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EQUITY RETURN AND SHORT-TERM INTERST RATE VOLATILITY: LEVEL EFFECTS AND ASYMMETRIC DYNAMICS

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Equity Return and Short-Term Interest Rate Volatility: Level Effects and Asymmetric Dynamics*

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Abstract

Evidence suggests that short-term interest rate volatility peaks with the level of short rates, while equity volatility responds asymmetrically to positive and negative shocks. We present an LM based test that distinguishes between level effects and asymmetry in volatility which is robust to the presence of unidentified nuisance parameters under the null. There is strong evidence of a level effect and asymmetric response in the relationship between S&P 500 Index returns and 3-month US Treasury Bills. The conditional covariance depends on the level of the short rate which has implications for hedging equity returns against short term interest rate movements.

Keywords: Level Effects; Asymmetry; LM Tests; Davies Problem; Nonlinear Granger Causality

J.E.L. Reference Numbers: C12; G12; E44

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This paper investigates the relationship between equity returns and short-term interest rates. Often, this relationship is examined in the context of one or other of two issues. First, the widespread theoretical and empirical evidence that suggests the volatility of short-term interest rates peaks with the level of the short-term rate; this is often referred to as the *levels effect*. Second, an *asymmetry* in volatility, that is the revision to the expected conditional volatility following a positive innovation does not equal the revision to expected volatility that occurs after a negative innovation of equal absolute magnitude. This asymmetry is associated in particular with equity returns; equity volatility is highest as prices trend downwards. A similar asymmetry is possible in interest rates. In this paper, however, we examine the impact of interest rate innovations on equity returns in a multivariate framework that allows for *both* levels effects and asymmetric responses to shocks.

Optimal inference about the conditional mean of a vector of returns requires that the conditional second moments be correctly specified. Whether neglected levels effects and/or asymmetries represents a specification error depends on whether these non-linearities are features of the data. However, a major difficulty in testing the null of no levels effect is the potential presence of an unidentified parameter under the null which causes such tests to have non-standard distributions. A contribution of this paper is to present, for the first time, a joint test for a level effect and asymmetry in volatility that is robust to the presence of the unidentified nuisance parameters under the null hypothesis. We use the results of this test to inform our conditional characterisation of the relationship between equity returns and short-term interest rates.

Our focus in the empirical section of the paper is a model of the joint distribution of US short-term interest rates and equity returns that allows for linear and non-linear causality and admits interaction within and across the conditional mean and conditional variance-covariance matrix. The bivariate GARCH-M models we propose allow us to test (i) the direction of causality between equity returns and short rates, (ii) whether the conditional variance of equity returns and short-term interest rates influence the conditional means of the series, (iii) whether shocks to short-term interest rates (equity returns) influence the conditional variance of equity returns (short term interest rates), (iv) whether positive and negative shocks to short-term interest rates (equity returns) have the same impact on the elements of the conditional variance-covariance matrix of equity returns and short-term interest rates, and (v) whether volatilities of equity returns and short-term interest rates are correlated with the level of the short-term interest rate.

A. Related Literature

There is a wide literature on the negative correlation between the nominal excess return on equity and the nominal interest rate.[1] Fama and Schwert (1977), for example, examine whether this negative correlation can be used to forecast periods where the expected excess return on equities is negative. Schwert (1981), Geske and Roll (1983) and Stulz (1986), inter alia argue that the negative correlation arises from the influence of inflation on equity returns and that this is proxied by the bill rate. Fama (1976), on the other hand, attributes changing risk premia in the term structure of bill rates to changing uncertainty about nominal interest rates (which is a proxy for inflation uncertainty). Campbell (1987) argues that there is information in shorter maturity debt instruments that is useful in predicting excess returns on both bonds and equities. In the same vein, Breen, Glosten and Jagannathan (1989) find that the one-month interest rate is useful in forecasting the sign and the variance of the excess return on equities. Glosten, Jagannathan and Runkle (1993) develop a GARCH-M model that allows the conditional volatility to respond differently to positive and negative innovations. Their model also includes the nominal short-term interest rate as a variable to predict the conditional variance of equity returns.

Widespread evidence also exists that suggests the volatility of equity returns is higher in a bear market than in a bull market. One potential explanation for such asymmetry in variance is the so-called 'leverage effect' of Black (1976) and Christie (1982). As equity values fall, the weight attached to debt in a firm's capital structure rises, ceteris paribus. This induces equity holders, who bear the residual risk of the firm, to perceive the stream of future income accruing to their portfolios as being relatively more risky. An alternative view is provided by the 'volatility-feedback' hypothesis of Campbell and Hentschel (1992). Assuming constant dividends, if expected returns increase when equity price volatility increases, then equity prices should fall when volatility rises. Nelson (1991), Engle and Ng (1993), Glosten, Jagannathan and Runkle (1993), Braun, Nelson and Sunnier (1995), Kroner and Ng (1995), Henry (1998), Henry and Sharma (1999), Engle and Cho (1999), and Brooks and Henry (2002), *inter alia*, provide evidence of time-variation and asymmetry in the variance-covariance structure of asset returns.

There is a also a large theoretical and empirical literature arguing that the volatility of short-term interest rates depends on the level of short-term in-

[1] See, for instance, Fama and Schwert (1977), Breen, Glosten and Jagannathan (1989), Keim and Stambaugh (1986), Ferson (1989), Campbell (1989), Campbell and Ammer (1993), Fama (1990), Schwert (1990), Shiller and Beltratti (1992) and Boudoukh, Richardson and Whitelaw (1994), *inter alia*.

terest rates. Chan, Karolyi, Longstaff and Sanders (1992) estimate a general non-linear short rate process which nests many of the short rate processes currently assumed in the literature. Using US data Chan et al. provide estimates of the level effect parameter that differs from the majority of the theoretical literature. Brenner, Harjes and Kroner (1996) show that the sensitivity of interest rate volatility to levels is substantially reduced when volatility is a function of both levels and unexpected shocks.

B. The Rest of the Study

This paper is organised as follows. The next section provides a brief survey of the literature. Section III develops the joint test for asymmetry and a level effect and reports the results of a Monte Carlo study of the small sample performance of the test. Section IV describes the data employed in our study. Section V introduces the multivariate GARCH-M models with level effects and reports the estimation, specification tests and hypothesis test results. Section VI summarizes and concludes.

I. Models of Short Term Interest Rates and Equity Returns

A. Short term interest rates

Consider the general non-linear short rate process, $\{r_t, t \geq 0\}$ proposed by Chan et al (1992)

$$dr = (\mu + \lambda r)\, dt + \phi r^{\delta} dW. \tag{1}$$

Here r represents the level of the interest rate, W is a Brownian motion and μ, λ and δ are parameters. The drift component of short term interest rates is captured by $\mu + \lambda r$ while the variance of unexpected changes in interest rates equals $\phi^2 r^{2\delta}$. While ϕ is a scale factor, the parameter δ controls the degree to which the interest rate level influences the volatility of short term interest rates.

The Chan et al (1992) model nests many of the existing interest rate models. For example when $\delta = 0$, (1) reduces to the Vasicek (1977) model, while $\delta = 1/2$ yields the Cox, Ingersoll and Ross (1985) model, see Chan et al (1992) *inter alia* for further details. Brenner, Harjes and Kroner (1996) argue that by allowing ϕ^2 to be a time varying function of the information set, Ω, one obtains a superior conditional characterisation of short term interest rate changes. Chan et al (1992), and Brenner, Harjes and Kroner (1996) *inter alia* consider the Euler-Maruyama discrete time approximation to (1) written as

$$\Delta r_t = \mu + \lambda r_{t-1} + \varepsilon_{r,t}, \tag{2}$$

where Ω_{t-1} represents the information set available at time $t-1$ and $E(\varepsilon_{r,t}|\Omega_{t-1})=0$. Letting $h_{r,t}$ represent the conditional variance of the short-term interest rate then $E\left(\varepsilon_{r,t}^2|\Omega_{t-1}\right) \equiv h_{r,t} = \phi^2 r_{t-1}^{2\delta}$. The sole source of conditional heteroscedasticity in (2) is through the squared level of the interest rate and thus excludes the information arrival process.

One common approach to capturing the effect of news on interest rate volatility is the GARCH(1,1) model

$$h_{r,t} = \alpha_0 + \beta h_{r,t-1} + \alpha_1 \varepsilon_{r,t-1}^2. \tag{3}$$

The innovation $\varepsilon_{r,t}$ represents a change in the information set from time $t-1$ to t and can be treated as a collective measure of news. In (3) only the magnitude of the innovation is important in determining $h_{r,t}$. Brenner, Harjes and Kroner (1996) extend (2) to allow for volatility clustering caused by information arrival using

$$\begin{aligned} \Delta r_t &= \mu + \lambda r_{t-1} + \varepsilon_{r,t}. \\ E(\varepsilon_{r,t}|\Omega_{t-1}) &= 0,\ E\left(\varepsilon_{r,t}^2|\Omega_{t-1}\right) \equiv h_{r,t} = \phi_t^2 r_{t-1}^{2\delta} \\ \phi_t^2 &= \alpha_0 + \alpha_1 \varepsilon_{r,t-1}^2 + \beta \phi_{t-1}^2 \end{aligned} \tag{4}$$

In high information periods when the magnitude of $\varepsilon_{r,t}$ is largest then the sensitivity of volatility to the level of short term interest rates is highest. Under the restriction $\alpha_1 = \beta = 0$, (4) collapses to (2) and volatility depends on levels alone. Furthermore when $\delta = 0$ then there is no levels effect.

An alternative approach to modelling volatility clustering and levels effects is the extended GARCH model

$$h_{r,t} = \alpha_0 + \alpha_1 \varepsilon_{r,t-1}^2 + \beta h_{r,t-1} + b r_{t-1}^{\delta}. \tag{5}$$

Under the null hypothesis $\alpha_1 = \beta = 0$, volatility depends on interest rate levels alone. If $b = 0$ then there is no levels effect, however under this null the parameter δ is unidentified and so tests of the null hypothesis $H_0 : b = 0$ will have a non-standard distribution, see Davies (1987) for further details. Henry and Suardi (2004b) present a test for the null of no levels effect which corrects for the Davies problem. Other authors test the null $H_0 : b = 0$ assuming δ is known, for instance Longstaff and Schwartz (1992) and Brenner, Harjes and Kroner (1996) assume $\delta = 1.0$ while Bekaert, Hodrick and Marshall (1997) assume $\delta = 0.5$.

B. Equity returns

Black and Scholes (1973) assume that equity prices are generated according to

$$ds = \theta dt + \sigma dW, \tag{6}$$

where θ and σ are parameters and W is a Weiner process. However, the differential equation (6) cannot accomodate the usual volatility clustering observed in financial time series. In addition to this volatility clustering phenomenon, the Black Scholes model of equity price changes does not allow for the fact that bear markets are more volatile than bull markets. Equity returns are said to display own variance asymmetry if

$$VAR\left[\Delta s_{t+1}|\Omega_t\right]|_{\varepsilon_{s,t}<0} - h_{s,t} > VAR\left[\Delta s_{t+1}|\Omega_t\right]|_{\varepsilon_{s,t}>0} - h_{s,t}. \tag{7}$$

Negative equity return innovations, $\varepsilon_{s,t} < 0$, lead to an upward revision of $h_{s,t}$, the conditional variance of returns. In the case of asymmetric volatility this increase in the expected conditional variance exceeds that for a shock of equal magnitude but opposite sign. Nelson (1991), Engle and Ng (1993), Glosten, Jagannathan and Runkle (1993) *inter alia* propose models to capture this asymmetry. The Glosten, Jagannathan and Runkle (1993) approach extends (3) using

$$h_{s,t} = \alpha_0 + \alpha_1 \varepsilon_{s,t-1}^2 + \beta h_{s,t-1} + \alpha_2 \eta_{s,t-1}^2. \tag{8}$$

Here $\eta_{s,t-1} = \min\left[0, \varepsilon_{s,t-1}\right]$. For positive values of α_2, negative innovations to equity returns lead to higher levels of volatility than would occur for a positive innovation of equal magnitude. The implication of (8) is that the size and sign of the innovation matters; bad news has more pernicious effects than good news if $\alpha_2 > 0$.

Engle and Ng (1993) present a test for a neglected asymmetric response to the sign and/or size of shocks in the second moments of conditionally heteroscedastic models. Define I_{t-1}^- as an indicator dummy that takes the value of 1 if $\varepsilon_{s,t-1} < 0$ and the value zero otherwise. The test for sign bias is based on the significance of ϕ_1 in

$$\upsilon_{s,t}^2 = \phi_0 + \phi_1 I_{t-1}^- + e_t, \tag{9}$$

where $\upsilon_{s,t}$ is the standardised residual of stock returns and e_t is a white noise error term. If positive and negative innovations to $\varepsilon_{s,t}$ impact on the conditional variance of Δs_t differently, then ϕ_1 will be statistically significant. It may also be the case that the source of the bias is caused not only by the sign, but also the magnitude or size of the shock. The negative size bias test is based on the significance of the slope coefficient ϕ_1 in

$$\upsilon_{s,t}^2 = \phi_0 + \phi_1 I_{t-1}^- \varepsilon_{s,t-1} + e_t. \tag{10}$$

Likewise, defining $I_{t-1}^{+} = 1 - I_{t-1}^{-}$, then the Engle and Ng (1993) joint test for asymmetry in variance is based on the regression

$$\upsilon_{s,t}^{2} = \phi_0 + \phi_1 I_{t-1}^{-} + \phi_2 I_{t-1}^{-}\varepsilon_{s,t-1} + \phi_3 I_{t-1}^{+}\varepsilon_{s,t-1} + e_t, \tag{11}$$

where e_t is a white noise disturbance term. Significance of the parameter ϕ_1 indicates the presence of *sign bias.* That is, positive and negative realisations of $\varepsilon_{s,t}$ affect future volatility differently to the prediction of the model. Similarly significance of ϕ_2 or ϕ_3 would suggest *size bias*, where not only the sign, but also the magnitude of innovation in return is important. A joint test for sign and size bias, based upon the Lagrange Multiplier Principle, may be performed as $T.R^2$ from the estimation of (11).

Glosten, Jagannathan and Runkle (1993) conclude that the level of the short term interest rate contains information that is useful in predicting future equity return volatility. Their full model may be written as

$$h_{s,t} = \alpha_0 + \alpha_1 \varepsilon_{s,t-1}^{2} + \beta h_{s,t-1} + \alpha_2 \eta_{s,t-1}^{2} + b r_{t-1}^{\delta}. \tag{12}$$

Glosten, Jagannathan and Runkle (1993) assume that $\delta = 1.0$. If this assumption is invalid then the evidence of non-linear causality from interest rates to equity return volatility must be considered tenuous. Secondly, this model imposes one-way non-linear causality from interest rates to equity returns. Should a feedback relationship exist then (12) would represent a misspecified model.

Of relevance here is Henry and Suardi's (2004a) discussion of the problems associated with testing for asymmetry in the face of a neglected levels effect. For example, they present Monte-Carlo evidence that the Engle and Ng (1993) tests for size and sign bias are prone to spuriously reject the null of no asymmetry in the face of an unparameterised levels effect. In the next section we develop a joint test for the presence of asymmetry and levels effects.

Finally, w note that in the event of non-linear causality more complex asymmetries may exist. For example, if the revision of the expected conditional variance of Δs_{t+1} differs across positive and negative interest rate innovations then expected conditional variance of Δs_t is said to display cross variance asymmetry

$$VAR\left[\Delta s_{t+1}|\Omega_t\right]|_{\varepsilon_{r,t}>0} - h_{s,t} > VAR\left[\Delta s_{t+1}|\Omega_t\right]|_{\varepsilon_{r,t}<0} - h_{s,t}. \tag{13}$$

Covariance asymmetry occurs if

$$COV\left[\Delta s_{t+1}, \Delta r_{t+1}|\Omega_t\right]|_{\varepsilon_{r,t}<0} - h_{rs,t} \neq COV\left[\Delta s_{t+1}, \Delta r_{t+1}|\Omega_t\right]|_{\varepsilon_{r,t}>0} - h_{rs,t} \tag{14}$$

or

$$COV\left[\Delta s_{t+1},\Delta r_{t+1}|\Omega_t\right]|_{\varepsilon_{s,t}<0}-h_{rs,t}\neq COV\left[\Delta s_{t+1},\Delta r_{t+1}|\Omega_t\right]|_{\varepsilon_{s,t}>0}-h_{rs,t}. \tag{15}$$

Brooks and Henry (2002), Brooks Henry and Persand (2003) and Henry, Olekalns and Shields (2004) *inter alia* capture time variation and asymmetric response to shocks in the variance covariance matrix using multivariate GARCH-M models. However the question of levels effects and asymmetric responses is largely unexplored.

II. A Lagrange Multiplier Test for Level Effects and Asymmetry

In developing a test for the joint null of asymmetry and levels effects an asymmetric GARCH model with a level effect provides a natural starting point:

$$\begin{aligned} \Delta r_t &= \varepsilon_t \\ \varepsilon_t|\Omega_{t-1} &\backsim N(0,h_t) \\ h_t &= \alpha_o+\alpha_1\varepsilon_{t-1}^2+\beta h_{t-1}+br_{t-1}^{\delta}+\alpha_2\eta_{t-1}^2 \end{aligned} \tag{16}$$

where $\beta+\alpha_1<1$, and β, $\alpha_i, b>0$ for $i=0,1$ and 2. If $\eta_{t-1}=\min(0,\varepsilon_{t-1})$ then negative innovations have a greater initial impact of magnitude $\alpha_1+\alpha_2$ on the volatility of the short rate change than a positive innovation of equal magnitude which has initial impact of size α_1. This model specification (16) is commonly employed in the empirical short rate literature (see Brenner et al., 1996; Christiansen, 2002; Ferreira, 2000; Bali, 1999). The asymmetric component introduced in the conditional variance specification takes on the Glosten, Jagannathan and Runkle (1993) asymmetric form. Despite the commonly observed asymmetric specification in (16), there is no reason why we should not test for a model where the volatility asymmetry stems from a positive rather than a negative innovation, in which case, $\eta_{t-1}=\max(0,\varepsilon_{t-1})$. In fact, the joint test for negative sign (and size) asymmetry and a level effect is trivial to extend to the case of positive sign (and size) asymmetry. Unlike equity returns, it is more likely that a positive innovation to the short rate may bring about higher volatility than a negative innovation of equal magnitude. For example, higher interest rates are associated with higher costs of borrowing funds in the credit market and may signal that the economy is over heated.

The level effect is captured by the dependence of the conditional volatility of the short rate change on the lagged short rate level. Its persistence is governed by the parameters b and δ.[2]

The null hypothesis we consider is that of a symmetric GARCH(1,1) while the alternative is an asymmetric GARCH(1,1) with a level effect. This may be formulated as follows

$$H_0 : \alpha_2 = b = 0$$

$$H_1 : \text{Either } \alpha_2 \text{ and/or } b \neq 0.$$

Sequential substitution for h_{t-1} and a first order Taylor series expansion about δ^* to linearise the level effect term (16) yields

$$\begin{aligned} h_t &= \sum_{i=1}^{t-1} \beta^{i-1}\alpha_o + \sum_{i=1}^{t-1} \beta^{i-1}\alpha_1\varepsilon_{t-i}^2 + \beta^{t-1}h_1 + \sum_{i=1}^{t-1} \beta^{i-1} b r_{t-i}^{\delta^*} (1 - \delta^* \ln r_{t-i}) \\ &\quad + \sum_{i=1}^{t-1} \beta^{i-1}\phi r_{t-i}^{\delta^*} \ln r_{t-i} + \sum_{i=1}^{t-1} \beta^{i-1}\alpha_2\eta_{t-i}^2 \end{aligned} \tag{17}$$

The null hypothesis of no level effect and no asymmetry may be reformulated as $H_0 : b = \phi = \alpha_2 = 0$ where $\phi = b\delta$. Under the assumption that the the residual ε_t is conditionally normally distributed, the Lagrange Multiplier test statistic under the null hypothesis is

$$\frac{1}{2}\left\{\sum_{t=1}^{T}\left[\frac{\varepsilon_t^2}{\tilde{h}_t} - 1\right]\left[\frac{1}{\tilde{h}_t}\frac{\partial h_t}{\partial \varpi}\right]\right\}'\left\{\sum_{t=1}^{T}\left[\frac{1}{\tilde{h}_t}\frac{\partial h_t}{\partial \varpi}\right]\left[\frac{1}{\tilde{h}_t}\frac{\partial h_t}{\partial \varpi}\right]'\right\}^{-1}\left\{\sum_{t=1}^{T}\left[\frac{\varepsilon_t^2}{\tilde{h}_t} - 1\right]\left[\frac{1}{\tilde{h}_t}\frac{\partial h_t}{\partial \varpi}\right]\right\} \tag{18}$$

where

$$\frac{\partial h_t}{\partial \varpi'} = \begin{bmatrix} \sum_{i=1}^{t-1} \hat{\beta}^{i-1} \\ \sum_{i=1}^{t-1} \hat{\beta}^{i-1} \varepsilon_{t-i}^2 \\ \sum_{i=1}^{t-1} \hat{\beta}^{i-1} \tilde{h}_{t-i} \\ \sum_{i=1}^{t-1} \hat{\beta}^{i-1} r_{t-i}^{\delta^*}(1 - \delta^* \ln r_{t-i}) \\ \sum_{i=1}^{t-1} \hat{\beta}^{i-1} r_{t-i}^{\delta^*} \ln r_{t-i} \\ \sum_{i=1}^{t-1} \hat{\beta}^{i-1} \eta_{t-i}^2 \end{bmatrix}';$$

$\tilde{h}_t$ is the conditional variance under the null of GARCH(1,1), ϖ' is the vector of parameters $(\alpha_0, \alpha_1, \beta, b, \phi, \alpha_2)$, and $\hat{\beta}$ is the estimated parameter β in the

[2] Implicitly the conditional mean of (16) is equivalent to $\Delta r_t = \mu + \lambda r_{t-1} + \varepsilon_{r,t}$ under the restriction $\mu = \lambda = 0$. This restriction is consistent with the evidence provided by Chan, Karolyi, Longstaff and Saunders (1992), Longstaff and Schwartz (1992), and Brenner, Harjes and Kroner (1996), *inter alia.*

GARCH(1,1) model. We show in the appendix that the LM test statistic (18) is asymptotically equivalent to $T \cdot R^2$ from the Outer Product Gradient auxiliary regression of

$$\left[\frac{\varepsilon_t^2}{\tilde{h}_t} - 1\right]$$

on X_t where

$$X_t' = \frac{1}{\tilde{h}_t}\begin{bmatrix} \sum_{i=1}^{t-1}\hat{\beta}^{i-1} \\ \sum_{i=1}^{t-1}\hat{\beta}^{i-1}\varepsilon_{t-i}^2 \\ \sum_{i=1}^{t-1}\hat{\beta}^{i-1}\tilde{h}_{t-i} \\ \sum_{i=1}^{t-1}\hat{\beta}^{i-1}r_{t-i}^{\delta^*}(1-\delta^*\ln r_{t-i}) \\ \sum_{i=1}^{t-1}\hat{\beta}^{i-1}r_{t-i}^{\delta^*}\ln r_{t-i} \\ \sum_{i=1}^{t-1}\hat{\beta}^{i-1}\eta_{t-i}^2 \end{bmatrix}. \tag{19}$$

Here T is the sample size and R^2 is the coefficient of determination from the regression (19). We refer to this test statistic as $LM(\delta^*)$ since it is computed using a set of theoretical values for $\delta^* = \{0, 0.5, 1, 1.5\}$. The test is approximately distributed as a Chi-square with three degrees of freedom, however we provide simulated critical values to allow for the approximation error.

Preliminary Monte Carlo experiments suggest that the empirical size of $LM(\delta^*)$ is significantly larger than the nominal size. This size distortion may result from a violation of the usual orthogonality conditions. The normalized residuals, $\tilde{v}_t \equiv \varepsilon_t/\sqrt{h_t}$ should be orthogonal to

$$\frac{1}{\tilde{h}_t}\left[\sum_{i=1}^{t-1}\hat{\beta}^{i-1}, \sum_{i=1}^{t-1}\hat{\beta}^{i-1}\varepsilon_{t-i}^2, \sum_{i=1}^{t-1}\hat{\beta}^{i-1}\tilde{h}_{t-i}\right], \tag{20}$$

but in practice exact orthogonality may not always hold because of the highly nonlinear structure of the model. In the event that these orthogonality conditions fail to hold, the empirical size of the test statistic may be distorted (see Engle and Ng , 1993, pp.1759). [3] To correct for the apparent upward bias in the empirical size of the test statistic, we employ the method introduced by Eitrheim and Teräsvirta (1996) and Engle and Ng (1993, pp. 1759). The procedure involves ensuring $\tilde{v}_t$ is orthogonal to (20). This is done by:

1. Regressing

$$\left[\frac{\varepsilon_t^2}{\tilde{h}_t} - 1\right]$$

[3] Another plausible reason for the observed upward bias in the test statistic's empirical size is due to the poor finite sample properties of the Outer Product Gradient regression based tests (see Davidson and MacKinnon (1993, pp. 477)).

on (20). The residuals from this regression, $\{\tilde{e}_t\}_{t=1}^{T}$, will by construction be orthogonal to (20).

2. Then regress $\tilde{e}_t$ on X_t specified in equation (19) and compute the regression R^2. The test statistic which is labelled $LM_1(\delta^*)$ is set equal to $T \cdot R^2$ and again is approximately distributed as a Chi-square with three degrees of freedom.

III. A Monte Carlo Experiment

A. The Simulated Size of the Test Statistics

To study the simulated size of the joint test statistic we generate data from the simple GARCH(1,1) process

$$\Delta r_t = \varepsilon_t \quad , \quad \varepsilon_t = \sqrt{h_t} \cdot v_t \qquad \text{where } v_t \backsim i.i.d.N(0,1) \tag{21}$$
$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta h_{t-1}$$

We examine the effect of increasing persistence in the conditional variance on the simulated size of the $LM(\delta^*)$ and $LM_1(\delta^*)$ test statistics. Following Engle and Ng's (1993) Monte Carlo study, we employ three sets of parameter values:

1. model H (for high persistence), where $(\alpha_0, \beta, \alpha_1) = (0.01, 0.9, 0.09)$ and $\alpha_1 + \beta = 0.99$

2. model M (for medium persistence), where $(\alpha_0, \beta, \alpha_1) = (0.05, 0.9, 0.05)$ and $\alpha_1 + \beta = 0.95$

3. model L (for low persistence), where $(\alpha_0, \beta, \alpha_1) = (0.2, 0.75, 0.05)$ and $\alpha_1 + \beta = 0.80$.

To mitigate the effect of start-up values in all the experiments, we discard the first 500 observations yielding samples of 500, 1000 and 3000 observations, drawn with 10,000 replications. Once the data have been generated, we estimate a GARCH(1,1) specification by maximizing the log-likelihood function using the Broyden, Fletcher, Goldfarb and Shanno (BFGS)[4] algorithm. The level effect test is then calculated on the resulting standardised residuals using the test statistics $LM(\delta^*)$ and $LM_1(\delta^*)$ for $\delta^* = \{0, 0.5, 1.0, 1.5\}$. Because of the highly non-linear structure of the models, in a small fraction of

[4]The BFGS algorithm with numerical derivatives is discussed in Judd (1988, pp. 114).

these replications, the convergence criterion is not satisfied. In such cases, new replications are added to ensure that there are 10,000 converged replications. To conserve space we report the results for the $LM_1(\delta^*)$. We note that there is some distortion in the empirical size of the uncorrected $LM(\delta^*)$ test regardless of the degree of persistence in the GARCH or the strength of the levels effect at all sample sizes. These results are available upon request from the authors.

-Table I about here-

The Monte Carlo evidence presented in Table I suggests that the corrected test, $LM_1(\delta^*)$, exhibits small size distortions for all data generating processes considered. However, for a sample of 3000 observations the empirical size of $LM_1(\delta^*)$ is close to the nominal size. The empirical size of the joint test statistic also appears to be invariant to the parameter value of δ^* used in the Taylor series approximation.[5]

B. The Simulated Power of the Test Statistics

The next Monte Carlo experiment examines the simulated power of the $LM_1(\delta^*)$ test. The data are generated according to

$$\begin{aligned} \Delta r_t &= \varepsilon_t \\ \varepsilon_t &= \sqrt{h_t} \cdot v_t \quad where\ v_t \backsim i.i.d.N(0,1) \\ h_t &= 0.005 + 0.7 \cdot h_{t-1} + 0.28 \cdot [|\varepsilon_{t-1}| - 0.23 \cdot \varepsilon_{t-1}]^2 + br_{t-1}^{\delta} \end{aligned} \tag{22}$$

A similar specification for the conditional variance equation was employed by Engle and Ng (1993).

The simulated power is illustrated for differing degrees of persistence in the level effect through changing the values of b and δ. The set of parameter values are $b = \{0.01, 0.5, 0.99\}$ and $\delta = \{0, 0.5, 1, 1.5\}$. The results for a sample of 3000 observations are reported in Tables IIA - IIC, respectively, using the simulated critical values, reported in Table 3 for different degrees of persistence in the GARCH structure.

-Tables IIA, IIB and IIC about here-

Across the different combinations of b and δ values the test rejects the null hypothesis of no asymmetry and no levels effect in at least 95% of simulations

[5] We perform further Monte carlo experiment for a mean reverting DGP of $\Delta r_t = \mu + \lambda r_{t-1} + \varepsilon_{r,t}$ where $\mu = 0$ and $\lambda = -0.01, -0.05, -0.10$. The results suggest that the empirical size of $LM_1(\delta^*)$ is not significantly altered. These results are available from the authors upon request.

for each data generating process. The joint test displays significant size adjusted power across all δ^* values considered.[6]

-Table III about here-

Empirical critical values reported in Table III are obtained from the empirical size of the corrected joint test statistic. It is worth noting that these values are relatively close to the relevant $\chi^2(3)$ variate indicating that the $\chi^2(3)$ may be a useful approximation to the true distribution of the test, especially for relatively large samples.

IV. Data Description

Equity prices and short-term interest rates were sampled at a weekly frequency over the period January 5, 1965 to November 04, 2003, yielding 2027 observations. The short term interest rate series is the U.S. three-month Treasury bill rate taken from the Federal Reserve Bank of St. Louis Economic Database. The Standard and Poor 500 (S&P 500) Composite equity price index was obtained from Datastream. Figure i plots the level and change in the U.S. three-month Treasury bill yield (Δr_t). Visual inspection of Figure i suggests that the short rate (i) is most volatile between 1979 and 1982 which includes the period of change in Federal Reserve monetary policy, (ii) that the volatility of Δr_t increases with the level of the short rate and (iii) that Δr_t displays volatility clustering.

-Figure i about here-

The equity return is constructed as $\Delta s_t = \ln(P_t/P_{t-1}) \times 100$ where P_t represents the level of the S&P500 index in period t. From Figure ii, it is evident that equity return displays volatility clustering. The sharp increase in volatility around the period of October 1987 coincides with the equity market crash.

-Figure ii about here-

Table IV presents summary statistics for the data series. There is strong evidence of a unit root in the levels P_t and r_t. However, the S&P 500 equity return and the change in short rate appear stationary. Both Δs_t and

[6] We perform sensitivity analysis for the empirical power of $LM_1(\delta^*)$ based on a mean reverting DGP of $\Delta r_t = \mu + \lambda r_{t-1} + \varepsilon_{r,t}$ where $\mu = 0$ and $\lambda = -0.01, -0.05, -0.10$. The results suggest that $LM_1(\delta^*)$ has high power and it is not affected by the stationarity property of the data. These results are available from the authors upon request.

Δr_t display strong evidence of excess kurtosis. The Bera Jarque (1982) test for the normality of Δs_t and Δr_t is significant. Engle's (1982) LM test for ARCH, performed using the squared residuals from a fifth order autoregression provides strong evidence of conditional heteroscedasticity in Δs_t and Δr_t.

-Table IV about here-

Engle and Ng (1993) sign and size bias tests results are reported in Table IV. For equity returns, the tests for negative sign, negative size and positive size biases are significant at the 10% level. The joint test for sign and size biases further confirms that equity return volatility responds asymmetrically to the sign and size of an innovation. In contrast, there is no statistical evidence supporting asymmetric volatility in the short rate change. The test for a level effect alone and the joint test for the null of no level effect and no asymmetry suggest that there is strong evidence that the volatility of Δr_t is dependent on the lagged short rate level. On the other hand, the null hypothesis of no level effect is satisfied for the equity returns data.

V. The Empirical Models

Given the evidence of conditional heteroscedasticity, asymmetry and level effects reported in Table IV, we model the joint data generating process underlying equity returns and changes in the short rate using a $VAR(m) - GARCH(p,q) - M$ model. The conditional mean of the model can be written as

$$
\begin{aligned}
Y_t &= \mu + \sum_{i=1}^{m} \Gamma_i Y_{t-i} + \Psi vech(H_t) + \varepsilon_t \qquad (23) \\
Y_t &= \begin{bmatrix} \Delta s_t \\ \Delta r_t \end{bmatrix}; \mu = \begin{bmatrix} \mu_s \\ \mu_r \end{bmatrix}; \Gamma_i = \begin{bmatrix} \Gamma_{i,s}^{(s)} & \Gamma_{i,r}^{(s)} \\ \Gamma_{i,s}^{(r)} & \Gamma_{i,r}^{(r)} \end{bmatrix}; \\
\Psi &= \begin{bmatrix} \psi_{1,s} & 0 & \psi_{1,r} \\ \psi_{2,s} & 0 & \psi_{2,r} \end{bmatrix}; \ \varepsilon_t = \begin{bmatrix} \varepsilon_{s,t} \\ \varepsilon_{r,t} \end{bmatrix}
\end{aligned}
$$

where '*vech*' is the column stacking operator of a lower triangular matrix. The test for the absence of linear Granger causality from Δr_t to Δs_t is just a test of the restriction $H_0 : \Gamma_{i,r}^{(s)} = 0$. Similarly, whether Δs_t linearly causes Δr_t simplifies to a test of the restriction $H_0 : \Gamma_{i,s}^{(r)} = 0$. The statistical significance of the GARCH specification in the conditional mean equation may also tested using the restrictions $H_0 : \psi_{i,s} = \psi_{i,sr} = \psi_{i,r} = 0 \ \forall \ i = 1, 2$.

Under the assumption $\varepsilon_t|\Omega_t \sim N(0, H_t)$, where

$$H_t = \begin{bmatrix} h_{s,t} & h_{sr,t} \\ h_{rs,t} & h_{r,t} \end{bmatrix}$$

the model may be estimated using maximum likelihood methods subject to the requirement that H_t be positive definite for all values of ε_t in the sample. Bollerslev and Wooldridge (1992) argue that, in the case of univariate GARCH models, asymptotically valid inference regarding normal QML estimates may be based upon robustified versions of the standard test statistics. The QML estimator for Multivariate GARCH models was shown to be strongly consistent by Jeantheau (1998), while Comte and Lieberman (2000) prove the asymptotic normality of the estimator.

To allow for the possibility of asymmetric responses to shocks we extend the BEKK approach of Engle and Kroner (1995) yielding

$$\begin{aligned} H_t &= C_o^{*'} C_o^* + A_{11}^{*'} \varepsilon_{t-1} \varepsilon_{t-1}^{'} A_{11}^* + B_{11}^{*'} H_{t-1} B_{11}^* + D_{11}^{*'} \xi_{t-1} \xi_{t-1}^{'} D_{11}^* \\ C_o^* &= \begin{bmatrix} c_{11}^* & c_{12}^* \\ 0 & c_{22}^* \end{bmatrix}; A_{11}^* = \begin{bmatrix} a_{11}^* & a_{12}^* \\ a_{21}^* & a_{22}^* \end{bmatrix}; B_{11}^* = \begin{bmatrix} b_{11}^* & b_{12}^* \\ b_{21}^* & b_{22}^* \end{bmatrix}; \\ D_{11}^* &= \begin{bmatrix} d_{11}^* & d_{12}^* \\ d_{21}^* & d_{22}^* \end{bmatrix}; \xi_{t-1} = \begin{bmatrix} \xi_{s,t-1} \\ \xi_{r,t-1} \end{bmatrix} = \begin{bmatrix} \min\{\varepsilon_{s,t-1}, 0\} \\ \max\{\varepsilon_{r,t-1}, 0\} \end{bmatrix}. \end{aligned} \tag{24}$$

We further propose two extensions to (24) to allow for levels effects. Firstly, consider the modified multivariate asymmetric GARCH-M model,

$$\begin{aligned} H_t &= C_o^{*'} C_o^* + A_{11}^{*'} \varepsilon_{t-1} \varepsilon_{t-1}^{'} A_{11}^* + B_{11}^{*'} H_{t-1} B_{11}^* + D_{11}^{*'} \xi_{t-1} \xi_{t-1}^{'} D_{11}^* + E_{11}^{*'} E_{11}^* r_{t-1}^{\delta} \\ E_{11}^* &= \begin{bmatrix} e_{11}^* & e_{12}^* \\ e_{21}^* & e_{22}^* \end{bmatrix}. \end{aligned} \tag{25}$$

Equation (25) captures an additive levels effect. However, a disadvantage of this approach is that, under the null hypothesis of no levels effect, the distribution of a test of $H_0 : e_{11}^* = e_{12}^* = e_{21}^* = e_{22}^* = 0$ will be non-standard because δ is unidentified under the null.

We may use (25) to test for non-linear Granger causality between equity returns and changes in the short term interest rate. For instance a test of the restriction $a_{12}^* = a_{21}^* = b_{12}^* = b_{21}^* = d_{12}^* = d_{21}^* = 0$ rules out spillovers in variance. If this null is satisfied then the non-linear causality occurs through the levels effect alone. The joint null of no non-linear causality and no levels effect implies the restriction $a_{12}^* = a_{21}^* = b_{12}^* = b_{21}^* = d_{12}^* = d_{21}^* = e_{11}^* = e_{21}^* = 0$. We employ Davies' (1987) upper bound approach to allow for the nuisance parameter δ which is unidentified under this null. It is straightforward to test for no variance and covariance asymmetry in Δs_t and Δr_t using $H_0 : d^*_{11} = d^*_{21} = 0$ and $H_0 : d^*_{12} = d^*_{22} = 0$, respectively. A joint test of the restriction $H_0 : d^*_{11} = d^*_{21} = d^*_{12} = d^*_{22} = 0$ examines whether H_t responds asymmetrically to positive and negative shocks to Δs_t and Δr_t.

An alternative approach is the multiplicative levels effect model ,

$$\begin{aligned} H_t &= \Phi_t \cdot r^{\delta}_{t-1} \\ \Phi_t &= C^{*'}_o C^*_o + A^{*'}_{11}\varepsilon_{t-1}\varepsilon^{'}_{t-1}A^*_{11} + B^{*'}_{11}\Phi_{t-1}B^*_{11} + D^{*'}_{11}\xi_{t-1}\xi^{'}_{t-1}D^*_{11}. \end{aligned} \quad (26)$$

Under the null of no levels effects $H_0 : \delta = 0$, $H_t = \Phi_t$ and (26) collapses to the asymmetric BEKK model (24). Notice that the three types of asymmetric response to shocks, namely own variance, cross variance and covariance asymmetry, are preserved in the time-varying variance-covariance matrix. In contrast to the additive level effects model, testing for no level effect in the multiplicative model is straightforward since under the null hypothesis $H_0 : \delta = 0$ there are no unidentified parameters.

VI. Results

A. Additive Level Effects Model

Table V summarizes the quasi-maximum likelihood estimates of the multivariate asymmetric GARCH model with additive level effects. The lag order of the VAR, m, is chosen using the Schwarz (1979) Information Criteria. A VAR(5) was considered optimal.

-Table V about here-

The conditional variance-covariance structure displays strong evidence of GARCH, asymmetry and a level effect. With the exception of $\hat{a}^*_{11}$ all estimated elements of the A^*_{11} matrix are significant. Similarly all the elements of B^*_{11} except $\hat{b}^*_{21}$ are significant. This, coupled with the significance of $\hat{d}^*_{11}$, suggests that equity return volatility is driven by news about interest rates, $\varepsilon_{r,t-1}$, bad news about equity returns, $\eta_{s,t-1}$, and lagged past conditional variances of Δr_t and Δs_t. On the other hand $h_{r,t}$ appears to be determined by $\varepsilon_{r,t-1}$, and $\varepsilon_{s,t-1}$. At the 5% level of significance there is no evidence of asymmetry as neither d^*_{21} nor d^*_{22} are significant.

-Table VI about here-

The multivariate asymmetric GARCH-M additive level effects model appears well specified. The residual diagnostic tests, reported in Table VI, show that the standardised residuals, $z_{i,t} = \varepsilon_{i,t}/\sqrt{h_{i,t}}$ where $i = r, s$, have a mean that is not significantly different from zero. Their variances are also approximately equal to one. The standardised residuals and the squared standardised residuals are free from up to fifth order serial correlation. However, the distributions of both residuals show evidence of negative skewness, while the distribution of the equity return residual is leptokurtic.[7]

-Table VII about here-

Table VII presents results from a series of hypothesis tests based on the additive model. A test of the null hypothesis of no linear Granger causality from Δs_t to Δr_t is satisfied for the data. Similarly, the null hypothesis of no reverse causuality is also satisfied at all usual levels of significance.

The conditional variance of stock returns, $h_{s,t}$ is individually significant and impacts negatively on Δr_t. However this result should be viewed with some caution as the null hypothesis of no GARCH -in-mean effects in the model as a whole is satisfied at the 5% level. There is strong evidence of non-linear Granger causality in the data. A Wald test for diagonality of the $A_{11}^*, B_{11}^*, D_{11}^*$, and E_{11}^* matrices is not satisfied for the data at any level of significance. In addition, the null hypothesis of a symmetric GARCH BEKK model is also rejected.

In testing the null of no level effects in the equity return conditional variance (that is $H_0 : e_{11}^* = e_{21}^* = 0$), δ represents an unidentified nuisance parameter. Our Likelihood Ratio test of this restriction employs the Davies (1987) upper bound significance approach to allow for these unidentified nuisance parameters under the null. Defining L_0 and L_1 as the value of the log-likelihood under the null and alternative respectively, $LR = 2(L_1 - L_0)$. Following Garcia and Perron (1996) if we assume that the likelihood ratio has a single peak, the significance of LR possesses an upper bound that is given by $\Pr(\chi_v^2 > LR) + 2^{(1-v/2)}(LR)^{v/2}\exp(-LR/2)/\Gamma(v/2)$ where v is the number of identified parameters under the alternative hypothesis and $\Gamma(.)$ denotes the gamma function. The evidence suggests that the short rate level influences the conditional variance of equity returns in a statistically significant fashion. Glosten, Jagannathan and Runkle (1993) set $\delta = 1$ in their model, this restriction is not satisfied at any usual level of confidence for our data.

Using a similar approach, the null of no level effects is not satisfied for $h_{r,t}$ at all usual significance levels. The null of no level effects in both equity

[7] Newey (1985) and Nelson (1991) conditional moment tests were also used to examine the orthogonality conditions implied by the model. The results, which are available upon request, suggest that the model provides an adequate conditional characterisation of the data.

returns and short rate conditional variance and covariance is also rejected at all standard levels of significance. The evidence suggests that the level of the short rate affects all elements of the variance-covariance matrix. When short-term interest rates are high, the volatility of short term interest rates and equity returns will be high, but $h_{rs,t}$ will also be high.

There is strong evidence of own variance and covariance asymmetry in equity returns. The null hypotheses $H_0 : d^*_{11} = 0$, and $H_0 : d^*_{11} = d^*_{21} = 0$ are not satisfied for the data. A test of the hypothesis $H_0 : d^*_{21} = 0$ provides no evidence that the conditional variance of equity returns is sensitive to the sign of the innovation in the short rate. The hypotheses $H_0 : d^*_{22} = 0$, and $H_0 : d^*_{12} = 0$ are satisfied for the data implying that short rates do not display own or cross variance asymmetry. The failure to reject the null of no short rate variance and covariance asymmetry further confirms the absence of asymmetric response to news in the conditional volatility of short rates.

Figure iii presents the estimated elements of H_t for the additive model. In October 1979 the US Federal Reserve switched policy from targetting the level of interest rates to targetting the growth of the monetary base. The impact of this policy switch is clear in Figure i and Figure iii; as the level of r_t increased, so did the volality of r_t. This is clear in both the raw data (Figure i) and the estimated elements of the H_t. The impact of the 1987 equity market crash on the raw data (Figure ii) and on h_{11} and h_{12} (Figure iii) is also apparent.[8]

Figure iv displays the estimated conditional correlation between Δr_t and Δs_t. The estimated correlation is calculated as $\hat{\rho}_{12} = \hat{h}_{12} / \left(\sqrt{\hat{h}_{11}} . \sqrt{\hat{h}_{22}} \right)$. With the exception of the period surrounding the 1987 crash $\hat{\rho}_{12} < 0$ for the pre - 1999 period as discussed by Fama and Schwert (1977), Breen, Glosten and Jagannathan (1989), Keim and Stambaugh (1986), Ferson (1989), Schwert (1990) and Boudoukh, Richardson and Whitelaw (1994), *inter alia*. However since 1999 $\hat{\rho}_{12} > 0$. This apparent change in the relationship is likely to have implications for managing short-term interest rate risk. This is particularly true given the evidence of levels effects and asymmetric responses in the estimated elements of H_t.

[8]Newey (1985) and Nelson (1991) conditional moment tests were also used to examine the model for sensitivity to the change in Federal Reserve policy setting and the 1987 Crash. The p-values for $E[(z^2_{r,t} - 1)CRASH] = 0$ and $E[(z^2_{s,t} - 1)CRASH] = 0$ are 0.37 and 0.45 respectively. On the other hand, the p-values for $E[(z^2_{r,t} - 1)FED] = 0$ and $E[(z^2_{s,t} - 1)FED] = 0$ are 0.28 and 0.19 respectively. Both $CRASH$ and FED are dummy variables capturing the stock market crash in 1987 and the Federal Reserve policy experiment in 1979-82. z_t is the standardised residuals. The results, which are available upon request, suggest that the model provides an adequate conditional characterisation of the data.

B. Multiplicative Level Effects Model

Table VIII presents the results for the multivariate asymmetric GARCH model with multiplicative level effects. Again a fifth order VAR was deemed optimal.

-Table VIII about here-

The estimated elements of A_{11}^*, B_{11}^*, and D_{11}^* suggest that $h_{r,t}$ depends on $\varepsilon_{s,t}$ and $\varepsilon_{r,t}$, but there is no real evidence of own or cross variance asymmetry in $h_{r,t}$. Conversely $h_{s,t}$ depends on bad news about equity returns, $\xi_{s,t}$. There is no evidence that $\varepsilon_{r,t}$ affects $h_{s,t}$. However the estimated value of δ suggests that all elements of the conditional variance-covariance matrix H_t are significantly affected by the level of the short rate.

-Table IX about here-

Table IX presents diagnostic tests for the multiplicative model. The standardised residuals have zero mean and unit variance. Both $z_{i,t}$ and $z_{i,t}^2$ for $i = r, s$ are free from up to fifth order serial correlation. Apart from some evidence that the standardised residuals deviate from normality, the multiplicative model appears to provide an adequate conditional characterisation of the data.

-Table X about here-

The test for no-causality between Δs_t and Δr_t is satisfied at five per cent significance level. The data also satisfies the null hypothesis of no-reverse causality at the same level of significance. There is also evidence of GARCH-in-mean, $h_{s,t}$ and $h_{r,t}$ are jointly significant. The evidence suggests that short rate volatility exhibits significant explanatory power for equity returns.

The null hypothesis of a diagonal conditional variance model is strongly rejected by the data. Similarly the null of no asymmetry is not satisfied at all usual significance levels. The test for no GARCH also supports the persistence in innovations in both the conditional variance of equity returns and short rate. Again, $h_{s,t}$ displays own variance asymmetry but only exhibits cross variance asymmetry at the ten per cent significance level. On the other hand $h_{r,t}$ clearly does not display own and cross variance asymmetry. The test for no short rate variance and covariance asymmetry further confirms the absence of asymmetric volatility in the short rate.

The null of no level effect is rejected at the five per cent significance level. The significance of the parameter δ suggests that the level of r_t exerts influence on the conditional variance and covariance of Δs_t and Δr_t. The Cox, Ingersoll and Ross (1985) single-factor model implies $\delta = 0.5$, while

Dothan's (1978) model, the Geometric Brownian Motion process of Black and Scholes (1973), and Brennan and Schwartz (1980) model imply $\delta = 1.0$. The Cox, Ingersoll and Ross (1980) model used to study variable rate (VR) securities implies $\delta = 1.5$. The evidence in Table 10 is inconsistent with each of these values.

Figure v presents the estimated elements of H_t obtained from the multipicative model. The impact of the change in Monetary Policy over 1979-1982 and the 1987 crash on the elements of H_t are apparent.[9] The conditional correlation between Δr_t and Δs_t, displayed in Figure vi, has been largely positive and appears far more volatilie than the pre-1999 sample. Again this would have implications for the ability of equity investors to cover their positions against the effects of unexpected movements in short-term interest rates

Taken together, the results in Tables V to X suggest strongly that a symmetric multivariate GARCH model that does not allow for a level effect would represent a misspecification of the data. In periods when interest rates were high, such a model would tend to underforecast the levels of the elements of H_t. Furthermore this model would produce biased forecasts in periods when equity markets were trending downwards. Our results point towards the importance of considering a levels effect and asymmetric volatility dynamics in modelling the time series evolution of the joint variance-covariance of stock returns and short-term interest rates. In particular, the potential biases arising from the use of a misspecified model could have serious implications for risk management strategies. We leave further exploration of these issues on the agenda for future research.

VI. Conclusion

How do interest rate innovations impact on equity returns? At first glance answering this question appears relatively straightforward. However, theory backed up with widespread empirical evidence sugests that interest rate volatility is positively correlated with the level of the short-term interest rate. Similar support exists to suggest that equity volatility is highest when equity prices are trending downwards. Any adequate attempt to investigate

[9] Newey (1985) and Nelson (1991) conditional moment tests were also used to examine the model for sensitivity to the change in Federal Reserve policy setting and the 1989 stock market crash. The p-values for $E[(z_{r,t}^2 - 1)CRASH] = 0$ and $E[(z_{s,t}^2 - 1)CRASH] = 0$ are 0.23 and 0.35 respectively, while the p-values for $E[(z_{r,t}^2 - 1)FED] = 0$ and $E[(z_{s,t}^2 - 1)FED] = 0$ are 0.17 and 0.41. Note that $CRASH$ and FED are dummy variables capturing the stock market crash in 1987 and the Federal Reserve policy experiment in 1979-82. z_t is the standardised residuals. The results, which are available upon request, suggest that the model provides an adequate conditional characterisation of the data.

the relationship between interest rate fluctuations and equity returns should address these complex non-linear dynamics. Failure to do so could represent a mispecification of the conditional characterisation of the data, yielding unreliable inference.

Detecting a level effect is a non trivial task given the potential presence of the unidentified exponent parameter under the null hypothesis. Similarly, detecting a level effect in the presence of unparameterised asymmetry is not straightforward. In this paper we develop an LM test for the joint null of no levels effect and no asymmetry which accomodates the nuisance parameter problem. Monte-Carlo evidence suggests that the test has impressive power for samples of 1000 observations or greater. However, there appear to be some size distortions, particularly for the smaller samples considered in our study. The tests provide evidence of a level effect in the sample of three-month US Treasury Bills examined in this study, but little evidence of asymmetry. Conversely there is strong evidence of asymmetry in the returns to the Standard and Poors 500 Index examined but little evidence of a level effect.

Two approaches are followed in this paper to parameterise the dependence of the conditional variance-covariance matrix of equity returns and the changes in the short term interest rate on the level of the short-term interest rate. The evidence from asymmetric multivariate GARCH-M models, with either additive or multiplicative level effects is consistent; a univariate model would represent a misspecification of the data. There is strong evidence of asymmetry to news about equities in equity volatility but no evidence that interest rate volatility responds asymmetrically to shocks to either series. There is strong evidence in support of a level effect in interest rate volatility, and some evidence that equity return volatility peaks as short-term interest rates peak. Furthermore the evidence suggests that the conditional covariance of changes in the short-term interest rate and equity returns depends on the level of the short rate and responds asymmetrically to news about equity returns.

Our estimates of the conditional correlation between equity returns and the short-term interest rate suggest that the sign of this correlation may have changed in 1999. The usual negative correlation, often attributed to the influence of inflation on equity returns is apparent until late 1998. However since 1999 our results suggest that the correlation has been largely positive. This change in sign may indicate an expectation of deflation, or that there may have been a change in the underlying relationship between equity returns and the short-term interest rate.

These results have implications for risk management. The ability to hedge equity portfolios against interest rate movements, which depends upon the conditional correlation between equity returns and short-term interest rate innovations, may be reduced when short-term interest rates are high and/or when equity prices are falling.

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Tables and Figures

Table I

Simulated Size of the Corrected Joint Test Statistic: Actual Rejection Frequencies When the Null is True

$$\begin{aligned} \Delta r_t &= \varepsilon_t \ , \ \varepsilon_t = \sqrt{h_t} \cdot v_t \\ v_t &\backsim i.i.d.N(0,1) \\ h_t &= a_0 + a_1 \varepsilon_{t-1}^2 + \beta h_{t-1} \end{aligned}$$

Persistence		H			M			L		
Sample Size		500	1000	3000	500	1000	3000	500	1000	3000
		Actual Rejection Frequencies (%)								
$\delta^* = 0.0$	1%	0.04	2.94	0.95	0.00	2.52	1.07	0.00	1.33	0.89
	5%	0.18	7.27	4.97	0.00	6.19	4.59	0.00	5.09	4.15
	10%	0.46	13.87	9.56	0.00	11.15	8.35	0.00	10.51	8.23
$\delta^* = 0.5$	1%	39.77	2.77	2.93	0.00	2.35	1.72	0.00	1.54	0.96
	5%	75.48	8.69	5.98	0.02	7.31	5.96	4.85	5.74	4.70
	10%	83.34	14.75	11.38	0.54	13.15	10.64	11.02	10.75	10.02
$\delta^* = 1.0$	1%	0.02	2.36	2.58	6.76	1.76	1.31	0.00	1.47	0.99
	5%	0.78	7.22	5.74	64.14	6.90	5.50	1.83	6.16	4.30
	10%	2.84	13.45	11.61	84.61	12.81	9.50	6.55	11.19	9.55
$\delta^* = 1.5$	1%	0.07	2.48	3.15	0.00	1.99	1.51	0.00	0.76	1.17
	5%	5.03	8.12	5.59	0.00	6.11	5.19	5.69	5.64	5.07
	10%	14.94	15.90	11.75	0.00	13.89	9.28	15.66	11.56	9.97

Notes: Figures in each cell represent actual rejection frequencies (%). Persistence Measures - H: $(\alpha_o, \beta, \alpha_1) = (0.01, 0.9, 0.09)$, M: $(\alpha_o, \beta, \alpha_1) = (0.05, 0.9, 0.05)$, L: $(\alpha_o, \beta, \alpha_1) = (0.2, 0.75, 0.05)$

Table IIA

Simulated Power of the Corrected Joint Test Statistic for Sample 3000 with GJR Asymmetry: Actual Rejection Frequencies When the Null is False

$$\begin{aligned} \Delta r_t &= \varepsilon_t \\ \varepsilon_t &= \sqrt{h_t} \cdot v_t \quad v_t \backsim i.i.d.N(0,1) \\ h_t &= 0.005 + 0.7 \cdot h_{t-1} + 0.28 \cdot [|\varepsilon_{t-1}| - 0.23 \cdot \varepsilon_{t-1}]^2 + br_{t-1}^{\delta} \end{aligned}$$

Level Effect		$b = 0.01, \delta = 0.5$			$b = 0.5, \delta = 0.5$			$b = 0.99, \delta = 0.5$		
		H	M	L	H	M	L	H	M	L
		Actual Rejection Frequencies (%)								
$\delta^* = 0.0$	1%	99.26	98.29	99.00	99.49	99.24	99.50	99.36	99.14	99.39
	5%	99.77	99.71	99.77	99.87	99.86	99.87	99.83	99.74	99.83
	10%	99.84	99.82	99.84	99.96	99.93	99.96	99.87	99.85	99.87
$\delta^* = 0.5$	1%	99.17	99.17	99.26	99.92	99.92	99.93	99.89	99.89	99.89
	5%	99.53	99.59	99.60	99.96	99.97	99.97	99.91	99.93	99.94
	10%	99.64	99.71	99.71	99.97	99.98	99.98	99.95	99.95	99.95
$\delta^* = 1.0$	1%	99.61	99.46	99.61	99.83	99.70	99.81	99.96	99.94	99.96
	5%	99.76	99.76	99.80	99.91	99.91	99.93	99.99	99.99	99.99
	10%	99.83	99.82	99.85	99.95	99.95	99.96	100	99.99	100
$\delta^* = 1.5$	1%	99.31	99.31	99.31	99.84	99.84	99.84	99.94	99.94	99.94
	5%	99.48	99.60	99.60	99.88	99.94	99.94	99.94	99.95	99.95
	10%	99.64	99.72	99.72	99.96	99.97	99.97	99.95	99.97	99.97

Table IIB

Simulated Power of the Corrected Joint Test Statistic for Sample 3000 with GJR Asymmetry: Actual Rejection Frequencies When the Null is False

$$\begin{aligned} \Delta r_t &= \varepsilon_t \\ \varepsilon_t &= \sqrt{h_t} \cdot v_t \quad v_t \backsim i.i.d.N(0,1) \\ h_t &= 0.005 + 0.7 \cdot h_{t-1} + 0.28 \cdot [|\varepsilon_{t-1}| - 0.23 \cdot \varepsilon_{t-1}]^2 + br_{t-1}^{\delta} \end{aligned}$$

Level Effect		$b = 0.01, \delta = 1.0$			$b = 0.5, \delta = 1.0$			$b = 0.99, \delta = 1.0$		
		H	M	L	H	M	L	H	M	L
		Actual Rejection Frequencies (%)								
$\delta^* = 0.0$	1%	98.61	98.49	98.93	99.64	9958	99.65	99.69	99.60	99.69
	5%	99.69	99.60	99.69	99.92	99.88	99.91	99.94	99.92	99.94
	10%	99.81	99.75	99.81	99.95	99.95	99.95	99.97	99.96	99.97
$\delta^* = 0.5$	1%	100	100	100	100	100	100	100	100	100
	5%	100	100	100	100	100	100	100	100	100
	10%	100	100	100	100	100	100	100	100	100
$\delta^* = 1.0$	1%	100	100	100	100	100	100	100	100	100
	5%	99.99	100	100	100	100	100	100	100	100
	10%	100	100	100	100	100	100	100	100	100
$\delta^* = 1.5$	1%	100	100	100	100	100	100	100	100	100
	5%	100	100	100	100	100	100	100	100	100
	10%	100	100	100	100	100	100	100	100	100

Table IIC

Simulated Power of the Corrected Joint Test Statistic for Sample 3000 with GJR Asymmetry: Actual Rejection Frequencies When the Null is False

$$\begin{aligned}\Delta r_t &= \varepsilon_t \\ \varepsilon_t &= \sqrt{h_t}\cdot v_t \quad v_t \backsim i.i.d.N(0,1) \\ h_t &= 0.005 + 0.7\cdot h_{t-1} + 0.28\cdot\left[|\varepsilon_{t-1}| - 0.23\cdot\varepsilon_{t-1}\right]^2 + br_{t-1}^{\delta}\end{aligned}$$

Level Effect		$b=0.01,\delta=1.5$			$b=0.5,\delta=1.5$			$b=0.99,\delta=1.5$		
		H	M	L	H	M	L	H	M	L
		Actual Rejection Frequencies (%)								
$\delta^*=0.0$	1%	93.61	92.10	93.68	95.20	94.30	95.26	95.31	94.61	95.38
	5%	97.18	96.61	97.11	97.56	97.21	97.51	97.82	97.47	97.78
	10%	98.09	97.79	98.11	98.28	98.07	98.30	98.60	98.33	98.61
$\delta^*=0.5$	1%	100	100	100	99.99	99.99	99.99	99.97	99.97	99.97
	5%	100	100	100	100	100	100	99.97	99.97	99.97
	10%	100	100	100	100	100	100	99.97	99.97	99.97
$\delta^*=1.0$	1%	100	100	100	99.97	99.97	99.97	99.99	99.99	99.99
	5%	100	100	100	99.99	99.99	99.99	100	100	100
	10%	100	100	100	99.99	99.99	100	100	100	100
$\delta^*=1.5$	1%	100	100	100	99.98	99.98	99.98	100	100	100
	5%	100	100	100	99.99	99.99	99.99	100	100	100
	10%	100	100	100	99.99	99.99	99.99	100	100	100

Table III

Empirical Critical Values for sample size 3000

Persistence		H	M	L
$\delta^* = 0.0$	1%	10.876	12.022	10.789
	5%	7.183	7.795	7.242
	10%	5.649	6.234	5.630
$\delta^* = 0.5$	1%	11.328	11.338	10.825
	5%	8.944	7.815	7.389
	10%	6.946	6.251	6.209
$\delta^* = 1.0$	1%	11.317	12.993	11.343
	5%	8.752	8.644	7.799
	10%	6.727	7.165	6.027
$\delta^* = 1.5$	1%	11.337	11.327	11.322
	5%	9.444	7.802	7.814
	10%	7.149	6.2463	6.251
χ^2 (3)	1%		11.3449	
	5%		7.81473	
	10%		6.25139	

Notes: See notes to Table I.

Table IV

Summary Statistics

Data Series	S&P 500 Return (Δs)	3-mth T-Bill Yield Change (Δr)
Mean	0.1244	-0.0014
Variance	5.1729	0.0665
Skewness	-0.9522	-0.5434
Kurtosis	16.9413	20.9800
ADF(5)	-18.0361	-17.2931
PP(5)	-48.0795	-40.6889
KPSS(μ)	0.2046	0.1129
KPSS(τ)	0.0851	0.0224
Jarque-Berra $\sim \chi^2(2)$	16713.39	27390.01
	[0.0000]	[0.0000]
ARCH(5)*	2.6558	61.0037
	[0.0212]	[0.0000]
Ljung-Box statistic $Q(5)^*$	0.2848	0.5167
	[0.9980]	[0.9920]
Engle and Ng's Asymmetry Tests		
Negative Sign	2.4566	-0.5644
	[0.014]	[0.5725]
Negative Size	-1.8355	-1.2734
	[0.0666]	[0.2030]
Positive Size	-2.1377	-0.0356
	[0.0327]	[0.9716]
Joint Test	6.8273	4.4458
	[0.0776]	[0.2172]
Level Effect Test $LM_0(\delta^*)$		
$\delta^* = 0.0$	0.0402	15.4085
	[0.9801]	[4.5090×10^{-4}]
$\delta^* = 0.5$	0.5016	17.9886
	[0.7782]	[1.2411×10^{-4}]
$\delta^* = 1.0$	0.5406	18.9526
	[0.7632]	[7.6648×10^{-5}]
$\delta^* = 1.5$	0.5822	19.7303
	[0.7474]	[5.1953×10^{-5}]
Joint Test for Asymmetry and Level Effects $LM_1(\delta^*)$		
$\delta^* = 0.0$	7.4257	15.5410
	[0.0595]	[0.0014]
$\delta^* = 0.5$	7.7837	18.0890
	[0.0507]	[4.2164×10^{-4}]
$\delta^* = 1.0$	7.8200	19.0395
	[0.0501]	[2.6830×10^{-4}]
$\delta^* = 1.5$	7.8283	19.8071
	[0.0490]	[1.8611×10^{-4}]

Note: ADF(5) and PP(5) include an intercept and trend in the regressions. Both tests have 1%, 5% and 10% critical values of -3.9642, -3.4128 and -3.1284 respectively. KPSS(μ) 1%, 5% and 10% critical values are 0.739, 0.463 and 0.347 respectively. KPSS(τ) 1%, 5% and 10% critical values are 0.216, 0.146 and 0.119 respectively. The figures in parentheses are p-values. ARCH tests are performed on the residuals from a fifth order autoregression.

Table V

Estimates of the Multivariate Asymmetric GARCH Additive Level Effects Model

Conditional Mean Equations							
	μ_s	Δs_{t-1}	Δs_{t-2}	Δs_{t-3}	Δs_{t-4}	Δs_{t-5}	
Δs_t	0.1259***	-0.0413	0.0183	0.0393***	-0.0145	-0.0342	
	(0.0443)	(0.0282)	(0.0224)	(0.0089)	(0.0349)	(0.0322)	
	Δr_{t-1}	Δr_{t-2}	Δr_{t-3}	Δr_{t-4}	Δr_{t-5}	$H_{s,t}$	$H_{r,t}$
	-0.3694	-0.0388	0.0270	-0.5238	-0.1097	0.0060	-0.4543
	(0.2595)	(0.6465)	(0.3150)	(0.4832)	(0.1628)	(0.0131)	(0.3101)

	μ_r	Δs_{t-1}	Δs_{t-2}	Δs_{t-3}	Δs_{t-4}	Δs_{t-5}	
Δr_t	0.0078**	0.0004	-0.0007	-0.0003	0.0002	-0.0005	
	(0.0036)	(0.0011)	(0.0011)	(0.0012)	(0.0015)	(0.0012)	
	Δr_{t-1}	Δr_{t-2}	Δr_{t-3}	Δr_{t-4}	Δr_{t-5}	$H_{s,t}$	$H_{r,t}$
	0.0517**	0.0060	0.0197	0.0988***	0.0455	-0.0009**	-0.0623
	(0.0250)	(0.0494)	(0.0354)	(0.0237)	(0.0318)	(0.0005)	(0.0800)

Conditional Variance-Covariance Structure

$$C_o^* = \begin{bmatrix} \begin{matrix}0.3604^{**}\\(0.1228)\end{matrix} & \begin{matrix}-0.0033\\(0.0086)\end{matrix} \\ 0 & \begin{matrix}1.23\times 10^{-7}\\(0.0050)\end{matrix} \end{bmatrix} \quad A_{11}^* = \begin{bmatrix} \begin{matrix}-0.0732\\(0.0646)\end{matrix} & \begin{matrix}0.0020^{**}\\(0.0008)\end{matrix} \\ \begin{matrix}-0.5110^{***}\\(0.1454)\end{matrix} & \begin{matrix}0.3708^{***}\\(0.0502)\end{matrix} \end{bmatrix} \quad B_{11}^* = \begin{bmatrix} \begin{matrix}0.9495^{***}\\(0.0185)\end{matrix} & \begin{matrix}0.0037\\(0.0033)\end{matrix} \\ \begin{matrix}2.6319^{***}\\(0.3131)\end{matrix} & \begin{matrix}-0.8937^{***}\\(0.0292)\end{matrix} \end{bmatrix}$$

$$D_{11}^* = \begin{bmatrix} \begin{matrix}0.3327^{***}\\(0.0388)\end{matrix} & \begin{matrix}0.0022^{*}\\(0.0012)\end{matrix} \\ \begin{matrix}-0.4031\\(0.3039)\end{matrix} & \begin{matrix}0.1874^{*}\\(0.0996)\end{matrix} \end{bmatrix} \quad E_{11}^* = \begin{bmatrix} \begin{matrix}-0.0099^{***}\\(0.0014)\end{matrix} & \begin{matrix}-0.0009^{***}\\(0.0002)\end{matrix} \\ \begin{matrix}0.0046^{***}\\(0.0014)\end{matrix} & \begin{matrix}0.0037^{***}\\(0.0002)\end{matrix} \end{bmatrix} \quad \delta = \begin{matrix}2.9374^{***}\\(0.2665)\end{matrix}$$

Note: Figures in parentheses ( ) are quasi maximimum likelihood robust standard errors. *, ** and *** indicate statistical significance at the 10%, 5% and 1% level respectively.

Table VI

Diagnostic Tests Results for Additive Level Effects Model

Standardized Residual Diagnostics					
	Mean	**Variance**	**Skewness**	**Kurtosis**	**Q(5)**
$z_{s,t}$	-0.0070	0.9992	-1.0274	11.1732	4.5450
	[0.7517]		[0.0000]	[0.0000]	[0.4739]
$z_{r,t}$	-0.0026	0.9947	-0.0268	3.2089	2.3656
	[0.9603]		[0.9103]	[0.0000]	[0.7966]
	Mean	**Variance**	**$Q^2(5)$**		
$z^2_{s,t}$	0.9987	13.1429	1.2347		
	[0.0000]		[0.9415]		
$z^2_{r,t}$	0.9942	5.1409	2.8840		
	[0.0000]		[0.7179]		

Notes: Marginal significance levels displayed as [.].

Table VII

Hypothesis Tests for the Multivariate Asymmetric GARCH Additive Level Effects Model

Mean Hypothesis Tests			
Test of linear Granger causality			
$H_0 : \Delta r_t \nrightarrow \Delta s_t$ $\quad \chi^2_{(5)} =$ 1.7248 [0.8859]		$H_0 : \Delta s_t \nrightarrow \Delta r_t$ $\quad \chi^2_{(5)} =$ 1.1727 [0.9475]	
Test of GARCH-in-mean Specification			
$H_0 : \psi_{i,s} = \psi_{i,r} = 0$ for $i = 1, 2$		$\chi^2_4 = 9.6248$	[0.0472]
Conditional Variance-Covariance Hypothesis Tests			
Diagonal Conditional Variance Model			
$H_0 : a^*_{ij} = b^*_{ij} = d^*_{ij} = e^*_{ij} = 0 \quad \forall\ i \neq j$ where $i, j = 1, 2$		P($\chi^2_{(8)} > 71.457$)= 0.0000	
Symmetric GARCH (BEKK Model)			
$H_0 : d^*_{ij} = e^*_{ij} = 0 \quad \forall\ i$ and j where $i, j = 1, 2$		P($\chi^2_{(8)} > 77.976$) = 0.0000	
No GARCH			
$H_0 : a^*_{ij} = b^*_{ij} = d^*_{ij} = e^*_{ij} = 0 \quad \forall\ i$ and j where $i, j = 1, 2$		P($\chi^2_{(16)} > 83.521$) = 0.0000	
Level Effects Tests			
Test for No Level Effects in H_t			
$H_0 : e^*_{ij} = 0 \quad \forall\ i$ and j where $i, j = 1, 2$	P($\chi^2_{(4)}$>35.413) = 0.0000		
Test for No Level Effects in $h_{s,t}$ and $h_{rs,t}$			
$H_0 : e^*_{21} = e^*_{11} = 0$	P($\chi^2_{(2)}$>21.922) = 0.0000		
Test for No Level Effects in $h_{r,t}$ and $h_{rs,t}$			
$H_0 : e^*_{12} = e^*_{22} = 0$	P($\chi^2_{(2)}$>25.351) = 0.0000		
Second Moment Asymmetry Tests			
Test for no own variance and covariance asymmetry:Δs_t			
$H_0 : d^*_{11} = d^*_{21} = 0$	$\chi^2_{(2)} = 85.5529$	[0.0000]	
Test for no own variance and/or covariance asymmetry:Δr_t			
$H_0 : d^*_{12} = d^*_{22} = 0$	$\chi^2_{(2)} = 7.4054$	[0.0247]	
Test for Own Variance Asymmetry			
$\Delta s_t : H_0 : d^*_{11} = 0$	$\chi^2_{(1)} = 73.5914$	[0.0000]	
$\Delta r_t \quad H_0 : d^*_{22} = 0$	$\chi^2_{(1)} = 3.5429$	[0.0598]	
Test for Cross Variance Asymmetry			
$\Delta s_t : H_0 : d^*_{21} = 0$	$\chi^2_{(1)} = 1.7599$	[0.1846]	
$\Delta r_t : H_0 : d^*_{12} = 0$	$\chi^2_{(1)} = 3.4684$	[0.0626]	

Note: Figures in parentheses [ ]are p-values. For 5% significance level, the critical value for Chi-square distribution with 1, 2 and 4 degrees of freedom are 3.841, 5.991 and 9.488 respectively.

Table VIII

Estimates of the Multivariate Asymmetric GARCH Multiplicative Level Effects Model

Conditional Mean Equations

	μ_s	Δs_{t-1}	Δs_{t-2}	Δs_{t-3}	Δs_{t-4}	Δs_{t-5}	
Δs_t	0.0935	-0.0420	0.0169	0.0453	-0.0104	-0.0301	
	(0.0619)	(0.0281)	(0.0220)	(0.0320)	(0.0305)	(0.0205)	
	Δr_{t-1}	Δr_{t-2}	Δr_{t-3}	Δr_{t-4}	Δr_{t-5}	$H_{s,t}$	$H_{r,t}$
	-0.3877*	-0.0577	0.0080	-0.4687**	-0.0946	0.0105	-0.3414**
	(0.2034)	(0.2214)	(0.1592)	(0.1977)	(0.1625)	(0.0104)	(0.1731)

	μ_r	Δs_{t-1}	Δs_{t-2}	Δs_{t-3}	Δs_{t-4}	Δs_{t-5}	
Δr_t	0.0061	0.0002	-0.0012	-0.0001	0.0008	-0.0002	
	(0.0032)*	(0.0012)	(0.0012)	(0.0012)	(0.0011)	(0.0013)	
	Δr_{t-1}	Δr_{t-2}	Δr_{t-3}	Δr_{t-4}	Δr_{t-5}	$H_{s,t}$	$H_{r,t}$
	0.0406*	0.0031	0.0212	0.0970***	0.0458**	-0.0008	-0.0807
	(0.0236)	(0.0237)	(0.0207)	(0.0244)	(0.0225)	(0.0005)	(0.0548)

Conditional Variance-Covariance Structure

$$C_o^* = \begin{bmatrix} \underset{(0.1233)}{0.4357^{***}} & \underset{(0.0038)}{-0.0096^{**}} \\ 0 & \underset{(0.0041)}{-0.0165^{***}} \end{bmatrix} \quad A_{11}^* = \begin{bmatrix} \underset{(0.0563)}{0.0059} & \underset{(0.0010)}{0.0004} \\ \underset{(0.1531)}{-0.3772^{**}} & \underset{(0.0551)}{0.3612^{***}} \end{bmatrix}$$

$$B_{11}^* = \begin{bmatrix} \underset{(0.0336)}{0.9128^{***}} & \underset{(0.0006)}{0.0008} \\ \underset{(0.0431)}{-0.0008} & \underset{(0.0206)}{0.8978^{***}} \end{bmatrix} \quad D_{11}^* = \begin{bmatrix} \underset{(0.0435)}{0.3787^{***}} & \underset{(0.0010)}{0.0012} \\ \underset{(0.2260)}{0.3980^{*}} & \underset{(0.1537)}{0.0095} \end{bmatrix} \quad \delta = \underset{(0.0155)}{0.0375^{***}}$$

Note: See notes to Table V.

Table IX

Diagnostic Tests Results for Multiplicative Level Effects Model

	Standardized Residual Diagnostics				
	Mean	**Variance**	**Skewness**	**Kurtosis**	**Q(5)**
$z_{s,t}$	-0.0051	0.9965	-1.0422	11.3610	5.1064
	[0.8184]		[0.0000]	[0.0000]	[0.4030]
$z_{r,t}$	0.0059	0.9926	0.0484	3.2599	2.8519
	[0.7885]		[0.3747]	[0.0000]	[0.7228]
	Mean	**Variance**	**$\mathbf{Q^2(5)}$**		
$z_{s,t}^2$	0.9960	13.2516	1.2135		
	[0.0000]		[0.9436]		
$z_{r,t}^2$	0.9921	5.1698	3.5286		
	[0.0000]		[0.6191]		

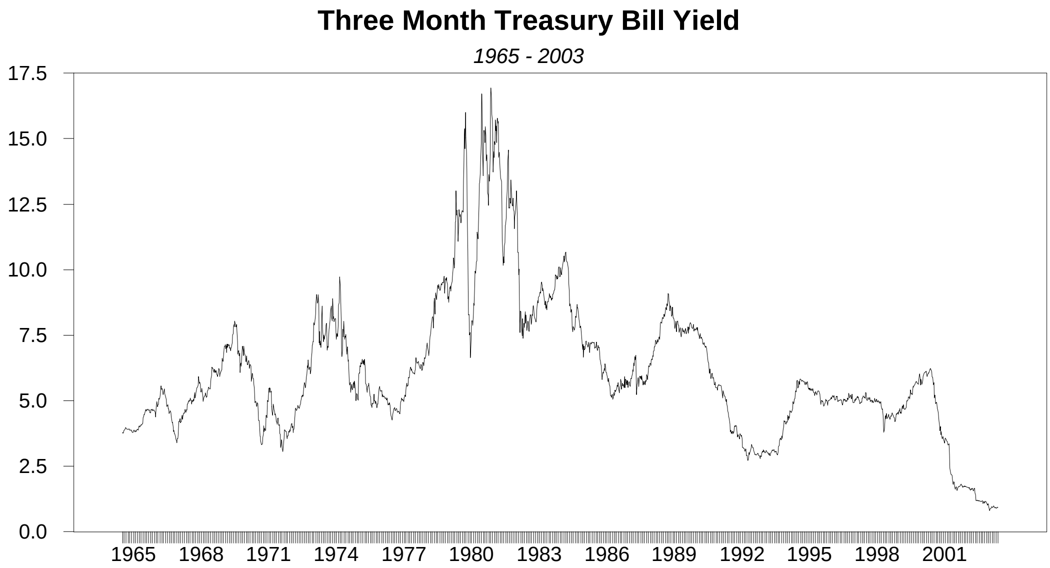
Notes: See notes to Table VI

Table X

Hypothesis Tests for the Multivariate Asymmetric GARCH Multiplicative Level Effects Model

Mean Hypothesis Tests			
Test of linear Granger			
$H_0 : \Delta r_t \nrightarrow \Delta s_t$	$\chi^2_{(5)} =$ 10.9973 [0.0514]	$H_0 : \Delta s_t \nrightarrow \Delta r_t$	$\chi^2_{(5)} =$ 1.8786 [0.8657]
Test of GARCH-in-mean Specification			
$H_0 : \psi_{i,s} = \psi_{i,r} = 0$ for $i = 1, 2$		$\chi^2_4 = 14.4555$	[0.0060]
Conditional Variance-Covariance Hypothesis Tests			
Diagonal Conditional Variance Model			
$H_0 : a^*_{ij} = b^*_{ij} = d^*_{ij} = 0 \quad \forall\, i \neq j$ where $i, j = 1, 2$		$\chi^2_{(6)}$=21.8287	[0.0013]
Symmetric GARCH (BEKK Model)			
$H_0 : d^*_{ij} = 0 \quad \forall\, i$ and j where $i, j = 1, 2$		$\chi^2_{(4)} = 78.1762$	[0.0000]
No GARCH			
$H_0 : a^*_{ij} = b^*_{ij} = d^*_{ij} = 0 \quad \forall\, i$ and j where $i, j = 1, 2$		$\chi^2_{(12)} = 13399.9768$	[0.0000]
Level Effects Tests			
Test for No Level Effects			
$H_0 : \ \delta = \ 0$	$\chi^2_{(1)} = 5.8634$		[0.0155]
Level Effects Tests for Theoretical Values of δ			
$H_0 : \ \delta = \ 0.5$	$\chi^2_{(1)} = 891.1654$		[0.0000]
$H_0 : \ \delta = \ 1.0$	$\chi^2_{(1)} = 3859.6689$		[0.0000]
$H_0 : \ \delta = \ 1.5$	$\chi^2_{(1)} = 8911.3740$		[0.0000]
Second Moment Asymmetry Tests			
Test for no own variance and covariance asymmetry:Δs_t			
$H_0 : d^*_{11} = d^*_{21} = 0$	$\chi^2_{(2)} = 76.1179$		[0.0000]
Test for no own variance and covariance asymmetry:Δr_t			
$H_0 : d^*_{12} = d^*_{22} = 0$	$\chi^2_{(2)} = 1.4004$		[0.4965]
Test for own variance asymmetry			
$\Delta s_t : H_0 : d^*_{11} = 0$	$\chi^2_{(1)} = 75.7113$		[0.0000]
$\Delta r_t : H_0 : d^*_{22} = 0$	$\chi^2_{(1)} = 0.0038$		[0.9510]
Test for Cross Variance Asymmetry			
$\Delta s_t : H_0 : d^*_{21} = 0$	$\chi^2_{(1)} = 3.1013$		[0.0782]
$\Delta r_t : H_0 : d^*_{12} = 0$	$\chi^2_{(1)} = 1.3173$		[0.2511]

Note: See notes to Table VII.



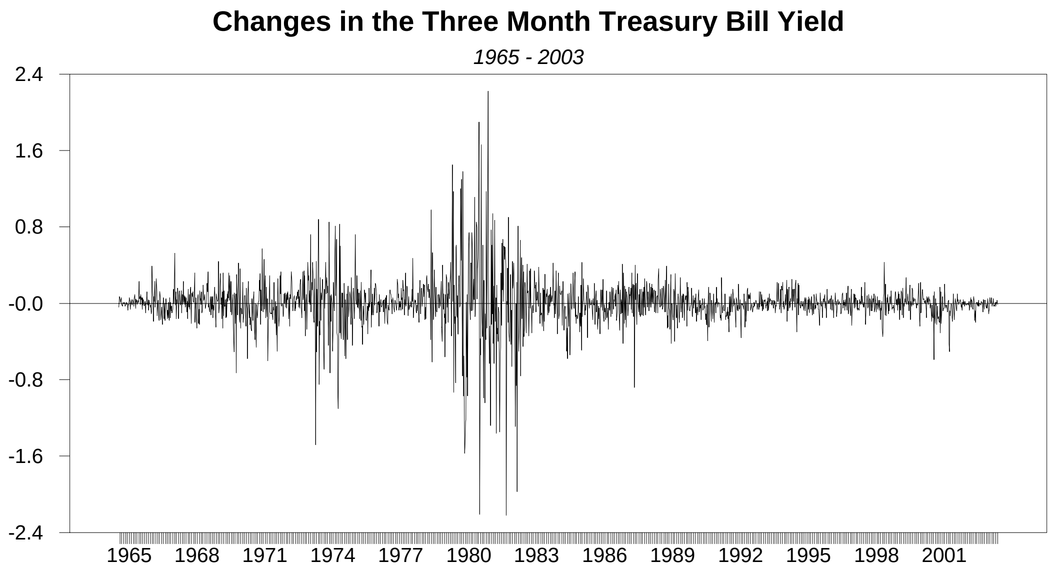


Figure i: r_t and Δr_t

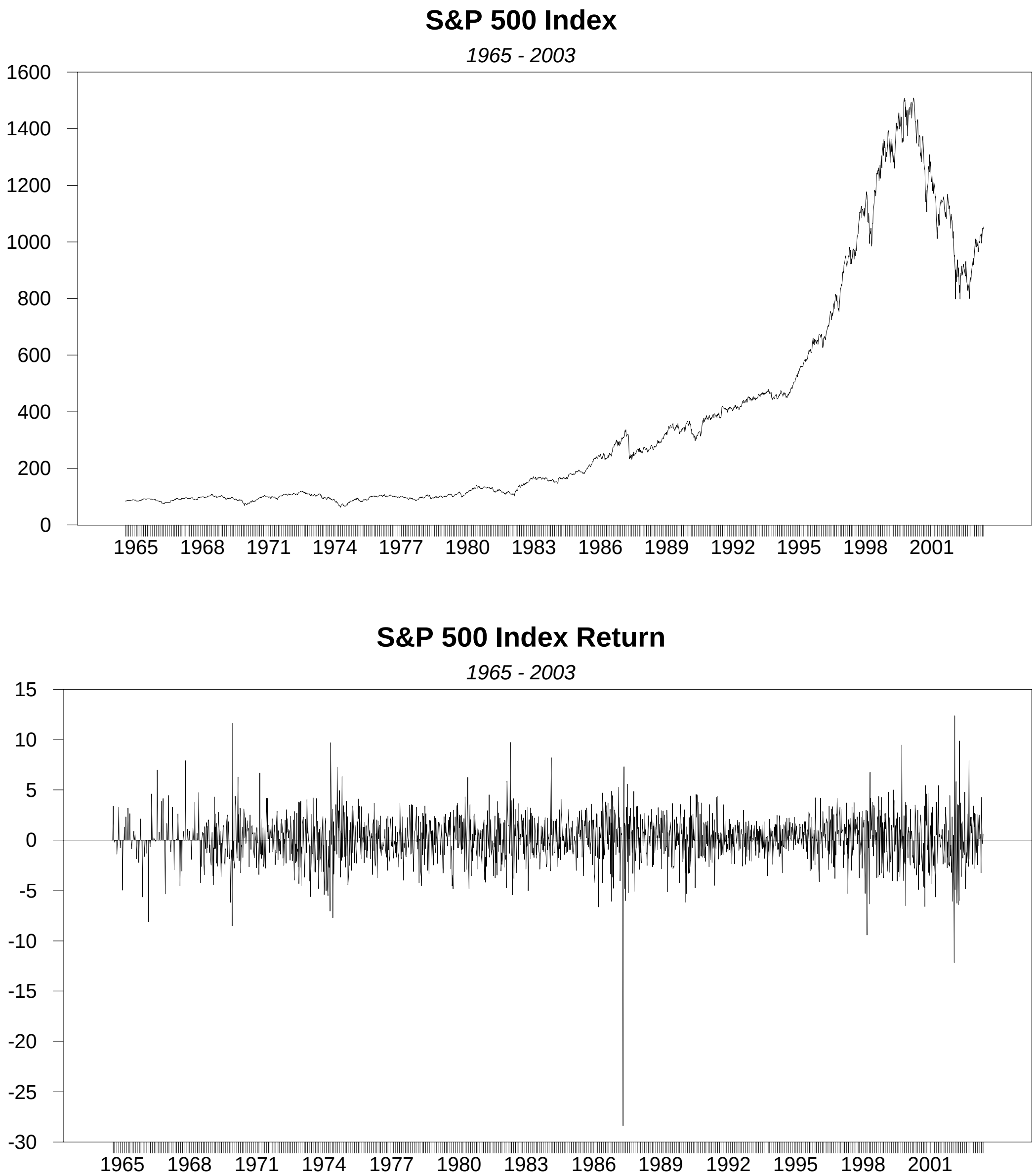


Figure ii: P_t and Δs_t

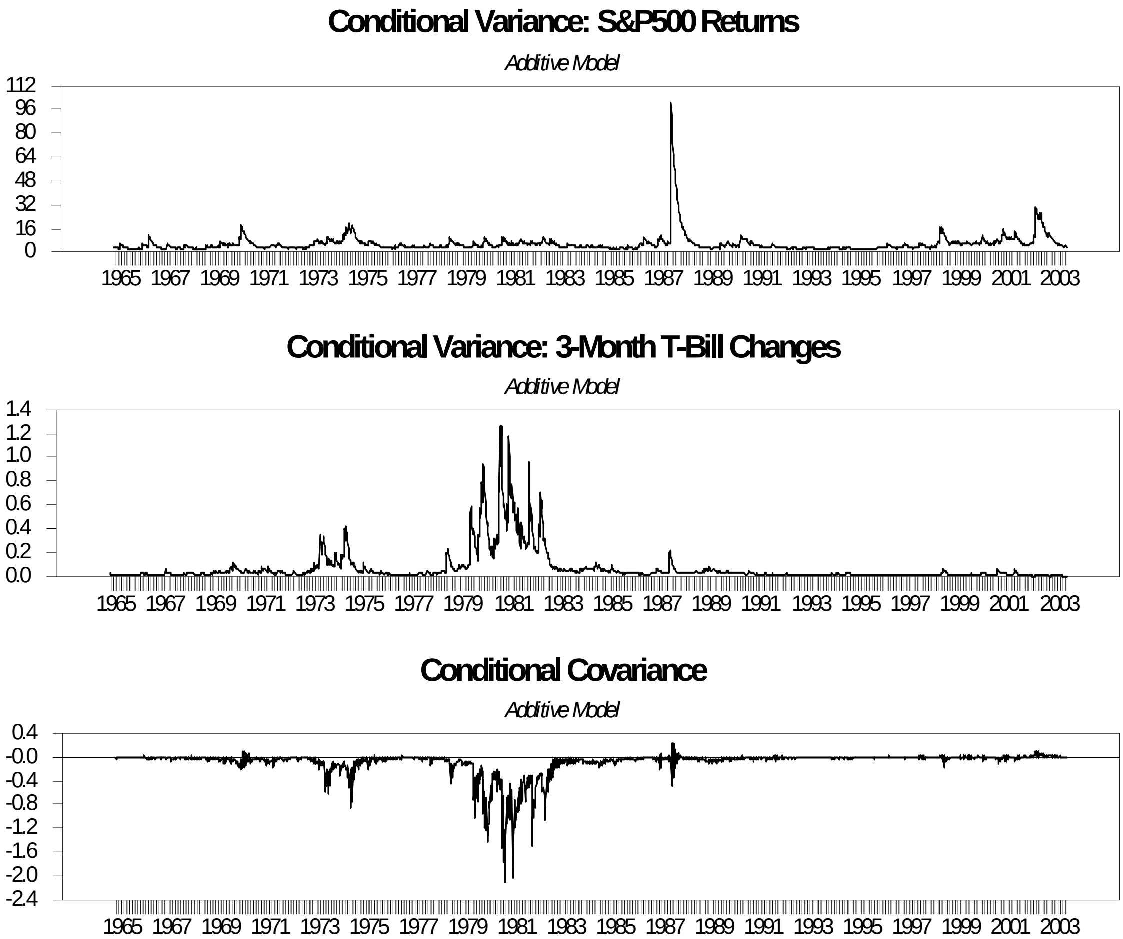


Figure iii: $\hat{h}_{11}, \hat{h}_{12}$ and $\hat{h}_{22}$.

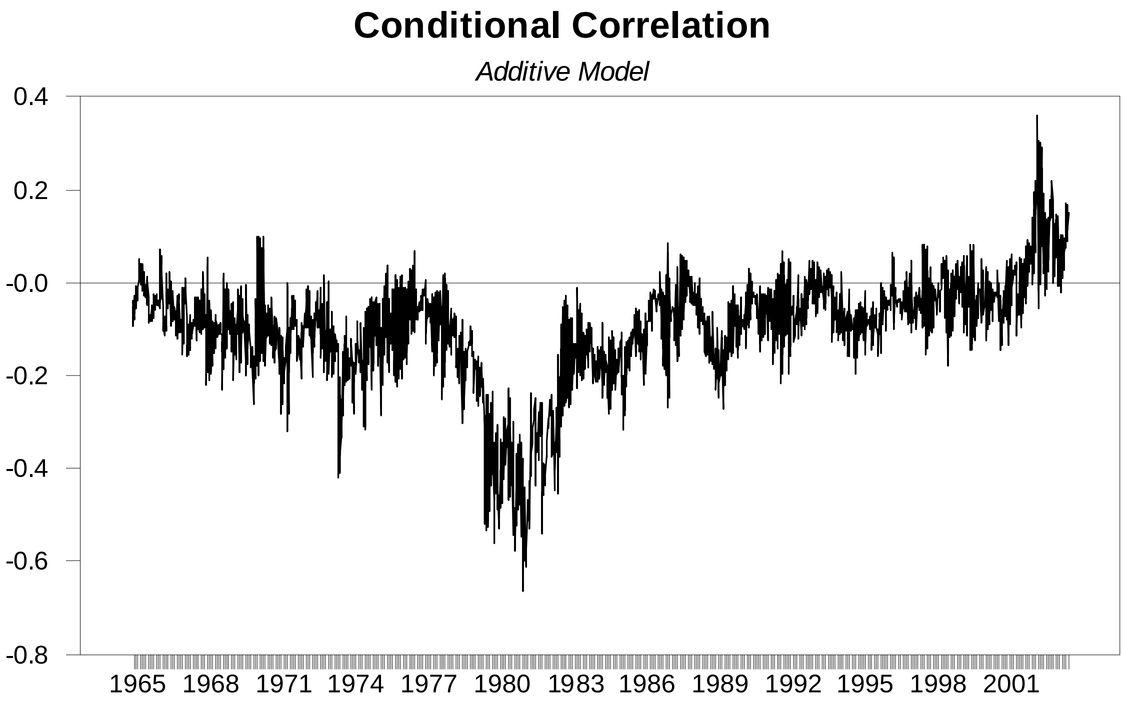


Figure iv: $\hat{\rho}_{12}$

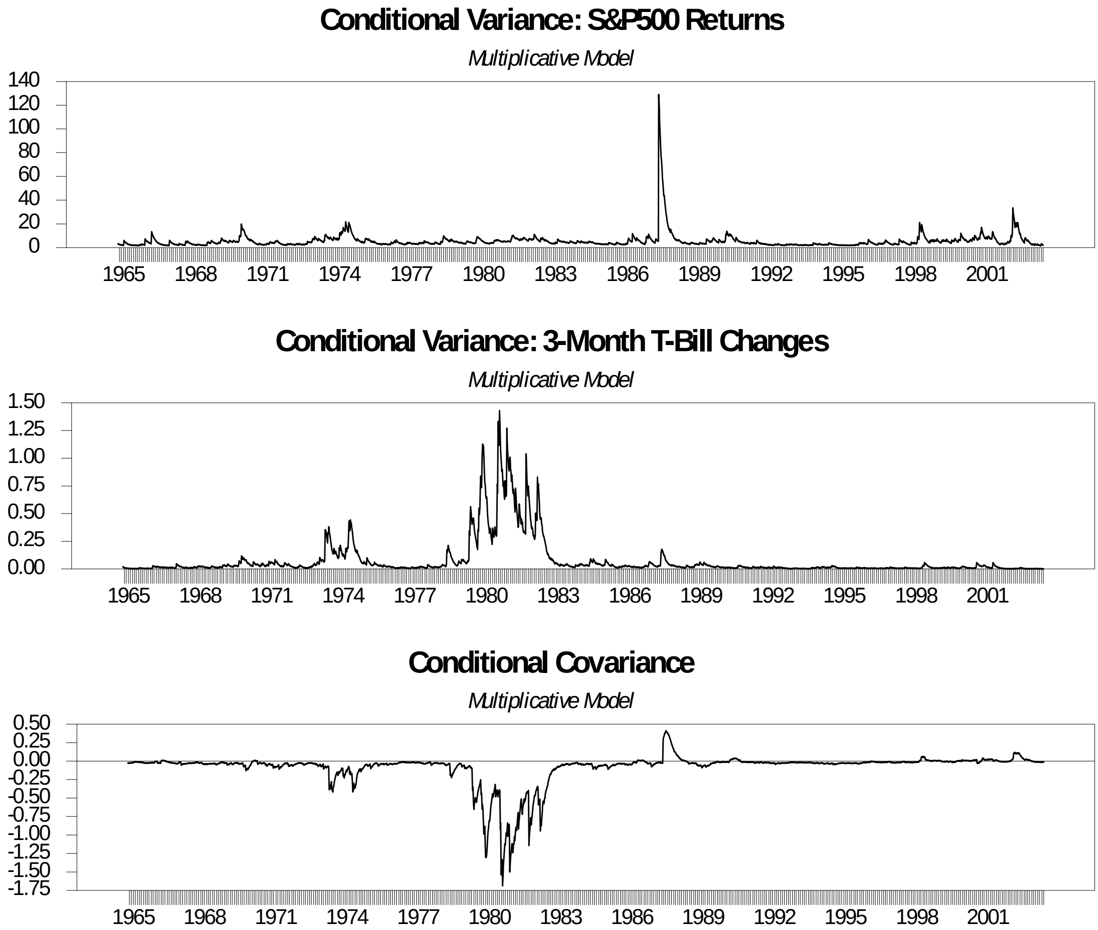


Figure v: $\hat{h}_{11}, \hat{h}_{12}$ and $\hat{h}_{22}$.

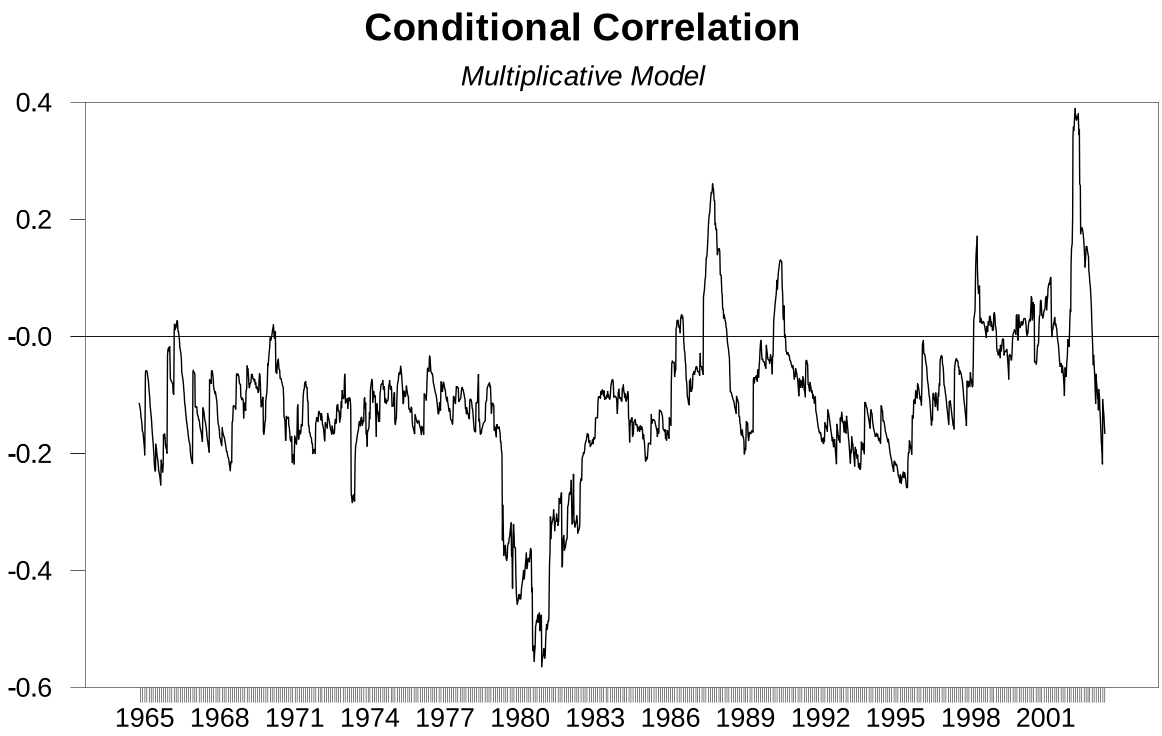


Figure vi: $\hat{\rho}_{12}$

Appendix

Derivation of asymptotically equivalent $T \cdot R^2$ statistic

Rewrite the Lagrange multiplier test statistic under the null hypothesis defined in equation (18)

$$\frac{1}{2}\left\{\sum_{t=1}^{T}\left[\frac{\varepsilon_t^2}{\tilde{h}_t}-1\right]\left[\frac{1}{\tilde{h}_t}\frac{\partial h_t}{\partial \varpi}\right]\right\}'\left\{\sum_{t=1}^{T}\left[\frac{1}{\tilde{h}_t}\frac{\partial h_t}{\partial \varpi}\right]\left[\frac{1}{\tilde{h}_t}\frac{\partial h_t}{\partial \varpi}\right]'\right\}^{-1}\left\{\sum_{t=1}^{T}\left[\frac{\varepsilon_t^2}{\tilde{h}_t}-1\right]\left[\frac{1}{\tilde{h}_t}\frac{\partial h_t}{\partial \varpi}\right]\right\}$$

as

$$LM(\delta^*) = \frac{1}{2}\mathbf{y}'\mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y} \tag{A1}$$

where $\mathbf{y}' = (y_1, ...y_T)$, $y_t = \left[\frac{\varepsilon_t^2}{\tilde{h}_t} - 1\right]$, $\mathbf{X}' = (x_1, ..., x_T)$ and $x_t = \left[\frac{1}{\tilde{h}_t}\frac{\partial h_t}{\partial \varpi'}\right]$.

Consider the coefficient of determination of the regression $\mathbf{y} = \mathbf{X}\boldsymbol{\theta} + \varepsilon$ where $\varepsilon' = (\varepsilon_1, ..., \varepsilon_T)$ and $\varepsilon_t \tilde{} N(0, \tilde{h}_t)$

$$\begin{aligned} R^2 &= \frac{\hat{\mathbf{y}}'\hat{\mathbf{y}} - T\bar{\mathbf{y}}^2}{\mathbf{y}'\mathbf{y} - T\bar{\mathbf{y}}^2} \\ &= \frac{\hat{\boldsymbol{\theta}}'\mathbf{X}'\mathbf{X}\hat{\boldsymbol{\theta}} - T\bar{\mathbf{y}}^2}{\mathbf{y}'\mathbf{y} - T\bar{\mathbf{y}}^2} \\ &= \frac{\mathbf{y}'\mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y} - T\bar{\mathbf{y}}^2}{\mathbf{y}'\mathbf{y} - T\bar{\mathbf{y}}^2}. \end{aligned} \tag{A2}$$

Let $y_t = \left[\frac{\varepsilon_t^2}{\tilde{h}_t} - 1\right]$ so that $\bar{\mathbf{y}} = E\left[\frac{\varepsilon_t^2}{\tilde{h}_t} - 1\right] = 0$ and $T^{-1}\mathbf{y}'\mathbf{y} = T^{-1}\sum_{t=1}^{T}\left[\frac{\varepsilon_t^2}{\tilde{h}_t} - 1\right]^2 \xrightarrow{p} 2$. Simplifying and multiplying both sides of equation (A2) by T gives

$$TR^2 = \frac{1}{2}\mathbf{y}'\mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y} = LM(\delta^*). \tag{A3}$$

This suggests that the $LM(\delta^*)$ test is asymptotically equivalent to TR^2 from an outer product regression of y_t on x_t. Given that the $LM_1(\delta^*)$ test statistic is χ^2 distributed with three degrees of freedom, the asymptotically equivalent TR^2 also follows a χ^2 variate with the same number of degrees of freedom.