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Discounting and the Social Time Preference Rate

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Abstract

This paper shows that the emphasis on a social time preference rate (defined as the sum of a pure time preference rate and the product of the elasticity of marginal valuation and the growth rate) in social evaluations where money values are discounted using the social time preference rate, is not advisable. It can give an entirely different, and arbitrary, ranking of alternative streams compared with the direct use of the pure time preference rate to discount 'social welfare' in each period (where social welfare is a – usually isoelastic – function of money values).

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1 Introduction

The aim of this paper is to draw attention to a problem associated with the use of the social time preference rate, which seems to have been overlooked in the literature on cost-benefit analysis and project appraisal. The social time preference rate, δ say, is a central concept in cost-benefit analysis involving the evaluation of alternative time profiles of, say, consumption. It is defined as the sum of the pure time preference rate, ρ, and the product of the elasticity of marginal valuation, ε, and the growth rate of consumption, g: hence $\delta = \rho + \varepsilon g$. It is therefore often stressed that an extreme assumption of zero pure time preference does not imply zero discounting, in view of the presence of the term εg.[1] The values of ρ and ε reflect the value judgements of an independent judge or policy maker.[2] Hence, cost-benefit analyses should in principle include a sensitivity analysis, involving the evaluation of time profiles for a range of values of ρ and ε.

The social time preference rate has in fact been the focus of considerable attention recently, partly because of its use by Stern (2006) in examining the economic effects of climate change and abatement policies.[3] Much attention has been given to the choice of values of ρ and ε.[4] Several of the critics have stressed the point made above, that the appropriate approach is not, as in Stern (2006), to impose particular values but to carry out sensitivity analyses, so that readers, with their own possibly different value judgements, can make up their own minds. However, the focus of the present paper is on the use of the social time preference rate itself.

The social welfare function, summarising the value judgements of the

[1] An arbitrary list of texts on cost-benefit analysis which discuss the social time preference rate includes Brent (1990, pp. 71-72, 92; 2003, pp. 166-168), Hanley and Spash (1993, pp. 128-130), Bateman *et al.* (2002, pp. 55-58), Layard and Glaister (1994, pp. 33-35), Dasgupta and Pearce (1972, pp. 141-143), Pearce and Ulph (1998) and Lind (1982, p. 89).

[2] Despite the inclusion of the word 'social', the social time preference rate is not an attribute of a society, in view of the well-known problem of aggregating preferences.

[3] See also the background paper by Hepburn (2006). Examples of criticisms of the discount rate used by Stern include Nordhaus (2006), Dasgupta (2006) and Carter *et al.* (2006).

[4] However, a discussion concentrating on g_t is by Weitzman (2007).

decision maker and underlying evaluations based on the social time preference rate, takes the form $\sum_{t=1}^{T} U(c_t)\left(\frac{1}{1+\rho}\right)^{t-1}$, where c_t represents consumption in period $t = 1, ..., T$ and $U(c_t)$ is the weight attached to c_t by the judge; the more concave is U, the greater the judge's aversion to variability over time (as distinct from time preference). This is discussed further in section 2, where it is shown that the concept of the time preference rate arises from consideration of the different question of determining the optimal time stream of c_t, subject to a wealth constraint. A comparison between the direct use of the social welfare function and the use of the social time preference rate is provided in section 3. It is argued that evaluations should be based directly on the form of welfare function mentioned above, rather than simply discounting values of c_t using the discount rate $\rho+\varepsilon g$. The potentially misleading nature of the latter approach is illustrated using a numerical example in section 4.

2 A Social Welfare Function

Suppose it is required to evaluate a time stream $c_1, ...c_T$ of consumption. For convenience c_t can be considered as the aggregate consumption in a society with constant population size and composition. Such an evaluation cannot avoid the use of value judgements, and the usual approach is to examine the implications of adopting a range of value judgements, using an additive Paretian social welfare function – representing the views of an independent judge – which takes the form:

$$W = \sum_{t=1}^{T} U(c_t)\left(\frac{1}{1+\rho}\right)^{t-1} \tag{1}$$

where $U(c_t)$ is the weight attached to period t's consumption by the judge, and ρ is the rate of pure time preference. The weighting function U is often called a utility function – although this terminology is misleading – and hence the time preference rate is sometimes also called a 'utility discount rate'.

Consideration of alternative value judgements regarding U is facilitated by the use of the iso-elastic form:

$$U(c_t) = \frac{c_t^{1-\varepsilon}}{1-\varepsilon} \tag{2}$$

The term $\varepsilon \neq 1$ measures the degree of constant relative aversion to variability on the part of the judge. Those who refer to U as a utility function typically refer to ε as the constant (absolute value of the) elasticity of marginal utility. Hence alternative value judgements – within the context of this class of welfare functions – can be examined by investigating W for a range of values of ε and ρ.

However, this is not always the way analyses proceed. Suppose that instead of considering an exogenous time stream of c_t, a 'social planner' has to determine the optimal time path by maximising a social welfare function of the form in (1). This is maximised subject to a budget constraint, which can be regarded as taking the simple form:

$$\sum_{t=1}^{T} \left(\frac{1}{1+r}\right)^{t-1} c_t = Y \tag{3}$$

where Y represents a measure of the present value of resources available for consumption over the period, and r is the rate of interest in a 'perfect' capital market. By forming the Lagrangean for this problem it can be shown that the first order condition for period t can be expressed as:

$$g_t = \frac{1}{\varepsilon}(r - \rho) \tag{4}$$

where g_t is the optimal proportional growth rate of consumption at t. This expression is known as the Euler equation for optimal consumption: it describes the time path of consumption for which (1) is maximised. This equation plays a prominent role in optimal growth models, where r can be regarded as being determined by, for example, the marginal product of capital – depending on the precise nature of the model considered.

Having obtained the result in (4) for optimal consumption, the standard approach is to move swiftly to the different context of cost-benefit evaluations, along the following lines. Another way of expressing the Euler equation is to rearrange (4) to give $r = \rho + \varepsilon g_t$. In the context of social evaluations of given time streams, it is this rearrangement of (4) that plays a substantial role. Considerable attention is given to the right hand side, which is called the social time preference rate, δ, and is thus defined as:

$$\delta = \rho + \varepsilon g_t \tag{5}$$

The social time preference rate can therefore vary over time, depending on the behaviour of g_t. In practice, it is often assumed that g, the growth rate of aggregate consumption, is constant. The latter is typically taken as the long run or average rate of growth over the relevant period. The value of δ is the discount rate used to evaluate the present value of the time stream of c_t, for $t = 1, ..., T$, rather than the time stream of $U(c_t)$. Hence δ is often called the 'consumption discount rate'. In the context of cost-benefit analyses where money values of an exogenous consumption stream are evaluated, then the social time preference rate, δ, does not need to be set equal to the market rate of interest, so that δ does not have to equal r. This in turn means that no degree of freedom is lost in the choice of parameters ρ and ε (for a given g). These two terms reflect the value judgements of the independent judge whose preferences are summarised by (1).

The social time preference rate is sometimes called the 'consumption discount rate' because it is applied to money values of consumption in each period, whereas the pure time preference rate is sometimes called the 'utility discount rate' because it is applied to weighted consumption values, with the weighting function described as a 'utility function'. In the literature, it is simply taken for granted that discounting money values according to the rate in (5) is appropriate, rather than starting from the more fundamental social welfare function. The following section therefore compares the two approaches.

3 Alternative Evaluation Methods

The standard approach in cost-benefit evaluations, discussed in the previous section, is to use a social time preference rate, as in (5), to discount money flows c_t over a specified period. This produces a 'social evaluation' using W^*, where:

$$W^* = \sum_{t=1}^{T} c_t \left(\frac{1}{1+\delta} \right)^{t-1} \tag{6}$$

with $\delta = \rho + \varepsilon g$. It is taken for granted that this function gives the same ranking of projects as does the social welfare function in (1). This section compares the two evaluation methods explicitly.

In comparing the two forms of evaluation, it is convenient to begin with the most favourable case, that is where consumption does in fact grow at the constant proportional rate, g. Hence $c_t = c_1 (1+g)^{t-1}$, for $t = 1, ..., T$, and substitution gives:

$$W = \sum_{t=1}^{T} \frac{\left\{c_1 (1+g)^{t-1}\right\}^{1-\varepsilon}}{1-\varepsilon} \left(\frac{1}{1+\rho}\right)^{t-1} \tag{7}$$

Rearrangement of this expression gives:

$$W = \sum_{t=1}^{T} \frac{c_1^{1-\varepsilon} (1+g)^{(t-1)-\varepsilon(t-1)}}{1-\varepsilon} \left(\frac{1}{1+\rho}\right)^{t-1} \tag{8}$$

and:

$$W = \frac{c_1^{-\varepsilon}}{1-\varepsilon} \sum_{t=1}^{T} c_1 (1+g)^{(t-1)} \left(\frac{(1+g)^{-\varepsilon}}{1+\rho}\right)^{t-1} \tag{9}$$

Furthermore, using the approximation $(1+\rho)(1+g)^{\varepsilon} = 1 + \rho + \varepsilon g$, this becomes:

$$W = \frac{c_1^{-\varepsilon}}{1-\varepsilon} \sum_{t=1}^{T} c_t \left(\frac{1}{1+\rho+\varepsilon g}\right)^{t-1} \tag{10}$$

and:

$$W = \frac{c_1^{-\varepsilon}}{1-\varepsilon} W^* \tag{11}$$

This final results demonstrates that it is not in fact correct to believe that W^*, obtained by discounting money values of consumption at the social time preference rate, coincides with W, obtained by discounting $U(c_t)$ at the pure time preference rate ρ.

For given ε, W^* automatically gives the same ranking as W only if $\varepsilon < 1$ and two consumption streams, with different growth rates, have the same initial value of consumption. Otherwise, inconsistencies can arise.

For example, suppose $\varepsilon = 2$, and two consumption streams A and B give values of $W_A = -50$ and $W_B = -100$. Hence using this criterion, stream A is judged to be superior to B. If $c_{A,1} = 10$, equation (11) shows that

$W_A^* = -100W_A = 5000$. if $c_{B,1} = 2$, $W_B^* = -4W_B = 400$, and the ranking by W^* agrees in this case. However, if the initial consumption values of the two streams are both equal to 2, then the values of W_A^* and W_B^* are 200 and 400 respectively, and the ranking is reversed.

Suppose instead that $\varepsilon = 0.5$, and $W_A = 80$ while $W_B = 50$, so that stream A is again judged to be superior to stream B. However, if $c_{A,1} = 4$ and $c_{B,1} = 16$, it can be seen that $W_A = \left(2/\sqrt{4}\right) W_A^*$ and W_A^* is also equal to 80, but $W_B^* = 50/(2/\sqrt{16}) = 100$: hence the ranking is reversed when W^* is used.

One way to view the comparisons is to recognise that if the 'utility' function is instead $U(c_t) = kc_t^{1-\varepsilon}$, with the constant $k = c_1^{\varepsilon}$, then the values of W and W^* are equal for any given consumption stream. This may seem like an innocent monotonic transformation of U. However, the social evaluation functions are essentially cardinal: they are not invariant to monotonic transformations of U, since they are expressed explicitly as additive functions of different U values. Furthermore, the function $kc_t^{1-\varepsilon}$ gives negative marginal utility if $\varepsilon > 1$, and so must be ruled out.

Furthermore, in practice it is likely that g is not constant over the relevant period. The assumption of constant g introduces a further 'error' when discounting money values using $\rho + \varepsilon g$, compared with discounting U with ρ. Allowance for changing growth rates is automatic in the latter case.

4 A Numerical Example

More complex comparisons may result from more variable time profiles, making the choice of alternative streams more sensitive to the choices of ε and ρ. Consider Figure 1, where time stream A results from a constant growth rate of 2.3 per cent (staring from 10 units), but profile B results from a fixed trend rate of growth (of 1.8 per cent, starting from 4 units) combined with a cyclical growth component having an amplitude of 5 per cent and a wavelength of 165 periods. From the multiple intersections, it is likely that stream B has the highest value of $W(C)$ for both low and high values of ρ, while stream A is likely to dominate for intermediate values, though the precise values again

again likely to be sensitive to the choice of ε. An example is given in Figure 2, for a value of the elasticity of marginal valuation, ε, of 0.6.

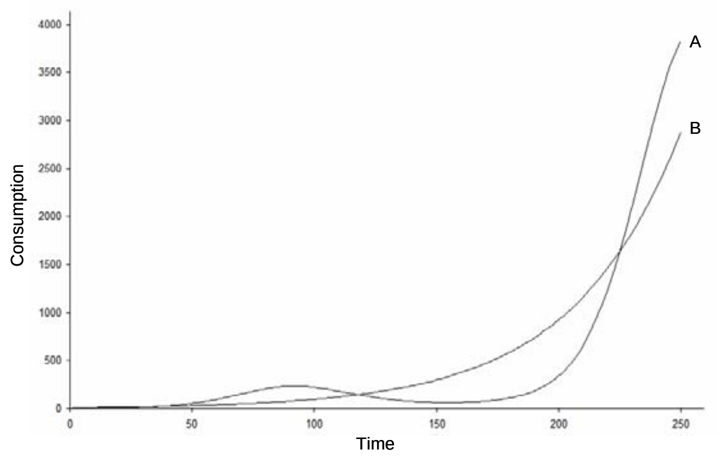

Figure 1: Alternative Time Profiles

Evaluations of the two time profiles using W^* are unlikely to give the same ranking. For example, Figure 3 shows the present value of the time streams of consumption shown in Figure 1, for $\varepsilon = 0.6$, using W^*, that is with money values discounted using the rate $\rho + \varepsilon g$ and with g set equal to the trend rate of growth. It can be seen that profile A dominates profile B for all values of ρ whereas, using the same value of $\varepsilon = 0.6$, comparisons of W depend significantly on the value of ρ used, as illustrated in Figure 2 above.

5 Conclusions

This paper has shown that the emphasis on a social time preference rate, expressed in terms of $\rho + \varepsilon g$, in social evaluations where money values are discounted using the social time preference rate, is not advisable. It can give an entirely different, and arbitrary, ranking of alternative streams compared with the discounting of U using ρ.

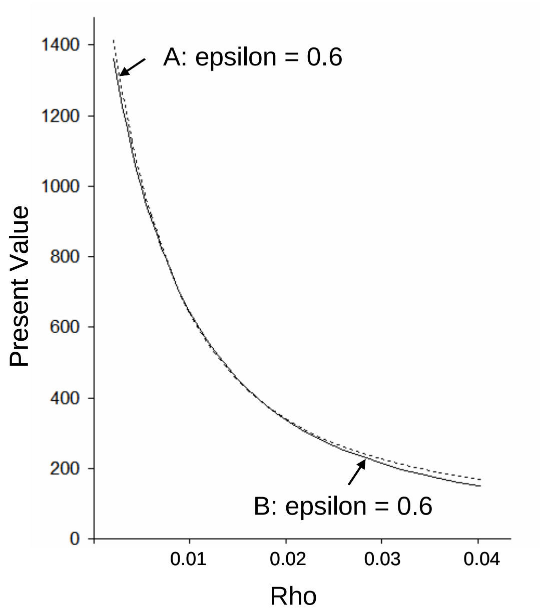

Figure 2: Rankings for Epsilon of 0.6

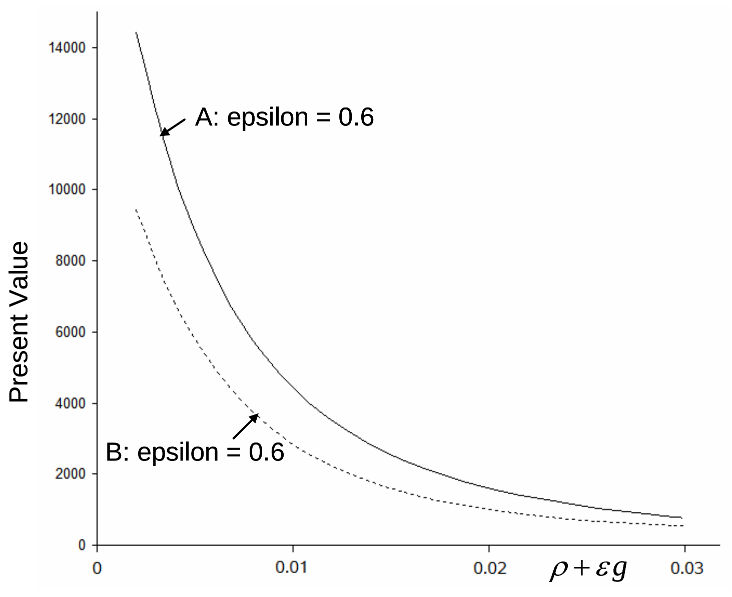

Figure 3: Comparisons Using the Social Time Preference Rate

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