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ESTIMATING STATE-CONTINGENT PRODUCTION FRONTIERS

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ESTIMATING STATE-CONTINGENT PRODUCTION FRONTIERS[1]

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Abstract

Chambers and Quiggin (2000) advocate the use of state-contingent production technologies to represent risky production and establish important theoretical results concerning producer behaviour under uncertainty. Unfortunately, perceived problems in the estimation of state-contingent models have limited the usefulness of the approach in policy formulation. We show that fixed and random effects state-contingent production frontiers can be conveniently estimated in a finite mixtures framework. An empirical example is provided. Compared to standard estimation approaches, we find that estimating production frontiers in a state-contingent framework produces significantly different estimates of elasticities, firm technical efficiencies and other quantities of economic interest.

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1. INTRODUCTION

One of the defining features of agricultural industries is the presence of *production risk*. Production risk refers to the unpredictable and uncontrollable characteristics of the physical environment (eg. lack of rainfall, pest infestations, natural disasters) that typically give rise to output shortfalls. A model that allows for this type of uncertainty is

$$(1) \qquad \ln Y = f(X_1, ..., X_K, \varepsilon) - u$$

where Y is output, ε is a random variable representing production risk, $X_1, ..., X_K$ are inputs that must be chosen before ε is revealed, and u is a non-negative variable representing technical inefficiency. By technical inefficiency we mean the inability of the firm to manage a chosen bundle of inputs to maximize output. Common causes of technical inefficiency include failure to perform production operations at exactly the right time (eg. planting or application of herbicides) and the sub-optimal assignment of personnel to specialised tasks. Special cases of model (1) can be found in both the uncertainty literature (eg Just and Pope, 1978; Newbery and Stiglitz, 1981) and the efficiency literature (eg Pitt and Lee, 1981; Kumbhakar, 1990; Battese and Coelli, 1992).

In a series of recent contributions, Chambers and Quiggin (1996, 1997, 2000, 2002, 2004) have investigated the ability of this model to explain important aspects of producer behaviour under uncertainty. For the purposes of their analyses they found it convenient to treat ε as a discrete random variable that assumes values in the set $\Omega = \{1, 2, ..., J\}$. The elements of this set correspond to different *states of nature* – combinations of rainfall, temperature, humidity and other factors that produce environments ranging from "very poor seasonal conditions" (state 1) to "excellent seasonal conditions" (state J). Chambers and Quiggin (2000) show that for each state of nature there exists a so-called *state-contingent production function* that , in the context of the production frontier in equation (1), can be written as

$$(2) \qquad \ln Y = f_j(X_1, ..., X_K) - u$$

This function specifies the output level realised when state $\varepsilon = j$ occurs. They have used this type of representation of the production technology to establish important theoretical results concerning input and output choices in the presence of risk. Unfortunately, perceived difficulties in estimating state-contingent frontiers have limited the usefulness of the approach in applied economic analysis and policy formulation.

Estimation of state-contingent frontiers is complicated by the fact that states of nature are usually unobserved. Although it is possible to collect data on rainfall, temperature, humidity and other determinants of states of nature, this data is usually highly aggregated across space and time and cannot be used to reliably discriminate between different states of nature at the farm level. In this paper we overcome the problem by treating ε as a latent variable and estimating the model using Bayesian methods.

The plan of the paper is as follows. In Section 2 we make the common simplifying assumption that $f_j(.)$ can be approximated by a function that is linear in the parameters. We then write a simple panel data version of (2) as a finite mixture model. In Section 3 we consider Bayesian estimation of this model under the assumption that the inefficiency term u is a fixed parameter. In Section 4 we reconsider the estimation problem under the alternative assumption that u is a random variable. In Section 5 we consider a more general state-contingent model that permits certain functions of the parameters to vary in economically-plausible ways. An empirical illustration is presented in Section 6 where we use Philippines rice data to estimate several state-contingent frontiers. Compared to more traditional methods, we find that estimating production technologies in a state-contingent framework produces significantly different estimates of expected output elasticities and measures of technical efficiency. The paper is concluded in Section 7 where we comment on the way state-contingent frontiers can be used to disentangle the effects of inefficiency and risk.

2. A FINITE MIXTURE REPRESENTATION

Let Y_{it} denote realised output and X_{kit} the amount of the k-th input used by firm i in period t ($k = 1, ..., K$; $i = 1, ..., N$; $t = 1, ..., T$). If $f_j(.)$ can be approximated by a function that is linear in the parameters we can specify a relationship between observed outputs and inputs of the form:

$$\text{(3)} \qquad \ln Y_{it} = \phi_j + \mathbf{x}_{it}'\alpha + v_{itj} - u_i \qquad \text{if } \varepsilon_{it} = j$$

where ϕ_j is a state-varying intercept parameter, $\mathbf{x}_{it}$ is an $M \times 1$ vector of (transformations of) inputs, α is a state-invariant $M \times 1$ vector of slope parameters, and $v_{itj} \sim N(0, h_j^{-1})$ is a normal random variable representing statistical noise (ie. the combined effects of measurement errors and errors arising from the use of approximating functional forms). The precision (inverse variance) of this random error h_j is assumed to be state dependent, and hence carries the subscript j. By using the subscript j on the intercept ϕ_j we are allowing expected log-output to vary across states of nature. However, by not using this subscript on the slope coefficients we are, among other things, keeping the elasticities of expected output with respect to inputs constant across states – this is a convenient but implausible assumption that will be relaxed later in the paper. By using the subscripts i and t on ε_{it} we are allowing states of nature to vary across both firms and time, thus allowing for localised weather conditions (eg. hailstorms) and contained outbreaks of disease (ie. outbreaks that may cause farms to be placed in quarantine). Finally, by using the single subscript i on u_i we are assuming the inefficiency effects are time-invariant – if this assumption is found to be too restrictive then generalising the model to account for time-varying inefficiency effects is straightforward using, for example, the framework developed by Kumbhakar (1990) and Battese and Coelli (1992, 1995).

After the introduction of probabilities for the realization of each state, $\pi_j = \Pr(\varepsilon_{it} = j)$, equation (3) becomes an example of a finite mixture model. Such models usually arise whenever a variable is observed under a finite number of different conditions (eg. the distribution of the height of adults reflects the mixture of males and females in the population; the distribution of agricultural output reflects the mixture of relatively poor, average and good seasons across firms and time). For an in-depth treatment of mixture models see McLachlan and Peel (2000). For a more concise Bayesian treatment see Koop (2003).

When estimating mixture models it is convenient to introduce dummy variables that indicate the mixture component from which each observation is drawn. In this paper we define the vector of dummy variables $\mathbf{d}_{it} = (d_{it1}, ..., d_{itJ})'$ where $d_{itj} = 1$ if $\varepsilon_{it} = j$ and $d_{itj} = 0$ otherwise. Then (3) can be written as

$$(4) \qquad \ln Y_{it} = \mathbf{d}_{it}'\phi + \mathbf{x}_{it}'\alpha + v_{it} - u_i$$

where $\phi = (\phi_1, ..., \phi_J)'$. The disturbance term is now $v_{it} \sim N(0, (\mathbf{d}_{it}'\mathbf{h})^{-1})$ where $\mathbf{h} = (h_1, ..., h_J)'$ is the vector of state-varying precision parameters.

Conditional on $\mathbf{u} = (u_1, ..., u_N)'$, the likelihood function for this model can be written

$$(5) \qquad p(\mathbf{y} \mid \alpha, \mathbf{h}, \phi, \mathbf{u}, \pi) = (2\pi)^{-NT/2} \prod_{i=1}^{N} \prod_{t=1}^{T} \left\{ \sum_{j=1}^{J} \pi_j \sqrt{h_j} \exp\left[-0.5 h_j (\ln Y_{it} - \phi_j - \mathbf{x}_{it}'\alpha + u_i)^2\right] \right\}$$

where $\pi = (\pi_1, ..., \pi_J)'$ and $\mathbf{y}$ is an NT-dimensional vector with elements $\ln Y_{it}$. There are two problems with this likelihood function that make estimation difficult. First, it is unbounded, implying the standard theory underpinning maximum likelihood estimation breaks down (see Koop, 2003, p.255). Second, more than one set of parameter values will yield the same likelihood, implying the parameters are unidentified. A solution to the first problem, and the one we adopt in this paper, is to estimate the model in a Bayesian framework using an informative prior. One of several solutions to the second problem is to impose identifying restrictions of the form

$$(6) \qquad E[\ln Y_{it} \mid \varepsilon_{it} = 1] \leq E[\ln Y_{it} \mid \varepsilon_{it} = 2] \leq \ ... \ \leq E[\ln Y_{it} \mid \varepsilon_{it} = J]$$

or, equivalently,

$$(7) \qquad \phi_1 \leq \phi_2 \leq \ ... \ \leq \phi_J.$$

These restrictions are known in the mixtures literature as *labelling restrictions*. In the current context they ensure that expected log-output increases as seasonal conditions improve. It is also possible to identify the parameters of the model using labelling restrictions on expected output (rather than log-output), the state probabilities or the state-dependent variances of the noise components. However, the rationale for imposing such restrictions may not be as appealing as the rationale underpinning (6) and/or, when expressed in terms of the parameters, they may not be as simple as the inequality constraints (7).

In the following sections we consider Bayesian estimation of the model under the assumptions that the inefficiency effects are either fixed or random. Following the work of Schmidt and Sickles (1984), these two competing assumptions have become commonplace in the efficiency literature.

3. FIXED EFFECTS

For the case where the u_i are treated as fixed effects it is convenient to parameterise the model in terms of $\delta_j = \phi_j - u_1$ and $\psi_i = u_1 - u_i$. Defining the dummy variable $m_{itk} = 1$ if $i = k$ and $m_{itk} = 0$ otherwise, equation (4) can then be written:

$$(8) \qquad \ln Y_{it} = \mathbf{d}_{it}'\delta + \mathbf{m}_{it}'\psi + \mathbf{x}_{it}'\alpha + v_{it}$$

where $\delta = (\delta_1, ..., \delta_J)'$, $\mathbf{m}_{it} = (m_{it2}, ..., m_{itN})'$ and $\psi = (\psi_2, ..., \psi_N)'$. Including only $N - 1$ individual dummy variables in (8) allows us to avoid a dummy variable trap (the N individual dummy variables and J state dummy variables are perfectly collinear).

For Bayesian analysis we define $\beta = (\delta', \psi', \alpha')'$ and adopt the independent but proper prior $p(\beta, \mathbf{h}, \pi) = p(\beta) \times p(\mathbf{h}) \times p(\pi)$ where

$$(9) \qquad p(\beta) \propto f_N(\beta \mid \underline{\beta}, \underline{\mathbf{V}}) \times I(\delta_1 \leq \delta_2 \leq \ ... \ \leq \delta_J)$$

$$(10) \qquad p(\mathbf{h}) = \prod_{j=1}^{J} f_G(h_j \mid \underline{s}_j^{-2}, \underline{\upsilon}_j)$$

and

$$(11) \qquad p(\pi) = f_D(\pi \mid \underline{\pi}).$$

The notation $f_N(.)$, $f_G(.)$ and $f_D(.)$ for the normal, gamma and dirichlet probability density functions (pdfs) is adopted from Koop (2003), and $I(.)$ is an indicator function that takes the value one if the argument is true and zero otherwise. The inequality constraints in (9) are simply a reparameterization of the inequality constraints in (7). We underscore some of the parameters in equations (9) to (11) to indicate they are parameters of the prior distribution to be chosen by the researcher – we will discuss the elicitation of prior parameters in the context of our empirical example in Section 6.

Bayes's Theorem is used to combine the priors (9) to (11) with the likelihood function (5). Because we use Gibbs sampling to estimate posterior quantities of interest, the mathematical form of the resulting posterior pdf is of less interest than the conditional posterior pdfs that can be derived from it. If we define $\mathbf{d} = (\mathbf{d}_{11}', ..., \mathbf{d}_{NT}')'$ we can write these conditional posteriors as:

$$(12)\quad p(\beta \mid \mathbf{y}, \mathbf{h}, \pi, \mathbf{d}) \propto f_N(\beta \mid \overline{\beta}, \overline{\mathbf{V}}) \times I(\delta_1 \leq \delta_2 \leq \ldots \leq \delta_J)$$

$$(13)\quad p(\mathbf{h} \mid \mathbf{y}, \beta, \pi, \mathbf{d}) = \prod_{j=1}^{J} f_G(h_j \mid \overline{s}_j^{-2}, \overline{\upsilon}_j)$$

$$(14)\quad p(\pi \mid \mathbf{y}, \beta, \mathbf{h}, \mathbf{d}) = f_D(\pi \mid \overline{\pi})$$

$$(15)\quad p(\mathbf{d} \mid \mathbf{y}, \beta, \mathbf{h}, \pi) = \prod_{i=1}^{N} \prod_{t=1}^{T} f_M(\mathbf{d}_{it} \mid 1, \overline{\mathbf{d}}_{it})$$

where $f_M(.)$ is the notation used by Koop (2003) for the multinomial pdf. We use overbars on some of the parameters to indicate they are parameters of posterior pdfs. Expressions for these parameters are provided in Appendix A.

The conditional posterior pdfs given by (12) to (15) can be used within a Gibbs sampler to obtain samples of observations on the unknown parameters. The Gibbs sampler is now routinely used for Bayesian analysis in problems involving latent variables – for details see Koop (2003). Simulating from the gamma, dirichlet and multinomial pdfs is reasonably straightforward using most software packages. Efficient sampling from the truncated normal pdf in (12) can be done using a mixture of normal rejection sampling and exponential rejection sampling (see Geweke, 1991). These samples can then be used to draw inferences concerning any quantities of interest. For example, observations on ψ can be used to draw inferences concerning the measure of relative technical efficiency

$$(16)\quad RTE_i = \exp(\psi_i - \max_k(\psi_k)).$$

4. RANDOM EFFECTS

As an alternative to the fixed effects model we now assume u_i in equation (4) is an exponential random variable (gamma with 2 degrees of freedom) with pdf

$$(17)\quad p(u_i \mid \lambda^{-1}) = f_G(u_i \mid \lambda, 2) = \lambda^{-1}\exp(-\lambda^{-1}u_i).$$

This assumption is a common one in the literature – see for example Koop and Steel (2001) – although other distributions such as the half normal or truncated normal have been used. For Bayesian analysis we redefine $\beta = (\phi', \alpha')'$ and adopt the prior $p(\beta, \mathbf{h}, \mathbf{u}, \pi, \lambda^{-1}) = p(\beta) \times p(\mathbf{h})$

$\times\, p(\pi) \times p(\mathbf{u}\,|\lambda^{-1}) \times p(\lambda^{-1})$ where $p(\beta)$, $p(\mathbf{h})$ and $p(\pi)$ are given by (9) to (11). In the case of $p(\beta)$, the dimensions of the prior parameters are suitably redefined and the inequality restrictions are expressed in terms of the ϕ_j. The other prior pdfs are given by

(18) $$p(\mathbf{u}\,|\,\lambda^{-1}) = \prod_{i=1}^{N} f_G(u_i\,|\,\lambda, 2)$$

and

(19) $$p(\lambda^{-1}) = f_G(\lambda^{-1}\,|\,-1/(\ln \tau^*), 2) \propto \exp\{(\ln \tau^*)/\lambda\}.$$

The hyperparameter τ^* in (19) is the researcher's prior estimate of median technical efficiency (Koop, Steel and Osiewalski 1995). Again, the posterior pdf is of less interest than the following conditional posterior pdfs:

(20) $$p(\beta\,|\,\mathbf{y}, \mathbf{h}, \pi, \mathbf{d}, \mathbf{u}, \lambda) \propto f_N(\beta\,|\,\overline{\beta}, \overline{\mathbf{V}}) \times I(\phi_1 \le \phi_2 \le \ldots \le \phi_J)$$

(21) $$p(\mathbf{h}\,|\,\mathbf{y}, \beta, \pi, \mathbf{d}, \mathbf{u}, \lambda) = \prod_{j=1}^{J} f_G(h_j\,|\,\overline{s}_j^{-2}, \overline{\upsilon}_j)$$

(22) $$p(\pi\,|\,\mathbf{y}, \beta, \mathbf{h}, \mathbf{d}, \mathbf{u}, \lambda) = f_D(\pi\,|\,\overline{\pi})$$

(23) $$p(\mathbf{d}\,|\,\mathbf{y}, \beta, \mathbf{h}, \pi, \mathbf{u}, \lambda) = \prod_{i=1}^{N}\prod_{t=1}^{T} f_M(\mathbf{d}_{it}\,|\,1, \overline{\mathbf{d}}_{it})$$

(24) $$p(\lambda^{-1}\,|\,\mathbf{y}, \beta, \mathbf{h}, \pi, \mathbf{d}, \mathbf{u}) = f_G(\lambda^{-1}\,|\,(N+1)/(\mathbf{u}'\mathbf{j}_N - \ln \tau^*), 2(N+1))$$

(25) $$p(u_i\,|\,\mathbf{y}, \beta, \mathbf{h}, \pi, \mathbf{d}, \lambda) \propto f_N(u_i\,|\,\mu_{ui}, \sigma_{ui}^2) \times I(u_i \ge 0)$$

where $\mathbf{j}_N$ is a vector of ones of length N. Expressions for the parameters of these conditional posterior pdfs are provided in Appendix B. Again, they can be used within a Gibbs sampler to obtain samples of observations on all unknown parameters, including the elements of $\mathbf{u}$. These elements are of particular interest because they can be used to calculate the measure of technical efficiency

(26) $$TE_i = \exp(-u_i).$$

5. STATE-VARYING SLOPE COEFFICIENTS

The state-contingent production frontier given by (3) has the desirable property that expected output is permitted to vary across states of nature. Specifically, if u_i is fixed then

$$(27) \qquad E(Y_{it} \mid \varepsilon_{it} = j) = \exp\{\delta_j + \mathbf{m}_{it}'\psi + \mathbf{x}_{it}'\alpha + 0.5h_j^{-1}\}$$

while if u_i is random

$$(28) \qquad E(Y_{it} \mid \varepsilon_{it} = j) = \left(\frac{1}{1+\lambda}\right)\exp\{\phi_j + \mathbf{x}_{it}'\alpha + 0.5h_j^{-1}\}$$

However, in both cases the elasticity of expected output in state j with respect to the k-th input,

$$(29) \qquad \eta_{jkit} = \frac{\partial \ln E(Y_{it} \mid \varepsilon_{it} = j)}{\partial \ln X_{kit}} = \frac{\partial(\mathbf{x}_{it}'\alpha)}{\partial \ln X_{kit}}$$

is state-invariant. This property may be implausible in some production contexts. For example, it rules out the possibility that marginal increases in irrigation water will increase expected crop output in a dry season but decrease expected crop output in a wet season. To allow for such possibilities, the slope coefficients in equations (3) and (4) must be permitted to vary with j. Allowing the slope coefficients to vary across states of nature also gives rise to more plausible properties concerning the variances of state-contingent outputs – inputs may become 'risk-increasing' in some states of nature and 'risk-decreasing' in others.

A model allowing both slope and intercept coefficients to vary across states of nature can be written in the form:

$$(30) \qquad \ln Y_{it} = \mathbf{d}_{it}'\phi + (\mathbf{x}_{it} \otimes \mathbf{d}_{it})'\alpha + v_{it} - u_i$$

where α is now a vector of length $MJ \times 1$. To solve the mixtures identification problem we encountered in Sections 4 and 5 it is convenient to scale the inputs so that $\mathbf{x}_{it} = \mathbf{0}$ at the variable means. Then the constraint:

$$(31) \quad E[\ln Y_{it} \mid \mathbf{x}_{it} = \mathbf{0}, \varepsilon_{it} = 1] \le E[\ln Y_{it} \mid \mathbf{x}_{it} = \mathbf{0}, \varepsilon_{it} = 2] \le \ \ldots \ \le E[\ln Y_{it} \mid \mathbf{x}_{it} = \mathbf{0}, \varepsilon_{it} = J]$$

is equivalent to the labelling restriction (7). Suitable priors for this model are straightforward generalisations of the priors discussed in Sections 4 and 5, and the conditional posterior pdfs are then generalisations of equations (12) to (15) and (20) to (25). The parameters of the conditional posterior pdfs for the random effects model are presented in Appendix C (for reasons that will become apparent in the following section, we are mainly interested in state-contingent random effects frontiers).

For input allocation under uncertainty and for assessing the optimality of a particular input bundle, the marginal expected product and marginal risk (defined as the derivative of the variance of output with respect to an input) of each input are of interest. In our case, where we have a state-contingent function with a firm-specific inefficiency term, there are four different marginal expected products and marginal risks that are potentially useful: there are those that are conditional on a specific inefficiency term u_i and a specific state j; those that are conditional on u_i, but not j; those that are conditional on j, but not u_i; and those that are conditional on neither u_i nor j. Expressions for the conditional and unconditional means and variances and their derivatives for both the state-varying intercept model and the state-varying slopes model are given in Appendix D. The values of these expressions depend on all the parameters and the levels of inputs. An example of the posterior pdf for one of them is given in the next section.

6. EMPIRICAL EXAMPLE

Villano et al (2004) investigate input-output relationships for a sample of rice farmers in the Tarlac region of the Philippines. These farmers have no access to irrigation and so output shortfalls are due to variations in both technical efficiency and seasonal conditions. In this paper we analyse a subset of the Villano et al data. Our sample comprises 352 observations on N = 44 rice farmers covering the T = 8 years from 1990/91 to 1997/98. The output variable is tonnes of freshly-threshed rice. Input variables are hectares planted (X_{1it}), person-days of hired and family labor (X_{2it}) and kilograms of fertilizer (X_{3it}). Descriptive statistics for the raw data are reported in Table 1.

We begin by assuming a translog functional form where

(32) $\mathbf{x}_{it} = [TR_{it}, \ln X_{1it}, \ln X_{2it}, \ln X_{3it}, 0.5(\ln X_{1it})^2, (\ln X_{1it})(\ln X_{2it}), (\ln X_{1it})(\ln X_{3it}), 0.5(\ln X_{2it})^2, (\ln X_{2it})(\ln X_{3it}), 0.5(\ln X_{3it})^2]'$

and TR_{it} is a time trend included to account for technical change. Prior to estimation the input variables were scaled to have unit means. Thus, when $TR_{it} = 0$ and all inputs are set to their mean values we have $\mathbf{x}_{it} = \mathbf{0}$. Among other things, this means the first-order coefficients in the model can be interpreted as elasticities of expected output evaluated at the input means, and the constraint (31) collapses to the labelling restriction (7).

For Bayesian analysis we must specify the parameters of the prior densities. It is convenient to start with the parameters of (11) and (19), namely $\underline{\pi}$ and τ^*. Since we have no prior information on the relative likelihoods of different states of nature we set $\underline{\pi} = \mathbf{j}_J$, implying each state is equally likely. Since we have no prior information concerning farm efficiency we follow Koop, Steel and Osiewalski (1995) and set the median of the prior efficiency distribution to $\tau^* = 0.875$.

To elicit the parameters of the prior density (10) we note from Table 1 that the range of $\ln Y_{it}$ is 5.85. Thus, we could expect any reasonable regression model to have errors that are less than 5.85/2 = 2.925 in absolute value. If a normally distributed random variable has precision 0.44 then 95% of values will lie in the interval ±2.94. This leads us to set $\underline{s}_j^{-2} = 0.44$. To ensure this prior information is given small weight relative to the data we set $\underline{\upsilon}_j = 0.01NT \approx 4$.

For the prior density (9) we consider $E(\ln Y_{it} \mid u_i = -\ln 0.875, \varepsilon_{it} = j, \mathbf{x}_{it} = \mathbf{0}) = \phi_j - 0.134$. This leads us to centre the prior distribution for ϕ_j at $q_j + 0.134$ where q_j denotes the $(2j-1)/2J$-th percentile of the sample observations on $\ln Y_{it}$. To ensure this prior information is given small weight relative to the data we use a prior variance of $100\underline{s}_j^2 = 225$. We use similar reasoning and our knowledge of economic theory and the Philippines rice industry to specify prior means and variances for the remaining elements of $\boldsymbol{\beta}$: the coefficient of TR_{it} measures the annual rate of growth in output and our prior pdf for this coefficient is centred at 0.02 with a variance of 0.15; the first-order slope coefficients are elasticities evaluated at the input means and their pdfs are centred at 0.5 with prior variances of 6.5; and the second-order slope coefficients must be close to zero if the translog function is to satisfy curvature properties implied by economic theory, so we centre their pdfs at zero with variances of 26. All these variances are large enough to ensure the joint prior density is very diffuse. Among other things, this means our empirical estimates are robust to large changes in the mean of the joint prior density.

In the remainder of this section we report and discuss estimates of parameters and other interesting characteristics of several stochastic frontiers. For purposes of comparison, we estimated a standard (ie. not state-contingent) fixed effects frontier (labelled FE) and a standard random effects frontier with exponentially distributed inefficiency effects (RE). The Gibbs samplers used for these models are special cases of those discussed in Sections 3 and 4 – details can be found in Koop and Steel (2001). We then specified J = 3 states corresponding to relatively "poor", "average" and "good" seasonal conditions and proceeded to estimate several state-contingent frontiers. The simplest of these are the fixed effects frontier discussed in Section 3 (SC-FE) and the random effects frontier discussed in Section 4 (SC-RE). A more general model is the state-contingent random effects frontier discussed in Section 5, where all the parameters are permitted to vary across states. We estimated this more general model twice, each time with different prior information. In the first case (SC-RE-*all*) we used the prior information already described. In the second (SC-RE-η>0), partly to illustrate the flexibility of the Bayesian approach, and partly to include genuine prior information, we estimated the frontier with additional prior information in the form of non-negativity constraints on land and labor elasticities.. We chose not to impose non-negativity constraints on the fertilizer elasticity because it is possible that higher rates of fertilizer application in dry seasons may burn the rice crop and lead to lower outputs. Estimation of the SC-RE-η>0 model required a trivial modification to the program used to estimate the SC-RE-*all* model.

The Gibbs sampling algorithms used to estimate the various models were programmed in GAUSS and used to generate stationary Markov chains of length 20,000. The results reported below are summary statistics for these chains, and include estimates of unknown parameters, state-probabilities and measures of technical efficiency.

Parameters

Parameter estimates for our six models are reported in Table 2. Estimated posterior means are reported in one block of six columns and estimated posterior standard deviations are reported in a second block.

The FE and RE parameter estimates suggest that rice output in the study region has been increasing at a rate of 1.4% to 1.7% per annum, and that the elasticity of expected output with respect to area (evaluated at the input means) is approximately 0.6. However, the SC-FE and SC-RE estimates suggest that productivity growth has been as low as 0.8% per annum and the area elasticity is less than 0.3. We conclude that estimating production technologies in the simplest of state-contingent frameworks can have a significant impact on estimates of (functions of) parameters of interest to economists.

Not surprisingly, we find that estimating more flexible state-contingent models can provide additional insights into the rate and nature of technical change. For example, the SC-RE-*all* and SC-RE-$\eta>0$ results provide evidence that technological developments have led to higher expected outputs in poor seasons and lower expected outputs in average seasons. Specifically, we estimate that poor-season expected outputs have been increasing at a rate of more than 2.5% per annum while average-season expected outputs have been decreasing at a rate of 1.3% to 1.4% per annum. These results may be partly due to the development of rice varieties that are lower-yielding but better able to tolerate extremes of temperature, humidity and rainfall.

Our SC-RE-*all* and SC-RE-$\eta>0$ results also point to relatively high elasticities in extreme seasonal conditions. For example, we estimate that a one percent increase in area planted will increase expected output by as much as 0.6% in either a poor or a good season, but will increase expected output by only 0.1% in an average season. Furthermore, the SC-RE-*all* results suggest that labor and fertilizer elasticities are negative in poor seasons. Negative elasticities are generally regarded as implausible, so the SC-RE-$\eta>0$ model constrains the labor and area elasticities to be nonnegative. We did not sign-constrain the fertilizer elasticity because we are aware that high rates of fertilizer application in very dry (ie. poor) seasons may decrease output. The SC-RE-$\eta>0$ results suggest that the fertilizer elasticity is only negative in poor seasons, and that it increases as seasonal conditions improve. The estimates of the sign-constrained labor elasticities also increase as seasonal conditions improve.

It is clear that the state-contingent models are flexible enough to produce qualitatively different estimates of technical change and output response across different states of nature – these models contain enough parameters to capture state-varying characteristics of the production technology. However, this flexibility comes at a cost – greater uncertainty associated with estimating larger numbers of parameters is reflected in higher estimated posterior standard deviations. In the most extreme case, estimated standard deviations for the SC-RE-*all* model are approximately five times larger than those for the RE model. Inequality constraints on the area and labor elasticities have the effect of lowering these estimated posterior standard deviations, highlighting one of the advantages of incorporating non-sample information into the estimation process.

Estimated posterior means and standard deviations give an incomplete picture of likely and unlikely values of the unknown parameters. A more complete picture is given by estimated marginal posterior pdfs such as those depicted in Figures 1 and 2. Both of these figures illustrate the effects of imposing inequality constraints. The effect of the labelling restriction (7) is illustrated in Figure 1 where we present SC-RE estimates of the marginal posterior pdfs of the intercept coefficients in each state of nature. The effect of imposing non-negativity constraints is illustrated in Figure 2 where we present the SC-RE-*all* and SC-RE-η>0 estimates of the marginal posterior pdf of the good-season labor elasticity (evaluated at the input means), and contrast them with the SC-RE posterior pdf of the labor elasticity.

State Probabilities

Estimates of unconditional state probabilities are reported at the bottom of Table 2. Results from the simpler state-contingent models (SC-FE and SC-RE) suggest that a randomly-selected farmer is roughly twice as likely to experience average seasonal conditions than good seasonal conditions. These estimates are reasonably precise – see Figure 3 where we present SC-RE estimates of the marginal posteriors. The more flexible state-contingent models (SC-RE-*all* and SC-RE-η>0) suggest that the probabilities of experiencing poor, average and good seasonal conditions are fairly similar, ranging from 0.31 to 0.36.

Table 3 reports estimates of state probabilities for three representative farmers in all eight time periods. The results from different models are similar, and provide evidence that different farmers may experience different seasonal conditions in the same time period. Using the SC-RE-η>0 results, for example, we see that farmer 2 almost certainly had a good season in period 4, while farmers 1 and 3 are more likely to have experienced average seasons.

Technical Efficiency

Table 4 reports estimated means and standard deviations of posterior pdfs for measures of technical efficiency. In the case of the fixed effects models we measure relative technical efficiency using (16); in the case of the random effects models we measure technical efficiency using (26).

Whether or not standard (ie. non-state-contingent) stochastic frontier models are estimated in a frequentist or Bayesian framework, efficiency estimates obtained from fixed effects models are often found to be significantly lower than estimates obtained from random effects models. This stylised fact is evident in Table 4 where the FE technical efficiency estimates are on average only half the size of the RE estimates. As noted in Koop and Steel (2001), the reason, in Bayesian terms, is that the fixed effects prior pdf for relative technical efficiency has probability mass concentrated towards zero (the use of a noninformative prior pdf for the ψ_i implies an informative prior pdf for RTE_i of the form $p(RTE_i) \propto 1/RTE_i$). In contrast, the random effects prior pdf for TE_i has probability mass concentrated closer to one.

It is apparent from Table 4 that estimating a fixed effects frontier in a state-contingent framework has very little impact on measures of relative technical efficiency. The SC-FE estimates of RTE_i are still implausibly low, suggesting the information contained in the (implausible) fixed effects prior pdf is still dominating the information contained in the data. Like many other frequentist and Bayesian researchers, our inability to obtain sensible results from a fixed effects model has caused us to focus our attention on models in which the inefficiency effects are treated as random.

In contrast with our experience with the fixed effects model, we find that estimating a random effects frontier in a simple state-contingent framework has a significant impact on measures of technical efficiency – the RE estimates reported in Table 4 average 0.86 while the SC-RE estimates average 0.93. Thus, our use of a simple state-contingent random effects model means that on average half of the output shortfall previously attributed to inefficiency can now be attributed to unfavourable states of nature, ie., risk. Estimating more flexible state-contingent models yields even higher estimates of technical efficiency – whether or not we impose sign constraints on the area and labor elasticities, the state-contingent models with state-varying slope coefficients yield technical efficiency estimates that range from 0.88 to 0.97, with an average of 0.95.

The results for individual firms provide stronger evidence of the importance of accounting for risk. In the case of farmer 34, for example, if we estimate the production technology in a non-state-contingent framework we obtain a technical efficiency estimate of only 0.55 (RE model); after accounting for risk we obtain a technical efficiency estimate of 0.91 (SC-RE-η>0 model). Thus, for this farmer the estimated output shortfall due to inefficiency (9%) is minor compared to the estimated output shortfall due to risk (34%).

The consequences of estimating random effects frontiers in different empirical frameworks are summarised in Figures 4 to 6 where we present estimated marginal posterior pdfs for the technical efficiencies of three representative firms. Firm 12 can be regarded as an above-average firm in terms of technical efficiency; the efficiency of firms 5 and 34 can be regarded as average and below-average respectively.

Other Quantities

One of the important advantages of the Bayesian estimation approach is that we can easily obtain finite sample results concerning any (possibly nonlinear) functions of the parameters. For example, using results in Appendix D, for the case of a random effects model with state-dependent slope coefficients, the marginal risk of input X_k, evaluated at the input means (where $\mathbf{x}_{it} = \mathbf{0}$), is

$$(33) \qquad \left.\frac{\partial \operatorname{var}(Y_{it})}{\partial X_{kit}}\right|_{\mathbf{x}_{it}=\mathbf{0}} = 2\sum_{j=1}^{J} \pi_j \alpha_{0kj} \kappa_j \left[(1+2\lambda)^{-1} \kappa_j \exp\{h_j^{-1}\} - (1+\lambda)^{-2} \theta_0 \right]$$

where $\kappa_j = \exp\{\phi_j + 0.5 h_j^{-1}\}$, $\theta_0 = \sum_{j=1}^{J} \pi_j \kappa_j$ and α_{0kj} is the coefficient of $\ln X_{kit}$.

For illustrative purposes, Figure 7 presents the SC-RE-η>0 estimated posterior pdf for the marginal effect given by (33), evaluated for k = 2 (labor). It is evident from this figure that there is high probability that labor is a 'risk-increasing' input.

6. CONCLUSION

For many years the standard tool for analysing relationships between agricultural inputs and outputs has been the simple production function. In the late 1970s, consideration of output shortfalls led some researchers to specify econometric models with heteroskedastic error terms representing risk (eg. Just and Pope 1978). At the same time, productivity researchers began specifying frontier models containing one-sided error terms representing technical inefficiency (eg. Pitt and Lee 1981). Only recently have economists attempted to construct econometric models that explicitly account for both inefficiency and risk (eg. Kumbhakar 2002).

One of the simplest and arguably most powerful theoretical frameworks for jointly analysing inefficiency and risk is the state-contingent framework recently popularised by Chambers and Quiggin (2000). However, there have been few if any attempts to empirically estimate state-contingent models in the economics literature, possibly because underlying 'states of nature' are unobserved and regarded as too difficult to quantify (eg. Rasmussen, 2004). In this paper we have shown how to overcome the problem by representing state-contingent models in a finite mixture framework.

We have used Bayesian methods to estimate several state-contingent production frontiers for Philippines rice farmers. Our results suggest that elasticities of expected output with respect to inputs vary significantly across states of nature. Moreover, estimating production frontiers in a state-contingent framework yields significantly higher estimates of technical efficiency. This is not surprising – standard (ie. non-state-contingent) stochastic frontier models decompose deviations from the frontier into inefficiency and noise, while state-contingent frontier models decompose these deviations into inefficiency, noise *and risk*. In the case of one farmer in our sample, we found that three-quarters of average estimated output shortfalls were due to unfavourable seasonal conditions (ie. risk) and only one quarter to inefficiency.

Our ability to decompose output shortfalls into inefficiency and risk components represents a step forward in the econometric analysis of agricultural production technologies. However, our methods also have application in areas outside agriculture. Indeed, our methods are likely to have application in every area of business and commerce – whether they are efficient or not, most firms carry some form of liability insurance, implying they operate in environments characterised by risk.

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APPENDIX A
Posterior Parameters for Fixed Effects Model

Define $\mathbf{z}_{it} = (\mathbf{d}_{it}', \mathbf{m}_{it}', \mathbf{x}_{it}')'$ and $y_{it} = \ln Y_{it}$. Then, the parameters for the conditional posterior pdfs given in equations (12) – (15) are given by

$$\overline{\mathbf{V}} = (\underline{\mathbf{V}}^{-1} + \sum_{i=1}^{N}\sum_{t=1}^{T}(\mathbf{d}_{it}'\mathbf{h})\mathbf{z}_{it}\mathbf{z}_{it}')^{-1}$$

$$\overline{\beta} = \overline{\mathbf{V}}(\underline{\mathbf{V}}^{-1}\underline{\beta} + \sum_{i=1}^{N}\sum_{t=1}^{T}(\mathbf{d}_{it}'\mathbf{h})\mathbf{z}_{it}y_{it})$$

$$\overline{\upsilon}_j = \sum_{i=1}^{N}\sum_{t=1}^{T} d_{itj} + \underline{\upsilon}_j$$

$$\overline{s}_j^2 = \frac{\sum_{i=1}^{N}\sum_{t=1}^{T} d_{itj}(y_{it} - \mathbf{z}_{it}'\beta)^2 + \underline{\upsilon}_j\,\underline{s}_j^2}{\overline{\upsilon}_j}$$

$$\overline{\pi} = \underline{\pi} + \sum_{i=1}^{N}\sum_{t=1}^{T} \mathbf{d}_{it}$$

$$\overline{\mathbf{d}}_{it} = \left[\frac{\pi_1 f_N(y_{it} \mid \delta_1 + \mathbf{m}_{it}'\boldsymbol{\psi} + \mathbf{x}_{it}'\boldsymbol{\alpha}, h_1^{-1})}{\sum_{j=1}^{J}\pi_j f_N(y_{it} \mid \delta_j + \mathbf{m}_{it}'\boldsymbol{\psi} + \mathbf{x}_{it}'\boldsymbol{\alpha}, h_j^{-1})}, \ldots, \frac{\pi_J f_N(y_{it} \mid \delta_J + \mathbf{m}_{it}'\boldsymbol{\psi} + \mathbf{x}_{it}'\boldsymbol{\alpha}, h_J^{-1})}{\sum_{j=1}^{J}\pi_j f_N(y_{it} \mid \delta_j + \mathbf{m}_{it}'\boldsymbol{\psi} + \mathbf{x}_{it}'\boldsymbol{\alpha}, h_j^{-1})}\right]'$$

APPENDIX B

Posterior Parameters for Random Effects Model with State-Invariant Slope Coefficients

Define $\mathbf{z}_{it} = (\mathbf{d}_{it}', \mathbf{x}_{it}')'$ and $y_{it} = \ln Y_{it}$. Then, the parameters for the conditional posterior pdfs in equations (20) – (25) are

$$\overline{\mathbf{V}} = (\underline{\mathbf{V}}^{-1} + \sum_{i=1}^{N}\sum_{t=1}^{T}(\mathbf{d}_{it}'\mathbf{h})\mathbf{z}_{it}\mathbf{z}_{it}')^{-1}$$

$$\overline{\beta} = \overline{\mathbf{V}}(\underline{\mathbf{V}}^{-1}\underline{\beta} + \sum_{i=1}^{N}\sum_{t=1}^{T}(\mathbf{d}_{it}'\mathbf{h})\mathbf{z}_{it}[y_{it} + u_i])$$

$$\overline{\upsilon}_j = \sum_{i=1}^{N}\sum_{t=1}^{T} d_{itj} + \underline{\upsilon}_j$$

$$\overline{s}_j^2 = \frac{\sum_{i=1}^{N}\sum_{t=1}^{T} d_{itj}(y_{it} + u_i - \mathbf{z}_{it}'\beta)^2 + \underline{\upsilon}_j \underline{s}_j^2}{\overline{\upsilon}_j}$$

$$\overline{\pi} = \underline{\pi} + \sum_{i=1}^{N}\sum_{t=1}^{T} \mathbf{d}_{it}$$

$$\overline{\mathbf{d}}_{it} = \left[\frac{\pi_1 f_N(y_{it} \mid \phi_1 + \mathbf{x}_{it}'\alpha - u_i, h_1^{-1})}{\sum_{j=1}^{J}\pi_j f_N(y_{it} \mid \phi_j + \mathbf{x}_{it}'\alpha - u_i, h_j^{-1})}, \ldots, \frac{\pi_J f_N(y_{it} \mid \phi_J + \mathbf{x}_{it}'\alpha - u_i, h_J^{-1})}{\sum_{j=1}^{J}\pi_j f_N(y_{it} \mid \phi_j + \mathbf{x}_{it}'\alpha - u_i, h_j^{-1})} \right]'$$

$$\sigma_{ui}^2 = \frac{1}{\sum_{t=1}^{T}\mathbf{d}_{it}'\mathbf{h}}$$

$$\mu_{ui} = \frac{\sum_{t=1}^{T}(\mathbf{d}_{it}'\mathbf{h})(\mathbf{d}_{it}'\phi) + \sum_{t=1}^{T}(\mathbf{d}_{it}'\mathbf{h})\mathbf{x}_{it}'\alpha - \sum_{t=1}^{T}(\mathbf{d}_{it}'\mathbf{h})y_{it} - \lambda^{-1}}{\sum_{t=1}^{T}\mathbf{d}_{it}'\mathbf{h}}$$

APPENDIX C

Posterior Parameters for Random Effects Model with State-Varying Slope Coefficients

Define $\mathbf{z}_{it} = (\mathbf{d}_{it}', (\mathbf{x}_{it}\otimes\mathbf{d}_{it})')'$ and $y_{it} = \ln Y_{it}$. Then, expressions for the conditional posterior pdf parameters $\overline{\upsilon}_j$, $\overline{s}_j^2$, $\overline{\boldsymbol{\pi}}$ and σ_{ui}^2 are the same as those given in Appendix B, with the dimension of $\boldsymbol{\beta}$ suitably modified. The remaining conditional posterior parameters are

$$\overline{\mathbf{V}} = [(\underline{\mathbf{V}}\otimes\mathbf{I}_J)^{-1} + \sum_{i=1}^{N}\sum_{t=1}^{T}(\mathbf{d}_{it}'\mathbf{h})\mathbf{z}_{it}\mathbf{z}_{it}']^{-1}$$

$$\overline{\beta} = \overline{\mathbf{V}}[(\underline{\mathbf{V}}\otimes\mathbf{I}_J)^{-1}\underline{\beta} + \sum_{i=1}^{N}\sum_{t=1}^{T}(\mathbf{d}_{it}'\mathbf{h})\mathbf{z}_{it}(y_{it} + u_i)]$$

The above expressions for $\overline{\mathbf{V}}$ and $\overline{\boldsymbol{\beta}}$ assume a prior covariance matrix for $\boldsymbol{\beta}$ that can be written as $\underline{\mathbf{V}}\otimes\mathbf{I}_J$. That is, the prior covariance matrices for the coefficient vectors for each state are identical. Our empirical work did employ such a prior covariance matrix, but a more general one can be used by simply replacing $\underline{\mathbf{V}}\otimes\mathbf{I}_J$ by a newly-defined $\underline{\mathbf{V}}$.

$$\mu_{ui} = \frac{\sum_{t=1}^{T}(\mathbf{d}_{it}'\mathbf{h})\mathbf{z}_{it}'\beta - \sum_{t=1}^{T}(\mathbf{d}_{it}'\mathbf{h})y_{it} - \lambda^{-1}}{\sum_{t=1}^{T}\mathbf{d}_{it}'\mathbf{h}}$$

$$\overline{\mathbf{d}}_{it} = \left[\frac{\pi_1 f_N(y_{it} \mid \phi_1 + (\mathbf{x}_{it}\otimes\mathbf{i}_1)'\alpha - u_i, h_1^{-1})}{\sum_{j=1}^{J}\pi_j f_N(y_{it} \mid \phi_j + (\mathbf{x}_{it}\otimes\mathbf{i}_j)'\alpha - u_i, h_j^{-1})}, \ldots, \frac{\pi_J f_N(y_{it} \mid \phi_J + (\mathbf{x}_{it}\otimes\mathbf{i}_J)'\alpha - u_i, h_J^{-1})}{\sum_{j=1}^{J}\pi_j f_N(y_{it} \mid \phi_j + (\mathbf{x}_{it}\otimes\mathbf{i}_j)'\alpha - u_i, h_j^{-1})}\right]'$$

where $\mathbf{i}_j$ is the j-th column of $\mathbf{I}_J$.

APPENDIX D

Conditional and Unconditional Means and Variances of Output and Their Derivatives

Random effects model with state-invariant slope coefficients

Letting $\kappa_j = \exp\{\phi_j + 0.5h_j^{-1}\}$, expressions for the different mean outputs can be written as

$$E(Y_{it} \mid u_i, \varepsilon_{it} = j) = \kappa_j \exp\{\mathbf{x}'_{it}\boldsymbol{\alpha} - u_i\}$$

$$E(Y_{it} \mid \varepsilon_{it} = j) = \left(\frac{\kappa_j}{1+\lambda}\right)\exp\{\mathbf{x}'_{it}\boldsymbol{\alpha}\}$$

$$E(Y_{it} \mid u_i) = \exp\{\mathbf{x}'_{it}\boldsymbol{\alpha} - u_i\} \times \sum_{j=1}^{J} \pi_j \kappa_j$$

$$E(Y_{it}) = \left(\frac{1}{1+\lambda}\right)\exp\{\mathbf{x}'_{it}\boldsymbol{\alpha}\} \times \sum_{j=1}^{J} \pi_j \kappa_j$$

For consideration of the derivatives it is convenient to let $\omega_{kit} = \partial(\mathbf{x}'_{it}\boldsymbol{\alpha})/\partial X_{kit}$. For example, in the case of our three-input translog model we define $\boldsymbol{\alpha}$ = (α_{TR}, α_{01}, α_{02}, α_{03}, α_{11}, α_{12}, α_{13}, α_{22}, α_{23}, α_{33})' and

$$\omega_{kit} = \frac{\partial(\mathbf{x}'_{it}\boldsymbol{\alpha})}{\partial X_{kit}} = \frac{1}{X_{kit}}\left(\alpha_{0k} + \sum_{\ell=1}^{3} \alpha_{k\ell} \log X_{\ell it}\right) \qquad \text{with } \alpha_{k\ell} = \alpha_{\ell k}$$

The marginal expected outputs can now be written as

$$\frac{\partial E(Y_{it} \mid u_i, \varepsilon_{it} = j)}{\partial X_{kit}} = \omega_{kit} E(Y_{it} \mid u_i, \varepsilon_{it} = j)$$

$$\frac{\partial E(Y_{it} \mid \varepsilon_{it} = j)}{\partial X_{kit}} = \omega_{kit} E(Y_{it} \mid \varepsilon_{it} = j)$$

$$\frac{\partial E(Y_{it} \mid u_i)}{\partial X_{kit}} = \omega_{kit} E(Y_{it} \mid u_i)$$

$$\frac{\partial E(Y_{it})}{\partial X_{kit}} = \omega_{kit} E(Y_{it})$$

The conditional and unconditional variances are given by

$$\mathrm{var}(Y_{it} \mid u_i, \varepsilon_{it} = j) = \left[E(Y_{it} \mid u_i, \varepsilon_{it} = j)\right]^2 \left(\exp\{h_j^{-1}\} - 1\right)$$

$$\operatorname{var}(Y_{it} \mid \varepsilon_{it} = j) = \left[E(Y_{it} \mid \varepsilon_{it} = j)\right]^2 \left(\frac{(1+\lambda)^2 \exp\{h_j^{-1}\}}{1+2\lambda} - 1 \right)$$

$$\operatorname{var}(Y_{it} \mid u_i) = \left[E(Y_{it} \mid u_i)\right]^2 \left[\frac{\sum_{j=1}^{J} \pi_j \kappa_j^2 \exp\{h_j^{-1}\}}{\left(\sum_{j=1}^{J} \pi_j \kappa_j \right)^2} - 1 \right]$$

$$\operatorname{var}(Y_{it}) = \left[E(Y_{it})\right]^2 \left[\frac{(1+\lambda)^2 \sum_{j=1}^{J} \pi_j \kappa_j^2 \exp\{h_j^{-1}\}}{(1+2\lambda)\left(\sum_{j=1}^{J} \pi_j \kappa_j \right)^2} - 1 \right]$$

The corresponding expressions for marginal risk are

$$\frac{\partial \operatorname{var}(Y_{it} \mid u_i, \varepsilon_{it} = j)}{\partial X_{kit}} = 2\omega_{kit} \left[E(Y_{it} \mid u_i, \varepsilon_{it} = j)\right]^2 \left(\exp\{h_j^{-1}\} - 1 \right)$$

$$\frac{\partial \operatorname{var}(Y_{it} \mid \varepsilon_{it} = j)}{\partial X_{kit}} = 2\omega_{kit} \left[E(Y_{it} \mid \varepsilon_{it} = j)\right]^2 \left(\frac{(1+\lambda)^2 \exp\{h_j^{-1}\}}{1+2\lambda} - 1 \right)$$

$$\frac{\partial \operatorname{var}(Y_{it} \mid u_i)}{\partial X_{kit}} = 2\omega_{kit} \left[E(Y_{it} \mid u_i)\right]^2 \left[\frac{\sum_{j=1}^{J} \pi_j \kappa_j^2 \exp\{h_j^{-1}\}}{\left(\sum_{j=1}^{J} \pi_j \kappa_j \right)^2} - 1 \right]$$

$$\frac{\partial \operatorname{var}(Y_{it})}{\partial X_{kit}} = 2\omega_{kit} \left[E(Y_{it})\right]^2 \left[\frac{(1+\lambda)^2 \sum_{j=1}^{J} \pi_j \kappa_j^2 \exp\{h_j^{-1}\}}{(1+2\lambda)\left(\sum_{j=1}^{J} \pi_j \kappa_j \right)^2} - 1 \right]$$

Random effects model with state-dependent slope coefficients

When α is state-dependent we replace α (and its components) by α_j in any expressions that are conditional on $\varepsilon_{it} = j$. Correspondingly, we define $\omega_{kitj} = \partial(\mathbf{x}_{it}'\boldsymbol{\alpha}_j)/\partial X_{kit}$. For the three-input translog model we have $\boldsymbol{\alpha}_j = (\alpha_{TRj}, \alpha_{01j}, \alpha_{02j}, \alpha_{03j}, \alpha_{11j}, \alpha_{12j}, \alpha_{13j}, \alpha_{22j}, \alpha_{23j}, \alpha_{33j})'$ so that

$$\omega_{kitj} = \frac{\partial(\mathbf{x}'_{it}\boldsymbol{\alpha}_j)}{\partial X_{kit}} = \frac{1}{X_{kit}}\left(\alpha_{0kj} + \sum_{\ell=1}^{3}\alpha_{k\ell j}\log X_{\ell it}\right) \qquad \text{where } \alpha_{k\ell j} = \alpha_{\ell kj}.$$

In addition, we set

$$\theta_{it} = \sum_{j=1}^{J}\pi_j\kappa_j\exp\{\mathbf{x}'_{it}\boldsymbol{\alpha}_j\}$$

and

$$\xi_{kit} = \frac{\partial\theta_{it}}{\partial X_{kit}} = \sum_{j=1}^{J}\pi_j\omega_{kit}\kappa_j\exp\{\mathbf{x}'_{it}\boldsymbol{\alpha}_j\}$$

For the expressions that are not conditional on $\varepsilon_{it} = j$ we then have:

$$E(Y_{it}\,|\,u_i) = \theta_{it}\exp\{-u_i\}$$

$$E(Y_{it}) = \theta_{it}(1+\lambda)^{-1}$$

$$\frac{\partial E(Y_{it}\,|\,u_i)}{\partial X_{kit}} = \xi_{kit}\exp\{-u_i\}$$

$$\frac{\partial E(Y_{it})}{\partial X_{kit}} = \xi_{kit}(1+\lambda)^{-1}$$

$$\mathrm{var}(Y_{it}\,|\,u_i) = \exp\{-2u_i\}\left[\sum_{j=1}^{J}\pi_j\kappa_j^2\exp\{2\mathbf{x}'_{it}\boldsymbol{\alpha}_j + h_j^{-1}\} - \theta_{it}^2\right]$$

$$\mathrm{var}(Y_{it}) = (1+2\lambda)^{-1}\sum_{j=1}^{J}\pi_j\kappa_j^2\exp\{2\mathbf{x}'_{it}\boldsymbol{\alpha}_j + h_j^{-1}\} - (1+\lambda)^{-2}\theta_{it}^2$$

$$\frac{\partial\,\mathrm{var}(Y_{it}\,|\,u_i)}{\partial X_{kit}} = 2\exp\{-2u_i\}\left[\sum_{j=1}^{J}\pi_j\omega_{kitj}\kappa_j^2\exp\{2\mathbf{x}'_{it}\boldsymbol{\alpha}_j + h_j^{-1}\} - \theta_{it}\xi_{kit}\right]$$

$$\frac{\partial\,\mathrm{var}(Y_{it})}{\partial X_{kit}} = 2(1+2\lambda)^{-1}\sum_{j=1}^{J}\pi_j\omega_{kitj}\kappa_j^2\exp\{2\mathbf{x}'_{it}\boldsymbol{\alpha}_j + h_j^{-1}\} - 2(1+\lambda)^{-2}\theta_{it}\xi_{kit}$$

Table 1. Descriptive Statistics

Variable		Mean	St. Deviation	Minimum	Maximum
Y	Rice Output	6.4664	5.0767	0.09	31.10
X_1	Area	2.1175	1.4514	0.20	7.00
X_2	Labor	107.20	76.646	8.00	436.00
X_3	Fertilizer	187.05	168.59	3.40	1030.9

Table 2. Parameters

Variable		State	Estimated Posterior Means						Estimated Posterior Standard Deviations					
			FE	RE	SC-FE	SC-RE	SC-RE-*all*	SC-RE-η>0	FE	RE	SC-FE	SC-RE	SC-RE-*all*	SC-RE-η>0
		1			1.245	1.392	1.118	1.112			0.154	0.086	0.200	0.196
1	Intercept	2	1.694	1.938	1.639	1.804	1.814	1.803	0.125	0.055	0.139	0.063	0.087	0.087
		3			2.098	2.214	2.082	2.079			0.150	0.076	0.108	0.103
		1					0.028	0.029					0.018	0.018
TR	Trend	2	0.017	0.014	0.013	0.008	-0.014	-0.013	0.008	0.008	0.009	0.009	0.016	0.016
		3					0.009	0.010					0.014	0.014
		1					0.615	0.434					0.296	0.239
ln X_1	Area	2	0.590	0.665	0.270	0.273	0.133	0.143	0.138	0.103	0.161	0.116	0.195	0.103
		3					0.561	0.369					0.471	0.241
		1					-0.333	0.107					0.242	0.094
ln X_2	Labor	2	0.047	0.125	0.037	0.169	0.024	0.119	0.101	0.092	0.114	0.101	0.184	0.090
		3					-0.106	0.184					0.347	0.144
		1					-0.199	-0.368					0.239	0.227
ln X_3	Fertilizer	2	0.124	0.192	0.013	0.113	0.112	0.049	0.071	0.061	0.082	0.064	0.124	0.110
		3					0.313	0.231					0.201	0.190
		1					0.011	0.006					0.339	0.344
0.5(ln X_1)^2	Area × Area	2	-0.810	-0.254	-0.558	-0.157	-0.408	-0.395	0.342	0.272	0.386	0.309	0.933	0.927
		3					-0.690	-0.158					1.795	1.416
		1					0.176	0.213					0.341	0.334
(ln X_1)(ln X_2)	Area × Labor	2	0.608	0.561	0.295	0.280	0.336	0.391	0.249	0.241	0.285	0.269	0.647	0.641
		3					0.700	0.379					1.237	1.154
		1					0.171	0.048					0.241	0.229
(ln X_1)(ln X_3)	Area × Fertilizer	2	0.057	-0.013	0.023	-0.014	-0.005	-0.048	0.168	0.161	0.190	0.177	0.456	0.415
		3					-0.529	-0.492					0.581	0.578
		1					-0.580	-0.366					0.510	0.489
0.5(ln X_2)^2	Labor × Labor	2	-0.531	-0.529	-0.299	-0.242	-0.145	-0.066	0.341	0.336	0.391	0.371	0.733	0.729
		3					-0.221	-0.408					1.332	1.296
		1					-0.288	-0.223					0.223	0.218
(ln X_2)(ln X_3)	Labor × Fertilizer	2	-0.335	-0.282	-0.253	-0.207	-0.181	-0.204	0.141	0.140	0.162	0.158	0.459	0.437
		3					0.555	0.637					0.495	0.484
		1					-0.172	-0.228					0.189	0.186
0.5(ln X_3)^2	Fertilizer × Fertilizer	2	0.196	0.222	0.113	0.141	0.152	0.145	0.073	0.069	0.083	0.077	0.109	0.106
		3					-0.395	-0.385					0.358	0.357
		1			6.015	5.545	5.810	5.648			1.016	0.940	1.082	1.084
Precision		2	9.379	9.022	9.707	9.927	8.513	8.528	0.771	0.745	1.358	1.330	1.298	1.286
		3			6.060	5.439	8.303	8.346			1.227	1.210	1.466	1.513
		1			0.317	0.334	0.312	0.306			0.049	0.049	0.060	0.063
State Probabilities		2			0.451	0.457	0.363	0.364			0.055	0.053	0.050	0.051
		3			0.232	0.208	0.325	0.330			0.045	0.047	0.058	0.062

Table 3. State Probabilities For Firms 1 to 3

Firm	Period	SC-FE			SC-RE			SC-RE-*all*			SC-RE-η>0		
		π_1	π_2	π_3	π_1	π_2	π_3	π_1	π_2	π_3	π_1	π_2	π_3
1	1	0.116	0.488	0.396	0.173	0.571	0.257	0.037	0.457	0.506	0.069	0.480	0.451
	2	0.147	0.549	0.304	0.206	0.609	0.186	0.064	0.561	0.376	0.066	0.566	0.368
	3	0.094	0.406	0.501	0.146	0.545	0.309	0.030	0.404	0.565	0.028	0.392	0.581
	4	0.151	0.564	0.285	0.214	0.608	0.179	0.081	0.545	0.374	0.066	0.529	0.406
	5	0.141	0.537	0.322	0.205	0.605	0.190	0.062	0.542	0.396	0.070	0.539	0.391
	6	0.445	0.506	0.049	0.506	0.453	0.041	0.397	0.549	0.054	0.407	0.540	0.053
	7	0.464	0.493	0.043	0.576	0.394	0.030	0.371	0.579	0.050	0.335	0.612	0.052
	8	0.159	0.560	0.281	0.171	0.585	0.244	0.159	0.445	0.396	0.111	0.421	0.469
2	1	0.161	0.561	0.278	0.151	0.556	0.293	0.045	0.362	0.594	0.051	0.384	0.566
	2	0.182	0.574	0.244	0.145	0.537	0.318	0.085	0.321	0.594	0.094	0.345	0.561
	3	0.093	0.410	0.497	0.097	0.412	0.491	0.023	0.155	0.821	0.023	0.165	0.812
	4	0.032	0.129	0.839	0.045	0.163	0.792	0.002	0.032	0.966	0.005	0.045	0.950
	5	0.054	0.246	0.701	0.067	0.265	0.668	0.019	0.120	0.861	0.013	0.092	0.895
	6	0.060	0.288	0.651	0.074	0.287	0.640	0.020	0.065	0.915	0.045	0.116	0.839
	7	0.138	0.507	0.355	0.134	0.522	0.344	0.075	0.231	0.694	0.047	0.200	0.754
	8	0.099	0.424	0.477	0.072	0.287	0.641	0.114	0.132	0.754	0.113	0.150	0.737
3	1	0.084	0.384	0.532	0.134	0.498	0.368	0.040	0.335	0.625	0.051	0.357	0.592
	2	0.083	0.372	0.545	0.151	0.541	0.307	0.079	0.335	0.586	0.045	0.317	0.638
	3	0.081	0.376	0.543	0.152	0.539	0.309	0.071	0.345	0.584	0.086	0.384	0.530
	4	0.125	0.506	0.370	0.204	0.596	0.201	0.125	0.483	0.392	0.135	0.511	0.354
	5	0.194	0.593	0.213	0.309	0.591	0.100	0.165	0.692	0.142	0.203	0.678	0.119
	6	0.094	0.412	0.494	0.121	0.485	0.394	0.060	0.292	0.648	0.036	0.257	0.707
	7	0.403	0.535	0.062	0.528	0.433	0.039	0.361	0.602	0.037	0.370	0.595	0.034
	8	0.086	0.381	0.533	0.117	0.467	0.416	0.100	0.217	0.683	0.098	0.232	0.670

Table 4. Technical Efficiencies

Firm	Estimated Posterior Means						Estimated Posterior Standard Deviations					
	FE	RE	SC-FE	SC-RE	SC-RE-*all*	SC-RE-η>0	FE	RE	SC-FE	SC-RE	SC-RE-*all*	SC-RE-η>0
1	0.343	0.813	0.350	0.906	0.947	0.950	0.097	0.093	0.096	0.075	0.050	0.047
2	0.552	0.942	0.497	0.952	0.965	0.965	0.140	0.049	0.119	0.044	0.034	0.034
3	0.362	0.762	0.366	0.892	0.942	0.943	0.092	0.099	0.087	0.082	0.054	0.053
4	0.388	0.926	0.365	0.945	0.952	0.952	0.115	0.059	0.112	0.049	0.045	0.046
5	0.412	0.876	0.339	0.895	0.940	0.939	0.108	0.081	0.088	0.082	0.055	0.056
6	0.439	0.899	0.431	0.937	0.954	0.957	0.104	0.073	0.119	0.057	0.044	0.041
7	0.464	0.937	0.384	0.935	0.956	0.962	0.131	0.052	0.110	0.057	0.043	0.037
8	0.377	0.893	0.386	0.939	0.958	0.960	0.111	0.073	0.115	0.054	0.040	0.039
9	0.422	0.905	0.328	0.893	0.941	0.942	0.114	0.069	0.088	0.083	0.055	0.054
10	0.445	0.910	0.383	0.921	0.944	0.945	0.120	0.066	0.100	0.066	0.051	0.052
11	0.251	0.650	0.277	0.865	0.882	0.889	0.069	0.091	0.083	0.099	0.094	0.091
12	0.954	0.949	0.813	0.952	0.965	0.966	0.098	0.047	0.189	0.047	0.036	0.035
13	0.329	0.849	0.351	0.933	0.950	0.946	0.096	0.088	0.107	0.058	0.047	0.051
14	0.376	0.897	0.367	0.938	0.960	0.960	0.106	0.072	0.106	0.055	0.038	0.038
15	0.485	0.802	0.444	0.865	0.928	0.930	0.092	0.101	0.112	0.101	0.067	0.066
16	0.468	0.937	0.488	0.963	0.969	0.970	0.120	0.052	0.141	0.036	0.031	0.029
17	0.538	0.929	0.530	0.949	0.963	0.963	0.142	0.057	0.128	0.047	0.036	0.036
18	0.799	0.953	0.932	0.971	0.954	0.957	0.180	0.042	0.126	0.029	0.046	0.043
19	0.493	0.906	0.515	0.949	0.965	0.965	0.125	0.068	0.120	0.047	0.034	0.034
20	0.545	0.942	0.490	0.949	0.953	0.953	0.143	0.049	0.121	0.047	0.045	0.045
:	:	:	:	:	:	:	:	:	:	:	:	:
34	0.199	0.552	0.227	0.789	0.895	0.906	0.058	0.087	0.072	0.129	0.095	0.088
35	0.455	0.939	0.382	0.938	0.960	0.958	0.130	0.050	0.109	0.055	0.038	0.040
36	0.295	0.765	0.315	0.903	0.945	0.943	0.084	0.096	0.092	0.077	0.051	0.053
37	0.314	0.812	0.350	0.929	0.938	0.945	0.091	0.095	0.110	0.063	0.058	0.053
38	0.446	0.941	0.440	0.958	0.956	0.959	0.128	0.049	0.135	0.039	0.043	0.040
39	0.543	0.941	0.511	0.957	0.959	0.961	0.128	0.050	0.120	0.040	0.040	0.038
40	0.360	0.837	0.377	0.926	0.946	0.949	0.092	0.094	0.110	0.066	0.052	0.049
41	0.290	0.753	0.320	0.901	0.940	0.942	0.085	0.095	0.095	0.077	0.055	0.055
42	0.390	0.932	0.371	0.951	0.951	0.951	0.116	0.056	0.117	0.045	0.046	0.047
43	0.404	0.924	0.339	0.930	0.952	0.958	0.114	0.060	0.099	0.061	0.046	0.041
44	0.299	0.797	0.311	0.917	0.943	0.943	0.086	0.095	0.094	0.070	0.054	0.054
Mean	0.422	0.863	0.414	0.926	0.948	0.950	0.109	0.074	0.111	0.062	0.049	0.047
Minimum	0.199	0.552	0.227	0.789	0.882	0.889	0.058	0.040	0.072	0.029	0.031	0.029
Maximum	0.954	0.954	0.932	0.971	0.969	0.970	0.180	0.106	0.189	0.129	0.095	0.091

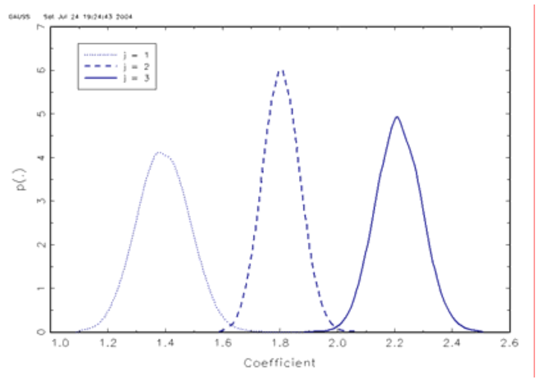

Figure 1. SC-RE Estimated Posteriors for ϕ_j

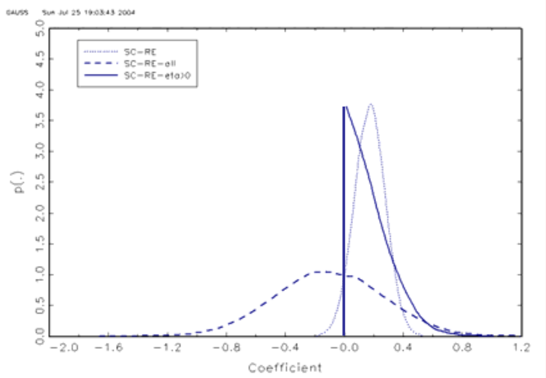

Figure 2. Estimated Posteriors for the Labor Elasticity in a Good Season (at Mean Inputs)

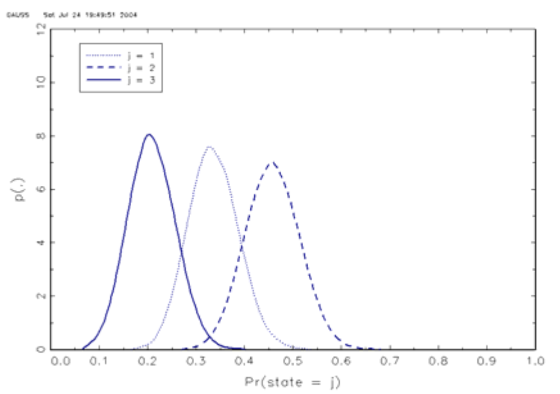

Figure 3. SC-RE Posteriors for π_j

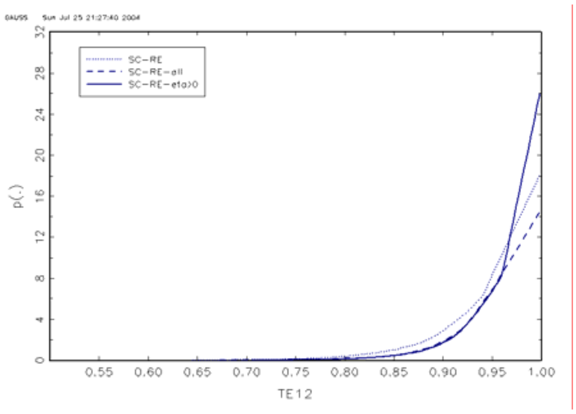

Figure 4. Estimated Posteriors for the Technical Efficiency of Firm 12

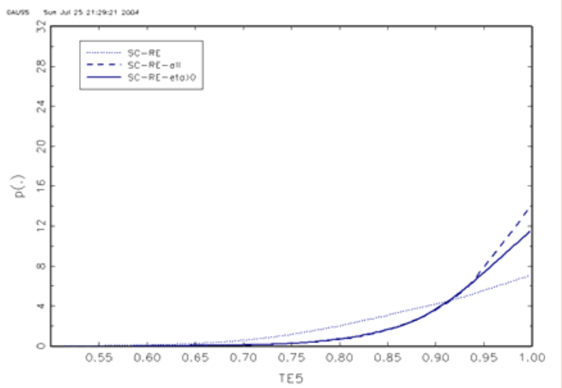

Figure 5. Estimated Posteriors for the Technical Efficiency of Firm 5

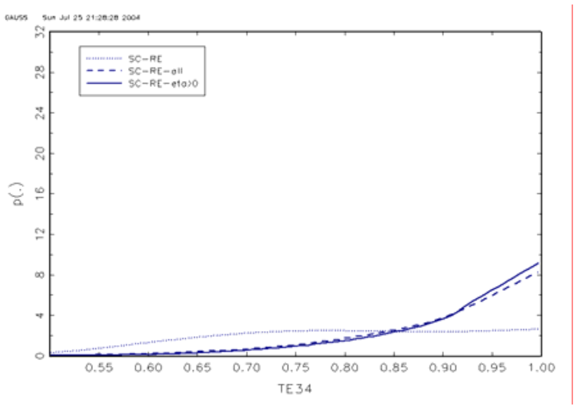

Figure 6. Estimated Posteriors for the Technical Efficiency of Firm 34

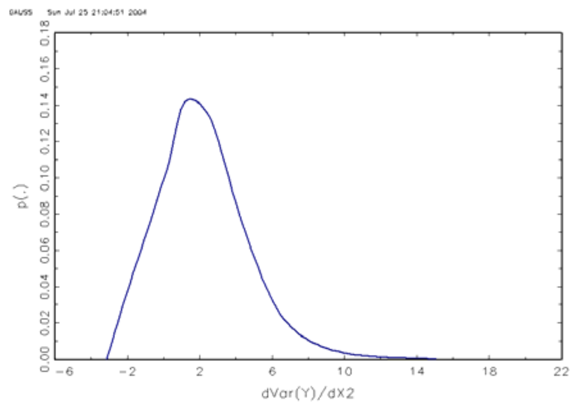

Figure 7. SC-RE-η>0 Posterior for $\partial \text{var}(Y_{it})/\partial X_{2it}$ evaluated at $\mathbf{x}_{it} = 0$.