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# Modelling Personal Income Taxation in Spain: Revenue Elasticities and Regional Comparisons 

By<br>John Creedy and José Félix Sanz-Sanz ${ }^{1}$


#### Abstract

This paper derives analytical expressions for the revenue elasticity of the Spanish personal income tax system, as applied to tax units and in aggregate. This is complicated by the schedular nature of the system, and the role of central and regional governments, along with the existence of a range of tax credits and eligible expenditures and deductions. Empirical estimates are obtained using a cross-sectional dataset which enables a number of important ancillary elasticities (relating to allowances and tax credits, and different income sources) to be estimated. It was found that there is considerable variation among tax units in the revenue elasticity, with highly (positively) skewed distributions. The nature of the distributions varies among regions of Spain, and the aggregate elasticities for each region were found to display some variation associated with income distribution differences. The national aggregate is found to be around 1.3. The paper also derives aggregate tax revenue as a function of characteristics of the distribution of taxable income in each region. This allows the sources of revenue differences among regions to be identified.


[^0]
## 1 Introduction

An important characteristic of any personal income tax structure is the elasticity of income tax revenue with respect to changes in gross income, when there are no adjustments to income thresholds or other discretionary changes to the tax structure. The revenue elasticity provides, at individual and aggregate levels, a measure of 'fiscal drag' arising from the failure to adjust income tax thresholds when incomes increase. Fiscal drag, or 'built-in flexibility', has implications for both the revenue and redistributive effects of taxation over the business cycle. ${ }^{2}$ This measure is also useful when considering the 'automatic stabilisation' properties of the tax system. ${ }^{3}$ For tax planning purposes it is also important to be able to estimate the implications for total income tax revenue of a range of exogenous changes.

The aim of this paper is to estimate the revenue elasticity properties of the Spanish personal income tax structure and to provide a method of examining the implications for total tax revenue of a number of changes, including changes in the distribution of income and in the tax structure itself. Although the focus of attention is the Spanish structure, the methods used are more widely applicable.

The Spanish tax system differs from that of many other countries and has undergone significant reforms, in additional to the type of base-broadening and rate-reducing changes which have been common in many other countries. ${ }^{4}$ In particular, income taxation (since 2002) is shared between Central and Regional Governments, consisting of 15 autonomous regions within the Common Territory. In addition, different tax rates and thresholds, and other rules influencing the difference between

[^1]gross and taxable income, apply to a range of income sources: this involves the use of a multi-schedular tax structure. There are numerous deductions, allowances and tax credits (at central and regional levels) which apply at various stages. A number of these elements depend on non-income as well as income characteristics of tax units. This complexity means that extensions need to be made to standard methods of obtaining revenue elasticities. ${ }^{5}$

The approach followed here is to derive an analytical expression for the revenue elasticity of tax units. This is shown to depend on a number of 'ancillary elasticities' which affect the way in which eligible expenditures and deductions, and tax credits, vary with unit income, along with the relative movements of each income source. A large cross-sectional sample of Spanish tax units is then used to estimate values of the ancillary elasticities, allowing for a substantial degree of heterogeneity whereby the elasticities differ according to total tax unit income, the demographic composition of the unit, the location (automonous region) and the income source. The aggregate revenue elasticity for each region and for the country as a whole is then obtained as a tax-share weighted sum of tax unit revenue elasticities, where the weights depend on the way in which each tax unit's income changes when total income changes.

Having examined revenue elasticities, this paper then turns to the derivation of an expression for aggregate tax revenue, depending on proportions of people (within each region) and proportions of total income between the income thresholds of the income tax function. In carrying out the aggregation process in a tractable manner, it is necessary to begin from a given distribution of taxable income. This contrasts with the approach used to calculating revenue elasticities, where the latter are related to changes in gross incomes. The approach allows the sources of differences in tax revenue among regions to be identified.

[^2]Section 2 provides a description of the Spanish personal income tax system and formulates analytical expressions for the tax liability of each tax unit. Revenue elasticities relating to each tax unit are derived in Section 3, which also provides some numerical illustrations of their variation with tax unit income. Section 4 turns to the empirical estimation of revenue elasticities. First it obtains the distribution over tax units, using the ancillary elasticity estimates. Second, aggregate revenue elasticities for each region are reported. Section 4 also considers the potential implications of alternative income dynamic processes which allow 'regression' away from or towards the geometric mean income. Aggregate tax revenue is then examined in more detail in Section 5. Brief conclusions are given in Section 6 .

## 2 The Tax Structure

This section describes the main elements of the personal income tax structure in Spain. The accounting period is the tax year, which corresponds to the calendar year. Subsection 2.1 provides a basic description of the structure as it applies to an individual tax unit, where the unit may consist of single individuals or married couples who decide to file jointly. In view of the operation of tax credits, several special cases need to be distinguished, as discussed in subsection 2.2.

### 2.1 Income Taxation of a Tax Unit

Let $y_{h i}$ denote the gross income of tax unit $h$ from source $i=1, \ldots, I$. In transforming from gross to taxable income, there are tax-deductible expenditures and non-income allowances. Let $E_{h i}$ denote the tax-deductible expenditure for unit $h$ relating to source $i$. In general these expenditures are expected to be a function of gross income: this is examined in more detail below. Non-income allowances for tax unit $h$ relating to source $i$ are denoted $A_{h i}$. Taxable income, $x_{h i}$ is given by:

$$
\begin{equation*}
x_{h i}=\max \left\{0, y_{h i}-E_{h i}-A_{h i}\right\} \tag{1}
\end{equation*}
$$

If the sum of actual tax-deductible expenditures and non-income allowances exceeds gross income for any income source, the unit effectively has 'losses' associated with
that source. ${ }^{6}$ A distinction can therefore be drawn between actual expenditures and those which are claimed in a year: in the following discussion, $E_{h i}$ refers to actual expenditures. A complication is that any 'losses' can be carried forward for a period of four years, to be deducted against future income for the same source. However, no allowance is made for this dynamic element on the grounds that the losses form a very small component of income, as shown in Appendix A.

The income tax structure has marginal tax rates $t_{k i}$ and thresholds $a_{k i}$ for $k=1, \ldots, K$, where $t_{k i}$ applies between $a_{k i}$ and $a_{k+1, i}$ (with $\left.a_{K+1, i}=\infty\right) .{ }^{7}$ In addition, as mentioned above, separate rates are imposed at the central and regional government levels, although the income thresholds are common. Letting superscripts $C$ and $R$ refer to central and regional rates respectively:

$$
\begin{equation*}
t_{k i}=t_{k i}^{C}+t_{k i}^{R} \tag{2}
\end{equation*}
$$

For a multi-step tax structure with $K$ steps, $T(x)=0$ for $a_{0}=0<x<a_{1}$, $T(x)=t_{1}\left(x-a_{1}\right)$ for $a_{1}<x<a_{2}$, and $T(x)=t_{1}\left(a_{2}-a_{1}\right)+t_{2}\left(x-a_{2}\right)$ for $a_{2}<x<a_{3}$, and so on. Then in general, if $a_{k}<x<a_{k+1}$, Creedy and Gemmell (2006, p. 25) show that:

$$
\begin{equation*}
T(x)=t_{k}\left(x-a_{k}^{\prime}\right) \tag{3}
\end{equation*}
$$

where:

$$
\begin{equation*}
a_{k}^{\prime}=\frac{1}{t_{k}} \sum_{j=1}^{k} a_{j}\left(t_{j}-t_{j-1}\right) \tag{4}
\end{equation*}
$$

Hence in the present context, if $a_{k i}<x_{h i}<a_{k+1, i}$, unit $h$ is in the $k$ th tax bracket for source $i$ and the following expressions describe income taxation at central and regional levels.

$$
\begin{align*}
& T_{i}^{C}\left(y_{h i} \mid a_{k i}<x_{h i}<a_{k+1, i}\right)=t_{k i h}^{C}\left(x_{h i}-a_{k i h}^{C}\right)  \tag{5}\\
& T_{i}^{R}\left(y_{h i} \mid a_{k i}<x_{h i}<a_{k+1, i}\right)=t_{k i h}^{R}\left(x_{h i}-a_{k i h}^{, R}\right) \tag{6}
\end{align*}
$$

[^3]The terms $a_{k i}^{\prime C}$ and $a_{k i}^{\prime R}$ are the corresponding thresholds such that tax liability in a multi-threshold tax structure can be expressed in terms of an equivalent single-rate structure. In writing the expressions (5) and (6) the marginal tax rate terms, $t$, along with the effective thresholds, $a^{\prime}$, need the $h$ subscripts, in order to clarify the point that the tax rates and thresholds indicated are those that apply to the tax unit in question, depending on the tax bracket into which the unit falls.

In addition, there are central and regional government non-refundable tax credits of $C_{C}$ and $C_{R}$. Total tax paid by unit $h$ is expressed as:

$$
\begin{equation*}
T\left(\sum_{i} y_{h i}\right)=\max \left\{0, \sum_{i=1}^{I} T_{i}^{C}\left(y_{h i}\right)-C_{C}\right\}+\max \left\{0, \sum_{i=1}^{I} T_{i}^{R}\left(y_{h i}\right)-C_{R}\right\} \tag{7}
\end{equation*}
$$

In addition, there are refundable tax credits, unrelated to income. However, it is argued that such refundable credits, since they can in principle be administered by a separate authority and their cost is unrelated to the income tax structure, should not be included where - as here - emphasis is on the revenue elasticity from the point of view of revenue growth and fiscal drag. This issue is discussed further in Appendix B.

The existence of non-refundable tax credits means that several cases must be distinguished. These are discussed in the following subsection.

### 2.2 Special Cases

Consider the most common situation where tax unit $h$ is such that $\sum_{i=1}^{I} T_{i}^{C}\left(y_{h i}\right)>C_{C}$ and $\sum_{i=1}^{I} T_{i}^{R}\left(y_{h i}\right)>C_{R}$. The expression given in (7) above for tax liability is thus simplified to:

$$
\begin{equation*}
T\left(\sum_{i} y_{h i}\right)=\sum_{i=1}^{I}\left\{T_{i}^{C}\left(y_{h i}\right)+T_{i}^{R}\left(y_{h i}\right)\right\}-\left(C_{R}+C_{C}\right) \tag{8}
\end{equation*}
$$

and:

$$
\begin{equation*}
T\left(\sum_{i} y_{h i}\right)=\sum_{i=1}^{I}\left\{t_{k i h} x_{h i}-\left(t_{k i h}^{C} a_{k i h}^{\prime C}+t_{k i h}^{R} a_{k i h}^{\prime R}\right)\right\}-\left(C_{R}+C_{C}\right) \tag{9}
\end{equation*}
$$

Furthermore, where $x_{h i}>0$ this becomes:

$$
\begin{equation*}
T\left(\sum_{i} y_{h i}\right)=\sum_{i=1}^{I}\left\{t_{k i h}\left(y_{h i}-E_{h i}-A_{h i}\right)-\left(t_{k i h}^{C} a_{k i h}^{\prime C}+t_{k i h}^{R} a_{k i h}^{\prime R}\right)\right\}-\left(C_{R}+C_{C}\right) \tag{10}
\end{equation*}
$$

A further simplification is available in view of the fact that the central and regional income thresholds are the same. Using the above expression for $a^{\prime}{ }_{k}$, it can be shown that:

$$
\begin{equation*}
t_{k i h}^{C} a_{k i h}^{C}+t_{k i h}^{R} a_{k i h}^{\prime R}=\sum_{j=1}^{k_{k i}} a_{j i}\left(t_{j i}-t_{j-1, i}\right) \tag{11}
\end{equation*}
$$

If, alternatively, $\sum_{i=1}^{I} T_{i}^{C}\left(y_{h i}\right)>C_{C}$ but $\sum_{i=1}^{I} T_{i}^{R}\left(y_{h i}\right)<C_{R}$, tax liability is thus:

$$
\begin{equation*}
T\left(\sum_{i} y_{h i}\right)=\sum_{i=1}^{I}\left\{T_{i}^{C}\left(y_{h i}\right)\right\}-C_{C} \tag{12}
\end{equation*}
$$

and if $x_{h i}>0$ this becomes:

$$
\begin{equation*}
T\left(\sum_{i} y_{h i}\right)=\sum_{i=1}^{I}\left\{t_{k i h}^{C}\left(y_{h i}-E_{h i}-A_{h i}\right)-\left(t_{k i h}^{C} a_{k i h}^{\prime}\right)\right\}-C_{C} \tag{13}
\end{equation*}
$$

with $t_{k i}^{C} a^{\prime C}{ }_{k i}=\sum_{j=1}^{k} a_{j i}\left(t_{j i}^{C}-t_{j-1, i}^{C}\right)$. Similarly, if $\sum_{i=1}^{I} T_{i}^{C}\left(y_{h i}\right)<C_{C}$ but $\sum_{i=1}^{I} T_{i}^{R}\left(y_{h i}\right)>C_{R}$, the above expressions apply with $C$ replaced by $R$.

## 3 Individual Revenue Elasticities

This section considers the tax revenue elasticity, measuring the extent to which tax revenue increases when gross income increases, at the level of the tax unit.
Consider the effect on tax paid by a tax unit of a small increase in gross income, arising from changes in each of the sources, which does not take the unit into a higher tax bracket. $^{8}$ First, define $T\left(\sum_{i} y_{h i}\right) \equiv T_{h}$ as the total tax paid by the unit. Furthermore, define $y_{h}=\sum_{i} y_{h i}$ as total gross income from all sources.

The change in tax paid by the unit when total gross income changes is given by:

[^4]\[

$$
\begin{equation*}
\frac{d T_{h}}{d y_{h}}=\sum_{i=1}^{I} \frac{\partial T_{h}}{\partial y_{h i}} \frac{\partial y_{h i}}{\partial y_{h}} \tag{14}
\end{equation*}
$$

\]

Hence:

$$
\begin{equation*}
\frac{y_{h}}{T_{h}} \frac{d T_{h}}{d y_{h}}=\sum_{i=1}^{I}\left(\frac{y_{h i}}{T_{h}} \frac{\partial T_{h}}{\partial y_{h i}}\right)\left(\frac{y_{h}}{y_{h i}} \frac{\partial y_{h i}}{\partial y_{h}}\right) \tag{15}
\end{equation*}
$$

In general denote the elasticity of A with respect to B using the notation $\eta_{A, B}$. Thus:

$$
\begin{equation*}
\eta_{T_{h}, y_{h}}=\sum_{i=1}^{I} \eta_{T_{h}, y_{h i}} \eta_{y_{h}, y_{h}} \tag{16}
\end{equation*}
$$

The elasticity of total tax paid by unit $h$ therefore depends on the way in which the individual components of income change when the unit's total gross income changes, determined by $\eta_{y_{h}, y_{h}}$.

Consider the component elasticity $\eta_{T_{h}, y_{h i}}$. Here it is not possible to obtain a component elasticity defined in terms of the revenue from a single source, because the non-refundable tax credits are related to total income tax rather than its components. If it were possible to distinguish revenue from each source, as for example $T_{h i}$, the elasticity $\eta_{T_{h}, y_{h}}$ could be expressed as a tax-share $\left(T_{h i} / T_{h}\right)$ weighted sum of the product of individual elasticities $\eta_{T_{i,}, y_{h i}}$ and $\eta_{y_{h_{i}}, y_{h}}$.

For those with positive taxable incomes in excess of the tax credits, and supposing that eligible expenditures and allowances change when income from source $q$ changes:

$$
\begin{equation*}
\frac{\partial T_{h}}{\partial y_{h q}}=t_{k q h}\left(1-\frac{\partial E_{h q}}{\partial y_{h q}}-\frac{\partial A_{h q}}{\partial y_{h q}}\right) \tag{17}
\end{equation*}
$$

This can be rewritten:

$$
\begin{equation*}
\frac{y_{h q}}{T_{h}} \frac{\partial T_{h}}{\partial y_{h q}}=\eta_{T_{h}, y_{h q}}=\frac{t_{k q} y_{h q}}{T_{h}}\left(1-\frac{\partial E_{h q}}{\partial y_{h q}}-\frac{\partial A_{h q}}{\partial y_{h q}}\right) \tag{18}
\end{equation*}
$$

The ratio $T_{h} / y_{h q}$ is the total tax paid by unit $h$ as a proportion of $h$ 's income from source $q$, which may be denoted by $A T R^{\prime}{ }_{h q}$ : the prime is added here as it is not the averate rate associated with source $q$. It can thus be interpreted as a kind of average
tax rate: if there were no distinction between income sources, it would be a standard average tax rate. The term $\partial T_{h} / \partial y_{h q}$ is the marginal tax rate, $M T R_{h q}$, relating to a change in income source $q$. The tax revenue elasticity for unit $h$ with respect to a change in income source $q$ is thus the ratio, $M T R_{h q} / A T R^{\prime}{ }_{h q}$, as in the standard result. Then it can be seen that:

$$
\begin{equation*}
\eta_{T_{h}, y_{h q}}=\frac{t_{k q h} y_{h q}}{T_{h}}-\eta_{E_{h q}, y_{h q}}\left(\frac{t_{k q h} E_{h q}}{T_{h}}\right)-\eta_{A_{n q}, y_{h q}}\left(\frac{t_{k q h} A_{h q}}{T_{h}}\right) \tag{19}
\end{equation*}
$$

The term $t_{k q h} E_{h q} / T_{h}$ represents the tax 'saved' at the margin from the existence of the deduction, $E_{h q}$, expressed as a ratio of total tax paid. Denote this by $\theta_{E, h q}$. A similar term, $\theta_{A, h q}$, can be defined relating to allowances. Furthermore, let $t_{\text {kqh }}=M I T R_{\text {hqh }}$, where the subscript $h$ is included as a reminder that the appropriate marginal rate depends on the specific situation facing the tax unit. The notation, including ' $I$ ', indicates that it is the marginal income tax rate, not the effective marginal tax rate, $\partial T_{h} / \partial y_{h q}$. The elasticity can therefore be written:

$$
\begin{equation*}
\eta_{T_{h}, y_{h q}}=\frac{M I T R_{h q}}{A T R_{h q}^{\prime}}-\eta_{E_{n q}, y_{h q}} \theta_{E, h q}-\eta_{A_{n q}, y_{h q}} \theta_{A, h q} \tag{20}
\end{equation*}
$$

In the special case where $E_{h q}$ and $A_{h q}$ are fixed, so that $\theta_{E, h q}=\theta_{A, h q}=0$, then of course MITR $_{h q}=M T R_{h q}=t_{k q} .{ }^{9}$

A further complication arises where the tax credits, $C_{C}$ and $C_{R}$, are not fixed, but depend on household characteristics. These credits are not connected with individual income sources, unlike the expenditures and allowances. Suppose instead that the tax credits depend on total income, $y_{h}$. The above elasticity is then further reduced by subtracting the term:

$$
\begin{equation*}
\left\{\eta_{C_{C h}, y_{h}}\left(\frac{C_{C h}}{T_{h}}\right)+\eta_{C_{R h}, y_{h}}\left(\frac{C_{R h}}{T_{h}}\right)\right\}\left(\frac{1}{\eta_{y_{h q}, y_{h}}}\right) \tag{21}
\end{equation*}
$$

[^5]Using the above property that $\eta_{T_{h}, y_{h}}=\sum_{i=1}^{I} \eta_{T_{h}, y_{h}} \eta_{y_{h i}, y_{h}}$, defining $A T R_{h}=T_{h} / y_{h}$ as the overall average tax rate facing the unit, and noting that $\eta_{y_{h i}, y_{h}}=\eta_{y_{h}, y_{h i}}^{-1}$ and, for example, $\eta_{a, b} \eta_{b, c}=\eta_{a, c}$, it can be shown that, for those taxpayers with $C_{C}>T_{h}^{C}$ and $C_{R}>T_{h}^{R}:$

$$
\begin{align*}
\eta_{T_{h} y_{h}} & =\sum_{i=1}^{I} \frac{t_{k i h}}{A T R_{h}} \\
& \times\left[\eta_{y_{h i} y_{h}} \frac{y_{h i}}{y_{h}}-\eta_{E_{h i} y_{h}} \frac{E_{h i}}{y_{h}}-\eta_{A_{h i y} y_{h}} \frac{A_{h i}}{y_{h}}-\left\{\eta_{C_{C h}, y_{h}} \frac{C_{C h}}{y_{h}}+\eta_{C_{R h}, y_{h}} \frac{C_{R h}}{y_{h}}\right\} \frac{1}{t_{k h}}\right] \tag{22}
\end{align*}
$$

This can also be written as:

$$
\begin{align*}
\eta_{T_{h} y_{h}}= & \sum_{i=1}^{I} \frac{t_{k i h}}{A T R_{h}}\left(\frac{y_{h i}}{y_{h}}\right) \eta_{y_{h h} y_{h}} \\
& -\sum_{i=1}^{I} \frac{t_{k i h}}{T_{h}}\left[\left(\eta_{E_{h h} y_{h}} E_{h i}+\eta_{A_{h i} y_{h}} A_{h i}\right)+\left(\eta_{C_{C h}, y_{h}} C_{C h}+\eta_{C_{R h}, y_{h}} C_{R h}\right) \frac{1}{t_{k i h}}\right] \tag{23}
\end{align*}
$$

If there were only one income source, then $y_{h i} / y_{h}=\eta_{y_{h i}, y_{h}}=1$ and the first term above would be simply the ratio of the marginal tax rate to the average tax rate facing the unit: this is the standard expression for the revenue elasticity. The second term shows the modifications arising from the eligible expenditures and allowances, which are involved in the transformation from gross to taxable income, and the central and regional tax credits. Special cases of this result apply for situations where tax credits are greater or equal than the tax liability after the application of the tax schedule.

### 3.1 Illustrative Examples

This subsection illustrates the way in which the tax revenue elasticity varies for individuals in Spain. Following the Spanish tax code operating in 2007, attention is concentrated on just two sources of income and on the effects of varying eligible expenditures, allowances and tax credits as gross income increases. The first income source includes: labour income; alimony; self employment income; income from property and income applications to shareholders coming from Corporations under the fiscal transparency regime (similar to S-Corporations in the USA). The second
income source includes: capital gains and any form of income derived from financial savings such as interest rates from bank accounts and deposits, share dividends, bond interest or any other type of yield earned from debt saving instruments. Incomes include both monetary compensations and fringe benefits.

The allowable tax deductions, $E$, are income related specific deductions which generally include a shortlist of necessary expenditures incurred in order to earn the relevant income. Good examples of this are the employee Social Security contributions and union membership fees for labour income, loan interest payments, maintenance costs or economic depreciation in the case of property income, or a restricted list of some operating expenses from savings or entrepreneurship. Together with this, $E$ entails the existence of a fixed labour-specific tax deduction of $4,000 €$ for earnings less than or equal to $9,000 €$. Notwithstanding, this tax deduction turns out to be income-decreasing for earnings between $9,000 €$ and $13,000 €$ and becoming fixed again at a reduced amount of $2,600 €$ for earnings of $13,000 €$ and above.

Allowances, A, incorporate non-specific tax allowances and deductions. This includes paid palimony, contributions to Pension Schemes and personal and family allowances. Examples of the latter are the allowances recognized for special circumstances such as age, disability or the existence of dependants (ancestors and/or descendants). These non-specific income allowances are normally capped and present some limitations for its application in terms of the taxpayer's income level and type of income. Finally, tax credits include all non-refundable tax relief enjoyed by the taxpayers in order to compute the final tax due after applying the tax schedules. For a detailed description of the specific quantities applied in year 2007 see Agencia Tributaria (2008), and for an evolution of all these concepts through time in the Spanish case, see Romero and Sanz-Sanz (2007).

The marginal rates and thresholds for the first income source are shown in Table 1. For the second source, tax is paid at fixed central and regional (marginal and average) rates of 0.111 and 0.069 .

Table 1 Tax Structure for Income Source 1

| Income Threshold <br> $(€$ s $)$ | Central Govt <br> MTR | Regional Govt <br> MTR | Total <br> MTR |
| :--- | :--- | :--- | :--- |
| 0 | 0.1566 | 0.0834 | 0.24 |
| 17,360 | 0.1827 | 0.0973 | 0.28 |
| 32,360 | 0.2414 | 0.1286 | 0.37 |
| 52,360 | 0.2713 | 0.1587 | 0.43 |

Four different cases, for parameters listed in Table 2, are illustrated. In each case a fixed ratio of income from the two sources is assumed, whereby source two is 10 per cent of source one. Case 1 takes the (unrealistic) extreme of fixed eligible expenses, allowances and credits. The following cases gradually introduce elasticities, assumed to be constant, so that Case 4 allows all deductions and credits to vary as income varies. For example, in obtaining the values of expenditures, and so on, the following specification was thus used:

$$
\begin{equation*}
E_{h i}=E_{0} y_{h i} \eta_{E_{i}, y_{i}} \tag{24}
\end{equation*}
$$

The various elasticities, such as $\eta_{E_{i}, y_{i}}$, are referred to here as 'ancillary elasticities', and their estimation for Spain is described in the following Section, with values reported in Appendix D. For estimation purposes, a major aim was to allow for as much population heterogeneity as possible. For present illustrative purposes the parameters in Table 2 are imposed, based on orders of magnitude obtained for the estimates.

Table 2 Alternative Parameters for Four Cases

|  | Case 1 | Case 2 | Case 3 | Case 4 |
| :--- | :--- | :--- | :--- | :--- |
| Source 1 |  |  |  |  |
| $\mathrm{E}_{0}$ | 3500 | 98 | 98 | 98 |
| Elasticity | 0 | 0.4 | 0.4 | 0.4 |
| $\mathrm{~A}_{0}$ | 5000 | 5000 | 7000 | 7000 |
| Elasticity | 0 | 0 | 0.005 | 0.005 |
| Source 2 |  |  |  |  |
| $\mathrm{E}_{0}$ | 35 | 0.3 | 0.3 | 0.3 |
| Elasticity | 0 | 0.8 | 0.8 | 0.8 |
| A $_{0}$ | 5000 | 5000 | 4750 | 4750 |
| Elasticity | 0 | 0 | 0.05 | 0.05 |
| Credits |  |  |  |  |
| C $_{\text {C }}$ | 1200 | 1200 | 1200 | 800 |
| Elasticity | 0 | 0 | 0 | 0.05 |
| C $_{\text {R } 0}$ | 550 | 550 | 550 | 13 |
| Elasticity | 0 | 0 | 0 | 0.4 |

The resulting variations in individual revenue elasticities are shown in Figures 1 to 4. Clearly the highest elasticity values are obtained when expenditures, deductions and credits are fixed. Tax unit elasticities can become extremely high where income is just above the tax threshold where units begin to pay tax: in the limit - right at the threshold - the elasticity is of course infinitely high because the denominator (the initial tax paid) is zero. This property influences the distribution of elasticities discussed in the following Section.

From Figure 1, no tax is paid until total income reaches approximately $16,775 €$, when income from the first source becomes subject to the regional government rate of 0.0834 and income from the second source is taxed at the regional government rate of 0.069 . At these levels, just above the threshold when the individual begins to pay tax, the revenue elasticity is very high. It then falls, until a total income of about $17,875 €$ is reached. At this point, the individual's incomes from both sources are taxed at both the central government and regional rates, so that the marginal tax rates applying to sources one and two are 0.24 and 0.18 respectively. On crossing into the higher marginal tax rate brackets, the revenue elasticity shoots up again, after which it declines steadily until reaching the next threshold.

When total income is about $28600 €$, the marginal tax rate applied to income from the first source becomes 0.28 (the combined central and regional rates), and a smaller
jump in the revenue elasticity is observed. The next income threshold is about 45,100 $€$ when income from the first source begins to be taxed at a combined rate of 0.37 . The effect is that the pattern of revenue elasticities displays the familiar 'saw tooth' pattern.

Figure 1 Variation in Individual Revenue Elasticity with Total Gross Income: Case 1


Figure 2 Variation in Individual Revenue Elasticity with Total Gross Income: Case 2


Case 2, where positive ancillary elasticities are introduced for eligible expenditures, the pattern is similar to that for Case 1 , although of course the effective income thresholds are different. Thus initially only regional government taxes are paid in relation to both income sources, then another threshold is reached where central government tax rates are also applied. The individual then gradually moves into the higher tax brackets relating to the first income source. Similar characteristics apply when, in Case 3, ancillary elasticities for allowances are also positive.

## Figure 3 Variation in Individual Revenue Elasticity with Total Gross Income: Case 3



Figure 4 Variation in Individual Revenue Elasticity with Total Gross Income: Case 4


Case 4, where all ancillary elasticities are positive, gives rise to a slightly different pattern. In this case the income level, of about $23,375 €$, at which the individual begins to pay tax involves paying only the central government rates of 0.1566 and 0.111 for the first and second income source respectively. Very soon after this, at the level of $23,650 €$, the individual pays tax on both income sources at the combined central and regional rates of 0.24 and 0.18 for the two sources. A kink, or 'tooth' arises in the revenue elasticity curve at the income of $34,100 €$, when income source one attracts the higher combined marginal tax rate of 0.28 . Then at $51,700 €$, the individual moves into the next tax bracket for this source, with a marginal rate of 0.37 . Movement to the top marginal tax bracket is not shown in the diagram.

## 4 Empirical Estimates

This section presents estimated values of the individual revenue elasticity as defined in equation (23). Results were obtained using the Personal Income Tax information reported for a sample of 896,390 Spanish tax units. The original dataset comes from a cross-sectional dataset from the Spanish Tax Agency for year 2002. The data were adjusted to tax year 2007 and the simulated personal income tax is the one that came into force in January 2007.

The first step was to compute the ancillary elasticities relating to the variation in expenditures and allowances. An important priority was to allow for as much heterogeneity as possible. For each of the 15 Autonomous Communities, the sample was split into subsamples according to 5 quantiles of total gross income, and within each quantile by the size of the tax unit. In the latter case three categories were used consisting of: one member; two members; and three or more members. Therefore, the total sample was divided into 225 subsamples $(15 \times 5 \times 3)$, and for each of these 225 subsamples the ancillary elasticities were obtained by running the following Tobit regression (where the sampling weight of each tax unit was taken into account):

$$
\begin{equation*}
\log z_{h}=\alpha+\eta_{z, y} \log y_{h}+\gamma \log Q_{h}+\varepsilon \tag{25}
\end{equation*}
$$

Where $z$ is the relevant variable for which the constant elasticity, $\eta_{z, y}$, with respect to total gross income is required (that is, $y_{1}, y_{2}, E_{1}, E_{2}, A_{1}, C_{C}$ and $C_{R}$ ), and the matrix $Q$ represents a set of dummy variables capturing the type of tax-return (joint or separate
filing), marital status (four categories) and type of main source of income (three categories).

As a consequence of the procedure described above 1,575 estimations were run (seven ancillary elasticities for each of the 225 subgroups). The tables in Appendix D report the ancillary elasticities for each region according to the quantile and the size of the tax unit. These Tables report the required elasticities as long as they are statistically significant at a significance level of 5 per cent - otherwise a zero is reported.

For the case of $\eta_{A_{2} y_{h}}$ the procedure was slightly different, as follows. The values of $A_{2}$ are positive only if the magnitude of $A_{1}$ has not been entirely absorbed by the first income source $y_{1}$. In those cases, the excess of $A_{1}$ can be transferred as an allowance to reduce the second source of income $y_{2}$. Thus $A_{2}$ is positive only for tax units for whom $y_{1}$ is sufficiently small not to absorb all its entitled $A_{1}$. In other words, tax units which are rich in income from source 1 will not enjoy any transfer and as a result they will have $A_{2}=0$. This fact is confirmed by the basic data as can be seen in the tables reported in Appendix D , which report the magnitude of $A_{2}$ by quantiles of $y_{1}$.

As a result, the ancillary elasticity $\eta_{A_{2} y_{n}}$ was calculated following the same procedure as for the other ancillary elasticities, but using the quintiles of $y_{1}$ instead of $y_{h}$. Specifically, $A_{2}$ exists only for the first two quintiles of $y_{1}$ and mainly in the first one, so that the elasticity is reported in greater detailed for the first quintile (divided into three household sizes) whereas the rest of the tabulated quintiles are taken together without discriminating by household size. As can be seen, $\eta_{A_{2} y_{h}}$ is zero for the last 4 quintiles of $y_{1}$ and negative for the first quintile regardless of household size and region. There are three exceptions: Andalucia; Castilla y León; and Cataluña present a strong positive elasticity for the second quintile.

To compute the remaining terms in equation (23) for each tax unit, the 2007 tax structure was applied to each tax unit in the sample. For each tax unit the appropriate values of $y_{1}, y_{2}, y_{t}, A_{1}, A_{2}, C_{C}, C_{r}$ and the marginal tax rates levied on each income source, $t_{k i h}$, as well as the weighted total marginal tax rate, $\frac{y_{1}}{y_{h}} t_{k 1 h}+\frac{y_{2}}{y_{h}} t_{k 2 h}$, and the average tax rate (ATR). The last segment of Appendix C reports some basic statistics for relevant variables, both for the whole country and for each of the Autonomous Communities (regions).

All the ingredients of $\eta_{T_{h} y_{h}}$ were thus available for each tax unit, and Summary measures of the distribution of individual elasticities are reported in Table 3 for each region.

Table 3 Quartiles of Individual Revenue Elasticities by Region and for The Whole Country

|  | Lower quartile | Median | Upper <br> quartile |
| :--- | :--- | :--- | :--- |
| National | 1.1214 | 1.4082 | 1.7761 |
| Andalucia | 0.9673 | 1.3004 | 1.6172 |
| Aragon | 1.1128 | 1.3819 | 1.8037 |
| Asturias | 1.1414 | 1.4731 | 1.9418 |
| Baleares | 1.0127 | 1.3865 | 1.9085 |
| Canarias | 1.0207 | 1.3504 | 1.7578 |
| Cantabria | 1.2341 | 1.5071 | 1.8395 |
| Castilla-Leon | 1.0710 | 1.3829 | 1.7248 |
| Castilla- |  |  |  |
| LaMancha | 1.0553 | 1.3905 | 1.7490 |
| Cataluña | 1.1874 | 1.3660 | 1.6667 |
| Valencia | 1.0806 | 1.3706 | 1.8863 |
| Extremadura | 0.9384 | 1.3016 | 1.6013 |
| Galicia | 0.9480 | 1.3297 | 1.8183 |
| Madrid | 1.2896 | 1.5507 | 1.8279 |
| Murcia | 1.1164 | 1.4429 | 1.9578 |
| Rioja | 1.1214 | 1.4082 | 1.7761 |

The distribution of individual revenue elasticities is of course highly skewed because those individuals who are just above an income threshold have very high revenue elasticities, as discussed in the previous Section.

Further details regarding the distribution of individual elasticities can be illustrated using 'box plots', as in Figures 5 to 7, which provide a graphical representation of the main characteristics of a given distribution. A box plot is formed by a box, two 'whiskers' and two 'fences', as follows. The right border of the box is the upper quartile; the left border is the lower quartile; and the line inside the box is the median. Hence the width of the box shows the inter-quartile range. The whiskers are the two horizontal lines to the left and right of the box which end in two vertical lines known as the fences. The right fence shows the highest value of the distribution that is smaller than or equal to the third quartile plus 1.5 times the inter-quartile range. The left fence shows the lowest value of the distribution that is greater than or equal to the first quartile minus 1.5 times the inter-quartile range. The box therefore indicates the dispersion and the skewness of the distribution.

Figure 5 Distribution of Individual Elasticities by Income Quintile and Size of Tax Unit: All Regions


BY SIZE OF TAX UNIT
all taxpayers


BY INCOME QUINTILES
total gross income taxpayers with positive tax liability


BY SIZE OF TAX UNIT
taxpayers with positive tax liability


Figure 6 Distribution of Individual Elasticities by Main Income and Marital

## Status: All Regions



Figure 7 Distribution of Individual Elasticities by Autonomous Community in Common Territory


The boxes on the left hand side of each figure refer to all tax units, and thus include all those with a zero tax liability. When the elasticities are classified by income quintiles, it can be seen that there are nurerous negative elasticities, many of which are large in absolute terms. These negative elasticities are associated mainly with tax units who pay small amounts of personal income tax but have low incomes and ancillary elasticities which are greater than unity; thus (some of) the eligible expenditures, allowances and tax credits increase by more than gross income. The dispersion is substantially affected by whether all tax units are included, or whether attention is restricted to those who pay positive amounts of personal income tax.

There is little variation in the dispersion of individual revenue elasticities, classified by tax unit size. Those whose main source of income is entrepreneurial income have a lower dispersion when only taxpayers are included, compared with the population of all tax units. This result is affected by the great ability of such tax units to claim substantial amounts of eligible expenditure and allowances.

### 4.1 The Aggregate Revenue Elasticity

Consider next the aggregate tax revenue elasticity, over $H$ tax units. Define $Y=\sum_{h=1}^{H} y_{h}$ and $T=\sum_{h=1}^{H} T_{h}$ as aggregate income and tax revenue respectively. Then:

$$
\begin{equation*}
\frac{d T}{d Y} \frac{Y}{T}=\sum_{h=1}^{H}\left(\frac{\partial T_{h}}{\partial y_{h}} \frac{y_{h}}{T_{h}}\right)\left(\frac{\partial y_{h}}{\partial Y} \frac{Y}{y_{h}}\right)\left(\frac{T_{h}}{T}\right) \tag{26}
\end{equation*}
$$

and:

$$
\begin{equation*}
\eta_{T, Y}=\sum_{h=1}^{H} \eta_{T_{h}, y_{h}} \eta_{y_{h}, Y}\left(\frac{T_{h}}{T}\right) \tag{27}
\end{equation*}
$$

The elasticity of aggregate revenue with respect to aggregate income is thus a taxshare weighted average of the product of individual revenue elasticities and the elasticity of individual income with respect to total income. Hence it depends not only the tax structure but on the extent to which individual incomes change when aggregate income changes. And, as shown above, the individual revenue elasticities depend on the extent to which the components of individuals' incomes change as each individual's income changes.

In order to show the relevance of taking into account the schedular design of the tax as well as the rules that affect the definition of the taxable income and the final tax due, it is of interest to consider alternative measures of the aggregate revenue elasticity. Allowing for progressively more flexibility or endogeneity of deductions and allowances, the following terms are used for the individual elasticities:

$$
\begin{gather*}
\eta_{l h}=\sum_{i=1}^{I} \frac{t_{k h}}{A T R_{h}}\left(\frac{y_{h i}}{y_{h}}\right)  \tag{28}\\
\eta_{2 h}=\sum_{i=1}^{I} \frac{t_{k i h}}{A T R_{h}}\left(\frac{y_{h i}}{y_{h}}\right) \eta_{y_{h i} y_{h}}  \tag{29}\\
\eta_{3 h}=\sum_{i=1}^{I} \frac{t_{k i h}}{A T R_{h}}\left(\frac{y_{h i}}{y_{h}}\right) \eta_{y_{h h i} y_{h}}-\sum_{i=1}^{I}\left(\frac{t_{k i h}}{T_{h}}\right)\left[\left(\eta_{E_{h i y_{h}}} E_{h i}+\eta_{A_{h i j} y_{h}} A_{h i}\right)\right] \tag{30}
\end{gather*}
$$

Along with (23), which gives, say, $\eta_{4 n}$.

Table $4 \quad$ Aggregate Revenue Elasticities: $\eta_{y_{h}, Y}=1$

|  | $\eta_{1}$ | $\eta_{2}$ | $\eta_{3}$ | $\eta_{4}$ |
| :--- | :--- | :--- | :--- | :--- |
| National | 2.0732 | 2.1010 | 1.6238 | 1.3516 |
| Andalucía | 2.2403 | 2.2444 | 1.6813 | 1.2231 |
| Aragón | 2.1577 | 2.0983 | 1.5823 | 1.3266 |
| Asturias | 2.1926 | 2.2358 | 1.7334 | 1.4369 |
| Baleares | 2.0425 | 2.0047 | 1.5878 | 1.3282 |
| Canarias | 2.1566 | 2.1250 | 1.6885 | 1.3235 |
| Cantabria | 2.1645 | 2.1011 | 1.5900 | 1.3164 |
| Castilla-León | 2.2493 | 2.2275 | 1.5390 | 1.2051 |
| Castilla-La Mancha | 2.3570 | 2.3310 | 1.7721 | 1.2842 |
| Cataluña | 1.9944 | 1.9501 | 1.5615 | 1.3668 |
| Valencia | 2.1714 | 2.1569 | 1.6281 | 1.3026 |
| Extremadura | 2.3600 | 2.2622 | 1.6661 | 1.0762 |
| Galicia | 2.1924 | 2.1721 | 1.6028 | 1.1985 |
| Madrid | 1.8614 | 2.0515 | 1.6300 | 1.5057 |
| Murcia | 2.2954 | 2.2725 | 1.8047 | 1.3189 |
| Rioja | 2.1905 | 2.1212 | 1.6275 | 1.3665 |

In obtaining results reported here, the assumption was made that $\eta_{y_{h}, Y}$ is unity; that is, all incomes move in the same proportion. The resulting aggregate elasticities are shown in Table 4. The elasticity $\eta_{1}$ assumes not only that all deductions and credits are fixed irrespective of income, but that the two sources of income remain in fixed proportions for all individuals. The second elasticity, $\eta_{2}$, uses information about the (cross-sectional) variation in income proportions to attribute an elasticity $\eta_{y_{n}, y_{h}}$ to each tax unit's income source. This has a relatively small effect on the revenue elasticity estimates. Larger effects are observed where eligible expenditures and deductions, and then tax credits, vary with tax unit income: in each case the aggregate revenue elasticity falls when the ancillary elasticities are used.

The revenue elasticities in the final column of Table 4 vary from just over 1.0 to about 1.5. The variation across regions arises from regional differences in gross incomes, since all regions face similar tax structures.

In general, the aggregate values are similar to those reported for a number of other countries. On the US, see Fries et al. (1982), Dye and McGuire (1991) and Ram(1991). UK results are reported in Johnson and Lambert (1989) and Creedy and Gemmell (2004a, 2006, pp. 113). Canadian estimates are given by King and McMorran (2002) ${ }^{10}$, and for New Zealand see Creedy and Gemmell (2004b, 2006, p.171). Lower elasticities of around 1, using time series methods, are given for Turkey by Kuştepeli and Şapçi (2006).

In considering the revenue elasticities reported above, it should be remembered that they relate to revenue changes associated with changes in gross incomes. Many empirical studies actually begin not from gross income but from taxable income; that is, measured income has already been adjusted for eligible expenditures and allowances, so that the tax function can be applied directly as a function of taxable income.

[^6]In the case of a single income source, where $x$ and $y$ are, as above, taxable and gross income, and tax paid is $T(x(y))$, then the revenue elasticity is $\eta_{T, y}=\left(\eta_{T, x}\right)\left(\eta_{x, y}\right)$. Furthermore, writing $x=y-D$, where $D$ refers to all allowances and deductions, it can be shown that:

$$
\begin{equation*}
\eta_{x, y}=\left(1-\frac{D}{y}\right)^{-1}\left(1-\frac{D}{y} \eta_{D, y}\right) \tag{31}
\end{equation*}
$$

Where $\eta_{D, y}$ is the elasticity of deductions with respect to gross income. It is clear from (33) that if $\eta_{D, y}<1$, then $\eta_{x, y}>1$ and the revenue elasticity with respect to gross income exceeds the revenue elasticity with respect to taxable income.

In the following Section, which turns to the modelling of aggregate tax revenue, it is shown that a large simplification is possible when taking taxable income as the basis.

### 4.2 Income Dynamics

The above estimates, in common with most studies, are obtained on the assumption that all incomes move together, so that $\eta_{y_{h}, Y}$ is equal to unity. In the absence of direct information on the dynamic process of relative income changes from year to year, it is possible to consider the sensitivity of results to an assumed degree of regression towards, or away from, the geometric mean. Following Creedy and Gemmell (2006), suppose income dynamics can be described by the relationship:

$$
\begin{equation*}
\eta_{y_{y_{h}, Y}}=1-(1-\beta)\left(\log y_{h}-E(\log y)\right) \tag{32}
\end{equation*}
$$

where $E(\log y)$ is the mean $\log$-income, or equivalently the logarithm of geometric mean income. The coefficient, $\beta$, therefore governs systematic movements within the income distribution. If $\beta<1$ there are systematic equalising relative movements whereby those below the geometric mean income experience relative larger increases than those above the geometric mean, when total income increases. A value of $\beta>1$ implies systematic disequalising income movements.

The effects on aggregate revenue elasticities of differential income changes are shown in Tables 5 and 6, which may be compared with Table 4.

Table $5 \quad$ Aggregate Elasticities: $\beta=0.9$

|  | $\eta_{1}$ | $\eta_{2}$ | $\eta_{3}$ | $\eta_{4}$ |
| :--- | :--- | :--- | :--- | :--- |
| National | 1.9480 | 1.9712 | 1.5054 | 1.2198 |
| Andalucia | 2.1274 | 2.1296 | 1.5763 | 1.0999 |
| Aragon | 2.0504 | 1.9930 | 1.4908 | 1.2216 |
| Asturias | 2.0856 | 2.1240 | 1.6314 | 1.3162 |
| Baleares | 1.9115 | 1.8804 | 1.4735 | 1.1989 |
| Canarias | 2.0382 | 2.0046 | 1.5786 | 1.2075 |
| Cantabria | 2.0536 | 1.9801 | 1.4814 | 1.1933 |
| Castilla-Leon | 2.1476 | 2.1293 | 1.4575 | 1.1062 |
| Castilla-LaMancha 2.2517 | 2.2303 | 1.6850 | 1.1769 |  |
| Cataluña | 1.8706 | 1.8327 | 1.4545 | 1.2495 |
| Valencia | 2.0523 | 2.0371 | 1.5170 | 1.1748 |
| Extremadura | 2.2580 | 2.1673 | 1.5848 | 0.9714 |
| Galicia | 2.0775 | 2.0660 | 1.5045 | 1.0815 |
| Madrid | 1.7203 | 1.8873 | 1.4778 | 1.3467 |
| Murcia | 2.1804 | 2.1536 | 1.6966 | 1.1882 |
| Rioja | 2.0842 | 2.0290 | 1.5471 | 1.2705 |

Table $6 \quad$ Aggregate Elasticities: $\beta=1.1$

|  | $\eta_{1}$ | $\eta_{2}$ | $\eta_{3}$ | $\eta_{4}$ |
| :--- | :--- | :--- | :--- | :--- |
| Nacional | 2.1984 | 2.2309 | 1.7422 | 1.4834 |
| Andalucia | 2.3532 | 2.3593 | 1.7862 | 1.3462 |
| Aragon | 2.2650 | 2.2037 | 1.6737 | 1.4317 |
| Asturias | 2.2995 | 2.3477 | 1.8353 | 1.5577 |
| Baleares | 2.1735 | 2.1290 | 1.7022 | 1.4575 |
| Canarias | 2.2750 | 2.2455 | 1.7985 | 1.4396 |
| Cantabria | 2.2755 | 2.2222 | 1.6986 | 1.4395 |
| Castilla-Leon | 2.3510 | 2.3257 | 1.6205 | 1.3040 |
| Castilla-LaMancha | 2.4623 | 2.4318 | 1.8592 | 1.3914 |
| Cataluña | 2.1181 | 2.0675 | 1.6686 | 1.4841 |
| Valencia | 2.2905 | 2.2766 | 1.7392 | 1.4305 |
| Extremadura | 2.4620 | 2.3570 | 1.7474 | 1.1810 |
| Galicia | 2.3074 | 2.2783 | 1.7011 | 1.3154 |
| Madrid | 2.0024 | 2.2157 | 1.7822 | 1.6647 |
| Murcia | 2.4103 | 2.3914 | 1.9128 | 1.4496 |
| Rioja | 2.2968 | 2.2134 | 1.7080 | 1.4625 |

Further detail of the effect of income dynamics can be seen in Figure 4, for the case of the national aggregate revenue elasticity. It can be seen that the elasticity varies linearly with $\beta$.

Figure $8 \quad$ Variation in Aggregate National Elasticity as $\beta$ Varies


The increase in the revenue elasticity as $\beta$ increases is associated with the resulting rise in income inequality as those below the geometric mean experience relatively smaller percentage income increases. The larger proportion of the population just above the lower income thresholds implies an increase in the number of tax units having larger revenue elasticities. The decline in the elasticities associated with the higher income groups is relatively small, as can be seen from the shapes of the elasticity profiles shown above. Hence, in aggregate the revenue elasticity increases with $\beta$.

The linearity of the schedule in Figure 8 can be seen by substituting (31) into (27), whereby:

$$
\begin{equation*}
\eta_{T, Y}=\sum_{h=1}^{H} \eta_{T_{h}, y_{h}}\left(\frac{T_{h}}{T}\right)-(1-\beta) \sum_{h=1}^{H} \eta_{T_{h}, y_{h}}\left(\frac{T_{h}}{T}\right)\left(\log y_{h}-E(\log y)\right) \tag{33}
\end{equation*}
$$

Although no direct evidence is available here, it is unlikely that $\beta$ deviates far from unity. For example, a value of $\beta=0.9$ would be considered low, implying for
example that if total income were to increase by 10 per cent, the lower quartile would increase by about 14 per cent whereas the upper quartile would increase by only about 3 per cent. This implies considerable 'regression towards the (geometric) mean'. ${ }^{11}$

## 5 Total Tax Revenue

This section turns to modelling aggregate tax revenue and its components. First, it is shown that in view of the complexity of the Spanish tax structure, it is not possible to express total revenue as a convenient function of proportions of people and proportions of total income within the tax brackets, or adjacent gross income thresholds. However, further progress can be made by taking taxable income as the basic distribution.

### 5.1 Aggregation over Individuals

Total revenue, $T$, is made up of revenue from the three categories discussed in subsection 2.2. Suppose there are $n_{C+R}$ units for whom $\sum_{i=1}^{I} T_{i}^{C}\left(y_{h i}\right)>C_{C}$ and $\sum_{i=1}^{I} T_{i}^{R}\left(y_{h i}\right)>C_{R}$, with $n_{C}$ units for whom $\sum_{i=1}^{I} T_{i}^{C}\left(y_{h i}\right)>C_{C}$ but $\sum_{i=1}^{I} T_{i}^{R}\left(y_{h i}\right)<C_{R}$, and a further $n_{R}$ for whom $\sum_{i=1}^{I} T_{i}^{C}\left(y_{h i}\right)<C_{C}$ but $\sum_{i=1}^{I} T_{i}^{R}\left(y_{h i}\right)>C_{R}$. Total tax revenue is thus expressed as:

$$
\begin{align*}
T= & \sum_{h \in n_{C+R}}\left[\sum_{i=1}^{I}\left\{T_{i}^{C}\left(y_{h i}\right)+T_{i}^{R}\left(y_{h i}\right)\right\}\right] \\
& +\sum_{h \in n_{C}}\left[\sum_{i=1}^{I} T_{i}^{C}\left(y_{h i}\right)\right]+\sum_{h \in n_{R}}\left[\sum_{i=1}^{I} T_{i}^{R}\left(y_{h i}\right)\right]  \tag{34}\\
& -\left(n_{C+R}+n_{C}\right) C_{C}-\left(n_{C+R}+n_{R}\right) C_{R}
\end{align*}
$$

Given the existence of the different income sources and the application of the nonrefundable tax credits to total income tax liability at central and regional levels (rather than applying to each source), and the existence of different tax structures for

[^7]different income sources, the above expression for total revenue cannot be reduced to a convenient expression in terms of characteristics of the distributions of component gross income sources.

However, further progress can be made by considering as the starting point, instead of the distribution of gross income, the distribution of taxable income, $x$. Indeed, as discussed in subsection 4.1, many studies of revenue elasticities take this variable as the 'given' distribution and define the elasticity of tax revenue with respect to changes in taxable income rather than gross income. For the $n_{C+R}$ taxpayers whose central and regional tax exceeds the relevant credits, equations (10) and (11) can be used to write their tax as:

$$
\begin{equation*}
T\left(\sum_{i=1} y_{h i}\right)=\sum_{i=1}^{I}\left\{t_{k i h}\left(y_{h i}-E_{h i}-A_{h i}\right)-\sum_{j=1}^{k_{i n}} a_{j i}\left(t_{j i}-t_{j-1, i}\right)\right\}-\left(C_{R}+C_{C}\right) \tag{35}
\end{equation*}
$$

which becomes:

$$
\begin{equation*}
T(Y)=\sum_{i=1}^{I}\left\{t_{k i h}\left(x_{h i}-a_{j i h}^{\prime}\right)\right\}-\left(C_{R}+C_{C}\right) \tag{36}
\end{equation*}
$$

where $a_{j i h}^{\prime}=\frac{1}{t_{k i h}} \sum_{j=1}^{k_{k i n}} a_{j i}\left(t_{j i}-t_{j-1, i}\right)$. If there are $N$ taxpayers (that is, whose tax liability is positive, so that $N=n_{C+R}+n_{R}+n_{C}$ ), the total revenue can be expressed as:

$$
\begin{equation*}
T(Y)=\sum_{h=1}^{N} \sum_{i=1}^{I}\left\{t_{k i h}\left(x_{h i}-a_{j i h}^{\prime}\right)\right\}-N\left(\bar{C}_{R}+\bar{C}_{C}\right) \tag{37}
\end{equation*}
$$

Where, as above, $x_{h i}$ is the taxable income for income source $i$ for tax unit $h$ (that is, $x_{h i}=y_{h i}-E_{h i}-A_{h i}$ ). The terms $\bar{C}_{C}$ and $\bar{C}_{R}$ denote the appropriate average value defined over taxpayers, remembering the tax schedule asymmetry whereby tax must be positive. The first term in (37) can be rewritten as:

$$
\begin{equation*}
\sum_{i=1}^{I} \sum_{h=1}^{N}\left\{t_{k i h}\left(x_{h i}-a_{j i h}^{\prime}\right)\right\} \tag{38}
\end{equation*}
$$

In the case of a single source of income, with a multi-step function, the tax per person can be expressed in terms of summary information about the distribution of taxable income, which determines the proportion of tax units falling into the various marginal
tax rate groups. ${ }^{12}$ For example, suppose that $F(x)$ denotes the distribution function of taxable income, $x>0$. Tax per unit is thus:

$$
\begin{equation*}
\sum_{k=1}^{K}\left[t_{k} \int_{a_{k}}^{a_{k+1}}\left(x-a_{k}^{\prime}\right) d F(x)\right] \tag{39}
\end{equation*}
$$

Define $F_{1}(x)$ as the first-moment distribution function, that is the proportion of total income of units below $x$, and introduce the general term $G\left(a_{k}\right)$, defined as:

$$
\begin{equation*}
G\left(a_{k}\right)=\left\{F_{1}\left(a_{k+1}\right)-F_{1}\left(a_{k}\right)\right\}-\frac{a_{k}^{\prime}}{\bar{x}}\left\{F\left(a_{k+1}\right)-F\left(a_{k}\right)\right\} \tag{40}
\end{equation*}
$$

The first term in curly brackets gives the proportion of total income between adjacent thresholds, and the second term in curly brackets is the number of tax units between those thresholds. The expression in (40) can also be written as:

$$
\begin{equation*}
G\left(a_{k}\right)=\left\{F\left(a_{k+1}\right)-F\left(a_{k}\right)\right\}\left[\frac{F_{1}\left(a_{k+1}\right)-F_{1}\left(a_{k}\right)}{F\left(a_{k+1}\right)-F\left(a_{k}\right)}-\frac{a_{k}^{\prime}}{\bar{x}}\right] \tag{41}
\end{equation*}
$$

The first term inside the square brackets of (41) is the slope of the Lorenz curve of the relevant distribution of income, between the two points associated with adjacent income tax thresholds. The Lorenz curve has a slope of 45 degrees at the arithmetic mean; that is, $\frac{d F_{1}(\bar{x})}{d F(\bar{x})}=1$. The second term in the square brackets is simply the ratio of the 'effective' threshold to arithmetic mean income. And of course the term in curly brackets in (41) is the proportion of people within the tax bracket.

Total revenue per person is thus:

$$
\begin{equation*}
\bar{x} \sum_{k=1}^{K} t_{k} G\left(a_{k}\right) \tag{42}
\end{equation*}
$$

Hence, for the case of several income sources, each with its own tax schedule, total tax revenue over all individuals and sources becomes:

$$
\begin{equation*}
T(Y)=N \sum_{i=1}^{I} \bar{X}_{i}\left[\sum_{k_{i}=1}^{K_{i}}\left\{t_{k_{i}} G_{i}\left(a_{k_{i}}\right)\right\}\right]-N\left[\left(\bar{C}_{R}+\bar{C}_{C}\right)\right] \tag{43}
\end{equation*}
$$

[^8]The first term in equation (43) can usefully be written in vector notation. Define the column vectors:

$$
G_{i}=\left[\begin{array}{c}
G_{i}\left(a_{1 i}\right)  \tag{44}\\
G_{i}\left(a_{2 i}\right) \\
\cdot \\
G_{i}\left(a_{K i}\right)
\end{array}\right]
$$

and:

$$
t_{i}=\left[\begin{array}{c}
t_{1 i}  \tag{45}\\
t_{2 i} \\
\cdot \\
t_{K i}
\end{array}\right]
$$

Then, if a prime indicates that the vector is written as a row vector:

$$
\begin{equation*}
\sum_{k_{i}=1}^{K_{i}}\left\{t_{k_{i}} G_{k_{i}}\left(a_{k_{i}}\right)\right\}=t_{i}^{\prime} G_{i} \tag{46}
\end{equation*}
$$

These values may be placed in a column vector, denoted $\left[t^{\prime} G\right]$. Then if $\bar{x}$ represents a column vector whose $i$ th element consists of the arithmetic mean, $\bar{x}_{i}$, then:

$$
\begin{equation*}
N \sum_{i=1}^{I} \bar{x}_{i}\left[\sum_{k_{i}=1}^{K_{i}}\left\{t_{k_{i}} G_{k_{i}}\left(a_{k_{i}}\right)\right\}\right]=N \bar{x}^{\prime}\left[t^{\prime} G\right] \tag{47}
\end{equation*}
$$

where, as before, a prime indicates transposition. This allows the effects of tax and income distribution changes to be easily examined.

### 5.2 Empirical Application

This subsection reports the values of the various terms involved in obtaining total tax revenue, derived in the previous subsection. First, Table 7 gives, for each region and for all regions combined, the number of individuals who pay positive amounts of tax, along with the arithmetic means of the two income sources. The final two columns of Table 7 show the arithmetic means of the central and regional tax credits, which together give the last term in equation (43). There are clearly substantial differences in arithmetic mean incomes among regions.

Table 7 Number of Taxpayers (with positive tax) and Arithmetic Means of Taxable Incomes and Tax Credits (€s)

|  | $N$ | $\bar{x}_{1}$ | $\bar{x}_{2}$ | $\bar{C}_{C}$ | $\bar{C}_{R}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| National | $12,229,939$ | $20,816.74$ | $2,220.59$ | $1,228.84$ | 646.27 |
| Andalucía | $1,772,425$ | $19,148.92$ | $1,686.22$ | $1,254.13$ | 660.46 |
| Aragón | 480,016 | $19,239.42$ | $2,338.24$ | $1,190.27$ | 626.56 |
| Asturias | 362,701 | $19,894.59$ | $1,648.66$ | $1,203.68$ | 634.44 |
| Baleares | 311,943 | $20,006.38$ | $2,313.47$ | $1,176.61$ | 619.04 |
| Canarias | 472,742 | $20,330.13$ | $1,439.27$ | $1,259.68$ | 661.04 |
| Cantabria | 181,796 | $19,546.91$ | $2,287.14$ | $1,220.81$ | 642.94 |
| Castilla-León | 790,965 | $18,619.11$ | $1,889.49$ | $1,208.71$ | 636.20 |
| Castilla-La Mancha | 469,160 | $17,565.85$ | $1,563.90$ | $1,216.59$ | 640.83 |
| Cataluña | $2,473,158$ | $22,266.55$ | $2,506.11$ | $1,225.06$ | 643.65 |
| Valencia | $1,391,005$ | $18,720.08$ | $2,278.14$ | $1,197.66$ | 630.46 |
| Extremadura | 244,384 | $17,149.91$ | $1,378.86$ | $1,190.61$ | 628.12 |
| Galicia | 730,355 | $18,520.83$ | $1,763.29$ | $1,161.04$ | 612.92 |
| Madrid | $2,131,743$ | $25,885.79$ | $3,096.36$ | $1,279.47$ | 670.80 |
| Murcia | 309,242 | $18,780.95$ | $1,729.87$ | $1,271.32$ | 669.52 |
| Rioja | 108,306 | $18,529.28$ | $2,540.61$ | $1,212.06$ | 637.13 |

The expression for aggregate revenue in each region requires the various proportions of people and proportions of income at each of the tax thresholds. This is simple for the second source of income, since the tax function is linear. For the first income source, Tables 8 and 9 report values of the first moment distribution, and the distribution function, respectively for the required income thresholds. These two tables thus together give three points along the Lorenz curve of the first source of income in each region. For example, for all regions combined there are approximately five per cent of tax units (those paying positive tax) above the top threshold for income source 1, and they are responsible for about 20 per cent of total income from that source. However, for Madrid, eight per cent of positive taxpayers are above the top threshold, and they are responsible for about 30 per cent of total income from source 1 .

Table 8 Proportions of Total Taxable Income Below Thresholds (First Income Source)

|  | $F_{1}\left(a_{2}\right)$ | $F_{1}\left(a_{3}\right)$ | $F_{1}\left(a_{4}\right)$ |
| :--- | :--- | :--- | :--- |
| National | 0.29019 | 0.63216 | 0.80771 |
| Andalucía | 0.33006 | 0.69497 | 0.85926 |
| Aragón | 0.32503 | 0.69809 | 0.86190 |
| Asturias | 0.31424 | 0.70250 | 0.86856 |
| Baleares | 0.32095 | 0.62923 | 0.80426 |
| Canarias | 0.29832 | 0.63865 | 0.82860 |
| Cantabria | 0.32786 | 0.68411 | 0.85049 |
| Castilla-León | 0.34005 | 0.71888 | 0.88787 |
| Castilla-La Mancha 0.38301 | 0.73908 | 0.89074 |  |
| Cataluña | 0.25939 | 0.59872 | 0.77946 |
| Valencia | 0.34565 | 0.68570 | 0.84315 |
| Extremadura | 0.39003 | 0.74727 | 0.89608 |
| Galicia | 0.34662 | 0.69615 | 0.86208 |
| Madrid | 0.19713 | 0.50417 | 0.70211 |
| Murcia | 0.34329 | 0.69763 | 0.86529 |
| Rioja | 0.36322 | 0.70585 | 0.87476 |

All that is required to obtain the values of $G$ are the values of the effective tax thresholds, $a^{\prime}{ }_{k}$. For central and regional rates combined, the relevant values are 2,$480 ; 9,748.11$ and $15,693.95$. The resulting values of $G$ are given in Table 10. From the analytical results derived above, for any tax bracket, multiplying $G$ by the relevant tax rate gives the ratio of tax raised by the bracket per capita to the total income per capita. As the tax rates are common across regions, comparisons of the extent of revenue within each region, arising from income source 1 , can be made by moving down the columns. The table shows the relative importance of the top tax bracket in Madrid, and the unimportance of the bottom tax bracket, compared with other regions. ${ }^{13}$ Finally total tax revenue is reported in Table 11.

[^9]Table $9 \quad$ Proportions of Taxpayers below Thresholds (First Income Source)

|  | $F\left(a_{2}\right)$ | $F\left(a_{3}\right)$ | $F\left(a_{4}\right)$ |
| :--- | :--- | :--- | :--- |
| National | 0.56074 | 0.86344 | 0.95491 |
| Andalucía | 0.59035 | 0.88808 | 0.96763 |
| Aragón | 0.58030 | 0.88792 | 0.96704 |
| Asturias | 0.55800 | 0.88570 | 0.96864 |
| Baleares | 0.60398 | 0.86906 | 0.95701 |
| Canarias | 0.57119 | 0.86188 | 0.95896 |
| Cantabria | 0.58835 | 0.88265 | 0.96492 |
| Castilla-León | 0.59488 | 0.89430 | 0.97353 |
| Castilla-La Mancha | 0.64104 | 0.90839 | 0.97528 |
| Cataluña | 0.52079 | 0.84388 | 0.94432 |
| Valencia | 0.61925 | 0.89186 | 0.96510 |
| Extremadura | 0.65168 | 0.91198 | 0.97600 |
| Galicia | 0.61687 | 0.89160 | 0.96861 |
| Madrid | 0.45962 | 0.79380 | 0.92136 |
| Murcia | 0.60652 | 0.89010 | 0.96956 |
| Rioja | 0.62178 | 0.89268 | 0.97164 |

Table 10 Values of G for each Tax Threshold (First Income Source According to Total Tax Schedule)

|  | $G\left(a_{1}\right)$ | $G\left(a_{2}\right)$ | $G\left(a_{3}\right)$ | $G\left(a_{4}\right)$ |
| :--- | :--- | :--- | :--- | :--- |
| National | 0.290193 | 0.305909 | 0.132714 | 0.158296 |
| Andalucía | 0.330057 | 0.326354 | 0.123800 | 0.114204 |
| Aragón | 0.325034 | 0.333407 | 0.123714 | 0.111220 |
| Asturias | 0.314239 | 0.347408 | 0.125427 | 0.106700 |
| Baleares | 0.320947 | 0.275424 | 0.132178 | 0.162015 |
| Canarias | 0.298323 | 0.304869 | 0.143401 | 0.139719 |
| Cantabria | 0.327856 | 0.318916 | 0.125351 | 0.121344 |
| Castilla-León | 0.340052 | 0.338948 | 0.127502 | 0.089822 |
| Castilla-La Mancha | 0.383015 | 0.318322 | 0.114534 | 0.087180 |
| Cataluña | 0.259386 | 0.303344 | 0.136769 | 0.181298 |
| Valencia | 0.345647 | 0.303937 | 0.119315 | 0.127588 |
| Extremadura | 0.390029 | 0.319601 | 0.112424 | 0.081951 |
| Galicia | 0.346616 | 0.312747 | 0.125395 | 0.111321 |
| Madrid | 0.197128 | 0.273870 | 0.148977 | 0.249379 |
| Murcia | 0.343290 | 0.316890 | 0.126414 | 0.109277 |
| Rioja | 0.363215 | 0.306374 | 0.127377 | 0.101215 |

Table 11 Estimated Total Tax Revenue (€s)

|  | Total Revenue | Central <br> Government. | Regional <br> Government. |
| :--- | :--- | :--- | :--- |
| National | $51,148,217,111$ | $33,061,697,229$ | $18,086,519,880$ |
| Andalucía | $6,155,788,630$ | $3,972,274,107$ | $2,183,514,522$ |
| Aragón | $1,776,894,526$ | $1,145,725,113$ | $631,169,413$ |
| Asturias | $1,352,992,058$ | $874,123,198$ | $478,868,860$ |
| Baleares | $1,271,754,868$ | $818,791,512$ | $452,963,356$ |
| Canarias | $1,810,355,523$ | $1,167,961,396$ | $642,394,127$ |
| Cantabria | $683,181,263$ | $440,400,756$ | $242,780,507$ |
| Castilla-León | $2,672,907,005$ | $1,725,380,941$ | $947,526,063$ |
| Castilla-La Mancha | $1,410,927,079$ | $910,996,715$ | $499,930,364$ |
| Cataluña | $11,679,310,225$ | $7,520,720,015$ | $4,158,590,210$ |
| Valencia | $4,981,828,923$ | $3,209,163,568$ | $1,772,665,355$ |
| Extremadura | $705,599,802$ | $456,083,010$ | $249,516,793$ |
| Galicia | $2,521,071,133$ | $1,627,923,016$ | $893,148,116$ |
| Madrid | $12,712,845,321$ | $8,281,203,401$ | $4,431,641,920$ |
| Murcia | $1,034,493,203$ | $667,339,810$ | $367,153,393$ |
| Rioja | $378,267,552$ | $243,610,671$ | $134,656,881$ |

For example, for all regions combined, the total income tax revenue, gross of the tax credits, is given from (47) by the total number of positive taxpayers multiplied by the term (where values for $G$ are rounded to two decimal places for convenience):

$$
\left.\left[\begin{array}{ll}
20,816 & 2,220
\end{array}\right]\left[\begin{array}{llll}
0.24 & 0.28 & 0.37 & 0.433
\end{array}\right]\left[\begin{array}{l}
0.29  \tag{48}\\
0.31 \\
0.13 \\
0.16
\end{array}\right]\right]
$$

When the total amount of tax credits per (positive) taxpayer, of $1,228+646$, is deducted from this result, the net tax per capita is obtained. Multiplying this value by the total number of taxpayers gives the value in the first row of Table 11. ${ }^{14}$ Results are obtained for the regions simply by changing the vector of arithmetic mean

[^10]incomes and the vector of $G$ values in (48), and then using the appropriate values of $N$ and the average tax credits.

The effects on gross tax revenue of changes in the average income from the second source, or changes in the relative dispersion of income from the first source (which changes the Lorenz curve and thus the $G$ values), or changes in the marginal tax rate structure, are thus easily examined using modifications to expressions of the form shown in (48). For example, elimination of the top marginal income tax bracket simply means that the row vector of tax rates has only three elements and the column vector of $G$ values is reduced to three elements with the third element replaced by 0.29 .

Changes in the tax thresholds have the effect of changing the $G$ values. Hence a 'ready reckoner' could be produced by replacing Tables 8 and 9 by larger tables giving values of the distribution function and first moment distribution function for a range of income levels. The introduction of additional tax brackets for the second income source could be accommodated by producing similar tables for that source.

The effects of change in the distribution of income within a region can be examined using the same kind of summary information. For example, if mean income increases, whereby incomes in a region are assumed to increase by the same proportion, this is equivalent to a reduction in the tax thresholds, so that information about the Lorenz curve (the $F$ and $F_{1}$ values) can be used to obtain the appropriate $G$ value. A change in inequality can be accommodated by specifying the way in which the Lorenz curve for the region changes.

The difficulty of dealing with the central and regional tax credits and thus aggregate net income tax revenue remains, as an analytical expression for aggregate credits has not been obtained.

As suggested above, the effects of changing only the tax rates are easily examined in this framework, as only the vector of marginal rates needs to be altered in expressions corresponding to (48). For example, the previous discussion has not allowed for the
small change in the tax rate structure in Madrid in 2007, making it unique among the Spanish regions. The income thresholds for the first income source are the same as in Table 1 above, and the central government rates are the same, but the marginal tax rates for Madrid became $0.0794,0.0943,0.1266$ and 0.1577 for the four income brackets. This involves a slight reduction in all the rates, with the largest reductions being for the first and second tax brackets. Given the nature of the distribution of income in Madrid for the first source, it is anticipated that this would have relatively little effect on total revenue. But in view of the differences among regions in their income distributions, the same could not be said of the other regions.

The effects on total tax revenue, and revenue within each region, if all regions were to adopt the Madrid structure, can easily be obtained using the information given above. The percentage changes in total (central plus regional) tax revenue and in the regional tax revenue alone are shown in Table 12. In producing these values, it was assumed that the average tax credits within each region remain unchanged. Clearly the poorer regions, where a much larger proportion of total income is obtained by those who fall into the first two tax brackets, would experience substantially larger reductions in tax revenue.

Table 12 Percentage Reduction in Tax Revenue from Adoption of Madrid's 2007 Tax Rate Structure

|  | Total <br> Revenue | Regional <br> Revenue |
| :--- | :---: | :--- |
| National | 1.7268 | 5.0096 |
| Andalucía | 1.9868 | 5.8113 |
| Aragon | 1.8754 | 5.4658 |
| Asturias | 1.9240 | 5.6338 |
| Baleares | 1.7081 | 4.9486 |
| Canarias | 1.8697 | 5.4546 |
| Cantabria | 1.8631 | 5.4260 |
| Castilla-Leon | 2.0137 | 5.8968 |
| Castilla-LaMancha | 2.1540 | 6.3276 |
| Cataluña | 1.6141 | 4.6696 |
| Valencia | 1.8701 | 5.4398 |
| Extremadura | 2.2008 | 6.4844 |
| Galicia | 1.9373 | 5.6687 |
| Madrid | 1.3815 | 3.9630 |
| Murcia | 2.0302 | 5.9393 |
| Rioja | 1.9282 | 5.6124 |
|  | 36 |  |

### 5.3 Regional Comparisons

The previous subsection considered the effects on all regions of adopting a different regional structure of marginal income tax rates. As explained above, it is possible to use the same basic approach to consider the effects of a range of changes in the taxable income distributions of each region. This is particularly useful in the present context where it is clear that different regions have different fiscal capacities. Such disparities in regional revenue-raising abilities are especially evident when, as here, progressive taxes are assigned partially to regional governments.

Central governments normally carry out some form of regional fiscal equalization. Under these circumstances, sound design of these inter-regional transfers requires a clear understanding of the precise sources of divergence of regional fiscal capacities. The present approach can thus contribute to the debate on regional transfers by clarifying precisely how regions differ with respect to the tax structure and the distribution of taxpayers. This is because equation (43) makes it evident that differences in revenue hinge on basically four factors: the number and distribution of taxpayers, the distribution of taxable incomes and the specific tax parameters that define the structure -marginal tax rates, tax bracket thresholds and average tax credits.

The present approach makes it possible to construct a matrix in which each region's tax revenue can be computed under the assumption that it shares one or more of the characteristics of other regions. Thus a ' 15 by 15 ' matrix is obtained such that each entry shows the revenue obtained by a row region, under the assumption that it has a particular characteristic of the column region. The leading diagonal of such a square matrix obviously shows the actual revenue obtained by the region. This matrix is augmented by an additional row and column for the country as a whole. Similarly, the information can be displayed in relative terms, showing the percentage differences in revenue which could be raised by each region, given different assumed characteristics (so that each corresponding leading diagonal element is zero).

To illustrate the kind of information which can be produced along these lines, Tables 13 to 15 report three such hypothetical ' 16 by 16 ' matrices for Spanish regions in the

2007 fiscal year. These were computed using the sample of tax-returns described in section 4 and, as before, refer to the fifteen Autonomous Communities of the Common Territory. Each matrix shows the relative impact on the revenue collection of the row region if it were to replicate the specific characteristic of the column region. Specifically, Table 13 presents the revenue impact of differences in arithmetic mean taxable incomes. Table 14 depicts the effects of differences in the form of the relative taxable income distributions; that is, the arithmetic means are unchanged but the proportions of people in each tax bracket, and the corresponding proportions of total taxable income within each bracket, are assumed to the those of the region in the columns. Finally, Table 15 shows the revenue consequences of simultaneous changes in both the arithmetic mean taxable income and the relative distributions of income. ${ }^{15}$

For example, Table 13 shows that if Andalucía were to have the same arithmetic mean taxable income as Aragon (a given percentage change in all incomes), it would have 4.17 per cent higher income tax revenue. However, from Table 14 if Andalucia were to have its actual arithmetic mean, but the same relative form of income distribution as Aragon, it would have slightly less revenue: there would be a reduction of 0.40 per cent. Table 15 indicates that if the distribution of taxable income in Andalucia were the precisely the same as in Aragon (in both absolute and relative terms), its revenue would be 3.77 per cent higher. In fact these effects are additive, so that the elements of Table 15 effectively equal the sum of the corresponding elements in Tables 13 and 14.

[^11]Table 13 Effects of Varying Arithmetic Mean Taxable Income (Row Region has Mean Income of Column Region)

|  | Nat. | And. | Arag. | Ast. | Bal. | Can. | Cant. | Cast.- <br> L. | Cast.- <br> LM. | Cat. | Val. | Extr. | Gal. | Mad. | Murc. | Rioja |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| National | 0 | -14.73 | -11.26 | -9.33 | -5.65 | -6.98 | -9.19 | -17.81 | -27.07 | 12.04 | -15.39 | -30.96 | -19.09 | 41.57 | -17.29 | -15.69 |
| Andalucia | 17.31 | 0 | 4.17 | 6.30 | 10.72 | 9.01 | 6.58 | -3.56 | -14.43 | 31.42 | -0.67 | -19.02 | -5.07 | 66.02 | -2.98 | -0.97 |
| Aragon | 12.32 | -3.91 | 0 | 2.00 | 6.15 | 4.54 | 2.26 | -7.25 | -17.44 | 25.55 | -4.54 | -21.74 | -8.67 | 57.99 | -6.70 | -4.82 |
| Asturias | 10.23 | -5.86 | -1.98 | 0 | 4.11 | 2.52 | 0.27 | -9.17 | -19.27 | 23.35 | -6.47 | -23.53 | -10.57 | 55.50 | -8.63 | -6.75 |
| Baleares | 5.80 | -9.34 | -5.77 | -3.79 | 0 | -1.38 | -3.64 | -12.50 | -22.00 | 18.16 | -10.01 | -26.01 | -13.81 | 48.49 | -11.96 | -10.31 |
| Canarias | 7.62 | -8.43 | -4.63 | -2.55 | 1.48 | 0 | -2.37 | -11.77 | -21.85 | 20.73 | -9.13 | -26.10 | -13.16 | 52.89 | -11.21 | -9.44 |
| Cantabria | 9.96 | -6.10 | -2.24 | -0.24 | 3.85 | 2.28 | 0 | -9.42 | -19.51 | 23.07 | -6.74 | -23.76 | -10.82 | 55.20 | -8.87 | -7.03 |
| Castilla-Leon | 21.17 | 3.60 | 7.87 | 9.98 | 14.51 | 12.71 | 10.31 | 0 | -11.04 | 35.50 | 2.96 | -15.69 | -1.54 | 70.60 | 0.58 | 2.67 |
| Castilla-LaMancha | 35.79 | 16.25 | 21.04 | 23.33 | 28.40 | 26.34 | 23.74 | 12.27 | 0 | 51.71 | 15.59 | -5.18 | 10.55 | 90.71 | 12.90 | 15.29 |
| Cataluña | -10.88 | -24.18 | -21.09 | -19.29 | -16.00 | -17.15 | -19.20 | -26.99 | -35.34 | 0 | -24.82 | -38.86 | -28.13 | 26.70 | -26.50 | -25.11 |
| Valencia | 17.50 | 0.66 | 4.71 | 6.80 | 11.09 | 9.44 | 7.06 | -2.81 | -13.38 | 31.24 | 0 | -17.84 | -4.28 | 64.91 | -2.24 | -0.30 |
| Extremadura | 42.51 | 22.23 | 27.21 | 29.57 | 34.85 | 32.69 | 30.02 | 18.11 | 5.38 | 59.02 | 21.56 | 0 | 16.33 | 99.47 | 18.76 | 21.26 |
| Galicia | 22.46 | 5.09 | 9.28 | 11.41 | 15.86 | 14.13 | 11.70 | 1.52 | -9.39 | 36.62 | 4.43 | -13.99 | 0 | 71.34 | 2.10 | 4.13 |
| Madrid | -30.66 | -41.48 | -39.04 | -37.47 | -34.87 | -35.68 | -37.49 | -43.81 | -50.61 | -21.79 | -42.09 | -53.46 | -44.73 |  | -43.39 | -42.37 |
| Murcia | 20.99 | 3.08 | 7.41 | 9.60 | 14.19 | 12.40 | 9.90 | -0.60 | -11.85 | 35.60 | 2.40 | -16.59 | -2.17 | 71.40 | 0 | 2.09 |
| Rioja | 17.96 | 0.91 | 5.05 | 7.11 | 11.49 | 9.76 | 7.42 | -2.59 | -13.29 | 31.87 | 0.28 | -17.81 | -4.08 | 65.94 | -2.02 | 0 |

Table 14 Effects of Varying Relative Incomes: Row Region has F and $F_{1}$ of Column Region

|  | Nat. | And. | Arag. | Ast. | Bal. | Can. | Cant. | Cast.- <br> L. | Cast.- <br> LM. | Cat. | Val. | Extr. | Gal. | Mad. | Murc | Rioja |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| National | 0 | -1.98 | -2.32 | -2.90 | 0.89 | -0.70 | -1.80 | -3.15 | -3.11 | 0.13 | -0.84 | -3.30 | -1.69 | 1.69 | -1.99 | -2.44 |
| Andalucia | 1.89 | 0 | -0.40 | -1.09 | 3.00 | 1.15 | 0.14 | -1.21 | -0.99 | 1.77 | 1.32 | -1.15 | 0.40 | 2.89 | 0.04 | -0.41 |
| Aragon | 2.18 | 0.37 | 0 | -0.65 | 3.21 | 1.47 | 0.51 | -0.77 | -0.57 | 2.07 | 1.61 | -0.73 | 0.74 | 3.17 | 0.41 | -0.02 |
| Asturias | 2.99 | 1.02 | 0.65 | 0 | 4.01 | 2.25 | 1.18 | -0.19 | -0.06 | 3.00 | 2.27 | -0.23 | 1.37 | 4.36 | 1.04 | 0.58 |
| Baleares | -0.93 | -2.76 | -3.10 | -3.70 | 0 | -1.61 | -2.61 | -3.88 | -3.77 | -0.91 | -1.61 | -3.93 | -2.44 | 0.38 | -2.74 | -3.17 |
| Canarias | 0.74 | -1.29 | -1.66 | -2.29 | 1.73 | 0 | -1.12 | -2.52 | -2.43 | 0.82 | -0.06 | -2.62 | -0.96 | 2.32 | -1.29 | -1.76 |
| Cantabria | 1.72 | -0.14 | -0.51 | -1.15 | 2.73 | 1.00 | 0 | -1.30 | -1.14 | 1.66 | 1.08 | -1.30 | 0.21 | 2.87 | -0.11 | -0.55 |
| Castilla-Leon | 2.96 | 1.17 | 0.77 | 0.07 | 4.11 | 2.22 | 1.29 | 0 | 0.28 | 2.74 | 2.51 | 0.14 | 1.60 | 3.65 | 1.24 | 0.81 |
| Castilla-LaMancha | 2.36 | 0.72 | 0.27 | -0.50 | 3.68 | 1.61 | 0.80 | -0.45 | 0 | 1.91 | 2.18 | -0.11 | 1.23 | 2.38 | 0.83 | 0.42 |
| Cataluña | -0.30 | -2.38 | -2.68 | -3.21 | 0.46 | -0.99 | -2.17 | -3.55 | -3.63 | 0 | -1.33 | -3.84 | -2.15 | 1.86 | -2.42 | -2.89 |
| Valencia | 0.45 | -1.26 | -1.64 | -2.31 | 1.53 | -0.25 | -1.15 | -2.38 | -2.12 | 0.26 | 0 | -2.26 | -0.87 | 1.16 | -1.20 | -1.62 |
| Extremadura | 2.30 | 0.74 | 0.27 | -0.53 | 3.68 | 1.55 | 0.79 | -0.42 | 0.10 | 1.74 | 2.24 | 0 | 1.28 | 2.01 | 0.87 | 0.47 |
| Galicia | 1.30 | -0.42 | -0.81 | -1.50 | 2.43 | 0.59 | -0.31 | -1.55 | -1.27 | 1.07 | 0.89 | -1.40 | 0 | 1.91 | -0.35 | -0.77 |
| Madrid | -2.98 | -5.22 | -5.47 | -5.91 | -2.43 | -3.65 | -4.96 | -6.39 | -6.68 | -2.39 | -4.33 | -6.92 | -5.10 | 0 | -5.32 | -5.81 |
| Murcia | 1.79 | -0.06 | -0.47 | -1.18 | 2.95 | 1.04 | 0.07 | -1.26 | -1.00 | 1.60 | 1.29 | -1.15 | 0.36 | 2.59 | 0 | -0.45 |
| Rioja | 2.05 | 0.35 | -0.04 | -0.72 | 3.16 | 1.34 | 0.46 | -0.78 | -0.49 | 1.82 | 1.63 | -0.63 | 0.76 | 2.66 | 0.41 | 0 |

Table 15 Effects of Varying Mean Taxable Income and its Distribution: Row Region has Distribution of Column Region

|  | Nat. | And. | Arag. | Ast. | Bal. | Can. | Cant. | $\begin{aligned} & \text { Cast.- } \\ & \text { L. } \end{aligned}$ | Cast.- <br> LM. | Cat. | Val. | Extr. | Gal. | Mad. | Murc. | Rioj. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| National | 0 | -16.30 | -13.18 | -11.99 | -4.75 | -7.66 | -10.72 | -20.19 | -28.76 | 12.38 | -15.78 | -32.54 | -20.16 | 45.86 | -18.72 | -17.39 |
| Andalucia | 19.69 | 0 | 3.77 | 5.20 | 13.96 | 10.44 | 6.74 | -4.71 | -15.06 | 34.65 | 0.63 | -19.63 | -4.66 | 75.11 | -2.92 | -1.32 |
| Aragon | 14.94 | -3.54 | 0 | 1.35 | 9.56 | 6.26 | 2.79 | -7.95 | -17.66 | 28.97 | -2.94 | -21.95 | -7.91 | 66.93 | -6.28 | -4.78 |
| Asturias | 13.49 | -4.85 | -1.34 | 0 | 8.15 | 4.87 | 1.43 | -9.23 | -18.86 | 27.42 | -4.26 | -23.12 | -9.19 | 65.08 | -7.57 | -6.08 |
| Baleares | 4.89 | -11.89 | -8.68 | -7.46 | 0 | -3.00 | -6.15 | -15.90 | -24.72 | 17.63 | -11.36 | -28.61 | -15.86 | 52.09 | -14.38 | -13.02 |
| Canarias | 8.39 | -9.47 | -6.05 | -4.75 | 3.19 | 0 | -3.36 | -13.74 | -23.12 | 21.96 | -8.89 | -27.27 | -13.69 | 58.65 | -12.12 | -10.67 |
| Cantabria | 11.97 | -6.23 | -2.74 | -1.42 | 6.67 | 3.42 | 0 | -10.58 | -20.14 | 25.80 | -5.64 | -24.37 | -10.54 | 63.19 | -8.93 | -7.45 |
| Castilla-Leon | 25.08 | 4.84 | 8.71 | 10.19 | 19.18 | 15.56 | 11.76 | 0 | -10.64 | 40.45 | 5.49 | -15.34 | 0.05 | 82.03 | 1.83 | 3.48 |
| Castilla-LaMancha | 40.13 | 17.39 | 21.74 | 23.40 | 33.51 | 29.44 | 25.17 | 11.95 | 0 | 57.41 | 18.12 | -5.28 | 12.01 | 104.13 | 14.01 | 15.86 |
| Cataluña | -11.00 | -25.48 | -22.71 | -21.66 | -15.22 | -17.81 | -20.53 | -28.95 | -36.56 | 0 | -25.02 | -39.92 | -28.91 | 29.75 | -27.63 | -26.46 |
| Valencia | 18.49 | -0.61 | 3.04 | 4.44 | 12.93 | 9.51 | 5.92 | -5.18 | -15.21 | 32.99 | 0 | -19.65 | -5.13 | 72.22 | -3.45 | -1.89 |
| Extremadura | 47.30 | 23.61 | 28.14 | 29.87 | 40.40 | 36.17 | 31.72 | 17.95 | 5.50 | 65.29 | 24.37 | 0 | 18.00 | 113.96 | 20.09 | 22.02 |
| Galicia | 24.51 | 4.69 | 8.48 | 9.93 | 18.74 | 15.19 | 11.47 | -0.05 | -10.46 | 39.55 | 5.32 | -15.06 |  | 80.26 | 1.75 | 3.36 |
| Madrid | -31.83 | -43.15 | -40.98 | -40.16 | -35.13 | -37.15 | -39.27 | -45.85 | -51.79 | -23.24 | -42.78 | -54.42 | -45.82 | 0 | -44.83 | -43.90 |
| Murcia | 23.48 | 3.04 | 6.95 | 8.44 | 17.53 | 13.87 | 10.03 | -1.85 | -12.60 | 39.01 | 3.69 | -17.34 | -1.80 | 81.01 | 0 | 1.66 |
| Rioja | 20.90 | 1.31 | 5.06 | 6.49 | 15.20 | 11.69 | 8.02 | -3.37 | -13.66 | 35.77 | 1.94 | -18.20 | -3.32 | 76.00 | -1.59 | 0 |

## 6 Conclusions

The aim of this paper was to derive analytical expressions for aggregate revenue and the revenue elasticity of the Spanish personal income tax system as applied to tax units and in aggregate. This was considerably complicated by the schedular nature of the system, the role of central and regional governments, along with the existence of a range of tax credits and eligible expenditures and deductions.

Empirical estimates of revenue elasticities were obtained using a large cross-sectional data set which enabled a number of important ancillary elasticities (relating to allowances and tax credits, and different income sources) to be estimated. The functional relationship between gross income and personal income taxation was examined, rather than starting from a given distribution of taxable income.

It was found that there is considerable variation among tax units in the revenue elasticity, with highly (positively) skewed distributions. Similarly, the aggregate elasticities for each region were found to vary, associated with variations in the income distributions. Variations were around a value of about 1.3.

Formal expressions for aggregate tax revenue were derived, in terms of the distribution of taxable income. It was possible to separate total revenue into components relating to the income tax structure and summary measures of the distribution of taxable income, in particular the proportions of taxpayers, and of total taxable income, in each tax bracket. It was thus possible to examine the sources of differences among regions.

It is suggested that the approach developed here is of value not only in understanding the fiscal drag and 'automatic stabilisation' properties of the personal tax structure, but in considering the factors - particularly the nature of the distribution of taxable income - affecting total tax revenue.

## Appendix A. Losses as a Proportion of Income

Section 2 refers briefly to the role of losses in the Spanish personal tax system. The first column of Table 16 reports negative (general) taxable income generated each tax year, as a proportion of the tax year's current taxable income. The second column contains the proportion of the negative taxable income, carried forward from the last four tax years, used to offset current (general) taxable income. It can be seen that the amount of negative taxable income generated each tax year is well below 1 per cent. Furthermore, the amount of carried-forward taxable income from the last four years to offset against current taxable income is even less relevant in relative terms, being well below 0.1 per cent. As with corporation losses, many losses are not used by the taxpayers and becoming 'stranded'. Hence, the loss asymmetry in the tax function is of little relevance in determining the aggregate tax liability.

Table 16 Negative Taxable Income as a Proportion of Total Taxable Income

| Tax Year | \% of generated <br> income | negative | taxable of used negative taxable income <br> (coming from previous four years) |
| :--- | :--- | :---: | :---: |
| 1997 | 0.18 | 0.06 |  |
| 1998 | 0.22 | 0.05 |  |
| $1999^{*}$ | less than 0.35 | 0.06 |  |
| $2000^{*}$ | less than 0.39 | 0.06 |  |
| $2001^{*}$ | less than 0.74 | 0.06 |  |
| 2002 | 0.55 | 0.07 |  |
| 2003 | 0.54 | 0.06 |  |
| 2004 | 0.48 | 0.06 |  |
| 2005 | 0.5 | 0.05 |  |
| 2006 | 0.4 | 0.06 |  |

Source: Memorias de la Administracion Tributaria (Tax Office's Annual Tax Report), several years.
$\left({ }^{*}\right)$ This figures includes not only negative taxable income but taxable income up to $3,000 €$

## Appendix B. The Treatment of Allowances

This appendix considers refundable and non-refundable tax allowances. Suppose there is a simple tax structure with a marginal rate of $t$ applied to income, $y$ in excess of $a$, and there is a 'refundable tax credit' of $b$. The term 'refundable' means that if income tax is less than $b$, the individual receives a payment (pays negative tax). The net tax paid is

$$
\begin{equation*}
T(y)=t(y-a)-b \tag{B1}
\end{equation*}
$$

The total expenditure on the refundable tax credit $b$ remains fixed, so long as the population size is fixed. Those with incomes between $a$ and $a+b / t$ pay some income tax but face an overall negative average tax rate.

For taxpayers, net income is $y(1-t)+a t+b$ and the tax-free threshold can be regarded as giving rise to a kind of tax credit worth at. This is a 'non-refundable tax credit', such that those with $y<a$ receive nothing.

The non-refundable credit is intimately connected with the income tax structure. It determines who pays a zero marginal income tax rate, and the size of the 'nonrefundable credit' is determined by the tax rate as well as $a$. The total 'tax expenditure' associated with the threshold $a$ varies as the tax rate and the income distribution changes: it increases as the number of people above the threshold increases.

Consider only values of income for which tax, net of $b$, is positive. The average tax rate is:

$$
\begin{equation*}
A T R=t\left(1-\frac{a}{y}\right)-\frac{b}{y} \tag{B2}
\end{equation*}
$$

The individual revenue elasticity is given by $M T R / A T R$ and is thus:

$$
\begin{equation*}
\eta=\frac{M T R}{A T R}=\left(1-\frac{a+b / t}{y}\right)^{-1} \tag{B3}
\end{equation*}
$$

Hence for those with positive net average tax rates, the elasticity is higher when $b$ is included (essentially because it lowers their average tax rate). A higher value of the refundable tax credit $b$ has the effect of raising the revenue elasticity.

Alternatively, it is possible simply to think of the two components of the structure separately. It could be said to combine an income tax with a tax-free threshold, and an unconditional transfer payment that is unrelated to income. Indeed, the refundable tax credit could be administered, without any change in net incomes, by an entirely separate agency and could be given a name (such as a 'basic income', or 'grant') that is unrelated to income taxation. In contrast, it would not be possible to separate the non-refundable tax credit from the income tax system.

Considering only the income tax system, the individual revenue elasticity is then:

$$
\begin{equation*}
\eta=\left(1-\frac{a}{y}\right)^{-1}=1+\left(\frac{a}{y-a}\right) \tag{B4}
\end{equation*}
$$

as conventionally obtained.

If interest is in using the revenue elasticity at a given income level as an indication of overall progressivity of taxes and transfers, then the refundable tax credit clearly increases progressivity of the tax and transfer system as a whole. Perhaps it is then desirable to include both components. ${ }^{16}$ But if concern is with the effect on tax revenue of inflation - fiscal drag - then it can be argued that allowance should be made only for non-refundable tax credits, and not refundable credits which, as suggested above, can be entirely separated from the income tax system, both conceptually and administratively.

[^12]
## Appendix C.

Table 17 Separate values of $\boldsymbol{G}$ for Central and Regional Governments

| Central Government Tax Schedule |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
|  | $G\left(a_{1}\right)$ | $G\left(a_{2}\right)$ | $G\left(a_{3}\right)$ | $G\left(a_{4}\right)$ |
| National | 0.290193 | 0.305909 | 0.132724 | 0.161007 |
| Andalucía | 0.330057 | 0.326354 | 0.123810 | 0.116320 |
| Aragón | 0.325034 | 0.333407 | 0.123724 | 0.113364 |
| Asturias | 0.314239 | 0.347408 | 0.125436 | 0.108673 |
| Baleares | 0.320947 | 0.275424 | 0.132188 | 0.164704 |
| Canarias | 0.298323 | 0.304869 | 0.143412 | 0.142245 |
| Cantabria | 0.327856 | 0.318916 | 0.125361 | 0.123590 |
| Castilla-León | 0.340052 | 0.338948 | 0.127512 | 0.091601 |
| Castilla-La Mancha 0.383015 | 0.318322 | 0.114543 | 0.088942 |  |
| Cataluña | 0.259386 | 0.303344 | 0.136780 | 0.184428 |
| Valencia | 0.345647 | 0.303937 | 0.119325 | 0.129922 |
| Extremadura | 0.390029 | 0.319601 | 0.112433 | 0.083703 |
| Galicia | 0.346616 | 0.312747 | 0.125405 | 0.113442 |
| Madrid | 0.197128 | 0.275025 | 0.149916 | 0.254015 |
| Murcia | 0.343290 | 0.316890 | 0.126424 | 0.111306 |
| Rioja | 0.363215 | 0.306374 | 0.127387 | 0.103131 |

Regional Government Tax Schedule

|  | $G\left(a_{1}\right)$ | $G\left(a_{2}\right)$ | $G\left(a_{3}\right)$ | $G\left(a_{4}\right)$ |
| :--- | :--- | :--- | :--- | :--- |
| National | 0.290193 | 0.305909 | 0.132695 | 0.153661 |
| Andalucía | 0.330057 | 0.326354 | 0.123782 | 0.110587 |
| Aragón | 0.325034 | 0.333407 | 0.123696 | 0.107555 |
| Asturias | 0.314239 | 0.347408 | 0.125408 | 0.103328 |
| Baleares | 0.320947 | 0.275424 | 0.132159 | 0.157417 |
| Canarias | 0.298323 | 0.304869 | 0.143380 | 0.135400 |
| Cantabria | 0.327856 | 0.318916 | 0.125333 | 0.117504 |
| Castilla-León | 0.340052 | 0.338948 | 0.127483 | 0.086780 |
| Castilla-La Mancha 0.383015 | 0.318322 | 0.114517 | 0.084169 |  |
| Cataluña | 0.259386 | 0.303344 | 0.136749 | 0.175948 |
| Valencia | 0.345647 | 0.303937 | 0.119298 | 0.123599 |
| Extremadura | 0.390029 | 0.319601 | 0.112407 | 0.078957 |
| Galicia | 0.346616 | 0.312747 | 0.125377 | 0.107694 |
| Madrid | 0.197128 | 0.271630 | 0.147188 | 0.241403 |
| Murcia | 0.343290 | 0.316890 | 0.126396 | 0.105809 |
| Rioja | 0.363215 | 0.306374 | 0.127358 | 0.097940 |

## Appendix D. Detailed Tables

This appendix reports summary information regarding the calculation of individual and aggregate elasticities reported above.

First, Tables 18 to 22 show ancillary elasticities estimated for a range of types of tax unit.

Tables 23 and 24 then examine the relevance of allowance transfers from income source 1 to source 2 . The first column reports the total number of tax units within the quintile in the region that enjoy the transfer whereas the second column exhibits the percentage of total allowance transfer within the region that is absorbed by the quintile of $y_{1}$. As shown, this allowance transfer seems to be quite regionally symmetric, as nearly for every region around 10 per cent of the taxpayers within the first quantile of $y_{1}$ enjoy the transfer of allowances and around 98 per cent of the total transferred allowance is enjoyed by tax units in the first quintile. This fact suggests that in computing the ancillary elasticity $\eta_{A_{2} y_{h}}$ attention must be focused on the distribution of $y_{1}$ and not of the total gross income $y_{h}$.

Finally, Tables 26-28 present basic summary statistics.

Table 18
Ancillary Elasticities

| 1.ANDALUCIA | $\eta_{y_{1} y_{h}}$ | $\eta_{y_{2} y_{h}}$ | $\eta_{E_{1} y_{h}}$ | $\eta_{E_{2} y_{h}}$ | $\eta_{A_{1} y_{h}}$ | $\eta_{C c y_{h}}$ | $\eta_{C r y_{h}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| q1_s1 | 1.8205 | 0.2058 | 1.3595 | 4.5981 | 27.3225 | 2.0410 | 2.0116 |
| q1_s2 | 2.4653 | 0.3790 | 2.4337 | 4.3717 | 3.4366 | 3.2165 | 3.1698 |
| q1_s3 | 2.1075 | 0.0000 | 1.7938 | 2.7451 | 3.3026 | 3.4651 | 3.4229 |
| q2_s1 | 1.1769 | 7.1982 | 0.5454 | 128.3250 | 87.9449 | 0.6522 | 0.6493 |
| q2_s2 | 1.1115 | -2.1130 | 1.7965 | 102.4565 | 3.7185 | 1.0644 | 1.1029 |
| q2_s3 | 1.0486 | 5.7543 | 0.7493 | 129.1173 | 3.3098 | 1.9123 | 1.8804 |
| q3_s1 | 0.7120 | 9.9772 | 0.0000 | 59.1907 | 19.3692 | 0.3040 | 0.2826 |
| q3_s2 | 1.1111 | 4.9313 | 1.1714 | 37.8552 | 2.5185 | 0.3718 | 0.3846 |
| q3_s3 | 1.0256 | 7.4778 | 0.8834 | 55.6681 | 1.8466 | 0.8578 | 0.8611 |
| q4_s1 | 0.9262 | 5.8885 | 1.3786 | 31.0950 | 18.3945 | 0.2010 | 0.1935 |
| q4_s2 | 0.8996 | 0.0000 | 0.9749 | 14.1905 | 2.0204 | 0.1908 | 0.1718 |
| q4_s3 | 0.9767 | 0.0000 | 0.0000 | 25.7131 | 1.3256 | 0.1989 | 0.1920 |
| q5_s1 | 0.6873 | 2.1406 | -0.3787 | -4.3432 | -1.6393 | 0.0514 | 0.0444 |
| q5_s2 | 0.7398 | 2.9049 | 0.0000 | 0.0000 | 1.3373 | 0.0000 | 0.0000 |
| q5_s3 | 0.9277 | 3.1408 | 0.2336 | -3.3722 | 1.5950 | 0.0799 | 0.0762 |
| 2.ARAGÓN | $\eta_{y_{1} y_{h}}$ | $\eta_{y_{2} y_{h}}$ | $\eta_{E_{1} y_{h}}$ | $\eta_{E_{2} y_{h}}$ | $\eta_{A_{1} y_{h}}$ | $\eta_{C c y_{h}}$ | $\eta_{C r y_{h}}$ |
| q1_s1 | 1.4250 | 0.5685 | 0.8685 | 3.9588 | 12.4536 | 1.9495 | 1.9208 |
| q1_s2 | 1.8843 | 0.6728 | 1.4421 | 5.0059 | 2.8221 | 2.8060 | 2.7626 |
| q1_s3 | 1.7934 | 0.4116 | 1.2803 | 3.0965 | 2.8225 | 2.8991 | 2.8628 |
| q2_s1 | 1.4505 | 0.0000 | 1.1728 | 47.3184 | 33.2372 | 0.4147 | 0.4751 |
| q2_s2 | 1.2646 | -1.8892 | 0.9436 | 32.7148 | 2.6282 | 1.4606 | 1.4190 |
| q2_s3 | 1.0805 | 0.0000 | 0.0000 | 32.1120 | 5.5508 | 1.1781 | 1.1590 |
| q3_s1 | 0.5632 | 3.8990 | -0.7661 | 38.4092 | 14.9674 | 0.2910 | 0.2683 |
| q3_s2 | 1.0306 | 0.0000 | 0.0000 | 38.9248 | 1.9822 | 0.3199 | 0.3874 |
| q3_s3 | 1.0834 | 2.8321 | 0.0000 | 24.6546 | 3.1928 | 0.6635 | 0.6756 |
| q4_s1 | 1.0271 | 0.0000 | 0.6986 | 19.7880 | 0.0000 | 0.0000 | 0.0000 |
| q4_s2 | 1.0262 | 0.0000 | 0.6376 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| q4_s3 | 1.0013 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| q5_s1 | 0.7619 | 2.0768 | 0.2393 | 0.0000 | 0.0000 | 0.0000 | 0.0601 |
| q5_s2 | 0.9181 | 1.8416 | 0.4902 | 0.0000 | 1.8270 | 0.0000 | 0.0000 |
| q5_s3 | 0.8714 | 1.8909 | 0.2026 | 0.0000 | 1.8559 | 0.0461 | 0.0451 |
| 3.ASTURIAS | $\eta_{y_{1} y_{h}}$ | $\eta_{y_{2} y_{h}}$ | $\eta_{E_{1} y_{h}}$ | $\eta_{E_{2} y_{h}}$ | $\eta_{A_{1} y_{h}}$ | $\eta_{C c y_{h}}$ | $\eta_{C r y_{h}}$ |
| q1_s1 | 1.7063 | 0.3353 | 1.3490 | 3.5916 | 14.7046 | 1.8832 | 1.8549 |
| q1_s2 | 2.0593 | 0.4367 | 1.8835 | 5.5782 | 2.9393 | 3.1504 | 3.1024 |
| q1_s3 | 1.9677 | 0.0000 | 2.1454 | 3.0272 | 2.2235 | 2.9185 | 2.8831 |
| q2_s1 | 1.3195 | 0.0000 | 1.3192 | 76.5031 | 51.2197 | 0.7538 | 0.7649 |
| q2_s2 | 0.9585 | 0.0000 | 1.2793 | 59.5610 | 3.6513 | 2.0431 | 2.0477 |
| q2_s3 | 1.1520 | 8.4528 | -1.0714 | 0.0000 | 1.5852 | 2.1234 | 2.1103 |
| q3_s1 | 1.0272 | 0.0000 | 0.0000 | 0.0000 | 15.0482 | 0.4849 | 0.4559 |
| q3_s2 | 0.8664 | 0.0000 | 0.0000 | 47.2858 | 2.1763 | 0.6929 | 0.6721 |
| q3_s3 | 0.8937 | 0.0000 | 0.0000 | 85.9305 | 2.2531 | 0.4859 | 0.5005 |
| q4_s1 | 1.0210 | 10.8294 | 0.0000 | 34.1867 | 0.0000 | 0.0000 | 0.0000 |
| q4_s2 | 0.9503 | 8.0631 | 0.0000 | 38.2689 | 0.0000 | 0.0000 | 0.0000 |
| q4_s3 | 1.0159 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| q5_s1 | 0.7110 | 3.7579 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| q5_s2 | 0.8777 | 1.6330 | 0.0000 | 0.0000 | 1.0350 | -0.0717 | -0.0733 |
| q5_s3 | 0.9737 | 2.6641 | 0.0000 | 0.0000 | 1.0127 | 0.0000 | 0.0000 |

Table 19 Ancillary Elasticities (Continued)

| 4.BALEARES | $\eta_{y_{1} y_{h}}$ | $\eta_{y_{2} y_{h}}$ | $\eta_{E_{1} y_{h}}$ | $\eta_{E_{2} y_{h}}$ | $\eta_{A_{1} y_{h}}$ | $\eta_{C c y_{h}}$ | $\eta_{C r y_{h}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| q1_s1 | 1.5501 | 0.2360 | 0.8848 | 6.3064 | 28.3043 | 1.9475 | 1.9192 |
| q1_s2 | 2.2556 | 0.3038 | 1.5877 | 6.8316 | 5.6941 | 2.8488 | 2.7982 |
| q1_s3 | 1.7156 | 0.0000 | 1.1882 | 0.0000 | 3.1893 | 2.8929 | 2.8557 |
| q2_s1 | 0.8698 | 0.0000 | 0.4517 | 138.9513 | 68.2686 | 0.5463 | 0.5543 |
| q2_s2 | 1.2602 | 0.0000 | 0.7924 | 84.6014 | 8.4081 | 1.4202 | 1.4313 |
| q2_s3 | 0.9036 | 0.0000 | 0.0000 | 129.4957 | 10.5477 | 1.1634 | 1.1501 |
| q3_s1 | 1.0517 | 0.0000 | 0.0000 | 0.0000 | 26.5219 | 0.5627 | 0.5286 |
| q3_s2 | 0.9889 | 3.6977 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| q3_s3 | 0.9223 | 6.9295 | 0.0000 | 60.5937 | 0.0000 | 0.5288 | 0.5251 |
| q4_s1 | 1.4391 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| q4_s2 | 1.2056 | 0.0000 | 0.0000 | 28.8982 | 0.0000 | 0.0000 | 0.0000 |
| q4_s3 | 0.9633 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| q5_s1 | 0.6188 | 1.9306 | 0.0000 | -7.6470 | -2.2827 | -0.1194 | -0.1141 |
| q5_s2 | 0.9829 | 1.5435 | 0.3900 | 0.0000 | 1.0245 | 0.0000 | 0.0000 |
| q5_s3 | 0.8599 | 1.7982 | 0.2007 | -2.7518 | 1.8752 | 0.0409 | 0.0403 |
| 5.CANARIAS | $\eta_{y_{1} y_{h}}$ | $\eta_{y_{2} y_{h}}$ | $\eta_{E_{1} y_{h}}$ | $\eta_{E_{2} y_{h}}$ | $\eta_{A_{1} y_{h}}$ | $\eta_{C c y_{h}}$ | $\eta_{C r y_{h}}$ |
| q1_s1 | 1.3350 | 0.0000 | 1.1848 | 2.4709 | 20.8826 | 2.2713 | 2.2398 |
| q1_s2 | 1.8006 | 0.0000 | 2.0868 | 3.3040 | 2.5293 | 3.2092 | 3.1628 |
| q1_s3 | 1.4746 | 0.0000 | 1.2656 | 2.5069 | 2.4717 | 2.8763 | 2.8407 |
| q2_s1 | 1.0215 | 11.8792 | 0.7669 | 133.9884 | 42.3304 | 0.3762 | 0.3697 |
| q2_s2 | 1.1385 | 4.1228 | 1.4386 | 80.1577 | 1.9472 | 1.2301 | 1.2621 |
| q2_s3 | 1.0029 | 10.4332 | 0.4987 | 113.8376 | 3.3765 | 1.6138 | 1.5915 |
| q3_s1 | 0.7362 | 7.7179 | 0.0000 | 50.2407 | 0.0000 | 0.0000 | 0.0000 |
| q3_s2 | 0.6380 | 9.2348 | 1.5675 | 31.9513 | 1.6762 | 0.2719 | 0.2516 |
| q3_s3 | 0.9330 | 7.1229 | 1.6649 | 42.7605 | 0.0000 | 0.6032 | 0.5976 |
| q4_s1 | 0.9548 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| q4_s2 | 1.1288 | 0.0000 | 0.0000 | 0.0000 | 1.5947 | 0.0000 | 0.0000 |
| q4_s3 | 1.0401 | 4.1661 | 0.0000 | 28.4397 | 1.6333 | 0.0000 | 0.0000 |
| q5_s1 | 0.6581 | 2.7895 | 0.0000 | -6.9937 | 0.0000 | 0.3120 | 0.3040 |
| q5_s2 | 0.8242 | 3.1597 | 0.2611 | 0.0000 | 0.7369 | 0.2615 | 0.2532 |
| q5_s3 | 0.9688 | 3.3904 | 0.3346 | -2.5319 | 1.3082 | 0.2918 | 0.2879 |
| 6.CANTABRIA | $\eta_{y_{1} y_{h}}$ | $\eta_{y_{2} y_{h}}$ | $\eta_{E_{1} y_{h}}$ | $\eta_{E_{2} y_{h}}$ | $\eta_{A_{1} y_{h}}$ | $\eta_{C c y_{h}}$ | $\eta_{C r y_{h}}$ |
| q1_s1 | 1.9938 | 0.4285 | 0.9121 | 2.1654 | 18.9311 | 1.9805 | 1.9485 |
| q1_s2 | 2.5854 | 0.5642 | 1.4734 | 5.1249 | 3.1040 | 2.4378 | 2.4003 |
| q1_s3 | 2.7109 | 0.0000 | 1.3822 | 1.6888 | 2.6639 | 3.2045 | 3.1640 |
| q2_s1 | 1.5120 | 2.7318 | 1.4593 | 63.5130 | 49.6894 | 0.6749 | 0.7105 |
| q2_s2 | 1.6249 | 0.0000 | 1.8295 | 53.8142 | 4.5355 | 1.8991 | 1.8483 |
| q2_s3 | 1.1322 | 3.6478 | 0.0000 | 64.7111 | 6.9505 | 1.7383 | 1.7117 |
| q3_s1 | 1.4992 | 0.0000 | 0.0000 | 30.0837 | 20.9058 | 0.0000 | 0.0000 |
| q3_s2 | 0.7369 | 0.0000 | 0.0000 | 43.7776 | 0.0000 | 0.0000 | 0.0000 |
| q3_s3 | 0.0000 | 5.3591 | 0.0000 | 51.3651 | 1.5421 | 0.7441 | 0.7404 |
| q4_s1 | 1.0063 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| q4_s2 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| q4_s3 | 0.9628 | 0.0000 | 0.0000 | 0.0000 | 3.0079 | 0.0000 | 0.0000 |
| q5_s1 | 0.9117 | 1.2343 | 0.8185 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| q5_s2 | 0.9941 | 2.4223 | 0.3900 | 0.0000 | 1.0926 | 0.0000 | 0.0000 |
| q5_s3 | 1.0225 | 2.2780 | 0.0000 | 0.0000 | 1.4441 | 0.1175 | 0.1134 |

Table 20 Ancillary Elasticities (Continued.)

| $\begin{aligned} & \text { 7.CASTILLA. } \\ & \text {-LEÓN } \end{aligned}$ | $\eta_{y_{1} y_{h}}$ | $\eta_{y_{2} y_{h}}$ | $\eta_{E_{1} y_{h}}$ | $\eta_{E_{2} y_{h}}$ | $\eta_{A_{1} y_{h}}$ | $\eta_{C c y_{h}}$ | $\eta_{C r y_{h}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| q1_s1 | 2.0242 | 0.6203 | 1.2294 | 3.6613 | 28.9743 | 1.8425 | 2.1527 |
| q1_s2 | 2.8639 | 0.6498 | 2.3222 | 4.3653 | 4.9284 | 2.7991 | 3.5656 |
| q1_s3 | 2.4881 | 0.4375 | 2.0019 | 3.7181 | 3.9891 | 2.7058 | 3.4365 |
| q2_s1 | 1.3332 | -1.0370 | 1.1798 | 44.3580 | 62.2967 | 0.5313 | 0.6043 |
| q2_s2 | 0.9414 | -3.1346 | 1.1729 | 29.2927 | 4.0597 | 1.3595 | 1.4475 |
| q2_s3 | 0.8350 | 0.0000 | 1.1134 | 37.3354 | 6.1049 | 1.4951 | 1.5473 |
| q3_s1 | 0.8839 | 3.3659 | 0.6456 | 26.6663 | 14.6671 | 0.3499 | 0.3304 |
| q3_s2 | 1.1416 | 2.6431 | 0.0000 | 26.8162 | 2.3996 | 0.3543 | 0.3827 |
| q3_s3 | 0.7732 | 3.8607 | 0.7678 | 49.4595 | 3.5170 | 0.6750 | 0.6881 |
| q4_s1 | 0.9590 | 2.4449 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| q4_s2 | 0.9923 | 0.0000 | 0.8889 | 18.1597 | 1.9230 | 0.0000 | 0.0000 |
| q4_s3 | 1.0194 | 2.8897 | 0.0000 | 15.0051 | 2.0646 | 0.0000 | 0.0000 |
| q5_s1 | 0.8441 | 1.8957 | 0.3332 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| q5_s2 | 0.7643 | 1.8933 | 0.4139 | 2.3468 | 2.5508 | -0.0437 | -0.0515 |
| q5_s3 | 0.9742 | 1.8554 | 0.5500 | 2.3777 | 2.8466 | 0.0721 | 0.0646 |
| 8.CASTILLA- <br> LA MANCHA | $\eta_{y_{1} y_{h}}$ | $\eta_{y_{2} y_{h}}$ | $\eta_{E_{1} y_{h}}$ | $\eta_{E_{2} y_{h}}$ | $\eta_{A_{1} y_{h}}$ | $\eta_{C \subset y_{h}}$ | $\eta_{C r y_{h}}$ |
| q1_s1 | 1.8479 | 0.4980 | 1.1269 | 3.9525 | 12.2662 | 1.9043 | 1.8786 |
| q1_s2 | 2.1594 | 0.6174 | 1.7816 | 5.1921 | 1.6619 | 3.6114 | 3.5554 |
| q1_s3 | 2.2881 | 0.2801 | 1.7177 | 1.7446 | 1.9827 | 3.6069 | 3.5632 |
| q2_s1 | 1.2166 | 5.9103 | 1.0560 | 78.0549 | 39.5604 | 0.6283 | 0.6509 |
| q2_s2 | 1.3730 | -3.8874 | 1.4940 | 58.2568 | 1.5127 | 1.2970 | 1.3161 |
| q2_s3 | 0.9865 | 2.9457 | 0.5633 | 66.0249 | 2.8651 | 1.5656 | 1.5430 |
| q3_s1 | 0.8097 | 0.0000 | 0.9213 | 37.1462 | 8.3192 | 0.2819 | 0.2537 |
| q3_s2 | 0.9003 | 3.7643 | 1.0943 | 30.0386 | 0.0000 | 0.2463 | 0.3010 |
| q3_s3 | 0.9475 | 5.7290 | 0.0000 | 27.1166 | 0.0000 | 0.7308 | 0.7263 |
| q4_s1 | 1.0243 | 0.0000 | 0.0000 | 0.0000 | 8.8109 | 0.0000 | 0.0000 |
| q4_s2 | 0.9769 | 4.5993 | 0.0000 | 0.0000 | 0.0000 | 0.1906 | 0.1870 |
| q4_s3 | 0.8302 | 3.5042 | 0.0000 | 0.0000 | 1.2409 | 0.1043 | 0.1031 |
| q5_s1 | 0.8559 | 2.7507 | 0.6308 | 0.0000 | 1.5616 | 0.1550 | 0.1463 |
| q5_s2 | 0.6728 | 1.7477 | 0.3617 | 0.0000 | 1.4584 | 0.0000 | 0.0000 |
| q5_s3 | 0.9516 | 2.4500 | 0.3482 | 1.6601 | 1.7194 | 0.0453 | 0.0480 |
| 9.CATALUÑ <br> A | $\eta_{y_{1} y_{h}}$ | $\eta_{y_{2} y_{h}}$ | $\eta_{E_{1} y_{h}}$ | $\eta_{E_{2} y_{h}}$ | $\eta_{A_{1} y_{h}}$ | $\eta_{C c y_{h}}$ | $\eta_{C r y_{h}}$ |
| q1_s1 | 1.8801 | 0.3372 | 1.3023 | 5.2325 | 36.5498 | 2.1046 | 2.5151 |
| q1_s2 | 2.4396 | 0.5427 | 2.1184 | 5.1343 | 5.4327 | 2.6641 | 3.3848 |
| q1_s3 | 2.0844 | 0.2930 | 1.7918 | 4.8056 | 4.7565 | 3.3997 | 4.3632 |
| q2_s1 | 1.2691 | 0.0000 | 0.7463 | 93.2560 | 84.8643 | 0.6192 | 0.6302 |
| q2_s2 | 1.1340 | 0.0000 | 0.7777 | 75.1066 | 6.6710 | 1.2067 | 1.3594 |
| q2_s3 | 1.2194 | 1.7079 | 0.8174 | 74.8974 | 10.6875 | 1.2028 | 1.2486 |
| q3_s1 | 1.0628 | 0.0000 | 0.7171 | 40.5051 | 18.1412 | 0.4119 | 0.3768 |
| q3_s2 | 1.5956 | 2.5038 | 0.5831 | 35.7114 | -2.2788 | 0.3287 | 0.3462 |
| q3_s3 | 0.8440 | 0.0000 | 0.0000 | 40.6745 | 3.9973 | 0.5346 | 0.5385 |
| q4_s1 | 0.6446 | 0.0000 | 1.1294 | 24.9068 | 0.0000 | 0.0000 | 0.0000 |
| q4_s2 | 0.8955 | 1.6495 | 0.9527 | 25.2876 | 0.0000 | 0.0000 | 0.0000 |
| q4_s3 | 1.1859 | 0.0000 | 1.2033 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| q5_s1 | 0.7671 | 1.5166 | 0.1348 | -4.3291 | -1.2435 | 0.0000 | 0.0000 |
| q5_s2 | 0.9682 | 1.3211 | 0.2996 | -3.0253 | 1.3964 | 0.0000 | 0.0000 |
| q5_s3 | 0.9387 | 1.4722 | 0.1951 | -6.5189 | 1.1371 | 0.1066 | 0.1035 |

Table 21 Ancillary Elasticities (Continued)

| 10. C.VALENCIANA | $\eta_{y_{1} y_{h}}$ | $\eta_{y_{2} y_{h}}$ | $\eta_{E_{1} y_{h}}$ | $\eta_{E_{2} y_{h}}$ | $\eta_{A_{1} y_{h}}$ | $\eta_{C c y_{h}}$ | $\eta_{C r y_{h}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| q1_s1 | 1.6081 | 0.3398 | 0.9860 | 6.7619 | 26.7854 | 2.1633 | 2.1305 |
| q1_s2 | 2.1051 | 0.4564 | 2.0451 | 5.6606 | 3.2977 | 3.1345 | 3.0837 |
| q1_s3 | 1.8299 | 0.0000 | 1.5441 | 6.7808 | 3.3960 | 2.8988 | 2.8634 |
| q2_s1 | 1.2415 | 0.0000 | 0.6937 | 100.8261 | 87.4532 | 0.4239 | 0.4558 |
| q2_s2 | 1.1155 | 0.0000 | 0.7722 | 85.2117 | 4.1657 | 1.3290 | 1.3561 |
| q2_s3 | 1.0600 | 0.0000 | 0.0000 | 114.9881 | 5.3258 | 1.8076 | 1.7702 |
| q3_s1 | 0.9468 | 8.2590 | 0.0000 | 28.9398 | 16.6372 | 0.2933 | 0.2741 |
| q3_s2 | 0.6716 | 5.6793 | 0.0000 | 37.0663 | 3.1720 | 0.2776 | 0.2495 |
| q3_s3 | 0.9361 | 0.0000 | 0.0000 | 56.8774 | 2.0000 | 0.6566 | 0.6684 |
| q4_s1 | 1.2862 | 0.0000 | 0.0000 | 27.3681 | 18.2219 | 0.0000 | 0.0000 |
| q4_s2 | 0.9905 | 0.0000 | 0.0000 | 0.0000 | 2.8309 | -0.1706 | 0.0000 |
| q4_s3 | 0.8963 | 4.4397 | 0.0000 | 17.1309 | 0.0000 | 0.0000 | 0.0000 |
| q5_s1 | 0.7139 | 2.4676 | 0.0000 | -3.3929 | -1.7737 | 0.0000 | 0.0000 |
| q5_s2 | 0.7814 | 2.1786 | 0.0000 | -1.5149 | 0.7139 | 0.0000 | 0.0000 |
| q5_s3 | 0.8982 | 2.3211 | 0.0000 | -2.5843 | 1.6074 | 0.0733 | 0.0700 |
| 11.EXTREMADURA | $\eta_{y_{1} y_{h}}$ | $\eta_{y_{2} y_{h}}$ | $\eta_{E_{1} y_{h}}$ | $\eta_{E_{2} y_{h}}$ | $\eta_{A_{1} y_{h}}$ | $\eta_{C c y_{h}}$ | $\eta_{C r y_{h}}$ |
| q1_s1 | 1.4454 | 0.3390 | 0.7411 | 5.0005 | 13.1230 | 1.8823 | 1.8561 |
| q1_s2 | 1.6215 | 0.5181 | 0.9240 | 5.0525 | 1.8199 | 4.0801 | 4.0165 |
| q1_s3 | 1.6483 | 0.3549 | 0.9142 | 3.2053 | 2.2451 | 3.8361 | 3.7859 |
| q2_s1 | 1.0989 | 0.0000 | 1.0289 | 84.9568 | 47.7531 | 0.3849 | 0.4066 |
| q2_s2 | 0.9708 | 0.0000 | 0.8909 | 68.8747 | 1.4870 | 1.3216 | 1.3499 |
| q2_s3 | 1.0784 | 0.0000 | 0.5995 | 88.8294 | 2.1742 | 1.8956 | 1.8590 |
| q3_s1 | 1.3294 | 0.0000 | 0.0000 | 0.0000 | 20.7554 | 0.4418 | 0.4166 |
| q3_s2 | 0.9508 | 0.0000 | 0.0000 | 40.1981 | 1.3586 | 0.4595 | 0.4900 |
| q3_s3 | 1.0379 | 0.0000 | 0.0000 | 42.0373 | 0.0000 | 0.7770 | 0.7942 |
| q4_s1 | 1.0298 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| q4_s2 | 0.6486 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| q4_s3 | 1.0333 | 0.0000 | 0.0000 | 27.0921 | 2.3585 | 0.2542 | 0.2442 |
| q5_s1 | 0.7147 | 3.1426 | 0.0000 | 0.0000 | 3.5106 | 0.0000 | 0.0000 |
| q5_s2 | 0.6916 | 1.9755 | 0.2927 | 0.0000 | 1.0546 | 0.0000 | 0.0000 |
| q5_s3 | 0.9408 | 1.9524 | 0.2207 | 0.0000 | 1.8815 | 0.0461 | 0.0440 |
| 12.GALICIA | $\eta_{y_{1} y_{h}}$ | $\eta_{y_{2} y_{h}}$ | $\eta_{E_{1} y_{h}}$ | $\eta_{E_{2} y_{h}}$ | $\eta_{A_{1} y_{h}}$ | $\eta_{C c y_{h}}$ | $\eta_{C r y_{h}}$ |
| q1_s1 | 1.8915 | 0.3832 | 1.4723 | 2.4911 | 25.7811 | 1.9746 | 1.9447 |
| q1_s2 | 2.2954 | 0.5274 | 2.4546 | 4.9655 | 3.4036 | 2.8354 | 2.7887 |
| q1_s3 | 2.1737 | 0.3427 | 2.7202 | 3.0592 | 3.0711 | 2.8585 | 2.8228 |
| q2_s1 | 1.4955 | 2.2678 | 0.8776 | 84.6358 | 78.9203 | 0.3947 | 0.4150 |
| q2_s2 | 1.4210 | -1.4378 | 0.9010 | 63.5349 | 5.5315 | 1.2477 | 1.3085 |
| q2_s3 | 1.1512 | 3.3239 | 0.0000 | 93.0862 | 6.6263 | 1.5838 | 1.5570 |
| q3_s1 | 1.0331 | 0.0000 | 0.0000 | 20.4439 | 27.7931 | 0.2989 | 0.2753 |
| q3_s2 | 0.8813 | 0.0000 | 0.0000 | 26.5955 | 4.6097 | 0.5034 | 0.5223 |
| q3_s3 | 1.0031 | 2.0815 | 1.1116 | 29.3046 | 2.2865 | 0.6458 | 0.6504 |
| q4_s1 | 1.2405 | 4.7003 | 1.3134 | 0.0000 | 14.8608 | 0.0000 | 0.0000 |
| q4_s2 | 1.0478 | 0.0000 | 0.0000 | 26.2959 | 0.0000 | 0.0000 | 0.0000 |
| q4_s3 | 0.9678 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.1471 | 0.1465 |
| q5_s1 | 0.6199 | 2.5121 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| q5_s2 | 0.7226 | 1.9098 | 0.0000 | 0.0000 | 1.0492 | 0.0000 | 0.0000 |
| q5_s3 | 0.9091 | 2.3956 | 0.0000 | 0.0000 | 1.5111 | 0.0790 | 0.0771 |

Table 22 Ancillary Elasticities (Continued)

| 13. MADRID | $\eta_{y_{1} y_{h}}$ | $\eta_{y_{2} y_{h}}$ | $\eta_{E_{1} y_{h}}$ | $\eta_{E_{2} y_{h}}$ | $\eta_{A_{1} y_{h}}$ | $\eta_{C c y_{h}}$ | $\eta_{C r y_{h}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| q1_s1 | 1.7376 | 0.0000 | 1.1176 | 5.8808 | 27.8641 | 2.0558 | 2.0289 |
| q1_s2 | 2.5448 | 0.0000 | 2.2251 | 3.8212 | 4.1030 | 2.4541 | 2.4164 |
| q1_s3 | 2.0223 | -0.4228 | 1.6699 | 4.1805 | 3.4917 | 2.4758 | 2.4464 |
| q2_s1 | 1.0451 | 14.3179 | 0.3857 | 125.6319 | 100.8233 | 0.6182 | 0.6380 |
| q2_s2 | 1.0806 | 0.0000 | 0.6029 | 71.0491 | 5.1413 | 1.6720 | 1.6147 |
| q2_s3 | 1.1965 | 12.7355 | 0.7417 | 100.3224 | 8.7103 | 1.5100 | 1.4975 |
| q3_s1 | 1.0674 | 21.7006 | 0.0000 | 57.6473 | 0.0000 | 0.0000 | 0.0000 |
| q3_s2 | 1.1147 | 0.0000 | 0.0000 | 67.5781 | 0.0000 | 0.3889 | 0.3978 |
| q3_s3 | 1.0286 | 17.0318 | 0.0000 | 71.4179 | 9.3086 | 0.6513 | 0.6314 |
| q4_s1 | 0.8175 | 0.0000 | 0.0000 | 0.0000 | 28.7706 | 0.0000 | 0.0000 |
| q4_s2 | 0.9013 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| q4_s3 | 1.0371 | 0.0000 | 0.0000 | 0.0000 | 5.8871 | 0.0000 | 0.0000 |
| q5_s1 | 1.0707 | 2.3522 | 0.5616 | -2.5481 | 0.6368 | 0.0232 | 0.0000 |
| q5_s2 | 0.9531 | 1.6480 | 0.2210 | -1.4988 | 1.1803 | -0.0217 | -0.0237 |
| q5_s3 | 1.0746 | 2.4575 | 0.0941 | -3.0772 | 1.4306 | 0.1167 | 0.1123 |
| 14. MURCIA | $\eta_{y_{1} y_{h}}$ | $\eta_{y_{2} y_{h}}$ | $\eta_{E_{1} y_{h}}$ | $\eta_{E_{2} y_{h}}$ | $\eta_{A_{1} y_{h}}$ | $\eta_{C c y_{h}}$ | $\eta_{C r y_{h}}$ |
| q1_s1 | 1.2846 | 0.2726 | 0.7871 | 5.4376 | 21.3876 | 2.1069 | 2.0783 |
| q1_s2 | 1.6568 | 0.5604 | 1.4539 | 6.4650 | 2.6695 | 3.6203 | 3.5734 |
| q1_s3 | 1.4503 | 0.2013 | 0.9833 | 4.1555 | 2.6057 | 3.2809 | 3.2422 |
| q2_s1 | 1.1156 | 5.7032 | 0.0000 | 131.3364 | 41.8318 | 1.1995 | 1.1719 |
| q2_s2 | 0.9684 | 0.0000 | 0.4542 | 103.9960 | 2.9609 | 1.9270 | 1.9741 |
| q2_s3 | 1.0671 | 5.6682 | 0.7225 | 150.6989 | 3.3626 | 1.9864 | 1.9590 |
| q3_s1 | 0.9972 | 0.0000 | 0.0000 | 72.4662 | 16.7221 | 0.0000 | 0.0000 |
| q3_s2 | 0.8422 | 5.6851 | 0.0000 | 50.1372 | 0.0000 | 0.5469 | 0.5347 |
| q3_s3 | 0.9439 | 9.0476 | 0.0000 | 53.4844 | 0.0000 | 0.6699 | 0.6719 |
| q4_s1 | 1.0394 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | -0.5200 | -0.5079 |
| q4_s2 | 0.7584 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| q4_s3 | 1.1301 | 0.0000 | 1.1018 | 0.0000 | 0.0000 | 0.2754 | 0.2631 |
| q5_s1 | 0.9410 | 2.4641 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| q5_s2 | 0.6966 | 3.1325 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| q5_s3 | 0.8923 | 2.1635 | 0.4289 | 0.0000 | 1.4163 | 0.0958 | 0.0941 |
| 15.-RIOJA | $\eta_{y_{1} y_{h}}$ | $\eta_{y_{2} y_{h}}$ | $\eta_{E_{1} y_{h}}$ | $\eta_{E_{2} y_{h}}$ | $\eta_{A_{1} y_{h}}$ | $\eta_{C c y_{h}}$ | $\eta_{C r y_{h}}$ |
| q1_s1 | 1.4331 | 0.5671 | 1.6222 | 2.5662 | 8.4924 | 2.0262 | 1.9905 |
| q1_s2 | 1.8920 | 0.8156 | 2.8636 | 5.9033 | 2.6889 | 3.5610 | 3.5082 |
| q1_s3 | 1.6264 | 0.3666 | 1.5478 | 0.0000 | 2.3944 | 3.5131 | 3.4652 |
| q2_s1 | 1.5141 | 0.0000 | 0.9115 | 45.8763 | 29.7247 | 0.4323 | 0.4426 |
| q2_s2 | 1.4904 | 0.0000 | 0.0000 | 23.9619 | 2.6452 | 1.3608 | 1.3231 |
| q2_s3 | 0.7462 | 0.0000 | 0.0000 | 0.0000 | 4.5320 | 0.9685 | 0.9226 |
| q3_s1 | 1.0765 | 0.0000 | 0.0000 | 0.0000 | 10.9141 | 0.0000 | 0.2662 |
| q3_s2 | 0.9849 | 0.0000 | 0.0000 | 33.5031 | 0.0000 | 0.4056 | 0.4270 |
| q3_s3 | 0.9967 | 5.1696 | 0.0000 | 30.6389 | 2.6807 | 0.4962 | 0.5029 |
| q4_s1 | 1.2425 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| q4_s2 | 0.9285 | 3.6108 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| q4_s3 | 0.9203 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| q5_s1 | 0.6524 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| q5_s2 | 0.7959 | 1.5261 | 0.0000 | 3.3390 | 1.2002 | 0.0000 | 0.0000 |
| q5_s3 | 0.8772 | 2.2746 | 0.0000 | 0.0000 | 1.9354 | 0.0000 | 0.0000 |

Table 23 Ancillary Elasticities (Continued)

|  | 1. ANDALUCIA | 2.ARAGÓN | 3.ASTURIAS | 4.BALEARES |
| :---: | :---: | :---: | :---: | :---: |
|  | $\eta_{A_{2} y_{h}}$ | $\eta_{A_{2} y_{h}}$ | $\eta_{A_{2} y_{h}}$ | $\eta_{A_{2} y_{h}}$ |
| q1y1_s1 | -2.3045 | -2.3926 | -1.8299 | -1.6309 |
| q1y1_s2 | -1.2318 | -1.0804 | -1.0348 | -1.3616 |
| q1y1_s3 | -1.2811 | -1.4364 | -1.1986 | -1.1720 |
| q2y1 | 27.8889 | 0.0000 | 0.0000 | 0.0000 |
| q3y1 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| q4y1 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| q5y1 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|  | 5. CANARIAS | 6.CANTABRIA | 7.CASTILLA-LEÓN | 8.CASTILLA-LAMANCHA |
|  | $\eta_{A_{2} y_{h}}$ | $\eta_{A_{2} y_{h}}$ | $\eta_{A_{2} y_{h}}$ | $\eta_{A_{2} y_{h}}$ |
| q1y1_s1 | -2.4492 | -2.9433 | -3.0592 | -3.1073 |
| q1y1_s2 | -1.5713 | -1.2506 | -1.5991 | -1.7577 |
| q1y1_s3 | -1.1071 | -1.3995 | -1.4823 | -1.4018 |
| q2y1 | 0.0000 | 0.0000 | 11.7329 | 0.0000 |
| q3y1 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| q4y1 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| q5y1 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|  | 9.CATALUÑA | 10. C. VALENCIANA | 11.EXTREMADURA | 12.GALICIA |
|  | $\eta_{A_{2} y_{h}}$ | $\eta_{A_{2} y_{h}}$ | $\eta_{A_{2} y_{h}}$ | $\eta_{A_{2} y_{h}}$ |
| q1y1_s1 | -1.7060 | -1.5217 | 0.0000 | -2.9982 |
| q1y1_s2 | -1.2424 | -1.3263 | -1.8460 | -2.2999 |
| q1y1_s3 | -1.4376 | -1.1180 | -1.7441 | -2.0975 |
| q2y1 | 19.5174 | 0.0000 | 0.0000 | 0.0000 |
| q3y1 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| q4y1 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| q5y1 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|  | 13.MADRID | 14.MURCIA | 15.RIOJA |  |
|  | $\eta_{A_{2} y_{h}}$ | $\eta_{A_{2} y_{h}}$ | $\eta_{A_{2} y_{h}}$ |  |
| q1y1_s1 | -0.8009 | -1.8647 | -3.0001 |  |
| q1y1_s2 | -0.7080 | -1.4009 | -1.2561 |  |
| q1y1_s3 | -0.6662 | -1.3354 | -2.0290 |  |
| q2y1 | 0.0000 | 0.0000 | 0.0000 |  |
| q3y1 | 0.0000 | 0.0000 | 0.0000 |  |
| q4y1 | 0.0000 | 0.0000 | 0.0000 |  |
| q5y1 | 0.0000 | 0.0000 | 0.0000 |  |

Table 24 Relevance of Allowance Transfer from Income Source 1 to Source 2: By Quintiles of Income Source 1

| 1.ANDALUCIA | \% Tax Unit with $A 2>0$ | \% over <br> total A2 | 2.ARAGÓN | \% Tax Unit with $A 2>0$ | \% over <br> total |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  | A2 |
| q1_y1 | 0.101 | 0.98 | q1_y1 | 0.096 | 0.97 |
| q2_y1 | 0.004 | 0.02 | q2_y1 | 0.006 | 0.03 |
| q3_y1 | 0.000 | 0.00 | q3_y1 | 0.001 | 0.00 |
| q4_y1 | 0.000 | 0.00 | q4_y1 | 0.000 | 0.00 |
| q5_y1 | 0.000 | 0.00 | q5_y1 | 0.000 | 0.00 |
| 3.ASTURIAS | \% Tax Unit with $A 2>0$ | \% over <br> total A2 | 4.BALEARES | \% Tax Unit with $A 2>0$ | \% over <br> total |
|  |  |  |  |  | A2 |
| q1_y1 | 0.104 | 0.94 | q1_y1 | 0.093 | 0.97 |
| q2_y1 | 0.010 | 0.05 | q2_y1 | 0.003 | 0.02 |
| q3_y1 | 0.002 | 0.01 | q3_y1 | 0.000 | 0.00 |
| q4_y1 | 0.000 | 0.00 | q4_y1 | 0.000 | 0.00 |
| q5_y1 | 0.000 | 0.00 | q5_y1 | 0.000 | 0.00 |
| 5.CANARIAS | \% Tax Unit with $\mathrm{A} 2>0$ | \% over total A2 | 6.CANTABRIA | \% Tax Unit with $\mathrm{A} 2>0$ | \% over <br> total |
|  |  |  |  |  | A2 |
| q1_y1 | 0.063 | 0.98 | q1_y1 | 0.088 | 0.97 |
| q2_y1 | 0.001 | 0.02 | q2_y1 | 0.007 | 0.02 |
| q3_y1 | 0.000 | 0.00 | q3_y1 | 0.000 | 0.00 |
| q4_y1 | 0.000 | 0.00 | q4_y1 | 0.000 | 0.00 |
| q5_y1 | 0.000 | 0.00 | q5_y1 | 0.000 | 0.00 |
| 7.CASTILLA- | \% Tax Unit | \% over | 8.CASTILLA- | \% Tax Unit | \% over |
| LEÓN | with $\mathrm{A} 2>0$ | total A2 | LA MANCHA | with $\mathrm{A} 2>0$ |  |
|  |  |  |  |  | A2 |
| q1_y1 | 0.094 | 0.97 | q1_y1 | 0.112 | 0.98 |
| q2_y1 | 0.006 | 0.03 | q2_y1 | 0.004 | 0.02 |
| q3_y1 | 0.001 | 0.00 | q3_y1 | 0.000 | 0.00 |
| q4_y1 | 0.000 | 0.00 | q4_y1 | 0.000 | 0.00 |
| q5_y1 | 0.000 | 0.00 | q5_y1 | 0.000 | 0.00 |

Table 25 Relevance of Allowance Transfer from Income Source 1 to Source 2: By Quintiles of Income Source 1 (Continued).

| 9.CATALUÑA | \% Tax Unit with A2>0 | \% over total A2 | 10.VALENCIA | \% Tax Unit with $\mathrm{A} 2>0$ | \% over total |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  | A2 |
| q1_y1 | 0.090 | 0.97 | q1_y1 | 0.081 | 0.97 |
| q2_y1 | 0.005 | 0.02 | q2_y1 | 0.005 | 0.03 |
| q3_yl | 0.001 | 0.00 | q3_yl | 0.000 | 0.00 |
| q4_yl | 0.000 | 0.00 | q4_y1 | 0.000 | 0.00 |
| q5_y1 | 0.000 | 0.00 | q5_yl | 0.000 | 0.00 |
| 11.EXTREMADURA | \% Tax Unit with A2>0 | \% over <br> total A2 | 12.GALICIA | \% Tax Unit with A2>0 | \% over <br> total |
|  |  |  |  |  | A2 |
| q1_y1 | 0.113 | 0.98 | q1_y1 | 0.086 | 0.97 |
| q2_y1 | 0.003 | 0.02 | q2_y1 | 0.005 | 0.02 |
| q3_yl | 0.000 | 0.00 | q3_yl | 0.001 | 0.00 |
| q4_y1 | 0.000 | 0.00 | q4_yl | 0.000 | 0.00 |
| q5_yl | 0.000 | 0.00 | q5_yl | 0.000 | 0.00 |
| 13.MADRID | \% Tax Unit | \% over | 14.MURCIA | \% Tax Unit | \% over |
|  | with A2>0 | total A2 |  | with A2>0 | total |
|  |  |  |  |  | A2 |
| q1_yl | 0.090 | 0.97 | q1_y1 | 0.107 | 0.95 |
| q2_y1 | 0.004 | 0.02 | q2_yl | 0.007 | 0.05 |
| q3_y1 | 0.000 | 0.00 | q3_y1 | 0.000 | 0.00 |
| q4_y1 | 0.000 | 0.00 | q4_yl | 0.000 | 0.00 |
| q5_y1 | 0.000 | 0.01 | q5_y1 | 0.000 | 0.00 |
| 15.RIOJA | \% Tax Unit | \% over |  |  |  |
|  | with A2>0 | total A2 |  |  |  |
| q1_yl | 0.116 | 0.98 |  |  |  |
| q2_y1 | 0.005 | 0.02 |  |  |  |
| q3_y1 | 0.000 | 0.00 |  |  |  |
| q4_y1 | 0.000 | 0.00 |  |  |  |
| q5_y1 | 0.000 | 0.00 |  |  |  |

Table 26 Basic Statistics for Key Tax Variables for Whole Country and for each Autonomous Community

| WHOLE COUNTRY |  |  |  | 1.ANDALUCIA |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Mean | Std_Dev | Max |  | Mean | Std_Dev | Max |
| $t_{k 1 h}$ | 0.2938 | 0.0771 | 0.43 | $t_{\text {k1h }}$ | 0.2818 | 0.0712 | 0.43 |
| $t_{k 2 h}$ | 0.1527 | 0.0646 | 0.18 | $t_{k 2 h}$ | 0.1424 | 0.0731 | 0.18 |
| $t_{k h \text { (uvighted) }}$ | 0.2871 | 0.0712 | 0.43 | $t_{k h \text { (weigbted) }}$ | 0.2775 | 0.0656 | 0.43 |
| ATR ${ }_{\text {h }}$ | 0.1235 | 0.0923 | 0.43 | $A^{\prime} R_{h}$ | 0.1040 | 0.0878 | 0.43 |
| ATR_TI ${ }_{h}{ }^{*}$ | 0.1503 | 0.1012 | 0.43 | ATR_TI ${ }_{h}{ }^{*}$ | 0.1277 | 0.0979 | 0.43 |
| $y_{h 1} / y_{h}$ | 0.9259 | 0.1902 | 1 | $y_{h 1} / y_{h}$ | 0.9374 | 0.1841 | 1 |
| $y_{h 2} / y_{h}$ | 0.0716 | 0.1846 | 1 | $y_{h 2} / y_{h}$ | 0.0587 | 0.1744 | 1 |
| $E_{h 1} / y_{h}$ | 0.2588 | 0.1665 | 1 | $E_{h 1} / y_{h}$ | 0.2720 | 0.1776 | 1 |
| $E_{h 2} / y_{h}$ | 0.0038 | 0.0262 | 1 | $E_{h 2} / y_{h}$ | 0.0021 | 0.0221 | 1 |
| $A_{h 1} / y_{h}$ | 0.0604 | 0.1084 | 1 | $A_{h 1} / y_{h}$ | 0.0783 | 0.1228 | 1 |
| $A_{h 2} / y_{h}$ | 0.0087 | 0.0858 | 1 | $A_{h 2} / y_{h}$ | 0.0095 | 0.0898 | 1 |
| $C_{C h} / y_{h}$ | 0.0615 | 0.0302 | 0.2612 | $C_{C h} / y_{h}$ | 0.0640 | 0.0303 | 0.1566 |
| $\begin{aligned} & C_{R h} / y_{h} \\ & \text { 2.ARAGÓN } \end{aligned}$ | 0.0325 | 0.0161 | 0.1293 | $\begin{aligned} & C_{R h} / y_{h} \\ & \text { 3.ASTURIAS } \end{aligned}$ | 0.0339 | 0.0162 | 0.0834 |
|  | Mean | Std_Dev | Max |  | Mean | Std_Dev | Max |
| $t_{\text {k1h }}$ | 0.2862 | 0.0721 | 0.43 | $t_{\text {k1h }}$ | 0.2853 | 0.0712 | 0.43 |
| $t_{k 2 h}$ | 0.1658 | 0.0485 | 0.18 | $t_{k 2 h}$ | 0.1544 | 0.0629 | 0.18 |
| $t_{k h \text { (veighted) }}$ | 0.2788 | 0.0657 | 0.43 | $t_{k h(\text { weighted) }}$ | 0.2803 | 0.0650 | 0.43 |
| $A^{\prime} R_{h}$ | 0.1161 | 0.0820 | 0.42 | $A^{\prime} R_{h}$ | 0.1136 | 0.0835 | 0.42 |
| $A T R_{-} T I_{h}{ }^{*}$ | 0.1443 | 0.0922 | 0.42 | $A T R_{-} T I_{h}{ }^{*}$ | 0.1427 | 0.0935 | 0.42 |
| $y_{h 1} / y_{h}$ | 0.9048 | 0.2055 | 1 | $y_{h 1} / y_{h}$ | 0.9236 | 0.1982 | 1 |
| $y_{h_{2}} / y_{h}$ | 0.0941 | 0.2034 | 1 | $y_{h 2} / y_{h}$ | 0.0736 | 0.1921 | 1 |
| $E_{h 1} / y_{h}$ | 0.2519 | 0.1627 | 1 | $E_{h 1} / y_{h}$ | 0.2668 | 0.1812 | 1 |
| $E_{h 2} / y_{h}$ | 0.0066 | 0.0323 | 1 | $E_{h 2} / y_{h}$ | 0.0041 | 0.0252 | 1 |
| $A_{h 1} / y_{h}$ | 0.0540 | 0.0992 | 1 | $A_{h 1} / y_{h}$ | 0.0593 | 0.1044 | 1 |
| $A_{h 2} / y_{h}$ | 0.0114 | 0.0980 | 1 | $A_{h 2} / y_{h}$ | 0.0115 | 0.0988 | 1 |
| $C_{C h} / y_{h}$ | 0.0613 | 0.0300 | 0.1566 | $C_{C h} / y_{h}$ | 0.0598 | 0.0311 | 0.1566 |
| $C_{R h} / y_{h}$ | 0.0324 | 0.0160 | 0.0834 | $C_{R h} / y_{h}$ | 0.0318 | 0.0167 | 0.0834 |

* $A T R_{\_} T I_{h}$ stands for the ratio of total tax due to total taxable income ( $T I$ ) i.e. $A T R_{-} T I_{h}=T I_{h} / y_{h}$

Table 27
Basic Statistics for Key Tax Variables for Whole Country and for each Autonomous Community (Continued)


Table 28 Basic Statistics for Key Tax Variables for Whole Country and for each Autonomous Community (Continued)


Table 29 Basic Statistics for Key Tax Variables for Whole Country and for each Autonomous Community (Continued)


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[^0]:    ${ }^{1}$ Department of Economics, The University of Melbourne and Universidad Complutense de Madrid, respectively. We are grateful for support from the Fundacion de las Cajas de Ahorros (FUNCAS).

[^1]:    ${ }^{2}$ The revenue elasticity is closely linked to one of the measures of progressivity proposed by Musgrave and Thin (1948), and the link with progressivity is examined further in Podder (1997). On a possible relationship between the elasticity and government expenditure, see Craig and Heins (1980) and Misiolek and Elder (1988).
    ${ }^{3}$ On automatic stabilisation aspects of revenue elasticity, see Pohjola (1985), van den Noord (2000) and Mabbett (2004). A reduced importance was predicted to follow the 'death of inflation', by Heinemann (2001).
    ${ }^{4}$ On the recent reforms, see OECD (2006).

[^2]:    ${ }^{5}$ Alternative methods include times series regressions and simulation. An early study of the US is Greytak and Thursby (1979). Important contributions were made in a series of papers by Hutton (1980) and Hutton and Lambert (1980, 1982a, 1982b, 1983, 1989). See also Caminada and Goudswaard (1996). For a survey of analytical properties, see Creedy and Gemmell $(2002,2006)$.

[^3]:    ${ }^{6}$ This creates a tax asymmetry similar to that associated with corporation taxation, where its role is much more significant.
    ${ }^{7}$ From 2007, there is an exception in that Madrid has a slightly different tax structure from that of the other regions. This minor difference is neglected here.

[^4]:    ${ }^{8}$ It is common not to allow for such transitions when using analytical expressions. However, when using a simulation approach which actually computes discrete income and tax changes, considerable care is needed because very large individual values, for a very small number of units, can distort the aggregate results.

[^5]:    ${ }^{9}$ The treatment of the relationship between allowances and income from each source is slightly simplified here and in the following subsection. However, as explained in Section 4, the full details are modelled when obtaining empirical values.

[^6]:    ${ }^{10}$ They found a large variation between 1994 and 1998 of between 1.8 and 2.9, but judged the 'underlying' value to be 1.4. For medium term revenue forecasting, they proposed values in the range 1 to 1.3.

[^7]:    ${ }^{11}$ Random variations in proportional income changes, in addition to the systematic regression, can - if sufficiently large - lead to an increase in overall inequality; see Creedy (1985).

[^8]:    ${ }^{12}$ For further discussion, see Creedy and Gemmell (2006).

[^9]:    ${ }^{13}$ Separate values of $G$ for Central and Regional tax schedules are shown in Appendix C.

[^10]:    ${ }^{14}$ As a useful check on the programming of the calculations, aggregate revenue was obtained both using the formulae and by simply adding all the individual tax unit amounts, giving exactly the same results.

[^11]:    ${ }^{15}$ In producing these results it has been assumed that average tax credits remain unchanged

[^12]:    ${ }^{16}$ However, measures of progressivity based on the Gini measure, such as Kakwani's measure of disproportionality, could not be produced because the Gini is not defined for negative values

