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OPTIMAL LAND CONVERSION AND GROWTH WITH UNCERTAIN BIODIVERSITY COSTS

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Optimal Land Conversion and Growth With Uncertain Biodiversity Costs*

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Abstract

An important characteristic defining the threat of environmental crises is the uncertainty about their consequences for future welfare. Random processes governing ecosystem dynamics and adaptation to anthropogenic change are important sources of prevailing ecological uncertainty and contribute to the problem of how to balance economic development against natural resource conservation. The aim of this study is to examine optimal growth subject to non-linear dynamic environmental constraints. In a two-sector exogenous growth framework we model a stochastic environmental good, exhibiting uncertain ecological responses to environmental change, and describe the economic and environmental trade-offs that ensue for a risk-averse social planner. Allowing for ecological risk tends to slow economic growth if environmental impacts are assumed to increase exponentially as the rate of disturbance increases. Taken in isolation the effects of ecosystem resilience and ecological uncertainty on the rate of natural resource development are ambiguous and depend on normative parameters such as the social planner's attitude to risk and rate of time preference.

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1 Introduction

The aim of this paper is to characterise the trade-offs between economic growth and natural resource conservation by determining the optimal rate of conversion of a natural reserve of land. Land conversion fuels consumption growth in the private sector, while simultaneously giving rise to uncertain environmental consequences, accounted for in terms of biodiversity benefits and modelled as non-linear stochastic processes.[1] By modelling uncertainty explicitly in this way and allowing for decision-making to take non-linear economic-ecological dynamics into account, we build on existing literature on environmental constraints to economic growth. Krautkraemer (1998) found that allowing for deterministic environmental constraints in a model with exogenous technological progress reduces steady-state economic growth. Analysing the conditions necessary to attain an ecologically sustainable and balanced growth path, Hofkes (1996) showed that these relate to production and substitution elasticities as well as the function describing the dynamic relationship between the economic and the ecological system. Barrett (1992) concluded that normative parameters such as concerns for preservation and the welfare of future generations determine to a great extent the degree of environmental sacrifice for economic growth.

The importance for biological diversity and ecosystem resilience of rates of change of land use follows from ecological studies on extinction debt by Tilman *et al.* (1994) and Seabloom *et al.* (2002), whereby habitat destruction results in an increased number of species populations for which the threshold conditions for survival are no longer met.[2] Without habitat restoration or

[1] A number of economic studies of ecosystems examine the problem of prevailing ecological uncertainty. On the simulation of non-linear and stochastic ecosystem dynamics, see Carpenter and Cottingham (1997) and Perrings and Walker (1995).

[2] A number of studies focusing on economic-ecological aspects of biodiversity conservation model biodiversity as deterministic species-area relationships without allowing for the dynamic interaction between disturbance caused by land conversion and *in situ* biodiversity; see for example Smulders *et al.* (2004) and Polasky *et al.* (2004). On the economics of species extinction see Swanson (1994), and on the disturbance aspect of logging on tropical forests, see Chapman and Fimbel (2001). Swallow (1990) explored the economics of depleting a nonrenewable natural resource which supports an interdependent and renewable natural resource.

species phenotypic evolution, such species face local extinction with a high probability in the near future. The time lag between the conversion of land and extinction debt becoming an observable cost of land conversion gives rise to uncertainty. In addition, Hanski and Ovaskainen (2002) argued that faster rates of land conversion are likely to result in greater extinction debts and loss of ecosystem resilience, with the precise magnitude of these effects also subject to greater uncertainty.[3] We model the ecological processes of extinction debt as the uncertain outcome of a race between demographic stochasticity and evolutionary rescue, expressed as a function of the rate of change to which the ecosystem is exposed.

In the model, extinction debt does not directly affect the production of a private good, which can be consumed or invested in the form of capital which, along with land, is required for production.[4] However, it has negative consequences for biodiversity, which is valued as a public good. In order to concentrate on the explicit modelling of ecological uncertainty, the benefits arising from the private consumption good are assumed to be known with certainty. The economic and environmental trade-offs facing a risk-averse social planner over an infinite horizon are examined. The concept of ecological risk modelled here is closely related to the notion of environmental option value – the value attributed to the option of preserving a given environmental resource in the face of uncertainty. This paper treats environmental option value as a risk premium in the sense of Ciccetti and Freeman (1971).

Section 2 constructs the two-sector growth model, which describes the interaction between private sector growth and a stochastically dynamic natural resource in the public good sector. Section 3 considers the two components which comprise the arguments in the social welfare function. The trade-offs and policy choices of a risk-averse social planner are examined in section 4, where sensitivity analyses are carried out using numerical examples. Conclusions are in section 5.

[3] See Perrings (1998) for an exposition on the link between ecosystem states and their resilience to altered environmental conditions.

[4] This differs from the analysis in Hofkes (1996), where the ecological resource plays a role in the private good production sector.

2 The Framework of Analysis

The model consists of three principal components. First, a public good sector consisting of a natural resource, reserve land, gives rise to biodiversity benefits. Second, a private good, which may be used for consumption or investment purposes, is produced in what may be thought of as an agricultural sector. The third component is a planner who maximises a social welfare function defined in terms of the expected present values of private and public good benefits. For the purpose of this model, human impact on biodiversity is defined as a function of the rate, v, at which reserve land is converted for use in the private sector. The planner is regarded as making an optimal choice of v. These three principal components are discussed in subsections 2.1 to 2.3.

The economy is initially endowed with capital of K_0 and easily accessible reserve land, $R_0 = 1$, which is rich in biodiversity. Biodiversity reserve land may be converted for use in production.[5] Let $L(t)$ denote the land stock in the private sector at time, t, used in the production of a private good, and let $R(t)$ denote the reserve land at t (with $R(0) = R_0$), following conversion at the proportional rate, v.[6] Hence at time t:

$$R(t) = e^{-vt} R_0 \tag{1}$$

and:

$$L(t) = 1 - e^{-vt} R_0. \tag{2}$$

2.1 The Private Good Sector

Production in the private (agricultural) sector gives rise to a single good, $X(t)$. Population and labour supply are assumed to remain constant, and the production function is Cobb-Douglas, whereby:

$$X(t) = AL(t)^{\alpha} K(t)^{1-\alpha} \tag{3}$$

[5] Natural resource development is motivated by domestic consumption and production growth as opposed to exploiting trade opportunities as in Smulders *et al.* (2004)

[6] The focus here is on biodiversity in the forested reserve and not, as in Pascual *et al.* (2003) on the biological diversity characterising agricultural land.

where $K(t)$ is the capital stock at time t and $0 < \alpha < 1$. The output of the private good sector at time t is divided into consumption, $C(t)$, and investment, $I(t)$, so that:

$$X(t) = C(t) + I(t). \tag{4}$$

The rate of capital depreciation is δ, where $0 < \delta < 1$. Thus, where $\dot{K} = dK(t)/dt$, the equation of motion of the capital stock is:

$$\dot{K} = I(t) - \delta K(t). \tag{5}$$

An optimum profile of investment and consumption must be determined, conditional on the rate of conversion of reserve land. The optimum time profile of consumption, given v, is defined as $\hat{C}(t)$ and is examined further in section 3.2 and the Appendix.

2.2 The Public Good Sector

Reserve land is essential for the conservation of biological diversity, which is valued for its existence and option value, including its role in the fight against diseases and pathogens now and in the future. Furthermore, biological diversity and resilience of ecosystems are known to be highly susceptible to alterations to the environment. Within this context the extent and speed of reserve land conversion and habitat loss are important. The former is important as it determines the base, $R(t)$, from which biodiversity benefits are drawn and the latter is valued for its role in determining population density, $D(t)$, via its effect on extinction debt and ecological resilience. In the context of this model, biodiversity is represented by a bundle of populations, exhibiting identical responses to environmental change over time.

Using Grummet and Stirzaker's (1992) approach to measure biological populations, the stock of biodiversity at time t, $P(t)$, is regarded as a product of the population density, $D(t)$, of a representative species and the size of biodiversity reserve land, $R(t)$, so that:

$$P(t) = D(t)R(t). \tag{6}$$

While $R(t)$ reflects the stock effect of reserve land conversion on biological diversity, $D(t)$ measures the ecological response to the loss of habitat and associated disturbance levels. This response may be viewed as the outcome of a race between two independent processes – demographic stochasticity and evolutionary rescue, following Gomulkiewicz and Holt (1995). Higher rates of land conversion, v, represent increased environmental disturbance associated with less time to adapt to these changes. This may cause the current environmental conditions to come dangerously close to the minimum threshold, leaving species more susceptible to demographic stochasticity and increasing the risk of extinction. In contrast, a counteracting ecological response exists, whereby ecosystem disturbances stimulate phenotypic evolution.[7]

Biologists have developed birth-death models capable of approximating such processes; see for example Grummet and Stirzaker (1992). Here a simple birth-death process serves as the basis for modelling the representative species population density in the public good sector over time. The probability of species extinction, μ, is assumed to increase with rising rates of land conversion for private good production, so that:

$$\mu = f(v). \tag{7}$$

The probability of evolutionary rescue is denoted by η. Assuming that the extinction effect dominates that of evolutionary rescue, η is assumed to be a proportion of μ, so that:

$$\eta = \beta\mu \tag{8}$$

and $0 < \beta \leq 1$. Using a Markov chain, Grummet and Stirzaker (1992) show that the expected population density at time t, $E(D_t)$, is a function of the initial level of density, which evolves at an exponential rate such that:

$$E(D_t) = D_0 e^{-(1-\beta)\mu t}. \tag{9}$$

If $\beta = 1$, the corresponding variance is given by:

$$V(D_t) = 2D_0\mu t \tag{10}$$

[7] Phenotypic evolution is the process by which a species, otherwise destined for extinction, may be rescued by its own phenotypic traits that allow it to cope with and ultimately thrive in its altered environment.

and if $\beta \neq 1$:

$$\begin{aligned} V(D_t) &= D_0 \frac{\mu+\eta}{\mu-\eta} \left\{1 - e^{-(\mu-\eta)t}\right\} e^{-(\mu-\eta)t} \\ &= D_0 \frac{1+\beta}{1-\beta} \left\{1 - e^{-\mu(1-\beta)t}\right\} e^{-\mu(1-\beta)t}. \end{aligned} \tag{11}$$

Both η and μ are functions of anthropogenic disturbance in the ecosystem, represented by the rate of land conversion, v. Hence, this variable provides the link between both sectors in the economy and is the means through which the social planner characterises the trade-offs between the private and the public good. Ecological uncertainty increases as both μ and β increase.

2.3 The Social Planner

The social planner's objective is to maximise a social welfare function by setting a single policy variable, the rate of land conversion. As described in the previous subsection, anthropogenic disturbances of biodiverse ecological systems increase the probability of species extinction and evolutionary rescue, but the magnitudes of these effects are stochastic. As a result, the net human impact on the environment is uncertain. While the social planner may be risk averse regarding the ecological good, consumption benefits of the private good are assumed to be known with certainty.

The planner is assumed to have constant relative risk aversion regarding the public good. Hence the contribution of the public good to social welfare at time t, $W_P(t)$, is defined by:

$$W_P(t) = \frac{P(t)^{1-\varepsilon}}{1-\varepsilon} \tag{12}$$

where ε is the degree of constant relative risk aversion, with $\varepsilon \geqslant 0$ and $\varepsilon \neq 1$.[8] This implies that the social planner's attitude toward losing a given percentage of species remains unchanged as the level of biodiversity in the ecosystem changes. A risk-averse social planner would choose to have less of the public good with certainty over having more with uncertainty. Milgrom

[8]When $\beta = 1$, then $W_P(t) = \log P(t)$.

and Roberts (1992) show that the certainty equivalent, $\hat{P}(t)$, can be expressed as:

$$\hat{P}(t) = E\left(P(t)\right) - \frac{\varepsilon}{2} V\left(P(t)\right) \tag{13}$$

where the second term on the right hand side of equation (13) represents the risk premium. Substituting (9) and (11) into (13) yields:

$$\widehat{P}(t) = R_0 D_0 e^{-[v+(1-\beta)f(v)]t} \left[1 - \frac{\varepsilon}{2}\left(\frac{1+\beta}{1-\beta}\right)\left(1 - e^{-(1-\beta)f(v)t}\right)\right]. \tag{14}$$

The variance used to arrive at (14) applies to all $\beta \neq 0$ and $\beta \neq 1$. The case of a risk-neutral social planner is characterised by $\varepsilon = 0$, where only the first term in (14) is relevant.

The social planner's goal is to maximise a social welfare function which has as arguments the present value of the certainty equivalent of the public good and the present value of consumption along the optimal growth path, so that:

$$W = W\left(W_C, W_P\right) \tag{15}$$

where:

$$W_P = \int_0^\infty e^{-rt} \hat{P}(t) dt \tag{16}$$

and:

$$W_C = \int_0^\infty e^{-rt} \hat{C}(t) dt \tag{17}$$

where r is the planner's rate of time preference. As defined above, $\hat{C}(t)$ represents output of the private good at period t along the optimal growth path, conditional on v. This way of expressing the welfare function allows for a clear trade-off between separate private and public benefits, and means that the functional form of the function is fully flexible. The two components, W_P and W_C, are examined in turn in the following section.

3 The Two Components of The SWF

This section derives convenient formulae for the two components, W_P and W_C, of the social welfare function in (15). Subsections 3.1 and 3.2 examine public and private good components in turn.

3.1 The Public Good

The present value, W_P, of the benefit (in terms of the certainty equivalent) arising the public good is given by substituting (14) into (16) and integrating, whereby:

$$W_P = \frac{R_0 D_0}{v + r + (1-\beta) f(v)} \left[1 - \frac{\varepsilon}{2} \left(\frac{1+\beta}{1-\beta} \right) \left(\frac{v + r + (1-\beta) f(v)}{v + r + 2(1-\beta) f(v)} \right) \right]. \tag{18}$$

Given that the probability of species extinction increases as the rate of land conversion increases, that is $f(v)' > 0$, the value of the risk premium is positive for all values of v as long as $0 < \beta < 1$. Accounting for ecological risk from land conversion therefore leads to lower public good utility over the full range of v.

The relationship between the probability of species extinction and the rate of land conversion is captured by the general functional form $f(v)$. A convenient specification of this function, in terms of a single parameter, is:

$$\mu = f(v) = v^{\theta} \tag{19}$$

where $\theta \geqq 1$, implying that the ecosystem is tolerant to small scale disturbance. Significant ecological response is triggered only as the disturbance, represented by the rate of land conversion, v, increases to high levels. The coefficient θ may be said to describe the 'resilience' of the system, as higher values of θ are associated with lower values of μ. Thus, the extinction debt incurred through land conversion is expected to be smaller the larger is θ.

3.2 Private Sector Growth

Given the choice of the constant rate of land conversion, v, it is necessary to determine the optimum time profile of consumption, denoted $\hat{C}(t)$. This is examined in detail in the Appendix, where it is shown that:

$$\hat{C}(t) = \frac{K}{L}\left[\left(\frac{r+\delta}{1-\alpha} - \delta\right)\left(1 - R_0 e^{-vt}\right) - vR_0 e^{-vt}\right] \tag{20}$$

and along the balanced growth path, the ratio of capital to land remains constant at the value given by:[9]

$$\frac{K}{L} = \left(\frac{1-\alpha}{r+\delta}\right)^{1/\alpha} A^{1/\alpha} \tag{21}$$

Substitution into equation (17) and integrating gives the contribution of the private good to social welfare as:

$$W_C = \frac{K}{L}\left[\left(\frac{r+\delta}{1-\alpha} - \delta\right)\left(\frac{1}{r} - \frac{R_0}{r+v}\right) - R_0 \frac{v}{r+v}\right] \tag{22}$$

where the capital-land ratio is again given by equation (21).

4 Trade-Offs and Policy Choice

This section examines the policy choice of v for the case where W in (15) is expressed as:

$$W = W_P^{\gamma} W_C^{1-\gamma} \tag{23}$$

This specification allows for a single parameter, γ, to describe the relative weight attached to biodiversity benefits, and rules out corner solutions where either no private good is produced or all land is immediately converted.

It is useful to illustrate the properties of the model using numerical examples. For convenience, a number of the parameters may be normalised, so that $R_0 = D_0 = A = 1$. Production in the private sector is assumed to be elastic with respect to land, with $\alpha = 0.7$. The model emphasises the trade-off between biodiversity, representing a significant value of national wealth, and agricultural production. Biodiversity hotspots, the most diverse and therefore most valuable natural areas, are predominantly situated in developing countries. In these countries, subsistence agriculture is still common,

[9] The classic exogenous growth model formulation with decreasing returns to scale in the private sector satisfies the Inada conditions and leads to the growth rates of private sector variables assymptotically approaching zero as t approaches infinity. For example it can be shown that $\frac{\dot{K}}{K} = \frac{\dot{L}}{L} = v\frac{R(t)}{L(t)}$.

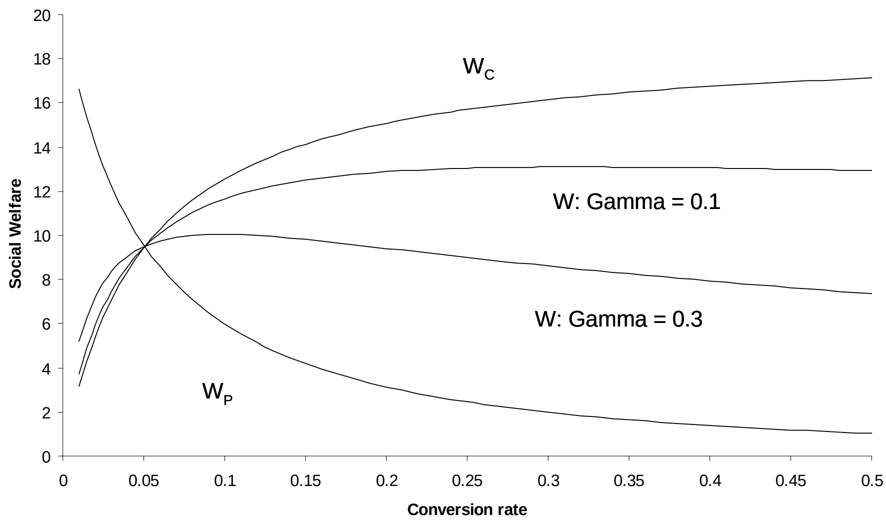

Figure 1: Social Welfare and Land Conversion Rate

with a high proportion of income linked to the area of land under cultivation as opposed to modern agriculture where a larger proportion of agricultural output is due to capital intensive inputs such as machinery and fertilizers. Capital is assumed to depreciate at the proportional rate, $\delta = 0.10$.

Empirical tests of the phenomenon of evolutionary rescue are rare, so little guidance is available regarding the value of the constant β in the probability of evolutionary rescue. Gomulkiewicz and Holt (1995) suggest low likelihoods of such an event occurring. Therefore, in the majority of cases examined below, β is set equal to 0.05, but the effects of varying β are also examined.

Normative variables, those depending on the planner's value judgements, include the weight attached to biodiversity, γ, the rate of time preference, r, and the constant degree of relative risk aversion, ε. It is therefore useful to consider the sensitivity of results to these variables.

First, an indication of the form of variations in the welfare components is given in Figure 1, obtained using $r = 0.05$, $\theta = 2.0$ and $\beta = 0.05$ for ecological resilience and $\varepsilon = 1.5$, with other parameters as indicated above. This value of r may be thought to be rlatively high, but sensitivity analyses are examined below. The component W_C increases as v increases, while

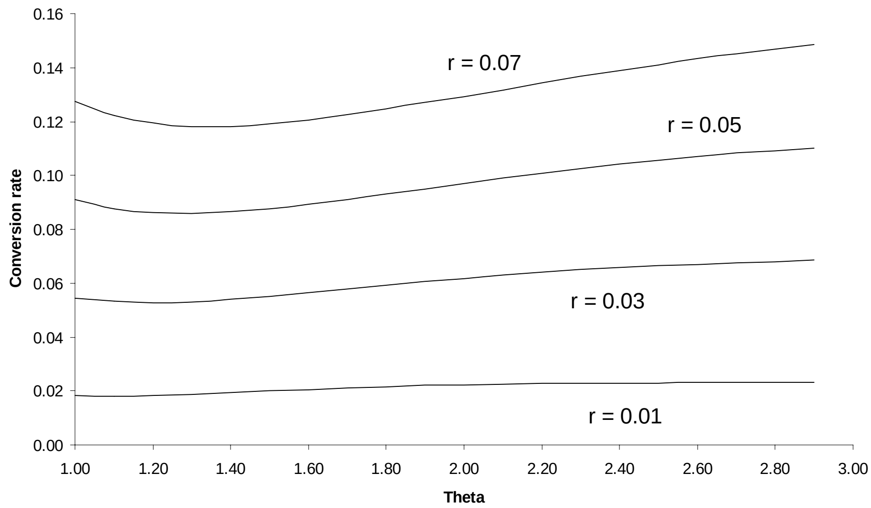

Figure 2: Optimal Conversion Rate and Theta

W_P decreases continuously. The value of social welfare, W, passes through the point of intersection of the two components and its global maximum coincides with the rate of land conversion the social planner optimally chooses to maximise the joint welfare from the public and the private good. Figure 1 shows the variation in total welfare for two values of γ, the weight attached to biodiversity benefits. The optimal value of v clearly increases as γ is reduced. It appears from the W profiles that the 'cost' of exceeding the optimal v is low, as each profile is relatively flat beyond its maximum. However, this arises because of the relatively high discount rate used; a lower discount rate substantially increases the cost of adopting an excessive conversion rate.[10]

Figure 2 shows how the optimal conversion rate varies as the degree of environmental resilience, θ, varies, for alternative discount rates. The optimal rate was obtained numerically, by searching for the value of v that maximises W. Other parameters are the same as those used for Figure 1, with $\gamma = 0.3$. Clearly, a higher discount rate produces a higher optimal conversion rate.

[10] The importance of interest rates within the context of economic growth has also been noted by Rubio and Renan (1998), who find constant land allocations to the private sector optimal in the long run for high discount rates.

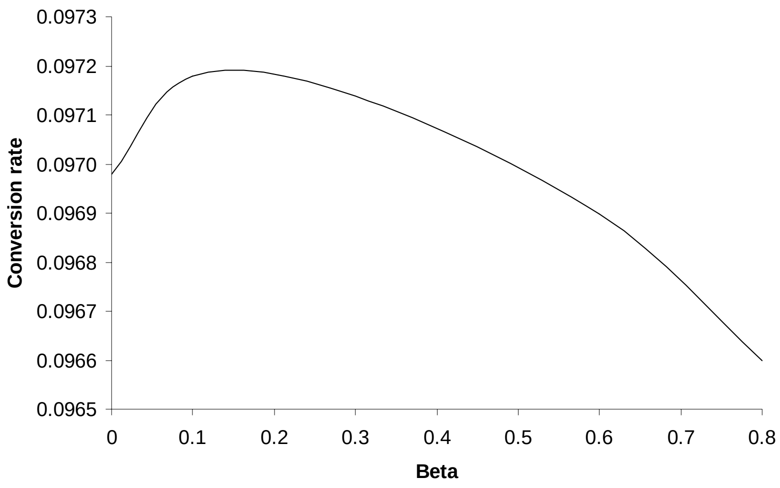

Figure 3: Optimal Conversion Rate and Beta

For the higher values of r and low $\theta \geq 1$, the optimal conversion rate falls very slightly as θ increases. This arises because of the link between μ and η, so that the increase in θ is associated with a reduction in η, which is more noticeable at lower values of θ. This reduction in the probability of 'rescue' produces the lower optimal conversion rate. This effect is less noticeable at lower discount rates, where a longer term perspective is taken. For higher values of resilience, further reductions in μ via increases in θ clearly dominate. This result supports the conjecture that more resilient ecosystems can cope with higher levels of disturbance before their ecological response, for example in terms of the size of the extinction debt, become prohibitive to further increase the rate of land conversion and consumption growth.

The effect of varying the probability of evolutionary rescue, η, relative to the probability of species extinction, μ, is shown by varying β in Figure 3, for other parameters as in Figure 1, with $\gamma = 0.3$. There is little variation in the optimal conversion rate as β is increased. At low values, the increase in η as β increases has a dominating effect on the conversion rate, which thus increases slightly. However, for higher β values, the effect on ecological uncertainty – the variance in equation (11) – dominates for a risk-averse planner, and the

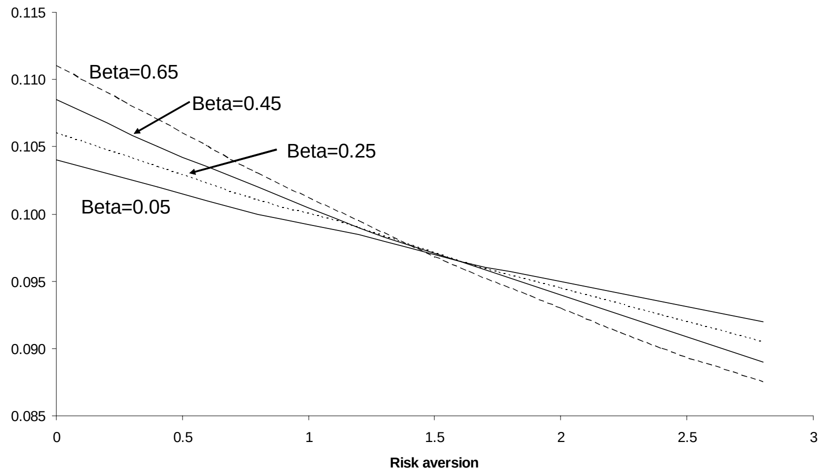

Figure 4: Optimal Conversion Rate and Risk Aversion: Alternative Betas

optimal conversion rate falls as β increases further. The suggestion here that higher values of β are associated with an optimal conversion rate that is more sensitive to risk aversion is shown in Figure 4. Again, other parameters are as in Figure 1, with $\gamma = 0.3$. The effect on optimal conversion rates of a 'living dead' species being rescued by phenotypic evolution with greater probability outweighs the increased uncertainty as long as the social planner exhibits levels of risk aversion below a given threshold. Risk aversion above the threshold implies that the added uncertainty surrounding the ecological response to land conversion leads the social planner to opt for a more prudent approach to natural resource development.

The effect on optimal conversion rates as risk aversion is increased, for alternative values of θ, is shown in Figure 5.[11] As expected, higher ecological resilience is associated with higher optimal conversion rates and a lower sensitivity to risk aversion.[12]

The relative risk aversion parameters considered here cover a wide range from risk neutrality to substantial aversion. Estimates from empirical and

[11] Again, other parameters are as in Figure 1, with $\gamma = 0.3$.

[12] For values of θ close to unity, not shown in the diagram, an increase in θ is associated, in view of the fixed value of β, with a reduction in η. This means that, for lower values of risk aversion, the optimal value of the land conversion rate actually increases slightly.

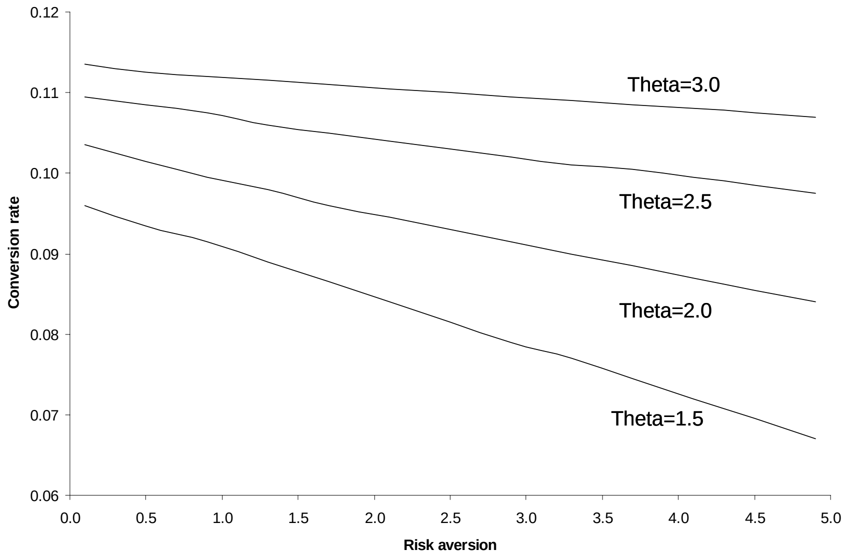

Figure 5: Conversion Rate and Risk Aversion: Alternative Thetas

experimental studies, ranging from 0.3 to 2.5 and reported by Dubois and Vukina (2003) and Johansson-Stenman *et al.* (2001), do not relate to environmental risks. Where such risks are studied, they are limited to the willingness to pay for risk reduction, as in Carson and Mitchell (2000), or environmental damage compensation; see Earnhart (2000). Benartzi and Thaler (1995) found that individuals act with more prudence if confronted with a one-off gamble, which is more relevant to the present context. Also, Johansson-Stenman *et al.* (2001) and Praag and Booij (2003) found that relative risk aversion increases with a longer time horizon.

5 Conclusions

The aim of this paper was to examine the optimal choice of land conversion where values, in a social welfare function, are attached both to consumption of a private good and uncertain biodiversity benefits arising from the unconverted land. Extinction debt, the ecological response to environmental disturbance, was modelled as a stochastic process, reflecting the considerable uncertainty involved. The dynamics in the private sector in terms of consumption and capital growth are governed by the rate of land conversion from biodiversity reserve into agricultural land. Environmental quality was assumed to appear only in the social welfare function, and not in the production function of the private sector good.

It is widely recognised that concern for the environment acts as a restraint on the rate of land conversion, and hence growth in the private sector.[13] This paper has concentrated on the role of ecological resilience to environmental disturbance, uncertainty and a planner's attitude to risk in determining the optimal balance between economic growth and ecological conservation. It was shown that none of these variables considered has an unequivocal effect on optimal conversion. Instead they are interlinked so that increases in ecological resilience and uncertainty may lead to faster or slower optimal natural resource development depending on the magnitude of other variables such as the social planner's attitude to risk and the discount rate applied. The analysis demonstrated the importance of having information on the relationship between land conversion, ecological resilience and the magnitude of the extinction debt, as changing the curvature of the ecological response function led in some instances to almost double the optimal conversion rate. The results were less sensitive to assumptions regarding the probability of evolutionary rescue, which is fortunate given the difficulties involved in testing for such processes.

Sensitivity analyses conducted to test the robustness of results showed that the model is sensitive to normative parameters characterising the social welfare function, such as the weight attached to biodiversity benefits, the degree of risk aversion and the choice of the discount rate. These results reinforce the argument of Barrett (1992, p. 279) that 'the fate of the natural environment depends as much on society's ethical views as on positive economic parameters'.

[13] See for example Gradus and Smulders (1993) who test this hypothesis using three different types of growth model.

Appendix: Growth in The Private Good Sector

This appendix derives the optimum growth characteristics of the private good, remembering that this is considered as conditional on the rate of land conversion. The objective is thus to derive the time profile of $C(t)$ to maximise:

$$\int_{0}^{\infty} e^{-rt} C(t) dt \tag{24}$$

which, given the conditions (3)-(5), is subject to:

$$\dot{K} = AK(t)^{1-\alpha} L(t)^{\alpha} - C(t) - \delta K(t) \tag{25}$$

where $K(0) = K_0$ and $\dot{K} = dK/dt$. To simplify notation, the time argument t is henceforth omitted from the control and state variables.

The current value Hamiltonian comprises of the integral in (24) and the capital constraint (25):

$$\tilde{H} = C + m(AK^{1-\alpha} L^{\alpha} - C - \delta K) \tag{26}$$

where $m = e^{rt}\lambda(s)$. The Hamiltonian conditions are (27-29):

$$\frac{\partial \tilde{H}}{\partial C} = 0 = 1 - m \tag{27}$$

$$\frac{\partial \tilde{H}}{\partial K} = -\dot{m} + rm = (1-\alpha) m A K^{-\alpha} L^{\alpha} - m\delta \tag{28}$$

where $\dot{m} = dm/dt$, and:

$$\frac{\partial \tilde{H}}{\partial m} = \dot{K} = AK(t)^{1-\alpha} L(t)^{\alpha} - C(t) - \delta K(t) \tag{29}$$

The transversality condition arising from the endpoint constraint is:

$$m(\infty) \geqslant 0 \quad \text{and} \quad \left[\hat{K}(\infty) - K_{\infty}\right] m(\infty) = 0 \tag{30}$$

where $\hat{K}$ is the capital stock that results from the optimal path at infinity.

It follows from equation (27) that the co-state variable, m, equals 1 for all t. Thus the capital constraint in equation (26) is binding at all time and $\dot{m} = 0$. Applying this result to (28) gives the capital-land ratio:

$$\frac{K}{L} = \left(\frac{1-\alpha}{r+\delta}\right)^{1/\alpha} A^{1/\alpha} \tag{31}$$

Along the optimal growth path, the ratio of capital to land remains constant.

Rearranging the third Hamiltonian condition (29) and appropriate substitution gives the optimal private good consumption, $\hat{C}(t)$, at any point in time along its transitional path towards the endpoint:

$$\hat{C} = \frac{K}{L}\left[\left(\frac{r+\delta}{1-\alpha} - \delta\right)\left(1 - R_0 e^{-vt}\right) - vR_0 e^{-vt}\right] \tag{32}$$

The term in the square brackets in equation (32) is the intertemporal opportunity cost of investment on consumption. The ratio in the first parenthesis in (32) determines the extent of the trade-off between investment and consumption. The effective discount rate in the numerator favours consumption over investment. The denominator represents the constraint on consumption through the investment requirement, which is higher the more elastic agricultural production is with respect to capital. The capital depreciation rate, δ affects consumption in much the same way: increased investment is necessary to offset increases in capital depreciation, leading to lower consumption levels along the optimal path. The two exponential terms represent the effect of the binding land constraint on consumption.

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