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Abstract

This paper examines the chance of winning a Tullock-contest when participants differ in both their talent and their attitude towards risk. For the case of CARA preferences, it is shown that the winning probability may be higher for a low-skilled agent with a low degree of risk aversion than for a high-skilled agent with a high degree of risk-aversion. Such an outcome often is undesirable. It will occur if and only if the agents' ratio of degrees of risk aversion is larger than their ratio of skill levels and the rent of the contest is sufficiently high.

JEL-Code: C72, D72.

Keywords: selection contest, asymmetric players, risk aversion.

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1 Introduction

May the best man win! Following this dictum, in many areas of life contests are used with the objective of selecting the most able. In business, entrepreneurs conduct job interviews or assessment centers in order to hire or promote the agent with the highest skills. In politics, elections are run where the candidates try to convince the voters of their abilities for holding office. In sports, athletes compete against each other in order to single out the strongest one.

But is it really the most gifted one who has the best chance of winning such a selection contest? Is the winner of the famous Tour de France really the most talented cyclist or simply the guy who fears least the consequences of doping? Is the president-elect indeed best suited for holding office or just the one who had the guts to invest more money into the campaign? Seemingly, an agent's success during a contest does not depend on her skills only but also on her attitude towards risk.

Despite this observation, the economic literature on asymmetric contests has mainly focussed on models where agents differ in just one dimension. In particular, the influence of skill heterogeneity and asymmetric risk aversion on the probability of winning a contest have not yet been analyzed simultaneously but only apart. On the one hand, Baik (1994) shows that, with respect to skills, the more talented agent has, ceteris paribus, the higher probability of winning the contest. On the other hand, authors like Skaperdas and Gan (1995) or Cornes and Hartley (2003) demonstrate for the case of constant absolute risk aversion that the less risk averse agent invests, ceteris paribus, more and has a higher probability of winning.

Given these results, now consider a contest between agent A, who is high skilled but highly risk averse, and agent B, who is low skilled but barely risk averse. Two natural questions arise: i) Who has the higher probability of winning the contest and who spends more effort? ii) How does the design of the contest, e.g. the amount of prize money, impact the winning probabilities and effort levels?

To study this topic more closely, I engage a simple model of a two-person Tullock-contest where agents differ in both their skill levels and their degree of constant absolute risk aversion.[1] Maximizing their expected payoff, the agents simultaneously choose the effort levels they invest in order to win the contest. It is shown that the agent's probability of winning is increasing (decreasing) in the own (opponent's) skill level but, starting from the sym-

[1] Note the difference to the models of Hvide (2002) and Hvide and Kristiansen (2003), where both players have the same attitude towards risk ex-ante but where risk taking is a strategic variable that is endogenously determined in the equilibrium of the contest.

metric benchmark, decreasing (increasing) in the own (opponent's) degree of risk aversion. One concludes that both the chance of winning and the effort level may be higher for a low-skilled agent with a low degree of risk aversion than for a high-skilled agent with a high degree of risk-aversion. These situations occur if and only if the agents' difference in risk aversion – measured by the ratio of degrees of risk aversion – is bigger than their difference in talent – measured by the ratio of skill levels – and the rent of the contest, i.e. the prize money, is sufficiently high.

In many situations, especially in selection contests, an outcome where the venturesome beats the gifted is undesirable.[2] The designer of the contest should therefore take measures in order to reduce the riskiness of the contest if she does not want to discriminate against risk averse agents. However, this is not a trivial task. As the analysis demonstrates, for example, if the agents' difference in risk aversion is larger than their difference in talent, the agents' winning probabilities will not be monotonic with respect to the contest rent. Put differently, there is some optimal prize money maximizing the chance of the gifted.

The remainder of this paper is organized as follows: The formal model is set up in Section 2 and used to derive the main results in Section 3. Finally, applications and possible extensions are discussed in Section 4.

2 General Framework of the Contest

In this section, I first introduce the basic assumptions used in the analysis of the contest game. I then consider the individual maximization problems and derive a general expression characterizing the winning probabilities in the equilibrium of this game. Using this expression, I finally show that the agent's probability of winning is increasing (decreasing) in the own (opponent's) skill level.

2.1 Basic assumptions

There are N agents participating in a winner-take-all contest competing for some rent $R > 0$. Each agent $i \in \{1, \ldots, N\}$ has an initial wealth endowment I_i and can spend some resources $x_i \in [0, I_i]$ in order to improve her probability of winning p_i. This probability is determined by the following contest success

[2] In the context of sales contests, Bono (2008) characterizes a situation where it might be desirable to promote less risk averse managers, since they exert, ceteris paribus, more effort.

function (CSF):

$$p_i := \frac{f_i(x_i)}{\sum_{j=1}^{N} f_j(x_j)}, \tag{1}$$

where $f_i : [0, I_i] \rightarrow \mathbb{R}_0^+$ is an increasing concave function of x_i satisfying $f_i(0) = 0$. For the sake of concreteness and ease of calculation, assume $N = 2$ and

$$f_i(x_i) := \theta_i x_i, \tag{2}$$

where $\theta_i > 0$ is a parameter expressing agent i's skill level. θ_i is also referred to as agent i's talent for the task required within the contest. Note that equation (2) states, reasonably enough, a complementarity between talent and effort, which is standard in the related literature (e.g. Baik, 1994).

Introducing risk aversion into the analysis of contests I follow the approach proposed by Skaperdas and Gan (1995) and Cornes and Hartley (2003), respectively, and assume that the preferences of agent i can be expressed by the following utility function which exhibits constant absolute risk aversion (CARA):

$$u_i(W_i) := -e^{-\alpha_i W_i}, \tag{3}$$

where α_i is agent i's constant degree of absolute risk aversion. I include the limit case of a risk-neutral player i with $u_i(W_i) = W_i$ into the analysis, and refer to this situation as one in which $\alpha_i = 0$.

2.2 From individual objectives to equilibrium

The contest is organized as a simultaneous move game. The players choose their effort levels x_i in order to maximize their expected utility Eu_i from consumption W_i, which equals $I_i - x_i + R$ if agent i wins the contest and $I_i - x_i$ otherwise. Hence, for $i, j \in \{1, 2\}, i \neq j$,

$$\begin{aligned} Eu_i &= p_i u_i(I_i - x_i + R) + (1 - p_i) u_i(I_i - x_i) \\ &= -\left[\frac{\theta_i x_i}{\theta_1 x_1 + \theta_2 x_2} e^{-\alpha_1(I_i - x_i + R)} + \frac{\theta_j x_j}{\theta_1 x_1 + \theta_2 x_2} e^{-\alpha_1(I_i - x_i)}\right]. \end{aligned}$$

For ease of notation, define

$$p_i' := \frac{\partial p_i}{\partial x_i} = \theta_i \frac{\theta_j x_j}{(\theta_1 x_1 + \theta_2 x_2)^2} \geq 0, \tag{4}$$

$$p_i'' := \frac{\partial^2 p_i}{\partial x_i^2} = -\frac{2\theta_i^2}{\theta_1 x_1 + \theta_2 x_2} \frac{\theta_j x_j}{(\theta_1 x_1 + \theta_2 x_2)^2} \leq 0, \tag{5}$$

$$\beta(\alpha) := \frac{\alpha}{1 - e^{-\alpha R}} > 0, \tag{6}$$

$$\delta(\alpha) := \frac{\alpha e^{-\alpha R}}{1 - e^{-\alpha R}} = e^{-\alpha R} \beta(\alpha) > 0. \tag{7}$$

Using the identity $e^X = \sum_{k=0}^{\infty} \frac{X^k}{k!}$ for any real X, it is easily verified that β is an increasing function of α and δ is a decreasing function of α (Skaperdas and Gan, 1995, supplementary appendix to Proposition 2). The first order condition (FOC) for an interior solution of agent i's maximization problem yields

$$p_i' = \beta(\alpha_i)(p_i e^{-\alpha_i R} + 1 - p_i). \tag{8}$$

Equation (8) implicitly defines the reaction function of agent i, i.e. her optimal effort x_i as a function of the opponent's effort x_j. Under the assumptions made, the rent seeking game has a unique Nash equilibrium in pure strategies, as shown by Cornes and Hartley (2003, Propostioin 3.3) and Yamazaki (2008), respectively. In order to compute this equilibrium under different parameter constellations, divide the FOC of agent 1 by the FOC of agent 2, and note that $\frac{p_1'}{p_2'} = \frac{x_2}{x_1}$. Denoting by $q := \frac{p_2}{p_1} = \frac{\theta_2 x_2}{\theta_1 x_1}$ the competitive balance of the contest,[3] this yields

$$\frac{\theta_1}{\theta_2} q = \frac{\beta(\alpha_1)}{\beta(\alpha_2)} \cdot \frac{e^{-\alpha_1 R} + q}{q e^{-\alpha_2 R} + 1}. \tag{9}$$

Equation (9) can be transformed into a quadratic equation for the competitive balance in equilibrium; as $\frac{\theta_2 \delta(\alpha_1)}{\theta_1 \delta(\alpha_2)} > 0$, only the positive root yields a feasible solution $q > 0$, i.e.

$$q = \sqrt{\frac{\theta_2 \delta(\alpha_1)}{\theta_1 \delta(\alpha_2)} + \left(\frac{\theta_1 \beta(\alpha_2) - \theta_2 \beta(\alpha_1)}{2\theta_1 \delta(\alpha_2)} \right)^2} - \frac{\theta_1 \beta(\alpha_2) - \theta_2 \beta(\alpha_1)}{2\theta_1 \delta(\alpha_2)}. \tag{10}$$

A value $q < 1$ indicates that agent 1's probability of winning exceeds the one of agent 2, i.e. $p_1 > p_2$, and vice versa for $q > 1$. Note that q depends, ceteris paribus, only on the ratio of skill levels but not on their exakt values.[4]

2.3 Comparative statics w.r.t. skill levels

As intuition suggests, being talented always redounds to the contestant's advantage. A formal proof of Proposition 1 can be found in Appendix A.

Proposition 1 *The agent's probability of winning is increasing (decreasing) in the own (opponent's) skill level, i.e.* $\frac{\partial q}{\partial \theta_1} < 0 < \frac{\partial q}{\partial \theta_2}$.

[3] Some authors use the difference in winning probabilities as an alternative measure of 'competitive balance' or 'closeness' of the contest (see e.g. Runkel, 2006a,b).

[4] For $\theta := \frac{\theta_2}{\theta_1}$, one has $q = \sqrt{\frac{\theta\delta(\alpha_1)}{\delta(\alpha_2)} + \left(\frac{\beta(\alpha_2) - \theta\beta(\alpha_1)}{2\delta(\alpha_2)}\right)^2} - \frac{\beta(\alpha_2) - \theta\beta(\alpha_1)}{2\delta(\alpha_2)}$. Hence, without loss of generality, θ_1 might be normalized to 1.

3 Specific Scenarios of Heterogeneity

In this section, I compare the outcome of the contest for different scenarios of heterogeneity among the contestants and derive a series of further comparative static results. I start with the case of symmetric, i.e. identical, players as a benchmark. I then analyze successively how the outcome changes if one introduces heterogeneity between the agents with respect to their skills only, their risk attitude only, and both skill and risk aversion.

3.1 The symmetric benchmark

In this subsection, I derive the equilibrium of the contest as well as its comparative statics properties for the case of identical players.

Proposition 2 *Suppose $\theta_1 = \theta_2 = \theta > 0$ and $\alpha_1 = \alpha_2 = \alpha \geq 0$.*

(a) The equilibrium is symmetric with equal winning probabilities $p_1^* = p_2^* = \frac{1}{2}$ (i.e. $q = 1$) and effort levels, which do not depend on the skill level θ:

$$x_1^* = x_2^* = x_{sym}^* := \begin{cases} \frac{R}{4} & \text{if} \quad \alpha = 0 \\ \frac{e^{\alpha R}-1}{2\alpha(e^{\alpha R}+1)} & \text{if} \quad \alpha > 0 \end{cases}$$

(b) The effort levels are increasing in the prize: $\frac{\partial x_{sym}^*}{\partial R} > 0$.

(c) The effort levels are decreasing in the degree of risk aversion: $\frac{\partial x_{sym}^*}{\partial \alpha} < 0$.

The proof can be found in Appendix A. Parts (a) and (b) of Proposition 2 are straightforward generalizations of the respective results in the case of risk-neutral players (Baik, 1994) and, as such, very intuitive: The intensity of competition among equal competitors does not depend on the (skill) level the contest takes place at, but is positively related to the rent offered.

Part (c) of Proposition 2 resolves the general ambiguity result of Konrad and Schlesinger (1997) for the case of preferences with CARA. Note that a higher degree of risk aversion has two opposing effects on the individual investment decision. On the one hand, there is the so called *gambling effect*: Since participation in the contest comes along with an uncertain payment, it may be regarded as a lottery, which the agents invest the less into the more risk averse they are. On the other hand, there is a so called effect of *self-protection*: By spending more effort, the players can reduce their probability of loosing the contest. Therefore, the more risk averse they are the more they invest. Under our assumptions, the gambling effect outweighs the effect of self protection.

Traditionally, the literature conducts the comparative statics with respect to the *dissipation rate*, which is defined as the fraction ρ of the rent that is 'wasted' in form of aggregate effort, i.e. $\rho := \frac{\sum_{j=1}^{N} x_j}{R}$.

Corollary 1 *For $\theta_1 = \theta_2 = \theta > 0$ and $\alpha_1 = \alpha_2 = \alpha > 0$, the equilibrium rent dissipation rate equals*

$$\rho_{sym} = \frac{e^{\alpha R} - 1}{\alpha R(e^{\alpha R} + 1)} < 1$$

and is decreasing in both the prize R and the degree of risk aversion α.

The proof can be found in Appendix A. As shown by Hillman and Samet (1987) in a more general framework, less than the full rent will be dissipated if agents are risk averse. The comparative statics of Corollary 1 are in line with the simulations run by Hillman and Katz (1984) for the case of logarithmic utilities and with the results in Long and Vousden (1987) for contests with rents that are divisible among agents.[5] Put differently, for the case of preferences with CARA, the intuition holds that less of the rent will be wasted if the agents are more risk averse or if the stakes are higher.

3.2 Asymmetric skills

In this subsection, I derive the equilibrium of the contest as well as its comparative statics properties for the case of players with the same degree of risk aversion but different skill levels.

Proposition 3 *Suppose, without loss of generality, $\theta_1 > \theta_2 > 0$ and $\alpha_1 = \alpha_2 = \alpha \geq 0$.*

(a) In equilibrium,

$$1 > q = \begin{cases} \frac{\theta_2}{\theta_1} & \text{if} \quad \alpha = 0 \\ \sqrt{\frac{\theta_2}{\theta_1} + \left(\frac{e^{\alpha R}}{2}\frac{\theta_1 - \theta_2}{\theta_1}\right)^2} - \frac{e^{\alpha R}}{2}\frac{\theta_1 - \theta_2}{\theta_1} & \text{if} \quad \alpha > 0, \end{cases}$$

i.e. the player with the higher skill level has the better probability of winning, $p_1^* > p_2^*$.

[5] They contrast, though, to the diametric result in Fabella (1992). However, Konrad and Schlesinger (1997, footnote 11) report that his "result is not correct as the paper contains several serious errors".

(b) (i) *Under risk neutrality ($\alpha = 0$), competitive balance is independent of the prize, i.e.* $\frac{\partial q}{\partial R} = 0$.

(ii) *In the case of risk aversion ($\alpha > 0$), the higher the prize, the better the winning probability of the more talented agent, i.e.* $\frac{\partial q}{\partial R} < 0$.

(c) *The higher the degree of risk aversion, the better the winning probability of the more talented agent, i.e.* $\frac{\partial q}{\partial \alpha} < 0$.

The proof can be found in Appendix A. Part (a) of Proposition 3 is a straightforward generalization of the respective result in the case of risk-neutral players (Baik, 1994) and, as such, very intuitive: The more talented agent has the better chance of winning and, according to Proposition 1, this probability is the higher the larger the gap in skills is. Part (b) of Proposition 3 shows that the neutrality result for risk neutral agents (Runkel, 2006a, Proposition 1 (b)) does not hold for risk averse players: A higher prize increases the 'riskiness' of the contest which is worse for the less skilled agent being more likely to loose. A similar intuition also drives the result in Part (c) of Proposition 3.

Can there also be said something about the equilibrium effort levels? If the agents are risk neutral, they will exert the same effort level $x_1^* = x_2^* = \frac{R\theta_1\theta_2}{(\theta_1+\theta_2)^2}$ in equilibrium, which will be maximal for equal skills $\theta_1 = \theta_2$ (Baik, 1994). Under risk aversion, however, the equilibrium effort levels will differ if and only if agents differ in skills. To see this, consider the relative effort $\xi := \frac{x_2^*}{x_1^*} = \frac{\theta_1}{\theta_2} q$ in equilibrium; then the following statements hold.

Corollary 2 *Suppose, without loss of generality, $\theta_1 > \theta_2 > 0$ and $\alpha_1 = \alpha_2 = \alpha > 0$.*

(a) *The higher the skill level of agent i the higher her relative equilibrium effort, i.e.* $\frac{\partial \xi}{\partial \theta_1} < 0 < \frac{\partial \xi}{\partial \theta_2}$.

(b) *In equilibrium, the agent with the higher skill level exerts more effort, i.e.* $x_1^* > x_2^*$.

The proof can be found in Appendix A. The results confirm the complementary character of effort and skill and back up the much cited anecdotal evidence for talent to come along with diligence.

3.3 Asymmetric risk aversion

In this subsection, I derive the equilibrium of the contest as well as its comparative statics properties for the case of players with the same skill level but different degrees of risk aversion.

Proposition 4 *Suppose, without loss of generality, $\theta_1 = \theta_2 = \theta > 0$ and $\alpha_1 > \alpha_2 \geq 0$.*

(a) In equilibrium,

$$q = \sqrt{\frac{\delta(\alpha_1)}{\delta(\alpha_2)} + \left(\frac{1}{2}\frac{\beta(\alpha_2) - \beta(\alpha_1)}{\delta(\alpha_2)}\right)^2} - \frac{1}{2}\frac{\beta(\alpha_2) - \beta(\alpha_1)}{\delta(\alpha_2)}.$$

(b) The player with the higher degree of risk aversion exerts less effort and has the smaller probability of winning, i.e. $x_1^* < x_2^*$ and $p_1^* < p_2^*$.

(c) The higher the prize, the smaller the winning probability of the more risk averse agent, i.e. $\frac{\partial q}{\partial R} > 0$.

The proof can be found in Appendix A. Part (b) of Proposition 4 reproduces the respective results in Skaperdas and Gan (1995, Proposition 2b), Cornes and Hartley (2003, Proposition 3.4), and Bono (2008, Proposition 1) for the specific framework at hand. The intuition here is similar to the corresponding result in the symmetric equilibrium. Since, under the assumptions made, the gambling effect outweighs the effect of self-protection, the less risk averse agent will spend more effort and thus have a higher chance of winning the contest. An analogous reasoning explains the result of Part (c). An increasing prize raises the 'riskiness' of the contest which is worse for the more risk averse agent. Figure 1 illustrates these results displaying the graph of $q(R)$ for $\theta_1 = 1$, $\theta_2 = 1$, $\alpha_1 = 1$, and $\alpha_2 = \frac{1}{4}$.

3.4 Asymmetric skills and risk aversion

In this subsection, I allow for both asymmetric skills and risk aversion. Focussing on the non-trivial cases, I assume that agent 1 has a higher skill level and, at the same time, a higher degree of risk aversion than agent 2. Therefore, agent 1 is called the *gifted* and agent 2 is called the *venturesome*. I compare the agents' winning probabilities as well as effort levels in equilibrium and conduct a comparative statics analysis with respect to the rent R.

Resuming the previous results, part (a) of Proposition 3 shows, on the one hand, that for some equal degree of risk aversion, agent 1 with the higher skill level has the higher chance of winning. On the other hand, Part (b) of Proposition 4 states that, in the case of equal skills, agent 1 will have a smaller winning probability if he has a higher degree of risk aversion. Therefore, whether the gifted or the venturesome has the better chance of winning will

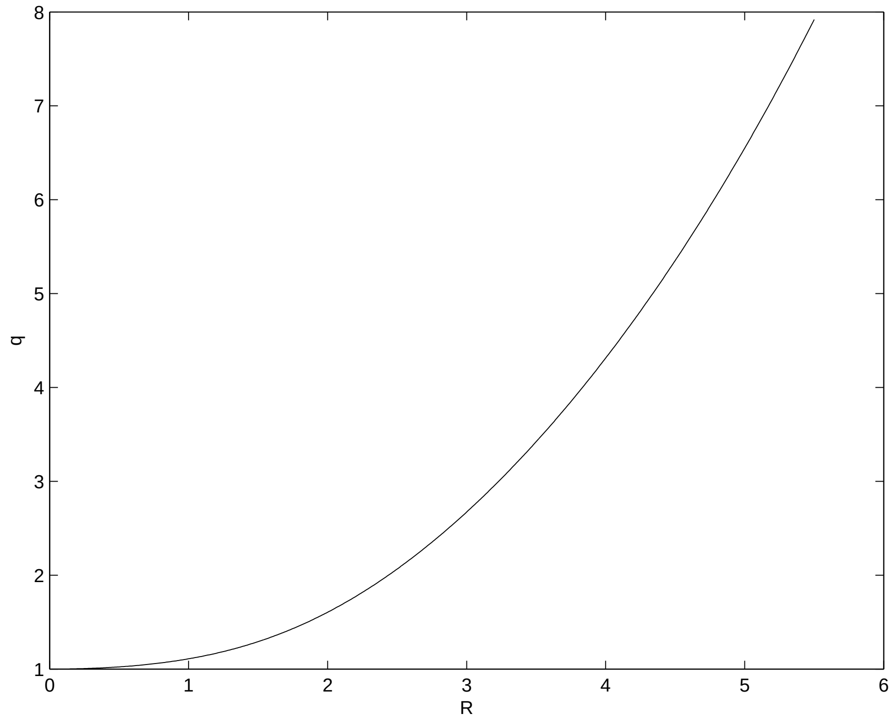

Figure 1: Asymmetric risk aversion

generally depend on the relative differences in skill and risk aversion. In particular, by the continuity of the relevant functions, the implicit functions theorem implies the following

Corollary 3 *There exists a set of parameter values with* $\theta_1 > \theta_2$ *and* $\alpha_1 > \alpha_2$ *such that* $q > 1$, *i.e.* $p_1^* < p_2^*$.

Corollary 3 characterizes a situation where the presence of heterogeneity with respect to the agents' attitude towards risk induces some kind of failure of the contest: The probability of selecting the agent with the lower skills is higher than the probability of selecting the agent with the higher skills. Put differently, the venturesome has a superior chance of beating the gifted.

Moreover, note that a higher rent on the one hand increases the chance of winning for the agent with the higher skill level (Proposition 3(c)), but on the other hand decreases the chance of winning for the agent with the higher degree of risk aversion (Proposition 4(c)). Consequently, from the viewpoint of contest design, it is a priori not clear how the prize of the contest should be chosen in order to minimize the winning probability of the venturesome for not ending up in the undesirable situation characterized by Corollary 3.

These remarks raise two questions: First, under what circumstances is the venturesome more likely to win than the gifted? Second, how does the prize money impact the chance for such an undesirable outcome? As the main result of this paper, these questions are answered by the following

Proposition 5 *Suppose $\theta_1 > \theta_2 > 0$ and $\alpha_1 > \alpha_2 > 0$.*

(a) If $\frac{\theta_1}{\theta_2} \geq \frac{\alpha_1}{\alpha_2}$, then

(i) $q < 1$ for all $R > 0$,

(ii) $\frac{\partial q}{\partial R} < 0$.

(b) If $\frac{\theta_1}{\theta_2} < \frac{\alpha_1}{\alpha_2}$, then there exist cut-off values $R_0, \tilde{R}_0, R_1, \tilde{R}_1$ with $R_0 \leq R_1$, $R_0 \leq \tilde{R}_0$, and $R_1 \leq \tilde{R}_1$ such that

(i) q is decreasing for all $R < R_0$,

(ii) $q < 1$ for all $R < R_1$,

(iii) $q \geq 1$ for all $R \geq \tilde{R}_1$,

(iv) q is increasing for all $R \geq \tilde{R}_0$.

The proof can be found in Appendix A. Part (a) of Proposition 5 shows that the winning probability for the gifted will always be higher than for the venturesome if the ratio of the agents' skill levels is at least as high as the ratio of their degrees of risk aversion. In this case, differences in skills predominate differences in risk behavior and, hence, the results resemble the case in which agents differ only in talent. Particularly, a higher prize increases the chance of winning for the agent with the higher skill level (as in Proposition 3(c)). Figure 2 illustrates these results displaying the graph of $q(R)$ for $\theta_1 = 1$, $\theta_2 = \frac{1}{2}$, $\alpha_1 = 1$, and $\alpha_2 = \frac{1}{2}$.

As part (b) of Proposition 5 shows, things will be slightly more complicated if the ratio of the agents' skill levels is smaller than the ratio of their degrees of risk aversion. In this case, again the winning probability for the gifted will be higher than for the venturesome (and increasing) if the rent is sufficiently small. However, if the rent exceeds a certain threshold, the opposite will be true: the winning probability for the venturesome will be higher than for the gifted (and increasing). Moreover, all simulations run confirm that $R_0 = \tilde{R}_0$ and $R_1 = \tilde{R}_1$. In fact, I could not falsify tightening part (b) of Proposition 5 as follows.

Conjecture 1 *If $\frac{\theta_1}{\theta_2} < \frac{\alpha_1}{\alpha_2}$, then there exist cut-off values $0 < R_0 < R_1$ such that*

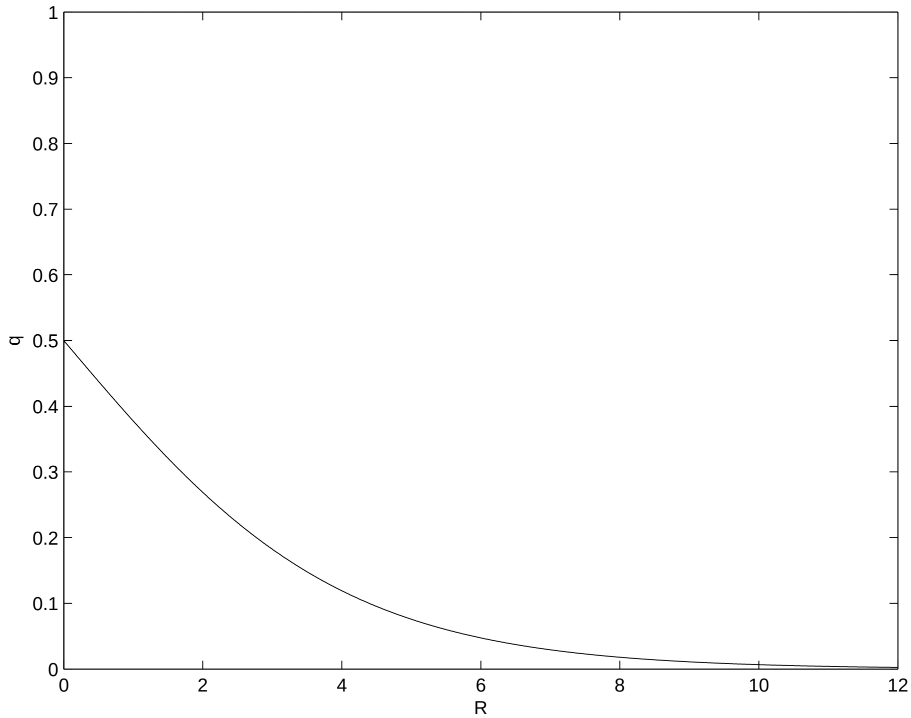

Figure 2: Predominance of differences in skills

(a) $q \geq 1 \Leftrightarrow R \geq R_1$ *(with equality for* $R = R_1$*),*

(b) $\frac{\partial q}{\partial R} \geq 0 \Leftrightarrow R \geq R_0$ *(with equality for* $R = R_0$*).*

To get an intuition for this result, note that rising stakes increase the riskiness of the contest as perceived by risk averse agents. Hence, for low rents, risk considerations do not play much of a role and skill differences are the predominant factor. However, as the rents increase, differences in risk aversion become more and more important. Consequently, from a certain threshold R_0 on, the winning probability of the venturesome starts to increase as the stakes become higher, and it even exceeds the winning probability of the gifted as the rent rises above R_1. Figure 3 illustrates these results displaying the graph of $q(R)$ for $\theta_1 = 1$, $\theta_2 = \frac{1}{2}$, $\alpha_1 = 1$, and $\alpha_2 = \frac{1}{4}$.

Similar results hold for the corresponding effort levels. In order to specify the respective properties, remember the definition of relative equilibrium effort $\xi := \frac{x_2^*}{x_1^*} = \frac{\theta_1}{\theta_2} q$.

Corollary 4 *Suppose* $\theta_1 > \theta_2 > 0$ *and* $\alpha_1 > \alpha_2 > 0$*.*

(a) If $\frac{\theta_1}{\theta_2} \geq \frac{\alpha_1}{\alpha_2}$*, then*

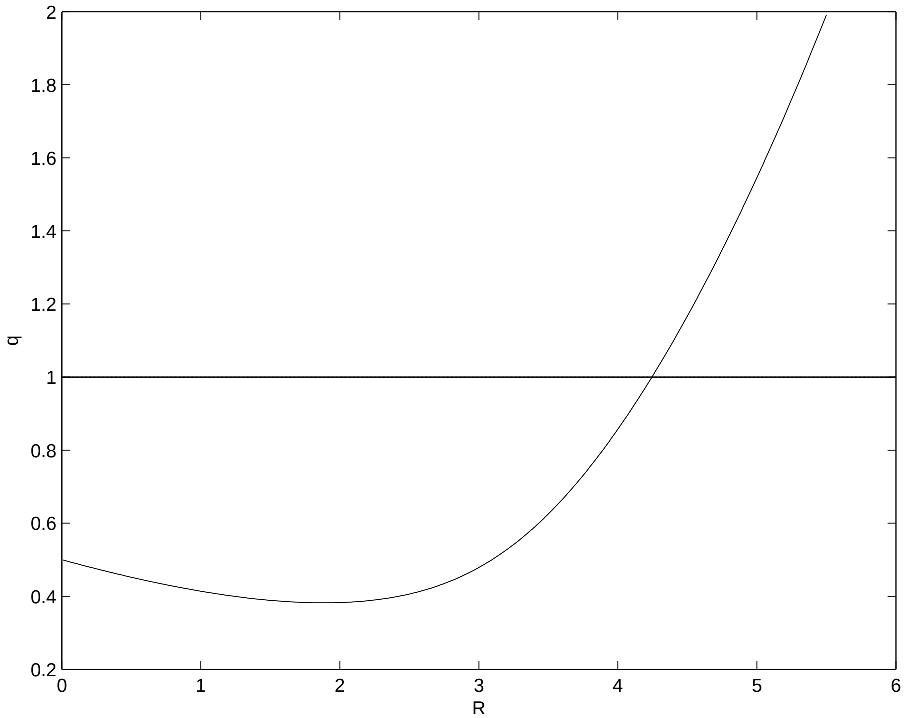

Figure 3: Predominance of differences in risk aversion

(i) $\frac{\partial \xi}{\partial R} < 0$,

(ii) $\xi < 1$ *for all* $R > 0$.

(b) If $\frac{\theta_1}{\theta_2} < \frac{\alpha_1}{\alpha_2}$ *and Conjecture 1 holds, then*

(i) $\frac{\partial \xi}{\partial R} \geq 0 \Leftrightarrow R \geq R_0$ *(with equality for* $R = R_0$*),*

(ii) there exists a cut-off value $\bar{R}$ *with* $R_0 < \bar{R} < R_1$ *such that* $\xi \geq 1 \Leftrightarrow R \geq \bar{R}$ *(with equality for* $R = \bar{R}$*).*

The proof can be found in Appendix A. In case that differences in skills predominate differences in risk behavior, again, the results resemble the case in which agents differ only in talent (c.f. Corollary 2): In equilibrium, the agent with the higher skill level exerts more effort. Moreover, since his winning probability increases as the rent rises, so does his relative equilibrium effort.

However, if the ratio of the agents' skill levels is smaller than the ratio of their degrees of risk aversion, the effort of the gifted will be higher than the effort of the venturesome if and only if the rent is sufficiently small. Since the winning probability of the venturesome is increasing in the rent if and only if $R > R_0$, the same must be true for his relative effort. Moreover, since the venturesome has a higher chance of winning than the gifted for all rents above R_1 despite his lower skills, his effort must exceed the one of the gifted from an even lower threshold $\bar{R}$ on.

In order to study how the thresholds R_0 and R_1 depend on the ratio of skills and the levels of risk aversion, I have run a large number of numerical simulations leading to the following

Conjecture 2 *Suppose* $\frac{\theta_1}{\theta_2} < \frac{\alpha_1}{\alpha_2}$ *and define* R_0 *and* R_1 *as in Conjecture 1. Then, ceteris paribus,* R_0 *and* R_1 *are locally*

(a) increasing functions of $\frac{\theta_1}{\theta_2}$,

(b) decreasing functions of α_1,

(c) increasing functions of α_2.

Though these statements have not yet been proven analytically, the intuition behind them is straightforward. As the skill ratio increases, the relative importance of differences in skills compared to differences in the levels of risk aversion increases. Consequently, the region in which the winning probability of the gifted exceeds that of the venturesome (is increasing) becomes larger. The same consequences arise as the gifted gets less risk averse or the venturesome gets more risk averse, since again the relative importance of differences in skills compared to differences in the levels of risk aversion increases.

4 Concluding Remarks

I have examined the chance of winning a Tullock-contest when participants differ in both their talent and their attitude towards risk. For the case of CARA preferences, it has been shown that the agent's probability of winning is increasing (decreasing) in the own (opponent's) skill level but, starting from the symmetric benchmark, decreasing (increasing) in the own (opponent's) degree of risk aversion. If the difference in skill levels is small while the difference in risk aversion is big, both the winning probability and the effort chosen are higher for a low-skilled agent with a low degree of risk aversion than for a high-skilled agent with a high degree of risk-aversion whenever the prize money is sufficiently high.

Put differently, there are situations in which the agent with the higher skill level might nevertheless be very likely to lose the contest if, at the same time, she exhibits a higher degree of risk aversion. Such an outcome often is undesirable, especially in the case of a selection contest. A selection contest aims at finding out who is most productive in fulfilling a certain task and, hence, wants to rank the agents according to their skills rather than attitude towards risk. Examples range from job interviews or promotion contests over political elections to tournaments in sports with its dictum: *May the best man win!*

Therefore, a selection contest should be designed in order to reduce the riskiness of the game. In the model at hand, the only parameter available for contest design is the contest prize.[6] It has been shown that if the agents' difference in risk aversion is bigger than their difference in talent, there is some finite prize money maximizing the chance of the gifted. Put differently, the model makes an argument for weakening the agents' incentives in the provision of effort in order to improve the performance of selection. To emphasize this point, in the following I briefly discuss the applications of the model to the fields of labor economics, political economy, and sport economics as well as the respective implications.

In a promotion contest, the rent R can be interpreted as the differential between the manager's wage levels before and after promotion. In situations where differences in the degrees of risk aversion predominate differences in skill levels, the analysis suggests that large wage differentials may lead to inefficient promotion decisions in the sense that the venturesome is promoted instead of the gifted. This gives rise to the following hypothesis: In industries with large wage differentials promoted managers are less risk averse than in industries with low wage differentials. At the same time, this reasoning makes an argument for weakening the managers' incentives by moderate wage differentials.[7]

In a political election, R may be interpreted as the rent from being elected and holding office. With a similar reasoning as above, high such rents might induce a high probability of electing the venturesome instead of the gifted. This could justify comparatively low salaries from political offices.

The basically positive character of our analysis allows to apply the model equally to alternative goals of the contest designer. In many sports contests, for example, the closeness of the game exerts a positive externality on some related markets. Hence, the organizer of the event may not aim at maximizing the chance of the gifted but try to design a competition that is as balanced as possible (see e.g. Runkel, 2006a). The model at hand answers the question how the prize money should be chosen in order to achieve this

[6] To extend the analysis in this direction, one could think of a framework in which the contest designer has more instruments at hand. Imagine, for example, a situation where the designer can influence the agents' effort costs (which are normalized to 1 in our model).

[7] Limiting of wages for top managers has been set highly on the political agenda in many countries after the financial crisis of 2008.

goal. Again, the optimal rent is finite.

Our analysis applies to a wide variety of real world situations. For instance, many scandals in business, politics, and sports lead to the impression that the agent's success during a selection contest is not always based on superior skills but the result of cheating. Sportsmen dope, politicians betray, managers bribe. In real life, besides plain effort, cheating is an illegal but often applied possibility for the agent to enhance the probability of winning. It is rather intuitive that the availability of a cheating technology has a strong impact on the riskiness of the contest. Hence, the model can be engaged in order to investigate how the availability of such a technology influences the relative winning probabilities of the gifted and the venturesome.

A Appendix

Proof of Proposition 1.

Since q depends, ceteris paribus, only on the ratio of skill levels $\theta := \frac{\theta_2}{\theta_1}$, it is sufficient to show that $\frac{\partial q}{\partial \theta} > 0$. Differentiating q with respect to θ and rearranging terms yields

$$\frac{\partial q}{\partial \theta} = \frac{2\delta_1\delta_2 + \theta\beta_1^2 - \beta_1\beta_2 + 2\delta_2\beta_1\sqrt{\cdot}}{(2\delta_2)^2\sqrt{\cdot}},$$

where I use the shortcuts $\beta_i := \beta(\alpha_i)$ and $\delta_i := \delta(\alpha_i)$ for $i \in \{1, 2\}$ as well as $\sqrt{\cdot} := \sqrt{\frac{\theta\delta_1}{\delta_2} + \left(\frac{\beta_2 - \theta\beta_1}{2\delta_2}\right)^2}$. Hence, $\frac{\partial q}{\partial \theta} > 0$ if and only if

$$2\delta_2\beta_1\sqrt{\cdot} > \beta_1\beta_2 - \theta\beta_1^2 - 2\delta_1\delta_2. \tag{11}$$

If the right hand side of inequality (11) is negative, the statement is obviously true; if it is non-negative, inequality (11) is equivalent to

$$\begin{aligned} 4\delta_2^2\beta_1^2\left(\frac{\theta\delta_1}{\delta_2} + \left(\frac{\beta_2 - \theta\beta_1}{2\delta_2}\right)^2\right) &> (\beta_1\beta_2 - \theta\beta_1^2 - 2\delta_1\delta_2)^2 \\ \Leftrightarrow \quad \beta_1\beta_2 &> \delta_1\delta_2 \\ \Leftrightarrow \quad \beta_1\beta_2 &> e^{-\alpha_1 R}\beta_1 e^{-\alpha_2 R}\beta_2 \\ \Leftrightarrow \quad 1 &> e^{-(\alpha_1+\alpha_2)R}. \end{aligned}$$

The last inequality is true for all positive values of R as long as at least one of the players is risk averse. For the case of both players being risk-neutral, see Proposition 3.

$\square$

Proof of Proposition 2.

Since the case of risk neutral agents is discussed extensively in the literature (see e.g. Baik, 1994), I concentrate on risk averse agents.

(a) For $\theta_1 = \theta_2 = \theta > 0$ and $\alpha_1 = \alpha_2 = \alpha > 0$ equation (10) immediately implies $q = 1$. Hence, the equilibrium is symmetric with equal winning probabilities $p_1^* = p_2^* = \frac{1}{2}$ and effort levels $x_1^* = x_2^* = x_{\text{sym}}^*$, which may be easily computed from the FOC (8).

(b) $\frac{\partial x_{\text{sym}}^*}{\partial R} = \frac{e^{\alpha R}}{(e^{\alpha R}-1)^2} > 0$.

(c) Using the identity $e^X = \sum_{k=0}^{\infty} \frac{X^k}{k!}$ for any real X, one verifies that

$$\frac{\partial x_{\text{sym}}^*}{\partial \alpha} = \frac{1 + 2\alpha R e^{\alpha R} - e^{2\alpha R}}{2\alpha^2 (e^{\alpha R} + 1)^2} < 0.$$

□

Proof of Corollary 1.

The equilibrium rent dissipation rate can be computed immediately from the equilibrium effort levels in Proposition 2. Denoting $A := \alpha R$ one calculates

$$\frac{\partial \rho_{\text{sym}}}{\partial A} = \frac{1 + 2Ae^A - e^{2A}}{A^2 (e^A + 1)^2} < 0,$$

where the last inequality is verified, again, using the identity $e^X = \sum_{k=0}^{\infty} \frac{X^k}{k!}$ for any real X. From this inequality it is also apparent that $\rho_{\text{sym}} < 1$.

□

Proof of Proposition 3.

(a) The case of risk neutral agents is due to Baik (1994). Hence, I concentrate once more on risk averse agents. For $\theta_1 > \theta_2 > 0$ and $\alpha_1 = \alpha_2 = \alpha > 0$, the value of q can be immediately computed from equation (10). Moreover, $\theta_1 - \theta_2 > 0$ and, hence,

$$q = \sqrt{\frac{\theta_2}{\theta_1} + \left(\frac{e^{\alpha R}}{2} \frac{\theta_1 - \theta_2}{\theta_1}\right)^2} - \frac{e^{\alpha R}}{2} \frac{\theta_1 - \theta_2}{\theta_1} < \sqrt{\frac{\theta_2}{\theta_1}} < 1.$$

(b) (i) Trivial.

(c) and (b) (ii) Again denoting $A := \alpha R$, one has to show that $\frac{\partial q}{\partial A} < 0$. Using $\theta_1 - \theta_2 > 0$, one easily verifies that this is equivalent to

$$\frac{e^A}{2}\frac{\theta_1 - \theta_2}{\theta_1} < \sqrt{\frac{\theta_2}{\theta_1} + \left(\frac{e^A}{2}\frac{\theta_1 - \theta_2}{\theta_1}\right)^2},$$

which is obviously true.

□

Proof of Corollary 2.

Suppose $\theta_1 > \theta_2 > 0$ and $\alpha_1 = \alpha_2 = \alpha > 0$.

(a) One has to show that $\frac{\partial \xi}{\partial \theta_1} < 0 < \frac{\partial \xi}{\partial \theta_2}$. One easily verifies that either inequality is equivalent to

$$1 + \frac{e^{2\alpha R}}{2}\frac{\theta_1 - \theta_2}{\theta_2} < \sqrt{\frac{\theta_1}{\theta_2}e^{2\alpha R} + \left(\frac{e^{2\alpha R}}{2}\frac{\theta_1 - \theta_2}{\theta_2}\right)^2},$$

which is true as $1 < e^{2\alpha R}$.

(b) Noting that $\xi_{|\theta_1 = \theta_2} = 1$ and applying part (a) one concludes $\xi < 1$, i.e. $x_1^* > x_2^*$, for all $\theta_1 > \theta_2$.

□

Proof of Proposition 4.

Suppose $\theta_1 = \theta_2 = \theta > 0$ and $\alpha_1 \geq \alpha_2 > 0$.

(a) The value of q can be immediately computed from equation (10).

(b) An elegant and rigorous proof of this result can be found in Cornes and Hartley (2003). Here, I first give a direct proof for sufficiently small differences in risk aversion along the lines of Skaperdas and Gan (1995, Proposition 2b). Put differently, it is shown that $\alpha_1 = \alpha_2 + \varepsilon$, with $\varepsilon > 0$ sufficiently small, implies $x_1^* < x_2^*$ and $p_1^* < p_2^*$. Later on, however, the general statement will be implied by Proposition 5 (c.f. Appendix A, Corollary 5).

Evaluating

$$\begin{aligned}\frac{\partial q}{\partial \alpha_1} &= \frac{1}{2}\left(\sqrt{\frac{\delta(\alpha_1)}{\delta(\alpha_2)}+\left(\frac{1}{2}\frac{\beta(\alpha_2)-\beta(\alpha_1)}{\delta(\alpha_2)}\right)^2}\right)^{-1} \\ &\quad \cdot\left[\frac{\delta'(\alpha_1)}{\delta(\alpha_2)}-\frac{1}{2}\frac{\beta'(\alpha_1)}{\delta(\alpha_2)}\frac{\beta(\alpha_2)-\beta(\alpha_1)}{\delta(\alpha_2)}\right]+\frac{1}{2}\frac{\beta'(\alpha_1)}{\delta(\alpha_2)}\end{aligned}$$

at $\alpha_1=\alpha_2=\alpha$ yields

$$\frac{\partial q}{\partial \alpha_1}_{|\alpha_1=\alpha_2=\alpha} = \frac{1}{2}\frac{\delta'(\alpha)+\beta'(\alpha)}{\delta(\alpha)} > 0 \tag{12}$$

which is positive, since $\delta(\alpha)>0$ and

$$\delta'(\alpha)+\beta'(\alpha) = \frac{e^{2\alpha R}-(1+2\alpha R e^{\alpha R})}{(e^{\alpha R}-1)^2} > 0.$$

The last inequality can be verified using the identity $e^X=\sum_{k=0}^{\infty}\frac{X^k}{k!}$ for any real X.

Applying the result (12) and noting that $q_{|\alpha_1=\alpha_2}=1$ for symmetric agents, one finds $\bar{\varepsilon}>0$ such that $\alpha_1=\alpha_2+\varepsilon$ implies $q>1$ and thus $p_1^*<p_2^*$ for all $0<\varepsilon<\bar{\varepsilon}$. However, as $\theta_1=\theta_2$, $p_1^*<p_2^*$ is possible if and only if $x_1^*<x_2^*$.

(c) The higher the prize, the smaller the winning probability of the more risk averse agent, i.e. $\frac{\partial q}{\partial R}>0$: The statement will be implied by Proposition 5 (c.f. Appendix A, Corollary 5).

□

Proofs of Proposition 5 and Corollary 4.

Before beginning the proofs, note that $\frac{\theta_1}{\theta_2}\geq\frac{\alpha_1}{\alpha_2}$ is equivalent to $\theta_1\alpha_2\geq\theta_2\alpha_1$ and that $\frac{\theta_1}{\theta_2}<\frac{\alpha_1}{\alpha_2}$ is equivalent to $\theta_1\alpha_2<\theta_2\alpha_1$. Furthermore, for brevity, the following definitions are made:

$$\begin{aligned} a := a(R) :=& \frac{\theta_2\delta(\alpha_1)}{\theta_1\delta(\alpha_2)} \\ =& \frac{\theta_2\alpha_1 e^{-\alpha_1 R}}{1-e^{-\alpha_1 R}}\cdot\frac{1-e^{-\alpha_2 R}}{\theta_1\alpha_2 e^{-\alpha_2 R}} \\ =& \frac{\theta_2\alpha_1}{\theta_1\alpha_2}\cdot\frac{e^{-\alpha_1 R}}{e^{-\alpha_2 R}}\cdot\frac{1-e^{-\alpha_2 R}}{1-e^{-\alpha_1 R}} > 0,\end{aligned} \tag{13}$$

$$
\begin{aligned}
b := b(R) :=& \frac{\theta_1\beta(\alpha_2) - \theta_2\beta(\alpha_1)}{2\theta_1\delta(\alpha_2)} \\
=& \frac{\theta_1\alpha_2}{1-e^{-\alpha_2 R}} \cdot \frac{1-e^{-\alpha_2 R}}{2\theta_1\alpha_2 e^{-\alpha_2 R}} - \frac{\theta_2\alpha_1}{1-e^{-\alpha_1 R}} \cdot \frac{1-e^{-\alpha_2 R}}{2\theta_1\alpha_2 e^{-\alpha_2 R}} \\
=& \frac{1}{2e^{-\alpha_2 R}} - \frac{1-e^{-\alpha_2 R}}{1-e^{-\alpha_1 R}} \cdot \frac{\theta_2\alpha_1}{2\theta_1\alpha_2 e^{-\alpha_2 R}} \\
=& \frac{1}{2e^{-\alpha_2 R}} \left[1 - \frac{1-e^{-\alpha_2 R}}{1-e^{-\alpha_1 R}} \cdot \frac{\theta_2\alpha_1}{\theta_1\alpha_2}\right],
\end{aligned} \tag{14}
$$

$$
\xi' := \frac{\partial \xi}{\partial R}, \quad \xi'' := \frac{\partial^2 \xi}{\partial R^2} \quad \text{for any function } \xi \in \{q, a, b, \beta(\alpha), \delta(\alpha)\}.
$$

Thus,

$$
q = \sqrt{\frac{\theta_2\delta(\alpha_1)}{\theta_1\delta(\alpha_2)} + \left(\frac{\theta_1\beta(\alpha_2) - \theta_2\beta(\alpha_1)}{2\theta_1\delta(\alpha_2)}\right)^2} - \frac{\theta_1\beta(\alpha_2) - \theta_2\beta(\alpha_1)}{2\theta_1\delta(\alpha_2)}
$$

can be written as

$$
q = \sqrt{a + b^2} - b
$$

and q' as

$$
\begin{aligned}
q' =& \frac{a' + 2bb'}{2\sqrt{a+b^2}} - b' \\
=& \frac{a'}{2\sqrt{a+b^2}} + \underbrace{\frac{b}{\sqrt{a+b^2}}}_{<1} \cdot b' - b' \\
=& \frac{a'}{2\sqrt{a+b^2}} + b' \cdot \underbrace{\left(\frac{b}{\sqrt{a+b^2}} - 1\right)}_{<0}.
\end{aligned} \tag{15}
$$

The proofs of Proposition 5 and Corollary 4 will repeatedly make use of the following limit result.

Lemma 1 *For $\theta_1 \geq \theta_2 > 0$, $\alpha_1 \geq \alpha_2 > 0$ and $R > 0$ the following hold:*

(a) $\lim_{R\to 0} q = \frac{\theta_2}{\theta_1}$.

(b) $\lim_{R\to 0} \xi = 1$.

As Lemma 1 shows, for very small rents, risk considerations do not play a role: The agents winning probabilities are solely determined by the skill ratio because their investments coincide.

Proof.

(a) First, the limits of a and b for $R \to 0$ are considered and afterwards the limit of q is calculated. For $R \to 0$ all $e^{-\alpha_i R}$ tend to 1, yielding

$$a = \frac{\theta_2 \alpha_1}{\theta_1 \alpha_2} \cdot \underbrace{\frac{e^{-\alpha_1 R}}{e^{-\alpha_2 R}}}_{\to 1} \cdot \frac{\overbrace{1 - e^{-\alpha_2 R}}^{\to 0}}{\underbrace{1 - e^{-\alpha_1 R}}_{\to 0}} \qquad \text{for } R \to 0.$$

Thus, l'Hôpital's rule can be applied. The sign $\stackrel{l'H}{=}$ indicates that, according to l'Hôpital's rule, the expression on the left of the sign is equal to the expression on the right of the sign.

$$\lim_{R \to 0} \frac{1 - e^{-\alpha_2 R}}{1 - e^{-\alpha_1 R}} \stackrel{l'H}{=} \lim_{R \to 0} \frac{\alpha_2 e^{-\alpha_2 R}}{\alpha_1 e^{-\alpha_1 R}} = \frac{\alpha_2}{\alpha_1}$$

and hence,

$$a = \frac{\theta_2 \alpha_1}{\theta_1 \alpha_2} \cdot \underbrace{\frac{e^{-\alpha_1 R}}{e^{-\alpha_2 R}}}_{\to 1} \cdot \underbrace{\frac{1 - e^{-\alpha_2 R}}{1 - e^{-\alpha_1 R}}}_{\to \frac{\alpha_2}{\alpha_1}} \longrightarrow \frac{\theta_2}{\theta_1} \qquad \text{for } R \to 0.$$

Analogously,

$$b = \frac{1}{2 \underbrace{e^{-\alpha_2 R}}_{\to 1}} - \underbrace{\frac{1 - e^{-\alpha_2 R}}{1 - e^{-\alpha_1 R}}}_{\to \frac{\alpha_2}{\alpha_1}} \cdot \frac{\theta_2 \alpha_1}{2 \theta_1 \alpha_2 \underbrace{e^{-\alpha_2 R}}_{\to 1}} \qquad \text{for } R \to 0,$$

yielding

$$\lim_{R \to 0} b = \frac{1}{2} - \frac{\alpha_2}{\alpha_1} \cdot \frac{\theta_2 \alpha_1}{2 \theta_1 \alpha_2} = \frac{1}{2} - \frac{1}{2} \cdot \frac{\theta_2}{\theta_1}.$$

Hence, for $R \to 0$,

$$\begin{aligned} q \longrightarrow & \underbrace{\sqrt{\frac{\theta_2}{\theta_1} + \frac{1}{4} - \frac{1}{2} \cdot \frac{\theta_2}{\theta_1} + \frac{1}{4} \cdot \frac{\theta_2^2}{\theta_1^2}}}_{(\frac{1}{2} + \frac{1}{2} \cdot \frac{\theta_2}{\theta_1})^2} - \frac{1}{2} + \frac{1}{2} \cdot \frac{\theta_2}{\theta_1} \\ = & \frac{1}{2} + \frac{1}{2} \cdot \frac{\theta_2}{\theta_1} - \frac{1}{2} + \frac{1}{2} \cdot \frac{\theta_2}{\theta_1} = \frac{\theta_2}{\theta_1}. \end{aligned}$$

(b) Using the definition of relative equilibrium effort $\xi := \frac{x_2^*}{x_1^*} = \frac{\theta_1}{\theta_2} q$, the statement follows immediately from part (a).

□

Proof of Proposition 5.

(a) Let $\frac{\theta_1}{\theta_2} \geq \frac{\alpha_1}{\alpha_2}$.

(i) I first show that $q < 1$. According to equation (14),

$$b = \frac{\theta_1 \beta(\alpha_2) - \theta_2 \beta(\alpha_1)}{2\theta_1 \delta(\alpha_2)} = \frac{1}{2e^{-\alpha_2 R}} \left[1 - \frac{1 - e^{-\alpha_2 R}}{1 - e^{-\alpha_1 R}} \cdot \frac{\theta_2 \alpha_1}{\theta_1 \alpha_2} \right] > 0.$$

For $\theta_1 > \theta_2$, $\alpha_1 > \alpha_2$ and with δ decreasing in α one has $a < 1$ according to (13). Therefore,

$$q = \sqrt{a + b^2} - b \leq \sqrt{a} + \sqrt{b^2} - b = \sqrt{a} < 1.$$

(ii) I now show that $q' < 0$. According to equation (15), this is the case if and only if

$$\frac{a'}{2\sqrt{a + b^2}} < b' \cdot \underbrace{\left(1 - \frac{b}{\sqrt{a + b^2}} \right)}_{>0}. \tag{16}$$

Using equations (6) and (7), one easily verifies that

$$\beta'(\alpha) = \delta'(\alpha) = -\beta(\alpha)\delta(\alpha) < 0 \tag{17}$$

and

$$\beta(\alpha) - \delta(\alpha) = \alpha.$$

Having said this, one computes

$$a' = \frac{\theta_2 \delta(\alpha_1)}{\theta_1 \delta(\alpha_2)} [\beta(\alpha_2) - \beta(\alpha_1)] < 0 \tag{18}$$

and

$$b' = \frac{\theta_1 \beta(\alpha_2)[\beta(\alpha_2) - \delta(\alpha_2)] - \theta_2 \beta(\alpha_1)[\beta(\alpha_2) - \delta(\alpha_1)]}{2\theta_1 \delta(\alpha_2)}. \tag{19}$$

The inequality in (18) is due to the fact that β is increasing in α. Hence, for $b' \geq 0$, (16) is trivially satisfied. Now, consider the case of $b' < 0$. Then (16) is equivalent to

$$\frac{a'}{2b'} > \sqrt{a + b^2} - b = q.$$

Since $q < 1$, it is sufficient to show $\frac{a'}{2b'} > 1$. Remembering $b' < 0$ and denoting $\beta_i := \beta(\alpha_i)$ and $\delta_i := \delta(\alpha_i)$ for $i \in \{1, 2\}$, this is equivalent to

$$\begin{array}{rrcl} & a' & < & 2b' \\ \Leftrightarrow & \theta_2\delta_1\beta_2 - \theta_2\delta_1\beta_1 & < & \theta_1\beta_2\beta_2 - \theta_1\delta_2\beta_2 - \theta_2\beta_1\beta_2 + \theta_2\delta_1\beta_1 \\ & \Leftrightarrow \quad 0 & < & 2\theta_2\delta_1\beta_1 - \theta_2\beta_1\beta_2 - \theta_2\delta_1\beta_2 + \theta_1\beta_2\underbrace{(\beta_2 - \delta_2)}_{\alpha_2} \\ & \Leftrightarrow \quad 0 & < & 2\theta_2\delta_1(\beta_1 - \beta_2) + \theta_2\beta_2\underbrace{(\delta_1 - \beta_1)}_{-\alpha_1} + \theta_1\beta_2\alpha_2 \\ & \Leftrightarrow \quad 0 & < & 2\theta_2\delta_1\underbrace{(\beta_1 - \beta_2)}_{>0} + \beta_2\underbrace{(\theta_1\alpha_2 - \theta_2\alpha_1)}_{\geq 0}. \end{array}$$

(b) Now let $\frac{\theta_1}{\theta_2} < \frac{\alpha_1}{\alpha_2}$. Note again that according to Lemma 1

$$\lim_{R \to 0} q = \frac{\theta_2}{\theta_1} < 1. \tag{20}$$

In the following it is shown that

$$\lim_{R \to \infty} q = \infty, \tag{21}$$

$$\lim_{R \to \infty} q' = \infty, \tag{22}$$

$$\lim_{R \to 0} q' = -\frac{(\alpha_1 + \alpha_2)\theta_2(\theta_1 - \theta_2)}{2\theta_1(\theta_1 + \theta_2)} < 0. \tag{23}$$

The results (20)–(23) imply the existence of values $R_0, \tilde{R}_0, R_1, \tilde{R}_1$ with $R_0 \leq R_1, \tilde{R}_0$ and $R_1 \leq \tilde{R}_1$ such that

$$\begin{array}{rrl} q \quad \text{is decreasing} & \text{for all} & R < R_0, \\ q \quad \text{is increasing} & \text{for all} & R \geq \tilde{R}_0, \\ q < 1 & \text{for all} & R < R_1, \\ q \geq 1 & \text{for all} & R \geq \tilde{R}_1. \end{array}$$

In order to show $R_0 = \tilde{R}_0$ and $R_1 = \tilde{R}_1$ and hereby prove Conjecture 1, it would be sufficient to demonstrate that q has at most one extremum. Though I have not been able to prove this result analytically, all simulations run confirm it.

Proof of equation (21): Using equation (13), $\alpha_1 > \alpha_2$ implies

$$a = \frac{\theta_2\alpha_1}{\theta_1\alpha_2} \cdot \underbrace{e^{R(-\alpha_1+\alpha_2)}}_{\to 0} \cdot \underbrace{\frac{1 - e^{-\alpha_2 R}}{1 - e^{-\alpha_1 R}}}_{\to 1} \longrightarrow 0 \qquad \text{for } R \to \infty.$$

Using equation (14), $\theta_1\alpha_2 < \theta_2\alpha_1$ implies

$$b = \frac{1}{2\underbrace{e^{-\alpha_2 R}}_{\to 0}} \underbrace{\left[1 - \frac{1-e^{-\alpha_2 R}}{1-e^{-\alpha_1 R}} \cdot \frac{\theta_2\alpha_1}{\theta_1\alpha_2}\right]}_{\to 1-\frac{\theta_2\alpha_1}{\theta_1\alpha_2}<0} \longrightarrow -\infty \qquad \text{for } R \to \infty.$$

Now, the limit in (21) follows from $q = \sqrt{a+b^2} - b$.

Proof of equation (22): From equation (18) one computes

$$a' = \frac{\theta_2\alpha_1}{\theta_1\alpha_2} \cdot e^{-(\alpha_1-\alpha_2)R} \cdot \underbrace{\left(\frac{-\alpha_1 + \alpha_2 + \alpha_1 e^{-\alpha_2 R} - \alpha_2 e^{-\alpha_1 R}}{\left(1-e^{-\alpha_1 R}\right)^2}\right)}_{=:H_{a'}}. \tag{24}$$

Obviously, $\lim_{R\to\infty} a' = 0$. Moreover, equation (13) implies $\lim_{R\to\infty} a = 0$. From equation (19) one computes

$$b' = e^{\alpha_2 R}\left[\frac{\theta_1\alpha_2}{2\theta_1} - \frac{\theta_2\alpha_1}{2\theta_1\alpha_2} \cdot \underbrace{\left(\frac{\alpha_2 - \alpha_2 e^{-\alpha_1 R} - \alpha_1 e^{-\alpha_1 R} + \alpha_1 e^{-(\alpha_1+\alpha_2)R}}{\left(1-e^{-\alpha_1 R}\right)^2}\right)}_{=:H_{b'}}\right]. \tag{25}$$

Since $\theta_1\alpha_2 < \theta_2\alpha_1$,

$$\lim_{R\to\infty} b' = \lim_{R\to\infty} e^{\alpha_2 R}\left[\frac{\theta_1\alpha_2}{2\theta_1} - \frac{\theta_2\alpha_1}{2\theta_1}\right] = -\infty.$$

Similarly, equation (14) implies

$$\lim_{R\to\infty} b = \lim_{R\to\infty} e^{\alpha_2 R}\left[\frac{\theta_1\alpha_2 - \theta_2\alpha_1}{2\theta_1\alpha_2}\right] = -\infty.$$

Using these results it is straightforward to see that

$$q' = \frac{a'}{2\sqrt{a+b^2}} - b'\left(1 - \frac{b}{\sqrt{a+b^2}}\right) \to \infty \quad \text{for} \quad R \to \infty.$$

Note that the term in brackets exceeds one for R sufficiently large.

Proof of equation (23): Equation (24) implies

$$\lim_{R\to 0} a' = \frac{\theta_2\alpha_1}{\theta_1\alpha_2} \cdot \lim_{R\to 0} H_{a'} = \frac{\theta_2(\alpha_2-\alpha_1)}{2\theta_1},$$

where L'Hospital's rule is applied twice in order to compute $\lim_{R\to 0} H_{a'}$. Similarly, equation (25) implies

$$\lim_{R\to 0} b' = \frac{\alpha_2}{2} - \frac{\theta_2\alpha_1}{2\theta_1\alpha_2} \cdot \lim_{R\to 0} H_{b'} = \frac{2\theta_1\alpha_2 - \theta_2\alpha_1 - \theta_2\alpha_2}{4\theta_1},$$

where, again, L'Hospital's rule is applied twice in order to compute $\lim_{R\to 0} H_{b'}$. Moreover, recall from the proof of Lemma 1 that

$$\lim_{R\to 0} a = \frac{\theta_2}{\theta_1}, \qquad \lim_{R\to 0} b = \frac{1}{2} - \frac{1}{2}\frac{\theta_2}{\theta_1}, \qquad \text{and} \qquad \lim_{R\to 0} \sqrt{a+b^2} = \frac{1}{2} + \frac{1}{2}\frac{\theta_2}{\theta_1}.$$

Given these results and using equation (15) one computes

$$\begin{aligned}
\lim_{R\to 0} q' =& \frac{\frac{\theta_2(\alpha_2-\alpha_1)}{2\theta_1}}{1+\frac{\theta_2}{\theta_1}} + \frac{2\theta_1\alpha_2 - \theta_2\alpha_1 - \theta_2\alpha_2}{4\theta_1} \cdot \left(\frac{\frac{1}{2} - \frac{1}{2}\cdot\frac{\theta_2}{\theta_1}}{\frac{1}{2} + \frac{1}{2}\cdot\frac{\theta_2}{\theta_1}} - 1 \right) \\
=& \frac{\frac{\theta_2(\alpha_2-\alpha_1)}{2\theta_1}}{\frac{\theta_1+\theta_2}{\theta_1}} + \frac{2\theta_1\alpha_2 - \theta_2\alpha_1 - \theta_2\alpha_2}{4\theta_1} \cdot \left(\frac{\frac{1}{2} - \frac{1}{2}\cdot\frac{\theta_2}{\theta_1} - \left(\frac{1}{2} + \frac{1}{2}\cdot\frac{\theta_2}{\theta_1}\right)}{\frac{1}{2} + \frac{1}{2}\cdot\frac{\theta_2}{\theta_1}} \right) \\
=& \frac{\frac{\theta_2(\alpha_2-\alpha_1)}{2}}{\theta_1+\theta_2} + \frac{2\theta_1\alpha_2 - \theta_2\alpha_1 - \theta_2\alpha_2}{4\theta_1} \cdot \left(\frac{-\frac{\theta_2}{\theta_1}}{\frac{\theta_1+\theta_2}{2\theta_1}} \right) \\
=& \frac{\frac{\theta_2(\alpha_2-\alpha_1)}{2}}{\theta_1+\theta_2} - \frac{2\theta_2 \cdot \frac{2\theta_1\alpha_2 - \theta_2\alpha_1 - \theta_2\alpha_2}{4\theta_1}}{\theta_1+\theta_2} \\
=& \frac{1}{\theta_1+\theta_2} \cdot \left(\frac{\theta_1\theta_2\alpha_2 - \theta_1\theta_2\alpha_1 - 2\theta_1\theta_2\alpha_2 + \theta_2^2\alpha_1 + \theta_2^2\alpha_2}{2\theta_1} \right) \\
=& -\frac{\theta_2(\theta_1-\theta_2)(\alpha_1+\alpha_2)}{2\theta_1(\theta_1+\theta_2)} < 0.
\end{aligned} \tag{26}$$

□

Proof of Corollary 4.

Recalling the definition of $\xi := \frac{x_2^*}{x_1^*} = \frac{\theta_1}{\theta_2} q$, the statements follow immediately from Proposition 5 and Lemma 1.

□

Completing the Proof of Proposition 4.

The following Corollary 5 extends Proposition 5 to the case in which agents are equally skilled thereby completing the proof of Proposition 4.

Corollary 5

Suppose Conjecture 1 holds. If $\theta_1 = \theta_2$ *and* $\alpha_1 > \alpha_2 > 0$ *then* $R_0 = R_1 = 0$.

Proof.

Note that for $\theta_1 = \theta_2$ one has $\theta_1\alpha_2 < \theta_2\alpha_1$. In Lemma 1 it has already been shown that q tends to $\frac{\theta_2}{\theta_1}$ for $R \to 0$, which equals one if $\theta_1 = \theta_2$. Given that Conjecture 1 holds, q has at most one extremum for $R \geq 0$. Hence, it is sufficient to show that q' tends to zero for $R \to 0$. However, this is obvious from (26).

$\square$

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