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Inequality Comparisons in a Multi-Period Framework: The Role of Alternative Welfare Metrics*

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Abstract

This paper considers the use of alternative welfare metrics in evaluations of income inequality in a multi-period context. Using Norwegian longitudinal income data, it is found, as in many studies, that inequality is lower when each individual's annual average income is used as welfare metric, compared with the use of a single-period accounting framework. However, this result does not necessarily hold when aversion to income fluctuations is introduced. Furthermore, when actual incomes are replaced by expected incomes (conditional on an initial period), using a model of income dynamics, higher values of inequality over longer periods are typically found, although comparisons depend on inequality and variability aversion parameters. The results are strongly influenced by the observed high degree of systematic regression towards the (geometric) mean, combined with a large extent of individual unexpected effects.

Keywords: income mobility, welfare evaluations, relative mobility process
JEL: D31; C23; I31

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1 Introduction

Evaluations of changes in the distribution of income must begin by deciding on a number of fundamental ingredients, each of which involves value judgements. First, a choice of 'welfare metric', concerning what is to be measured for each unit of analysis, must be made. Secondly, a decision is needed regarding the time period of analysis. Third, the unit of analysis itself must be chosen. Finally, the form of 'social evaluation function', which encapsulates further explicit distributional value judgements, has to be specified. The present paper explores the use of alternative welfare metrics in a multi-period context, using the individual as the basic unit of analysis and an additive, individualistic Paretean social welfare function reflecting belief in the 'principle of transfers' (whereby a transfer from relatively rich to poor individuals, leaving their rankings unchanged, is considered an improvement). The welfare metrics are based on alternative income concepts rather than, say, consumption or utility measures which allow for variations in the value of leisure time.

Consideration of a multi-period context necessarily introduces the role of income mobility. This implies that inequality of income measured over a longer period is lower than that in the highest single year.[1] A further argument concerns comparative static changes: if higher annual income inequality is associated with increased relative income mobility, it is possible that inequality of income measured over several years is lower. Hence, longer-period inequality may fall, and welfare might increase, despite the rise in annual income inequality: this is referred to as a 'mobility offsetting' argument. However, the welfare metric could allow for other effects.[2] For example, if there is imperfect substitutability of incomes over time (Atkinson, Bourguignon and Morrisson, 1992) and individuals are averse to income variability, the offsetting argument is weakened (Creedy and Wilhelm, 2002).

The discussion has so far been in terms of *ex-post* income measures. Hence, yet another approach is to attempt to allow explicitly for the uncertainty associated with mobility by constructing a welfare metric based on an *ex-ante* income measure. This in turn requires the use of a model of expectations formation based on observed income dynamics. The association between mobility and ex-ante income uncertainty has also

[1] Conditions under which inequality is lower than in all years are examined by Creedy (1997).

[2] The question of whether income mobility represents equality of opportunity, as in Bénabou and Ok (2001a), is not considered here.

been stressed by Parker and Rougier (2001), Gottschalk and Spolaore (2002)[3] and Ben-Shahar and Sulganik (2008).[4]

This paper presents results where expected income is derived by estimating an autoregressive model of income dynamics. A closed-form expression for expected income, conditional on initial income, is obtained. Thus a 'rational expectations' approach is used, whereby individuals are assumed to form expectations based on the dynamic model of incomes and associated parameter estimates. The model specifies individual income in a given period as a function of the relative distance from the geometric mean of a previous period's income, an individual fixed effect and a stochastic component. The social welfare function, and hence distributional value judgements, examined are based on the Atkinson (1970) inequality index. To illustrate the framework, longitudinal data for individuals in Norway over the period 1993–2005 are used.

In Section 2 the data and the Atkinson index are briefly described. Section 3 presents results using *ex-post* welfare metrics. Section 4 introduces *ex-ante* income uncertainty and presents a procedure for using expected future incomes in the welfare metric. Section 5 summarises the main findings.

2 Data and Inequality Measurement

2.1 The Data

The data used below come from *Income Statistics for Persons and Families in Norway 1993–2005* (Statistics Norway, 2006). These data contain register-based information on the whole population, derived primarily from information retrieved from all income tax returns in the Directorate of Taxes' Register of Personal Tax-Payers.

The income measure is annual income after tax. Thus income is defined as labour income, plus positive capital income, plus net capital gains, plus transfers minus direct taxes. This is the definition used in all official income statistics. Negative capital income (interest paid on mortgages) is not included in the definition because there is no

[3] They present a decomposition analysis where the extent to which future incomes depend on current income is separated from effects due to rank reversals. For other decompositions, see Ruiz-Castillo (2004), Van Kerm (2004) and Jenkins and Van Kerm (2006).

[4] Other studies involving mobility and long-term incomes include, for example, Shorrocks (1978a), Chakravarty, Dutta and Weymark (1985), Fields (2010) and Hungerford (2011). For surveys of mobility, see Atkinson, Bourguignon and Morrisson (1992), Maasoumi (1998) and Fields and Ok (1999).

corresponding income from housing in the statistics. Estimates of income mobility are typically sensitive to persons entering and leaving the labour market. Hence, persons under age 26 and above 65 are excluded, and those with an income below 100NOK[5] in any year are excluded. The effects of inflation have been removed by deflating all incomes to the 1998 level using the consumer price index.

2.2 The Atkinson Inequality Measure

Let individual i's income (the welfare metric, ignoring the time period at this point) be denoted y_i, for $i = 1, ..., n$. The Atkinson inequality measure is based on the use of an additive social welfare function, $W = W(y_1, ..., y_n)$ of the form:

$$W = \sum_{i=1}^{n} U(y_i) \tag{1}$$

where $U(y_i)$ is the weight attached to y_i, and is specified as:

$$U(y_i|\,\varepsilon) = \begin{cases} \frac{y_i^{1-\varepsilon}}{1-\varepsilon} & \text{for} \quad \varepsilon \neq 1 \\ \ln y_i & \text{for} \quad \varepsilon = 1 \end{cases} \tag{2}$$

Hence $\varepsilon \geq 0$ captures the concavity of U, corresponding to the aversion to relative inequality. Let y_{EDE} denote the equally distributed equivalent income, that is, the income which, if obtained by each person, gives the same social welfare as the actual distribution. Hence, the equally distributed equivalent is given by:

$$y_{EDE} = \begin{cases} \left(\frac{1}{n}\sum_{i=1}^{n} y_i^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}} & \text{for} \quad \varepsilon \neq 1 \\ \exp\left(\frac{1}{n}\sum_{i=1}^{n} \ln y_i\right) & \text{for} \quad \varepsilon = 1 \end{cases} \tag{3}$$

Atkinson's index of inequality, I, is the proportional difference between the arithmetic mean, $\bar{y}$, and y_{EDE}, so that:

$$I = \frac{\bar{y} - y_{EDE}}{\bar{y}} \tag{4}$$

and I reflects the 'wastefulness of inequality'. Clearly, $y_{EDE} = \bar{y}(1 - I)$ which, as the 'abbreviated welfare function', illustrates the nature of the 'trade-off' between 'equity', $1 - I$, and efficiency, $\bar{y}$.

[5] This is equivalent to US$15.50 using 2005 exchange rate.

3 Alternative Ex-post Evaluations

Figures 1 and 2 show, for the period 1993–2005 and for two inequality aversion parameters, the time profiles of inequality and the equally distributed equivalent. The period may be divided into two periods of equal length, 1994–99 and 2000–05.[6] The first period reflects a relatively stable degree of inequality while the second period displays more variability, initially decreasing and then increasing steadily. The increase appears to be associated largely with changes in the distribution of dividend income. In particular, the year 2001, when annual inequality was relatively low, is an exception because a temporary tax on dividend income was in place. The steady rise in annual equally distributed equivalent income is associated with growth in annual average income, although this was relatively small during 2000–01.

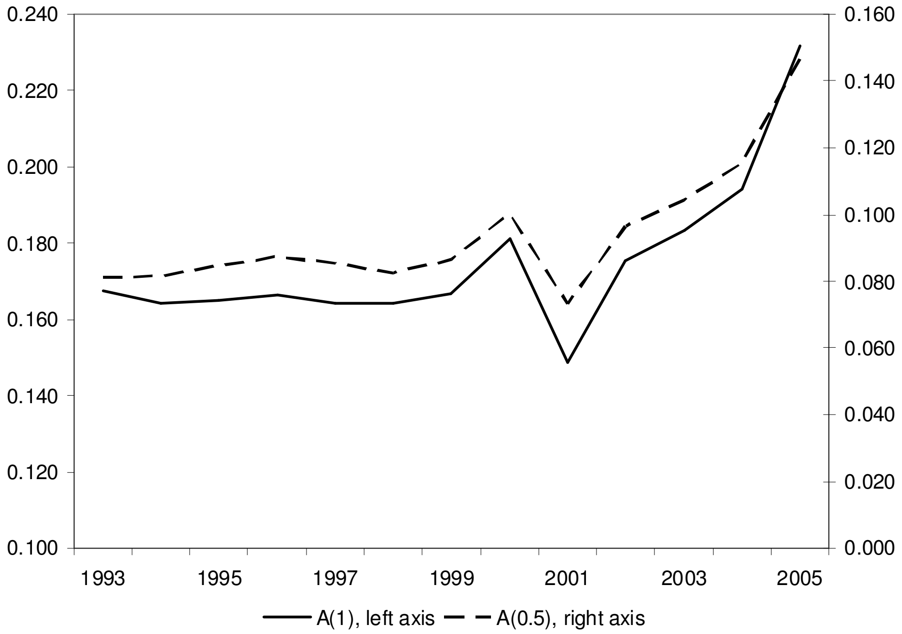

Figure 1: Inequality of Annual Income: Atkinson Index 1993-2005, $\varepsilon = 0.5$ and $\varepsilon = 1$

A time subscript must now be added to each individuals' income. For convenience, the following ignores discounting. Consider first an *ex-post* evaluation over T periods

[6]Although 1993 is not therefore used in inequality comparisons, this year is used as a base year when estimating ex-ante incomes.

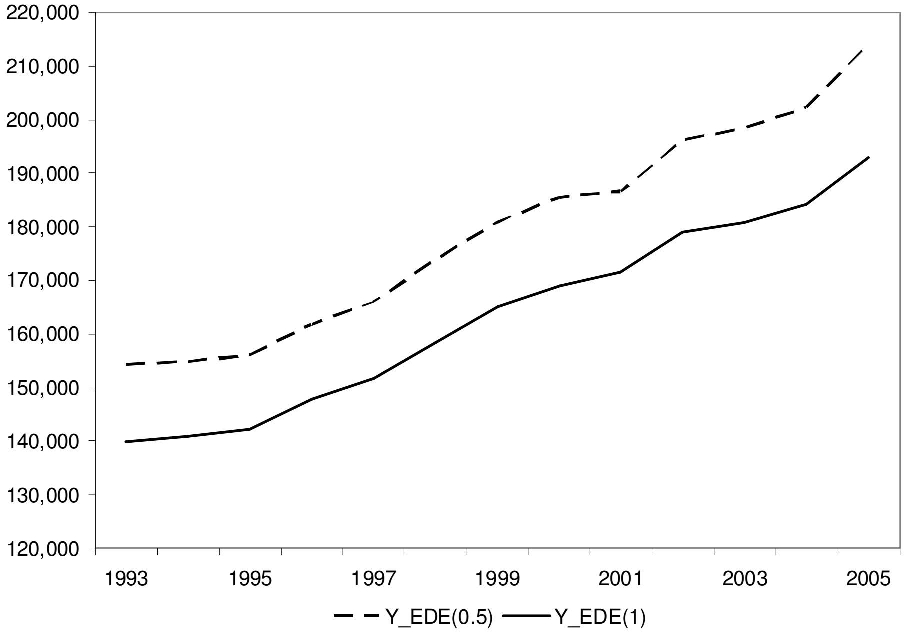

Figure 2: Equally Distributed Equivalent Annual Income: 1993-2005

which uses as welfare metric for each individual the annual average income, $\bar{y}_i = \frac{1}{T}\sum_{t=1}^{T} y_{it}$. Hence the welfare function is not actually concerned with the way in which any individual's income is distributed over the time period, and thus may be said to reflect a lack of concern for the nature of the mobility process. For the period 1994–2005, the use of the annual average as welfare metric for each individual gives Atkinson inequality measures of 0.076, 0.134 and 0.210 respectively for values of ε of 0.5, 1.0 and 1.5. These each reflect value judgements which tolerate substantial leaks in making equalising transfers.[7] These values may be compared with the annual average inequality measures of 0.099, 0.181 and 0.298 respectively, showing that the use of a longer period whereby individual incomes are averaged is in this case equalising.

Further details for the two sub-periods are shown in Table 1. For the second period, the absolute reduction in inequality when using annual average income as the welfare metric compared with annual average inequality, is double the reduction obtained for the first period. The inequality reducing effects of using a longer accounting period,

[7] For example, if 1 unit is taken from A to make a transfer to B, where A is twice as rich as B, a transfer of 0.5 units leaves social welfare unchanged if $\varepsilon = 1$. This falls to 0.25 units if $\varepsilon = 2$.

Table 1: Annual Average Income Inequality and Inequality with Annual Average as Welfare Metric

	Period 1994–1999			Period 2000–2005		
	$\varepsilon = 0.5$	$\varepsilon = 1.0$	$\varepsilon = 1.5$	$\varepsilon = 0.5$	$\varepsilon = 1.0$	$\varepsilon = 1.5$
Inequality: Annual average of single year values						
$\bar{I}_y$	0.084	0.165	0.263	0.108	0.188	0.307
Using each individual's annual average as welfare metric						
$I_{\bar{y}}$	0.071	0.134	0.183	0.082	0.147	0.228

mentioned above, therefore appears to be greater in a period when annual inequality is generally increasing.[8] However,the percentage reduction in inequality is larger in the first period for $\varepsilon = 1.5$. This arises because the very high degree of inequality aversion places more emphasis on the low end of the income distribution. Table 1 does not report standard errors because the data cover the whole population.

The income mobility which produces the increasing annual inequality is thus also responsible for reducing the inequality of a multi-period income measure (each person's annual average) below average annual inequality. However, mobility may not necessarily be seen as beneficial from an individual's point of view. It may also be seen as an undesirable source of economic instability. For example, individuals may for some reason be unable to smooth consumption over time when facing income fluctuations and they might be averse to such variability in income. Imperfections of capital markets or other constraints may prevent individuals from smoothing consumption over time; see Atkinson, Bourguignon and Morrison (1992).[9]

It may therefore be desired to allow, in the welfare metric, for an aversion to income variability, as suggested by Creedy and Wilhelm (2002); see also Jarvis and Jenkins (1998) on the disutility of income volatility. This can be done, in an *ex-post* context,

[8] This of course differs from the comparative static argument discussed in the introduction, and examined in detail by Creedy and Wilhelm (2002) in terms of inequality and social welfare.

[9] Shorrocks (1978a) argues that mobility is always desirable, whereas Chakravarty, Dutta and Weymark (1985) establish a no-mobility hypothetical benchmark from which they can distinguish between desirable and undesirable mobility. Like King (1983), they make use of the equally distributed equivalent idea. The difficulties of establishing a reasonable social welfare understanding of income mobility is discussed by Atkinson (1981), Dardanoni (1993) and Fields (2010).

by using instead of $\bar{y}_i$ a welfare metric, $\widetilde{y}_i$, defined as:

$$\widetilde{y}_i = \frac{1}{T} \sum_{t=1}^{T} \frac{y_{it}^{1-\gamma}}{1-\gamma} \tag{5}$$

The parameter γ measures the degree of aversion to variability of income over time, and the same parameter is assumed to apply to all individuals. As *ex-post* values are used, the aversion coefficient, γ, is not interpreted in terms of risk aversion: this is discussed in Section 4 below. The relative values of ε and γ determine whether inequality aversion (of the judge whose value judgements are represented by the welfare function) is high enough to overcome the individuals' aversion to income variability over time. When aversion to income variability is high relative to inequality aversion, a more 'static' society is preferred, in which income is more stable at the 'cost' of higher inequality of multi-period income.

Table 2: Atkinson Inequality Measures with Aversion to Income Fluctuations

	Period 1994–1999			Period 2000–2005		
	$\varepsilon = 0.5$	$\varepsilon = 1.0$	$\varepsilon = 1.5$	$\varepsilon = 0.5$	$\varepsilon = 1.0$	$\varepsilon = 1.5$
$\gamma = 0$	0.071	0.134	0.217	0.082	0.147	0.289
$\gamma = 0.5$	0.085	0.148	0.236	0.098	0.161	0.311
$\gamma = 2.0$	0.130	0.206	0.349	0.144	0.217	0.375
$\gamma = 3.0$	0.151	0.235	0.427	0.166	0.244	0.499

Table 2 shows the extent to which the values in Table 1 are increased when aversion to intertemporal fluctuations is introduced. In order eliminate the effect of general income growth over time, incomes were adjusted so that average annual income is constant (and equal to the overall mean) in each period. Hence the inequality values in the first row of Table 2 differ somewhat from those in the final row of Table 1. The inequality differences between the first and second sub-periods are less influenced by aversion to income variability over time. The absolute differences for $\gamma = 0$ and positive γs are similar in the two sub-periods.

4 An Ex-ante Perspective

4.1 The Welfare Metric

The suggestion that relative income mobility is associated with uncertainty leads to the idea that an alternative evaluation may be based on an ex-ante measure, rather than ex-post incomes as in the previous section. For example, Shorrocks (1978b, p.1016) argues that 'interest in mobility is not only concerned with movement but also predictability'. Furthermore, the uncertainty aspect of mobility is emphasised by contributions which see mobility in terms of future opportunities, as in Bénabou and Ok (2001a), or account for origin independence, as in Gottschalk and Spolaore (2002).

The approach considered here is to replace the above welfare metric with one defined in terms of expected incomes, conditional on income in a specified period, $E\left(y_{it}\right|y_{i0})$, so that (5) is replaced by:

$$E\left(\widetilde{y}_i\right) = \frac{1}{T}\sum_{t=1}^{T}\frac{E\left(y_{it}\right|y_{i0})^{1-\gamma}}{1-\gamma} \tag{6}$$

Here the parameter, γ, can be interpreted in terms of risk aversion. In a one commodity setting and with indifference with respect to the timing of risk, risk aversion is the inverse of the elasticity of intertemporal substitution. Thus resistance to intertemporal substitution, or variability aversion, is closely related to risk aversion.

Application of this approach therefore requires knowledge of the conditional expectation of future incomes. The following subsection proposes a measure of expected income obtained by modelling the income process.

4.2 Modelling Income Dynamics

Consider a dynamic process containing both a stochastic component and a component in which changes depend on the position of individuals relative to the geometric mean; see also Creedy (1985) and Creedy and Wilhelm (2002). As before, y_{it} denotes individual i's income in period t, and let μ_t denote the mean of logarithms in period t, with $m_t = \exp\left(\mu_t\right)$ as the geometric mean. The income process can be written as:

$$y_{it} = \left(\frac{y_{it-1}}{m_{t-1}}\right)^{\beta}\exp\left(\mu_t + v_i + \eta_{it}\right) \tag{7}$$

Table 3: Parameter Estimates for the Income Mobility Process

Method and Parameter	All years	1994-1999	2000-2005
LSDV			
β	0.452	0.279	0.227
σ_η	0.276	0.280	0.282
AB-GMM			
β	0.492	0.477	0.351
σ_η	0.415	0.308	0.316
GMMSYS			
β	0.476	0.486	0.387
σ_η	0.419	0.309	0.313

where the stochastic component consists of an individual-specific effect, v_i, and a random component, η_{it}, assumed to be independent of income, with a zero mean and variance in each period of σ_η^2. Equation (7) can be rewritten as:

$$(\log y_{it} - \mu_t) = \beta \left(\log y_{it-1} - \mu_{t-1}\right) + v_i + \eta_{it} \qquad (8)$$

The autoregressive parameter, β, captures variations in income that decline more slowly over time. In other words it reflects movements in income that, while not permanent, tend to persist for several years. Suppose also that in this simple income process, the autoregressive parameter and income variance is common for all individuals, and heterogeneity in the process is represented through the individual fixed effect (individual fixed level relative to the mean) and the error term.

Table 3 reports results of using several estimators. These include the least squares dummy variables (LSDV), and generalised method of moments (GMM) estimators as in Arellano and Bond (1991) and Blundell and Bond (1998). Because the lagged dependent variable is correlated with the error term, it has been shown that the use of LSDV result in biased estimates. Anderson and Hsiao (1981) suggested first eliminating the fixed effect by taking first differences, and then using y_{t-2} as instrument for Δy_t. However, this does not exploit all the relevant moment conditions so it is not the efficient GMM estimator. Arrelano and Bond (1991) derived all of the relevant moment conditions from the dynamic panel data model to be used in GMM estimation. This estimator is known as the Arrelano-Bond GMM estimator. Other instruments have been suggested by a succession of researchers, such as the Arellano and Bover/Blundell and Bond system estimator, which uses moment conditions in which lagged first differences of the dependent variable are instruments for the level equation. In practice, it is difficult to find good instruments for the first-differenced lagged dependent variable, which can itself create problems for the estimation. Kiviet (1995) shows that panel data models using instrumental variable estimation often lead to poor finite sample efficiency and bias. Also, tests shows that none of the methods reject the assumption of no autocorrelation in first differenced errors.

However, the major aim of the present paper is not to estimate a dynamic model, otherwise a more sophisticated econometric framework, with a richer specification of error terms and heterogeneous parameters, would have been used. Here, it is required to produce a simple model of individuals' expectations of future incomes. Table 3 show that a common result for all specifications is that the estimated value of β is higher in the first sub-period than in the second sub-period, while estimate of σ_η^2 is higher in the second sub-period. The lower degree of regression towards the mean and lower variance in the first period implies lower income mobility, but also higher predictability of future incomes. Conversely, the parameter values imply higher mobility and less predictability in the second sub-period. In the following subsection, reported results are based on the Arrelano-Bond GMM estimator.

4.3 Inequality Using the Ex-ante Welfare Metric

In order to obtain measures for the contribution of the estimated income process to the overall ex-ante welfare evaluation, a closed-form expression for expected income as a function of income in the initial period, $E\left(y_{it}|y_{i0}\right)$, is required. It is shown in the Appendix that:

$$E\left(y_{it}|y_{i0}\right) = \exp\left\{\mu_t + \beta^t\left(\log y_{i0} - \mu_0\right) + v_i\left(\frac{1-\beta^t}{1-\beta}\right) + \frac{1-\beta^{2t}}{2\left(1-\beta^2\right)}\sigma_\eta^2\right\} \tag{9}$$

where estimates of individual fixed-effects, v_i, are obtained using their sample counterparts. The corresponding equally distributed equivalent in terms of expected income, $EDE_{E(y|y_0)}$, can be expressed as:

$$EDE_{E(y|y_0)} = \left[\frac{1}{n}\sum_{i=1}^{n}\left(\frac{1}{T}\sum_{t=1}^{T}E\left(y_{it}|y_{i0}\right)^{1-\gamma}\right)^{\frac{1-\varepsilon}{1-\gamma}}\right]^{\frac{1}{1-\varepsilon}} \tag{10}$$

from which, given the arithmetic mean, the Atkinson measure can be obtained in the usual way. In this case it depends on the degree of regression towards the mean, the income variance, the degree of aversion to inequality and the degree of aversion to fluctuations in income. When $\beta < 1$, the initial (relative) position is given less weight over time, while the role of the individual specific position is increasing over time. Expected income is also increasing over time. This is due to the particular loglinear functional form chosen for the income process.

Table 4: Atkinson Inequality Indices for Expected Income

	Period 1994–1999			Period 2000–2005		
	$\varepsilon = 0.5$	$\varepsilon = 1.0$	$\varepsilon = 1.5$	$\varepsilon = 0.5$	$\varepsilon = 1.0$	$\varepsilon = 1.5$
$\gamma = 0$	0.004	0.068	0.135	0.109	0.165	0.218
$\gamma = 0.5$	0.037	0.095	0.165	0.150	0.208	0.246
$\gamma = 2.0$	0.132	0.221	0.417	0.250	0.367	0.492
$\gamma = 3.0$	0.166	0.394	0.513	0.290	0.481	0.603

The inequality measures for the *ex-ante* welfare metric are shown in Table 4, where again any effects of income growth are eliminated by maintaining arithmetic mean constant. These may be compared with Table 2. For the sub-period 1994–99, inequality is lower for all values of ε examined and for the variability aversion coefficients of 0 and 0.5. For the very high values of γ of 2.0 and 3.0, inequality is higher when the ex-ante measure is used, particularly for the high inequality aversion coefficient. The estimated value of β is rather low while that of σ_η^2 is high. The considerable variability implied by the high σ_η^2 would produce increasing annual inequality over time, without the low value of β, implying considerable regression towards the mean. In the expression for $E(y_{it}|y_{i0})$, the effects of terms involving powers of β rapidly become insignificant. Expected incomes are dominated by the high σ_η^2 which, for the high mobility-aversion cases, implies a higher measured inequality. From (9), setting all terms involving β^t and β^{2t} to zero and rearranging gives:

$$\log\{E(y_{it}|y_{i0}) - \mu_t\} = \frac{v_i}{1-\beta} + \frac{\sigma_\eta^2}{2(1-\beta^2)} \tag{11}$$

Hence the variance of logarithms of expected income soon becomes $\sigma_v^2/(1-\beta)^2$, where σ_v^2 is the variance of the fixed effect in the autoregressive income-generation equation.

Therefore for higher σ_v^2 and lower β, as in the second sub-period considered, inequality of expected values is quickly increasing towards a relatively high value.

In the *ex-post* case, there is less inequality than anticipated as a result of the regression towards the mean. For the second sub-period, the role of unanticipated, but systematically equalising, mobility is even greater and β is lower. Hence the *ex-ante* welfare metric produces higher inequality, for nearly all combinations of variability aversion and inequality aversion parameters, than for the *ex-post* metric. The exceptions are for the combination of low variability aversion with very high inequality aversion. Also, the inequality differences between the two sub-periods are maintained or increased when moving from the *ex-post* to the *ex-ante* perspective.

It may be argued that it is difficult to interpret the results for the two periods in Tables 3 and 4 in terms of different income processes because they begin with different initial distributions. For this reason two sensitivity analyses were carried out. First, it was assumed simply that $(\log y_{i0} - \mu_0) = 0$ for all individuals (so that the fixed effect is the only individual variation). Second, the same initial distribution was used in both periods (hence, the second period process was estimated using the initial 1993 distribution. Unreported results show that the results are not sensitive to the choice of initial distributions. This lack of sensitivity is likely to arise because of the high degree of regression towards the mean.

Also, in (8) there are no other explanatory variables, such as age, family composition and education, but it is well known that income is systematically related to age. Also, education influences the overall income level as well as the age-income profile. Family composition is another variable that may affect individual income, especially for women. These variables have not been used because they substantially complicate the prediction of future incomes. While age is straightforward to predict, predicting future family composition is less so. Education is challenging too, as the specification already accounts for a fixed effect. Thus, fixed effects soak up much of the explanatory power of variables that are either time-invariant or close to time-invariance. Regressions were in fact run using other explanatory variables (such as age, family composition and education) which were found to lower the estimated autoregression coefficient somewhat in all specifications, but did not change the overall result. This suggests that the main differences between income mobility in the two periods' is due to genuine income dynamics rather than, for instance, substantial differences in family dynamics. Comparisons were also made using alternative income definitions. Labor income yields similar estimates for the autoregression coefficient, but exhibits a much larger variance. For gross income there is less regression towards the mean than for the two other income definitions (that is, higher β), and the difference between the two periods is larger. Also, as expected, the standard deviation of gross income is higher than for income after tax.

5 Conclusions

The aim of this paper has been to consider the use of alternative welfare metrics in evaluations of income inequality when a multi-period income measure is used, and hence relative income mobility plays a crucial role in influencing the relationship between short- and long-period inequality. One basic approach, most commonly adopted in income distribution studies, is to base measures on *ex-post* magnitudes. Using Norwegian longitudinal income data, it was found, as in many studies, that income inequality is lower when each individual's annual average income is used as welfare metric, compared with the use of a single-period accounting framework. However, the longer accounting period can produce both lower and higher inequality than annual measures, depending on the assumed degree of aversion to income fluctuations over time.

The second approach took as its starting point the argument that relative income mobility introduces uncertainty about future incomes, so that it may be desired to evaluate inequality using an *ex-ante* approach. To this end, a regression model of income dynamics was used in order to generate individuals' expected values of future income, conditional on actual income in a specified initial period. The use of expected incomes was found generally to produce higher values of inequality over longer periods, although again comparisons depend on the assumption regarding the aversion to income inequality of the social welfare function, and aversion to income fluctuations on the part of individuals. The results were strongly influenced by the observed high degree of systematic regression towards the (geometric) mean, combined with a large extent of random proportional income changes. The distinction between expected and unexpected mobility was thus found to be important.

In the choice of welfare metric there is of course no single 'correct' approach, and the contribution of the economist is to investigate the implications of adopting alternative value judgements. The present paper is therefore in this spirit of extending the range of value judgements which can be examined.

Appendix A: Expected Income

This appendix derives the expected value of an individual's income, conditional on income in an earlier period. Define z_t, dropping individual subscripts, as:

$$z_t = (\log y_t - \mu_t) \tag{A.1}$$

Inserting z_t in equation (8) gives:

$$z_t = \beta z_{t-1} + v + \eta_t \tag{A.2}$$

Backwards induction yields:

$$\begin{aligned} z_t &= v + \eta_t + \beta\left(v + \eta_{t-1}\right) + \beta^2\left(v + \eta_{t-2}\right) + ... + \beta^{k-1}\left(v + \eta_{t-k+1}\right) + \beta^t z_{t-k} \\ &= \beta^t z_{t-k} + \sum_{s=0}^{k-1} v\beta^s + \xi_t \end{aligned} \tag{A.3}$$

where:

$$\xi_t = \eta_t + \beta\eta_{t-1} + \beta^2\eta_{t-2} + ... + \beta^{k-1}\eta_{t-k+1} \tag{A.4}$$

Using $y_t = \exp\left(z_t + \mu_t\right)$ from (A.1):

$$y_t = \exp\left\{\mu_t + v\left(\frac{1-\beta^k}{1-\beta}\right) + \xi_t + \beta^t\left(\log y_{t-k} - \mu_{t-k}\right)\right\} \tag{A.5}$$

The variance of $\log y_t$ is thus equal to $Var\left(\xi_t\right)$, which, if η is normally distributed, is from (A.4) given by a weighted sum of normal variables. This is also normally distributed, with $Var\left(\sum_i a_i X_i\right) = \sum_i a_i^2 Var X_i$. Hence the variance of $\log y_t$ is:

$$Var\left(\sum_{s=0}^{k-1} v\beta^s\right) = \sigma_\eta^2 \sum_{s=0}^{k-1} \beta^{2s} = \frac{1-\beta^{2k}}{1-\beta^2}\sigma_\eta^2 \tag{A.6}$$

In general, if the variable x is lognormally distributed with mean and variance of logarithms of m and s^2 respectively, then $E\left(x\right) = \exp\left(m + \frac{s^2}{2}\right)$. Taking expectations of y, gives:

$$E\left(y_t | y_{t-k}\right) = \exp\left\{\mu_t + \beta^t\left(\log y_{t-k} - \mu_{t-k}\right) + v\left(\frac{1-\beta^k}{1-\beta}\right) + \frac{1-\beta^{2k}}{2\left(1-\beta^2\right)}\sigma_\eta^2\right\} \tag{A.7}$$

This is the result given in (9) above.

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