperative outcome (denoted M for monopoly, for which $x^M = (\lambda + 2\beta)^{-1}(\alpha + u - \theta)$) where joint profits are maximized under full information. This is so since the market solution does not internalize the payoff externalities and therefore if $\beta \neq 0$ it will produce an expected output $E[\tilde{x}^*] = \alpha(\beta + \lambda)^{-1}$ which is too high (low) with strategic substitutes (complements) in relation to the optimal $E[x^M] = \alpha(2\beta + \lambda)^{-1}$. Furthermore, the market solution does not internalize the information externalities. At the *internal team* (IT) benchmark, joint profits are maximized and information externalities internalized with decentralized strategies.¹³ The question is whether the market solution allocates the correct weights (from the players' collective welfare viewpoint) to private and public information. We show that the answer to this question is qualitatively similar to the one derived when analyzing the *total surplus* team benchmark.

As before, it can be seen that the internal team-efficient solution minimizes, over the restricted strategies, the expected loss L with respect to the full information cooperative outcome x^M , and that

¹³ Indeed, when $\beta = 0$ there are no externalities (payoff or informational) and the internal team and market solutions coincide.

$$L = \left((2\beta + \lambda) E \left[(\tilde{x} - x^M)^2 \right] + \lambda E \left[(x_i - \tilde{x})^2 \right] \right) / 2.$$

The first term in the sum corresponds to allocative inefficiency in the average quantity, which is proportional to $E \left[(\tilde{x} - x^M)^2 \right]$, and the second term to productive inefficiency, which is proportional to $E \left[(x_i - \tilde{x})^2 \right]$.

It can be checked that the form of the internal optimal team strategy is $x_i = (\lambda + \beta)^{-1} \left(p - (\gamma s_i + (1 - \gamma) E[\theta | z]) \right)$ where $\gamma = (\lambda + \beta)a$ (while at the market solution we have that $\gamma = \lambda a$). The loss at any candidate internal team solution (which internalizes the payoff externality and for which $E[\tilde{x}] = \alpha(2\beta + \lambda)^{-1}$) will depend only on the response to private information a since at this candidate solution we have $E \left[(\tilde{x} - x^M)^2 \right] = (1 - (\lambda + \beta)a)^2 / (\tau(2\beta + \lambda)^2)$ and $E \left[(x_i - \tilde{x})^2 \right] = a^2 / \tau_\varepsilon$.

This yields a strictly convex L as a function of a . As before, changing a has opposite effects on both sources of the loss. Now the internal team solution optimally trades off the sources of the loss with respect to the responsiveness to private information among decentralized strategies which internalize payoff externalities.

In this case at the market solution there is both an information (IE) and a payoff (PE) externality, even with full information the market solution is not efficient (i.e. cooperative). The impact of the externalities on the response to private information can be assessed similarly as before. The market takes the public statistic z or p as given while the internal team solution takes into account both the impact on public informativeness (IE) and on payoffs (PE):

$$\begin{aligned} \frac{\partial E[\pi_i]}{\partial a} = & E \left[\underbrace{(p - MC(x_i)) \left(\frac{\partial x_i}{\partial a} \right)_{z \text{ ct.}}}_{\text{Market}} \right] + \\ & E \left[\underbrace{(p - MC(x_i)) \left(\frac{\partial x_i}{\partial z} \frac{\partial z}{\partial a} \right)}_{\text{IE}} \right] + \\ & E \left[\underbrace{x_i \left(\frac{\partial p}{\partial \tilde{x}} \frac{\partial \tilde{x}}{\partial a} \right)}_{\text{PE}} \right]. \end{aligned}$$

The market term is null at the market solution and the sum of the IE and PE terms can be evaluated as follows:

$$\left. \frac{\partial E[\pi_i]}{\partial a} \right|_{a=a^*} = -\beta a^* \left(c^* \lambda \sigma_\varepsilon^2 + (c^* \beta - 1)^2 \sigma_\theta^2 \right).$$

It is worth noting that while, as before, $\text{sgn}\{\text{IE}\} = \text{sgn}\{-\beta c^*\}$ we have that $\text{sgn}\{\text{PE}\} = \text{sgn}\{-\beta\}$ since $(c^* \beta - 1)^2 \sigma_\theta^2 > 0$, and therefore the PE term will call for a lower (higher) response to private information with strategic substitutes (complements) than the market solution. If $\beta > 0$ there is adverse selection and a high price indicates high costs. If, say, costs are high ($\theta - \bar{\theta} > 0$) then an increase in a will increase p ($\frac{\partial p}{\partial \tilde{x}} \frac{\partial \tilde{x}}{\partial a} = -\beta(c\beta - 1)(\theta - \bar{\theta}) > 0$ since at the market solution $c\beta - 1 < 1$) while x_i will tend to be low (since at the market solution $E[(\theta - \bar{\theta})x_i] = a\sigma_\theta^2(c\beta - 1) < 0$). This means that if $\beta > 0$, $\text{PE} < 0$ and a must be reduced. Similarly, we have that $\text{PE} > 0$ if $\beta < 0$. The results on PE are in line with the results obtained by Angeletos and Pavan (Section 6.5, 2007) with exogenous public signals (and therefore no information externality).¹⁴ We will see how the effect of the informational externality term may overturn this result when $c < 0$.

The next proposition characterizes the response to private information.

Proposition 5. *Let $\tau_\varepsilon > 0$. Then the internal team problem has a unique solution with*

$$(\lambda + \beta)^{-1} > a^{\text{IT}} > 0, \text{ and } \text{sgn}\{a^* - a^{\text{IT}}\} = \text{sgn}\left\{\beta \left(c^* \lambda \sigma_\varepsilon^2 + (c^* \beta - 1)^2 \sigma_\theta^2 \right)\right\}.$$

If $c^* \geq 0$ then $\text{sgn}\{a^* - a^{\text{IT}}\} = \text{sgn}\{\beta\}$. Therefore, as before, under strategic complements ($\beta < 0$), there is too little response to private information, $a^* < a^{\text{IT}}$. Indeed, the characterization yields the same qualitative result as in the previous section if $c^* > 0$: too much or too little response to private information in the presence of (respectively) strategic substitutability or strategic complementarity. In this case, however, if agents use Cournot strategies (i.e., if $c^* = 0$) then the market is not

¹⁴ Note also that in Angeletos and Pavan (2007) there is no noise in the payoff function while there is in our case.

internal team-efficient. This should not be surprising when one considers that, when $c^* = 0$, there is no information externality yet the payoff externality is not internalized, as agents set a quantity that is too large (small) under strategic substitutability (complementarity). If $\beta > 0$ and $c^* < 0$, then $c^* \lambda \sigma_\varepsilon^2 + (c^* \beta - 1)^2 \sigma_\theta^2 > 0$ for c^* close to zero or sufficiently negative (τ_u large). Only for intermediate values of c^* we have $c^* \lambda \sigma_\varepsilon^2 + (c^* \beta - 1)^2 \sigma_\theta^2 < 0$ and $a^\Pi > a^{\text{LE}}$. With strategic substitutes the market will bias the solution more towards putting too high a weight on private information since we may have $c^* \lambda \sigma_\varepsilon^2 + (c^* \beta - 1)^2 \sigma_\theta^2 > 0$ even if $c^* < 0$.

This is the same qualitative result concerning the response to private information as derived previously using the total surplus team benchmark—with the following proviso: when $c^* < 0$, it need not be the case that there is too little response to private information.

Remark 4. The weights to private information in the internal team and market solutions are, respectively, $\gamma^\Pi = (\lambda + \beta)a^\Pi$ and $\gamma^* = \lambda a^*$. It is easy to see that for τ_u small enough (and $\tau_\theta > 0$) we have that $\gamma^\Pi > \gamma^*$. The same result applies when $\beta > 0$ and $c^* \lambda \sigma_\varepsilon^2 + (c^* \beta - 1)^2 \sigma_\theta^2 < 0$ in which case $a^\Pi > a^*$ and therefore $\lambda \gamma^\Pi > (\lambda + \beta)\gamma^* > \lambda \gamma^*$.

6. Other interpretations of the model and applications.

In this section we extend the interpretation of the model to other applications.

6.1 *Investment complementarities.* In this case, $\beta < 0$ and we have strategic complementarity among investment decisions of the agents. The marginal benefit of investing is $p = \alpha + u - \beta \tilde{x}$, and the cost is $C(x_i) = \theta x_i + (\lambda/2)x_i^2$. The shock to the marginal benefit (u) can be understood as a shock to demand, while the shock to costs (θ) can be viewed as a productivity shock. Agents condition their decisions on the marginal benefit of investment p , derived, for example, from the public signals

on macroeconomic data released by the government (which in turn depend on the aggregate activity level). This description need not be taken literally and is simply meant to capture the reduced form of a dynamic process. For example, consider competitive firms deciding about investment in the presence of macroeconomic uncertainty as represented by the random variable θ , which affects profitability. In predicting θ , each firm has access to a private signal as well as to public information, consisting of aggregate past investment figures compiled by a government agency. Data on aggregate investment incorporates measurement error and, at each period, a noisy measure of the previous period's aggregate investment is made public.¹⁵ Proposition 4 indicates then that at the market solution agents respond too little to private information. This result is in line with the case of exogenous public information (Angeletos and Pavan, section 6.2, 2007). This should be not surprisingly since the informational and the scarcity index role of the public statistic are aligned in this case.

6.2 Monopolistic competition. The model applies also to a monopolistically competitive market with quantity-setting firms; in this case, either $\beta > 0$ (goods are substitutes) or $\beta < 0$ (goods are complements). Firm i faces the inverse demand for its product, $p_i = \alpha + u - \beta\tilde{x} - (\lambda/2)x_i$, and has costs θx_i . Each firm uses a supply function that is contingent on its own price: $X(s_i, p_i)$ for firm i . It follows then that observing the price p_i is informationally equivalent (for firm i) to observing $p \equiv \alpha + u - \beta\tilde{x}$.

Under monopolistic competition, the total surplus function (consistent with the differentiated demand system) is slightly different:

$$TS = (\alpha + u - \theta)\tilde{x} - \left(\beta\tilde{x}^2 + (\lambda/2) \int_0^1 x_i^2 di \right) / 2.$$

Here the market is not efficient under complete information because price is not equal to marginal cost. Each firm has some residual market power. The results of Section 4 do not apply but those of Section 5 apply when firms collude. It is interesting to note

¹⁵ For example, quarterly data on national accounts are subject to measurement error. Rodríguez-Mora and Schulstad (2007) show how government announcements regarding GNP growth affect growth via aggregate investment.

then that, if agents cannot use contingent strategies and there is no information externality issue (as in, e.g., cases of Cournot or Bertrand competition), Angeletos and Pavan (section 6.5, 2007) argue that the strategic complementarity case would exhibit excessive response to private information (the opposite of what occurs with endogenous public information) and that strategic substitutability would exhibit insufficient response to private information (in contrast with the case for endogenous public information, where either excessive or insufficient response to private information is possible).

6.3 Demand schedule competition. Let a buyer of a homogenous good with unknown ex post value θ face an inverse supply $p = \alpha + u + \beta \tilde{y}$, where $\tilde{y} = \int_0^1 y_i di$ and y_i is the demand of buyer i . The buyer's net benefit is given by $\pi_i = (\theta - p) y_i - (\lambda/2) y_i^2$, where λy_i^2 is a transaction or opportunity cost (or an adjustment for risk aversion). The model fits this setup if we let $y_i \equiv -x_i$. Some examples follow.

Firms purchasing labor. A firm purchases labor whose productivity θ is unknown—say, because of technological uncertainty—and faces an inverse linear labor supply (with $\beta > 0$) and quadratic adjustment costs in the labor stock. The firm has a private assessment of the productivity of labor, and inverse supply is subject to a shock. In particular, the welfare analysis of Section 4 applies letting $y_i \equiv -x_i$.

Traders in a financial market. Traders compete in demand schedules for a risky asset with liquidation value θ and face a quadratic adjustment cost in their position (alternatively, the parameter λ proxies for risk aversion). Each trader receives a private signal about the liquidation value of the asset. There are also behavioral traders: those who trade according to the elastic aggregate demand $(\alpha + u - p)/\beta$, where u is random. When $\beta > 0$, the behavioral agents are “value” traders who buy (sell) when the price is low (high). When $\beta < 0$, the behavioral agents are “momentum” traders who buy (sell) when the price is high (low).¹⁶ Our inverse

¹⁶ Genotte and Leland (1990) interpret the case $\beta < 0$ as program traders following a portfolio insurance strategy. Asness, Moskowitz, and Pedersen (2009) study empirical returns of value and

supply follows from the market-clearing equation. It is worth noting that behavioral value (momentum) traders induce strategic substitutability (complementarity) in the actions of informed traders.

In the financial market interpretation of the model, if momentum traders predominate ($\beta < 0$) then the slope of excess demand $\Xi' = -(\beta^{-1} + \hat{c})$ is positive. Less price-sensitive “momentum” traders (a more negative β) decreases the weight given to the private information of rational traders, and increases the informativeness of prices: $\tau = \tau_\theta + \beta^2 a^2 \tau_u$ (increasing $|\beta|$ increases $\beta^2 a^2$). Less price-sensitive “momentum” traders are associated with shallow markets when $\beta < 0$ since then market depth is increasing with β . If “value” traders predominate ($\beta > 0$) then less price sensitivity (higher β) decreases complementarity and, as before, decreases the weight given to the private information of rational traders while increasing the informativeness of prices.

If the behavioral traders are momentum traders ($\beta < 0$), then prices always contain too little information (from the collective viewpoint of informed traders). If the behavioral traders are value traders ($\beta > 0$) then the opposite occurs in the usual case of downward-sloping demand schedules for informed traders, which obtain when the volume of *behavioral trading* is large (low τ_u). When the volume generated by behavioral traders is small (high τ_u), demand schedules are upward sloping and prices may contain too little information. This happens for intermediate values of τ_u within its high-value region.

Asset auctions. Consider the auction of a financial asset for which (inverse) supply is price elastic: $p = \alpha + \beta \hat{y}$ with $\beta > 0$, where \hat{y} is the total quantity bid. The liquidation value θ of the asset may be its value in the secondary market (say, for a central bank liquidity or Treasury auction). The marginal valuation of a bidder is

momentum strategies. Hendershott and Seasholes (2009) find that program trading accounts for almost 14% of the average daily market volume at the NYSE in 1999-2005 and that program traders lose money on average. See Barberis and Thaler (2003) for a survey of behavioral biases.

decreasing in the amount bid.¹⁷ Each bidder receives a private signal about θ , and there are noncompetitive bidders who bid according to u/β . This setup yields $\hat{y} = \tilde{y} + u/\beta$, where \tilde{y} is the aggregate of competitive (informed) bids, and an effective inverse supply for the competitive bidders: $p = \alpha + u + \beta\tilde{y}$.

From the collective viewpoint of competitive bidders prices contain too much information in the usual case of downward-sloping demand schedules, which obtain when the volume of *noncompetitive bidding* is large (low τ_u). When the volume generated by noncompetitive bids is small (high τ_u), demand schedules are again upward sloping and prices may contain too little information for intermediate values of τ_u within its high-value region.

Double auction with noise traders. The model can also accommodate, as a limit case of the example just given, a double auction with noise traders demanding a random amount u . Suppose that noise traders bid $(\alpha + \hat{u} - p)/\beta$ with $\hat{u} = \beta u$. Then $\beta^{-1}(\alpha + \beta u - p) \rightarrow u$ as $\beta \rightarrow \infty$, and market clearing yields $u + \tilde{y} = 0$. It is then immediate from Proposition 1 that, in the limit as $\beta \rightarrow \infty$, $a = \tau_\varepsilon \lambda^{-1} (\tau_\varepsilon + \tau_\theta + \tau_u a^2)^{-1}$, $\hat{b} = 0$, and $\hat{c} = \tau_\varepsilon \lambda^{-1} (\tau_\varepsilon + \tau_u a^2)^{-1} > 0$. In this case, equilibrium schedules always have their natural (“right”) slope. Given a diffuse prior ($\tau_\theta = 0$), we have $\hat{c} = a$ and the equilibrium strategy is $X(s_i, p) = a(p - s_i)$, with trader i supplying or demanding according as the price is (respectively) larger or smaller than the private signal.

7. Concluding remarks.

Rational expectations equilibria (linear Bayesian equilibria) are not total surplus team-efficient even when the allowed allocations share certain properties with the market equilibrium (i.e., both are linear in information). The reason is that, in general, the market does not internalize the informational externality when prices convey information. The result is that the market does not trade off optimally non-

¹⁷ A justification for the case of liquidity auctions is given in Ewerhart, Cassola, and Valla (2009).

fundamental price volatility with the dispersion of individual actions. Only in exceptional circumstances (i.e., when the information externality vanishes) does the market get it right and strikes the optimal trade off between volatility and dispersion. Under strategic substitutability, prices will tend to convey too little information when the informational role of prices prevails over its index-of-scarcity role, or will convey too much information in the opposite case. Under strategic complementarity, such as in the presence of a network good, prices always convey too little information. The inefficiency of the market solution opens the door to the possibility that more precise public or private information will lead to an increased welfare loss. This is the case when the market already calls for a too large response to private information, then more precise private information exacerbates the problem.

These results extend to the internal team benchmark, in which the players' collective welfare is taken into account, as long as the index-of-scarcity role of prices prevails over their informational role. When this is not the case, the amount of information in prices may be above or below the welfare benchmark. It follows that received results on the optimal relative weights to be placed on private and public information (when the latter is exogenous) may be overturned when the informational role of the price conflicts with its index of scarcity role and the former is important enough.

Several extensions are worth considering. Examples include exploring tax-subsidy schemes to implement team-efficient solutions along the lines of Angeletos and Pavan (2009); and studying incentives to acquire information (as in Vives 1988; Burguet and Vives 2000; Hellwig and Veldkamp 2009; Myatt and Wallace 2012; Llosa and Venkateswaran 2012; Colombo et al. 2012).

Appendix

Proof of Proposition 1: From the posited strategy $X(s_i, z) = b - as_i + cz$, where $z = u + \beta a \theta$ and $1 - \beta c \neq 0$, we obtain that $p = \alpha - \beta b + (1 - \beta c)z$. From the first-order condition for player i we have

$$X(s_i, z) = \lambda^{-1} (\alpha - \beta b + (1 - \beta c)z - E[\theta | s_i, z]).$$

Here $E[\theta | s_i, z] = \gamma s_i + (1 - \gamma)E[\theta | z]$ with $\gamma = \tau_\varepsilon (\tau_\varepsilon + \tau)^{-1}$, $E[\theta | z] = \beta \tau_u a \tau^{-1} z$ (recall that we have normalized $\bar{\theta} = 0$), and $\tau = \tau_\theta + \beta^2 a^2 \tau_u$ from the projection theorem for Gaussian random variables. Note that $E[\theta | s_i, z] = \gamma s_i + hz$ where $h = \beta a \tau_u (\tau_\varepsilon + \tau)^{-1}$. Identifying coefficients with $X(s_i, z) = b - as_i + cz$, we can immediately obtain

$$a = \frac{\gamma}{\lambda} = \frac{\tau_\varepsilon}{\lambda(\tau_\varepsilon + \tau)}, \quad c = \frac{1 - h}{\beta + \lambda} = \frac{1}{\beta + \lambda} - \frac{\beta a \tau_u}{(\beta + \lambda)(\tau_\varepsilon + \tau)}, \quad \text{and} \quad b = \frac{\alpha}{\beta + \lambda}.$$

It follows that the equilibrium parameter a is determined as the unique (real), of the following cubic equations, that is positive and lies in the interval $a \in (0, \tau_\varepsilon \lambda^{-1} (\tau_\theta + \tau_\varepsilon)^{-1})$:

$$a = \frac{\tau_\varepsilon}{\lambda(\tau_\varepsilon + \tau_\theta + \beta^2 a^2 \tau_u)} \quad \text{or} \quad \beta^2 \tau_u a^3 + (\tau_\varepsilon + \tau_\theta)a - \lambda^{-1} \tau_\varepsilon = 0$$

and

$$c = \frac{1}{(\beta + \lambda)} - \frac{\beta \lambda \tau_u a^2}{(\beta + \lambda) \tau_\varepsilon}.$$

It is immediate from the preceding equality for c that $c < (\beta + \lambda)^{-1}$ (since $a \geq 0$) and that $1 - \beta c > 0$ (since $\beta + \lambda > 0$); therefore,

$$\beta c = \frac{\beta}{\beta + \lambda} - \frac{\beta^2 a \tau_u}{(\beta + \lambda)(\tau_\varepsilon + \tau)} < 1.$$

It follows that

$$X(s_i, p) = \hat{b} - as_i + \hat{c}p,$$

where $\hat{b} = b(1 - \lambda\hat{c})$, $b = \alpha/(\beta + \lambda)$, and $\hat{c} = c/(1 - \beta c)$ with $1 + \beta\hat{c} > 0$. From the equilibrium expression for $c = (\beta + \lambda)^{-1}(1 - \beta\lambda\tau_u a^2 \tau_\varepsilon^{-1})$ we obtain the expression for $\hat{c} = (c^{-1} - \beta)^{-1}$. ♦

Claim 1. *Linear equilibria in strategies with bounded means and with uniformly (across players) bounded variances yield linear equilibria of the schedule game for which the public statistic function is of type $P(\theta, u)$.*

Proof: If for player i we posit the strategy

$$x_i = \hat{b}_i + \hat{c}_i p - a_i s_i$$

then the aggregate action is given by

$$\tilde{x} = \int_0^1 x_i di = \hat{b} + \hat{c}p - a\theta - \int_0^1 a_i \varepsilon_i di = \hat{b} + \hat{c}p - a\theta,$$

where $\hat{b} = \int_0^1 \hat{b}_i di$, $\hat{c} = \int_0^1 \hat{c}_i di$, and $a = \int_0^1 a_i di$ (assuming that all terms are well-defined). Observe that, according to our convention on the average error terms of the signals, $\int_0^1 a_i \varepsilon_i di = 0$ a.s. provided that $\text{var}[a_i \varepsilon_i]$ is uniformly bounded across agents (since $\text{var}[\varepsilon_i] = \sigma_\varepsilon^2$, it is enough that a_i be uniformly bounded). In equilibrium, this will be the case. Therefore, if we restrict attention to candidate linear equilibria with parameters a_i uniformly bounded in i and with well-defined average parameters \hat{b} and \hat{c} , then $\tilde{x} = \hat{b} + \hat{c}p - a\theta$ and the public statistic function is of the type $P(\theta, u)$. ♦

Proof of Proposition 2: (i) From the equation determining the responsiveness to private information a , $\beta^2 \tau_u a^3 + (\tau_\varepsilon + \tau_\theta)a - \lambda^{-1} \tau_\varepsilon = 0$, it is immediate that a decreases with τ_u , τ_θ , β^2 and λ , that a increases with τ_ε . Note that $\text{sgn}\{\partial a / \partial \beta\} = \text{sgn}\{-\beta\}$. As τ_u ranges from 0 to ∞ , a decreases from $\lambda^{-1} \tau_\varepsilon (\tau_\theta + \tau_\varepsilon)^{-1}$ to 0.

(ii) As τ_u ranges from 0 to ∞ , the responsiveness to public information c goes from $(\beta + \lambda)^{-1}$ to $-\infty$ (resp. $+\infty$) if $\beta > 0$ (resp. $\beta < 0$). The result follows since, in equilibrium,

$$c = \frac{1}{\beta + \lambda} - \frac{\beta \lambda \tau_u a^2}{(\beta + \lambda) \tau_\varepsilon} = \frac{1}{\beta + \lambda} - \frac{1}{(\beta + \lambda) \beta} \left(\frac{1}{a} - \lambda \left(1 + \frac{\tau_\theta}{\tau_\varepsilon} \right) \right)$$

and $a \rightarrow 0$ as $\tau_u \rightarrow \infty$. It follows that $\text{sgn}\{\partial c / \partial \tau_u\} = \text{sgn}\{-\beta\}$ because $\partial a / \partial \tau_u < 0$.

Similarly, from the first part of the expression for c we have $\text{sgn}\{\partial c / \partial \tau_\theta\} = \text{sgn}\{\beta\}$ since $\partial a / \partial \tau_\theta < 0$. Furthermore, with some work it is possible to show that, in equilibrium,

$$\begin{aligned} \frac{\partial c}{\partial \tau_\varepsilon} &= (\beta + \lambda)^{-1} \lambda \beta \tau_u \tau_\varepsilon^{-1} a \left(2 \frac{a \lambda - 1}{\lambda (\tau_\theta + \tau_\varepsilon + 3a^2 \beta^2 \tau_u)} + a \tau_\varepsilon^{-1} \right) \quad \text{and} \\ \text{sgn} \left\{ 2 \frac{a \lambda - 1}{\lambda (\tau_\theta + \tau_\varepsilon + 3a^2 \beta^2 \tau_u)} + a \tau_\varepsilon^{-1} \right\} &= \text{sgn} \{ a \lambda \tau_\theta - 2 \tau_\varepsilon + 3 a \lambda \tau_\varepsilon + 3 a^3 \beta^2 \lambda \tau_u \} \\ &= \text{sgn} \{ -2 a \lambda \tau_\theta + \tau_\varepsilon \} \\ &= \text{sgn} \{ \beta^2 \tau_u \tau_\varepsilon^2 + 4 \lambda^2 \tau_\theta^2 (\tau_\varepsilon - \tau_\theta) \}. \end{aligned}$$

Hence we conclude that $\text{sgn}\{\partial c / \partial \tau_\varepsilon\} = \text{sgn}\{\beta(\beta^2 \tau_u \tau_\varepsilon^2 + 4 \lambda^2 \tau_\theta^2 (\tau_\varepsilon - \tau_\theta))\}$. Since

$\hat{c} = (c^{-1} - \beta)^{-1}$, it follows that \hat{c} goes from λ^{-1} to $-\beta^{-1}$ as τ_u ranges from 0 to ∞ ,¹⁸ $\text{sgn}\{\partial \hat{c} / \partial \tau_u\} = \text{sgn}\{-\partial \hat{c} / \partial \tau_\theta\} = \text{sgn}\{-\beta\}$, and $\text{sgn}\{\partial \hat{c} / \partial \tau_\varepsilon\} = \text{sgn}\{\partial c / \partial \tau_\varepsilon\}$. It is then immediate that $1 + \beta \hat{c}$ is decreasing in τ_u and increasing in τ_θ .

(iii) Price informativeness $\tau = \tau_\theta + \beta^2 a^2 \tau_u$ is increasing in τ_ε (since a increases with τ_ε) and also in τ_u (since $a = \lambda^{-1} \tau_\varepsilon (\tau_\varepsilon + \tau)^{-1}$ and a decreases with τ_u). Using the expression for $\partial a / \partial \tau_\theta$ we have that

$$\frac{\partial \tau}{\partial \tau_\theta} = 1 + 2 \beta^2 \tau_u a \frac{\partial a}{\partial \tau_\theta} = 1 - \frac{2 \beta^2 a^2 \tau_u}{\tau_\theta + \tau_\varepsilon + 3 a^2 \beta^2 \tau_u} = \frac{\tau_\theta + \tau_\varepsilon + a^2 \beta^2 \tau_u}{\tau_\theta + \tau_\varepsilon + 3 a^2 \beta^2 \tau_u} > 0.$$

Furthermore,

¹⁸ Note that if $\beta < 0$ and $\beta + \lambda > 0$ then $\lambda^{-1} < -\beta^{-1}$.

$$\frac{\partial \tau}{\partial \beta} = \tau_u \left(2\beta a^2 + 2\beta^2 a \frac{\partial a}{\partial \beta} \right) = 2\beta a \tau_u \left(a - \frac{2a^4 \tau_u \lambda \beta^2}{1 + 2a^3 \tau_u \lambda \beta^2} \right) = \frac{2\beta a^2 \tau_u}{1 + 2a^3 \tau_u \lambda \beta^2},$$

and therefore $\text{sgn}\{\partial \tau / \partial \beta\} = \text{sgn}\{\beta\}$.

(iv) From $x_i = \lambda^{-1} [p - E[\theta | s_i, z]]$ and $E[\theta | s_i, z] = \gamma s_i + (1 - \gamma) E[\theta | z]$ we obtain $x_i - \tilde{x} = \lambda^{-1} \gamma (s_i - \theta) = \lambda^{-1} \gamma \varepsilon_i$ and, noting that $\gamma = \lambda a$ we conclude that $E[(x_i - \tilde{x})^2] = a^2 \sigma_\varepsilon^2$. The results then follow from the comparative statics results for a in (i). ♦

Claim 2. $E[\theta | s_i, z] = \gamma s_i + h z$ with $h = \lambda \beta \tau_\varepsilon^{-1} \tau_u a^2$, $\partial |h| / \partial \tau_\theta < 0$, $\partial |h| / \partial \tau_u > 0$ and $\text{sgn}\{\partial |h| / \partial \beta\} = \text{sgn}\{\beta\}$.

Proof: From $h = \beta a \tau_u (\tau_\varepsilon + \tau)^{-1}$ in the proof of Proposition 1 it is immediate that $h = \lambda \beta \tau_\varepsilon^{-1} \tau_u a^2$. We have that $\partial |h| / \partial \tau_\theta < 0$ since $\partial a / \partial \tau_\theta < 0$; $\partial |h| / \partial \tau_u > 0$ since $\partial \tau / \partial \tau_u > 0$ and therefore $\partial (\tau_u a^2) / \partial \tau_u > 0$. Finally, we have that in equilibrium

$$\frac{\partial c}{\partial \beta} = -\frac{1}{(\lambda + \beta) \tau_\varepsilon} \left(\frac{\tau_\varepsilon + \lambda^2 \tau_u a^2}{\lambda + \beta} + \frac{4a^5 \lambda^3 \tau_u^2 \beta^2}{1 + 2a^3 \tau_u \lambda \beta^2} \right) < 0,$$

and from $c = (1 - h)(\beta + \lambda)^{-1}$ we can obtain $\partial h / \partial \beta > 0$, and therefore, $\text{sgn}\{\partial |h| / \partial \beta\} = \text{sgn}\{\beta\}$. ♦

Proof of Proposition 3: Note first that $\partial^2 E[\text{TS}] / \partial^2 b < 0$ and $\partial^2 E[\text{TS}] / \partial^2 c < 0$ whenever $\beta + \lambda > 0$. Given that $\partial x_i / \partial b = 1$, and $\partial x_i / \partial c = z$, we can optimize with respect to b and c to obtain

$$\begin{aligned} \frac{\partial E[\text{TS}]}{\partial b} &= E[(p - \text{MC}(x_i))] = 0, \\ \frac{\partial E[\text{TS}]}{\partial c} &= E[(p - \text{MC}(x_i))z] = 0, \end{aligned}$$

where $p = \alpha + u - \beta \tilde{x}$ and $\text{MC}(x_i) = \theta + \lambda x_i$. The constraint $E[p - \text{MC}(x_i)] = 0$ is equivalent to $b = \alpha / (\beta + \lambda)$ and $E[(p - \text{MC}(x_i))z] = 0$ is equivalent to

$c = c(a) \equiv \frac{1}{\beta + \lambda} - \frac{\beta a \tau_u (1 - \lambda a)}{\tau(\beta + \lambda)}$. Those constraints are also fulfilled by the market

solution since the first-order condition (FOC) for player i is $E[p - MC(x_i) | s_i, z] = 0$, from which it follows, according to the properties of Gaussian distributions, that $E[p - MC(x_i)] = 0$, and $E[(p - MC(x_i))z] = 0$ (as well as $E[(p - MC(x_i))s_i] = 0$).

It follows that the form of the team optimal strategy is $x_i = \lambda^{-1} [p - (\gamma s_i + (1 - \gamma) E[\theta | z])]$ where $\gamma = \lambda a$. We have that $\tilde{x} = \lambda^{-1} [p - (\gamma \theta + (1 - \gamma) E[\theta | z])]$ and that $\tilde{x} - x^o = (1 - \gamma)(\theta - E[\theta | z]) / (\beta + \lambda)$ and, since $\tau = (\text{var}[\theta | z])^{-1}$ we obtain $E[(\tilde{x} - x^o)^2] = (1 - \lambda a)^2 / (\tau(\beta + \lambda)^2)$. We know that $E[(x_i - \tilde{x})^2] = a^2 / \tau_\varepsilon$.

Let $WL = E[TS^o] - E[TS]$. Similarly as in the proof of Proposition 3 in Vives (2011) we can obtain, using an exact Taylor expansion of total surplus around the full information first best allocation x^o , that

$WL = \left((\beta + \lambda) E[(\tilde{x} - x^o)^2] + \lambda E[(x_i - \tilde{x})^2] \right) / 2$. It follows that

$$WL(a) = \frac{1}{2} \left(\frac{(1 - \lambda a)^2}{(\tau_\theta + \tau_u \beta^2 a^2)(\beta + \lambda)} + \frac{\lambda a^2}{\tau_\varepsilon} \right),$$

which is easily seen strictly convex in a

and with a unique solution $\lambda^{-1} > a^T > 0$. (Note that $\lambda^{-1} < a$ is dominated by $a = \lambda^{-1}$ and that $a < 0$ is dominated by $-a > 0$. Furthermore, it is immediate that $WL'(0) < 0$ and therefore $a > 0$ at the solution.)

The impact of a on $E[TS]$ is easily characterized (noting that $\partial E[TS] / \partial c = 0$ and therefore disregarding the indirect impact of a on $E[TS]$ via a change in c):

$$\begin{aligned}\frac{\partial E[\text{TS}]}{\partial a} &= E\left[(p - \text{MC}(x_i))\left(\frac{\partial x_i}{\partial a}\right)_{z \text{ ct.}}\right] + E\left[(p - \text{MC}(x_i))\left(\frac{\partial x_i}{\partial z} \frac{\partial z}{\partial a}\right)\right] \\ &= E[(p - \text{MC}(x_i))(-s_i + c\beta\theta)]\end{aligned}$$

given that $(\partial x_i / \partial a)_{z \text{ ct.}} = -s_i$, $\partial x_i / \partial z = c$ and $\partial z / \partial a = \beta\theta$.

Evaluating $\partial E[\text{TS}] / \partial a$ at the LE, where $E[(p - \text{MC}(x_i))s_i] = 0$, we obtain that

$\partial E[\text{TS}] / \partial a = c\beta E[(p - \text{MC}(x_i))\theta]$. Now, because

$$E[(p - \text{MC}(x_i))s_i] = E[(p - \text{MC}(x_i))\theta] + E[(p - \text{MC}(x_i))\varepsilon_i] = 0,$$

it follows that

$$\begin{aligned}E[(p - \text{MC}(x_i))\theta] &= -E[(p - \text{MC}(x_i))\varepsilon_i] \\ &= E[\text{MC}(x_i)\varepsilon_i] = E[(\theta + \lambda x_i)\varepsilon_i] = -\lambda a^{\text{LE}} \sigma_\varepsilon^2 < 0\end{aligned}$$

since ε_i is independent of all the model's other random variables and since $a^{\text{LE}} > 0$ when $\tau_\varepsilon > 0$. Hence

$$\text{sgn}\left\{\left.\frac{\partial E[\text{TS}]}{\partial a}\right|_{a=a^*}\right\} = \text{sgn}\{\text{IE}\} = \text{sgn}\{-\beta c^*\},$$

and this equals $\text{sgn}\{a^T - a^*\}$ because $E[\text{TS}]$ is single-peaked for $a > 0$ with a maximum at a^T . ♦

Proof of Proposition 4. The welfare loss at the team-efficient solution is given by $\text{WL}(a^T)$, which is decreasing in τ_ε, τ_u and τ_θ since WL is decreasing in τ_ε, τ_u and τ_θ for a given a and $\text{WL}'(a^T) = 0$. With respect to the market solution we have that

$$\frac{d\text{WL}}{d\tau_\theta}(a^*) = \frac{\partial \text{WL}}{\partial a} \frac{\partial a^*}{\partial \tau_\theta} + \frac{\partial \text{WL}}{\partial \tau_\theta},$$

where $\frac{\partial a^*}{\partial \tau_\theta} = -\frac{a}{\tau_\theta + \tau_\varepsilon + 3a^2\beta^2\tau_u}$ and a^* solves $\beta^2\tau_u a^3 + (\tau_\varepsilon + \tau_\theta)a - \lambda^{-1}\tau_\varepsilon = 0$.

Given that

$$\text{WL} = \frac{1}{2} \left(\frac{(1 - \lambda a)^2}{(\tau_\theta + \tau_u \beta^2 a^2)(\beta + \lambda)} + \frac{\lambda a^2}{\tau_\varepsilon} \right),$$

it is possible to show that

$$\frac{dWL}{d\tau_\theta}(a^*) < 0 \text{ if and only if } \frac{\tau_\theta + \tau_u \beta^2 a^2}{\tau_\varepsilon} > -\frac{2\beta + \lambda}{\lambda},$$

which is always true since $2\beta + \lambda > 0$. Exactly the same condition holds for $dWL(a^*)/d\tau_u < 0$. Furthermore, we can show that $dWL(a^*)/d\tau_\varepsilon < 0$ if and only if $\beta - \lambda \leq \frac{\tau_u \beta^2 a^*}{\tau_\theta} (a^* (\beta + \lambda) + 2) + (\beta + \lambda) \frac{\tau_\varepsilon}{\tau_\theta}$. It follows that WL will be increasing in τ_ε for $\beta > \lambda$ and $\tau_\varepsilon/\tau_\theta$ or τ_u small enough. ♦

Proof of Proposition 5: It proceeds in a parallel way to the proof of Proposition 3. Note first that $\partial^2 E[\pi_i]/\partial^2 b < 0$ and $\partial^2 E[\pi_i]/\partial^2 c < 0$ whenever $2\beta + \lambda > 0$. Given that $\pi_i = px_i - C(x_i)$, $p = \alpha + u - \beta \tilde{x}$, $\partial x_i/\partial b = 1$, and $\partial x_i/\partial c = \partial \tilde{x}/\partial c = z$ and $\partial p/\partial \tilde{x} = -\beta$ we can optimize with respect to b and c to obtain

$$\begin{aligned} \frac{\partial E[\pi_i]}{\partial b} &= E[(p - MC(x_i)) - \beta x_i] = 0, \\ \frac{\partial E[\pi_i]}{\partial c} &= E[(p - MC(x_i))z - \beta x_i z] = 0. \end{aligned}$$

where $MC(x_i) = \theta + \lambda x_i$. The constraint $E[(p - MC(x_i)) - \beta x_i] = 0$ is equivalent to $b = \alpha/(2\beta + \lambda)$; we can also check that $E[(p - MC(x_i))z - \beta x_i z] = 0$ is equivalent to $c = c^\Pi(a)$, where

$$c^\Pi(a) = \frac{1}{2\beta + \lambda} - \frac{\beta a \tau_u (1 - (\lambda + \beta)a)}{\tau (2\beta + \lambda)} \quad \text{and} \quad \tau = \tau_\theta + \beta^2 \tau_u a^2.$$

Note that due to payoff externalities ($\partial p/\partial \tilde{x} = -\beta$) the expressions for b and for c are different than in the market solution. It follows that the form of the internal team optimal strategy is $x_i = (\lambda + \beta)^{-1} [p - (\gamma s_i + (1 - \gamma)E[\theta | z])]$ where $\gamma = (\lambda + \beta)a$. We have that $\tilde{x} = (\lambda + \beta)^{-1} [p - (\gamma \theta + (1 - \gamma)E[\theta | z])]$ and that $\tilde{x} - x^M = (1 - \gamma)(\theta - E[\theta | z])/(2\beta + \lambda)$ and, since $\tau = (\text{var}[\theta | z])^{-1}$ we obtain $E[(\tilde{x} - x^M)^2] = (1 - (\lambda + \beta)a)^2 / (\tau (2\beta + \lambda)^2)$. We have that $E[(x_i - \tilde{x})^2] = a^2/\tau_\varepsilon$.

Let $L = E[\pi_i^M] - E[\pi_i]$. Similarly as before we can obtain that

$$L = \left((2\beta + \lambda) E\left[(\tilde{x} - x^M)^2 \right] + \lambda E\left[(x_i - \tilde{x})^2 \right] \right) / 2. \text{ It follows that}$$

$$L = \frac{1}{2} \left(\frac{(1 - (\lambda + \beta)a)^2}{(\tau_\theta + \tau_u \beta^2 a^2)(2\beta + \lambda)} + \frac{\lambda a^2}{\tau_\varepsilon} \right),$$

which is easily seen strictly convex in a and with a unique solution $(\lambda + \beta)^{-1} > a^{\text{IT}} > 0$. (Note that $(\lambda + \beta)^{-1} < a$ is dominated by $a = (\lambda + \beta)^{-1}$ and that $a < 0$ is dominated by $-a > 0$. Furthermore, it is immediate that $L'(0) < 0$ and therefore $a > 0$ at the solution.)

The impact of a on $E[\pi_i]$ is easily characterized (noting that $\partial E[\pi_i] / \partial c = 0$ and therefore disregarding the indirect impact of a on $E[\pi_i]$ via a change in c):

$$\begin{aligned} \frac{\partial E[\pi_i]}{\partial a} &= E \left[\underbrace{(p - MC(x_i)) \left(\frac{\partial x_i}{\partial a} \right)_{z \text{ ct.}}}_{M} \right] + \\ &E \left[\underbrace{(p - MC(x_i)) \left(\frac{\partial x_i}{\partial z} \frac{\partial z}{\partial a} \right)}_{IE} \right] + \\ &E \left[\underbrace{x_i \left(\frac{\partial p}{\partial \tilde{x}} \frac{\partial \tilde{x}}{\partial a} \right)}_{PE} \right] \\ &= E \left[(p - MC(x_i)) (-s_i + c\beta\theta) - \beta(c\beta - 1)\theta x_i \right] \end{aligned}$$

given that $(\partial x_i / \partial a)_{z \text{ ct.}} = -s_i$, $\partial x_i / \partial z = c$, $\partial z / \partial a = \beta\theta$, $\partial p / \partial \tilde{x} = -\beta$ and $\partial \tilde{x} / \partial a = (c\beta - 1)\theta$. Evaluating $\partial E[\pi_i] / \partial a$ at the equilibrium, where $E[(p - MC(x_i))s_i] = 0$, we obtain

$$\frac{\partial E[\pi_i]}{\partial a} = \beta E \left[c(p - MC(x_i))\theta - (c\beta - 1)\theta x_i \right].$$

As in the last section, we have $E[(p - MC(x_i))\theta] = -\lambda a \sigma_\varepsilon^2 < 0$ and, recalling that $\bar{\theta} = 0$, it is easily checked that $E[\theta x_i] = a \sigma_\theta^2 (c\beta - 1)$. At the equilibrium we have therefore¹⁹

$$\frac{\partial E[\pi_i]}{\partial a} = -\beta a^* (c^* \lambda \sigma_\varepsilon^2 + (c^* \beta - 1)^2 \sigma_\theta^2).$$

Since $E[\pi_i]$ is single-peaked for $a > 0$ and has a unique maximum at $a^{\text{IT}} > 0$ and $a^* > 0$, it follows that

$$\text{sgn}\{a^{\text{IT}} - a^*\} = \text{sgn}\left\{\left.\frac{\partial E[\pi_i]}{\partial a}\right|_{a=a^*}\right\} = \text{sgn}\left\{-\beta (c^* \lambda \sigma_\varepsilon^2 + (c^* \beta - 1)^2 \sigma_\theta^2)\right\}. \blacklozenge$$

¹⁹ Note also that at the equilibrium $c\beta - 1 < 0$.

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