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HUMAN CAPITAL, FINANCIAL INNOVATIONS AND GROWTH: A THEORETICAL APPROACH

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Abstract

This paper examines the symbiosis between financial development and human capital accumulation in generating endogenous growth. We develop a theoretical model where human capital is a key factor in the creation of financial innovations, resulting in financial development which in turns facilitates the acquisition of new human capital. Comparing the steady state solutions of the decentralized model with those of a hypothetical social planner reveals two sources of divergence between the solutions. In addition, we explore the comparative statics and transitional dynamics of these models, and examine their key implications for policy-makers.

KEYWORDS: Economic Growth, Finance, Human Capital
JEL CODES: G20, I20, O31, O41

1 Introduction

The role of human capital in producing sustained economic growth has been widely studied since the emergence of New Growth Theory in the mid-to-late 1980s. The most cited paper in this body of work is Lucas (1988), who in turn drew on much earlier work by Arrow (1962) and Uzawa (1965). In this class of theoretical growth models, a greater level of productivity in human capital production leads to a higher steady-state growth rate. More recently, a broad literature on finance and growth (surveyed in Levine (1997)) has surfaced. Financial development, it is argued, influences economic growth by removing borrowing constraints, by improving the management of risk, by facilitating information acquisition and resource allocation, by enabling the monitoring of managers and the exerting of corporate control, by mobilizing savings, and by encouraging efficiency-enhancing specialization. Many empirical studies have found that variables indicating the extent of financial development are positively correlated with overall growth rates, total factor productivity and factor accumulation. For example, Benhabib and Spiegel (2000) find that financial development variables such as the ratio of liquid liabilities of the financial sector to GDP (a proxy for the overall size of the formal financial intermediary sector) and the ratio of deposit-money bank domestic assets to depost-money bank assets plus central-bank domestic assets (a variable King and Levine (1993) believe emphasizes the risk-sharing and information services offered by banks) have statistically significant effects on investment in human capital.

In this paper, we aim to bring these two strands of work together in a three-sector growth model with endogenous human capital accumulation and financial innovation. In our model, human capital is a key input in creating financial innovations, resulting in financial development which in turns facilitates the acquisition of new human capital. Financial innovations result in a larger aggregate stock of financial products, which improves the intermediation process in transforming the savings of household into productive investment by firms. In addition, financial innovations facilitate the financing of human capital acquisition, raising the rate at which new human capital is produced. Although previous papers do not model the financial sector in the context of a macroeconomic growth model, they have investigated the link between borrowing constraints, human capital accumulation and growth. In contrast to papers such as Japelli and Pagano (1994) which argue that borrowing constraints promote growth by encouraging savings, de Gregorio (1996) presents an OLG model of a small open economy where borrowing constraints have negative effects on growth because the inability of individuals to borrow against future income reduces the incentives for human capital accumulation. Christou (1993) develops a neoclassical growth model with borrowing constraints and obtains similar results by simulating the model. Buiter and Kletzer (1995) make the same argument using a model where individuals must self-finance their training costs, whereas de Gregorio (1996) assume that education is free and focus on the trade-off between working and studying. Finally, Barro, Mankiw, and Sala-I-Martin (1995) discuss the implications of borrowing constraints in the financing of education for convergence of income across countries.

Our model suggests that productivity parameters in the financial innovations and human capital sectors affect the steady-state growth rate of the economy, as does the magnitude of the spillover effect from the stock of financial products on human capital accumulation, highlighting the symbiotic relationship between the two sectors. The solutions to the competitive, decentralized version of the model and that of a hypothetical social planner reveal multiple sources of divergence. We also examine the transitional dynamics of the model and discuss its ramifications for policy-making.

This paper is organized as follows: the next section presents a brief primer on the nature and types of financial innovations. In section 3, we present our three-sector growth model incorporating financial innovations and human capital accumulation, analyze the decentralized and social planner versions of this model, present their steady-state solutions, and explore their comparative statics and transitional dynamics. Section 4 looks at the policy implications arising from the model while Section 5 concludes.

2 Financial Innovations and the Financial Sector

2.1 Financial Innovations

We now discuss in a little detail the characteristics, types, and benefits of financial innovations. Llewellyn (1992) believes that the ultimate criterion when judging financial innovation is the extent to which it increases the efficiency of financial intermediation in particular and the functions of the financial system in general. Moreover, he asserts that it is possible to draw a parallel between financial innovations and similar processes in other industries. A computer hardware company, for example, seeks to enhance its competitive position in the marketplace by offering fundamentally new products, or by improving upon the technical characteristics of existing products, and by combining into one machine the characteristics of various existing machines. In the process, the basic function of 'computing' becomes more efficient. Similarly for financial firms, which can invent a brand new class of products, modify existing products, or combine the characteristics of several different products, thereby making financial intermediation more efficient. However, Llewellyn points out that a fundamental difference between financial innovations and technological innovations is that there are no protective patents in the financial industry. In finance, the characteristics of innovation are immediately visible and can be almost simultaneously copied by competitors. Why then do financial innovations still occur? Vaaler (2001) argues that the 1980's and 1990's saw a flurry of new financial products and services despite such innovations being costly to develop by pioneers and easy to imitate by rivals. Examining one class of financial innovations (asset-backed securities) and one particular innovator (Citicorp), he suggests that the paradox might be explained by examining cumulative first-mover performance effects across related product and geographic market contexts.

Llewellyn's (1992) analysis of the nature of financial innovations reveals two central aspects:

1. The creation of new financial instruments, techniques and markets;

2. The unbundling of the separate characteristics and risks of individual instruments and their reassembly in different combinations. Through a process of "spectrum filling", financial innovations can theoretically produce a range of instruments which encompasses all possible permutations of characteristcs. This moves the financial system closer to the Arrow-Debreu ideal where all transactors can ensure for themselves delivery of goods and services in all future contingencies. Arrow and Debreu (1954) demonstrate that the existence of risk can be an impediment to the efficient allocation of resources unless there exists a complete set of contingent commodity markets. In principle, therefore, the creation of new instruments moves closer to an approximation of the number of 'states of nature'.

2.1.1 Types of Finanical Innovations

According to Llewellyn, the plethora of characteristics of a financial product (which is a financial innovation made in the past) include:

1. Price risk, that is, the extent to which the price of an asset or liability may change;

2. Earnings risk, such as the difference between equity and loan contracts;

3. Credit risk, that is, the possibility of a default;

4. Pricing formula;

5. Conversion characteristics, such as the extent and cicumstances in which the instrument can be converted into something else;

6. Size of the facility;

7. Exchange rate risk;

8. Discretion, or the extent to which the instrument allows either the issuer or holder to exercise a discretion, for example an options contract;

9. Hedging facility, or the extent to which an instrument enables risks to be avoided. Examples here include forward contracts.

Bearing these characteristics in mind, one possible classification system for financial innovations developed by BIS (1986) separates them into:

1. Risk-transferring innovations, which either reduce the risk (price risk or credit risk) inherent in a particular instrument or alternatively enable the holder to protect against a particular risk;

2. Liqudity-enhancing innovations, such as securitized assets which enable loans to be sold in a secondary market which offers the lending institution the capacity to change the structure of its portfolio. Credit-generating innovations

widen the access to particular credit markets and may increase the total volume of credit.

3. Equity-generating innovations, which have the effect of giving an equity characteristc (where the rate of return on the asset is determined by the performance of the issuer) to assets where the nature of the debt-servicing commitment is predetermined, for example,.a debt-equity swap.

2.1.2 Benefits of Financial Innovations

Finally, Llewellyn (1992) lists some of the benefits of financial innovations:

a. The costs of financial intermediation may be reduced as they give borrowers access to a wider range of markets and facilities and allow different institutions to exploit their comparative advantage.

b. New instruments facilitate arbitrage between markets in different countries and instruments and in principle erode pricing anomalies, thus reducing market imperfections.

c. Some instruments widen the range of hedging possibilities and enable risks to be protected against.

d. Some instuments allow risks to be priced and to be shifted to those willing and able to absorb them.

e. Many instruments allow risks to be unbundled separately and "sold". If correctly priced, this enables the financial system to allocate resources more efficiently.

One can discuss the efficiency of the financial system in two different ways: *structural efficiency* (the range of choice offered in the system and its adaptability to changing cirumstances and preferences of users) and *allocative efficiency* (the ability of the system to price risks accurately and to allocate funds to where the risk-adjusted rates of return are highest.)

2.2 The Financial Sector: Innovators and Intermediaries

The financial sector in our model comprises financial innovators and financial intermediaries. The former produce new financial "blueprints" (products and services) using labor that is channelled away from the production of the final consumption good. As discussed above, these "blueprints" may include innovations such as ATMs, phone and internet banking, derivatives of existing financial products (including new types of options), initial public offerings (IPOs) of companies and anything which enables funds to be channelled more effectively from savers (households) to borrowers (firms seeking to raise capital to finance the purchase of new plant and equipment). We denote the stock of financial products (that is, old financial innovations) as τ.

Analogous to the Romer (1990) specification of the real R&D sector, the development of the financial sector is characterized by an ever-expanding variety of financial products. For simplicity, there is no "creative destruction" of existing financial products by successively superior products. However, the existing stock of financial innovations/products affect the production of new financial ideas according to

$$\dot{\tau}(t) = F(u_\tau(t)H(t))^\lambda \tau(t)^\phi,$$

where $\dot{\tau}$ denotes the quantity of financial innovations per unit time, u_τ is the fraction of aggregate human capital allocated to the financial sector, and F, λ, and ϕ are constants constrained to lie on the [0,1) interval. The idea is of a spillover effect from each financial innovation: financial innovators may build upon the ideas of other innovators to create a differentiated or an all-new financial product.

Financial intermediaries, on the other hand, are responsible for intermediating funds between borrowers and lenders. Borrowers are producers of the final consumption good while lenders are households with savings. The efficiency at which savings can be transformed into productive investment is specified to be dependent on the existing stock of financial innovations/products per adjusted unit of human capital (τ/H^κ, which we will label as ξ, $0 < \kappa < 1$), which proxies for the state of development and sophistication of the financial sector. The capital accumulation function hence looks like:

$$\begin{aligned} \dot{K}(t) &= \xi(t)(Y(t) - C(t)) - \delta K(t), \\ \xi(t) &= \frac{\tau(t)}{H(t)^\kappa}, \\ Y(t) &= AK(t)^\alpha (u_Y(t) H(t))^{1-\alpha}. \end{aligned}$$

where Y denotes output, K is the stock of capital, H is the stock of human capital, A is a (constant) technological parameter, u_Y is the share of human capital devoted to final goods production.[1], and δ is the rate at which capital depreciates. By including κ in our measure of transformative efficiency ξ, we are acknowledging that some financial innovations may be rivalrous (such as the creation of each new IPO, which may benefit from the knowledge gained from previous IPOs but nevertheless requires new labor to be expended in order

[1] Pagano (1993) specifies the saving-investment relationship as $\phi S = I$, where $1 - \phi$ is the flow of saving 'lost' in the process of financial intermediation. This (exogenous, in his case) fraction goes to banks as the "spread between lending and borrowing rates, and to securities brokers and dealers as commissions, fees and the like" (pp. 614-615).

to tailor it to the needs of individual firms) while others are not (such as a new financial instrument, which may in fact benefit from "thick market" effects as it becomes more widely traded). By restricting κ to lie strictly between 0 and 1, we are saying that *in the aggregate,* financial innovations or products are neither fully rivalrous nor fully non-rivalrous.[2]

In the steady state, τ/H^{κ} must be constant by definition. Therefore, the rate of financial innovations in the steady state must equal κ times the growth rate of human capital. Why does the number of financial products continually increase in the steady state even when all savings are completely transformed into investment? We argue that as per-capita income continues to rise (at a constant rate) in the steady state, so does the volume of funds that has to be intermediated. Due to the rivalrous nature of some financial products and services, this rising volume results in congestion and decreased efficiency in the financial sector unless more financial products are devised to alleviate the strain on it. Loosely speaking, resources such as human capital must continue to be directed to the financial sector as it services an expanding economy.

2.2.1 The Financial Sector and Human Capital Accumulation

The financial sector affects the accumulation of human capital in the following way: the stock of financial products τ (which proxies for the level of financial development) affects the rate at which new human capital is generated $\dot{H}$.

$$\dot{H}(t) = D\left(u_H(t)H(t)\right)^{\eta}\tau(t)^{\beta} - \delta_H H(t),$$

where u_H is the share of human capital devoted to new human capital production, δ_H is the rate at which human capital depreciates, and η and β are elasticity parameters constrained to take on values between 0 and 1. For simplicity, we assume that human capital production is relatively human capital intensive so that we can omit physical capital in the production function. This human capital accumulation equation collapses to the Lucas (1988) specification if $\eta = 1$ and $\beta = 0$. As discussed in the introduction, however, there exists a significant body of research indicating that financial development enhances human capital accumulation by removing borrowing constraints which hitherto prevented poorer households from accessing capital markets to finance the acquisition of human capital. It is reasonable, then, to believe that $\beta > 0$. We make the assumption that these benefits on human capital accumulation are a by-product of financial development and that financial intermediaries cannot extract the rents associated with these benefits.

[2]If $\kappa = 1$, then all financial products are strictly rivalrous; if $\kappa = 0$, then all financial products are strictly non-rivalrous, so that the efficiency of financial intermediation is dependent only on the stock of financial products and independent of population size.

3 The Model

In this section, we take an in-depth look into the structure of the decentralized version of the model. We explain the roles played by its key components: the final goods sector, the human capital production sector, the financial sector (incoporating financial innovators and financial intermediaries) and households, and examine their interactions. We then lay out the optimization problem faced by a hypothetical social planner, and proceed to show its steady-state solution. Finally, we examine the comparative statics of this model as well as its transitional dynamics.

3.1 The Decentralized Model

3.1.1 Final Goods Production.

The final goods sector produces the consumption good Y using a Cobb-Douglas technology to combine human capital H_Y (equal to $u_Y H$, where u_Y is the share of human capital, H, devoted to final goods production) and capital K:

$$Y = K^{\alpha} H_Y^{1-\alpha}. \tag{1}$$

A representative final goods producer thus solves the following profit maximization problem:

$$\max_{u_Y, K} \pi_Y = Y - w_Y H_Y - r_K K, \tag{2}$$

where w_Y is the wage in the final goods sector and r_K is the rental price of capital charged by financial intermediaries. The price of the final goods has been normalized to unity.

The first-order conditions require that the wage and rental price of capital be equal to the value of their marginal products:

$$w_Y = (1-\alpha)\frac{Y}{u_Y H}, \tag{3}$$

$$r_K = \frac{\alpha Y}{K}. \tag{4}$$

3.1.2 Human Capital Accumulation

As explained in Section 2.2.1, human capital accumulation is assumed to take the following form:

$$\dot{H}(t) = \widetilde{D}(u_H H)^{\eta} - \delta_H H,$$

$$\widetilde{D} \equiv D\tau^{\beta}. \tag{5}$$

where u_H is the share of aggregate human capital devoted to new human capital production, and δ_H is the rate at which human capital depreciates. In the decentralized model, the spillover effect from financial innovation on human capital accumulation is not internalized by individual agents.

Non-profit human capital producers (such as schools and universities) charge housholds $P_H \cdot \dot{H}$ to cover the cost of hiring educators, $w_H u_H H$, at every point in time. Positive externalities from financial development on human capital accumulation are not compensated. An alternative specification, not explored here, is to have human capital producers (private schools/universities) maximize profits $\pi_H = P_H \cdot \dot{H} - w_H u_H H$ but return these profits to households (say, through scholarships).

3.1.3 The Financial Sector

As in Chou and Chin (2001), the financial sector is composed of financial innovators and financial intermediaries-*cum*-venture capitalists. The former are responsible for producing financial innovations, τ, which then determines the degree of sophistication of the financial sector, proxied by ξ (equal to the ratio τ/H^{κ}, or the number of financial innovations per adjusted unit of human capital). A greater value of ξ allows more efficient intermediation between lenders (households) and borrowers (intermediate goods producers), resulting in a higher percentage of savings being transformed into useful capital.

Financial innovators are monopolists who make extra-normal profits by producing new financial products, using human capital as input, according to the production function

$$\dot{\tau} = \widetilde{F}\left(u_\tau H\right)^{\lambda}, \tag{6}$$

where $\widetilde{F} \equiv F\tau^{\phi}$. As in the human capital sector, financial innovators do not internalize the spillover effect from the existing stock of financial products. They therefore treat $\widetilde{F}$ as exogenously given.

The profit of a representative financial innovator, to be maximized by its choice of u_τ, is

$$\pi_\tau = P_\tau \dot{\tau} - w_\tau u_\tau H, \tag{7}$$

where P_τ is the price of each financial innovation. With these substitutions, the first order condition implies that

$$P_\tau = \frac{w_\tau u_\tau}{\lambda \gamma_\tau^* \tau}. \tag{8}$$

From this equation, we see that the price of each financial innovation is a function of the marginal factor cost of labor in the financial innovations sector. This equation may also be interpreted as an inverse demand function for τ.

Downstream in the financial sector, financial intermediaries purchase innovations from financial innovators (which, in the real world, are probably sister divisions of the same financial firms) and use them in transforming savings into productive investment as well as in the funding of real R&D activities. The financial intermediaries derive their income from: (a) charging the R&D firms the rate R_τ to finance their production of new designs; and (b) by charging firms in the (real) intermediate sector a higher interest rate (r_K) for renting capital than it pays out to households for their savings (r_V). The interest rate differential, $r_K - r_V$, may be thought of as the commission charged for intermediating funds. For simplicity, we assume that financial intermediation requires no human capital input. Financial intermediaries make zero profits as this sector is assumed to be perfectly competitive.

In each period, the representative financial intermediary ensures that revenues received from the final goods sector equal the cost of acquiring deposits from households and purchasing new products from financial innovators:

$$r_K K = r_V K + P_\tau \dot{\tau}. \tag{9}$$

3.1.4 Households

Finally, to close the model, we examine the consumption decision of households. As usual, we assume that this decision may be characterized by a representative consumer maximizing an additively separable utility function subject to a dynamic budget constraint. We use a conventional CRRA utility function and assume that households are ultimate owners of all capital and shareholders of final goods firms, financial intermediaries and financial innovators. The optimization problem is thus:

$$\max_{c, u_Y, u_\tau} \int_0^\infty e^{-\rho t} \frac{C^{1-\theta} - 1}{1-\theta} dt, \tag{10}$$

subject to

$$\begin{aligned} \dot{V} &= r_V K + w_Y u_Y H + w_\tau u_\tau H + w_H u_H H \\ &\quad - P_H \dot{H} + \pi_\tau - C, & (11) \\ \dot{K} &= \xi \dot{V}, & (12) \end{aligned}$$

where $\dot{V}$ represents the flow of households' stock of assets (that is, saving), and π_τ is the monopolistic profits from the financial innovators. The monopolistic profits of financial innovators, π_τ, equal to revenue $P_\tau \dot{\tau}$ minus labor costs $w_\tau u_\tau H$, are paid out to households who are also shareholders of these firms. In equilibrium, wages are equal across all labor markets, i.e. $w_Y = w_\tau = w_H = \bar{w}$.

Using these facts and substituting the assumption $P_H \dot{H} = w_H u_H H$ made in Section 3.1.2 and the intermediation condition $r_K K + R_\tau \tau = r_V K + P_\tau \dot{\tau}$ into (11) and (12) reduces the intertemporal budget constraint to

$$\dot{K} = \xi \left(r_K K + \bar{w} u_Y H - C \right). \tag{13}$$

We can show that the price of financial innovations is determined by the following arbitrage equation:

$$\xi r_K = \frac{\dot{V}}{P_\tau \tau} + \frac{\dot{P}_\tau}{P_\tau} \tag{14}$$

The opportunity cost to a financial intermediary of purchasing a financial innovation, $\xi r_K P_\tau$, must be equal to the average flow of savings intermediated by a unit of financial product, $\dot{V}/\tau$, and the associated capital gain, $\dot{P}_\tau$.

The solutions for the steady-state levels of u_H and u_τ, the shares of labor devoted to the human capital production sector and the financial innovations sector respectively, are shown in Appendix A. Using numerical simulations, we can demonstrate that their steady-state levels are lower in the decentralized model compared to their counterparts in the social planner's solution, the focus of the next sub-section. The sources of divergence are the positive externalities flowing from existing financial products to financial innovations and human capital production (which are only internalized by the social planner).

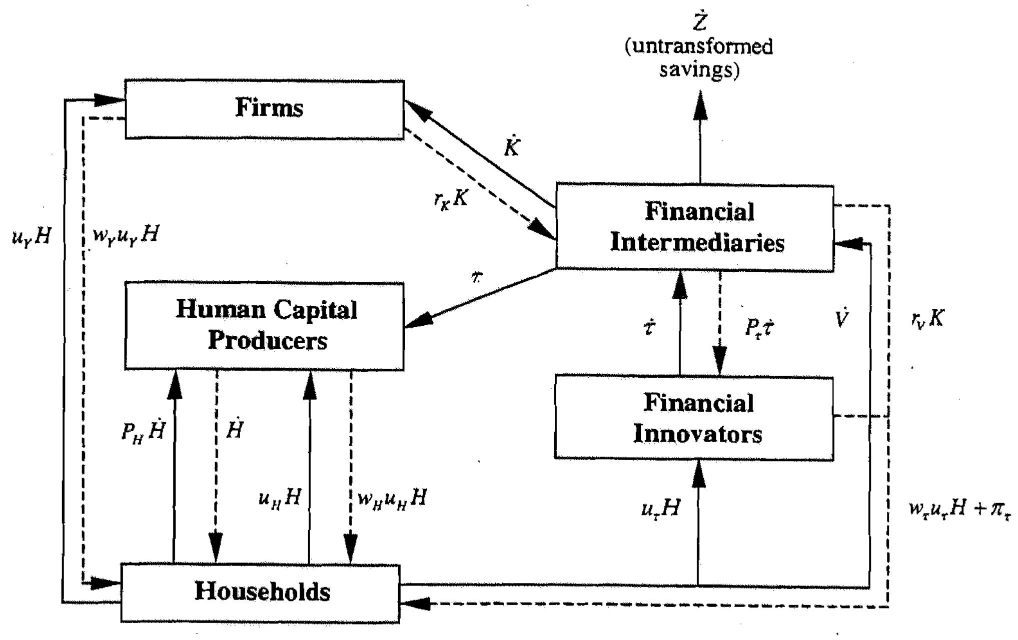

Figure 1: Flowchart of the Economy

3.2 The Social Planner's Model

3.2.1 Model Set-Up

The hypothetical social planner solves the following optimization problem:

$$\max_{C(t),u_Y(t),u_\tau(t)} U_0 = \int_0^\infty \frac{C(t)^{1-\theta}-1}{1-\theta} e^{-\rho t} dt, \tag{15}$$

subject to

$$\begin{aligned}
\dot{K}(t) &= \xi(t)\left[AK(t)^\alpha (u_Y(t)H(t))^{1-\alpha} - C(t)\right] - \delta_K K(t), & (16)\\
\dot{\tau}(t) &= F(u_\tau(t)H(t))^\lambda \tau(t)^\phi, & (17)\\
\dot{H}(t) &= D(u_H(t)H(t))^\eta \tau(t)^\beta - \delta_H H(t), & (18)\\
u_H(t) &= 1 - u_Y(t) - u_\tau(t) & (19)\\
\xi(t) &\equiv \tau(t)/L(t)^\kappa. & (20)
\end{aligned}$$

where the variables are as defined in the decentralized model. Note that $\alpha \in (0,1)$, $\{u_Y(t), u_\tau(t), u_H(t)\} \in [0,1]$ $\forall t$ and $\{\theta, \rho, \delta_K, \delta_H, n\} > 0$. To make the model as general as possible, we again allow the financial innovations sector and human capital production sector to take on any type of scale of production at this stage. The only requirements are that $\{\lambda, \eta\} \in (0,1]$ and $\{\phi, \beta\} \in [0,1]$.

As is standard, the model is solved using optimal control methods. Dropping time subscripts, the Hamiltonian is

$$\begin{aligned}
\mathbf{H} \equiv\ & \frac{C^{1-\theta}-1}{1-\theta}e^{-\rho t} + \nu\left\{\xi\left[K^\alpha A(u_y H)^{1-\alpha} - C\right] - \delta_K K\right\} & (21)\\
& + \mu F u_\tau^\lambda H^\lambda \tau^\phi + \pi\left[D(u_H H)^\eta \tau^\beta - \delta_H H\right],
\end{aligned}$$

where ν, μ, and π are the co-state variables corresponding to the state variables K, τ, and H respectively. The control variables are C, u_Y, and u_H. Unlike the competitive model, the social planner internalizes the spillovers from existing financial products on financial innovations, as well as the spillovers from financial products to human capital production.

The first-order conditions are

$$\begin{aligned}
\frac{\partial \mathbf{H}}{\partial C} &= C^{-\theta}e^{-\rho t} - \nu\xi = 0, & (22)\\
\frac{\partial \mathbf{H}}{\partial u_y} &= \nu\xi A K^\alpha (1-\alpha) u_Y^{-\alpha} H^{1-\alpha} - \pi D \eta v_H^{\eta-1} H^\eta \tau^\beta = 0, & (23)\\
\frac{\partial \mathbf{H}}{\partial u_\tau} &= \mu F \lambda u_\tau^{\lambda-1} H^\lambda \tau^\phi - \pi D \eta u_H^{\eta-1} H^\eta \tau^\beta = 0, & (24)
\end{aligned}$$

$$
\begin{aligned}
-\dot{\nu} &= \frac{\partial \mathbf{H}}{\partial K} = \nu\left(\xi A\alpha K^{\alpha-1}\left(u_Y H\right)^{1-\alpha} - \delta_K\right), && (25)\\
-\dot{\mu} &= \frac{\partial \mathbf{H}}{\partial \tau} = \nu H^{-\kappa}\left(AK^{\alpha}\left(u_Y H\right)^{1-\alpha} - C\right) + \mu F\phi\left(u_\tau H\right)^{\lambda}\tau^{\phi-1} \\
&\quad +\pi D\left(u_H H\right)^{\eta}\beta\tau^{\beta-1}, && (26)\\
-\dot{\pi} &= \frac{\partial \mathbf{H}}{\partial H} = \nu\xi AK^{\alpha}u_Y^{1-\alpha}(1-\alpha)H^{-\alpha} + \mu F u_\tau^{\lambda}\lambda H^{\lambda-1}\tau^{\phi} \\
&\quad +\pi(Du_H^{\eta}\eta H^{\eta-1}\tau^{\beta} - \delta_H), && (27)
\end{aligned}
$$

and the transversality conditions are

$$
\begin{aligned}
\lim_{t\to\infty}\nu\left(t\right)K\left(t\right) &= 0, && (28)\\
\lim_{t\to\infty}\mu\left(t\right)\tau\left(t\right) &= 0, && (29)\\
\lim_{t\to\infty}\pi\left(t\right)H\left(t\right) &= 0. && (30)
\end{aligned}
$$

3.2.2 Solving the Model

First define the physical capital to human capital ratio, K/H, as k, and the consumption to human capital ratio, C/H, as c. We characterize the balanced growth path of the model as one where these two ratios and ξ (the stock of financial products per adjusted unit of human capital, τ/H^{κ}) are constant in the steady state. In addition, the shares of human capital allocated to the final goods, financial innovations and human capital production sectors (u_Y, u_τ and u_H respectively) are also constant in the steady state. On the balanced growth path, output per unit of human capital, Y/H, is fixed so the growth rate of human capital is also the growth rate of output in the economy.

Before deriving the steady state solutions of c, k, ξ, u_Y and u_τ, we now show that the steady state properties of the model imply restrictions on κ and β, the parameters indicating the average degree of rivalry in financial innovations and the elasticity of spillovers from financial products on human capital production.

Since $\dot{\xi}/\xi = 0$ in the steady state and $\xi \equiv \tau/H^{\kappa}$, $\gamma_\tau = \kappa\gamma_H$, where $\gamma_\tau \equiv \dot{\tau}/\tau$ and $\gamma_H = \dot{H}/H$. In addition, from $\dot{\tau} = F u_\tau^{\lambda} H^{\lambda}\tau^{\phi}$, we have $\dot{\tau}/\tau = F u_\tau^{\lambda} H^{\lambda}\tau^{\phi-1}$. Taking the logarithms of the latter equation and differentiating both sides with respect to time, and using the fact that u_τ and γ_τ are constant in the steady state, yields $\gamma_\tau = \lambda\gamma_H/\left(1-\phi\right)$. Therefore

$$\kappa = \lambda/(1-\phi). \tag{31}$$

Moreover, as $\dot{H} = D\,(u_H H)^\eta \tau^\beta - \delta_H H$, it follows that $\gamma_H = D u_H^\eta H^{\eta-1}\tau^\beta - \delta_H$. Taking the growth rates of both sides of this equation and recalling that u_H and γ_H are constant in the steady state yields $\gamma_H = \beta\gamma_\tau/(1-\eta)$. Reconciling the two equations linking γ_τ and γ_H implies the restriction

$$\begin{aligned} \beta &= (1-\phi)(1-\eta)/\lambda \\ &= \frac{1-\eta}{\kappa}. \end{aligned} \tag{32}$$

The steady-state solutions for the five unknowns k, c, ξ, u_Y and u_τ are obtained by transforming the first-order conditions into five equations asserting that $\dot{k}/k = \dot{c}/c = \dot{\xi}/\xi = \dot{u}_Y/u_Y = \dot{u}_\tau/u_\tau = 0$. These equations may be written as:

$$\begin{aligned} \xi(Ak^{\alpha-1}u_Y^{1-\alpha} - c/k) &= \gamma_H + \delta_K, & (33)\\ \xi A\alpha k^{\alpha-1}u_Y^{1-\alpha} &= \rho + \theta\gamma_H + \delta_K, & (34)\\ F u_\tau^\lambda \xi^{\phi-1} &= \kappa\gamma_H, & (35)\\ \eta D u_H^{\eta-1}\xi^\beta - \delta_H &= \xi A\alpha k^{\alpha-1}u_Y^{1-\alpha} - \delta_K, & (36)\\ \eta D u_H^{\eta-1}\xi^\beta - \delta_H &= \frac{\lambda}{1-\alpha}\frac{F u_\tau^\lambda \xi^{\phi-1}}{Ak^{\alpha-1}u_Y^{1-\alpha}}\cdot\frac{u_Y}{u_\tau}\left(Ak^{\alpha-1}u_Y^{1-\alpha} - \frac{c}{k}\right) & \\ &\quad + F u_\tau^\lambda \xi^{\phi-1}\left(\phi + \frac{\lambda\beta}{\eta}\frac{u_H}{u_\tau}\right) & \\ &\quad + (1-\kappa)\gamma_H, & (37) \end{aligned}$$

where $\gamma_H = D u_H^\eta \xi^\beta - \delta_H$ and $u_H = 1 - u_Y - u_\tau$.

By further manipulating these five equations, we can show that the steady-state solution to u_H, u_H^*, is obtained implicitly from

$$\rho + \theta\gamma_H^* = \frac{\alpha\lambda\kappa\gamma_H^*(\gamma_H^* + \delta_\kappa)}{(1-\alpha)(\rho + \theta\gamma_H^* + \delta_\kappa)}\frac{1 - u_\tau^* - u_H^*}{u_\tau^*} + \kappa\gamma_H^*\left(\phi + \frac{\lambda\beta}{\eta}\frac{u_H^*}{u_\tau^*}\right) + (1-\kappa)\gamma_H^*, \tag{38}$$

where

$$\gamma_H^* = \frac{\rho + (1-\theta)\delta_H}{\frac{\eta}{u_H^*} - \theta} - \delta_H, \tag{39}$$

and

$$u_\tau^* = \left(\frac{\kappa\gamma_H^*}{F}\right)^{1/\lambda}\left(\frac{\rho + \theta\gamma_H + \delta_H}{\eta D}\right)^{\frac{1}{1-\eta}} u_H^*. \tag{40}$$

This last equation then yields u_{τ}^* as well as $u_Y^* = 1 - u_{\tau}^* - u_H^*$. In addition, we can sequentially obtain

$$\xi^* = \left(\frac{F u_{\tau}^{*\lambda}}{\kappa \gamma_H^*}\right)^{\frac{1}{1-\phi}}, \tag{41}$$

$$k^* = \left(\frac{\xi^* A \alpha}{\rho + \theta \gamma_H^* + \delta_K}\right)^{\frac{1}{1-\alpha}} u_Y^*, \tag{42}$$

$$c^* = \frac{\rho + \theta \gamma_H^* + \delta_\kappa - \alpha(\gamma_H^* + \delta_\kappa)}{\alpha} \cdot \frac{k^*}{\xi^*}. \tag{43}$$

3.2.3 Comparative Statics

Due to the complexity of the analytical solutions, we utilize simulation techniques to investigate the comparative statics of the model. Specifically, we analyze the impact of a change in θ, ρ, β, F, and D on the three shares of labor u_{τ}^*, u_Y^* and u_H^*, as well as the growth rate of the economy,.γ_H^*. The comparative statics are performed with respect to a particular parameter holding the other parameters constant. They should be interpreted relative to the base model with the following set of baseline values:

ρ	θ	α	δ_K	λ	ϕ	ϵ	β	δ_H	κ	A	D	F
0.02	1.5	1/3	0.05	2/3	0.2	2/3	0.4	0.05	5/6	0.35	0.2	0.1445

The results are presented in Fig.2 and Fig.3 and are summarized in the box below:

	u_H^*	u_{τ}^*	u_Y^*	γ_H^*
ρ	↓	↓	↑	↓
θ	↓	↓	↑	↓
β	↓	↓	↑	↑
F	↓	↑	↓	↑
D	↓	↑	↓	↑

An increase in the discount rate ρ predictably results in a reallocation of human capital to the final goods sector from the human capital production and financial innovations sectors. Since these two sectors are the engines of growth in the economy, the steady-state growth rate, γ_H^*, declines. An increase in θ, the measure of risk aversion or preference for consumption smoothing, produces the same qualitative effects as an increase in θ.

A rise in β, which measures the elasticity of spillovers from financial development on human capital accumulation, raises the steady-state growth rate of the economy, as do increases in D and F, the productivity parameters in the human capital and financial innovations production functions respectively.

Increases in D and F also result in an reallocation of human capital from the human capital and final goods sectors into the financial innovations sector.

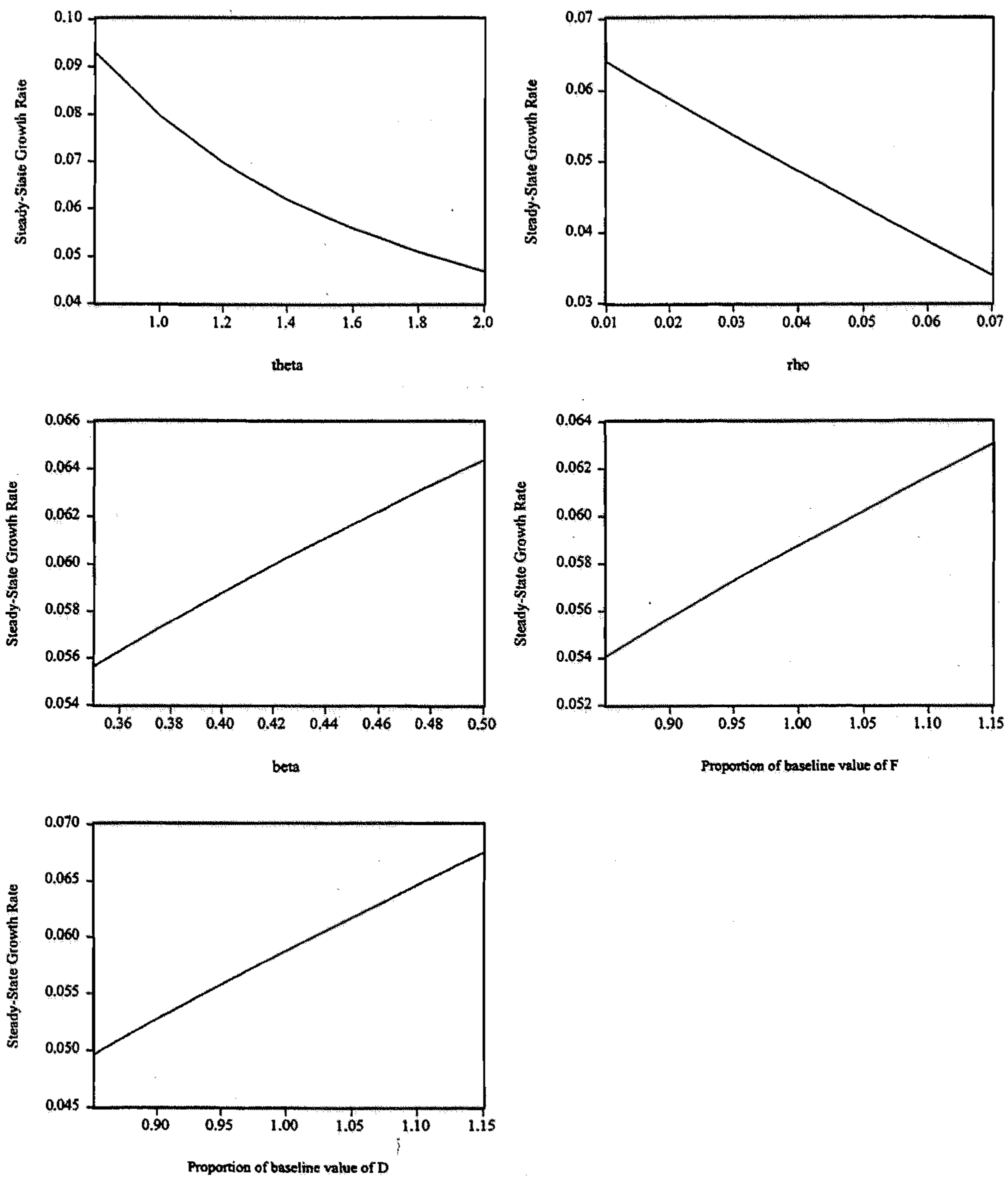

Figure 2: Simulated Comparative Statics for the Steady-State Growth Rate of the Economy

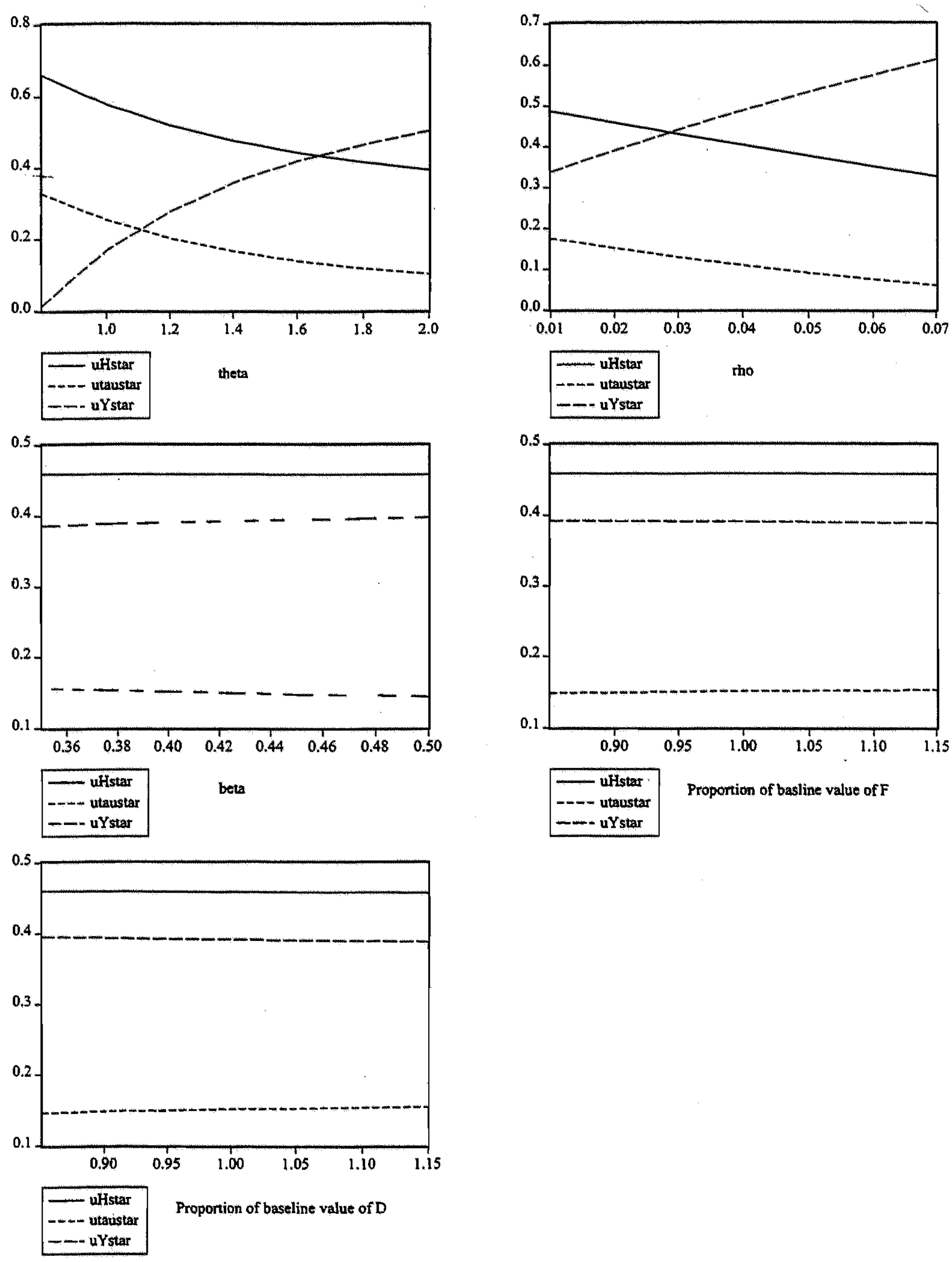

Figure 3: Simulated Comparative Statics for the Steady-State Shares of Labor Allocated to the 3 Sectors

3.2.4 Transitional Dynamics

The large dimensionality of the model (with 3 control variables and 3 state variables) necessitates the use of numerical methods when investigating its transitional dynamics. Specifically, we convert the model from continuous to discrete time and use the "shooting" method (implemented in a C-language computer program) to guess the magnitude of the jumps in the control variables c, u_Y, and u_τ occuring in the instant a shock impacts the system. "Correct" jumps ensure the system moves along the stable manifold until the new steady state is reached while incorrect jumps lead to dynamic paths which eventually violate the transversality conditions. In this section, we report the response of the state and control variables to a positive innovation in F. These are illustrated in Figure 6, which for clarity's sake is not drawn to scale.

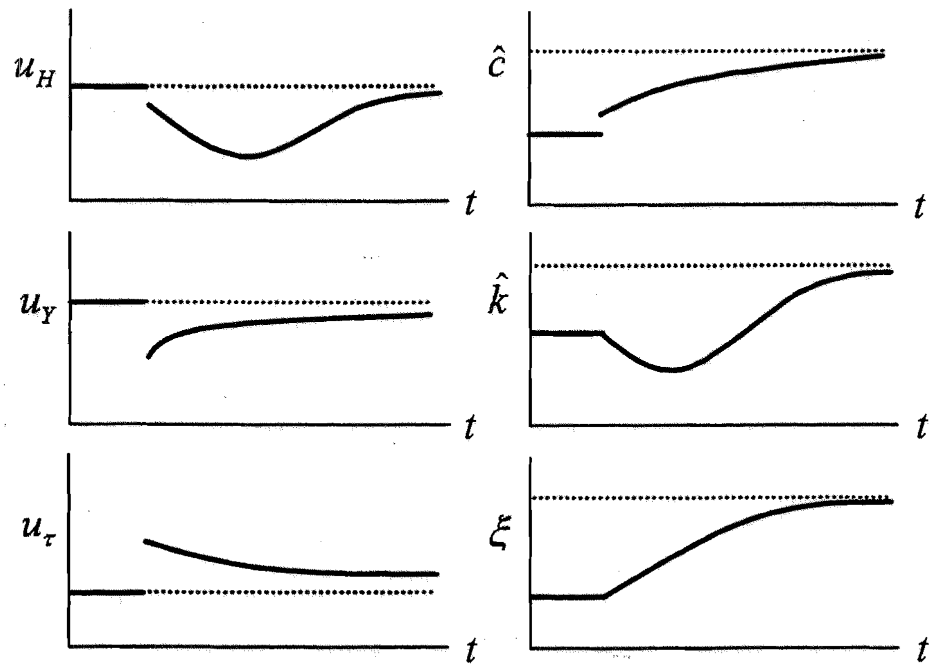

Figure 4: Impulse-Response Functions for an Increase in F

A rise in F raises the marginal product of labor of financial innovators, causing the share of labor in the financial sector, u_τ, to jump upwards. Conversely, this causes the share of labor in the final goods and human capital sectors, u_Y and u_H, to jump downwards in order for the marginal productivity of labor in these sectors to match that of the financial innovators. (Recall that both final goods and human capital production exhibit diminishing returns with respect to human capital.) u_Y and u_τ then slowly converge to their final levels; the change in F results in a steady-state reallocation of labor from the final goods sector and the human capital production sector to the financial innovations sector. The share of labor in the human capital production sector, u_H, at first continues to decline after the initial jump as u_Y recovers faster than the decline in u_τ, and then rises gradually back to its original level.

By affecting the marginal product of labor of financial innovators, the increase in F raises the wages received by households, causing consumption to jump up instantaneously. As more labor is channelled into the financial innovations sector and less into the final goods sector, capital accumulation declines until the increased efficiency of the financial intermediaries (due to rising ξ, the stock of financial products per adjusted unit of human capital, arising from u_τ being above its old steady state level) increases the rate of accumulation of capital and hence output. Consumption eventually reaches its new, higher steady-state level.

4 Policy Implications

Our comparative statics exercise shows that increases in the productivity parameters in the human capital and financial innovations sectors, D and F, raise the steady-state growth rate of the economy. The steady-state growth rate is also positively related to the magnitude of the spillover effect from financial development on human capital accumulation. This suggests that policymakers should try to encourage the development of financial products that are particularly effective in helping households finance the direct costs and opportunity costs of schooling. They should also endeavor to make the educational sector more efficient and productive.

Deregulation of the financial sector may lead to increased productivity of financial innovators (captured in our model by a rise in F), which raises the steady-state growth rate of the economy. (We can show that when F is too low, the economy may never achieve a 100 per cent transformation of savings into investment, i.e. $\xi < 1$ in the steady state.) Similarly, opening the financial sector of a less developed economy to leading-edge financial firms from advanced countries will enable a transfer of financial expertise from these countries to the less developed one, allowing the latter to raise its F parameter and thereby attaining a higher steady-state growth rate. This effect is not to be confused with the issue of increasing capital flows between countries.

Finally, the divergence of the decentralized solution from the planner's in this model suggests a reason for the desirability of mergers in the finance industry. As firms in the industry become fewer in numbers but stronger in market power, they begin to internalize the spillovers from current to future financial innovations. (If the industry consists of only one monopolistic firm, then it would in effect behave like the social planner with regards to such externalities.) This may account for the recent consolidations and mergers observed in the financial sector.

5 Conclusion

In this paper, we explored the inter-relationship between financial development and human capital accumulation. Financial development is brought about by financial innovations, a human capital-intensive activity, which improves the efficiency of financial intermediation. The rise in efficiency of financial intermediation in turn increases the pace of human capital accumulation. In the real world, for example, financial development alleviates borrowing constraints which may have previously prevented some housholds from acquiring human capital.

We used a formulation of the financial sector first demonstrated in Chou and Chin (2001) in the context of a growth model with endogenous technological progress. Our financial sector comprises financial innovators and financial intermediaries. Financial innovators utilize human capital and the existing catalog of financial products to develop new financial products and services. Financial intermediaries then purchase these innovations to improve their efficiency in transforming household savings into productive investment by firms. Financial innovations also have a positive spillover effect on human capital accumulation, with the aggregate stock of financial products affecting the rate at which new human capital is produced. We then explored the interactions between final goods firms, the human capital production sector, financial innovators, financial intermediaries, and households, explaining in detail the objective function and constraints faced by each entity in our model of the macroeconomy.

Comparing the solution to this decentralized, competitive model with that of a hypothetical social planner revealed two sources of divergence: the share of human capital allocated to the financial sector is lower in the decentralized model because financial innovators do not internalize the positive externalities of current financial innovations on future innovations and on human capital accumulation. This suggests that governments should perhaps play an active role in encouraging financial development, and that mergers and consolidations in the finance industry may enable firms to take into greater account the spillover from one financial innovation to the next. In addition, our comparative statics exercise suggests that policmakers should try to raise the efficiency and productivity of the financial innovations and human capital sectors, possibly through the deregulation of the financial sector and by reducing the amount of bureaucratic red-tape in the educational sector. Finally, policymakers should also encourage the development of financial products that assist in the financing of human capital accumulation.

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A The Decentralized Model

The Hamiltonian is

$$\begin{aligned} \mathbf{H} \equiv\ & \frac{c^{1-\theta}-1}{1-\theta}e^{-(\rho-n)t} + \nu\xi\left(r_K K + \bar{w}u_Y H - C\right) \\ & +\mu\widetilde{F}\left(u_\tau H\right)^{\lambda} + v\widetilde{D}\left(u_H H\right)^{\eta}. \end{aligned} \tag{44}$$

We can show that the solution to u_H^* is obtained implicitly from

$$\rho+\theta\gamma_H^* = \frac{\alpha\lambda\kappa\gamma_H^*(\gamma_H^*+\delta_\kappa)}{(1-\alpha)(\rho+\theta\gamma_H^*+\delta_\kappa)}\frac{1-u_\tau^*-u_H^*}{u_\tau^*}+(1-\kappa)\gamma_H^*, \tag{45}$$

where

$$\gamma_H^* = \frac{\rho+(1-\theta)\delta_H}{\frac{\eta}{u_H^*}-\tau}-\delta_H, \tag{46}$$

and

$$u_\tau^* = \left(\frac{\kappa\gamma_H^*}{F}\right)^{1/\lambda} \left(\frac{\rho + \theta\gamma_H + \delta_H}{\eta D}\right)^{\frac{1}{1-\eta}} u_H^*. \tag{47}$$

As in the planner's model, this equation then yields u_τ^*, as well as $u_Y^* = 1 - u_\tau^* - u_H^*$. In addition, we can sequentially obtain

$$\xi^* = \left(\frac{F u_\tau^{*\lambda}}{\kappa\gamma_H^*}\right)^{\frac{1}{1-\phi}}, \tag{48}$$

$$k^* = \left(\frac{\xi^* A\alpha}{\rho + \theta\gamma_H^* + \delta_K}\right)^{\frac{1}{1-\alpha}} u_Y^*, \tag{49}$$

$$c^* = \frac{\rho + \theta\gamma_H^* + \delta_\kappa - \alpha(\gamma_H^* + \delta_\kappa)}{\alpha} \cdot \frac{k^*}{\xi^*}. \tag{50}$$

The difference between the decentralized model and the planner's version is the absence of the term $-\kappa\gamma_H^*\left(\phi + \frac{\lambda\beta}{\eta}\frac{u_H^*}{u_\tau^*}\right)$ from the right-hand side of equation (38). As stated in Section 3.1.4, our simulations show that u_τ^* is smaller in the decentralized model for all reasonable choices of parameter values.

Note that depreciation is dropped here for simplicity.

B Mathematical Notation

C = consumption
ρ = subjective discount rate
θ = coefficient of risk-aversion in the utility function
δ = rate of depreciation
t = time
K = physical capital
H = human capital
u_Y = share of human capital devoted to production of final consumption good
u_τ = share of human capital devoted to production of financial innovations
u_H = share of human capital devoted to production of new human capital
τ = stock of financial innovations
$\xi \equiv \tau/H^\kappa$ = efficiency of intermediation between savings and investment
$c \equiv C/H$ = consumption to human capital ratio
$k \equiv K/H$ = physical capital to human capital ratio
$\widehat{k} \equiv K/AL$ = technology-augmented capital-labor ratio
γ_τ^* = steady-state growth rate of the stock of financial innovations
γ_H^* = steady-state growth rate of human capital
A = (constant) technology parameter

w_j = wage rate in sector j
r_V = interest rate on transformed savings earned by households
r_K = interest rate paid by financial intermediaries by borrowers (firms)
π_τ = profits earned by a financial innovator
P_τ = price of a financial product
P_H = price of one unit of human capital
α = capital's share of income generated in final goods production
λ = elasticity of financial innovation production with respect to human capital
ϕ = elasticity of financial innovation production with respect to the existing stock of financial products
κ = a measure of the average degree of rivalry in financial products
η = elasticity of human capital production with respect to existing human capital
β = elasticity of human capital production with respect to the stock of financial innovations

Financial Liberalization and Foreign Talent

(Preliminary Notes)

December 11, 2001

1 Financial Sector

A representative firm in the financial sector produces financial innovations according to the following function:

$$\dot{\tau} = \tilde{F} a^{\varepsilon\lambda} \left[L_\tau^\lambda + \left(a^{1-\varepsilon} L_f \right)^\lambda \right], \tag{1}$$

where $\tilde{F} \equiv F\tau^\phi$, $L_\tau \equiv u_\tau L$, $L_f \equiv \Omega L_\tau$, $\{a, b\} > 1$, $\varepsilon \in [0,1]$ and $\lambda \in (0,1)$. a denotes the relative productivity of foreign workers compared to domestic workers while ε measures the degree of skill diffusion from foreign workers to domestic workers. When $\varepsilon = 1$ full diffusion takes place so that domestic workers become as productive as foreign ones. Ω denotes the ratio of foreign workers to domestic workers employed in the financial sector. The financial innovator seeks to

$$\max_{L_\tau, L_f} \quad \pi_\tau = P_\tau \tilde{F} a^{\varepsilon\lambda} \left[L_\tau^\lambda + \left(a^{1-\varepsilon} L_f \right)^\lambda \right] - w_\tau L_\tau - w_f L_f, \tag{2}$$

where $w_f = b w_\tau$. The first-order conditions $\partial \pi_\tau / \partial L_\tau = 0$ and $\partial \pi_\tau / \partial L_f$ yield the following wage equations for the two types of workers:

$$w_\tau = \frac{\lambda P_\tau \tilde{F} a^{\varepsilon\lambda}}{L^{1-\lambda}}, \tag{3}$$

$$w_f = \frac{\lambda P_\tau \tilde{F} a^{\lambda}}{L^{1-\lambda}}. \tag{4}$$

Given the equilibrium condition $w_\tau = w_Y = \bar{w}$, we can obtain an equation for P_τ through equation (3) and then substitute it into equation (4) to get

$$\Omega = \left[\frac{a^{(1-\varepsilon)\lambda}}{b} \right]^{\frac{1}{1-\lambda}}.$$

Substituting the equations for P_τ and Ω into equation (2) yields the following equilibrium profits for the financial sector:

$$\pi_\tau = \frac{(1-\lambda)\, \bar{w} u_\tau L}{\lambda} \left[1 + b\Omega \right]. \tag{5}$$

where $b\Omega \equiv (a^{1-\varepsilon}/b)^{\frac{\lambda}{1-\lambda}}$. In the steady state, we need to write the price of each financial innovation as

$$p_\tau \equiv \frac{P_\tau}{L^{1-\kappa}} = \frac{\bar{w} u_\tau \left[1 + b\Omega \right]}{\lambda \gamma_\tau \xi}, \tag{6}$$

where $\kappa = \lambda / (1-\phi)$, $\gamma_\tau \equiv \dot{\tau}/\tau$ and $\xi \equiv \tau / L^\kappa$. Note that at every point in time, the following intermediation condition must hold:

$$r_K K = r_V K + P_\tau \dot{\tau}. \tag{7}$$

2 Final Goods Sector

A representative firm in the final goods sector seeks to

$$\max_{L_Y,K} \quad \pi_Y = AK^{\alpha}L_Y^{1-\alpha} - r_K K - w_Y L_Y, \tag{8}$$

where $L_Y \equiv u_Y L$ and $\alpha \in (0,1)$. The first-order conditions $\partial\pi_Y/\partial K = 0$ and $\partial\pi_Y/\partial L_Y$ yield the following equations respectively:

$$\begin{aligned} r_K &= \alpha A k^{\alpha-1} u_Y^{1-\alpha}, && (9)\\ w_Y &= \bar{w} = (1-\alpha)\, A k^{\alpha} u_Y^{-\alpha}, && (10) \end{aligned}$$

where $k \equiv K/L$.

3 Domestic Households

A representative domestic worker seeks to

$$\max_{c_d,u_Y} \quad U_{d,0} \equiv \int_0^{\infty} \frac{c_d^{1-\theta_1}-1}{1-\theta_1} e^{-(\rho_1-n)t} dt, \tag{11}$$

where $c_d \equiv C_d/L$, subject to

$$\begin{aligned} \dot{K}_d &= \tilde{\xi}\left(r_v K_d + w_Y u_Y L + w_\tau u_\tau L - C_d\right), && (12)\\ \dot{\tau} &= \tilde{F} a^{\varepsilon\lambda} (1+b\Omega)\left(u_\tau L\right)^{\lambda}, && (13)\\ r_K K &= r_V K + P_\tau \dot{\tau}, && (14)\\ K &= K_d + K_f, && (15)\\ 1 &= u_Y + u_\tau, && (16) \end{aligned}$$

where $\tilde{\xi} \equiv \tau/\tilde{L}^{\kappa}$, $\tilde{L} \equiv L + L_f = (1+\Omega u_\tau) L$, $K_d \equiv k_d L$ and $K_f \equiv k_f L_f = k_f \Omega u_\tau L$.

4 Foreign Households

A representative foreign worker seeks to

$$\max_{c_f} \quad U_{f,0} \equiv \int_0^{\infty} \frac{c_f^{1-\theta_2}-1}{1-\theta_2} e^{-(\rho_2-\tilde{n})t} dt, \tag{17}$$

where $c_f \equiv C_f/L_f$ and $\tilde{n} \equiv \dot{u}_\tau/u_\tau + n$, subject to

$$\begin{aligned} \dot{K}_f &= \tilde{\xi}\left(r_v K_f + w_f L_f - C_f\right), && (18)\\ r_K K &= r_V K + P_\tau \dot{\tau}, && (19)\\ K &= K_d + K_f, && (20)\\ 1 &= u_Y + u_\tau. && (21) \end{aligned}$$

5 Optimal Control Problem with respect to Domestic Households

$$
\begin{aligned}
\mathbf{H}_d \equiv\ & \frac{c_d^{1-\theta_1}-1}{1-\theta_1} e^{-(\rho_1-n)t} \\
& +\nu_d \tilde{\xi} \left[r_K K_d - \frac{w_\tau u_\tau L\,(1+b\Omega)}{\lambda} \frac{K_d}{K_d+K_f} + w_Y u_Y L + w_\tau u_\tau L - C_d \right] \\
& +\mu \tilde{F} a^{\varepsilon\lambda} (1+b\Omega)\,(u_\tau L)^\lambda . \qquad (22)
\end{aligned}
$$

The control variables are c_d and u_Y, the state variables are K_d and τ, and the costate variables are ν_d and μ. The first-order conditions for the control variables $\partial \mathbf{H}_d / \partial c_d = 0$ and $\partial \mathbf{H}_d / \partial u_Y = 0$ yield the following equations respectively:

$$
\frac{\dot{c}_d}{c_d} = -\frac{1}{\theta_1} \left(\rho_1 + \frac{\dot{\nu}_d}{\nu_d} + \frac{\dot{\tilde{\xi}}}{\tilde{\xi}} \right), \qquad (23)
$$

$$
\frac{\nu_d}{\mu} = \frac{\lambda \tilde{F} a^{\varepsilon\lambda} (1+b\Omega)\,(u_\tau L)^\lambda}{\tilde{\xi} \left[\frac{\kappa \Omega u_\tau}{1+\Omega u_\tau} \dot{V}_d + \frac{\bar{w} u_\tau L (1+b\Omega)}{\lambda} \left(\frac{K_d}{K}\right)^2 \right]}, \qquad (24)
$$

where $\dot{V}_d = r_v K_d + w_Y u_Y L + w_\tau u_\tau L - C_d$. The first-order conditions for the state variables are given by equations (12) and (13). The first-order conditions for the costate variables $\partial \mathbf{H}_d / \partial K_d = -\dot{\nu}_d$ and $\partial \mathbf{H}_d / \partial \tau = -\dot{\mu}$ yield the following equations respectively:

$$
-\frac{\dot{\nu}_d}{\nu_d} = \tilde{\xi} \left[r_K - \frac{\bar{w} u_\tau L\,(1+b\Omega)}{\lambda} \frac{K_f}{K^2} \right], \qquad (25)
$$

$$
-\frac{\dot{\mu}}{\mu} = \frac{\lambda \tilde{F} a^{\varepsilon\lambda} (1+b\Omega)\,(u_\tau L)^\lambda \tau^{\phi-1}}{\frac{\kappa \Omega u_\tau}{1+\Omega u_\tau} \dot{V}_d + \frac{\bar{w} u_\tau L (1+b\Omega)}{\lambda} \left(\frac{K_d}{K}\right)^2} \dot{V}_d . \qquad (26)
$$

Finally, the transversality conditions dictate that

$$
\lim_{t\to\infty} K_d(t)\,\nu_d(t) = 0, \qquad (27)
$$

$$
\lim_{t\to\infty} \tau(t)\,\mu(t) = 0. \qquad (28)
$$

6 Optimal Control Problem with respect to Foreign Households

$$\mathbf{H}_f \equiv \frac{c_f^{1-\theta_2}-1}{1-\theta_2}e^{-(\rho_2-\tilde{n})t} + \nu_f\tilde{\xi}\left[r_K K_f - \frac{w_\tau u_\tau L\left(1+b\Omega\right)}{\lambda}\frac{K_f}{K_d+K_f} + w_f L_f - C_f\right], \quad (29)$$

where the control variable is c_f, the state variable is K_f, and the costate variable is ν_f. The first-order condition for the control variable $\partial \mathbf{H}_f / \partial c_f = 0$ yields the following equation:

$$\frac{\dot{c}_f}{c_f} = -\frac{1}{\theta_2}\left(\rho_2 + \frac{\dot{\nu}_f}{\nu_f} + \frac{\dot{\tilde{\xi}}}{\tilde{\xi}}\right). \quad (30)$$

The first-order condition for the state variable is given by equation (18). The first-order condition for the costate variable $\partial \mathbf{H}_f / \partial K_f = -\dot{\nu}_f$ yields the following equation:

$$-\frac{\dot{\nu}_f}{\nu_f} = \tilde{\xi}\left[r_K - \frac{\bar{w}u_\tau L\left(1+b\Omega\right)}{\lambda}\frac{K_d}{K^2}\right]. \quad (31)$$

Finally, the transversality conditions dictate that

$$\lim_{t\to\infty} K_f\left(t\right)\nu_f\left(t\right) = 0. \quad (32)$$

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