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ABSTRACT
In the paper, we simulate a heterogeneous-agent version of the wage-posting model as derived by Montgomery (1991) with homogeneous workers and differently-productive employers. Wage policy of particular employer is positively correlated with employer's productivity level and the wage policy of the competitor. However, it is a less productive employer whose wage posting could also outweigh the posting of a more productive employer, though only temporarily.

JEL Classification: C7, C15, D8, J3.
Keywords: Job-search model; Wage posting; Heterogeneous agents; Numerical optimization.

INTRODUCTION

In the paper, we simulate a heterogeneous-agent wage-posting model of Montgomery (1991). Employers in the model announce their wage posts in a repeated non-cooperative game and workers apply to vacancies. All bargaining power is given to employers while workers only direct their job search according to their preferences. Workers in the model have complete knowledge about all posted wages and can make one job application. They do it uncooperatively which could lead to the situation where both apply to the same employer while leaving the other vacancy with no applications.

All job-postings in the model offer the same expected benefits to workers, should they apply. For instance, vacancies that offer lower wages attract smaller number of job applicants, thus raising each applicant's likelihood of being hired. Both employers in the model are differently-productive and their productivities develop according to the geometric Brownian motion. Employers in the model condition their wage policies according to the wage posted by each other and in relation to their own productivity levels.

Simulation results show that it is less productive employer whose productivity level is the main loadstar in the wage-setting process, while the wage policy of the more productive employer much less persistently concurs with his productivity level. It could also happen, although only temporarily, that it is a less productive employer whose wage-offer could be higher than that of the more productive employer.

In addition, we test the effects of intrinsic costs related to uncertainty, which employers face when changing the alternative, on the behavior of the model. This means that employer is not prone to changing current alternative, especially when it is expected that the benefit of adopting a new alternative would be relatively small. Rubinstein (1998) defines such behavior by the tradeoff between complexity and efficiency of alternatives, where agents (employers in our case) prefer efficient and simple alternatives.[1] The inclusion of intrinsic costs into the decision-making does not change the behavior of the model significantly and only smoothes the wage policy.

The paper proceeds as follows. Model is derived in Chapter 2, and simulations are performed in Chapter 3. Chapter 4 summarizes simulation results, and the last chapter concludes.

THE MODEL

The model resembles a simple 2×2 case from Montgomery (1991). Suppose that the labor market consists of two identical and anonymous workers and two differentially productive employers, each having one vacancy. Then both employers non-cooperatively post wage w as to maximize their profit π. Both workers observe the postings and uncooperatively apply each to one job application. Employer that

[1] For the literature review on the role of behavioral studies on the decision-making, see Hirshleifer (2001).

receives at least one application employs a worker at the promised wage w, and starts producing the output v. Job posting without an application remains vacant.

Under a non-cooperative repeated game-setting workers apply to the first employer with probability p and to the second employer with probability $(1-p)$.[2] If both workers apply to the same vacancy, then the employer randomly chooses one applicant while leaving the other unemployed. The following output matrix applies:

	1	2
1	$\frac{1}{2}w_1 / \frac{1}{2}w_1$	w_1 / w_2
2	w_2 / w_1	$\frac{1}{2}w_2 / \frac{1}{2}w_2$

The game has three Nash equilibria $\left\{(w_1, w_2),(w_2, w_1),\left(\left(\frac{1}{2}w_1,\frac{1}{2}w_2\right),\left(\frac{1}{2}w_1,\frac{1}{2}w_2\right)\right)\right\}$. In the first case worker 1 applies to employer 1 and worker 2 applies to employer 2, in the second case worker 1 applies to employer 2 and worker 2 applies to employer 1, and in a mixed strategy both workers apply to both employers with the probability 0.5. Solving for p in a mixed Nash equilibrium yields:

$$p = \frac{2w_1 - w_2}{w_1 + w_2} \qquad (1).$$

Employers can increase the probability of receiving an application to the vacancy by offering higher wage. That is to be expected in our case of differently-productive employers, where an empty vacancy of the more productive employer is more costly. However, higher wages attract more job applications thus lowering the probability for an individual worker to getting a higher-paid job thus preserving the equality of both wages' expected values.

A filled vacancy at the employer i produces a product of $v_{i,t}$. $v_{i,t}$ is normally distributed random variable that develops in time around 0.5 by a geometric Brownian motion with known drift μ and known variance σ^2 and is defined as

$$v_{i,t} = 0.5 + \mu t + \sigma Z_{i,t}.$$

$Z_{i,t} \sim N\left(\mu t, \sigma^2 t\right)$ is a Wiener process. Over time both employers observe output realizations so they can regularly update their beliefs about its value and solve their maximization problem:

$$\max_{w_i} \pi_i = (v_i - w_i)\left(1-(1-p(w_i))^2\right), \text{ given } v_i(\cdot) \qquad (2).$$

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[2] As workers are assumed to be homogeneous they must apply with the same probability to particular vacancy if they are to maximize their expected benefit.

Second expression in (2) depicts the probability that the employer i receives at least one job application. Using (1) for p_i and inserting it into the profit-maximization equation (2), alters the maximization problem:[3]

$$\max_{w_i} \pi_i = \left(v_i - w_i\right)\left(\frac{3w_{-i}\left(2w_i - w_{-i}\right)}{\left(w_i + w_{-i}\right)^2}\right), \text{given } v_i\left(\cdot\right), w_{-i}\left(\cdot\right) \qquad (3).$$

Solving (3) for both employers gives us their reaction functions, representing their best responses to each other's wage posting strategy:

$$R_1\left(w_2\right) = \frac{w_2\left(w_2 + 4v_1\right)}{5w_2 + 2v_1} \qquad (4),$$

$$R_2\left(w_1\right) = \frac{w_1\left(w_1 + 4v_2\right)}{5w_1 + 2v_2} \qquad (5).$$

Wage posted by a particular employer is in a positive correlation with the wage posted by other employer and employer's productivity level.

SIMULATIONS

The model consists of two employers $i = \{1, 2\}$ each of whom solves his own optimization problem as given in (3). All the decision-making in an algorithm is iterated forward in time for $t = 1, 2, ..., 1000$. To eliminate the dependency of results to initial conditions we discarded first hundred realizations, thus leaving additional 900 for further study. Both employers simultaneously condition their wage selections in time t according to each other's selection in time $t-1$ and their own productivity levels in time t, respectively. To fit the algorithm we rewrite the optimization problem as given by (3):

$$w_{i,t} = \arg\max_{w_{i,t}^k} \pi_{i,t} = \left(v_{i,t} - w_{i,t}^k\right)\left(\frac{3w_{-i,t-1}\left(2w_{i,t}^k - w_{-i,t-1}\right)}{\left(w_{i,t}^k + w_{-i,t-1}\right)^2}\right), \text{given } v_{i,t}\left(\cdot\right), w_{-i,t-1}\left(\cdot\right) \qquad (3a).$$

The decision function to be maximized in each period t is continuous, concave, and differentiable on the defined convex set $D \in (0,1)$. This means it has a unique maximum that is easily found by a line-search optimization. The decision-making algorithm works as follows:[4]

Initialization: choose initial $w_{i,t}^0$ and other parameter values

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[3] $-i$ stands for the not i employer.

[4] The algorithm is not optimized for speed.

Step 1: evaluate maximization function $\pi_{i,t}\left(w_{i,t}^{k}\right)$.

Step 2: $w_{i,t}^{k+1} = w_{i,t}^{k} + step; step = 1E-8$.

Step 3: if $\pi_{i,t}\left(w_{i,t}^{k+1}\right) - \pi_{i,t}\left(w_{i,t}^{k}\right) > 0$, go to step 1, else go to step 4.

Step 4: quit iteration and report $w_{i,t}^{k}$ as the optimum wage posted by employer i in time t, $w_{i,t}$, and proceed with the $t+1$.

To allow for the intrinsic costs of changing current wage policy, the logistic (Fermi) probability function is used as a driving mechanism to the employers' wage-setting process:

$$F\left(w_{i,t} \leftarrow w_{i,t-1}\right) = \left(1 + \exp\left[-\left|w_{i,t-1} - w_{i,t}\right|\kappa^{-1}\right]\right)^{-1} \quad (6).$$

Probability that the wage is regularly updated in each period t is a function of wage differential and $\kappa \in (0,1)$. Parameter κ defines the susceptibility to change the wage policy and the smaller the parameter the larger the probability that employer follows his own optimal strategy and vice versa. The rule to adopt a new wage in each period becomes: if $ran < F\left(w_{i,t} \leftarrow w_{i,t-1}\right)$ then $w_{i,t} = w_{i,t}$, else $w_{i,t} = w_{i,t-1}$, with $ran \sim U(0,1)$ being an i.i.d. random number. Simulations are performed for $\kappa = \{\downarrow 0, 0.01, 0.25\}$ and the results are averaged over 5 realizations.

Figure 1 depicts the development of productivity levels for both employers for the 900 observations we kept. As can be seen from the figure, geometric Brownian motion is set such as to allow only for minor changes in productivity of each employer; parameters are set to $\mu = 0.005$ and $\sigma^2 = 0.002$. To preserve the mutual comparability of simulation results we used these same productivity dataset in all simulation repetitions.

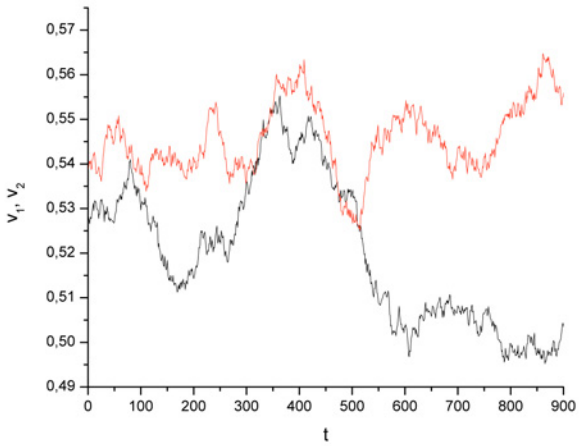

FIGURE 1: Geometric Brownian motion realizations. Black line depicts the development of $v_{1,t}$ and red line depicts the development of $v_{2,t}$.

RESULTS

Simulation results are depicted on Figure 2. The figure depicts the iteration process of the wage policy under $\kappa = \{\downarrow 0, 0.01\}$. For $\kappa = \{0.01, 0.25\}$ simulations produced equal results. That is why we excluded the latter from the further study.

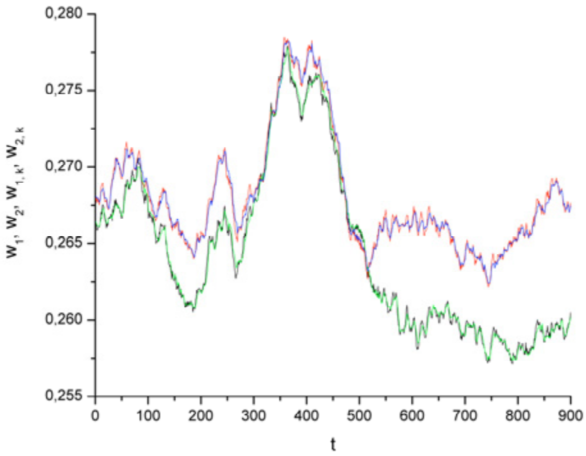

FIGURE 2: Optimal wage time-path. $black: w_1\,(red: w_2)$ is the wage rate of employer 1 (2) when $\kappa \downarrow 0$, $green: w_{1,\kappa}\,(blue: w_{2,\kappa})$ is the wage rate of employer 1 (2) when $\kappa = 0.01$

It is as expected that both heterogeneous employers post different wage offers. We have mentioned that higher wages increase the probability of receiving at least one job application. That is why we would expect that more productive employer whose vacancy is more costly posts higher wage offer. However, in some cases of minor productivity differences when a stochastic factor could suddenly change the productivity order of both employers, a less productive employer might offer higher wage for a very short time. It is Shi (2006) who also noted that in case of very small productivity differentials a less productive worker might get a higher wage offer (although in her case for other reasons).

We found from the simulations that the less productive employer dictates the wage policy for both, irrespective of the κ value. Partial correlation coefficients show that less productive employer entirely manages his wage policy according to his productivity level $\rho_{w_1,v_1} = 0.97$, but much less upon the productivity level of his more productive counterpart, $\rho_{w_1,v_2} = 0.14$. Many researchers predict a positive relation between wages and job productivity levels, as, for instance, Shimer (2005) or Abowd et al. (1999).

The behavior of the more productive employer is, however, different. His wage policy correlates mainly with the other employer's wage policy, $\rho_{w_2,w_1} = 0.89$ (and consequently also with his productivity level, $\rho_{w_2,v_1} = 0.76$), and much less with his own productivity level, $\rho_{w_2,v_1} = 0.56$.

Such conclusion makes sense. A profit-maximizing employer, in order to minimize his wage-bill, offers slightly larger wage than his lower-productive counterpart, irrespective of his own productivity level, insofar the productivity advantage over the other is large enough.

Now, let us have a closer look at the influence of κ on how the optimal wages of both employers from our model develop in time. It is clear from Figure 2 and Figure 3 that $w_{\cdot,\kappa\downarrow 0}$ is very close to $w_{\cdot,\kappa=0.01}$.

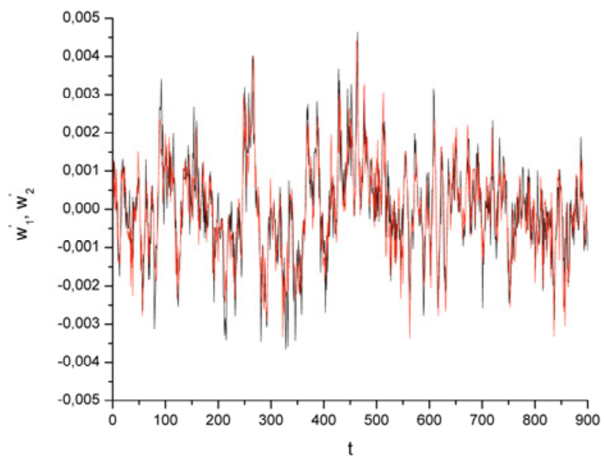

FIGURE 3: Relative difference in the wage policy for $\kappa = \{\downarrow 0, 0.01\}$. Red line shows $w_2' = \left(\left(w_{2,t}^{\kappa=0.01} - w_{2,t}^{\kappa\downarrow 0}\right) / w_{2,t}^{\kappa\downarrow 0}\right)$ and black line shows $w_1' = \left(\left(w_{1,t}^{\kappa=0.01} - w_{1,t}^{\kappa\downarrow 0}\right) / w_{1,t}^{\kappa\downarrow 0}\right)$.

The differences for w_1 and w_2 as shown in Figure 3 are low with mean and median being close to zero and show the correlation coefficient of $\rho_{w_2',w_1'} = 0.77$. The max-min spread is close to 0.8% and the difference from the optimal policy is largest for w_1, ranging from -0.37% to 0.46%. Such small deviations from the optimal wage setting are mostly due to the minor changes in productivity. In case of small changes the Fermi function entails that on average only every second "optimal" decision is accepted.

The inclusion of the intrinsic costs in the form of the Fermi function narrowed both employers' wage policy bands for 1.6% in case of w_1 and for 3.3% in case of w_2. Such smoothing is something we would expect because what the Fermi function does in the algorithm is affecting only the decision of the particular employer either to move from the current "optimal" wage to the new "optimal" wage or not. Therefore, the solution of the original model without κ characterizes an outer wage bound, which could only be narrowed.

CONCLUSION

In the paper, we test a 2×2 heterogeneous-agent version of the wage posting model as derived by Montgomery (1991). Homogeneous workers in the model are allowed to make only one job application to differently-productive employers. Simulations predict that the wage-policy is in a positive relation to the employer's productivity level. However, simulation results also indicate that it is a less productive employer, who can post higher wage than more productive employer, though only temporarily. The inclusion of intrinsic costs only smoothes the wage-path, with negligible effects to the basic simulation results. The max-min spread deviation from the "optimal" wage-posting is less than 1 percent. Such a low difference is basically due to the minor changes in productivity of the employers.

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