Competitive Markets, Collective Decisions and Group Formation

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Abstract

We consider a general equilibrium model where groups operating in a competitive market environment can have several members and make efficient collective consumption decisions. Individuals have the option to leave the group and make it on their own or join another group. We study the effect of these outside options on group formation, group stability, equilibrium existence, and equilibrium efficiency.

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1 Introduction

Concurrent interest in the formation, composition, stability, and decision making of households or, more generally, socio-economic groups requires a formal framework that incorporates the allocation of commodities to consumers and of people to households. We are going to analyze a general equilibrium model with multi-member households where such a dual allocation is brought about by three interacting mechanisms, each operating at a particular level of aggregation: Individual decisions are made to join or leave households. Collective decisions within households determine the consumption plans of household members. Competitive exchange across households achieves a feasible allocation of resources. Clearly, the three mechanisms interact. The household structure (that is the partition of the population into households) and the attractiveness of alternative households affect market prices and the allocation of resources among consumers. Conversely, market prices and the implied market opportunities influence the formation and stability of households. An economic theory of pure exchange among households ought to account for these interdependencies.

When dealing with household formation, one of the most critical modeling assumptions is how much choice between households an individual has. Here we consider a finite pure exchange economy with variable household structure and focus on two types of outside options available to household members. We first develop the concept of a **competitive equilibrium with free exit** (CEFE) where household members have one type of outside option, the "exit option" (EO): an individual may decide to leave its household and become single if this is to its advantage. Then we develop the concept of **competitive equilibrium with free household formation** (CEFH) which adds a second type of outside option, the "joining option" (JO): an individual may decide to leave its household and get accepted by another household or individual if this benefits all members of the resulting enlarged household.

The choice of threat points in households has been examined in a number of papers surveyed in Bourguignon and Chiappori (1994). This literature suggests that it is difficult to identify threat points empirically. Therefore, in our theoretical analysis, we start with the exit option as the narrowest view advanced in the literature on how individuals behave in households.[1] Then we are broadening the set of outside options.

Our approach follows the seminal contribution of Becker (1973) who has demonstrated that an inquiry into the determinants of and connections between sociological and economic choices can be very productive. We use a different model and address different questions. For instance, household-specific externalities play an important role in our approach. In contrast, Becker's model avoids consumption externalities in a unique way, by introducing a "household good", the sole explicit consumption good which is non-tradable, yet perfectly divisible within each household and does not cause any consumption externalities.

Our investigation of interacting allocation and decision mechanisms begins with Gersbach and Haller (2001) where we follow Haller (2000) and incorporate the collective rationality concept of Chiappori (1988, 1992) into a general equilibrium framework. There we perform welfare analysis with a variable household structure, but no outside options.[2] An allocation consists of a household structure and an allocation of commodities to individual consumers. A **competitive equilibrium** is defined accordingly. A household resides in a competitive market environment and makes efficient collective consumption decisions for its members. This setting has allowed us to study the interaction between two allocation mechanisms: collective decisions and competitive markets. This basic model will be amended in the present paper, introducing the two types of outside options, EO and JO, which constitute elements of a third allocation mechanism, the individual choice of household membership. Our conclusions are three-fold.

[1]In particular, Chiappori, Fortin, and Lacroix (2002) provide an empirical justification of divorce as a threat point. They study the effect of a "divorce laws index" on intra-household decisions. This index measures how favorable are state divorce law provisions to women and ranges between 1 and 4 in the sample, with a mean of 2.48 and a standard error of 0.88. It can be viewed as a rough proxy for the value of the exit option available to women. Chiappori *et al.* use 1988 PSID data of 1618 US households where both spouses have positive hours of work and are between 30 and 60 years old. They estimate that a one point increase in the divorce laws index induces husbands to transfer an additional $4,310 of nonlabor income to their wives. This suggests a considerable impact of the exit option on the intra-household decision process, given that the average nonlabor income per household was approximately $8,000.

[2]Corresponding equilibrium existence results can be found in Gersbach and Haller (1999).

First, we establish a neutrality theorem which asserts that in the absence of externalities, the set of CEFE is identical to the set of CEFH and equal to the set of (traditional) competitive equilibria when all individuals act and trade individually. Therefore, if group or household formation does not create any group or consumption externalities, individuals remain powerless in the sense that every individual can fare no better and no worse as a member of a multi-member household than as an individual market participant. The exit threat is sufficient for this to hold and adding more outside options affects neither equilibrium existence nor equilibrium welfare under these particular circumstances.

Second, suppose that more outside options, say addition of JO, eliminate some but not all competitive equilibria with free exit. One might conjecture that more stringent equilibrium conditions make the surviving equilibria "stronger" or "better", having passed more tests than the eliminated ones. It turns out that this conclusion is premature if "better" means "Pareto-superior": A surviving equilibrium can be weakly Pareto-dominated by an eliminated one. This kind of result suggests that the availability and awareness of more outside options can be socially harmful. It can destabilize households and, therefore, the household structure. However, the availability and people's awareness of more outside options need not always be socially harmful. The welfare comparison, in the sense of Pareto, of competitive equilibria with free exit which are also competitive equilibria with free household formation and those which are not, can go either way.

Third, we establish existence of non-trivial CEFE. We also find that the additional outside option, JO, can eliminate all competitive equilibria with free exit. Whereas there exists a competitive equilibrium with free exit under standard assumptions, there need not exist a competitive equilibrium with free household formation under the same assumptions. Still, competitive equilibria with free household formation exist in many instances. One example is the case of pure group exteralities and a unique optimal household structure with respect to group preferences. Another example is the case of one private good and group externalities such that household formation can be reduced to a two-sided matching problem. But we also provide a counter-example, Example 6, with two private goods and household formation reducible to a two-sided matching problem. In Example 6, stable matchings and market clearing cannot be achieved simultaneously. This kind of market failure is notably absent from the vast majority of the matching literature where markets are inactive and relative prices are irrelevant, simply because there exits at most one tradable commodity.

Our model is related to the club literature and the literatures on hedonic coalitions, matching, assignment games, and multilateral bargaining. The innovative approach to club theory taken by Ellickson et al. (1999, 2001) resembles ours in that it also deals with the allocation of individuals to groups (clubs, households) and the allocation of commodities to individuals. In our model, only the household at large is subject to a budget constraint and not necessarily each member. In contrast, club theory assumes that every club member is subject to an individual budget constraint. Both in household and in club models, individuals (indirectly or directly) participate in anonymous and competitive global markets and consider themselves exposed to market conditions on which they have no influence. One of the distinguishing features of the collective rationality assumption on households is that individuals do enjoy influence at the local or household level while they are without influence in the global market place. The precise relationship of our model to the club literature has been discussed in detail in the introduction and subsection 5.3 of Gersbach and Haller (2001).

We would further like to stress that Example 6 puts the traditional literature on matching into perspective. Namely, most of the work on hedonic coalitions [e.g. Greenberg (1978), Bogomolnaia and Jackson (2002), Banerjee *et al.* (2001)], matching [e.g. Gale and Shapley (1962), Alkan (1988), Roth and Sotomayor (1990)], assignment games [e.g. Shapley and Shubik (1972), Roth and Sotomayor (1990)], and multilateral bargaining [e.g. Rochford (1984), Crawford and Rochford (1986), Bennett (1988, 1997)] focuses on group formation and lacks competitive markets for commodities. Consequently, this literature fails to observe that in general, stable matchings and market clearing cannot be achieved simultaneously.[3]

The paper is organized as follows: In the next section, we introduce the model. In section 3, we define and discuss the equilibrium concepts. Welfare properties of CEFE and CEFH are studied in section 4. Existence issues are addressed in sections 5 and 6. We conclude with section 7. Lenghtier proofs are collected in an appendix, section 8.

[3]A noteworthy exception are Drèze and Greenberg (1980) who combine the concepts of individual stability and price equilibrium, but confine the analysis of their most comprehensive model to an instructive example.

2 Consumer Characteristics and Allocations

In this section, we describe the basic structure of the model: consumers, household structures, commodities, endowments, allocations, and preferences.

Consumers and Household Structures. We consider a finite population of **consumers**, represented by a set $I = \{1, \dots, n\}$. A generic consumer is denoted i or j. The population I is partitioned into households[4], i.e. there exists a partition P of I into non-empty subsets. We call any such partition P a **household structure in** I. A generic household is denoted h or g. Relative to P, we use the following terminology regarding $i \in I$ and $h \subseteq I$, $h \neq \emptyset$:

"household h exists" or "household h is formed"	iff $h \in P$;
"i belongs to h" or "individual i is a member of household h"	iff $i \in h$.

If P consists of H households, we frequently label them $h = 1, \dots, H$, provided this causes no confusion. We treat the household structure as an object of endogenous choice. Households are endogenously formed so that some household structure P is ultimately realized. Consequently, our **consumer allocation space** is $\mathcal{P}$, the set of all household structures in I.

Commodities. There exists a finite number $\ell \geq 1$ of commodities. Thus the commodity space is $\mathbb{R}^\ell$. Each commodity is formally treated as a private good, possibly with externalities in consumption. Consumer $i \in I$ has consumption set $X_i = \mathbb{R}^\ell_+$ so that the **commodity allocation space** is $\mathcal{X} \equiv \prod_{j \in I} X_j$. Generic elements of $\mathcal{X}$ are denoted $\mathbf{x} = (x_i)$, $\mathbf{y} = (y_i)$. Commodities are denoted by superscripts $k = 1, \dots, \ell$. For a potential household $h \subseteq I$, $h \neq \emptyset$, set $\mathcal{X}_h = \prod_{i \in h} X_i$, the consumption set for household h. $\mathcal{X}_h$ has generic elements $\mathbf{x_h} = (x_i)_{i \in h}$. If $\mathbf{x} = (x_i)_{i \in I} \in \mathcal{X}$ is a commodity allocation, then consumption for household h is the restriction of $\mathbf{x} = (x_i)_{i \in I}$ to h, $\mathbf{x_h} = (x_i)_{i \in h}$.

Endowments. For a potential household $h \subseteq I, h \neq \emptyset$, its **endowment** is a commodity bundle $\omega_h \in \mathbb{R}^\ell$ given by the sum of the endowments of all participating individuals: $\omega_h = \sum_{i \in h} \omega_{\{i\}}$. The **social endowment** is given

[4] While we stick to the suggestive term "household", a broader interpretation as socio-economic group or simply group would be quite appropriate in many instances, in particular since as a rule we do not impose any restrictions on household or group size, respectively.

by

$$\omega_S \equiv \sum_{h \in P} \omega_h. \tag{1}$$

Note that the social endowment is independent of the household structure, $\omega_S = \sum_{i \in I} \omega_{\{i\}}$.

Allocations. An **allocation** is a pair $(\mathbf{x}; P) \in \mathcal{X} \times \mathcal{P}$ specifying the consumption bundle and household membership of each consumer. We call an allocation $(\mathbf{x}; P) \in \mathcal{X} \times \mathcal{P}$ **feasible**, if

$$\sum_{i \in I} x_i = \omega_S. \tag{2}$$

After the specification of individual preferences, by means of utility representations, an allocation determines the welfare of each and every member of society.

Consumer Preferences. In principle, a consumer might have preferences on the allocation space $\mathcal{X} \times \mathcal{P}$ and care about each and every detail of an allocation. For individual $i \in I$, we assume that i has preferences on $\mathcal{X} \times \mathcal{P}$ represented by a utility function $U_i : \mathcal{X} \times \mathcal{P} \longrightarrow \mathbb{R}$.

In the following, we shall make the **general assumption** that an individual does not care about the features of an allocation beyond the boundaries of his own household. If a particular household structure is given, he is indifferent about the affiliation and consumption of individuals not belonging to his own household. Condition HSP is a formal expression of this assumption, with a slight abuse of notation.

(HSP) Household-Specific Preferences:

$U_i(\mathbf{x}; P) = U_i(\mathbf{x_h}; h)$ for $i \in h$, $h \in P$, $(\mathbf{x}; P) \in \mathcal{X} \times \mathcal{P}$.

The general assumption HSP is justifiable on the grounds that we want to design a model where multi-member households play a significant allocative role. HSP still admits a lot of flexibility. For example, it permits various kinds of consumption externalities within households. Suitable externalities may prevent the formation of certain households, even though we are not explicitly restricting household size. In the sequel, we shall in particular exploit the occurrence of pure group externalities that depend solely on the persons belonging to a household. Pure group externalities can capture all aspects of the benefits of human beings living together. They can represent, for instance, the emotional benefit from living together with other persons in the same household or the opportunity for receiving advice. To formulate the latter externalities, define $\mathcal{H}_i \equiv \{h \subseteq I | i \in h\}$ for $i \in I$. $\mathcal{H}_i$ is the set of potential households of which i would be a member.

(PGE) Pure Group Externalities: For each consumer i, there exist functions $U_i^c : X_i \to \mathbb{R}$ and $U_i^g : \mathcal{H}_i \to \mathbb{R}$ such that $U_i(\mathbf{x_h}; h) = U_i^c(x_i) + U_i^g(h)$ for $\mathbf{x_h} \in \mathcal{X}_h, h \in \mathcal{H}_i$.

PGE assumes that one can additively separate the pure consumption effect $U_i^c(x_i)$ from the pure group effect $U_i^g(h)$. A very special case is the **absence of externalities**, corresponding to $U_i^g \equiv 0$. At the other extreme lies the purely hedonic case, with $U_i^c \equiv 0$ or $\ell = 0$, studied by Banerjee *et al.* (2001) and Bogomolnaia and Jackson (2002).

All of our examples with the exception of Example 6 feature pure group externalities. But one should emphasize that despite their prominent role, our analysis is not confined to the case of pure group externalities. See in particular Propositions 3, 4, and 6.

3 The Equilibrium Concepts

Among the several conceivable ways to formulate an equilibrium state of a model with variable household structure, we define an equilibrium of commodities and consumers as a price system together with a household structure and a feasible resource allocation such that:

- a household chooses an efficient consumption schedule for its members, subject to the household budget constraint;
- markets clear;
- no individual has an incentive to leave a household and to participate as an individual in the market at the going prices.

These three conditions define a competitive equilibrium with free exit. We shall further allow for a second outside option:

- no individual can leave a household and get accepted by another household by proposing a feasible allocation for the enlarged household which makes everybody in this newly formed household better off at the going prices.

Adding the second option defines a competitive equilibrium with free household formation.

3.1 Definitions

In order to define the equilibrium concepts formally, we consider a household $h \in P$ and a price system $p \in \mathbb{R}^{\ell}$. For $\mathbf{x_h} = (x_i)_{i \in h} \in \mathcal{X}_h$,

$$p * \mathbf{x_h} \equiv p \cdot \left(\sum_{i \in h} x_i \right)$$

denotes the expenditure of household h on household consumption plan $\mathbf{x_h}$ at the price system p. As p and $\mathbf{x_h}$ are of different dimension for multi-member households, we use the $*$-product in lieu of the familiar inner product. Then h's **budget set** is defined as

$$B_h(p) = \{\mathbf{x_h} \in \mathcal{X}_h \ : \ p * \mathbf{x_h} \leq p \cdot \omega_h\}.$$

We next define the **efficient budget set** $EB_h(p)$ as the set of $\mathbf{x_h} \in B_h(p)$ with the property that there is no $\mathbf{y_h} \in B_h(p)$ such that

$U_i(\mathbf{y_h}; h) \geq U_i(\mathbf{x_h}; h)$ for all $i \in h$;

$U_i(\mathbf{y_h}; h) > U_i(\mathbf{x_h}; h)$ for some $i \in h$.

Further define a **state** of the economy as a triple $(p, \mathbf{x}; P)$ such that $p \in \mathbb{R}^{\ell}$ is a price system and $(\mathbf{x}; P) \in \mathcal{X} \times P$ is an allocation, i.e. $\mathbf{x} = (x_i)_{i \in I}$ is an allocation of commodities and P is an allocation of consumers (a household structure, a partition of the population into households). A state $(p, \mathbf{x}; P)$ is a **competitive equilibrium with free exit (CEFE)** if it satisfies the following conditions:

1. $\mathbf{x_h} \in EB_h(p)$ for all $h \in P$.
2. $\sum_i x_i = \omega_S$.
3. There is no $h \in P$, $i \in h$ and $y_i \in B_{\{i\}}(p)$ such that

   $U_i(y_i; \{i\}) > U_i(\mathbf{x_h}; h)$.

Finally a **competitive equilibrium with free household formation (CEFH)** is a CEFE $(p, \mathbf{x}; P)$ that also satisfies:

4. There are no h and $g \in P$, $i \in h$ and $\mathbf{y_{g\cup\{i\}}} \in B_{g\cup\{i\}}(p)$ such that

   $U_j(\mathbf{y_{g\cup\{i\}}}; g \cup \{i\}) > U_j(\mathbf{x_g}; g)$ for all $j \in g$;
   $U_i(\mathbf{y_{g\cup\{i\}}}; g \cup \{i\}) > U_i(\mathbf{x_h}; h)$.

3.2 Discussion

Condition 1 reflects collective rationality in the sense of Chiappori (1988, 1992), in contrast to the traditional "unitary" model where households are treated like single consumers. Efficient choice by the household refers to the individual consumption and welfare of its members, not merely to the aggregate consumption bundle of the household. Condition 2 requires market clearing. Conditions 1 and 2 alone define a **competitive equilibrium** $(p, \mathbf{x})$, given household structure P, discussed and studied in Haller (2000) and Gersbach and Haller (2001).

In addition, we impose condition 3 that no individual wants to leave a household and participate as a one-member household in the market at the going equilibrium prices. Condition 3 constitutes an individual rationality or voluntary participation (membership) constraint. Conditions 1 to 3 together define a **competitive equilibrium with free exit**.

Conditions 1 to 4 together define a **competitive equilibrium with free household formation**. Condition 4 requires that no individual can leave a household and can propose a feasible consumption allocation to the members of a new household, created by the individual and another already existing household, which makes everybody in the new household better off at the going equilibrium prices. Condition 4 still presumes that changes of the household structure are the result of individuals leaving a household and proposing a better allocation to an already existing one- or multi-person household. Condition 4 already appears in the earlier literature on coalition formation, beginning with Greenberg (1978) and Drèze and Greenberg (1980) who have introduced the concept of individually stable equilibrium where a coalition partition is individually stable if it is immune to individual movements which benefit the moving player and do not hurt any member of the group she joins.[5] Finally, our paper is related to the influential work of Hirschman (1970) who has considered the comparative efficiency of the exit and voice options as mechanisms of recuperation. In the absence of externalities, the exit option limits power within households in the sense that a person cannot achieve higher utility as a multi-member household than as an individual market participant. This follows from our first proposition.

One could think of even stronger conditions in the tradition of the matching literature (see Roth and Sotomayor 1990 for surveys) where two persons can break away from two different matches and form a new match. But it has been argued in other contexts, that the divorce threat and thus the exit option alone describes the behavior of individuals in multi-person households; see Bourguignon and Chiappori (1994) for a summary of this debate. Our condition 4 lies between these two perspectives on how individuals decide whether to leave a household. It proves sufficient to put the existence of equilibria with free household formation into question and it is just restrictive enough to make the normative issue how more outside options affect welfare an interesting one.

As it is formulated, condition 4 requires that all members must want the newly formed household $g \cup \{i\}$ with the proposed commodity allocation. Alternatively, one might require that none of the members of the previous household g be opposed to forming the new household, i.e. the inequalities pertaining to $j \in g$ become weak. The two formulations are equivalent under many, but not all circumstances.

[5]Among recent contributions to that literature using a similar condition are Banerjee, Konishi and Sönmez (2001), Jehiel and Scotchmer (2001), and Bogomolnaia and Jackson (2002). In our work we combine coalition formation, collective decisions and competitive markets.

4 Equilibrium Welfare

4.1 Inefficacy of Outside Options

We are interested in the individual's possibilities of achieving higher utility levels by participating in a particular household rather than acting and trading individually or participating in other households. One might conjecture that particular household members with high bargaining power could use the household to obtain more consumption. We commence by examining group formation when there are no externalities (i.e. there is absence of consumption and group externalities). We establish the following neutrality theorem.

Proposition 1 (Neutrality Theorem)
Suppose absence of externalities and continuity and local non-satiation of consumer preferences. Consider $(p; \mathbf{x}) \in \mathbb{R}^\ell \times \mathcal{X}$ *and any household structure* P*. Then the following three assertions are equivalent:*

(i) $(p, \mathbf{x}; P)$ *is a competitive equilibrium with free household formation.*

(ii) $(p, \mathbf{x}; P)$ *is a competitive equilibrium with free exit.*

(iii) $(p, \mathbf{x})$ *is a traditional competitive equilibrium where each agent acts and trades individually.*

The proof is given in the appendix. Proposition 1 asserts that in the absence of any externalities, free exit implies that a consumer can fare no better and no worse as a member of a multi-member household than as an individual market participant. If, in spite of free exit, some individuals enjoy higher utility levels as household members than they would obtain individually, some sort of externality has to be present. Proposition 1 also states that in the absence of any externalities, the set of competitive equilibria with free household formation is essentially equal to the set of (traditional) competitive equilibria when all individuals act and trade individually.

Proposition 1 extends the "Irrelevance Proposition" of Gersbach and Haller (2001). It conforms with intuition, but still requires a proof and has important implications for the role of outside options available to individuals in households operating in competitive commodity markets. If there are no externalities, adding more outside options for agents is irrelevant for consumption and utility allocation and hence for welfare. Suppose e. g. that all multi-person households take their decisions according to a Nash bargaining solution. Then, Proposition 1 says that it is irrelevant whether such bargaining takes place with outside options of the first type (exit) only or with outside options of both types (exit and possibly joining another consenting household). Equally important is the observation that adding more outside options does not impair the stability of households if externalities are absent. The downside is that household formation becomes pointless under these circumstances.

The working of the neutrality theorem is now illustrated by an example. In the example, it is shown that only an equal split of bargaining power between the members of a two-person household is consistent with a CEFE or CEFH. Different distributions of bargaining power are consistent with a market equilibrium where the household structure is fixed and exit is not an option. But those market equilibria violate condition 3 or condition 4 in the above definition of CEFE and CEFH. The example follows.

Example 1. Let $\ell = 2$, $I = \{1, 2, 3\}$. Preferences are represented by $U_i(\mathbf{x_h}; h) = u_i(x_i) = u_i(x_i^1, x_i^2)$ where x_i^k denotes the quantity of good k $(k = 1, 2)$ consumed by individual i. Specifically, we assume

$$\begin{aligned} U_1(x_1^1, x_1^2) &= \ln x_1^1, \\ U_2(x_2^1, x_2^2) &= \ln x_2^2, \\ U_3(x_3^1, x_3^2) &= \frac{1}{2}\ln x_3^1 + \frac{1}{2}\ln x_3^2. \end{aligned}$$

We further assume the individual endowments

$$w_1 = (0, \frac{1}{2}),\ w_2 = (0, \frac{1}{2}),\ w_3 = (1, 0).$$

Commodity prices are normalized so that $p_1 = 1$.

Consider first the household structure $P^0 = \{\{1\}, \{2\}, \{3\}\}$. It is obvious that there exists a unique market equilibrium $(p^0, \mathbf{x^0}; P^0)$, given by:

$$p^0 = (1, 1),$$

$$
\begin{aligned}
x_1^0 &= (\frac{1}{2}, 0), \\
x_2^0 &= (0, \frac{1}{2}), \\
x_3^0 &= (\frac{1}{2}, \frac{1}{2}).
\end{aligned}
$$

Consider next the household structure $P^* = \{\{1,2\},\{3\}\}$. Suppose that household $g = \{1,2\}$ maximizes a utilitarian social welfare function

$$
\begin{aligned}
W_h &= \alpha U_1(x_1) + (1-\alpha) U_2(x_2) \\
&= \alpha \ln x_1^1 + (1-\alpha) \ln x_2^2, \qquad 0 < \alpha < 1,
\end{aligned}
$$

subject to the budget constraint $x_1^1 + p_2 x_2^2 = p_2$. α can be interpreted as the weight of individual 1 in household g. Similarly, $1-\alpha$ is the weight of individual 2 in household g. The excess demand vectors of the households g and $h = \{3\}$, denoted by z_g and z_h, are given by

$$
\begin{aligned}
z_g &= (\alpha p_2, -\alpha), \\
z_h &= (-\frac{1}{2}, \frac{1}{2p_2}).
\end{aligned}
$$

A market equilibrium without exit $(p^*, \mathbf{x}^*; P^*)$ would require

$$
\begin{aligned}
p^* &= (1, \frac{1}{2\alpha}), \\
x_1^* &= (\frac{1}{2}, 0), \\
x_2^* &= (0, 1-\alpha), \\
x_3^* &= (\frac{1}{2}, \alpha).
\end{aligned}
$$

At prices p^*, individuals $i = 1,2$ could obtain the following consumption vectors x_1^s and x_2^s by leaving household g:

$$
\begin{aligned}
x_1^s &= \Big(\frac{1}{4\alpha}, 0\Big), \\
x_2^s &= \Big(0, \frac{1}{2}\Big).
\end{aligned}
$$

Except for $\alpha = \frac{1}{2}$, either $U_1(x_1^s) > U_1(x_1^*)$ or $U_2(x_2^s) > U_2(x_2^*)$ and, hence, $(p^*, \mathbf{x}^*; P^*)$ is a competitive equilibrium with exit only for $\alpha = \frac{1}{2}$. In this case, $x_1^* = x_1^0$ and $x_2^* = x_2^0$. Similarly $(p^*, \mathbf{x}^*; P^*)$ is a competitive equilibrium with free household formation if and only if $\alpha = \frac{1}{2}$. ••

4.2 Optimality of CEFE

The present paper focuses on the interaction of three allocation mechanisms: group formation, collective decisions within groups and competitive market exchange between groups. Which allocations qualify as optimal or efficient depends on how much freedom a social planner is granted in allocating resources and people. If a social planner can allocate both commodities and consumers, we obtain unconstrained or full Pareto optimality. Accordingly, an allocation $(\mathbf{x}; P)$ is called **fully Pareto-optimal** or an **optimum optimorum**, if "there is no better one", i.e. if

(i) $(\mathbf{x}; P)$ is feasible and

(ii) there is no feasible allocation $(\mathbf{x}'; P')$ satisfying $(U_i(\mathbf{x}'; P'))_{i \in I} > (U_i(\mathbf{x}; P))_{i \in I}$.

Denote by $\mathcal{M}^*$ the set of fully Pareto-optimal allocations. If all utility functions are continuous in consumption, $\mathcal{M}^*$ is not empty [Gersbach and Haller (2001)].

It is obvious that competitive equilibrium allocations with free exit need not be fully Pareto-optimal. Suppose e.g. that there are large gains from forming a two-person household because two individuals, say agent 1 and 2, have positive pure group externalities. No further externalities are present in the economy. Moreover, suppose that both agents have the same endowments and the same consumption preferences. A competitive equilibrium with free exit can have every person live in a single-person household. This equilibrium is, however, Pareto inefficient. Agent 1 and 2 could form a two-person household with a household excess demand function equal to the sum of individual excess demand functions. Hence, equilibrium prices and consumption allocation would remain as if all persons lived in single-person households. Hence, agent 1 and 2 would be better off while all other individuals receive the same utility. The example suggests that the lack of appropriate outside options causes inefficiency of CEFE. It also suggests that a pair of CEFE can be Pareto-rankable.[6] In the next section we discuss CEFH.

[6] It is an open question under which circumstances at least one fully Pareto-optimal equilibrium allocation exists.

4.3 Welfare Implications of Adding JO

We have seen that adding outside options is irrelevant if there are no externalities. Now we are going to examine the consequences of adding more outside options in the presence of externalities. Clearly the additional requirement can eliminate some of the competitive equilibria with free exit. But which ones? The good ones, the bad ones, all or none? We shall demonstrate by means of examples that each of the four conceivable alternatives is indeed possible.

We have already seen that under the hypothesis of Proposition 1, none of the equilibria is eliminated. In section 5, we consider examples where all equilibria are eliminated. In this subsection we demonstrate the other two possibilities. Let us first examine an example that exhibits a pair of weakly Pareto-rankable competitive equilibria with free exit where the inferior one is also a competitive equilibrium with free household formation whereas the superior one is not. Subsequently, we modify the example so that the superior competitive equilibrium with free exit turns out to be a competitive equilibrium with free household formation while the inferior equilibrium is eliminated by the additional requirement.

In both examples, the prospect of a tiny surplus share induces a currently single person to form a two-person household, leaving most of the surplus to the new partner. With a population of three people, this leads to the break-up of any existing two-person household and formation of a new one if the opportunity arises, that is if the joining option becomes available. In contrast, the members of a three-person household have no other household to join in a population of three people; thus the three-person household remains unaffected by the introduction of the joining option. In the examples one can suitably alter the household structure associated with a Pareto-superior competitive equilibrium with free exit by varying (primarily) the per capita surplus in three-person households.

Example 2. Let $I = \{1, 2, 3\}$ and $\ell = 1$. For a household h, let the endowment be $\omega_h = |h|$. Let preferences have utility representations of the form

$$U_i(\mathbf{x_h}; h) = a(|h|) \cdot x_i$$

for consumer i in household h where $a(1) = 2, a(2) = 8, a(3) = 5$. Since there is only one good and preferences are strictly monotone, we can set $p = 1$. First consider the competitive equilibrium with free exit $E^1 = (p; (1,1,1); \{\{1\},\{2,3\}\})$ with utility allocation $(2,8,8)$. Next consider the CEFE $E^2 = (p; (0.4, 1.3, 1.3); \{I\})$ with utility allocation $(2, 6.5, 6.5)$. Then E^1 weakly Pareto-dominates E^2. The inferior equilibrium is also a CEFH, since there is no other household to join. However, the superior equilibrium is not a CEFH. Namely individual 2 can propose to consumer 1 to form household $\{1,2\}$ with consumption $y_1 = 1/2$, $y_2 = 3/2$ which makes both better off. ••

Example 3. Let again $I = \{1,2,3\}$ and $\ell = 1$. Modify the previous example by setting $a(1) = 1, a(2) = 8, a(3) = 6$. Take E^1 as before, now with utility allocation $(1,8,8)$. Set $E^2 = (p; (1/5, 7/5, 7/5); \{I\})$ which is an efficient CEFH, with utility allocation $(1.2, 8.4, 8.4)$. Here E^1 is strictly dominated by E^2 and is not a CEFH. ••

The preceding examples highlight the ambivalent implications of adding more outside options for everybody in a society. There exist constellations where everybody is worse off. Nevertheless, there are clear circumstances where adding more outside options is not detrimental to welfare, where in fact some equilibria with free household formation are fully Pareto-optimal. For the purpose of describing such a situation, let us call $P \in \mathcal{P}$ an **optimal household structure**, if there exists a feasible $\mathbf{x}$ such that $(\mathbf{x}; P)$ is a fully Pareto-optimal allocation, i.e. $(\mathbf{x}; P) \in \mathcal{M}^*$. Then we obtain:

Proposition 2 *Suppose pure group externalities, that is* $U_i(\mathbf{x_h}; h) = U_i^c(x_i) + U_i^g(h)$ *for* $\mathbf{x_h} \in \mathcal{X}_h, h \in \mathcal{H}_i, i \in I$. *If*

(i) $(p, \mathbf{x})$ *is a competitive equilibrium of the pure exchange economy represented by* $(U_i^c, \omega_{\{i\}})_{i \in I}$, *where all* $U_i^c, i \in I$, *satisfy local non-satiation, and*

(ii) P *is the unique optimal household structure based solely on group preferences represented by* $U_i^g, i \in I$,

then the allocation $(\mathbf{x}; P)$ *is fully Pareto-optimal and the state* $(p, \mathbf{x}; P)$ *is a CEFH.*

The proof is given in the appendix. Proposition 2 means that free household formation will never destroy all Pareto-optimal allocations if there is a single optimal household structure based on group preferences alone. In that case, the equilibrium condition 4 of CEFH tends to eliminate some inefficient equilibria associated with inefficient household structures. The latter occurs in the following example: Adding the second type of outside options leads to the reshuffling of an inefficient household structure that can prevail as long as only the first type of outside options is available to individuals. Once the joining option becomes available as well, one individual joins another household which leads to the establishment of the optimal household structure.

Example 4. Let $I = \{1, 2, 3\}$ and $\ell = 1$. For a household h, the endowment is $w_h = |h|$. Preferences are represented by utility functions $U_i, i \in I$, and given as follows:

$$\begin{array}{rcll} U_i(\mathbf{x_h}; h) & = & x_i + k & \text{if } h = \{1, 2\} \\ U_i(\mathbf{x_h}; h) & = & x_i - k & \text{if } |h| = 3 \\ U_i(\mathbf{x_h}; h) & = & x_i & \text{otherwise} \end{array}$$

The group externalities satisfy $1 > k > 0$. Since there is only one commodity, we can set $p = 1$. Note that there exists a uniquely determined optimal household structure $P^* = \{\{1, 2\}, \{3\}\}$, based on pure group preferences alone. However, there also exists the CEFE

$$E^1 = (p, (1, 1, 1); \{\{1, 3\}, 2\})$$

with a different household structure and utility allocation $(1, 1, 1)$. The respective equilibrium allocation is, for instance, dominated by the fully Pareto-optimal allocation

$$((1 - k/2, 1 - k/2, 1 + k); \{\{1, 2\}, \{3\}\})$$

with utility allocation $(1 + k/2, 1 + 1/2, 1 + k)$. Moreover, E^1 is not a CEFH since the first and second individual could form a new household providing higher utility for both. Indeed, it is obvious that any allocation $(\mathbf{x}; P)$ with $P \neq P^*$ cannot be a competitive equilibrium allocation with free household formation. ••

5 Existence with EO

In this section we establish the existence of competitive equilibria with free exit. For that purpose, we denote by $P^0 = \{\{1\}, \ldots, \{n\}\}$ the household structure where all households are singletons and formulate a first equilibrium existence theorem.

Proposition 3 *(Trivial Equilibria) Suppose for all $i \in I$:*

(i) $\omega_i \gg 0$.

(ii) $U_i(x_i; \{i\})$ *is continuous, strictly monotone and concave in* x_i.

Then there exists a competitive equilibrium with free exit of the form $(p; \mathbf{x}; P^0)$.

PROOF. As an immediate consequence of Proposition 1 in Gersbach and Haller (1999) or as a corollary of the proof of Proposition 4 given in the appendix, we obtain existence of a price system p and an allocation $\mathbf{x}$ so that conditions 1 and 2 for a competitive equilibrium with free exit are satisfied. We need not check condition 3, since all individuals are already in one-person households which renders the exit option irrelevant. Q.E.D.

The proposition asserts the existence of trivial competitive equilibria with exit where everybody is single and is not exposed to externalities. We also know that under the provisions of the neutrality theorem, any household structure qualifies as equilibrium household structure, provided there is an equilibrium. Otherwise, for multi-member households to exist in equilibrium, there ought to be some incentive for multi-member household formation, some advantage from living in a larger household that prevents its members from leaving.

A priori, a large group or, to be precise, a non-single household h offers an advantage to its members if at any given price system, the group can afford consumption plans for its members that make each member better off than the member's optimal choice as a single consumer — which is captured by inequalities of the form (3) below. If preferences are assumed convex and continuous in household consumption, then under certain additional assumptions, Debreu's (1952) social equilibrium approach to the equilibrium existence problem proves most suitable.[7] Essentially it suffices to assume that every member i of multi-member household h prefers the consumption plan $\mathbf{x_h} = (x_j)_{j \in h}$ for the household to consuming the individual component x_i of $\mathbf{x_h}$ as a single person. But one crucial step in the social equilibrium approach is the restriction to truncated budget sets. This technicality makes the formal definition of the Large Group Advantage (LGA) condition below more complicated and elaborate, since one has to make sure that the inequalities (3) can be met even if household h is restricted to a truncated budget set. Formally, this requirement is captured by the following conditions 1-3. To this end, we restrict prices to the price simplex

$$\Delta = \left\{ p \in \mathbb{R}^{\ell}_{+} : \sum_{k=1}^{\ell} p^k = 1 \right\}.$$

We denote the relative interior of Δ by Δ^o. Further let us choose $k > 0$ so that the social endowment ω_S belongs to the cube $Q = [0, k]^{\ell}$. Set $K = [0, 2k]^{\ell}$.

(LGA) Large Group Advantage: *We say that a multi-member household h has large group advantage, if:*

1. *Every member $i \in h$ has a demand function $x_i^0(\cdot)$, where $x_i^0(p)$ denotes the demand of consumer i when trading individually from the endowment $\omega_{\{i\}}$ at prices $p \in \Delta^o$.*

2. *For every price system $p \in \Delta$, there exists a non-empty, compact and convex set $X_h(p) \subseteq B_h(p) \cap K^h$ which depends continuously on p.*

3. *For all $p \in \Delta^o$ and $\mathbf{x_h} \in B_h(p) \cap K^h$: $\mathbf{x_h} \in X_h(p)$ iff*

$$U_i(\mathbf{x_h}; h) - U_i(x_i^0(p); \{i\}) \geq \delta_i(p) \tag{3}$$

with some threshold $\delta_i(p) \geq 0$ holds for all $i \in h$.

To illustrate that the key condition 3 of LGA is non-vacuous, let us present two alternative assumptions on a multi-member household h that will yield

[7] We shall elaborate later on an alternative approach relying on the "excess demand lemma" which proves successful under different assumptions, including specific positive externalities of the separable type within household h. Without separability and purely positive externalities, the social equilibrium approach of Debreu is more promising.

condition 3 when supplemented with suitable further assumptions: (i) Sufficiently bounded individual demands so that $(x_i^0(p))_{i \in h} \in B_h(p) \cap K^h$. An example is given by the utility representation $U_i(x_i; \{i\}) = \min\{x_i^k | k = 1, \dots, \ell\}$. (ii) Group preferences which strictly dominate consumption preferences, e.g. $U_i(\cdot\, ; h) \geq 0$ and $U_i(\cdot\, ; \{i\}) < 0$ for $i \in h$. In this case, (3) becomes trivial. An example with $U_i(\cdot\, ; \{i\}) < 0$ is given by $U_i(x_i; \{i\}) = -\sum_k \exp(-x_i^k)$. The somewhat extreme cases (i) and (ii) have the virtue of being simple and transparent. In the appendix we show:

Proposition 4 *(Non-Trivial Equilibria) Suppose:*

(i) $\omega_h \gg 0$ *for all* $h \in \mathcal{H}$.

(ii) $U_i(\mathbf{x_h}; h)$ *is continuous and concave for all* $i \in h, h \in \mathcal{H}$.

(iii) $U_i(x_i; \{i\})$ *is strictly monotone for all* $i \in I$.

(iv) There exist a household $h \in \mathcal{H}$ *with* $1 < |h| < n$*, which has large group advantage (LGA), and a member* $j \in h$ *whose preferences are strictly monotonic in own consumption and who is not imposing any negative consumption externalities on other household members.*

Then there exists a competitive equilibrium with free exit of the form $(p, \mathbf{x}; P)$ *with* $P \neq P^0$*. More specifically,* $h \in P$ *for some* h *satisfying (iv).*

The proposition basically states that as soon as two or more agents can gain from living together in a household, non-trivial equilibria with free exit and a multi-member household exist. Needless to say that one can also impose conditions so that only small groups are viable in an equilibrium with free exit.

As mentioned in footnote 7, with different assumptions an approach relying on the "excess demand lemma" proves successful. Specifically, one makes certain standard assumptions (including strict concavity of the functions $U_i(\cdot\, ; \{i\})$) in combination with particular positive externalities of the separable type within household h. Two special cases of the latter are positive pure group externalities [PGE restricted to household h] on the one hand and positive separable pure consumption externalites [SEP of Haller (2000) restricted to household h] on the other hand. The proof is similar to that of Proposition 3 in Gersbach and Haller (1999).

6 Existence with EO and JO

In this section we take up the challenging question whether and under which circumstances competitive equilibria with free household formation exist. We start with the observation that Proposition 2 lends itself to an existence result.

Proposition 5 *Suppose pure group externalities, that is* $U_i(\mathbf{x_h}; h) = U_i^c(x_i) + U_i^g(h)$ *for* $\mathbf{x_h} \in \mathcal{X}_h, h \in \mathcal{H}_i, i \in I$. *If*

(i) $\omega_S \gg 0$, *each of the functions* $U_i^c, i \in I$, *is continuous, strictly increasing and strictly quasi-concave, and*

(ii) P *is the unique optimal household structure based solely on group preferences represented by* $U_i^g, i \in I$,

then a fully Pareto-optimal CEFH exists.

PROOF. By Proposition 17.C.1 of Mas-Colell *et al.* (1995), there exists a competitive equilibrium $(p, \mathbf{x})$ of the pure exchange economy represented by $(U_i^c, \omega_{\{i\}})_{i \in I}$ if (i) holds. By Proposition 2 above, the allocation $(\mathbf{x}; P)$ is fully Pareto-optimal and the state $(p, \mathbf{x}; P)$ is a CEFH if (i) and (ii) hold. Q.E.D.

Next we make the important observation that there are constellations where competitive equilibria with free household formation need not exist, where all conceivable household structures are destabilized by outside options of the second type (JO). Therein lies the challenge.

6.1 Non-Existence of Equilibria with Free Household Formation: An Example

We are going to present an example that exhibits pure group externalities. There is also a single consumption good. Consequently, in equilibrium no trade occurs across households, but utility can be (imperfectly) transferred between members of the same household. For each household, one can determine the feasible utility allocations for its members. Hence by varying the consumptive utility function u and the group externality parameters k and ϵ of the example, one can generate an entire family of hedonic coalition games in the sense of Drèze and Greenberg (1980). A transferable utility game results if (and only if) u is affine linear.

In the specific four-person example, surplus comparisons reveal that households of any size are unstable. A four-person household is unstable because it generates negative surplus and at least one of its members can fare better going single. A three-person household is unstable, since at least one member can benefit from forming a two-person household with the currently single individual and appropriate most of the surplus. This is possible because in a two-person household, the maximum per capita surplus is not much less than in a three-person household. But two coexisting two-person households do not constitute a stable configuration either, since at least one person will have an incentive to switch households. This is the case because every three-person household includes at least one person with a preference for three-person households so that the maximum per capita surplus in such a household exceeds the maximum per capita surplus in a two-person household. Finally, two singles can always benefit from forming a two-person household. Thus a household proves unstable regardless of size.

Example 5. Let $I = \{1, 2, 3, 4\}$ and $\ell = 1$. For a household h, the endowment is $w_h = |h|$. Preferences are represented by utility functions $U_i, i \in I$, and given as follows:

$$
\begin{aligned}
U_i(\mathbf{x_h}; h) &= u(x_i) && \text{if } h = \{i\} && (4)\\
U_i(\mathbf{x_h}; h) &= u(x_i) + k && \text{if } |h| = 2 && (5)\\
U_i(\mathbf{x_h}; h) &= u(x_i) + k && \text{if } |h| = 3,\ i = 1, 2 && (6)\\
U_i(\mathbf{x_h}; h) &= u(x_i) + k + \varepsilon && \text{if } |h| = 3,\ i = 3, 4 && (7)\\
U_i(\mathbf{x_h}; h) &= u(x_i) - k && \text{if } |h| = 4 && (8)
\end{aligned}
$$

The group externalities satisfy $k > 0$ and $k \geq \varepsilon \geq 0$. The function u is continuous and strictly increasing. It satisfies $u(1) \geq u(0) + k$. Since there is only one good, we can set $p = 1$.

We first consider the case $\varepsilon = 0$. Then, there exists a CEFH, namely

$$E^1 = (p, (1, 1, 1, 1), \{\{1, 2\}, \{3, 4\}\})$$

with utility allocation $(u(1)+k, u(1)+k, u(1)+k, u(1)+k)$. Since the population is homogeneous, there exist two other equilibria with the same utility allocation and household structures $\{\{1,3\},\{2,4\}\}$ and $\{\{1,4\},\{2,3\}\}$, respectively. No other equilibria with free household formation exist. For instance, the household structure $\{\{1,2,3\},4\}$ cannot be part of an equilibrium, since at least one individual in the household $\{1,2,3\}$ can propose to agent $i=4$ to form a two-person household which makes both individuals better off. Specifically, the individual leaving $\{1,2,3\}$ can offer $i=4$ a consumption level $u^{-1}(u(1)-k+\delta)$ for some small δ, $k>\delta>0$. Agent 4's utility will be $u(1)+\delta$ and therefore larger than in the candidate equilibrium. The deviating agent obtains a utility

$$u(2-u^{-1}(u(1)-k+\delta))+k$$

which exceeds the utility of at least one member in the household $\{1,2,3\}$ since $\delta<k$.

Next let us consider the case $\varepsilon>0$ where ε is sufficiently small. We claim that no CEFH exists. Consider first the candidate equilibrium E^1. Individual 2 could join $\{3,4\}$ by proposing the household allocation:

$$x_g=(x_2,x_3,x_4)=(3-2u^{-1}(u(1)-\varepsilon), u^{-1}(u(1)-\varepsilon), u^{-1}(u(1)-\varepsilon)) \quad (9)$$

which yields the utility allocation

$$(u(3-2u^{-1}(u(1)-\varepsilon))+k, u(1)+k, u(1)+k) \quad (10)$$

and makes agent 2 better off while the utility of individuals 3 and 4 remains constant. Hence, E^1 cannot be a CEFH. A similar argument applies mutatis mutandis for any other household structure with two two-person households. Furthermore, by essentially the same agument as before, no CEFH can exist with a three-person or four-person household. Finally, if everybody were alone, two persons could form a household and both be better off. Therefore, no CEFH exists. ••

The interesting feature of the example is that a small change of the externalities destroys the existence of a competitive equilibrium with free household formation. It is obvious that the existence problem in the example can be overcome by taking a specific number of replica of the original economy. In the example three replica would allow all individuals preferring a three-person household over a two-person household to be member of a three-person household while other individuals could live in two-person households. Later, however, we will see that enlarging the economy through replication cannot restore existence under all circumstances.

6.2 Existence with One Commodity

Having established the possibility of non-existence, we next identify circumstances in which a competitive equilibrium with free household formation exists. We first provide several simple existence results when trade of consumption goods does not matter, because there is only one commodity. Subsequently, the more challenging case of more than one commodity is considered.

6.2.1 Hedonic Coalitions

If there is only one commodity, the model resembles a game with "hedonic coalitions" à la Drèze and Greenberg (1980) where trade and transfers among coalitions are prohibited. Their concept of *individually stable equilibrium* (i.s.e.) is slighty stronger than our notion of competitive equilibrium with free household formation (CEFH). Their Example 3.1 and our Examples 5 and 6 are all instances of non-existence of a CEFH (and by implication, an i.s.e.).

The construction of Examples 1 and 2 generalizes and yields a first immediate existence result for CEFH. Let $I = \{1, \ldots, n\}$ and $\ell = 1$. Further, let preferences have utility representations of the form

$$U_i(\mathbf{x_h}; h) = A(h) \cdot x_i \tag{11}$$

for consumer i in household h where the externality coefficient $A(h) > 0$ represents a multiplicatively separable group externality within household h — which is ordinally equivalent to a pure group externality. One obtains as an immediate result:

Proposition 6 *If P is a household structure such that*

$$A(h) \cdot \omega_h \geq \sum_{i \in h} A(\{i\}) \cdot \omega_{\{i\}}$$

for all $h \in P$, then there exists a competitive equilibrium with free exit with household structure P. If in particular, the inequality holds for $h = I$, then there exists a competitive equilibrium with free household formation where the household I is formed.

6.2.2 Two-sided Matching

Next we deal with the existence of competitive equilibria with free household formation in the *marriage market*. The marriage market has been a prominent application of the two-sided matching approach [see Roth and Sotomayor (1990)]. Gale and Shapley (1962) have shown in their seminal paper that there always exists a stable matching for any marriage market. Many subsequent contributions have demonstrated the robustness of this classic result. We have already pointed out that condition 4 in our definition of a competitive equilibrium with free household formation (CEFE) is weaker than the stability condition in the matching literature [see Roth and Sotomayor (1990)] which requires that a matching be not blocked by any individual or pair of agents forming a new match. Therefore, this literature promises to provide further existence results in our context. Indeed, the existence results carry over from the matching literature to our framework when there is only one commodity.

To state such a result in our context we consider a simple marriage market as follows. We suppose $\ell = 1$ and that the population is divided into two non-empty, finite and disjoint sets, M and F: $M = \{m_1, \ldots, m_m\}$ is the set of men, and $F = \{f_1, \ldots, f_n\}$ is the set of women. We assume that each individual has some endowments, $\omega_i > 0$ and $\omega_j > 0$, respectively. The *preferences of men* are given by

$$U_i(\mathbf{x_h}; h) = x_i \quad \text{if } h = \{m_i\} \tag{12}$$

$$U_i(\mathbf{x_h}; h) = x_i + g_{ij} \quad \text{if } h = \{m_i, f_j\} \tag{13}$$

$$U_i(\mathbf{x_h}; h) = x_i - \bar{g} \quad \text{in all other cases} \tag{14}$$

We assume $\bar{g} > 0$ and $0 \leq g_{ij} \leq \omega_i$ for any potential couple $\{m_i, f_j\}$. The *preferences of women* are defined accordingly. We call such preferences **pure group externalities of the matching type**.

Such a marriage market where utility can be freely transferred within a household by an appropriate allocation of commodities and no trade through markets occurs, can be viewed as a generalized assignment game. We obtain:

Proposition 7 *Suppose $\ell = 1$ and pure group externalities of the matching type. Then a competitive equilibrium with free household formation exists.*

PROOF. Because of the exit condition 3 and $\bar{g} > 0$, we only have to consider single person households or matches between a man and a woman as potential households in a CEFH. Since our free household formation condition 4 is weaker than the stability condition in the matching literature we can rely on the existence proofs for the generalizations and variations of the assignment model provided by Shapley and Shubik (1972), Quinzii (1984), Gale (1984) and Alkan and Gale (1990); see also Roth and Sotomayor (1990).

Let us check the essential assumptions as they are formulated in Alkan and Gale (1990), for example. Let us hypothetically extend the domain of U_i to negative consumption – which will not occur in equilibrium. Then the range of the utility function is all of $\mathbb{R}$, since $U_i(x_i)$ is unbounded above and below. Moreover, for any couple, the corresponding Pareto-frontier in utility space is linear. Hence, we can apply Theorem 1 of Alkan and Gale (1990) which establishes existence of a core payoff and, consequently, of a CEFH. Q.E.D.

6.3 Non-Existence in the Marriage Market

When investigating the stable matching problem in our framework, where not only individuals are matched through the market but also commodities are traded and collective household decisions are taken, one encounters a number of new problems.

We have seen that the existence results of the matching literature are applicable in our framework provided that there is only one commodity. With several commodities, however, households may actively trade in the market. Consequently, what is feasible for a household depends on market prices. This price-dependence tends to undermine existence, even if households are restricted to singles and heterosexual marriages. Although our equilibrium conditions 3 and 4 are weaker than the standard stability condition for the marriage market, the existence result for the special case $\ell = 1$ does not carry over to the multiple goods case as the following example demonstrates. In the three-person example, externalities are confined to the one female individual whom we shall call Anita. She experiences a positive group externality and a negative consumption externality (with respect to the second good) when living with a partner. For any constant consumption of the partner, the consumption externality becomes less severe as Anita's consumption of the second good increases.

If everyone is single, the market clearing price of the second good happens to be low. Then at the going prices, Anita can afford enough own consumption of the second good so that the positive group externality dominates and she and another individual can both benefit from forming a two-person household. Now suppose Anita belongs to a two-person household with one single person remaining. Then the market is cleared at a high relative price of the second good. Therefore, as Anita can afford too little consumption of the second good, the negative consumption externality dominates, and she is better off alone. Hence for any given household structure, the market clearing prices are such that Anita can benefit from a change of household.

Example 6. Let $\ell = 2$ and $I = \{1, 2, 3\}$ where the first two individuals are male and $i = 3$ is Anita, the only female. The individual endowments are given by:

$$w_1 = (0, 1), w_2 = (0, 1), w_3 = (1, 1).$$

Preferences are represented by utility functions of the form $U_i(\mathbf{x_h}; h)$. Specifically,

$$\begin{array}{rcll} U_1(\mathbf{x_h}; h) & = & \ln x_1^2, & \text{if } h = \{1\}, \{1, 3\}; \\ U_2(\mathbf{x_h}; h) & = & \ln x_2^2, & \text{if } h = \{2\}, \{2, 3\}; \\ U_3(\mathbf{x_h}; h) & = & \alpha \ln x_3^1 + (1 - \alpha) \ln x_3^2, & \text{if } h = \{3\}; \\ U_3(\mathbf{x_h}; h) & = & \alpha \ln x_3^1 + (1 - \alpha) \ln(\max\{0, x_3^2 - k x_i^2\}) + g, & \text{if } h = \{3, i\}, i = 1, 2. \end{array}$$

where $0 < \alpha < 1$, x_i^j denotes the quantity of good j ($j = 1, 2$) consumed by individual i, and we adhere to the convention $\ln 0 = -\infty$.

Living in a two-person household with partner $i = 1$ or partner $i = 2$ provides the third individual with a positive group externality ($g > 0$). She suffers, however, from a negative consumption externality ($1 > k > 0$). We further assume that living in a three-person household or in $h = \{1, 2\}$ creates enormous negative group externalities and will never be chosen. Hence our model is of the matching type where the only conceivable household structures consist of single-person and two-person households.

Commodity prices are normalized so that $p_1 = 1$. Consider first the household structure $P^\circ = \{\{1\}, \{2\}, \{3\}\}$. It is obvious that there exists a unique competitive equilibrium $(p^0, \mathbf{x^0})$ relative to P° given by:

$$\begin{aligned} p^0 &= (1, p_2^0) \\ x_1^0 &= (0, 1) \\ x_2^0 &= (0, 1) \\ x_3^0 &= (1, 1) \end{aligned}$$

To determine the market clearing price, we observe that the demand x_3^2 is given by

$$x_3^2 = (1 - \alpha)(1 + p_2)/p_2.$$

Therefore market clearing, $x_3^2 = 1$, yields $p_2^0 = \frac{1-\alpha}{\alpha}$. At the going equilibrium prices $i = 3$ could propose to $i = 1$ to form the household $h = \{1, 3\}$ by offering $i = 3$ one unit of commodity 2. The remaining problem of individual 3 is

$$\begin{aligned} &\max \left\{ \alpha \ln x_3^1 + (1 - \alpha) \ln(\max\{0, x_3^2 - k\}) + g \right\} \\ &\text{s.t. } x_3^1 + p_2^0 x_3^2 = 1 + p_2^0. \end{aligned}$$

The solution is

$$\hat{x}_3^2 = (1 - \alpha)(1 + p_2^0)/p_2^0 + \alpha k \quad = 1 + \alpha k, \tag{15}$$

$$\hat{x}_3^1 = 1 + p_2^0 - p_2^0 x_3^2 \quad = 1 - (1 - \alpha)k \tag{16}$$

which yields utility

$$U_3(\mathbf{\hat{x}_h}; h) = \alpha \ln(1-(1-\alpha)k)+(1-\alpha) \ln(1-(1-\alpha)k)+g = \ln(1-(1-\alpha)k)+g.$$

Suppose that we choose parameters (k, g) such that

$$\ln(1 - (1 - \alpha)k) + g > 0.$$

Then $(p^0, \mathbf{x^0}; \mathrm{P}^\circ)$ is not a competitive equilibrium with free household formation because $h = \{1,3\}$ will be formed at equilibrium prices.

Consider next the household structure $P^* = \{\{1,3\},\{2\}\}$. Consider household $h = \{1,3\}$. The maximal utility the third individual can achieve, subject to 1's outside options, is attained when individual $i = 1$ consumes one unit of the second commodity. The remaining problem of individual 3 is as in the case before. Therefore we obtain the demand for the second commodity as

$$x_3^2 = (1-\alpha)(1+p_2)/p_2 + \alpha k.$$

But to be in equilibrium now, markets must clear again. Hence $x_3^1 = 1$, $x_3^2 = 1$ which requires equilibrium prices $p_2^* = \frac{1-\alpha}{\alpha(1-k)}$. The utility of individual 3 is

$$U_3(\mathbf{x}^*_\mathbf{h}; h) = (1-\alpha)\ln(1-k) + g.$$

Since there exist values of α such that

$$\ln(1-(1-\alpha)k) > (1-\alpha)\ln(1-k),$$

e. g. $\alpha = \frac{1}{2}$, we can fix such an α and choose parameter constellations (k, g) such that

$$U_3(\hat{\mathbf{x}}_\mathbf{h}; h) > 0 > U_3(\mathbf{x}^*_\mathbf{h}; h).$$

Since individual 3 can always achieve utility $U_3 = 0$ by living as a one-person household and consuming her endowments, we conclude that under the suitably chosen parameter constellation $(p^*, \mathbf{x}^*; P^*)$ is not a competitive equilibrium with free household formation: agent 3 prefers to be single at the going market prices. However, we have established before that agent 3 prefers to form a two-person household at the market prices which would obtain if everybody were single. Since individuals 1 and 2 are completely interchangeable, we conclude that no CEFH exists. ••

This example shows that active trade across households poses a challenge with regard to existence of stable outcomes not only for us, but also for the traditional matching literature. The hypotheses of the example and of Proposition 7 differ in two respects. First, there are several commodities. Second, there are no longer pure group externalities of the matching type. This begs the question whether existence of a CEFH can be obtained, if there are several commodities, but pure group externalities of the matching type prevail. In the most general form of the latter case, the population is partitioned into men and women; preferences are represented by $U_i(\mathbf{x_h}; h) = U_i^c(x_i) + U_i^g(h)$ such that based on the group preferences given by U_i^g alone, individual i strictly prefers staying single or forming a two-person household with a member of the opposite sex ("marriage") to any other household. Under these circumstances, the following proposition holds whose proof is straightforward.

Proposition 8 *Suppose the general case of pure group externalities of the matching type. If*

(i) $(p, \mathbf{x})$ *is a competitive equilibrium of the pure exchange economy represented by* $(U_i^c, w_{\{i\}})_{i \in I}$ *and*

(ii) P *is a stable matching with respect to pure group preferences,*

then the state $(p, \mathbf{x}; P)$ *is a CEFE.*

According to the classical result of Gale and Shapley (1962), condition (ii) can always be satisfied. Under standard assumptions on consumer characteristics, condition (i) holds as well and, consequently, a CEFE with stable matching exists. Mohemkar-Kheirandish (2001) shows, among other things, that under additional assumptions a CEFE of the form described in the proposition is also a CEFH. He assumes each U_i^c concave, strictly monotone and continuously differentiable on $\mathbb{R}^\ell_{++}$ so that the first order approach applies; each $w_{\{i\}}$ strictly positive; all males of the same type with strict preference for marriage; all females of the same type with strict preference for marriage; an equal number of males and females. Needless to say that a CEFE of the form suggested by Proposition 8 happens to be a CEFH, if the assumptions of Proposition 1 hold. Furthermore, such a CEFE turns out to be a CEFH whenever the strong assumption (ii) of Proposition 2 holds.

However, in general a CEFE of the form described in the last proposition need not be a CEFH. To see this, it suffices to consider a population consisting of one male and one female, where the male has a slight preference (in terms of the utility difference) for staying single and the female has a strong preference for being married. Let the corresponding (absolute) utility differentials be ϵ for the male and Δ for the female. Then the stable matching with respect to pure group preferences requires both to remain single. Now suppose they have identical and strictly positive endowments and identical consumption preferences of the Cobb-Douglas type. Then the competitive equilibrium in (i) is a no trade equilibrium. If ϵ is sufficiently small and Δ is sufficiently large, they can both benefit from getting married and shifting some consumption from the female to the male — which shows our claim.

Additional examples of non-existence appear in the literature on hedonic coalitions and matching. Example 4 of Bogomolnaia and Jackson (2002), the example of Alkan (1988) and the roommate example of Gale and Shapley (1962) all constitute purely hedonic cases that differ from marriage models. Our Example 5 does not belong to the marriage category either. It shares features of matching and assignment games due to the presence of a consumption good and pure group externalities. Our Example 6 is reminiscent of Example 3.3 in Drèze and Greenberg (1980), despite the fact that the latter is not a marriage model. Their common feature consists in the interaction of household formation and commodity allocation. The striking feature of Drèze and Greenberg's example is the absence of externalities. It is driven by household-specific (coalition-specific) endowments w_h with $w_h \neq \sum_{i \in h} w_{\{i\}}$ for some households h.

6.4 Discussion

Non-existence of a competitive equilibrium with certain properties renders the discussion of equilibrium household structures and equilibrium welfare obsolete. There are several possible responses to the non-existence problem.

First, the model might be misspecified. For instance, the modeling might be too parsimonious. While household stability cannot be achieved on purely economic grounds, given the two types of outside options depicted here, a full account of all the forces that stabilize – or destabilize – households might restore equilibrium.[8] Furthermore, the market for marriages may be more competitive than reflected in our equilibrium concept.

Second, one might suspect that price-taking is too restrictive. If only con-

[8]For example, household-specific human capital can serve as a bond among household members, as a referee has pointed out.

sumers could freely recontract without regard to market prices, then the economy would settle in an equilibrium state in the sense of Edgeworth, that is a core allocation. Indeed, the full core which allows for the reallocation of consumers and commodities, happens to be non-empty in Example 6. However, Gersbach and Haller (1999) contains a three-person example where gender does not matter and the full core turns out to be empty.

Third, non-existence of equilibrium may capture an important feature of reality. Let us recapitulate the essence of Example 6. Individuals may find it optimal to split at the going market prices in order to reduce negative consumption externalities. But at equilibrium prices of the changed household structure, individuals may find it optimal to form a two-person household in order to benefit from group externalities, because they can buy more of those goods which generate less consumption externalities. The marital status of the woman in the example affects her market opportunities and vice versa. Therefore, the woman may simply go through a sequence of marriage, divorce, marriage, divorce, etc., which constitutes an example of sequential monogamy, possibly with breaks. A dynamic approach suggests itself for future analysis.

Finally, non-existence may simply be a small number or integer problem that goes away when the population is large enough. For instance, non-existence in Example 5 disappears after suitable replication. Insofar, non-existence may be considered merely an artifact of the particular example. However, the problem is more intricate. Non-existence in Example 6 does not vanish under replication, not even asymptotically. The reason is that sizeable (relative to the economy) groups of consumers of the same type keep moving simultaneously into or out of households. To end on a positive note, sufficient dispersion of consumer characteristics will restore existence. If each agent is replaced not by identical clones, but by similar yet non-identical copies, then at certain prices, some of the females may wish to remain single while others may wish to stay in two-person households and, consequently, the household structure may end up to be stable. This certainly works in Example 6. This is not to say that existence would never become a problem if only consumer characteristics were well dispersed.

7 Conclusions

In this paper, we have studied a general equilibrium model where households operate in a competitive market environment, can have several members and make efficient collective consumption decisions. Our main concern has been the impact (on household stability and equilibrium efficiency) of introducing outside options. Our approach differs from partial equilibrium analyses which has produced countless theoretical and empirical studies of household related issues, involving numerous economic sub-disciplines and touching upon topics as diverse as fertility, mortality, demography, population dynamics, marriage and matching, status, income, poverty, nutrition, health, public transfers, education, social capital, human capital, employment, development, welfare, demand and supply, and so forth. Each of the sub-disciplines has developed its own rich body of theories and accumulated a host of empirical work. We think that applications of our model might provide a complimentary general equilibrium perspective on these issues.

Moreover, suitable extensions of the basic framework might address further pertinent issues that arose during the course of our analysis which we shall briefly revisit. While the current investigation is devoted to the allocative consequences of efficient decisions at the household level, an inquiry into the allocative consequences of inefficient household decisions could be fruitful as well. Suppose some households are making mistakes by the standards of collective rationality. Then one might address the causes of these mistakes or study their allocative consequences. For some thoughts related to the second issue we refer to the preliminary research on general equilibrium models with inefficient households in Gersbach and Haller (2002).

Public policy issues in a variety of areas might be addressed within the current framework. We provide three examples. The first area consists of policies that directly affect outside options. For instance, laws governing the right to divorce, child support and marital property upon divorce influence directly the attractiveness of exercising outside options. Suppose future research shows that, as a rule, granting more outside options to individuals promotes social efficiency. Then divorce-related property rights should not depend on the presence of other adults in the new households of the ex-spouses. However, some of our examples suggest that the joining option tends to destabilize households and to eliminate superior equilibria. If these negative effects turn out to be the rule, then taking the joining option should possibly be discouraged, for instance by granting less generous property rights to those who exercise it.

The second area comprises policies that influence consumption externalities. For instance, taxes that can affect consumption externalities may also affect the stability of households. Consider a two-person household which is formed because of positive group externalities. But one non-smoking member suffers from negative consumption externalities, because the partner is smoking. A sufficiently large cigarette tax reduces smoking and, depending on the elasticity of cigarette demand, may increase or reduce consumption of the remaining goods in the household. In the former case, a cigarette tax can enhance the stability of the particular household — which might serve as an additional argument in support of such a "sin tax". In the latter case, the non-smoker might prefer to leave the household. Thus a heavy "sin tax" may also destabilize certain households.

The last category includes taxes and transfers to and from households and their members. Consider for instance the case of female labor supply. A sizeable fraction of women do not work outside of their home when living with partners, but presumably would go to work if they were single — unless they went on welfare instead. Whether or not such corner solutions occur depends among other things on household decisions regarding care for children but very likely also on how a second household income is treated with respect to taxation and transfers. High marginal tax rates on second household incomes occur naturally when taxation is progressive, only total household income is taxed and tax codes do not distinguish between multi-person and single-person households. Such tax systems tend to promote the aforementioned corner solutions. They may also make the exit option excessively attractive, conceivably with undesirable consequences. Hence they can influence both the allocation of resources and the composition of households.

The policy questions have no straightforward answers. A general equilibrium approach helps capture relevant aggregate effects and spillovers across households and markets.

8 Appendix

Proof of Proposition 1:

Step 1:

We show that $(p, \mathbf{x}; P)$ is a competitive equilibrium with free exit if and only if $(p, \mathbf{x}; P^0)$ is a competitive equilibrium with free exit where $P^0 = \{\{i\} : i \in I\}$.

Suppose now that $(p, \mathbf{x}; P^0)$ is a CEFE. Recall that absence of externalities and local non-satiation is assumed. Hence, by the first welfare theorem, $\mathbf{x}$ is Pareto-optimal — regardless of the household structure. We claim that

$$\mathbf{x_h} \in EB_h(p) \text{ for any potential household } h. \tag{17}$$

Clearly, $px_i \leq p\omega_i$ for all i, hence $p * \mathbf{x_h} \leq p\omega_h$, i.e. $\mathbf{x_h} \in B_h(p)$ for all potential households h. Suppose $\mathbf{x_h} \notin EB_h(p)$ for some h. Then there exists $\mathbf{y_h} \in B_h(p)$ with

$$U_i(y_i) \geq U_i(x_i) \text{ for all } i \in h;$$
$$U_j(y_j) > U_j(x_j) \text{ for some } j \in h.$$

Equilibrium and local non-satiation imply

$$py_i \geq p\omega_i \text{ for all } i \in h;$$
$$py_j > p\omega_j \text{ for some } j \in h.$$

Hence $p * \mathbf{y_h} > p\omega_h$, contradicting $\mathbf{y_h} \in B_h(p)$. Therefore, (17) has to hold which implies the first condition of a competitive equilibrium with free exit. Further observe that the second and third defining conditions of a competitive equilibrium with free exit are trivially met here. Hence $(p, \mathbf{x}; P)$ is a CEFE.

Suppose next that $(p, \mathbf{x}; P)$ is a competitive equilibrium with free exit. Because of local non-satiation, $p \gg 0$. Because of continuity, we can then choose for each $i \in I$ a utility maximizer x_i^0 in $B_{\{i\}}(p)$, pertaining to the event that consumer i is acting individually and trading from his endowment ω_i at prices p. Since $(p, \mathbf{x}; P)$ is a CEFE,

$$U_i(x_i) \geq U_i(x_i^0) \text{ for all } i \in I.$$

We claim that

$$U_i(x_i) = U_i(x_i^0) \text{ for all } i \in I. \tag{18}$$

Suppose not. Then there exists a household $h \in P$ such that

$$\begin{aligned} U_i(x_i) &\geq U_i(x_i^0) \text{ for all } i \in h, \\ U_j(x_j) &> U_j(x_j^0) \text{ for some } j \in h. \end{aligned}$$

Hence, some individuals $j \in h$ cannot afford x_j when trading from ω_j at prices p. Hence, $p \cdot x_j > p \cdot \omega_j$. For all individuals i we have $p \cdot x_i \geq p \cdot \omega_i$. Summing up all individual budget constraints yields

$$p * \mathbf{x}_h = p \cdot \left(\sum_{i \in h} x_i \right) > p \cdot \left(\sum_{i \in h} \omega_i \right) = p \cdot \omega_h$$

which, however, violates the budget constraint of household h. Hence, $U_i(x_i) = U_i(x_i^0)$, $i \in I$. Because of local non-satiation, (18) implies

$$px_i \geq px_i^0 = p\omega_i \text{ for all individuals } i.$$

We further claim that

$$x_i \in B_{\{i\}}(p) \text{ for all } i \in I. \tag{19}$$

Suppose not. Then there exists a household $h \in P$ such that

$$\begin{aligned} px_i \geq p\omega_i \text{ for all } i \in h, \\ px_j > p\omega_j \text{ for some } j \in h, \end{aligned}$$

leading once more to a violation of the household's budget constraint. Hence (19) must hold. (18) and (19) imply that $(p, \mathbf{x}; P^0)$ is a CEFE.

Step 2:

We show that if $(p, \mathbf{x}; P^0)$ is a competitive equilibrium with free exit where $P^0 = \{\{i\} : i \in I\}$, then for any $P \in \mathcal{P}$, $(p, \mathbf{x}; P)$ is also a competitive equilibrium with free household formation.

Now let $(p, \mathbf{x}; P^0)$ be a CEFE and P be any feasible household structure. Because of the absence of externalities and local non-satiation, the first welfare theorem applies and $\mathbf{x}$ is Pareto-optimal regardless of the household structure. From step 1 we know that $(p, \mathbf{x}; P)$ is a CEFE and

$$\mathbf{x_h} \in EB_h(p) \text{ for any potential household } h. \tag{20}$$

We want to show that $(p, \mathbf{x}; P)$ is also a CEFH. Suppose not. Hence, there exist two households g and h in P and $i \in h$ and a consumption allocation $\mathbf{y_{g \cup \{i\}}}$ in $B_{g \cup \{i\}}(p)$ such that

$U_i(y_i) > U_i(x_i)$ and

$U_j(y_j) \geq U_j(x_j)$ for all $j \in g$.

Local non-satiation implies

$p\, y_i > p\, \omega_i = p\, x_i$ and

$p\, y_j \geq p\, \omega_j = p\, x_j$ for all $j \in g$.

Hence, individual i cannot afford y_i when trading from ω_i at prices p. For all individuals $j \in g$ we have $p \cdot x_j \geq p \cdot \omega_j$. Summing up all individual budget constraints yields

$$p * \mathbf{y_{g \cup \{i\}}} = p \cdot y_i + p \cdot \sum_{j \in g} x_j > p \cdot \omega_i + p \cdot \sum_{j \in g} \omega_j = p \cdot \omega_g + p \cdot \omega_i$$

which, however, violates the budget constraint of household $g \cup \{i\}$.

Hence, we obtain a contradiction unless $(p, \mathbf{x}; P)$ is a CEFH. Q.E.D.

Proof of Proposition 2:

Suppose the state $(p, \mathbf{x}; P)$ satisfies (i) and (ii). For $i \in I$, let $P(i)$ denote the corresponding element of P, i.e. the household to which i belongs. We first claim that there do not exist any $i \in I$ and $h \in \mathcal{H}_i$ with $U_i^g(h) > U_i^g(P(i))$. For otherwise, there would exist an optimal household structure based solely on group preferences, P^* such that $U_i^g(P^*(i)) \geq U_i^g(h) > U_i^g(P(i))$ and, therefore, $P^* \neq P$, contradicting (ii). Moreover, we observe that $\mathbf{x}$ is a Pareto-optimal allocation of the pure exchange economy $(U_i^c, \omega_{\{i\}})_{i \in I}$ because consumers are locally non-satiated.

Consider now any feasible allocation $(\mathbf{y}; P')$ and $i \in I$. Suppose $U_i(\mathbf{y_{P'(i)}}; P'(i)) > U_i(\mathbf{x_{P(i)}}; P(i))$. Then we claim that $U_j(\mathbf{y_{P'(j)}}; P'(j)) < U_j(\mathbf{x_{P(j)}}; P(j))$ for some $j \in I$. From above we have $U_i^g(P'(i)) \leq U_i^g(P(i))$.

Hence $U_i^c(y_i) > U_i^c(x_i)$. This implies $U_j^c(y_j) < U_j^c(x_j)$ for some $j \in I$, since $\mathbf{x}$ is a Pareto-optimal consumption allocation of the pure exchange economy $(U_i^c, \omega_{\{i\}})_{i \in I}$. Further $U_j^g(P'(j)) \leq U_j^g(P(j))$. Hence the claim follows. This shows that the allocation $(\mathbf{x}; P)$ is fully Pareto-optimal.

Next we prove that the state $(p, \mathbf{x}; P)$ is a competitive equilibrium with free household formation. Suppose not. Hence, there exist a household $h \in P$ and an individual $i \in h$ such that either i is better off as a single or there exists a household $g \in P$ which i can join and where the utility of all members of the newly created household $g \cup \{i\}$ can be improved. We concentrate on the latter case. The case when individual i forms a one-person household is similar.

Let $\mathbf{y_{g \cup \{i\}}} \in B_{g \cup \{i\}}(p)$ be an allocation in the newly created household $g \cup \{i\}$ which makes everybody in this household better off. Since $U_j^g(g \cup \{i\}) \leq U_j^g(P(j))$, $U_j^c(y_j) > U_j^c(x_j)$ has to hold for each $j \in g \cup \{i\}$. But since $(p, \mathbf{x})$ is a competitive equilibrium of the pure exchange economy $(U_i^c, \omega_{\{i\}})_{i \in I}$, we have $py_j > p\omega_j$ for all $j \in g \cup \{i\}$. Therefore, $p * \mathbf{y_{g \cup \{i\}}} > p \cdot \omega_{g \cup \{i\}}$, contradicting $\mathbf{y_{g \cup \{i\}}} \in B_{g \cup \{i\}}(p)$. Q.E.D.

Proof of Proposition 4:

We start from the proof of Proposition 1 in Gersbach and Haller (1999) and introduce exit options. By (iv), we can choose a potential household $h \in \mathcal{H}$ with $1 < |h| < n$ and large group advantage. Let us choose such a household h and corresponding $\delta_i(p), (i, p) \in h \times \Delta^0$, and $X_h(p), p \in \Delta$, with the properties stipulated by LGA. Consider the household structure

$$\overline{P} = \Big\{h\Big\} \bigcup \Big\{\{i\} : i \notin h\Big\}.$$

We claim that there exists a competitive equilibrium with free exit $(p, \mathbf{x}; \overline{P})$. In the following we take the desired household structure $\overline{P}$ as given. It remains to show the existence of a pair $(p, \mathbf{x})$ so that $(p, \mathbf{x}; \overline{P})$ constitutes a competitive equilibrium with free exit. A first crucial step in the argument is to show that with suitably chosen reduced budget sets the resulting market excess demand relation is non-empty-valued, convex-valued, u.h.c., and satisfies the strong form of Walras' law. In a second step, we obtain a market clearing price system $p \in \Delta^o$ and a respective feasible allocation $\mathbf{x}$ for the hypothetical economy with reduced budget sets. In a final step, we are going to show that, indeed, $(p, \mathbf{x}; \overline{P})$ is a competitive equilibrium with free exit.

Step 1. We consider household h maximizing, for each $p \in \Delta$, its aggregate welfare W_h defined as

$$W_h(\mathbf{x_h}) = \sum_{i \in h} U_i(\mathbf{x_h}; h)$$

on its restricted budget set $X_h(p)$. Because of LGA, $X_h(p)$ is convex, compact and non-empty. W_h is continuous and concave. Hence the set of aggregate welfare maximizers is non-empty, convex and compact. Consequently $D_h(p)$, the household's aggregate demand set, is non-empty, convex and compact. Moreover, the constraint correspondence $X_h(\cdot)$ is continuous. Therefore, by the Maximum Theorem (Ellickson (1993; Th. 5.47)), the demand correspondence $D_h(\cdot)$ is u.h.c.

For each one-person household $\{i\}, i \notin h$, let the household maximize, for each $p \in \Delta$, its utility $U_i(x_i; \{i\})$ on the truncated budget set $B_{\{i\}}(p) \cap K$ which is non-empty, convex and compact. Hence $D_{\{i\}}(p)$, the set of utility maximizers is non-empty, convex and compact. Since $\omega_i \gg 0$, the constraint correspondence $B_{\{i\}}(\cdot) \cap K$ is continuous. Again by the Maximum Theorem (Ellickson (1993; Th. 5.47)), the demand correspondence $D_{\{i\}}(\cdot)$ is u.h.c.

For household h, the presence of a consumer $j \in h$ whose preferences are strictly monotonic in his own consumption and who does not impose any negative externalities on other household members, implies budget exhaustion. For all consumers $i \notin h$, strict monotonicity of preferences implies budget exhaustion by household $\{i\}$.

Aggregation across households in $\overline{P}$ yields that $\Phi(\cdot)$, the market excess demand relation resulting from reduced budget sets is non-empty-valued, convex-valued, u.h.c., and satisfies the strong form of Walras' law.

Step 2. By Theorem 6.37 of Ellickson (1993), there exists a pair $(p, \mathbf{z}) \in \Delta \times \mathbb{R}^\ell$ with

(a) $\mathbf{z} \in \Phi(p)$ and

(b) $\mathbf{z} \leq 0$ and $\mathbf{z} = 0$ whenever $p \gg 0$.

Condition (a) means that

$$\mathbf{z} = \sum_{g \in \bar{P}} \mathbf{d_g} - \omega_S$$

where $\mathbf{d_g} \in D_g(p)$ for each $g \in \overline{P}$. A standard argument shows that for each $i \notin h$, if $\mathbf{d_{\{i\}}}$ maximizes i's utility on the truncated budget set $B_{\{i\}}(p) \cap K$, then it is also a utility maximizer on the non-truncated budget set $B_{\{i\}}(p)$. But then strict monotonicity of i's preferences requires $p \gg 0$. By assumption, $\overline{P}$ admits at least one single-person household. Therefore, by condition (b), $\mathbf{z} = 0$. Let us write x_i for $\mathbf{d_{\{i\}}}$ from now on.

Step 3. It remains to deal with household h. By definition, we have $\mathbf{d_h} = \sum_{i \in h} x_i$ where $\mathbf{x_h} = (x_i)_{i \in h}$ maximizes W_h on $X_h(p)$. We have to show that $\mathbf{x_h}$ is an efficient collective choice of household h under its budget constraint, i.e. $\mathbf{x_h} \in EB_h(p)$, and that nobody wants to leave the household at the going prices.

Maximizing W_h on $X_h(p)$ is the same as maximizing W_h on $B_h(p) \cap K^h$, subject to the additional constraint (3) for all $i \in h$. We claim that if $\mathbf{x_h}$ maximizes W_h on $B_h(p) \cap K^h$ subject to the constraints (3), then $\mathbf{x_h}$ is an efficient collective choice of household h with respect to the truncated budget set $B_h(p) \cap K^h$, without further qualifications. Namely, for some consumer in h to do better at $\mathbf{y_h} \in B_h(p) \cap K^h$ than at $\mathbf{x_h}$ without making anybody else in h worse off, would increase the value of W_h and, hence, would require $\mathbf{y_h} \notin X_h(p)$. But then by LGA, there is some other consumer i in h who violates (3) at $\mathbf{y_h}$ and, therefore, is worse off at $\mathbf{y_h}$ than at $\mathbf{x_h}$, a contradiction.

After having shown that $\mathbf{x_h}$ is an efficient collective choice of household h with respect to the truncated budget set $B_h(p) \cap K^h$, we claim next $\mathbf{x_h} \in EB_h(p)$. This follows from a routine argument as in the case of single-person households. We finally claim that no household member has an incentive to leave at the going prices. But this follows immediately from the fact that $\mathbf{x_h} \in X_h(p)$. For the latter fact implies that each household member i satisfies (3) and thus $U_i(\mathbf{x_h}; h) \geq U_i(x_i^0(p); \{i\})$ for all $i \in h$. Hence, no individual wants to exit household h and $(p, \mathbf{x}; \overline{P})$ is a competitive equilibrium with free exit as asserted. Q.E.D.

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