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**Horizontal Mergers with Free Entry
in Differentiated Oligopolies**

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Abstract

Antitrust authorities view the possibility of entry as a key determinant of whether a proposed merger will be harmful to society. This paper examines the effects of horizontal mergers in models of non-localized, differentiated Bertrand oligopoly that allow for free entry. The analysis of the long run effects of mergers in differentiated products markets raises issues that are significantly different from those in the short run or in homogeneous products markets due to the introduction of new varieties. Our analysis reveals that determining the properties of consumer preferences is crucial to the antitrust analysis of mergers in differentiated products markets. Specifically, we show that if the demand system satisfies the Independence from Irrelevant Alternatives (IIA) property and if the number of firms is treated as a continuous variable, mergers in differentiated products markets have no long run effect on consumer welfare. Moreover, in this case, marginal cost savings are to a large extent irrelevant to the consumer welfare effects of mergers. If the number of firms is treated as a discrete variable, fixed or marginal cost savings are a necessary condition for mergers to have zero or positive effect on consumer welfare. Using the example of linear demand, we show that if the demand system does not satisfy the IIA property, mergers in differentiated products markets can harm consumer welfare in long run equilibrium. Moreover, the amount of harm increases with consumers' taste for variety.

JEL classification: L13, L22, L41, K21

Keywords: Horizontal mergers; free entry; product differentiation; independence from irrelevant alternatives; antitrust policy

1 Introduction

Antitrust authorities around the world regard the possibility of entry as an important determinant of whether a proposed merger will be harmful to consumer and social welfare and, hence, whether they will allow it to proceed. For instance, the Horizontal Merger Guidelines issued by the antitrust authorities in the United States state that they consider "whether entry would be timely, likely, and sufficient either to deter or to counteract the competitive effects of concern."¹ Similar statements appear in the European Commission's horizontal merger guidelines and the Merger Guidelines of the Australian Consumer and Competition Commission (ACCC).²

The reason that entry is considered important in merger analysis is that even if a merger were to increase market power and prices in the industry, if it is possible subsequently for other firms to enter the market, such entry may push the prices back down, possibly to their pre-merger levels or below.³ This argument raises several important questions. First, can mergers induce entry? Second, if a merger induces entry, will it still be privately profitable? Third, and most importantly, if entry is possible, what conclusions can we draw about the implications of mergers for consumer welfare?

The goal of this paper is to analyze these questions in a theoretical model of non-localized product differentiation and Bertrand competition. Imperfect substitutability is a common and significant feature of many markets in which mergers occur. Moreover, the long run effects of mergers in differentiated products markets are substantially more complex than their effects in the short run or in homogeneous products markets.

The short run effects of mergers in differentiated products markets have been studied

¹See Section 0.2 of the United States Merger Guidelines available at www.ftc.gov/bc/docs/horizmer.htm.

²See paragraph 68 of the European Commission's horizontal merger guidelines available at europa.eu/scadplus/leg/en/lvb/l26107.htm and p. 48 of the ACCC Merger Guidelines available at www.accc.gov.au/content/index.phtml/itemId/304397.

³Deneckere and Davidson (1985), Farrell and Shapiro (1990), and Werden and Froeb (1994) analyze the impact of mergers in models without entry and conclude that mergers must be harmful to consumers unless they result in productive efficiencies. Deneckere and Davidson (1985) consider a symmetrically differentiated Bertrand market. Farrell and Shapiro (1990) assume a model of Cournot competition with homogeneous products. Werden and Froeb (1994) consider a logit demand model with asymmetrically differentiated firms and Bertrand competition.

by Deneckere and Davidson (1985), who show that mergers without marginal cost savings reduce consumer welfare. This is because they cause all prices to rise and to become asymmetric since the merging firms charge higher prices than the outsider firms. The asymmetry harms consumers because they have a taste for variety and, therefore, prefer to spread their consumption evenly over the differentiated products.

The long run effects of mergers in homogeneous products markets have been analyzed by Spector (2003) and Davidson and Mukherjee (2006). Spector (2003) shows that without technological synergies, mergers would be unprofitable if entry reduced prices to their pre-merger level. Hence, any profitable merger must harm consumer welfare.⁴ Davidson and Mukherjee (2006) show that mergers generating marginal cost savings have no effect on consumer welfare. This is because the marginal cost savings allow the merger to remain profitable even though entry reduces prices to their pre-merger level.

In differentiated goods markets, the effect of entry is not only to decrease prices, but also to introduce additional variety which is of benefit to consumers. Taste for variety plays a central role in the analysis of the long run effects of mergers because it affects the amount of entry that occurs, the market power that firms enjoy, and the disutility consumers get from asymmetric consumption. Hence, although it is clear that mergers that do not result in marginal cost reductions harm consumers without entry, this is by no means clear if the merger is followed by entry.

We first analyze the long run welfare effects of mergers with demand systems that satisfy the Independence from Irrelevant Alternatives (IIA) property. Two commonly used examples of demand systems that satisfy the IIA property are the logit and CES demand systems. We find that if the number of firms is treated as a continuous variable, there are no long run effects of mergers on consumer welfare.⁵ The benefits of merger-induced

⁴Both Spector (2003) and Davidson and Mukherjee (2006) show that with symmetric entry costs and no technological synergies, there can be no profitable mergers in homogeneous products markets in the long run. Hence, Spector (2003) assumes idiosyncratic entry costs to allow for profitable mergers in a long run equilibrium. Since Davidson and Mukherjee (2006) allow for marginal cost savings, they assume symmetric entry costs.

⁵Since consumer welfare is unaffected by the merger and the non-merging firms earn zero profits in a long run equilibrium, the social welfare effects of the merger depend only on its profitability.

entry exactly offset the harm caused by the merging firms' higher prices. Moreover, our analysis reveals that as long as demand satisfies the IIA property, marginal cost savings are to a large extent irrelevant to the consumer welfare effects of mergers. We also show that this welfare result can generalize to some cases with multi-product firms and to some cases where firms initially have asymmetric market shares.

From the analysis of mergers where the number of firms is treated as a continuous variable, it is possible to derive the implications for the more realistic case where the number of firms is discrete. The two main results we have are the following. First, without fixed or marginal cost savings, a merger may still be profitable because of the integer constraint. However, since this implies insufficient entry, the merger must harm consumer welfare. Second, the impact on consumer welfare of a profitable merger that generates either fixed or marginal cost savings depends solely on the operation of the integer constraint. This result implies that from the perspective of an antitrust authority or a court, fixed cost savings alone can be an indicator that a merger may benefit consumers. To our knowledge, this point has not been made in the literature to date.

To explore the long run effects of mergers in demand systems that do not satisfy the IIA property, we next consider two commonly used linear demand systems. In the first one, due to Shubik (1980), we show that mergers without marginal cost savings must always harm consumer welfare if the number of firms is treated as a continuous variable. We show, however, that this is because in Shubik's (1980) linear demand system consumers derive no benefit from new variety. We argue that this makes Shubik's (1980) system inappropriate for analyzing the long run effects of mergers in differentiated products markets as it rules out by assumption the variety effect that distinguishes differentiated products markets from homogeneous products markets.

We then analyze a linear demand system due to Ottaviano and Thisse (1999) that does feature taste for variety. Using numerical simulations, we show that in this context mergers without marginal cost savings can harm consumer welfare. Moreover, the magnitude of the harm caused by the merger in Ottaviano and Thisse's (1999) demand system depends on

consumers' taste for variety. This is in contrast with demand systems that satisfy the IIA property, where consumers' taste for variety has no impact on the effect of mergers.

Two closely related papers that also consider the effects of mergers in differentiated goods markets with entry are Werden and Froeb (1998), and Cabral (2003). Werden and Froeb (1998) consider the likelihood and effects of merger-induced entry using a logit framework with symmetric and asymmetric firms. They assume that there can be at most two merging firms and one entrant. Using simulations, they show that both consumer and social welfare may rise as a result of logit mergers with entry. However, they illustrate that most mergers are unlikely to induce entry and that most mergers would be unprofitable if they did induce entry. Cabral (2003) analyzes the effect of mergers with entry using a Salop-type localized competition model. He considers a market with two single-product incumbents and one potential entrant. His results reveal that if the market is sufficiently large, the merger induces entry and causes consumer welfare to rise. However, if the merger results in marginal cost savings, entry may be deterred and hence the merger may harm consumer welfare.

Our paper differs from both of these papers in important ways. While Cabral (2003) considers a localized competition model, we consider models of non-localized competition.⁶ Although Werden and Froeb (1998) also consider a model of non-localized competition, our approach differs in that we study the welfare effects of mergers by solving for the long run equilibrium with markets and mergers of any size. In comparison with Werden and Froeb (1998), our results indicate that especially with mergers involving multi-product firms and/or large mergers, one should not necessarily expect the integer constraint to result in zero entry or insufficient entry to counteract the negative effects of the merger. Moreover, our analysis applies generally to all demand systems that satisfy the IIA property. Finally, by comparing mergers in demand systems that satisfy the IIA property with mergers in demand systems that do not, we show that in differentiated products markets, determining the properties of consumer preferences is a crucial first step in the antitrust analysis of mergers.

⁶As Cabral (2003) also notes, markets in real life are probably characterized by both localized and non-localized competition.

We proceed in the following way. After presenting the model in Section 2, we consider in Section 3 the long run effects of mergers in demand systems that satisfy the IIA property, treating the number of firms first as a continuous and then as a discrete variable. We then turn our attention in Section 4 to demand systems that do not satisfy the IIA property, and consider Shubik's (1980) and Ottaviano and Thisse's (1999) linear demand systems. Finally, we conclude in Section 5 by summarizing the implications of our results and suggesting avenues for future research.

2 Model

Assume there are initially N firms in a market featuring product differentiation and free entry. The firms face identical marginal and fixed costs, denoted by c and F respectively, and they compete in prices. Firm i faces demand $q_i(N, \mathbf{p})$, where \mathbf{p} is the vector of prices, and earns profits

$$\pi_i = (p_i - c) q_i(N, \mathbf{p}) - F, \quad i = 1, \dots, N.$$

We assume that the market is initially in a long run equilibrium. This implies that there are potential entrants which initially find it unprofitable to enter the market. In the following analysis, to avoid integer problems, we first treat the number of firms as a continuous variable.⁷ Hence, in the long run equilibrium, all active firms earn zero profits before the merger. We then draw inferences about the equilibrium where the number of firms is discrete based on the continuous N analysis.

The timing of events is as follows. In Stage 1, an exogenously determined group of M of the N firms which are active in the pre-merger equilibrium merge. The merger creates a multi-product firm. The merging firms continue to produce the same products, but they make their pricing decisions jointly. In Stage 2, the potential entrants decide whether to enter and the incumbent outsider firms decide whether to exit.⁸ Let E stand for the number of entrants. The entrants produce goods symmetrically differentiated from the N

⁷This is standard in the literature. See Vives (2002) and Mankiw and Whinston (1986).

⁸Non-merging firms are referred to as outsider firms.

incumbent firms' goods.⁹ In Stage 3, all the firms in the market compete by choosing their prices. Throughout the text, we use the subscripts m , o , and nm to refer to the merging firms, the outsiders, and the case of no merger respectively.

From Deneckere and Davidson's (1985) short run analysis, we know that the outsider firms make higher profits than the merging firms in equilibrium if the merger does not generate cost savings. This is because the outsiders face a lower level of competition than each of the merged firms. This implies that all mergers are unprofitable in the long run without cost savings. With entry, the equilibrium profits of the outsider firms are driven down to zero. Since the merged firms earn less than the outsider firms, their profits must be less than zero. Hence, in the following analysis, we assume that the merger results in a sufficient level of cost savings to make it profitable to merge.¹⁰

To determine the long run impact of mergers on consumer and social welfare it is necessary to be more specific about consumers' preferences. We start the analysis in the next section by considering demand systems which satisfy the IIA property.

3 IIA and Long Run Effects of Mergers

We first show that mergers in demand systems that satisfy the IIA property do not affect consumer welfare in the long run if N is treated as a continuous variable. We then analyze the effect of mergers on social welfare and in the case of discrete N in Sections 3.1 and 3.2 respectively.

Suppose the representative consumer has an indirect utility function of the form

$$v(\mathbf{p}, y) = V(\bar{P}(\mathbf{p}), y) \quad (1)$$

where y is income and \bar{P} is a function of the vector of prices \mathbf{p} . This representation of the indirect utility function assumes weak separability of prices from income, which is a less

⁹We rule out any type of strategic behavior by the incumbents to deter entry, such as limit pricing and brand proliferation.

¹⁰Our analysis in Section 3 applies whether or not mergers generate marginal cost savings. If mergers do not result in marginal cost savings, then we assume they generate a sufficient level of fixed cost savings to make them profitable. In Section 4, we assume that mergers result in fixed cost savings only.

restrictive assumption than quasilinearity. By Roy's Identity, demand for firm i 's good is

$$q_i = -\frac{\frac{\partial V(\bar{P}, y)}{\partial p_i}}{\frac{\partial V(\bar{P}, y)}{\partial y}}. \quad (2)$$

We make the following assumptions which are implied by utility maximization.

Assumption 1 $\frac{\partial V}{\partial y} > 0$.

Assumption 2 $\frac{\partial V}{\partial p_i} < 0$ for all i .

Moreover, since we are interested in the case of gross substitutes, we assume

Assumption 3 $\frac{\partial q_i}{\partial p_j} > 0$ for all $i \neq j$.

Finally, we assume that the demand system satisfies the IIA property.

Assumption 4 $\frac{\partial}{\partial p_k} \left(\frac{q_i}{q_j} \right) = 0$ for all $i, j \neq k$.

Hence, in the context of continuous demand, the IIA property means that the ratio of quantities demanded of any two goods is independent of the existence or price of a third good.¹¹ Using (2) to substitute for q_i and q_j in the expression in Assumption 4 gives us

$$\frac{\partial}{\partial p_k} \left(\frac{\frac{\partial V}{\partial p_i}}{\frac{\partial V}{\partial p_j}} \right) = \frac{\partial}{\partial p_k} \left(\frac{\frac{\partial \bar{P}}{\partial p_i}}{\frac{\partial \bar{P}}{\partial p_j}} \right) = 0.$$

Goldman and Uzawa (1964) show that this condition implies a functional structure of the form

$$\bar{P} = G \left(\sum_i f_i(p_i) \right),$$

where $G(\cdot)$ is a monotonically increasing function of one variable and $f_i(p_i)$ is any function of p_i .¹² Hence, we can re-write (1) as

$$v(\mathbf{p}, y) = V \left(G \left(\sum_i f_i(p_i) \right), y \right),$$

¹¹A further implication of this property that makes it an attractive simplifying assumption in merger simulations is that the cross-price elasticities with respect to the price of a third good are the same for all goods in the market. See Hausman (1975).

¹²See Theorem 1 on p. 389 of Goldman and Uzawa (1964) for the proof.

which implies that consumer welfare remains unchanged if $\sum_i f_i(p_i)$ is unchanged after the merger.

To show that $\sum_i f_i(p_i)$ is unchanged after the merger, we first show that each outsider firm's profit function is monotonic in $\sum_{j \neq i} f_j(p_j)$.

Lemma 1 *The profits earned by an outsider firm i change monotonically with the value of $\sum_{j \neq i} f_j(p_j)$. Moreover, $\text{sign} \left\{ \frac{\partial \pi_i^*}{\partial \sum_{j \neq i} f_j(p_j)} \right\} = \text{sign} \left\{ -\frac{\partial v}{\partial \sum_{j \neq i} f_j(p_j)} \right\}$, where π_i^* is the maximum value of firm i 's profits.*

Proof. See Appendix A. ■

Lemma 1 establishes a connection between an outsider firm i 's profits and consumer welfare. The connection is that if the cumulative effect of any price changes by the rival firms and any entry that takes place as a result of the merger decreases an outsider firm i 's profit, it increases consumer welfare, and vice-versa. This result has immediate implications for the welfare analysis of the long run effects of mergers. These implications are summarized in Proposition 1.

Proposition 1 *If the demand system satisfies Assumptions 1-4, then mergers do not have any long run impact on consumer welfare.*

Proof. For continuous N , the long run equilibrium condition implies that an outsider firm i earns the same profit before and after the merger. By Lemma 1, we know that π_i^* changes monotonically with $\sum_{j \neq i} f_j(p_j)$, which implies that $\sum_{j \neq i} f_j(p_j)$ is the same before and after the merger. Hence, the best response of firm i must also be the same before and after the merger. This implies that $\sum_j f_j(p_j)$ does not change with the merger. Since (1) is a function of $\sum_j f_j(p_j)$ and y only, consumer welfare is unchanged by the merger in a long run equilibrium. ■

It is worth emphasizing the generality of the result stated in Proposition 1. First, even if a merger results in marginal cost savings, Proposition 1 continues to apply as long as these marginal cost savings do not induce all outsider firms to exit the market (i.e., as long

as there are still some outsider firms which continue to make zero profits).¹³ The result depends on the monotonicity of outsider firms' profits in $\sum_{j \neq i} f_j(p_j)$. This monotonicity does not rely on the actual values of the merging firms' prices.

Second, Proposition 1 does not require the $f_j(p_j)$ function to be symmetric among *all* the firms in the market. Although the IIA property implies identical cross-price elasticities, it does not imply that all varieties are equally attractive to the consumers and all firms have equal market shares in equilibrium.¹⁴ However, the result does require at least one outsider firm to earn zero profits both before and after the merger. This would be satisfied if the entrants and at least one of the outsider firms were symmetric.¹⁵

Third, Proposition 1 applies even if there are multi-product firms in the pre-merger equilibrium (perhaps as a result of previous mergers), and even if the merger involves multi-product firms. It is sufficient for the result stated in Proposition 1 to hold that there are some single-product outsiders in the pre-merger equilibrium, and that the entrants are single-product firms.

Fourth, the size of the merger plays no role in the consumer welfare result stated in Proposition 1. Hence, although large mergers induce more entry, more entry is required to offset the negative effects of large mergers. In demand systems which satisfy the IIA property, these two effects exactly offset each other.

Finally, the consumer welfare result does not require quasilinearity. The existence of income effects does not change the analysis so long as the indirect utility function is separable in income.¹⁶

The IIA property expressed in Assumption 4 is common to some demand systems that

¹³This result parallels the result of Davidson and Mukherjee (2006) who find that mergers which result in marginal cost savings in homogeneous products markets have no long run effect on consumer welfare.

¹⁴For instance, consider the asymmetric logit model studied by Werden and Froeb (1994). They assume that the conditional indirect utility of consumer i associated with the choice of product j is given by $U_{ij} = \alpha_j - \beta p_j + e_{ij}$.

¹⁵This will not always be the case, but it may apply in many industries where there are one or two "special" firms facing competition from other symmetrically differentiated firms.

¹⁶An important example of an IIA system with income effects is the symmetric Almost Ideal Demand System (AIDS). See Epstein and Rubinfeld (2001) for an argument for the use of a calibrated version of the AIDS system in merger analysis.

are frequently used in the literature.¹⁷ Consider, for example, the following logit demand system derived by Anderson et al., (1992).¹⁸

$$q_i = \frac{\exp\left(\frac{-p_i}{\mu}\right)}{\sum_j \exp\left(\frac{-p_j}{\mu}\right) + \exp\left(\frac{V_0}{\mu}\right)},$$

where μ represents consumers' taste for variety and V_0 is the net benefit a consumer receives from consuming the outside option. Alternatively, consider the following CES demand system.

$$q_i = \frac{p_i^{-\frac{(\mu+1)}{\mu}}}{\sum_j p_j^{-\frac{1}{\mu}}},$$

where again μ represents consumers' taste for variety. Both of these demand systems satisfy Assumptions 1-4. Hence, we can state the following corollary.

Corollary 1 *In logit and CES demand systems if the number of firms, N , is a continuous variable, mergers have no long run effect on consumer welfare.*

We analyze the effects of mergers with logit and CES demand systems in more detail in Appendix B and C respectively. We show that although the consumer welfare results are the same, they are driven by different effects. In the CES case, the benefits of increased variety are exactly offset by a reduction in the total quantity consumed of the differentiated products whereas in the logit case, the benefits of increased variety are exactly offset by a reduction in the quantity consumed of the numeraire.

3.1 Social Welfare

Defining social welfare as the sum of consumer welfare and profits, we can immediately see the implications of the above analysis for social welfare. Since there is no change in consumer welfare with IIA preferences, the change in social welfare reduces to the change in the profit levels. Because the outsiders are subject to the zero profit condition, this reduces

¹⁷ Anderson et al., (1992) make this point about the logit demand system on p. 42 and about the CES demand system on p. 86.

¹⁸See Ch. 7.4 in Anderson et al., (1992).

further to the change in profits for the merged firms. Hence, if the merger is profitable, social welfare rises, and if the merger is unprofitable, social welfare falls.

From Deneckere and Davidson (1985) we know that the merging firms earn less than the outsiders because of the free-rider effect. Hence, without fixed or marginal cost savings, the merger is unprofitable, and if it were to occur anyway, social welfare would fall as a result of the merger. If, however, the merger generates sufficiently large cost savings, the merger is profitable and social welfare rises.

3.2 Discrete N

It is possible to draw conclusions from the continuous N analysis for the discrete N case. The long run equilibrium condition for discrete N before the merger is that

$$\pi_{nm}(N) \geq 0 > \pi_{nm}(N+1).$$

After the merger, the condition is

$$\pi_o(N+E) \geq 0 > \pi_o(N+E+1).$$

In the case of discrete N , it is possible to have a profitable merger even without fixed cost savings because there may be insufficient entry. However, any profitable merger that does not generate fixed cost savings must harm consumers. This is because if the merger is profitable for the merging firms, it must also increase the profits of the outsiders due the free rider effect. In other words, it must be the case that

$$\pi_o(N+E) > \pi_{nm}(N).$$

For this to happen there must be less entry than that described in the analysis above such that in the post-merger long run equilibrium

$$p_o > p_{nm}.$$

Since consumer surplus is unchanged in the continuous case, and since there is less entry and higher prices in the discrete case, consumer welfare must fall.

If, however, the merger results in fixed or marginal cost savings, it is possible to have a profitable merger that benefits consumers. This could occur through the operation of the integer constraint. That is, suppose in the pre-merger equilibrium the integer constraint operates to give outsider firms profits that are higher than the profits they earn in the post-merger equilibrium. If this were the case, the amount of merger-induced entry would be more than sufficient to counteract the negative effects of the merger. Hence, consumer welfare would rise as a result of the merger.

It is clear, then, that cost savings are a necessary but not sufficient condition for consumer welfare to rise. This result has interesting implications for the antitrust treatment of mergers. It is unreasonable to expect an antitrust authority to have sufficient knowledge to predict the operation of the integer constraint. This means that, from the perspective of an antitrust authority or a court, cost savings generated by a merger are an indication that the merger may benefit consumers. Importantly, however, from a consumer welfare perspective, it is irrelevant whether these are fixed or marginal cost savings. Hence, the analysis provides a potential justification for favorable antitrust treatment of mergers generating fixed cost savings on the basis of consumer welfare analysis without resorting to social surplus arguments. Fixed cost savings may be relevant to this assessment, even though they do not directly affect prices.¹⁹

This discrete N analysis has implications for Werden and Froeb's (1998) conclusion that mergers are unlikely to induce entry. Especially in the case of mergers between multi-product firms or large mergers, the amount of entry that is induced in a continuous N long run equilibrium is likely to be larger and, hence, the integer constraint will operate to allow for a strictly positive amount of entry. In such cases, there is no strong *a priori* reason to expect the integer constraint to operate in a way that results in insufficient entry. It could equally result in more than sufficient entry to counteract the negative effects of the merger.

¹⁹This is especially important if antitrust treatment of mergers relies upon a consumer welfare analysis. For instance, under s 50 of the Trade Practices Act (1975) in Australia, once a merger dispute reaches the courts, it is not possible to justify the merger based on social welfare analysis.

4 Long run effects of mergers with linear demand

Since differentiated products demand systems do not universally satisfy the IIA property, it is important to consider the long run effects of mergers outside the class of demand systems that satisfy the IIA property. Hence, in this section we analyze the effects of mergers using linear demand systems. Since our goal is to show that mergers may harm consumer welfare in demand systems that do not satisfy the IIA property, we consider mergers that do not generate marginal cost savings.

Linear demand systems are important to consider for two reasons. First, they can be understood as first-order Taylor approximations to a larger class of non-linear demand systems. If firms are boundedly rational, they may actually use such an approximation to make their pricing decisions. If this is the case, using a linear demand system is not an approximation of their behavior at all.²⁰ Second, linear demand systems are frequently used in the literature. The following analysis shows that mergers with linear demand systems can lead to substantially different welfare conclusions from those that satisfy the IIA property.²¹

One of the commonly used linear demand systems in differentiated products literature is due to Shubik (1980). For example, Deneckere and Davidson (1985) use this linear demand system in their analysis of short run effects of mergers. In Appendix E, we show that in this system all mergers without marginal cost savings harm consumers in the long run. This is because consumers do not value variety in Shubik (1980). To see this, it is necessary to provide a precise definition of taste for variety. We follow Benassy's (1996) definition of taste for variety because of its intuitive appeal and its generality. In this definition, taste for variety is given by

$$T = \frac{U(q, N)}{U(Nq, 1)},$$

where $U(q, N)$ is the utility that a consumer derives from consuming q units of N symmetrically differentiated varieties. Thus, the numerator is the utility a consumer derives from

²⁰See Ottaviano and Thisse (1999) for a discussion of this point.

²¹The results based on the IIA property derived above may also change if it is assumed, as in Section 4.4 of Dixit and Stiglitz (1975) and Benassy (1996), that variety has a public good characteristic. We show this in Appendix D.

consuming a certain total quantity of the differentiated products, divided symmetrically amongst the N varieties in the market. The denominator represents the utility the consumer derives from consuming the same total quantity of a single variety. Hence, if $T = 1$, consumers have no taste for variety.

Applying this definition to Shubik's (1980) demand system, we get $T = 1$. This means that although consumers have what might be called a love of symmetry, they have no taste for variety. That is, they prefer to spread consumption over all available varieties, but they derive no additional utility from an increase in the number of available varieties. At symmetric prices, the consumer's utility is similar to the utility of a consumer in a homogeneous products market in that it does not depend on the number of varieties consumed. However, at asymmetric prices, consumer behavior is similar to that of a consumer in a differentiated products market in that they choose to consume positive quantities of the different varieties.

To see the implications of the fact that $T = 1$ in this demand system for the effect of variety on welfare, we show in Appendix E that at symmetric prices, utility depends on total quantity only and total quantity depends on the symmetric price level only. Therefore, introducing a new variety at a price above the incumbents' symmetric price level harms consumer welfare. In this sense, variety is a public bad in Shubik's (1980) demand system.²²

Consumers' attitudes towards variety in Shubik's (1980) demand system play a central role in the consumer welfare-reducing effects of mergers. Without a love of symmetry we would have a model with homogeneous products, in which case a merger would have no effect on prices and hence would not induce entry. However, because consumers have no corresponding love of variety, they do not benefit from the new varieties except insofar as they mitigate the price increasing effects of the merger. Since entry fails to completely reverse these price increases, consumers are harmed by the merger.

What distinguishes the welfare analysis of mergers in differentiated products markets from homogeneous products markets is the benefit that *new* variety brings to consumers.

²²Two related problems that make Shubik (1980) undesirable to use in a model with entry are its violation of WARP with respect to variety and the discontinuity of the indirect utility function. We elaborate on these in Appendix F.

Although Shubik's (1980) demand system features product differentiation, it lacks this important property. Without taste for variety, the only benefit that entry can bring is more intense price competition. Hence, Shubik's (1980) demand system is not sufficient to capture the welfare-improving effects of merger-induced entry in differentiated products markets.

4.1 Ottaviano and Thisse's (1999) linear demand system

In this section, we consider the consumer welfare effects of mergers in a linear demand system due to Ottaviano and Thisse (1999), where consumers do display a taste for variety. We show that this linear demand system still produces consumer welfare results which differ substantially from the IIA case.

Suppose the representative consumer's preferences can be described by the following utility function.

$$U(q_0; q_i, i \in \{1, \dots, N\}) = K + \alpha \sum_{i=1}^N q_i - \frac{\beta}{2} \sum_{i=1}^N q_i^2 - \frac{\gamma}{2} \sum_{i=1}^N \sum_{j \neq i}^N q_i q_j + q_0, \quad (3)$$

where $\alpha > 0$ and $\beta > \gamma > 0$. q_i stands for the quantity of variety i and q_0 stands for the quantity of the numeraire good. α is a measure of the size of the market for the differentiated goods. The absolute value of the fourth term in (3) increases as consumption becomes more symmetric. Hence, as γ increases, the representative consumer would have a preference for concentrating consumption. On the other hand, the absolute value of the third term in (3) increases as consumption becomes more concentrated. Hence, as β increases, the representative consumer would have a preference for smoothing consumption.

Applying Benassy's (1996) definition of taste for variety in the context of this quadratic utility function allows us to identify a condition on the parameters γ and β . Suppose Q is the total quantity consumed. If the consumer consumes only a single variety, (3) becomes

$$U = K + \alpha Q - \frac{\beta}{2} Q^2 - q_0. \quad (4)$$

If the consumer consumes N varieties in quantity q each such that $Q = Nq$, (3) becomes

$$U = K + \alpha Nq - \frac{\beta}{2} Nq^2 - \frac{\gamma}{2} N(N-1)q^2 - q_0. \quad (5)$$

The consumer has taste for variety whenever (5) dominates (4). Substituting for $Q = Nq$ in (4) and simplifying yields

$$\beta > \gamma.$$

Hence, as γ gets smaller for a given value of β , or as β gets larger for a given value of γ , the preference for variety gets stronger.

This quadratic utility function generates demand functions of the form

$$q_i = \frac{1}{\beta + \gamma(N-1)} \left[\alpha - \frac{\beta + \gamma(N-2)}{(\beta - \gamma)} p_i + \frac{\gamma}{(\beta - \gamma)} \sum_{j \neq i} p_j \right], \quad i \in [1, N]. \quad (6)$$

Since we are considering single-product firms and each firm produces a different variety, N represents both the number of varieties and the number of firms in the market. Note that when $\gamma = 0$, the demand for good i becomes independent of the prices of the other goods in the market. Moreover, as $(\beta - \gamma)$ increases, q_i becomes less sensitive to changes in own price and the prices of other goods. Hence, as taste for variety increases, firm i 's market power increases.

Assuming $c = 0$ without loss of generality, each firm i maximizes

$$\pi_i = p_i \cdot q_i(p_i) \quad (7)$$

with respect to p_i taking the prices charged by the other firms as given. Since all firms are symmetric before a merger takes place, solving for the equilibrium price level yields

$$p_{nm} = \alpha \left[\frac{(\beta - \gamma)}{\gamma(N-3) + 2\beta} \right]. \quad (8)$$

Equilibrium quantity can be expressed in terms of equilibrium price as follows.

$$q_{nm} = \frac{p_{nm}}{(\beta - \gamma)} \left[\frac{\beta + \gamma(N-2)}{(\beta + \gamma(N-1))} \right].$$

Substituting for these equilibrium price and quantity levels in (7) yield

$$\pi_{nm} = \alpha^2 \frac{(\beta - \gamma)(\beta + \gamma(N-2))}{(\gamma(N-3) + 2\beta)^2 (\beta + \gamma(N-1))} - F \quad (9)$$

as the equilibrium profit level for each firm. Note that $p_{nm} \rightarrow 0$ and $\pi_{nm} \rightarrow -F$ as $\beta \rightarrow \gamma$. That is, when the representative consumer has no taste for variety, competition drives prices and profits down to zero. Hence, unlike in the case of Shubik (1980), without a taste for variety, we have a model of homogeneous goods.

If M firms merge and set their prices collectively, the first order conditions yield

$$p_o = a \left[\frac{(\beta - \gamma)}{\gamma(N + E + M - 3) + 2\beta} \right] + \left[\frac{\gamma}{\gamma(N + E + M - 3) + 2\beta} \right] Mp_m$$

as the best response function of the outside firms and

$$p_m = a \left[\frac{(\beta - \gamma)}{2(\beta + \gamma(N + E - M - 1))} \right] + \left[\frac{\gamma}{2(\beta + \gamma(N + E - M - 1))} \right] (N + E - M)p_o$$

as the best response function of the merging firms. Solving these best response functions simultaneously gives us the equilibrium prices. Substituting the equilibrium prices into the demand functions yields the following equilibrium quantities as functions of the equilibrium prices.

$$\begin{aligned} q_o &= \frac{p_o}{(\beta - \gamma)} \left[\frac{\beta + \gamma(N + E - 2)}{(\beta + \gamma(N + E - 1))} \right] \\ q_m &= \frac{p_m}{(\beta - \gamma)} \left[\frac{\beta + \gamma(N + E - M - 1)}{(\beta + \gamma(N + E - 1))} \right]. \end{aligned}$$

To compare the prices charged by the outsiders before and after the merger, we use the long run equilibrium condition. Using the fact that

$$\pi_o = \frac{p_o^2}{(\beta - \gamma)} \left[\frac{\beta + \gamma(N + E - 2)}{(\beta + \gamma(N + E - 1))} \right] - F = \frac{p_{nm}^2}{(\beta - \gamma)} \left[\frac{\beta + \gamma(N - 2)}{(\beta + \gamma(N - 1))} \right] - F = \pi_{nm},$$

we get

$$\frac{p_o^2}{p_{nm}^2} = \frac{(\beta + \gamma(N - 2))(\beta + \gamma(N + E - 1))}{(\beta + \gamma(N - 1))(\beta + \gamma(N + E - 2))}.$$

It is straightforward to see that this ratio is < 1 and hence $p_o < p_{nm}$. This is in contrast with the demand systems that satisfy the IIA property, where, as we show in the proof of Proposition 1, $p_o = p_{nm}$. In this linear demand system, the outsider firms charge a price below the pre-merger equilibrium price because although they face less intense competition

from the merging firms, they face tougher competition overall due to the new entrants. Using the long run equilibrium condition, this implies $q_o > q_{nm}$.

Having solved for firm behavior, we would like to determine the effect of the merger on consumers. We begin by showing that in this demand system, the total quantity consumed of the differentiated products increases as a result of the merger.²³ This result may seem surprising since one of the primary antitrust concerns with mergers is that they reduce total output. In this case, total output rises because although entry results in business stealing, some of the demand for the entrants' products is "stolen" away from the numeraire. Hence, entry expands the market for the differentiated products.

Unfortunately, due to the intractability of the equations, we have not been able to show analytically the net effect of a merger on consumer welfare in this demand system. However, we have undertaken some numerical analysis which illustrates that mergers can harm both consumer and social welfare in the long run. Although this is not a general proof, we have not found any combination of parameter values for which consumer and social welfare is not harmed.

Since mergers are of most concern to antitrust authorities when they occur in relatively concentrated industries, consider the following parameter values which generate $N = 8.37$: $\alpha = 70$, $\beta = 20$, $\gamma = 5$, and $F = 15$. Table 1 reports the consumer welfare effects of mergers of varying sizes. Column 3 of Table 1 shows the pre-merger consumer welfare level, which is also equal to the pre-merger social welfare level because firms earn zero profits. As can be seen from the table, the change in consumer welfare is always negative and deteriorates as the size of the merger increases.²⁴

²³See Appendix G for the proof. In contrast, total quantity consumed of the differentiated products remains unchanged in the logit demand system and decreases in the CES and Shubik's (1980) linear demand systems.

²⁴The change in social welfare depends on the amount of fixed cost savings that the merger generates. For sufficiently large fixed cost savings the merger may improve social welfare.

Table1: Welfare effects of mergers in a linear demand system

M	E	CW_{nm}	ΔCW
2.07	0.11	216.93	-0.72
4.06	0.66	216.93	-3.68
6.05	1.61	216.93	-7.92
8.03	2.89	216.93	-12.37

Table 2 illustrates the impact of a change in β on the consumer welfare effects of mergers of a given size. For fixed values of γ , β captures consumers' taste for variety. The parameter values are the same as before except that we set $M = 0.6N$. Again, the change in consumer welfare is always negative. Interestingly, it is non-monotonic in β . To see why, note that β has two kinds of effect on firms' profits. The first effect is that, as can be seen from (3), a higher β corresponds to a higher taste for variety and, hence, higher market power for the firms. The second effect is that an increase in β increases the relative attractiveness of the numeraire compared with the differentiated products. The numerical analysis suggests that for low values of β , the first effect dominates. Hence, an increase in β results in increases in N and E . Higher market power also reduces the consumer welfare level in the pre-merger equilibrium. The net result is that an increase in β exacerbates the negative effects of the merger.

For high values of β , the second effect of β on firm profits dominates. This negative effect causes both N and E to decrease with β . The results indicate that for high values of β , consumer welfare in the pre-merger equilibrium still decreases with β , but the negative consumer welfare effects of the merger diminish with β .

Table 2: Impact of taste for variety in a linear demand system

β	N	E	CW_{nm}	ΔCW
6.14	4.23	0.57	393.29	-2.30
10.30	6.64	0.96	314.97	-4.56
20.91	8.42	1.06	210.31	-6.66
30.45	8.36	0.90	153.44	-5.04
40	7.64	0.67	111.62	-3.91

5 Conclusion

Antitrust authorities regard the possibility of entry as an important determinant of whether a proposed merger will be harmful to consumer and social welfare. We have analyzed whether entry can mitigate the negative effects of mergers in non-localized, differentiated products markets with Bertrand competition.

We show that if the demand system satisfies the IIA property and the number of firms is treated as a continuous variable, mergers in differentiated products markets have no long run effect on consumer welfare regardless of whether the merger generates marginal cost savings. If the number of firms is treated as a discrete variable, then mergers can harm or improve consumer welfare in the long run depending on whether there is insufficient or more than sufficient merger-induced entry due to the operation of the integer constraint. However, we show that if a profitable merger does not generate any cost savings, it must harm consumer welfare. Hence, cost savings, either fixed or marginal, are a positive indicator of the consumer welfare effects of mergers.

If the demand system does not satisfy the IIA property, mergers in differentiated products markets can harm consumer welfare in a long run equilibrium. Hence, our analysis reveals that unlike in the case of homogeneous products markets, determining the properties of consumer preferences is crucial to the antitrust analysis of mergers in differentiated products markets. The choice of the demand system to be used in the antitrust analysis can affect not only the predicted quantitative effects of a merger, but also the qualitative effects. This implies care should be taken when choosing a demand system to simulate the long run effects of mergers because the model selection can essentially pre-determine the results.

Moreover, it is worth emphasizing that when analyzing the long run effects of mergers in differentiated products markets, it is important to start with a demand system where consumers benefit from new variety. This is because in differentiated goods markets, the effect of entry is not only to decrease prices, but also to introduce additional variety which is of benefit to consumers. We have argued that this makes Shubik's (1980) system inap-

appropriate for analyzing the long run effects of mergers in differentiated products markets since it rules out by assumption the variety effect that distinguishes differentiated products markets from homogeneous products markets.

In conclusion, our analysis reveals that the long run effects of mergers in differentiated products markets are substantially more complex than their effects in the short run or in homogeneous products markets. For future research, we suggest that more systematic research is required into the modelling of taste for variety. Since consumer attitudes towards variety can have profound effects on the analysis of important policy questions, it would be valuable to develop a theory of taste for variety that can explain the relationship between the structure of the various functional representations of consumer preferences and primitive consumer preferences in relation to the addition of new varieties. Once this relationship is better understood, researchers and analysts will be better equipped to choose and defend their choices of demand systems.

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Appendix

A Proof of Lemma 1

Each outsider firm i solves the following problem.

$$\max_{p_i} \pi_i = (p_i - c) q_i \left(p_i, \sum_{j \neq i} f_j(p_j), y \right) - F.$$

Let π_i^* be the maximum value of this function. By the envelope theorem we have

$$\frac{\partial \pi_i^*}{\partial \left(\sum_{j \neq i} f_j(p_j) \right)} = \left(p_i^* \left(\sum_{j \neq i} f_j(p_j) \right) - c \right) \cdot \frac{\partial q_i^* \left(p_i^* \left(\sum_{j \neq i} f_j(p_j) \right), \sum_{j \neq i} f_j(p_j), y \right)}{\partial \left(\sum_{j \neq i} f_j(p_j) \right)}, \quad (1)$$

where p_i^* is the solution to firm i 's maximization problem taking the prices of all the other firms as given. Hence, π_i^* is monotonic in $\sum_{j \neq i} f_j(p_j)$ if q_i is monotonic in $\sum_{j \neq i} f_j(p_j)$.

Note that

$$\frac{\partial q_i}{\partial p_j} = \frac{\partial q_i}{\partial \left(\sum_k f_k(p_k) \right)} \cdot \frac{\partial f_j(p_j)}{\partial p_j}.$$

Since

$$\frac{\partial \left(\sum_k f_k(p_k) \right)}{\partial \left(\sum_{k \neq i} f_k(p_k) \right)} = 1,$$

we have

$$\frac{\partial q_i}{\partial p_j} = \frac{\partial q_i}{\partial \left(\sum_{k \neq i} f_k(p_k) \right)} \cdot \frac{\partial f_j(p_j)}{\partial p_j}. \quad (2)$$

From Assumption 3, we know that this expression is > 0 . Hence, q_i is monotonic in $\sum_{k \neq i} f_k(p_k)$ if $f_j(p_j)$ is monotonic in p_j .

Assumption 2 states that

$$\frac{\partial V}{\partial \left(\sum_j f_j(p_j) \right)} \cdot \frac{\partial f_i(p_i)}{\partial p_i} < 0$$

for all p_i, p_j . This implies

$$\text{sign} \left\{ \frac{\partial f_i(p_i)}{\partial p_i} \right\} = \text{sign} \left\{ \frac{\partial f_j(p_j)}{\partial p_j} \right\}$$

for all values of p_i, p_j .

Since $f_i(p_i)$ and $f_j(p_j)$ share no common arguments and since their slopes have the same sign as each other regardless of the values of p_i and p_j , they must be monotonic functions.

Using (2) we can conclude that q_i changes monotonically with $\sum_{j \neq i} f_j(p_j)$ and, therefore, π_i^* changes monotonically with $\sum_{j \neq i} f_j(p_j)$.

We next prove that $\text{sign} \left\{ \frac{\partial \pi_i^*}{\partial \sum_{j \neq i} f_j(p_j)} \right\} = \text{sign} \left\{ -\frac{\partial v}{\partial \sum_{j \neq i} f_j(p_j)} \right\}$. (1) implies that

$$\text{sign} \left\{ \frac{\partial \pi_i^*}{\partial \sum_{j \neq i} f_j(p_j)} \right\} = \text{sign} \left\{ \frac{\partial q_i^* \left(p_i^* \left(\sum_{j \neq i} f_j(p_j) \right), \sum_{j \neq i} f_j(p_j), y \right)}{\partial \left(\sum_{j \neq i} f_j(p_j) \right)} \right\}.$$

By Assumption 3 we have

$$\frac{\partial q_i^* \left(p_i^* \left(\sum_{j \neq i} f_j(p_j) \right), \sum_{j \neq i} f_j(p_j), y \right)}{\partial p_j} = \frac{\partial q_i^* \left(p_i^* \left(\sum_{k \neq i} f_k(p_k) \right), \sum_{k \neq i} f_k(p_k), y \right)}{\partial \left(\sum_{k \neq i} f_k(p_k) \right)} \cdot \frac{\partial f_j(p_j)}{\partial p_j} > 0$$

and by Assumption 2 we have

$$\frac{\partial v}{\partial p_j} = \frac{\partial v}{\partial \left(\sum_{k \neq i} f_k(p_k) \right)} \cdot \frac{\partial f_j(p_j)}{\partial p_j} < 0.$$

Since these imply

$$\text{sign} \left\{ \frac{\partial q_i^* \left(p_i^* \left(\sum_{k \neq i} f_k(p_k) \right), \sum_{k \neq i} f_k(p_k), y \right)}{\partial \left(\sum_{k \neq i} f_k(p_k) \right)} \right\} = \text{sign} \left\{ -\frac{\partial v}{\partial \left(\sum_{k \neq i} f_k(p_k) \right)} \right\},$$

we have

$$\text{sign} \left\{ \frac{\partial \pi_i^*}{\partial \sum_{j \neq i} f_j(p_j)} \right\} = \text{sign} \left\{ -\frac{\partial v}{\partial \sum_{j \neq i} f_j(p_j)} \right\}.$$

B Logit Demand

The specific logit demand framework we consider is that with an outside alternative, as set out in Anderson et al., (1992), Ch. 7.4.²⁵ This is a discrete choice model where heterogeneous consumers choose to consume a single unit of output from one of a number of firms each producing a symmetrically differentiated product. Indirect utility for an individual consuming good i , conditional on that individual choosing to consume good i , is

$$\tilde{V}_i = y - p_i + \varepsilon_i,$$

where y stands for the income level and ε_i represents an idiosyncratic match value between the consumer and good i . The ε_i 's are identically, independently, and double exponentially distributed. Conditional indirect utility for the numeraire is

$$\tilde{V}_0 = y + V_0 + \varepsilon_0.$$

Given ε_i and the prices, individuals choose to consume exactly one unit of the good that yields the highest indirect utility. Normalizing the number of consumers to unity, the expected demand function of firm i is the probability that a consumer would choose to purchase its product. As shown in Anderson et al., (1992), this demand function is given by

$$q_i = \frac{\exp\left(\frac{-p_i}{\mu}\right)}{\sum_j \exp\left(\frac{-p_j}{\mu}\right) + \exp\left(\frac{V_0}{\mu}\right)}, \quad (3)$$

where μ is proportional to the variance of ε_i . Hence, μ is a measure of consumer heterogeneity, or from a representative consumer perspective, taste for variety.

²⁵Anderson et al., (1992) show that both the logit and CES systems can be derived as representative consumer, random utility, and spatial models. We analyze the logit system in a random utility framework and the CES system in a representative consumer framework, as is standard in the literature.

Before the merger, firm i solves

$$\max_{p_i} (p_i - c) q_i,$$

where q_i is given in (3). The first order condition gives us²⁶

$$p_i = c + \frac{\mu}{(1 - q_i)}. \quad (4)$$

In the long run equilibrium,

$$(p_i - c) q_i = F.$$

Substituting this into the expression in (4) shows that the long run pre-merger price is

$$p_{nm} = c + \mu + F. \quad (5)$$

Since the first order condition is a function of the firm's own price and quantity only, the operation of the long run equilibrium condition makes the consideration of the strategic interactions between the firms irrelevant to the determination of the long run equilibrium price. In this sense, the long run equilibrium with a logit demand system has characteristics of a monopolistically competitive market, despite the fact that the firms do take into account the impact of their prices on the market price index, as shown in the denominator of (3).

After the merger, the first order condition for the outsiders' maximization problem is identical to that given in (4) because it is a function of own price and quantity only. This implies that in the post-merger long run equilibrium $p_o = p_{nm}$ for all $N + E - M$ outsiders. If the outsiders charge the same prices and earn the same profits before and after the merger, they must produce the same quantities too. Due to joint profit maximization, the merging firms charge a higher price. Hence, we get $p_m > p_o = p_{nm}$.

To analyze the impact of the merger on consumer welfare, we can apply Roy's Identity to (3). Integrating (3) we get

$$CW = \mu \ln \left[\sum_{i=1} \exp \left(\frac{-p_i}{\mu} \right) + \exp \left(\frac{V_0}{\mu} \right) \right].$$

²⁶ As shown by Anderson et al., (1992) on p. 222, the second order condition holds wherever the first order condition does, implying that the profit function is strictly quasiconcave.

Evaluating this expression at the pre-merger and post-merger long run equilibria gives us the change in consumer welfare.

$$\Delta CW = \mu \left[\ln \left[(N + E - M) \exp \left(\frac{-p_o}{\mu} \right) + M \exp \left(\frac{-p_m}{\mu} \right) + \exp \left(\frac{V_0}{\mu} \right) \right] - \ln \left[N \exp \left(\frac{-p_{nm}}{\mu} \right) + \exp \left(\frac{V_0}{\mu} \right) \right] \right]. \quad (6)$$

To show that $\Delta CW = 0$, we can use the fact that $q_{nm} = q_o$. From (3) we get

$$\frac{\exp \left(\frac{-p_{nm}}{\mu} \right)}{N \exp \left(\frac{-p_{nm}}{\mu} \right) + \exp \left(\frac{V_0}{\mu} \right)} = \frac{\exp \left(\frac{-p_o}{\mu} \right)}{(N + E - M) \exp \left(\frac{-p_o}{\mu} \right) + M \exp \left(\frac{-p_m}{\mu} \right) + \exp \left(\frac{V_0}{\mu} \right)}.$$

Setting $p_o = p_{nm}$ on the right hand side makes the numerators equal to each other. We get

$$N \exp \left(\frac{-p_{nm}}{\mu} \right) = (N + E - M) \exp \left(\frac{-p_o}{\mu} \right) + M \exp \left(\frac{-p_m}{\mu} \right). \quad (7)$$

Substituting this expression in (6) gives us the result that the merger has no impact on consumer welfare.

We can decompose the effect of the merger in the following way. Consider first what happens to total quantity. Change in total quantity is given by

$$\Delta Q = (N + E - M) q_o + M q_m - N q_{nm}.$$

Using (7) and solving for E yields

$$E = M \left(1 - \exp \left(-\frac{p_m - p_{nm}}{\mu} \right) \right). \quad (8)$$

Substituting in for E from (8) and simplifying we get

$$\Delta Q = M q_{nm} \left[-\exp \left(-\frac{p_m - p_{nm}}{\mu} \right) + \frac{\exp \left(\frac{-p_m}{\mu} \right)}{\exp \left(\frac{-p_{nm}}{\mu} \right)} \right] = 0.$$

Hence, after the merger, with no change in total quantity, the same amount of total consumption is distributed over more goods. Since $p_m > p_o = p_{nm}$, this implies that total expenditure increases. However, although consumption is asymmetric after the merger in that $q_m < q_o = q_{nm}$, when pre-merger and post-merger consumption of all $N + E$ goods is considered, consumption is more symmetric post-merger. Since consumers have heterogeneous tastes, they benefit from the increased variety available to them because they are able to find a better match.²⁷ The consumer welfare result above implies that this benefit

²⁷See Anderson et al., (1992), Ch. 7.4 for a technical discussion.

from reduced asymmetry is exactly offset by the increase in total expenditure.

C CES Demand

In this section, we consider a quasilinear utility function with a CES subutility. CES demand functions are closely related to logit demand functions. In fact, as Anderson et al., (2001) show, the two demand systems can be modeled as special cases of a more general demand system. We show in this section that although both the logit and CES demand systems yield the same consumer welfare result, the reasons behind the result in the two models are different.

Direct utility for the representative consumer in this framework is

$$U = (\mu + 1) \ln \left(\sum q_i^{\frac{1}{\mu+1}} \right) + q_o, \quad (9)$$

where μ again represents the taste for variety. That is, as μ increases, consumers prefer spreading their consumption over the available varieties in the market.

The representative consumer maximizes utility given in (9) subject to the budget constraint

$$\sum_{i=1}^N p_i q_i + q_o = 1$$

where, without loss of generality, income is set equal to 1. This optimization problem leads to demand functions of the form

$$q_i = \frac{p_i^{-\frac{\mu+1}{\mu}}}{\sum p_j^{-\frac{1}{\mu}}}. \quad (10)$$

Firm i maximizes $(p_i - c) q_i$. The pre-merger first order condition is

$$\frac{1}{(\mu + 1)} - 1 + (p_i - c) \left(\frac{1}{p_i} - \frac{q_i}{(\mu + 1)} \right) = 0.$$

Applying the long run equilibrium condition we get

$$p_{nm} = \frac{c(\mu + 1)}{(1 - F)}. \quad (11)$$

Once again, after the merger, the first order condition for the outsiders is identical to their first order condition before the merger. Hence, in the post-merger long run equilibrium, $p_o = p_{nm}$.

To get an expression for the consumer welfare function, we can substitute the demand functions specified in (10) into the direct utility function specified in (9). This gives us

$$CW = 1 + \mu \ln \left(\sum p_i^{-\frac{1}{\mu}} \right).$$

Evaluating this consumer welfare function at the pre-merger and post-merger long run equilibria we get

$$\Delta CW = \mu \left[\ln \left((N + E - M) p_o^{-\frac{1}{\mu}} + M p_m^{-\frac{1}{\mu}} \right) - \ln \left(N p_{nm}^{-\frac{1}{\mu}} \right) \right]. \quad (12)$$

Again, using the fact that $q_{nm} = q_o$, we can show that $\Delta CW = 0$. We have

$$\frac{p_{nm}^{-\frac{1}{\mu}}}{N p_{nm}^{-\frac{1}{\mu}}} = \frac{p_o^{-\frac{1}{\mu}}}{(N + E - M) p_o^{-\frac{1}{\mu}} + M p_m^{-\frac{1}{\mu}}}.$$

Since $p_{nm} = p_o$, this implies

$$N p_{nm}^{-\frac{1}{\mu}} = (N + E - M) p_o^{-\frac{1}{\mu}} + M p_m^{-\frac{1}{\mu}}. \quad (13)$$

Substituting in (12) gives us the result. Hence, the long run change in consumer welfare is as in the logit case.²⁸

However, in the CES case, the decomposition of the effect of the merger on consumer welfare is different from the logit case. To see this, note that the change in total quantity can be expressed as

$$\Delta Q = (N + E - M) q_o + M q_m - N q_{nm}.$$

We can solve for the equilibrium number of entrants, E , from (13).

$$E = M \left(1 - \left(\frac{p_{nm}}{p_m} \right)^{\frac{1}{\mu}} \right). \quad (14)$$

²⁸With a similar analysis, Anderson et al., (1997) show that the privatization of a public social surplus maximizing firm in a mixed oligopoly with a CES demand system has no long run effect on consumer welfare.

Substituting yields

$$\Delta Q = M q_{nm} \left(\frac{p_{nm}}{p_m} \right)^{\frac{1}{\mu}} \left(\frac{p_{nm}}{p_m} - 1 \right)$$

This expression < 0 since $p_{nm} < p_m$.

Using (10) we can show that total expenditure on the differentiated products, both pre-merger and post-merger, is

$$\sum p_i \frac{p_i^{-\frac{(\mu+1)}{\mu}}}{\sum p_j^{-\frac{1}{\mu}}} = \frac{\sum p_i^{-\frac{1}{\mu}}}{\sum p_j^{-\frac{1}{\mu}}} = 1.$$

Hence, in the CES case, consumption of the differentiated goods falls and consumption of the numeraire remains unchanged.²⁹ This implies that in the CES case the benefits of increased variety is exactly offset by the reduction in the total quantity consumed of the differentiated products, whereas in the logit system the benefits of increased variety is exactly offset by the reduction in the quantity of the numeraire.

D Variety as a public good

The results based on the IIA property derived above may change if it is assumed, as in Section 4.4 of Dixit and Stiglitz (1975) and Benassy (1996), that variety has a public good characteristic. Suppose indirect utility depends directly on the number of varieties available, not just indirectly through the level of consumption of those varieties. The simplest way to model this would be to incorporate it into a CES framework, which yields

$$U = N^\phi + (\mu + 1) \ln \left(\sum q_i^{\frac{1}{(\mu+1)}} \right) + q_o$$

where $\phi > 0$. As Dixit and Stiglitz (1975) argue, this may make sense if variety is valuable to accommodate potential future changes in consumer tastes or if individuals prefer to consume goods that are different to those consumed by other individuals in order to maintain a sense of individuality.

Because of the way N^ϕ enters the utility function, it does not affect optimal consumer or firm behavior. Hence, it has a public good characteristic. This implies that all of the

²⁹Total expenditure is always constant in CES demand systems.

market outcomes we derived in the CES analysis above still hold. Consumer welfare is given by

$$V = 1 + N^\phi + \mu \ln \left(\sum p_i^{-\frac{1}{\mu}} \right).$$

Because the market outcomes are the same as in the CES case, change in consumer welfare reduces to

$$(N + E)^\phi - N^\phi > 0.$$

Hence, the public good characteristic of variety makes mergers desirable for consumers in the long run because the entry they induce provides additional value to consumers beyond the benefits from consuming new varieties.

E Shubik's (1980) linear demand system

In Shubik's (1980) linear demand system, the representative consumer has a utility function of the form

$$U = V \sum q_i - \frac{1}{2} \left(\sum q_i \right)^2 - \frac{N}{2(1+\gamma)} \left[\sum q_i^2 - \frac{1}{N} \left(\sum q_i \right)^2 \right] + q_0. \quad (15)$$

Maximizing this utility function subject to the consumer's budget constraint yields the following demand function for firm i 's product.³⁰

$$q_i = \frac{1}{N} \left[\alpha - p_i - \gamma \left(p_i - \frac{1}{N} \sum_{j=1}^N p_j \right) \right]. \quad (16)$$

Before the merger, firm i maximizes $\pi_i = (p_i - c) q_i$. Assuming $c = 0$ without loss of generality and solving the optimization problem of the firm yields

$$p_{nm} = \frac{\alpha}{2 + \gamma \frac{N-1}{N}}$$

³⁰In Deneckere and Davidson's (1985) version of this system, the demand function does not have the $1/N$ term before the brackets. This makes no difference in their analysis because they do not consider entry. Davidson and Mukherjee (2006) use this same system without the $1/N$ term and do consider entry. Hence, their system generates qualitatively different long run equilibrium behavior from the formulation considered here.

as the pre-merger equilibrium price. We can then express the equilibrium quantity in terms of the equilibrium price in the following way.

$$q_{nm} = p_{nm} \left(\frac{N + \gamma(N - 1)}{N^2} \right). \quad (17)$$

After the merger, the outsider and merging firms set

$$p_o = \alpha \left(\frac{2N + \gamma(2N - M)}{4N + 2\gamma(3N - M - 1) + \gamma^2 \left(\frac{N-M}{N} \right) (2N + M - 2)} \right)$$

and

$$p_m = \alpha \left(\frac{2N + \gamma(2N - 1)}{4N + 2\gamma(3N - M - 1) + \gamma^2 \left(\frac{N-M}{N} \right) (2N + M - 2)} \right)$$

respectively. These imply that the equilibrium quantities can be expressed in terms of equilibrium prices as

$$q_o = p_o \left(\frac{N + E + \gamma(N + E - 1)}{(N + E)^2} \right) \quad (18)$$

and

$$q_m = p_m \left(\frac{N + E + \gamma(N + E - M)}{(N + E)^2} \right).$$

In the long run, $p_{nm}q_{nm} = p_oq_o$. Substituting for q_{nm} and q_o from (17) and (18) respectively, and cross-multiplying gives us

$$\frac{p_{nm}^2}{p_o^2} = \left(\frac{N^2 (N + E + \gamma(N + E - 1))}{(N + \gamma(N - 1))(N + E)^2} \right).$$

It is straightforward to show that this expression is < 1 . Hence, in a long run equilibrium, $p_o > p_{nm}$. Since joint profit maximization implies $p_m > p_o$, we can conclude that

$$p_m > p_o > p_{nm}.$$

In other words, all prices rise in the long run as a result of the merger. This implies that entry drives profits down to their original level more quickly than it drives prices down to

their original level.³²

To analyze the long run effect of the merger on consumer welfare, consider the consumer's utility as given in (15) at a symmetric equilibrium. We get

$$U = VNq - \frac{1}{2}(Nq)^2 + q_0. \quad (19)$$

This depends only on the total quantity consumed of the differentiated products and not on the number of varieties that total quantity is spread over. The long run effect of the merger on consumer welfare can be analyzed in three steps. First, holding prices constant at p_{nm} , consider an increase in the number of firms to $N + E$. This causes no change in total quantity since, as can be seen from (16), total quantity at a symmetric equilibrium is equal to $\alpha - p$.³³ Such an increase in the number of varieties causes no change in consumer welfare since neither the prices nor total quantity has changed. Second, consider an increase in the prices of all of the $N + E$ varieties to p_o . Clearly, this step harms consumer welfare. Finally, increase the prices of the merging firms to p_m . This step must also decrease consumer welfare. Hence, both total quantity and consumer welfare fall following a merger with entry.

F Entry in Shubik's (1980) demand system

To see that Shubik's (1980) demand system violates the principles underpinning the Weak Axiom of Revealed Preference (WARP), let \mathbf{q}^0 be the bundle of goods demanded by the consumer facing the price vector $\mathbf{p}^0 = (p_1, \dots, p_N)$ where $p_1 = p_2 = \dots = p_N = p$. Now let \mathbf{q}^1 be the bundle demanded by the consumer facing the price vector $\mathbf{p}^1 = (p_1, \dots, p_N, p_{zero})$ where the prices of goods 1 to N are the same as in \mathbf{p}^0 and p_{zero} is the price at which the consumer demands exactly zero from the $N + 1$ th firm. Note that \mathbf{q}^1 is affordable at the original price vector, \mathbf{p}^0 , because it contains zero units of the $N + 1$ th good, and the prices

³²This point can be most immediately seen by comparing this demand system with the demand system used in Davidson and Mukherjee (2006). The prices are identical in the two systems for any N , but the profit level is lower and decreases more rapidly with N in this demand system.

³³Recall that, holding prices constant, total quantity in the CES system is also invariant to the number of firms in a symmetric equilibrium.

of the other N goods are unchanged. Also, \mathbf{q}^0 is affordable at the new price vector, \mathbf{p}^1 , for the same reason.

The principles behind WARP then require these two bundles to be the same. However, in Shubik's demand system they are not. Demand for good i in the first bundle is

$$q_i^0 = \frac{1}{N} (\alpha - p).$$

Demand for good i in the second bundle is

$$q_i^1 = \frac{1}{N+1} \left[\alpha - p - \gamma \left(\frac{p - p_{N+1}}{N+1} \right) \right].$$

Solving for the value of p_{N+1} at which $q_{N+1} = 0$ gives

$$p_{zero} = \left[\frac{\alpha(N+1) + pN\gamma}{N(1+\gamma) + 1} \right]. \quad (20)$$

Solving for the p_{N+1} value such that $q_i^0 = q_i^1$ gives

$$p_{N+1} = (N+1) \left(\frac{\alpha + p(N\gamma - 1)}{N\gamma} \right),$$

which is clearly not the same as p_{zero} given in (20). This means that despite the fact that \mathbf{q}^1 is affordable at \mathbf{p}^0 and \mathbf{q}^0 is affordable at \mathbf{p}^1 , they are not the same bundle. In other words, \mathbf{q}^0 is revealed preferred to \mathbf{q}^1 and vice-versa. This result implies that in this demand system, the mere introduction of a new variety, even one that consumers choose not to consume due to its high price, alters consumers' preferences for the incumbent varieties. This would not be the case in a demand system that satisfies the principles behind WARP.

To see how this violation of the principles behind WARP relates to consumers' lack of taste for variety, note that in any model of product differentiation where the principles of WARP are satisfied, consumer welfare increases with the introduction of a new variety at a price at which consumers choose to consume a positive amount of that variety.³⁴ This implies that consumers' preferences satisfy Benassy's definition of taste for variety. Hence,

³⁴This is true unless the direct utility function depends on N directly. For example, consider the utility function specified in Appendix D. If variety were modeled as a public bad in this demand system, it would continue to satisfy WARP, but it would not necessarily feature a taste for variety.

a demand system where consumers' preferences do not satisfy Benassy's definition of taste for variety must necessarily violate WARP.

A related problem with this demand system is that the indirect utility function is discontinuous at p_{zero} . To see this, let us fix the prices of the incumbent varieties at the symmetric price p and consider how indirect utility changes with p_{N+1} . At $p_{N+1} = p$, the indirect utility level with $N + 1$ goods is equal to the indirect utility level with N goods. This is because, as established in Appendix E, when prices are symmetric and held constant at p , total quantity does not vary with the number of firms and direct utility depends on total quantity only. For $p < p_{N+1} < p_{zero}$, the indirect utility function is strictly decreasing in p_{N+1} . This implies that if a firm enters the market with a price higher than the symmetric price p , it harms consumer welfare despite the fact that consumers choose to consume a positive amount of the new variety. This is connected to consumers' lack of taste for variety. At $p_{N+1} = p_{zero}$, there is a jump in the indirect utility function since for $p_{N+1} > p_{zero}$ the $N + 1$ th variety is no longer in the market and the indirect utility function has the same value as it does when $p_{N+1} = p$.³⁵

G Merger's effect on total quantity in Ottaviano and Thisse's (1999) demand system

We would like to show that

$$(N + E - M)q_o + Mq_m - Nq_{nm} > 0. \quad (21)$$

Since this expression becomes intractable after substituting for the expressions of q_o , q_m and q_{nm} , we proceed in the following way.

Step 1: We first show that the quantity demanded of an outsider firm's variety at price p_{nm} is the same whether there are N firms charging p_{nm} , or N firms charging p_{nm} and a further E firms charging p_{zero} , where p_{zero} is the price at which consumers demand exactly

³⁵Removing the $1/N$ factor, as in Davidson and Mukherjee (2006) does not solve either one of these problems.

zero quantity from each of the E firms. To see this, consider the following three functions.

$$\widehat{q}_i(p_i) = \left[\frac{1}{\beta + \gamma(N-1)} \right] \left[\alpha - \frac{\beta + \gamma(N-2)}{(\beta - \gamma)} p_i + \frac{\gamma(N-1)}{(\beta - \gamma)} p_{nm} \right], \quad (22)$$

$$\bar{q}_i(p_i) = \left[\frac{1}{\beta + \gamma(N+E-1)} \right] \left[\alpha - \frac{\beta + \gamma(N+E-2)}{(\beta - \gamma)} p_i + \frac{\gamma}{(\beta - \gamma)} ((N-1)p_{nm} + Ep_{zero}) \right] \quad (23)$$

and

$$\bar{\bar{q}}_j(p_j) = \left[\frac{1}{\beta + \gamma(N+E-1)} \right] \left[\alpha - \frac{\beta + \gamma(N+E-2)}{(\beta - \gamma)} p_j + \frac{\gamma}{(\beta - \gamma)} (Np_{nm} + (E-1)p_{zero}) \right].$$

The first equation gives the quantity demanded for the variety produced by an outsider firm i if it has $N-1$ rivals each charging p_{nm} . Clearly, $\widehat{q}_i(p_{nm}) = q_{nm}$. The second equation gives the quantity demanded for the variety produced by an outsider firm i if it has $N-1$ rivals each charging p_{nm} and a further E rivals charging p_{zero} . The third equation gives the quantity demanded for the variety produced by an entrant firm j if it has N rivals each charging p_{nm} and a further $E-1$ rivals charging p_{zero} . It is straightforward to show that the value of p_{zero} that makes $\widehat{q}_i(p_{nm}) = \bar{q}_i(p_{nm}) = q_{nm}$ is the same value that sets $\bar{\bar{q}}_j(p_{zero}) = 0$.³⁶ This value is

$$p_{zero} = \frac{\alpha(\beta - \gamma) + \gamma N p_{nm}}{(\beta + \gamma(N-1))}.$$

Step 2: We can use (6) in the paper to get the following expressions for q_o and q_m .

$$q_o = \left[\frac{1}{\beta + \gamma(N+E-1)} \right] \left[\alpha - \frac{\beta + \gamma(N+E-2)}{(\beta - \gamma)} p_o + \frac{\gamma}{(\beta - \gamma)} ((N+E-M-1)p_o + Mp_m) \right] \quad (24)$$

and

$$q_m = \left[\frac{1}{\beta + \gamma(N+E-1)} \right] \left[\alpha - \frac{\beta + \gamma(N+E-2)}{(\beta - \gamma)} p_m + \frac{\gamma}{(\beta - \gamma)} ((N+E-M)p_o + (M-1)p_m) \right]. \quad (25)$$

Note that (21) is equivalent to

$$(N+E-M)q_o + Mq_m - Nq_{nm} - E\bar{\bar{q}}_j(p_{zero}) > 0$$

³⁶Note that this implies that Ottaviano and Thisse's (1999) demand system satisfies the principles underpinning WARP.

since $\bar{q}_j(p_{zero}) = 0$. After substituting for q_o , q_m and q_{nm} using (24), (25) and $\bar{q}_i(p_{nm})$ respectively, this inequality becomes

$$(\beta - \gamma) [Np_{nm} + Ep_{zero} - (N + E - M)p_o - Mp_m] > 0.$$

Given that $\beta > \gamma$, this condition means that total quantity increases after the merger if

$$Np_{nm} + Ep_{zero} > (N + E - M)p_o + Mp_m. \quad (26)$$

Step 3: To see that this inequality always holds in a long run equilibrium, consider the following function, which gives the demand for an outsider firm i after the merger and entry.

$$\tilde{q}_i(p_i) = \left[\frac{1}{\beta + \gamma(N + E - 1)} \right] \left[\frac{\alpha - \frac{\beta + \gamma(N + E - 2)}{(\beta - \gamma)} p_i}{+ \frac{\gamma}{(\beta - \gamma)} ((N + E - M - 1)p_o + Mp_m)} \right]. \quad (27)$$

We know from the long run equilibrium condition that $p_o < p_{nm}$. This implies that $\tilde{q}_i(p_i)$ is tangent to the average cost curve of a typical outsider at a lower price and higher quantity than the price and quantity at which $\hat{q}_i(p_i)$ is tangent to the average cost curve. Since the average cost curve is strictly decreasing and strictly convex, we have $\left| \frac{\partial \tilde{q}_i(p_i)}{\partial p_i} \right| > \left| \frac{\partial \hat{q}_i(p_i)}{\partial p_i} \right|$. Moreover, we can see from (23) and (27) that $\bar{q}_i(p_i)$ and $\tilde{q}_i(p_i)$ have the same slope. Hence, since $\bar{q}_i(p_i)$ is parallel to $\tilde{q}_i(p_i)$ and intersects $\hat{q}_i(p_i)$ at p_{nm} , it must be that $\bar{q}_i(p_i)$ has a higher intercept (on both axes) than $\tilde{q}_i(p_i)$. $\bar{q}_i(p_i) - \tilde{q}_i(p_i) > 0$ implies that

$$(N - 1)p_{nm} + Ep_{zero} > (N + E - M - 1)p_o + Mp_m.$$

Since, as noted above, $p_o < p_{nm}$, we can add p_o to the right hand side of this equality and p_{nm} to the left hand side, and still preserve the inequality. We get

$$Np_{nm} + Ep_{zero} > (N + E - M)p_o + Mp_m,$$

which is the same condition as (26).