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Quality Gaps

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Summary. In this paper I consider a monopolistic screening model with continuum of types when the type set is a disconnected subset of the real line. I prove that the product line remains connected provided that the gap in the type space is sufficiently small. I also use the results to show that the inverse screening problem may be ill-defined.

Keywords and Phrases: screening, product line, quality gap.

JEL Classification Numbers: C0, D8

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1 Introduction

In this paper I revisit the monopolistic screening model of Mussa and Rosen (1978). Screening models first attracted researches in the middle of the seventies. The pioneering paper in the area is Mirrlees (1971). It uses a monopolistic screening model to study the question of the optimal taxation in presence of private information on the side of the taxpayers.

In the following years monopolistic screening models found numerous applications in economics. For examples, see the papers by Adams and Yellen (1976), Mussa and Rosen (1978), Mirman and Sibley (1980), Baron and Myerson (1982), Sappington (1983), Maskin and Riley (1984), and Laffont, Maskin, Rochet (1987), among others.

These papers could be differentiated on two dimensions: the economic application they consider and the way they capture the private information. Private information was captured in the literature in three different ways: assuming a discrete set of types (e.g. Adams and Yellen, 1976), a unidimensional continuum of types (the majority of the papers in the literature), and a multi-dimensional continuum of types (e. g. Laffont, Maskin, Rochet, 1987). The reader is referred to Mas-Colell, Whinston, and Green (1995) for a textbook treatment of the model with a discrete type space, Fudenberg and

Tirole (1992) for a treatment of the model with a unidimensional continuum of types, and to Basov (2005) for an up-to-date description of the mathematical techniques, the main economic results, and the literature review of the multi-dimensional models.

In this paper I consider a model with a unidimensional continuum of types. For the concreteness of the exposition I concentrate on the case when a monopolist produces an indivisible good of different qualities. The consumers are interested in buying at most one unit of the good and have different tastes for the quality, which is their private information.

A common feature of all models with a continuum of types is that the type space is assumed to be connected. From the economic point of view this assumption implies that though tastes are heterogenous, the consumers cannot be divided into different distinct groups. In this paper I, on the contrary, assume that the consumers can be divided into two distinct groups, for example, high income and low income consumers. The marginal rate of substitution between quality and money for the representatives of the groups is always higher than for the representatives of the other. However, there is some taste heterogeneity within each group. Formally, this is modelled by assuming that the type space is a union of two disconnected segments.

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Generalization for more than two groups is straightforward.

The first main result of the paper is:

(R1) *The optimal product line is connected provided that the taste gap between two groups is sufficiently small.*

The result implies that if the groups are not sufficiently differentiated then if the monopolist offers two qualities in the equilibrium, she will also offer all the intermediate qualities. This result generalizes one of the most celebrated results in the models with connected type space: the connectedness of the product line. For those models it is a direct consequence of the *continuity* of the allocation as a function of the type. In the unidimensional the continuity of allocation was first proven by Mussa and Rosen (1978). The statement of the result in the multidimensional case can be found in Rochet and Chone (1998). For a proof in this case, see Carlier and Lachand-Robert (2001).

The main driving force behind the first result is the second order implementability condition, which states that to be implementable the allocation of quality should be weakly increasing. To understand the intuition, imagine that the type space is a union of two non-intersecting segments. If there is a quality gap then the consumers on the two segments of the type space should be served independently of each other. This implies that the con-

sumers on the right end of the left segment are served efficiently, while the allocation on the left end of the right segment is downward biased. For a sufficiently small gap between the segments, this will contradict the second order implementability constraint.

The second main result investigates the behavior and the regularity properties of the optimal tariff. I impose enough regularity on the data of the problem to rule out the need for the ironing procedure of Mussa and Rosen (1978). I also assume that the cost of production and the density of types are sufficiently well behaved. Under these conditions for a connected type space the optimal tariff will be continuously differentiable. If the types space is disconnected, however, the following result holds:

(R2) *Let the type space be disconnected, while the optimal product line be connected. Then the optimal tariff has $n-1$ kinks, where n is the number of the components of the type space.*

The above results can be used to study the inverse screening problem, i. e. the problem of restoring the types distribution from the observed tariff and product line. Solving the inverse screening model is important for evaluating the deadweight loss generated by the existence of the private information, which could be vital for a decision whether the monopoly (e. g.

an electricity or gas company) should be regulated. The third main result of the paper is:

(R3) *Assume that the optimal tariff has a kink. Then the inverse screening problem is ill-defined.*

As we will see below, if the optimal tariff has a kink not only it is impossible to find the unique types distribution that generates the given tariff, it is even impossible to restore the types space. To understand the reason for this note that at the kink point the subdifferential of the tariff is not single-valued. Therefore, the allocation is not invertible, i. e. a convex set of types will opt for the same quality. One cannot, however, infer uniquely the distribution of types over this convex set, or even whether there are any consumers whose types fall within it. We will see below that assuming that the type space connected when in fact it is not can lead to a serious overestimation of the fraction of the low types and, as a result, of the welfare costs of imperfect information.

It is worth noting that the lack of differentiability of the tariff is a necessary and sufficient condition for the inverse screening problem to be *essentially* ill-defined. Indeed, if the tariff is continuously differentiable the reader can easily convince herself that it is possible to determine uniquely the set

of the consumers who are served at equilibrium and the distribution of types over the participation region. It is also possible to determine the measure of the exclusion region. Though the exact distribution of the consumers over the exclusion region cannot be found, it will not affect any variables of economic interest (e. g. the monopolist's profits or the dead-weight losses). Therefore, I will say that in this case the inverse screening problem is essentially well defined.

The paper is organized in the following way. Section 2 introduces the general model. In Section 3 I solve some numerical examples. Section 4 discusses the inverse screening problem. Section 5 concludes.

2   The model

Consider a continuum of consumers each of whom is interested in buying at most one unit of an indivisible good. Different units of the good may, however, differ in quality, x. The marginal rate of substitution between quality and money, α, does not depend on quality but differs across the consumers, i. e. the utility has a form

$$u(\alpha, x, t) = \alpha x - t, \tag{1}$$

where t is the amount paid to the monopolist. I assume that α is private information of the consumer. However, it is common knowledge that α is distributed on $\Omega = [0, a] \cup [b, d]$ according to a density $f(\cdot)$. We assume that f is twice differentiable and strictly positive. Define the virtual type

$$v(\alpha) = \alpha - \frac{1 - F(\alpha)}{f(\alpha)}, \tag{2}$$

and assume that it is strictly increasing in α. This assumption will allow us to reduce the second order implementability condition that requires $x(\cdot)$ to be non-decreasing (see, Mussa and Rosen,1978) to a requirement

$$x(b) \geq x(a). \tag{3}$$

I also assume that

$$v(a) \geq 0, \tag{4}$$

otherwise all types on $[0, a]$ should be excluded from the contract and the problem reduces to the standard Mussa and Rosen (1978) problem. It will

also be useful to define:

$$\widetilde{v}(\alpha) = \alpha - \frac{F(a) - F(\alpha)}{f(\alpha)}. \tag{5}$$

It is straightforward to check that if $v(\cdot)$ increases in α, so does $\widetilde{v}(\cdot)$.

The utility of the outside option is the same across the consumers and is normalized to be zero. The cost of production is convex in quality and additive in quantity and is given by a twice differentiable, strictly increasing, convex function $c(x)$. I assume that

$$c(0) = c'(0) = 0. \tag{6}$$

The above consideration can be summarized by the following model. The monopolist selects a measurable function $t : R \to R$ to solve

$$\max_{t(\cdot)} \int_0^1 (t(x(\alpha)) - c(x(\alpha)))f(\alpha)d\alpha, \tag{7}$$

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subject to

$$x(\alpha) \quad \in \quad \arg\max(u(\alpha, x) - t(x)) \tag{8}$$

$$\max(u(\alpha, x) - t(x)) \quad \geq \quad 0. \tag{9}$$

The first of these constraints is known as the incentive compatibility constraint and states that for any tariff the consumers make the optimal choice, while the second is the individual rationality constraint, which states that consumers obtain at least as much utility from the purchase of the monopolist's product as they will from the outside option.

Define the consumer's surplus by:

$$s(\alpha) = \max(u(\alpha, x) - t(x)). \tag{10}$$

Note that equation (10) and the generalized envelope theorem (Milgrom and Segal, 2002) imply:

$$s'(\alpha) = x(\alpha) \tag{11}$$

for almost all $\alpha \in (0, a) \cup (b, d)$.[1] Moreover, the second order implementability

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[1] One should invoke the general formulation of the envelope theorem by Milgrom and Segal, since the optimal tariff may fail to be differentiable.

constraint can be expressed by requiring that $s(\cdot)$ is convex. Let us define the relaxed problem by

$$\max_{s(\cdot)} \int_{\Omega} (\alpha s'(\alpha) - s(\alpha) - c(s'(\alpha)))f(\alpha)d\alpha \qquad (12)$$

$$s.t. s(0) \geq 0, \ s'(b) \geq s'(a), \ s(b) \geq (b-a)s'(a) + s(a) \qquad (13)$$

Derivatives in the end points are defined by:

$$s'(a) = \lim_{\alpha \to a-0} s'(\alpha), \ s'(b) = \lim_{\alpha \to b+0} s'(\alpha). \qquad (14)$$

The first constraint here is the participation constraint. Condition (11), already embodied into the objective (12), is the first order incentive compatibility constraint between types α and $\alpha - d\alpha$, the third of the constraints (13) is the incentive compatibility constraint between types b and a, and the second of the constraints (13) is the consequence of the fact that any implementable allocation is weakly increasing. Our next objective is to solve problem (12)-(13) and demonstrate that its solution is convex, i. e. it is in fact the solution to the complete problem.

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Carrying out integration by parts one obtains:

$$\max_{s(\cdot)} \int_0^a (\widetilde{v}(\alpha)s'(\alpha) - c(s'(\alpha)))f(\alpha)d\alpha + \int_b^d (v(\alpha)s'(\alpha) - c(s'(\alpha)))f(\alpha)d\alpha$$
$$-F(a)s(0) - s(b)(1 - F(b)) \tag{15}$$

$$s.t. s(0) \geq 0, \ s'(b) \geq s'(a), \ s(b) \geq (b-a)s'(a) + s(a). \tag{16}$$

Note that the surplus function enters into the monopolist's objective only through its derivative and terms $s(0)$ and $s(b)$. Therefore, the monopolist first should solve:

$$\max_{s(\cdot)} \int_0^a (\widetilde{v}(\alpha)s'(\alpha) - c(s'(\alpha)))f(\alpha)d\alpha + \int_b^d (v(\alpha)s'(\alpha) - c(s'(\alpha)))f(\alpha)d\alpha$$
$$s.t. \ s'(b) \geq s'(a).$$
$$\tag{17}$$

The solution to this problem is defined up to function, which is piecewise constant on $[0, a]$ and $[b, d]$. The monopolist should then choose the constants to ensure that $s(0) = 0$ and $s(b) = (b - a)s'(a) + s(a)$. The first order

conditions for this problem are:

$$
\begin{cases}
\frac{\partial}{\partial \alpha}(\widetilde{v}(\alpha) - c'(s'(\alpha))f(\alpha)) = 0 \text{ for } \alpha \in (0, a) \\
\\
\frac{\partial}{\partial \alpha}(v(\alpha) - c'(s'(\alpha))f(\alpha)) = 0 \text{ for } \alpha \in (b, d) \\
\\
d = c'(s'(d)) \\
\\
a - c'(s'(a)) = \lambda \\
\\
\lambda \geq 0, \ s'(b) \geq s'(a), \ \lambda(s'(b) - s'(a)) = 0.
\end{cases}
\tag{18}
$$

The first two equations are Euler equations, the third is the usual "no distortion at the top" condition. To understand the last two conditions, note that if the constraint $s'(b) \geq s'(a)$ is slack then $\lambda = 0$ and one obtains $a = c'(s'(a))$ as it should be for a free boundary problem. Otherwise, $\lambda > 0$ and $s'(a) = s'(b)$, i. e. one obtains a fixed boundary problem.

Solving system (18) and returning to variable $x(\cdot)$, one can finally write

the solution in the following form:

$$
\begin{cases}
c'(x(b)) = v(b) \text{ for } \alpha \in [b, d] \\[2mm]
c'(x(\alpha)) = \max(0, \tilde{v}(\alpha) + \min(0, c'(x(b)) - a) \text{ for } \alpha \in [0, a] \\[2mm]
s(\alpha) = \displaystyle\int_0^\alpha x(t)dt \text{ for } \alpha \in [0, a] \\[2mm]
s(\alpha) = s(a) + x(a)(b - a) + \displaystyle\int_b^\alpha x(t)dt \text{ for } \alpha \in [b, d] \\[2mm]
t(x) = \max_{\alpha \in \Omega}(\alpha x - s(\alpha))
\end{cases}
\qquad . \qquad (19)
$$

Note that allocation in (19) is everywhere increasing and therefore, implementable. In practice, one has first to find the allocation on the right segment of the type space, use it to calculate the allocation on the left segment, and then proceed calculating the surplus in the reverse order. Clearly, the procedure easily generalizes for more than two disconnected segments.

An important feature of the solution is that if the monotonicity constraint $x(b) \geq x(a)$ binds the optimal tariff will have a kink at $x = x(a) = x(b)$. This happens because the first order conditions for the consumer problem imply that both types a and b belong to the subdifferential of the convex function $t(\cdot)$ at point x. In fact $\partial t(x) = [a, b]$, where ∂ stands for the subdifferential.

Let us analyze the second equation in the system (19) at point $\alpha = a$. At

that point it implies:

$$c'(x(a)) = \max(0, a + \min(0, c'(x(b)) - a). \qquad (20)$$

Note that the first equation in the system (19) implies $c'(x(b)) < b$. Therefore, for a sufficiently close to b

$$\min(0, c'(x(b)) - a) = c'(x(b)) - a, \qquad (21)$$

which implies

$$x(a) = x(b). \qquad (22)$$

The latter implies that the product line is connected. To generate a gap b should satisfy:

$$v(b) \geq a. \qquad (23)$$

For example, if the distribution of types is uniform this condition would imply:

$$b - a \geq d - b, \qquad (24)$$

i. e .the length of the right segment should exceed the size of the gap in the

type space.

3   Some numerical examples

In this section I am going to consider two numerical examples of the screening model with a disconnected type space. In the first example the resulting product line will contain a gap, while in the second one it will be connected.

Example 1. Assume that the cost of production is given by:

$$c(x) = \frac{x^2}{2} \tag{25}$$

and the distribution of types is uniform on $[0,1] \cup [3,4]$, i. e.

$$f(\alpha) = \begin{cases} \frac{1}{2}, & \text{for } \alpha \in [0,1] \cup [3,4] \\ 0, & \text{otherwise} \end{cases} \tag{26}$$

with a corresponding c.d.f.

$$F(\alpha) = \begin{cases} 0, & \text{for } \alpha \leq 0 \\ \frac{\alpha}{2}, & \text{for } \alpha \in (0, 1] \\ \frac{1}{2}, & \text{for } \alpha \in (1, 3] \\ \frac{\alpha - 2}{2}, & \text{for } \alpha \in (3, 4] \\ 1, & \text{for } \alpha > 4. \end{cases} \qquad (27)$$

According to our procedure, let us first calculate the allocation at segment $[3, 4]$. In this region

$$v(\alpha) = 2\alpha - 4, \qquad (28)$$

therefore

$$x(\alpha) = v(\alpha) = 2\alpha - 4. \qquad (29)$$

Note that

$$x(3) = 2 > x^{eff}(1) = 1, \qquad (30)$$

therefore the monotonicity constraint is not binding and on $[0, 1]$ the optimal allocation is given by:

$$x(\alpha) = \max(0, \widetilde{v}(\alpha)), \qquad (31)$$

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where $\widetilde{v}(\alpha)$, defined by (5) can be shown to be:

$$\widetilde{v}(\alpha) = 2\alpha - 1. \tag{32}$$

Therefore,

$$x(\alpha) = \max(2\alpha - 1, 0). \tag{33}$$

The exclusion region is now given by:

$$\Omega_0 \equiv \{\alpha \in \Omega : s(\alpha) = 0\} = [0, \frac{1}{2}]. \tag{34}$$

The surplus function on $[0, 1]$ can be found by integrating (11) subject to $s(0) = 0$. To do this, first note that (11) and (33) imply that $s(\cdot)$ is constant on $[0, 1/2]$, therefore $s(1/2) = 0$ and

$$s(\alpha) = \max(\alpha^2 - \alpha + \frac{1}{4}, 0) \tag{35}$$

for $\alpha \in [0, 1]$. Surplus on the set $[3, 4]$ is given by:

$$s(\alpha) = \alpha^2 - 4\alpha + C, \tag{36}$$

where constant C can be found from the condition that the type $\alpha = 3$ is indifferent between her contract and that of the type 1, i. e.

$$s(3) = 3x(1) - t(1) = 2x(1) + s(1) = \frac{9}{4}, \tag{37}$$

which implies that

$$C = \frac{21}{4}. \tag{38}$$

Therefore, the surplus is given by:

$$s(\alpha) = \begin{cases} \max(\alpha^2 - \alpha + \frac{1}{4}, 0), & \text{for } \alpha \in [0, 1] \\ \alpha^2 - 4\alpha + \frac{21}{4}, & \text{for } \alpha \in [3, 4] \end{cases}. \tag{39}$$

The optimal tariff can be now found as:

$$t(x) = \max(\alpha x - s(\alpha)) \tag{40}$$

and is given by:

$$t(x) = \begin{cases} \frac{x^2 + 2x}{4}, & \text{for } x \leq 1 \\ \frac{x^2 + 8x - 5}{4}, & \text{for } x \geq 2 \end{cases}. \tag{41}$$

The optimal product line is $[0, 1] \cup [2, 4]$.

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Example 2. Assume that the cost of production is given by:

$$c(x) = \frac{x^2}{2} \tag{42}$$

and the distribution of types is uniform on $[0,1] \cup [3/2, 5/2]$, i. e.

$$f(\alpha) = \begin{cases} \frac{1}{2}, & \text{for } \alpha \in [0,1] \cup [3/2, 5/2] \\[2ex] 0, & \text{otherwise} \end{cases} \tag{43}$$

with a corresponding c.d.f.

$$F(\alpha) = \begin{cases} 0, & \text{for } \alpha \leq 0 \\[1ex] \frac{\alpha}{2}, & \text{for } \alpha \in (0,1] \\[1ex] \frac{1}{2}, & \text{for } \alpha \in (1, \frac{3}{2}] \\[1ex] \frac{2\alpha-1}{4}, & \text{for } \alpha \in (\frac{3}{2}, \frac{5}{2}] \\[1ex] 1, & \text{for } \alpha > 5/2. \end{cases} \tag{44}$$

According to our procedure, let us first calculate the allocation on segment $[3/2, 5/2]$. In this region

$$v(\alpha) = 2\alpha - \frac{5}{2}, \tag{45}$$

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therefore

$$x(\alpha) = v(\alpha) = 2\alpha - \frac{5}{2}. \tag{46}$$

Note that

$$x(\frac{3}{2}) = \frac{1}{2} < x^{eff}(1) = 1, \tag{47}$$

therefore the monotonicity constraint is binding and on $[0, 1]$ the optimal allocation is given by:

$$x(\alpha) = \max(0, \widetilde{v}(\alpha) - \frac{1}{2}), \tag{48}$$

where $\widetilde{v}(\alpha)$, defined by (5) can be shown to be:

$$\widetilde{v}(\alpha) = 2\alpha - 1. \tag{49}$$

Therefore,

$$x(\alpha) = \max(2\alpha - \frac{3}{2}, 0). \tag{50}$$

The exclusion region is now given by:

$$\Omega_0 \equiv \{\alpha \in \Omega : s(\alpha) = 0\} = [0, \frac{3}{4}]. \tag{51}$$

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Note that the allocation on $[0, 1]$ in this example is below that in the previous one and the exclusion region is bigger. I. e. the presence of the group of consumers with slightly higher marginal rates of substitution between the quality and money depresses the market for the consumers located on $[0, 1]$, while the presence of consumers with significantly higher marginal rates of substitution has no effect.

The surplus function on $[0, 1]$ can be found by integrating (11) subject to $s(0) = 0$. To do this, first note that (11) and (33) imply that $s(\cdot)$ is constant on $[0, 3/4]$, therefore $s(3/4) = 0$ and

$$s(\alpha) = \max(\alpha^2 - \frac{3}{2}\alpha + \frac{9}{16}, 0) \qquad (52)$$

for $\alpha \in [0, 1]$. Surplus on the set $[3/2, 5/2]$ is given by:

$$s(\alpha) = \alpha^2 - \frac{5}{2}\alpha + C, \qquad (53)$$

where constant C can be found from the condition that the type $\alpha = 3/2$ is indifferent between her contract and that of the type 1, i. e.

$$s(\frac{3}{2}) = \frac{3}{2}x(1) - t(1) = \frac{1}{2}x(1) + s(1) = \frac{5}{16}, \qquad (54)$$

which implies that

$$C = \frac{29}{16}. \tag{55}$$

Therefore, the surplus is given by:

$$s(\alpha) = \begin{cases} \max(\alpha^2 - \frac{3}{2}\alpha + \frac{9}{16}, 0), & \text{for } \alpha \in [0, 1] \\ \\ \alpha^2 - \frac{5}{2}\alpha + \frac{29}{16}, & \text{for } \alpha \in [\frac{3}{2}, \frac{5}{2}] \end{cases}. \tag{56}$$

The optimal tariff can be now found as:

$$t(x) = \max(\alpha x - s(\alpha)) \tag{57}$$

and is given by:

$$t(x) = \begin{cases} \frac{x^2 + 3x}{4}, & \text{for } x \leq \frac{1}{2} \\ \\ \frac{x^2 + 5x - 1}{4}, & \text{for } x > \frac{1}{2} \end{cases}. \tag{58}$$

Note that, in agreement with the general observation made in the previous section, the tariff has a kink at $x = 1/2$. The optimal product line is connected and given by the segment $[0, 5/2]$.

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4　The inverse screening problem

Suppose an economist observes the tariff and the product line, i. e. the set of qualities offered on the market. She is reasonably well informed about the production process, i. e. is able to come up with a reasonable estimate of the cost function. She also knows the structure of the consumers' preferences, i. e. she knows that the utility has form (1). The inverse screening problem is to come up with a type distribution that justifies the observed tariff and product line.

Knowing type distribution may be important, especially for the policy makers, since it determines the size of efficiency losses due to the private information on the side of the consumers. If the right tail of the distribution is thick then the efficiency loss is relatively small. On the other hand, if the left tail of distribution is thick the efficiency losses are large and the government might consider buying the business from the monopolist and pricing the goods of different qualities at the production cost. It turns out, however, that the inverse screening problem may be ill defined, i. e. the solution to it is in general not unique.

For example, consider the following situation. Assume that

$$c(x) = \frac{x^2}{2} \tag{59}$$

the product line is $[0, 5/2]$ and the tariff is given by:

$$t(x) = \begin{cases} \frac{x^2 + 3x}{4}, & \text{for } x \leq \frac{1}{2} \\ \frac{x^2 + 5x - 1}{4}, & \text{for } x > \frac{1}{2} \end{cases} \tag{60}$$

This is, of course, exactly the product line and the tariff found in the previous section. Therefore, it can be rationalized assuming that the types are distributed uniformly on $[0, 1] \cup [3/2, 5/2]$. The same tariff and product line could, however, be rationalized by assuming that

$$F(\alpha) = \begin{cases} 0, & \text{for } \alpha \leq 0 \\ \frac{2\alpha}{3}, & \text{for } \alpha \in (0, 1] \\ \frac{6\alpha - 4}{3(2\alpha - 1)}, & \text{for } \alpha \in (1, \frac{3}{2}] \\ \frac{2\alpha + 7}{12}, & \text{for } \alpha \in (\frac{3}{2}, \frac{5}{2}] \\ 1, & \text{for } \alpha > 5/2. \end{cases} \tag{61}$$

Note that this distribution does not have gaps and the optimal allocation has

a bunch $x = 1/2$ for types $[1, 3/2]$. For this distribution $2/3$ of the total mass is concentrated at segment $[0, 1]$ against only $1/2$ for the original distribution. Moreover, here $1/2$ of the consumers are not served in equilibrium against $3/8$ before. Though (61) is not the only distribution with a connected type space that rationalizes tariff (60), all such distributions will have $2/3$ of their mass on $[0, 1]$. This is because if the type space is connected

$$v(\alpha) = c'(x(\alpha)) = x(\alpha) \tag{62}$$

over set of types for which the equilibrium is separating. Solving the consumer's maximization problem with tariff (60) one obtains:

$$x(\alpha) = 2\alpha - \frac{3}{2} \tag{63}$$

for $\alpha < 1$. Using (2) one obtains the following differential equation for $F(\cdot)$:

$$\alpha - \frac{1 - F}{F'} = 2\alpha - \frac{3}{2}, \tag{64}$$

which should be solved subject to $F(0) = 0$. Its unique solution is:

$$F(\alpha) = \frac{2}{3}\alpha \tag{65}$$

for $\alpha \in (0, 1]$.

To understand the reason for non-identifiability of the distribution of types let us first start with a consumer problem:

$$\max(\alpha x - t(x)), \tag{66}$$

where $t(\cdot)$ is given by (60). The first order condition is:

$$\alpha \in \partial t(x), \tag{67}$$

where $\partial t(\cdot)$ denotes the subdifferential of the convex function $t(\cdot)$. Calculating the subdifferential and inverting the relationship one obtains:

$$x(\alpha) = \begin{cases} 2\alpha - \frac{3}{2}, & \text{for } \alpha \in (0, 1] \\ \frac{1}{2}, & \text{for } \alpha \in (1, \frac{3}{2}] \\ 2\alpha - \frac{5}{2}, & \text{for } \alpha \in (\frac{3}{2}, \frac{5}{2}] \end{cases} . \tag{68}$$

There are two sources of non-identifiability. First, though $x(\alpha)$ is continuous it is not invertible. Therefore, though the product line is connected, one cannot conclude that the type space is. In fact, as we saw above, this allocation can be produced with a disconnected product line. Second, even if we are willing to assume that the product line is connected, we cannot say whether bunching already occurs in the relaxed problem (as it will be for distribution (61)) or is a result of the ironing procedure applied to the non-monotone solution of the relaxed problem. From the point of view of estimating the efficiency loss, the first problem is probably more serious, since c.d.f.s of all the distributions with the connected type space coincide on $[0, 1]$ and attach bigger mass to the left tail than those with a disconnected type space. Therefore, assuming connected type space may lead to a significant overestimation of efficiency losses.

Note that non-invertability of $x(\cdot)$ is due to non-differentiability of $t(\cdot)$, i. e. to the fact that $\partial t(x)$ fails to be a singleton. If, on the contrary, $t(\cdot)$ is differentiable than the first order conditions for the consumer problem can be inverted for all $x > 0$. Therefore, one can determine the set of types that are served at equilibrium and use Euler-Lagrange equations to calculate the probability density function over this set. Though one still

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cannot determine uniquely the distribution of types over the exclusion region, it does not affect any variables of economic interest. Therefore, in that case the inverse screening problem is well-defined.

5   Conclusions

In this paper I considered a monopolistic screening model with continuum of types when the type set is a disconnected subset of real line. I proved that the product line remains connected provided that the gap in the type space is sufficiently small. I also use the results to show that the inverse screening problem may be ill-defined. If the tariff has a kink then not only the *distribution* of types cannot be found but even the type space cannot be determined. Assuming that the types space connected, while in fact it is not, can lead to a significant overestimation of the welfare losses due to private information on the side of consumers. Therefore, if the knowledge of distribution of the consumer tastes is important for policy design, one could not rely on the observable market information and has to conduct additional surveys.

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