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Measurement of Business Cycles

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Abstract

We describe different ways of measuring the business cycle. Institutions such as the NBER, OECD and IMF do this through locating the turning points in series taken to represent the aggregate level of economic activity. The turning points are determined according to rules that either come from a parametric model or are non-parametric. Once located information can be extracted on cycle characteristics. We also distinguish cases where a single or multiple series are used to represent the level of activity.
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Measurement of business cycles provides a reference point against which macroeconomic theories and policy discussion can be assessed. The process requires an operational definition of a cycle, criteria to distinguish business cycles from other forms of fluctuation, procedures to detect the presence of a business cycle, and methods to measure its features. A central theme of this entry is that good measurement should not prejudge the nature of the phenomena under investigation. Moreover, it should produce statistics which are informative about features of interest and which can be formally analysed.

Defining and detecting cycles

In their classic work *Measuring Business Cycles,* Burns and Mitchell (BM) (1946) define *specific cycles* in a series y_t in terms of *turning points* in its sample path. This tradition has been central to work at the NBER and other institutions such as the IMF(2002) and OECD (See www.oecd.org/std/cli.).

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When it came to discussing the business cycle BM simply referred to y_t as the *level* of aggregate economic activity, although in this entry we will regard it as the *log* of economic activity as the turning points in the level and the log of economic activity are the same. When Mintz (1969,1972) had trouble finding turning points in the level of activity in surging economies such as West Germany, this led her to first extract a permanent component p_t from y_t and to then study turning points in $z_t = y_t - p_t$. The resulting *growth cycle* in z_t has many forms depending on the method used to extract the permanent component. Others such as the Economic Cycle Research Institute (ECRI), have studied turning points in the differenced data Δy_t. A generalization of this explored by Kedem (1980, 1996) and Harding (2003) is to study turning points in $\Delta^r y_t$.

At the time Mitchell began his work the alternative way of thinking about cycles (or oscillations) was to view y_t as composed of periodic components represented by sine and cosine waves i.e.

$$y_t = \sum_{j=1}^{m} \alpha_j \cos \lambda_j t + \beta_j \sin \lambda_j t, \tag{1}$$

where λ_j is the frequency of the $j'th$ oscillation. If $m = 1$ there would be a single periodic cycle. The problem with this way of looking at cycles was that few economic time series showed evidence of periodicity. To overcome that problem α_j and β_j were allowed to vary stochastically over time. Specifically, they were treated as uncorrelated random variables with zero mean and variance σ_j^2. This formulation meant that y_t had to be a stationary random variable and so could not be applied to the levels of variables such as GDP (unlike turning point analysis). However, in this form one can measure the importance of the j^{th} periodic cycle by looking at the ratio of σ_j^2 to the variance of y_t and it is the basis of spectral analysis. Such a perspective has increasingly been referred to as studying fluctuations rather than cycles, since the focus of attention is upon the variance of y_t.

To understand the difference between these alternative ways of measuring cycles take the special case where $\lambda_1 = 0$ and there is another frequency λ_2. Then

$$\begin{aligned} y_t &= \alpha_2 \cos \lambda_2 t + \beta_2 \sin \lambda_2 t + \alpha_{1t}, \\ &= y_t' + \alpha_{1t}. \end{aligned} \tag{2}$$

Now there are certainly turning points in the series y_t' and the period between them is determined by λ_2. In contrast, the turning points in y_t will also be affected by the random variable α_{1t}, and thus may be very different to those in y_t'. Information about cycles gathered from spectral analysis concerns the nature of turning points in y_t' and not y_t.To give a more concrete illustration of this point suppose that the model for y_t is of the form

$$y_t = 1.4y_{t-1} - .53y_{t-2} + e_t.$$

Then the periodic cycle in y_t can be isolated by setting $e_t = 0$ to get y_t'. Using the dating methods of an institution like NBER, the turning points in y_t' are 22 quarters apart, as could also be discovered by computing the roots of $(1 - 1.4L + .53L^2) = 0$. However, applying the same methods to y_t, one finds that the turning points in y_t will be on average 12 quarters apart. A further disadvantage of the periodic cycle approach is that the data needs to be filtered to render it stationary before analysis proceeds and, as Cogley (2006) observes, the filters most commonly used by macroeconomists can introduce spurious periodic cycles, thereby blurring the picture.

Locating turning points

To locate turning points in a series it is necessary to define what these are and to provide some way of recognizing them in a given data set. An obvious solution is to use the idea that peaks (troughs) are local maxima (minima) in the series y_t. Hence, if $\vee_t$ ($\wedge_t$) are binary variables taking the value of unity where there is a peak (trough) at t and zero otherwise, applying the proposed definition gives

$$\begin{aligned} \vee_t &= \mathbf{1}\left(y_t < y_{t\pm j},\, 1 \le j \le k\right) && (3) \\ \wedge_t &= \mathbf{1}\left(y_t > y_{tj\pm},\, 1 \le j \le k\right). && (4) \end{aligned}$$

In equations (3) and (4) $1(A)$ is the indicator function taking the value unity if the event A is true and zero otherwise. Of course this still leaves one with the need to describe the interval over which the local maxima or minima are said to occur i.e. a choice needs to be made regarding k. To replicate the main features of Burns and Mitchell's specific cycle dating procedures it is necessary to set $k = 5$ for monthly data or $k = 2$ for quarterly data.

This is not the end of the choices that need to be made when locating turning points, but the others do not relate to the location of local maxima and minima. Rather they concern the question of whether one should eliminate some of the local turns in deciding on a final set of turning points. Mostly these extra restrictions are imposed as phase length constraints, where phases are the periods of expansions and contractions between turning points. Thus NBER dating procedures require that completed phase and complete cycles durations last longer than 5 and 15 months respectively. These are generally referred to as censoring operations. Whether turning points should be censored depends on the objectives of the research. If the objective is to match NBER business cycle dates then censoring is essential. But if the researcher is pursuing other objectives such censoring may not be necessary. Censoring turning points makes it much harder to formally analyse the statistics produced and this may provide an important reason for not imposing them.

BM acknowledged that the final set of dates they selected for turning points reflected considerable amounts of judgement and incorporated specific information about economic activity at particular dates. Today, academic economists are primarily interested in the average characteristics of the cycle, and so it may well be that automated methods of turning point detection become attractive. In the early post WWII period many of the procedures used by BM were codified, producing an expert system for locating turning points. Ultimately, Bry and Boschan (1973), produced an algorithm and FORTRAN program (called BB here) that largely replicated this expert system. Subsequently Mark Watson (1994) implemented this algorithm in the language GAUSS, and that code is available at http://www.wws.princeton.edu/mwatson.

There were three key components to the BB algorithm. The first was to engage in some smoothing of the series and to find an initial set of turning points using equations (3) and (4) with $k = 5$. The second was to eliminate enough of these turning points so as to ensure that expansion and contraction phases exceed five months in duration, while completed cycles exceed fifteen months in duration. The third component was to ensure that peaks and troughs alternate by deleting multiple sequential occurrences of these. That was done through the application of various rules, such as choosing between two peaks based on which has the higher value of y_t.

Although BB were interested in analysing monthly data they suggested a method for working with quarterly data that involved treating the observations on each of the months in a quarter as one third of the quarterly value. A variant of BB was developed by Harding and Pagan (2002) and called BBQ. It omitted the smoothing in the BB algorithm but retained the three key principles of the BB algorithm. It also set $k = 2$ and made the minimum phase and cycle lengths be two and five quarters respectively. Faster recursive algorithms for locating turning points have been developed by Artis et al. (2004) and James Engel. Engel's computer programs are called MBBQ. They are written in MATLAB and GAUSS and are available at www.ncer.edu.au.

Model based procedures for defining and locating turning points

The procedures above do not require any knowledge of the data generating process for y_t. An alternative approach is to adopt a model of Δy_t and use this to locate turning points. To date the models used are *parametric* and generally feature two regimes. Perhaps the best known parametric model is that of Hamilton (1989), where the growth rate is treated as a Markov Switching (MS) process of the form $\Delta y_t = \mu_0(1-\xi_t) + \mu_1\xi_t + e_t$. Here μ_j are the growth rates in the two regimes, and these are indexed by a latent binary state, ξ_t, while e_t is a normally distributed zero mean error term. Here μ_0 is the growth rate of the low growth state and μ_1 is the high growth rate. Sometimes the restriction $\mu_0 < 0$ is also imposed. The model is completed by specifying the transition probabilities of moving from $\xi_{t-1} = 0$ or 1 to $\xi_t = 1$ or 0. The model can be made more complex with extra dynamics, different variances in each regime, allowing the transition probabilities to depend on some observable data etc. This parametric model is used to compute the conditional probability, $\Pr[\xi_t = 1|A_t]$, where A_t is either all or a sub-set of the growth rates $\{\Delta y_j\}_{j=1}^{T}$. Thus the estimate of $\Pr[\xi_t = 1|A_t]$ is a function of whatever growth rates are in A_t. Generally this probability will be a non-linear function of the elements in A_t although a linear function can be quite a good approximation – see Harding and Pagan (2003) for an example.

The cycle is then associated with a binary variable S_t that takes the value one in expansion and zero in contraction. A rule is used to construct S_t by comparing the estimated probability of being in the high growth state with some critical value. Hamilton chose .5, and most of those using the technique have followed suit. Consequently, if $\Pr[\xi_t = 1|A_t] > .5$, an expansion is signified and S_t is set to unity. If the criterion is not satisfied S_t is set to zero. Notice that the ξ_t are not the phase states; the latter are S_t. They are simply a device for producing some non-linear structure in Δy_t, although often one can think of the outcomes for ξ_t as signifying a low or high growth period. The correlation between S_t and ξ_t may be very low. Many applications of this methodology have now been made and the MS model that one chooses seems to vary a lot with the series it is being applied to. The simple one described above rarely works satisfactorily.

In most instances a decision about the utility of the method is made by comparing the business cycle states produced by the rule based on the magnitude of $\Pr[z_t = 1|A_t] > .5$ with those found by turning point methods. Because of the latter comparison one has to ask what the advantages there are in using a model to locate turning points. Chauvet and Piger (2003) claim that an advantage of the model-based approach is that it allows an investigator to forecast turning points in real time. There is some truth to this but it is exaggerated. Since forecasts can be found for any such model, they could be passed through any chosen dating algorithm to determine the predicted phases.

Measuring Cycle Features

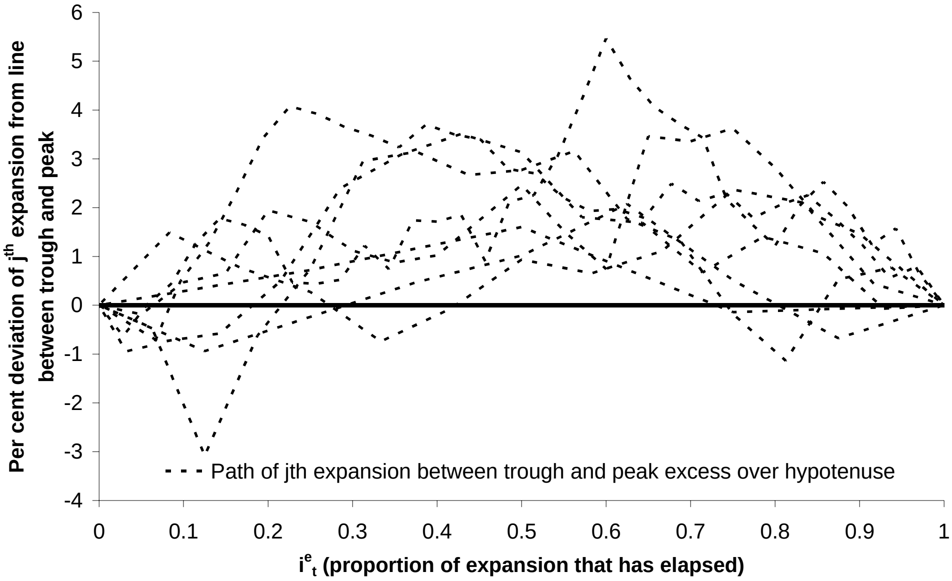
Turning points segment time series into phases. An expansion phase runs from the trough to the next peak. A contraction runs from a peak to the next trough. In what follows it is easiest to just describe the derivation of information on expansions.

The two most basic statistics related to phases are duration and amplitude. The *duration* of an expansion is the number of periods of time between the trough and next peak. The *amplitude* of an expansion measures the change in y_t from trough to the next peak. In many cases y_t is the log of some variable such as GDP or industrial production i.e. $y_t = ln(Y_t)$, and the amplitude has a natural interpretation as the approximate percentage change in Y_t between trough and peak.

Duration and amplitude form two sides of a triangle. Connecting the trough and peak produces the hypotenuse. If $y_t = ln(Y_t)$ then the hypotenuse represents the path followed by a variable that exhibits a constant growth rate during an expansion. With this in mind it is instructive to inspect the actual path followed by the data, and to compare that path with the constant growth path represented by the hypotenuse. Figure 1 shows how US expansion paths have deviated from the constant growth rate path in the post WWII period. The important feature evident in this figure is that the growth rate of GDP is not constant over the expansion phase and typically is highest in the first half of an expansion.

While comparisons such as that in Figure 1 are visually informative there is also a need for statistics that summarise the average shape of phases. Sichel (1994) divided expansions into three stages, computed the average growth rate for each stage, and showed graphs of these, as well as providing formal statistical tests of equality of the growth rates in each stage. Harding and Pagan (2002) compare the cumulated gain in an expansion with what it would have been if growth had been constant throughout the phase. This comparison was motivated by the idea mentioned above that a plot of y_t

Figure 1: Deviation of sample paths from hypotenuse, United States GDP during expansions in the post WWII period



against t during an expansion would look like a triangle if growth had been constant. The area of such a triangle would be one half the product of the amplitude and duration. If growth was not constant the area under the path actually followed by activity during the expansion would differ from the triangle. Thus a comparison of the two areas provides a measure of the extent of departure from a constant growth scenario. The evidence seems to be that expansions do not feature constant growth in some countries like Australia, the US and the UK, but do so in many European countries. The shape analysis is interesting since a linear process for Δy_t will produce phases that, on average, have constant growth rates. So a failure to see this signals the need for a non-linear process for Δy_t. The shape analysis also provides a useful tool for testing whether non-linear models produce realistic business cycles.

All of the methods for summarizing business cycle information can be applied to growth cycles and to data that has undergone higher order differencing. In addition Sichel (1993) suggested tests for "deepness" and "steepness" in the growth cycle that were effectively tests for symmetry in the densities of z_t and Δz_t.

Using Multivariate Information in Defining and Detecting Business Cycles

Burns and Mitchell's famous definition of a business cycle " Business cycles are a type of fluctuation found in the aggregate economic activity of nations... a cycle consists of expansions occurring at about the same time in many economic activities, followed by similarly general ... contractions.." has two aspects to it. One points to the need to identify aggregate economic activity, and the other to the fact that there should be synchronization across many series during the phases of a business cycle. They commented that GDP was a suitable index of economic activity although others, such as Moore and Zarnovitz (1986), have preferred a weighted average of several series rather than a single one. However, since data on GDP was not available to Burns and Mitchell, for either the time period or frequency in which they were interested, it is natural that they placed more emphasis upon the second component of their definition when discussing the business cycle.

This second component emphasises synchronization of the cycles in the specific series taken to represent economic activity. Burns and Mitchell took the turning points in many series and then extracted a *reference cycle* by determining those dates which peaks and troughs "clustered around". So a primary task is to be able to measure the tightness of the clusters. At the end of the process one also wishes to know how synchronized each of the specific cycles are with the cycle in the aggregate.

Harding and Pagan (2006) develop procedures to measure the tightness of clusters of turning points and the degree of synchronization of cycles through concordance indices that measure the fraction of time spent in the same phase. They apply those procedures to the series referred to by the NBER when dating the business cycle, and find that the turning points in those series are tightly clustered together. Harding (2003) finds that between March 1949 and September 2001 there is a concordance of 0.96 between the NBER business cycle states and the cycle obtained by locating turning points in US GDP.

Automated Construction of the Reference Cycle

To automate the calculation of the reference cycle requires some rules which will distill the specific cycle turning points into a single set of turning points. To determine what these rules might be one could look at the NBER Business Cycle Dating Committee procedure. They have a similar *modus* *operandi* to Burns and Mitchell as seen in their discussion about dating the 2001 recession at http://www.nber.org/cycles/recessions.html. However, one rarely gets a precise description either of how their decisions are made or the series used in that process. In addition it seems as if the series which have been most influential in decisions may have been different at different periods in time. The clearest description of the procedures for aggregating turning points in a set of series to create a reference cycle is in Boehm and Moore (1984), who explain how NBER methods were used when establishing a reference cycle for Australia. Their description can be taken as authoritative because Moore was a pivotal figure in the NBER Business Cycle Dating Committee for many years. Moore and Zarnowitz (1986) also provides information on methods used by NBER in dating the business cycle.

Given the fact that the process for establishing the reference cycle is a little vague, it should not be surprising that there have been few attempts at producing automated dating algorithms to establish it from multivariate series. Harding and Pagan (2006) construct an algorithm to replicate the NBER procedures described by Boehm and Moore (1984). They obtained the "clustering parameter" which is essential to measuring the tightness of turning point clusters by looking at Boehm and Moore's spreadsheets. The resulting algorithm produced a reference cycle that matched the Australian version established by Boehm and Moore quite well. Subsequently, it was tested on US data, and was able to produce quite a good replication of the reference cycle for that country, even though the clustering parameter had been calibrated with Australian data.

Model based procedures for defining detecting and extracting a reference cycle

Recently, academic economists have used parametric models to construct a coincident index and the reference cycle from n multivariate series $\Delta y_{1t}, ..., \Delta y_{nt}$. A common element to all approaches is to write Δy_{jt} as a function of a common component Δf_t and idiosyncratic components u_{jt} $(j = 1, ..., n)$. Hence a simple representation would be $\Delta y_{jt} = a_j \Delta f_t + u_{jt}$. The f_t is often thought of as the coincident index of the business cycle. Of course there may be more than one f_t but, ultimately, we can think of combining then together to form as a single variable. There are then many ways that models for Δf_t and u_{jt} might be specified, depending upon how strong the assumptions are that one wishes to make about the nature of f_t and u_{jt}. Often Δf_t is given an MS form e.g. Chauvet and Piger (2003). Depending on what these assumptions are will determine how an estimate of f_t is to be made. Stock and

Watson (1991) and Chauvet (1998) represent different approaches. In some instances one can avoid specifying precise parametric models for f_t and u_{jt}, restricting them only to be in a general class. Forni et al (2001)'s dynamic factor approach is the main representative of this latter technique. The main issue with these approaches is that the coincident index and reference cycle obtained are conditioned on the assumptions made about the data generating process. For that reason these approaches cannot provide a neutral measurement of the reference cycle.

Conclusion

Although widely used in official circles, Burns and Mitchell's methods of measuring cycles through turning points has been less popular in academia. But this has changed in recent years. There are a number of reasons why the methods have become increasingly attractive. First, information about the nature of the cycle phases can be generated, and this shape information proves important when trying to construct models of economic activity. Second, the literature now contains expert systems for locating turning points, and these have been coded into various computer languages, thereby eliminating the judgemental aspect of the method. Nevertheless, the automatically generated turning points have been quite good approximations to those found via judgement. Third, the ability to produce simulated data from parametric models means that such information can be passed through the algorithms for locating turning points to produce simulated distributions for the statistics that summarize the features of the cycle. Fourth, the emerging mathematics literature on crossing points provides a natural foundation on which to build a distribution theory for Burns and Mitchell's methods. Fifth, there is now a large literature on parametric methods for locating turning points and measuring cycles. This latter literature can readily be linked to the non-parametric turning point approach of investigators such as Burns and Mitchell, as seen in Harding and Pagan (2003).

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